Exercise session 3*

for the lecture "Adding Geometry: Euclidean Random Assignment Problems (ERAPs) and extensions, featuring a crash course on point processes"

Nesin Mathematics Village, Turkey, 29/07/2024–4/08/2024

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Exercise 1 (✱✱) (Hole probability) Consider a Poisson Point Process (PPP) $\mathbf{X} = (X_i\,;\, i \geq 1)$ of points in the plane with intensity measure $\mathrm{Leb}_{\mathbb{R}^2}$. Let D_1 be the unit disk, C_1 the unit circle, and P_n a regular n-gon inscribed in C_1 for $n \geq 3$. Let $\mathcal{R}_n = D_1 \setminus P_n$ (see Fig. 1 for $n = 5$).

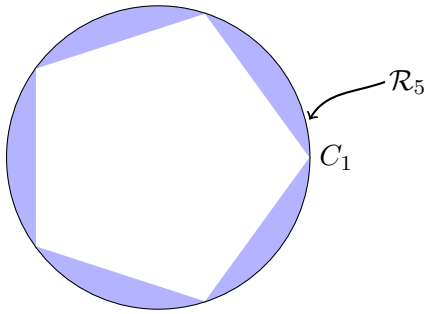

Fig. 1: The region $\mathcal{R}_5$ is shaded in blue.

✎ What is the smallest n s.t. the probability that no point of $\mathbf{X}$ falls into $\mathcal{R}_n$ is greater than $\frac{1}{2}$?

Exercise 2 (✱✱) (Quadratic ERAP on the unit interval) Consider the ERAP with uniform disorder over the unit interval $[0, 1]$ with $p = 2$.

✎ Taking inspiration from Exercise 1 of Session 2, prove that

$$\mathbb{E}[\mathcal{H}_{\mathrm{opt}}] = \frac{1}{3}\frac{n}{n+1}\,.$$

*Latest version (July 31, 2024) available electronically at: https://matteodachille.github.io/teaching

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