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HYDROPHYSICAL PROCESSES

Applying Modern Methods of Statistical Physics to Describe Fluctuations of Soil Moisture Reserve

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Abstract—The application of methods of modern theory of Brownian motion to the calculation of fluctuations of soil moisture reserve is considered. Analytical dependences of the correlation time of soil moisture reserve fluctuations and its mean on the characteristics of external stochastic and deterministic impacts are obtained for a nonlinear hydrological model of moisture reserve formation based on Langevin equation and generalized Langevin relationships. Variations of the low-frequency part of the spectrum of fast synoptic variables (the difference between precipitation and evaporation) because of interaction with moisture reserve fluctuations is examined. A new effect—an increase in the spectrum in the intermediate domain of time intervals between characteristic times—is explained by the time of variations of synoptic fluctuations and the time of variations of fluctuations of soil moisture reserve. The problem of determining a non-steady response of the mean soil moisture reserve to fluctuation of moistening regime is used to demonstrate the potentialities of a new apparatus of statistical physics—fluctuation theorems.

Keywords: soil moisture reserve, Langevin equation, fluctuations, correlation functions, spectrum

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Dynamic–stochastic models are used to solve some hydrological problems, such as modeling variation regimes river runoff, the levels of flow-through and drainless water bodies, soil moisture reserve, etc. [7, 9, 12, 14–16]. Such models are based on stochastic differential (finite-difference) equations with random perturbations (forces) and/or coefficients. When such perturbations within time of variation of the variables under consideration can be regarded as uncorrelated with respect to time, they are approximated by a delta-correlated random process, i.e., white noise [13].

The coefficients of stochastic differential (finite-difference) equations are commonly determined by appropriate treatment of empirical observational data on the processes involved in the models. In this case, there is no guarantee that the values of those coefficients will not change with possible climate changes. Indeed, several empirical procedures, which were successfully used in hydrometeorological forecasts in the 1960s–1970s, started yielding incorrect results in the 1980s, likely because of global warming [10]. This requires more attention to be paid to the possible procedures of derivation of stochastic differential equations for the description of hydrological cycle components and, in particular, modern methods of nonequilibrium statistical mechanics to be used.

From the viewpoint of statistical mechanics, stochastic differential equations with random forces delta-correlated in time are Langevin equations [4] for the random evolution of a set of slow variables $\mathbf{Y}$ (hereafter, bold-type roman characters are used to denote vectors, matrices, or operators), interacting with faster variables $\mathbf{X}$. In such cases, characteristic times τ_X and τ_Y for each subsystem of variables—fast and slow—are radically different ($\tau_X \ll \tau_Y$). In such cases, only slow variables are roughly described, while at the characteristic times of evolution of slow variables, the fast ones are specified only by their statistics determined for the values of $\mathbf{Y}(t)$ for each time t.

If the system of equations describing variations of slow $Y_i(t)$ and fast $X_\alpha(t)$ variables comprises the following equations:

$$\frac{dY_i(t)}{dt} = U_i(\mathbf{X}(t), \mathbf{Y}(t)), \quad i = 1, \ldots, N_Y, \tag{1a}$$

$$\frac{dX_\alpha(t)}{dt} = u_\alpha(\mathbf{X}(t), \mathbf{Y}(t)), \quad \alpha = 1, \ldots, N_X, \tag{1b}$$

then, in its brief description, any function of system variables is divided into a slow component, adapted on the average to the current state $\mathbf{Y}(t)$, and a deviation from it $F(\mathbf{X}(t), \mathbf{Y}(t)) = \langle F|\mathbf{Y}(t)\rangle + (F - \langle F|\mathbf{Y}(t)\rangle) = \langle F|\mathbf{Y}(t)\rangle + \delta F(\mathbf{X}(t), \mathbf{Y}(t))$ (hereafter, the angular brackets denote statistical averaging over the ensemble of realizations). In this case, the term *adapted* on the average implies the averaging only over fast variables with a specified probability density of solution (1b) at fixed $\mathbf{Y} = \mathbf{Y}(t)$, $\langle F|\mathbf{Y}\rangle = \int_X F(\mathbf{X}, \mathbf{Y})\rho_S(\mathbf{X}, \mathbf{Y})\, d\mathbf{X}$. The residue $\delta\mathbf{U}(t)$ is considered a rapidly fluctuating random process. K. Hasselmann [24] was the first to propose the application of such procedure to the right-hand parts of equations (1a), with $\delta\mathbf{U}(t)$ interpreted as a random force delta-correlated over time. This enables the equations (1a) to be considered as Langevin equations, for which a well-developed mathematical apparatus is available [4]. According to K. Hasselman, the slow variables are referred to as climatic, and the faster ones, as weather.

In this study, we consider soil moisture reserve W, i.e., the amount of moisture in a column of active soil layer with unit cross-section, as a climatic variable. The soil moisture content is an important hydrological component, which has a strong effect on the formation of river runoff and groundwater resources in a watershed. The characteristic time of moisture reserve relaxation (τ_E in formula (10)) is several months [12, 31], which is far in excess of the characteristic time of synoptic fluctuations of hydrometeorological processes that form moisture reserves (the latter time is several days). This feature allows us to apply the procedure of adaptation on the average.

A feature taken into account in the modeling of variations of moisture reserves was that the equation describing such variations contain a nonlinearity caused by the presence or absence of surface runoff, depending on the moistening regime.

Variations of moisture reserve can have a considerable effect on atmospheric variables, such as surface temperature and humidity [18–21]. Since the latter, in their turn, influence variations in moisture reserve, the system may show collective effects, resulting in changes in the spectra of atmospheric variables.

LANGEVIN EQUATIONS FOR SOIL MOISTURE RESERVES: DERIVATION BY METHOD OF PROJECTION OPERATORS

A method used to derive Langevin equations for slow variables in the initial system of equations (1) is based on the method of projection operators proposed by H. Mori [27, 28]. In this method, any function of the solution of Cauchy problem for the system of equations (1a)–(1b) is written as a function of the initial values of variables with the help of evolution operator $\mathbf{M}$:

$$F(t) = F(\mathbf{X}(t), \mathbf{Y}(t)) = e^{t\mathbf{M}}F(0),\qquad \mathbf{M} = u_\alpha \frac{\partial}{\partial X_\alpha} + U_i \frac{\partial}{\partial Y_i}. \tag{2}$$

In (2), after the application of operator $\exp(t\mathbf{M})$ to $F(\mathbf{X}, \mathbf{Y})$ we should set $((\mathbf{X}, \mathbf{Y}) = (\mathbf{X}(0), \mathbf{Y}(0))$, and the operator exponent should be calculated by formal expansion in Taylor series.

In the space of initial states, we introduce Mori projection operator $\mathbf{P}$:

$$\mathbf{P}F = \iint d\mathbf{Y}d\mathbf{X}F(\mathbf{X},\mathbf{Y})\delta(\mathbf{Y} - \mathbf{Y}(0))\rho_S(\mathbf{X}|\mathbf{Y}) = \langle F|\mathbf{Y}(0)\rangle,\quad \mathbf{P}^2 = \mathbf{P}. \tag{3}$$

We wrote (3) in expanded form to emphasize that operation (3) can be determined for any moment, so that

$$e^{t\mathbf{M}}\mathbf{P}F(0) = e^{t\mathbf{M}}\langle F|\mathbf{Y}(0)\rangle = \langle F|\mathbf{Y}(t)\rangle. \tag{4}$$

In that case, $\mathbf{Y}(t)$ is an exact solution of the initial system (1). For operator $\mathbf{P}$, we introduce an additional projection operator $\mathbf{Q} = \mathbf{I} - \mathbf{P}$ ($\mathbf{I}$ is unit operator). $\mathbf{Q}$ projects the motion of the system onto the complementary subspace ($\mathbf{PQ} = 0$) of deviations from the quasi-average values of variables.

Now, we apply to the right-hand part of (1a) $U_i(0) = \langle U_i|\mathbf{Y}(0)\rangle + \delta U_i(\mathbf{X}(0), \mathbf{Y}(0))$ the Mori operator identity $\exp(t(\mathbf{A} + \mathbf{B})) = \exp(t\mathbf{A}) + \int_0^t d\tau \exp[(t - \tau)](\mathbf{A} + \mathbf{B})\, \mathbf{B}\exp(\tau\mathbf{A})$, setting $\mathbf{A} = \mathbf{QM}$, $\mathbf{B} = \mathbf{PM}$. Considering (4) we finally receive the so-called generalized Langevin equation [27, 28]:

$$\frac{d\mathbf{Y}(t)}{dt} = \langle \mathbf{U}|\mathbf{Y}(t)\rangle + \int_0^t d\tau e^{(t-\tau)\mathbf{M}}\mathbf{PM}e^{\tau\mathbf{QM}}\delta\mathbf{U}(0) + e^{t\mathbf{QM}}\delta\mathbf{U}(0). \tag{5}$$

The division of the right-hand part of (1a), obtained by identity transformations, was made to rewrite it in the form convenient for applying approximations based on a priori knowledge or assumptions regarding the behavior of solutions of the initial system of equations (1a), (1b). The first term is the quasi-stationary (at fixed $\mathbf{Y}(t)$) conditional mean value of the rate of change in the slow variables, adapted to $\mathbf{Y}(t)$.

The second term in (5)—a memory integral—describes the contribution of the finiteness of the mean response delay time of fast variables to changes in slow variables; it depends on t and the values of $\mathbf{Y}(\tau \le t)$, while the dependence on $\mathbf{X}(0)$ in it disappears. The memory integral is of the order of τ_X/τ_Y. With the upper integration limit in the memory integral replaced by $t = +\infty$, equation (5) becomes Langevin equation, which formally contains no delay [28]:

$$\frac{d\mathbf{Y}(t)}{dt} = \langle \mathbf{U}_e|\mathbf{Y}(t)\rangle + f_Y(t),\qquad f_Y(t) = e^{t\mathbf{QM}}\delta\mathbf{U}(0),\quad \langle f_Y(t)|\mathbf{Y}(0)\rangle = 0, \tag{6a}$$

$$\langle \mathbf{U}_e|\mathbf{Y}(t)\rangle = \langle \mathbf{U}|\mathbf{Y}(t)\rangle + \int_0^\infty d\tau\langle \mathbf{M}f_Y(\tau)|\mathbf{Y}(t)\rangle. \tag{6b}$$

The first summand is a random force, whose mean, whatever probability distribution function of the initial values of the type $\rho(0) = \rho_y(\mathbf{Y}(0))\rho_S(\mathbf{X}(0)|\mathbf{Y}(0))$, is zero ($\rho_S$ is the stationary distribution density of the fast variables at fixed values of slow variables, ρ_y is the distribution density of slow variables). This follows from the equality $\mathbf{P}\exp(t\mathbf{QM})\mathbf{Q} = 0$.

Now let us consider fluctuations in soil moisture reserve W for the worm period (in the absence of snow or ice) under stochastic short-period perturbations of moisture fluxes between the soil and the atmosphere. As the basis, we take a simplified integral zero-dimensional model of moisture balance in the active soil layer [3, 4, 7, 9]:

$$\frac{dW(t)}{dt} = -E(t) + P(t) - R(t), \quad (7)$$

where E is evaporation, P is precipitation, R is surface runoff, and t is time. All variables, including moisture reserves (the amount of moisture, evaluated by the depth of equivalent water layer) refer to a column of active soil layer with a unit cross-section.

Since the time of change in soil moisture reserve τ_W is much greater than the lifetime of individual weather perturbations τ_a, we can use (6a) and (6b) and neglect the moisture reserve—precipitation feedback to rewrite equation (7) as a stochastic differential equation:

$$\frac{dW(t)}{dt} = -\langle E|W(t)\rangle + \langle P|W(t)\rangle - R(t) + \Delta P(t) - \Delta E(t), \quad (8)$$

where $\Delta P(t)$ and $\Delta E(t)$ are deviations of the current values from the quasi-averaged values (4).

In (8), the short-period fluctuations of heat input to the active layer because of synoptic variations of precipitation and evaporation: $\Delta F_W(t) = \Delta \mathrm{P}(t) - \Delta E(t)$ are identified separately. For $\Delta F_W(t)$, we take an approximation by delta-correlated random process with zero mean and a correlation function

$$K_F(t, t_1) = \langle \Delta F_W(t) \Delta F_W(t_1)\rangle = 2D_W \delta(t - t_1). \quad (9)$$

For the evaporation, adapted on the average to the current state of W, we take M.I. Budyko's parameterization [2]: $\langle E|W\rangle = E_0 \dfrac{W}{W_0}$ at $W \le W_0$ and $(\langle E|W\rangle = E_0$ at $W > W_0$. Here E_0 is evaporativity, W_0 is the value of soil moisture reserve corresponding to saturation with respect to evaporation. The adapted part of precipitation at this stage will be assumed independent of the current moisture reserve and set equal to the average over the region $\langle P\rangle$.

In the following sections, we will cancel this assumption in the description of the joint evolution of the hydrological regime of atmosphere and soil.

With such approximation, Langevin equation (8) for W can be transformed into

$$\frac{dW}{dt} = -\frac{1}{\tau_E}(W - W_E) - R + \Delta F_W(t), \quad \tau_E = \frac{W_0}{E_0}, \quad W_E = W_0\frac{\langle P\rangle}{E_0}. \quad (10)$$

A bucket model [7, 18–20] is used for surface runoff. In this model, the runoff is zero when soil moisture reserve is less than some critical value with respect to runoff: $\dfrac{dW}{dt} = P - E$ $(W < W_C)$. Once this value is attained, the excessive moisture inflow into a soil column will be discharged into surface runoff, the soil moisture reserve remaining unchanged: $\dfrac{dW}{dt} = 0$, $R = P - E$ $(W = W_C, P > E)$. This model is a limiting case of a model with finite relaxation time of soil moisture reserve to the critical value W_C due to runoff $R = \lambda_R H(W - W_C)$ at $\lambda_R \to \infty$ ($H(x)$ is Heaviside function, equal to zero at negative argument and unit otherwise).

Equation (10) for runoff parameterization with finite λ_R can be rewritten by introducing potential $U(W)$ [7] and assuming, for simplicity, $W_C = W_0$:

$$\frac{dW}{dt} = -\frac{\partial U}{\partial W} + \Delta F_W(t), \quad U(W) = \frac{1}{2\tau_E}(W - W_E)^2 + \lambda_R \int_{W_0}^{W} H(W' - W)\,dW'. \quad (11)$$

The potential U is introduced to make more convenient the representation of the stationary distribution density $\rho_{WS}(W) = C\exp\left[-\dfrac{U}{D_W}\right]$ for the solution of Fokker-Planck equation (12), where, for its calculation, we can set $U(W) = \dfrac{1}{2\tau_E}(W - W_E)^2 + \lambda_R(W - W_0)H(W - W_0)$.

In conclusion, of this section, we give some estimates. In the absence of runoff, equation (10) is linear. The variance of deviations $\Delta W(t) = W(t) - W_E(t)$ from the mean stationary W_E is $\sigma_W^2 = D_W\tau_E = \sigma_f^2\tau_f\tau_E$, where σ_f^2 and τ_f are the variance and the correlation time of random moisture inflow, respectively. If we assume the latter equal to the synoptic fluctuations of precipitation, whose probability distribution density can be described by the exponential distribution $\rho_P(P) = \langle P\rangle^{-1}\exp(-P/\langle P\rangle)$ [30], we obtain $\sigma_W = \langle P\rangle(\tau_P\tau_E)^{1/2}$. If $\tau_f \approx 1.5$ day, $\tau_E \approx 3$ months, and $\langle P\rangle$ 2–3 mm/day [12, 18], we have $\sigma_W \approx 3$ cm, which is in good agreement with observational data [12].

FOKKER-PLANCK EQUATION: EVALUATING AVERAGE SURFACE RUNOFF FOR A DYNAMIC–STOCHASTIC MODEL

We can write a kinetic equation for the probability density $\rho_W(W, t)$, corresponding to the stochastic differential equation (11). Neglecting the effect of random force cumulants higher than the second (Gaussian approximation), we obtain the Fokker-Planck equation [13]:

$$\frac{\partial \rho_W}{\partial t} = \frac{\partial}{\partial W}\left(\rho_W \frac{\partial U}{\partial W}\right) + \frac{\partial^2}{\partial W^2}(D_W \rho_W). \tag{12}$$

Next, we will consider the case of constant effect of random impacts. If we do not consider the regimes with desert climate, we obtain, as the limit at $\lambda_R \to \infty$ (instantaneous discharge of moisture excess through runoff), the stationary solution of (12) can be written as an expression, which is formally defined in the domain $(-\infty, W_0]$ [7, 9]:

$$\rho_{WS}(W) = C\exp\left[-\frac{(W - W_E)^2}{2D_E}\right], \quad D_E = D_W \tau_E,$$
$$C^{-1} = \int_{-\infty}^{W_E} \exp\left[-\frac{1}{2D_E}(W - W_E)^2\right] dW. \tag{13}$$

In (13), W_E, $D_E = D_W\tau_E$, and τ_E are the mean, variance, and relaxation time of fluctuations in soil moisture reserves in a steady state in the absence of runoff.

For the mean $\langle W\rangle = W_S$ and variance $\langle(\Delta W)^2\rangle = \sigma_{WS}^2$ of a state, which is steady on the average, we have formulas [7, 10]

$$W_s = W_E - \left(\frac{D_E}{2}\right)^{1/2} \frac{1}{F_R(\Pi_R)},$$
$$\sigma_{WS}^2 = D_E \tau_E \left(1 + \frac{\Pi_R}{F_R(\Pi_R)} - \frac{1}{2F_R^2(\Pi_R)}\right), \tag{14}$$

where $F_R(\Pi_R) = \frac{\sqrt{\pi}}{2} e^{\Pi_R^2} \operatorname{erfc}(\Pi_R)$, $\operatorname{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \times \int_0^x dz \exp(-z^2)$, and erfc($x$) is the erfc complementary probability integral [1].

The important dimensionless parameter in the theory

$$\Pi_R = \left(\frac{\langle P\rangle}{E_0} - 1\right)\left(\frac{W_0 E_0}{2D_W}\right)^{1/2} = (W_E - W_0)(2D_E)^{-1/2} \tag{15}$$

changes its sign with the passage from the regime with insufficient moistening $\left(\frac{\langle P\rangle}{E_0} < 1\right)$ to the regime with excessive moistening $\left(\frac{\langle P\rangle}{E_0} > 1\right)$. According to (14), the rate of this passage for the average characteristics of moisture reserve fluctuations is determined by the fluctuation rate. For instance, this rate can determine the smoothness of the dependence of surface runoff on precipitation [9].

Substituting

$$\langle R\rangle = \langle P\rangle - E_0 \frac{\langle W\rangle}{W_0}$$

in the averaged balance relationship (7), and evaluating $\langle W\rangle = W_S$ from (14), in the simple dynamic–stochastic model under consideration, we have for surface runoff:

$$\frac{\langle R\rangle}{E_0} = \left(\frac{D_W}{2E_W W_W}\right)^{1/2} \frac{1}{F_R(\Pi_R)}. \tag{16}$$

As follows from (16) and the definition of Π_R, the average runoff does not vary at the proportional changes in the rate of random synoptic fluctuations of sources D_W and the critical soil moisture reserve W_0.

At large positive Π_R (the asymptotic at erfc($x \gg 1$)1) [1]) and sufficiently large excess of precipitation over evaporativity $F_R \approx (2\Pi_R)^{-1}$, equation (16) also reaches its asymptotic: $\langle R\rangle = \langle P\rangle - E_0$. Otherwise, at small precipitation, the runoff tends to zero. In the transitional zone between the regimes of insufficient and excessive moistening, synoptic fluctuations facilitate a smooth passage, which can be referred to as statistical smoothing, as distinct from the interpolation formula for average surface runoff proposed by M.I. Budyko [4].

Budyko's formula, in the regime of insufficient moistening additionally takes into account the possible formation of surface runoff due to showers $R_i = \mu P \approx 0.2P$. For the stochastic model (10), as shown in [9], this leads to changes in parameters in the initial Langevin equation to

$$E_0^{(m)} = E_0 + \mu\langle P\rangle,$$
$$D_W^{(m)} = D_W\left(1 - \mu\frac{\langle W\rangle}{W_0}\right)^2, \quad W_E^{(m)} = W_E^{(m)}\frac{E_0}{E_0^{(m)}},$$

where the superscript (m) denotes the appropriate modified variable, μ is a numerical parameter.

With the values of parameters of the hydrological model (evaporativity, normal field capacity, and precipitation correlation time), characteristic of land in temperate latitudes, the stochastic model adequately reproduces the interpolation dependence of surface runoff on precipitation, found by M.I. Budyko [9]. It is worth mentioning that the stochastic version bears additional information about the relationship between the first and second moments of fluctuations in hydrological balance components. The analysis of such dependences is of use in modeling subgrid-scale processes in general circulation and climate models of intermediate complexity.

AN ANALYTICAL MODEL FOR CALCULATING THE CORRELATION TIME OF FLUCTUATIONS IN SOIL MOISTURE RESERVE

Processing of observational data on fluctuations in soil moisture reserves in the territory of the former USSR [31] has shown that the time correlation function of those fluctuations for the warm season can be adequately approximated by the exponential function $\langle \Delta W(t + \tau)\Delta W(t)\rangle = K_{WS}(\tau) = \sigma^2_{WS}\exp(-\tau/\tau_W)$. As before, the subscript S identifies the characteristics of stationary on the average fluctuations. A similar result was obtained in numerical experiments with GFDL general circulation model in Princeton University [18–20]. According to equation (10), this is what should have been expected in the absence of runoff, and we have $\tau_W \approx \tau_E$. This theoretical estimate was generally confirmed by observational data processing [31].

The numerical modeling of fluctuations of W by equation (10) with the use of Monte Carlo method [18] revealed a nonmonotonic dependence of the obtained values of τ_W on E_0. Far enough from the regime with soil saturated with moisture, according to (10), we obtain that tau $\tau_W \sim 1/E_0$, i.e., it increases with decreasing E_0. However, as the system approaches the regime of excessive moistening, τ_W passes through a maximum and, with a further decrease in E_0, τ_W starts decreasing. This is because of the growing role of the discharge of excessive moisture into runoff in the effective relaxation of anomalies in soil moisture reserve, as compared with the role of evaporation.

To describe this effect analytically, we will use the nonlinear model (11). For the correlation function of solution (10), we have the equation [7, 10]:

$$\frac{dK_{WS}(\tau > 0)}{d\tau} = -\langle U'(t+\tau)\Delta W(t)\rangle, \quad U' = \frac{\partial U}{\partial W}.$$

Because of the nonlinearity of (11), this equation will not be closed, since the right-hand part includes moments of higher order, whose determination implies an infinite chain of equations.

However, numerical experiments [18] and observational data processing [31] suggest that the correlation function decreases exponentially, so approximate analytical methods can be used even in the nonlinear case. The method of quasi-equilibrium approximation [17] is one of such methods. The main idea of this method is the replacement of the one-time and two-time probability distribution densities of solutions (11) by functions of a specified form, whose parameters are determined from the condition that the means and the correlation functions of exact and quasi-equilibrium distribution functions are to coincide. For stationary in the average fluctuations in quasi-equilibrium approximation, we have [17:

$$\frac{dK_{WS}(\tau > 0)}{d\tau} = -\frac{\langle U'(t)\Delta W(t)\rangle_S}{\sigma^2_{WS}} K_{WS}(\tau). \quad (17)$$

Therefore, in the quasi-equilibrium approximation, $\tau_W = \sigma^2_{WS}\langle U'(W)W\rangle_S^{-1}$. For any stochastic model of type (11), in which the boundary condition $\rho_{WS}(\pm\infty) = 0$ is introduced, the relationship

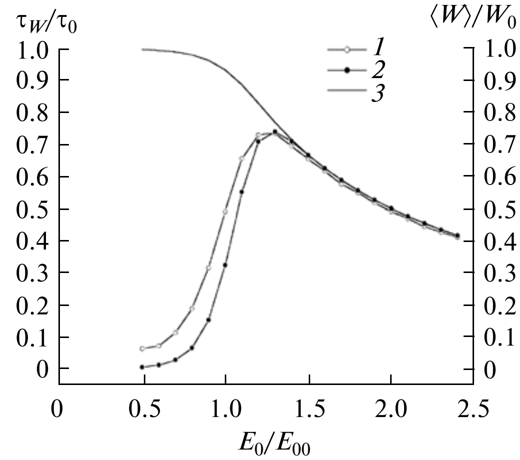

Fig. 1. Dimensionless correlation time τ_W/τ_0vs. E_0/E_{00} at $\langle P\rangle = E_{00}$. (*1*) Calculation using a finite-difference model, precipitation fluctuations have the exponential distribution; (*2*) estimation by quasi-equilibrium approximation; (*3*) estimated mean moisture reserves (right scale), standardized by its critical value.

$$\tau_W = \frac{\sigma^2_{WS}}{D_W}. \quad (18)$$

holds in a quasi-equilibrium approximation [7].

In the case of linear model, this relationship has been known as $\sigma^2_{WS} = D_W\tau_W$. For a nonlinear one-dimensional system, it can be proved by integration by parts in (17) during its averaging, taking into account that $\rho_{SW}(W) = C\exp(-U/D_W)$ tends to zero at the ends of the infinite interval. With the passage $\lambda_R \to \infty$ in (18), we determine τ_W via σ^2_{WS} with the help of (14).

To check the applicability of the quasi-equilibrium approximation for the description of dependence of τ_W on external parameters, the values of τ_W obtained by finite-difference approximation of model (10) with runoff parameterization by bucket model were compared with estimates based on formulas (14) and (18). The random sources were taken to be synoptic fluctuations of precipitation, not correlated at times of the order of $\Delta t = 1–2$ days; this interval will be taken as a time unit (conventional day). Next, we specify those fluctuations by a generator of independent random numbers with exponential distribution. Figure 1 gives the results of calculation of dependence of τ_W on E_0 at fixed values of mean precipitation at critical moisture reserve. We normalize the values of E and $\langle P\rangle$ by some characteristic value E_{00} and introduce $\tau_0 = W_0/E_{00}$, remembering that τ_0 is measured in conventional days—a time interval close to an actual day, within which, fluctuations in precipitation can be assumed uncorrelated. When determining relationships between the dimensionless $\langle R\rangle/E_{00}$, $\langle P\rangle/E_{00}$, $\langle E\rangle/E_{00}$, and τ_W/τ_0, without loss of generality, we can set $E_{00} = 1$ and $W_0 = \tau_0$. We choose the value $\tau_0 = 60$, corresponding to $\tau_E = 2$–4 months, $\langle P \rangle$ was assumed to equal E_{00}.

From Fig. 1, it can be seen that the sign of the derivative of the correlation time as a function evaporativity changes as early as $E_0 \approx 1.25\langle P \rangle$ ($\langle P \rangle = E_{00}$). According to Fig. 1 (curve *3*), the mean soil moisture reserve at those values accounts for 0.7 of the critical one. At $E_0 = \langle P \rangle$, this ratio is 0.8, and the value of τ_W, calculated by stochastic model, is about half of its estimate by the value of τ_E fro linear model in the absence of runoff. The comparison with empirical data on the ratio between the mean and critical moisture reserve [31] for the territory of the former USSR shows that the domains with $\langle W \rangle > W_0$ cover a considerable portion of high and temperate latitudes.

Data in Fig. 1 show that the quasi-equilibrium approximation (18) adequately reproduces the dependence of the correlation time of moisture reserve fluctuations on external conditions at a qualitative level, even when the distribution of synoptic fluctuations of moisture inputs is far from Gaussian.

THE DETERMINATION OF NONSTEADY RESPONSE OF SOIL MOISTURE RESERVE TO THE EXTERNAL IMPACT WITH THE USE OF NONLINEAR FLUCTUATION-DISSIPATIVE RELATIONSHIPS

Formulas (14) enable us to determine, for the simplified model under consideration, an average steady response W (not necessarily linear) to a specified constant perturbation of external factors (e.g., average precipitation or potential evaporation). However, actually, the external conditions can vary (e.g., within a year). If those changes are smooth, the response can be assumed quasi-stationary. However, in the case of fast changes in a nonlinear system in the presence of noises, the mean response can deviate much from the steady one. Considering that during the time of external perturbation, the regime of moistening in (10) can change, to determine the nonsteady response $\langle W(t) \rangle$ to changes in the external conditions is a nontrivial problem.

Now we will consider the mean deviations of a set of current values of variables $\mathbf{X}(t)$ from their equilibrium quasi-steady means $\langle \mathbf{X}(t) \rangle_S = \mathbf{X}_S$: $\Delta_S \mathbf{X}(t) = \mathbf{X}(t) - \mathbf{X}_S(t)$ at the given change in the external control parameter $\lambda(t)$. In this case, the equilibrium means are determined by the stationary probability distribution function $\rho_S(\mathbf{X}, \lambda(t))$, determined from a kinetic equation (e.g., by (11) for soil moisture reserve), under the assumption that $\lambda(t)$ (e.g., mean precipitation) will remain constant indefinitely long.

Determining the relationship between the characteristics of nonequilibrium, nonstationary state of the system and its equilibrium characteristics at the current values of specified external perturbation is the focus of a new branch of statistical physics—stochastic thermodynamics, which has been developing in recent years [22, 23, 29]. The core of the theory consists of the so-called fluctuation theorems (FT) or fluctuation relationships. One such relationship, which we will use below, establishes dependence between $\rho_S(\mathbf{X}, \lambda(t))$ and a functional of the trajectory of the random process in previous moments, irrespective of the time behavior of the external control parameter $\lambda(t)$.

A generalized potential $\varphi(\mathbf{X}, \lambda(t)) = -\ln \rho_s(\mathbf{X}, \lambda(t))$ is introduced. In the case of integral model (10), (11), $\varphi(\mathbf{X}, \lambda(t)) = U/D_W$ (at $D_W = \text{const}$). For each possible trajectory of the process, the potential $w(t) = \int_0^t \lambda'_\tau(\tau)\varphi'_\lambda(\mathbf{X}(\tau), \lambda(\tau))d\tau$ is introduced (an analogue of mechanical work), where strokes imply time differentiation, denoted by a subscript.

With averaging over all possible trajectories $\mathbf{X}(\tau < t)$, we have [22]:

$$\begin{aligned} g(\mathbf{X},t) &= \langle \delta(\mathbf{X}(t) - \mathbf{X})\exp\{-w(t)\}\rangle \\ &\equiv \exp[-\varphi(\mathbf{X},\lambda(t))] = \rho_S(\mathbf{X},\lambda(t)). \end{aligned} \tag{19}$$

In (19), $\delta(\mathbf{X})$ is Dirac delta function; the absence of an argument at a function implies that, unlike the current value on the trajectory, we mean a pre-specified value of a vector. Unlike the probability density $\rho(\mathbf{X}, t) = \langle \delta(\mathbf{X}(t) - \mathbf{X}) \rangle$, function $g(\mathbf{X}, t)$ is referred to as a weighed probability density [22]. The relationship (19) was proved for nonstationary Markov processes, which include the solution of (10), (11), as well as Hamiltonian systems [22]. It is valid for the case where at the initial moment, the system was in a stationary state.

Functional of the type (19) can be transformed by methods developed in [13] to find a characteristic functional of random processes. As the result, for example, for soil moisture reserve, we can obtain

$$\langle W(t) \rangle - W_S(\lambda(t)) = \int_0^t dt_1 \lambda'_t(t_1) K_{W,S}(t,t_1), \tag{20}$$

where correlator $K_{W,S}(t, t_1)$ is a standard fragment of the theory of linear response [10, 26]:

$$\begin{aligned} K_{W,S}(t,t_1) &= \left\langle W(t)\varphi'_\lambda(t_1) \right\rangle_S \\ &= -\left\langle F(t)\frac{\partial \ln \rho_{WS}(W(t_1),\lambda(t_1))}{\partial\lambda(t_1)} \right\rangle_S. \end{aligned} \tag{21}$$

Hereafter, the subscript S at two-time correlation functions implies that they are evaluated under the condition that at $t_1 < t$, the system was in a stationary state at $\lambda = \lambda(t_1)$.

The fluctuation–dissipation relationship (FDR) (20) differs from those taken in [5, 11, 25, 29], first, in that the deviations in it are taken from the current rather than initial quasi-stationary mean; and second, in that the factor before the function is not the external impact but its time derivative. However, the main feature is that the derivation of (20) does not assume the external impact to be small. Thus, (20) and (21) are nonlinear FDR.

Next, we will limit ourselves to the case of constant intensity of the random forces and take $W_E = \langle P\rangle/E_0$ as the control parameter $\lambda(t)$. It can be shown that (14) and (15) yield

$$\frac{\partial \ln \rho_S}{\partial W_E} = -\frac{\partial \varphi}{\partial W_S} = \frac{1}{D_S}(W - W_S) = \frac{\Delta_S W}{D_E}. \quad (22)$$

For the statistically nonstationary case, the quasi-equilibrium approximation (17), (18) becomes [17]

$$\frac{d\langle \Delta W(t)\Delta W(t_1)\rangle}{dt} = -\frac{1}{\tau_W(t)}(t)\langle \Delta W(t)\Delta W(t_1)\rangle. \quad (23)$$

Next, differentiating (2) with respect to time and considering (21)–(23), we obtain a linear differential equation for $\langle \Delta_S W(t)\rangle$:

$$\frac{d\langle \Delta_S W(t)\rangle}{dt} = -\frac{1}{\tau_W(t)}\langle \Delta_S W(t)\rangle - \frac{dW_E(t)}{dt}\frac{\sigma^2_{WS}(t)}{D_E},$$

whose solution is

$$\langle \Delta_S W(t)\rangle = -\int_0^t dt_1 \frac{dW_E(t_1)}{dt_1}\frac{\sigma^2_{WS}(t_1)}{D_E}\exp\{-[(\Lambda(t) - \Lambda(t_1)]\},$$
$$\Lambda(t) = \int_0^t dt\tau_W^{-1}(\tau)d\tau. \quad (24)$$

Let us apply the obtained relationships (14), (15), (18), and (24) to evaluate the average response of soil moisture reserve to variations in average precipitation. Assuming other parameters constant, we normalize the moisture reserve by its critical value, thus introducing dimensionless moisture reserve $w = W/W_0$. Formula (15) for dimensionless parameter Π_R becomes $\Pi_R = (w_E - 1)(2d_E)^{-1/2}$, where d_E is the dimensionless intensity of fluctuations in soil moisture reserve in the absence of runoff.

We assume that, at the initial moment $t = 0$, the system was in a statistically average stationary state with $\langle w(0)\rangle = 0.5$. The further calculations should be regarded as containing, in particular, the averaging over the statistical ensemble of initial anomalies. The issue of the average response to the specified initial anomaly for linear model was discussed, for example, in [12]. According to Fig. 1, such initial value of the mean soil moisture reserve corresponds to the regime of insufficient moistening far from the threshold, starting from which, the further increase in precipitation leads to formation of runoff. Let us normalize time t by $\tau_E(0) = \langle P(0)\rangle/E_0 = t_0$ by introducing $\tau = t/t_0$.

Let us consider the case, where, starting from moment $t = 0$, the mean precipitation increases linearly with time, resulting in linear growth in w_E from $w_E(0) = w_S(0)$ to $w_E(2t_0) = w_F$. The estimated response of the mean soil moisture reserve $\delta_0\langle w(t)\rangle = \langle w(t)\rangle - w_S(0)$ to an increase in the mean precipitation at $\delta_0 w_E = w_E(2t_0) - w_E(0) = 1.5$ and at different values of d_E is given in Fig. 2 by full lines. For the sake of comparison, we also give a dashed line corresponding to an idealized linear quasi-stationary increase in $w_S(t) = w_E(t)$ until saturation; next, we assume $w_S(t) = 1$. The dashed lines show the deviations of quasi-stationary values $\delta_0 w_S(t) = w_S(t) - w_S(0)$.

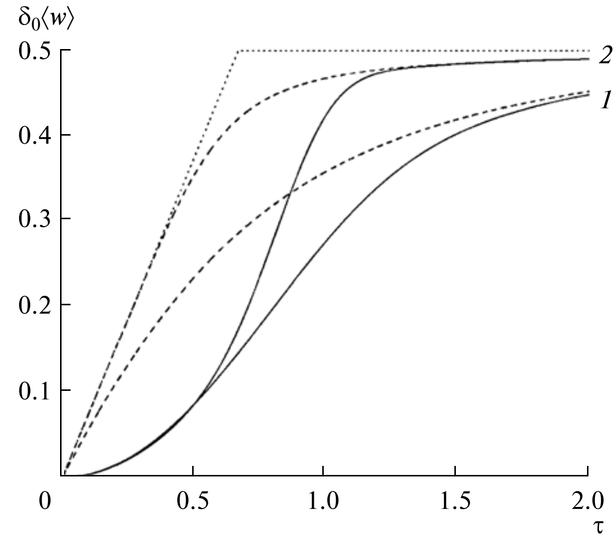

Fig. 2. The response of dimensionless mean moisture reserve $\delta_0\langle w(\tau)\rangle = \langle w(\tau)\rangle - w_S(0)$ to an increase in mean precipitation at $\delta_0 w_F = 1.5$ and different values of d_E, estimated by (14), (15), (18), and (24) (full lines); deviations of quasi-stationary values of $\delta_0 W_S = W_S(\tau) - W_S(0)$ (dashed lines). (*1*) $d_E = 0.1$, (*2*) $d_E = 0.01$; $\delta_0 w(\tau) = \delta_0 w_E(t)$ until the regime $\langle P\rangle = E_0$ is attained (dashed line).

At the given scenario, as can be seen from the knee on the dashed line in Fig. 2, the system passes from nonlinear regime with insufficient moistening to nonlinear regime with excessive moistening. In this case, the maximal difference can be seen near the boundary between the regimes, though within the domain of insufficient moistening.

Figure 2 also demonstrates the effect of the intensity of fluctuations on the width of the transition zone. The passage becomes smoother with increasing d_E, though the maximal deviation of the mean nonstationary response from the quasi-stationary mean decreases.

CHANGES IN THE FLUCTUATION SPECTRUM OF ATMOSPHERIC HUMIDITY DURING ITS INTERACTION WITH SOIL MOISTURE: THE METHOD OF EQUIVALENT STOCHASTIC SYSTEMS

The low-frequency fluctuations of soil moisture reserves caused by the response to short-period atmospheric impacts, in their turn, can cause, low-fre-

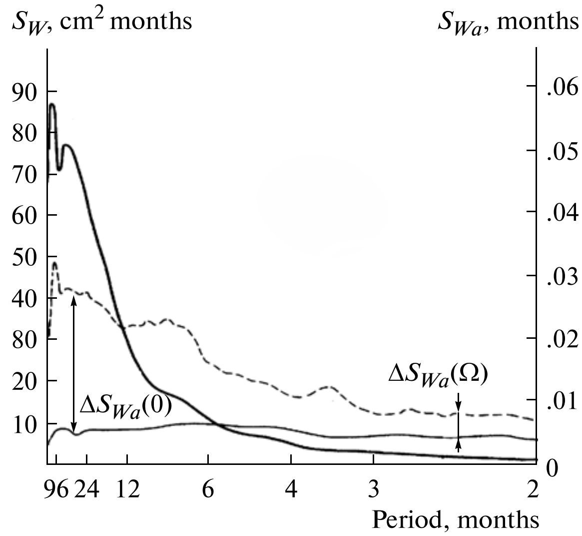

Fig. 3. Fluctuation spectra of the relative humidity of surface air S_{Wa} estimated by calculations on general circulation model [19, 20]. The noninteractive experiment is shown by the dashed line, the interactive one, by the full line; for comparison, the full thick line shows the spectrum of soil moisture reserve S_W (the right scale).

quency atmospheric variations, by modulating water vapor fluxes into the atmosphere.

The earliest numerical experiments [18–20] aimed to reveal the role of soil moisture content in atmospheric variations were carried out with the general atmospheric and oceanic circulation model in Geophysical Hydrodynamic Lab (Princeton, USA). In those experiments, soil moisture content was calculated using an integral model of moisture reserve evolution, i.e., moisture balance equation or top soil layer (7) with evaporation E parameterized by M.I. Budyko method and the runoff R, by "bucket model." The numerical experiment was divided into two stages. At the first stage (interactive experiment), a complete system of equations, approximating the interaction between the atmosphere and soil moisture reserves W, was integrated. The calculations were carried out for 50 years—a period long enough for reliably determining the first and second statistical moments given the annual course of oceanic surface temperature. At the second stage, the mean values of moisture reserve in each model cell (with their annual variations taken into account) were taken as specified external parameters and the atmospheric model alone was integrated over 25 years. Thus, for the mean of any variable F, we have $\langle F\rangle^{(n)} = \langle F|\langle W^{(i)}\rangle\rangle$. Hereafter, the superscripts (i) and (n) refer to the interactive and noninteractive experiments, respectively.

Figure 3 gives the results of processing the spectra of absolute air humidity according to data in [18–20], which illustrate the joint effect of the intensification of atmospheric variations because of the interaction between atmosphere and soil moisture during coordinated coevolution (interactive regime), compared with the case where soil moisture content is fixed (noninteractive regime). The spectra were averaged over the territory of the latitudinal belt between 36° N and 54° N in North America. In Fig. 3, arrows show the differences $\Delta S_{Wa}(0)$ between the spectra of relative humidity near zero frequency in experiments and the difference between spectrum sizes in the intermediate frequency domain—the spectral plateau $\Delta S_{Wa}(\Omega)$. An important feature of the spectra given in Fig. 3 is the increase in the spectral power of fluctuations in atmospheric humidity for the interactive experiment in the domain of spectral plateau: $S(\Omega\colon \tau_W \ll \omega \ll \tau_a) \approx \text{const}$.

To calculate the modification of the spectra of low-frequency fluctuations in the fast subsystem (atmosphere) during its interaction with fluctuations in the slow subsystem (soil moisture reserve), we can use the method of *equivalent stochastic systems* [6, 10]. In this method, the generalized Langevin equation (5) for slow variables **Y** is supplemented by a separation of fast variables **X** by analogy with the right-hand part of (5). As the result, we obtain *generalized Langevin relationships* [6]. With some general assumptions, equation (1b) for fast variables can be replaced by a stochastic differential equation with equivalent random force $\mathbf{f}_X(t)$, determined by variations in $\mathbf{X}_0(t)$ proper at $\mathbf{Y} = \text{const}$ (noninteractive experiment):

$$\frac{d\mathbf{X}(t)}{dt} = -\mathbf{B}_0\mathbf{X}(t) + \mathbf{A}\mathbf{Y}(t) + \mathbf{f}_X(t),$$
$$\mathbf{f}_X(t) = \frac{d\mathbf{X}_0(t)}{dt} + \mathbf{B}_0\mathbf{X}_0(t). \quad (25)$$

The matrices $\mathbf{B}_0$ and **A** in (25) are determined from the statistics of the solution of the problem of determining the response of $\delta_Y\langle\mathbf{X}(t)\rangle$ to anomalies in **Y**. The system thus constructed is equivalent to the original system, though only in the low-frequency domain.

To reproduce the effects revealed in plots in Fig. 3, let us supplement equation (7) for soil moisture reserve (in the regime of insufficient moistening) by an equation for integral moisture content in an atmospheric column W_a with the assumption that advective precipitation can also be parameterized according to the "bucket model" with some critical value of W_{ac}. For horizontal advection of moisture in the atmosphere D_h, let us express the parameterization in terms of the background value of atmospheric moisture content W_f: $D_h = -\lambda_D(W_a - W_f) + f_D$, where λ_D is a numerical factor. The random inflows of heat advection f_D are specified by a random-number generator with a uniform distribution. Thus, we have constructed a maximally coarse equivalent stochastic system for the description of moisture interaction between the atmosphere and soil. However, this system allows interactive and noninteractive experiments to be carried out and compared.

The results of such comparison for spectra at a realistic choice of model parameters are given in Fig. 4. In

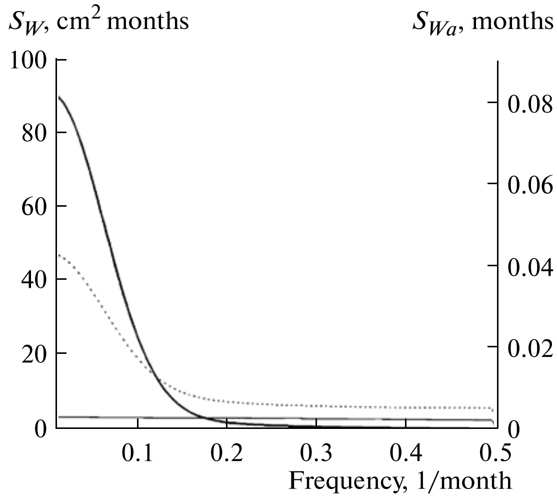

Fig. 4. Fluctuation spectra of relative air humidity W_a/W_{ac} by the results obtained using the stochastic model. The dashed line shows the noninteractive experiment, the full line, the interactive one; for comparison, the full thick line shows the spectrum of soil moisture reserve S_W calculated by the stochastic model (the right scale).

calculations with equivalent stochastic system, it was accepted that $W_f = 0.85W_{ac}$, $W_{ac} \approx 1$ cm, and time discretization was 3 days.

The comparison of Figs. 3 and 4 shows the climate model and the constructed stochastic model of interaction between moisture fluctuations in the atmosphere and soil to be in good agreement. In both models, the spectrum of fluctuations of the relative humidity in the intermediate frequency domain in the interactive experiment was about twice as large as that in the noninteractive experiment. At low frequencies, the increase in the spectrum of atmosphere relative humidity in the interactive experiment in stochastic model (more than 10 times) is greater than that in the hydrodynamic model (about 5 times).

CONCLUSIONS

The examples considered in the paper are limited to studying the behavior of fluctuations in soil moisture reserve. The spatially distributed fluctuations, which are of great importance for runoff determining, are not considered. Nevertheless, the presented methods allow multidimensional generalizations, and they have been formulated for solutions of multidimensional generalized Langevin equation (5). Those methods can also be applied to the case where finite-difference or spectral representations of fields (not necessarily moisture reserve) are used.

The determination of non-steady-state response of hydrological characteristics to external impacts with the help of fluctuation relationships of the type (19)–(20) should be regarded as promising, considering that the runoff characteristics generally show pronounced seasonal variations. The use of nonlinear methods of statistical physics can be of advantageous.

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