Spanning trees and forests via (two flavors of) matrices

Matteo D'ACHILLE

matteo.d-achille@univ-lorraine.fr

December 22, 2025

Project description: How does one count the number of spanning trees of a graph, or more generally of spanning forests? And how does one describe a "typical" object in these classes? A classical method dating back to Kirchhoff uses determinants of certain discrete laplacians of the graph, and can be used to deduce the shape of typical spanning forests [2]. But another method, that works for rectangular pieces of $\mathbb{Z}^2$ as well as more generic graph that have "layers", consists in applying the method of transfer matrices, another classic from both statistical physics and combinatorics. The spectrum of these matrices seems to have particular spectral property, that are conjectural at this point, and might be related to recent discoveries in spectral analysis of formal operators [1]. We propose that the candidate discovers both these methods, test the conjecture on various graphs, and try to see if the theories are connected in some - still unknown - manner.

Keywords: graph theory, combinatorics, spanning trees, spanning forests, random trees, spectral analysis, statistical physics, probability, local limits.

Candidate profile: You are a motivated Master 2 student in Mathematics, with a solid background in probability theory and/or combinatorics. Basic coding skills (especially in Python and/or Sage) are expected; no prior knowledge of the subject is required.

What we offer: The project can last **between 4** and **6 months** with a flexible starting date in **April 2026**.

Application: We require a detailed CV (including a complete transcript of grades), a cover letter explaining your interest in the project, and the names of one or two permanent professors who are willing to write a recommendation letter. The application material should be sent via email to the two above emails before **January 31st 2026**.

Mentors: Matteo D'Achille is Associate Professor at IECL, Université de Lorraine. He is specialized in probability with a focus on random geometry, statistical mechanics and random assignment problems. Paul Melotti is Associate Professor at IMO, Université Paris-Saclay, currently on gap year, who offers some punctual interactions with the project. He is specialized in probability theory, combinatorics and discrete geometry.

References

[1] R. Kenyon, M. Kontsevich, O. Ogievetskii, C. Pohoata, W. Sawin, S. Shlosman, *The miracle of integer eigenvalues*, Funct. Anal. Its Appl. 58, 182–194 (2024). Available at: `https://arxiv.org/pdf/2401.05291`

[2] **M. D'Achille**, N. Enriquez, **P.Melotti**, *Local limit of massive spanning forests on the complete graph*, (2024). Available at: `https://arxiv.org/pdf/2403.11740`

Last updated: December 22, 2025