Planning and multimodal transportation

Discrete Optimization and Decision Making - Final project

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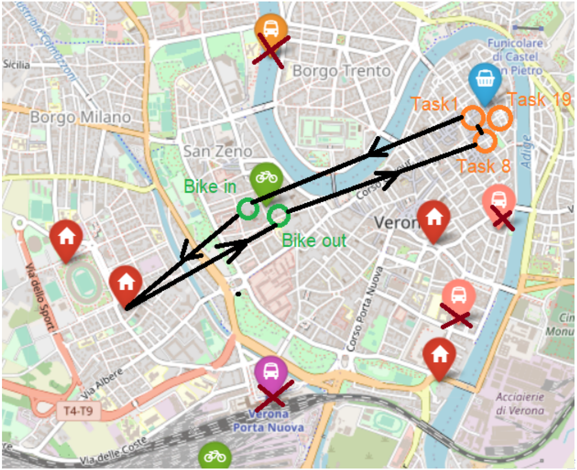
Starting remarks

Some aspects of the problem description haven't been included:

- Bus mode of transportation
- Number of bikes at the bike stops

Starting remarks

Some changes with respect to the notation used in the problem's description have been introduced



Variables

Binary variables describing choices regarding the route to take, the tasks to perform.

- $w_{n,i,j}$ for $n \in N$ and $i, v \in V$: binary, 1 if person n walks from node i to node j and 0 otherwise.
- $c_{n,i,j}$ for $n \in N$ and $i, v \in V$: binary, 1 if person n bikes from node i to node j and 0 otherwise.
- $z_{n,i}$ for $n \in N$ and $i \in V$: binary, 1 if person n visits node i and 0 otherwise.
- s_t for $t \in T$: binary, 1 if task t is done as special and 0 otherwise.

Variables

Continuous variables describing the evolution of time.

- u_i for $i \in H \cup P$: continuous, time place i is reached by the person visiting it.
- v_i for $n \in N$ and $i \in H \cup P$: continuous, time place i is left by the person visiting it.
- $t_{n,i}$ for $n \in N$ and $i \in B$: continuous, time person n visits bike stop i.

Variables

Binary variables needed to deal with the number of people in a place at the same time.

- $y1_{s,t}$ for $s, t \in T$: binary, 1 if task s starts before task t starts.
- $y2_{s,t}$ for $s, t \in T$: binary, 1 if task s ends before task t starts.

Constraints

The model contains many constraints which are more thouroughly presented in the report. Some important examples are:

- If a person visits a task place, they only do it once.

$$\sum_{j \in V} w_{n,i,j} + c_{n,i,j} = z_{n,i} \ \forall n \in N \ \forall i \in T$$

$$\sum_{j \in V} w_{n,j,i} + c_{n,j,i} = z_{n,i} \ \forall n \in N \ \forall i \in T$$

Constraints

- If a person enters a task place biking they leave biking, if they enter walking they leave walking.

$$\sum_{j \in V} w_{n,i,j} = \sum_{j \in V} w_{n,j,i} \ \forall n \in N \ \forall i \in T$$

$$\sum_{j \in V} c_{n,i,j} = \sum_{j \in V} c_{n,j,i} \ \forall n \in N \ \forall i \in T$$

- A task can't be done as special if it isn't special.

$$s_i \leq \mathbb{1}_{\{i \in T^*\}}$$

Constraints

- Constraint on the number of mode of transportation changes for each person. The number of changes is computed as the number of bike-leaving and bike-taking places visited in total.

$$\sum_{i \in B} z_{n,i} \leq K_n \; \forall n \in N$$

Constraints

- These constraints regulate the evolution of time when travelling between houses and task places. We only consider this constraints for places that person n can go to, so the places of their tasks (T_n) and their house (n).

$$v_i - u_j + t^{w}_{i,j} \leq tottime(1 - w_{n,i,j}) \; \forall n \in N \; \forall i,j \in T_n \cup \{n\}$$

$$v_i - u_j + t^{c}_{i,j} \leq tottime(1 - c_{n,i,j}) \; \forall n \in N \; \forall i,j \in T_n$$

Constraints

- These constraints impose that the task has to be started in the time window and that the person stays in the task place enough time to complete it.

$$u_t \geq a_t * z_{n^*,t} - tottime * s_t \ \forall t \in T$$

$$u_t \leq b_t * z_{n^*,t} + tottime * s_t \ \forall t \in T$$

$$v_t \geq u - t + \tau_t * z_{n^*,t} + \tau_t^+ * s_t$$

Constraints

- The following constraints enforce the definition of $y1$ and $y2$: $y1_{i,j}$ is 1 if and only if task i starts before task j starts, $y2_{i,j}$ is 1 if and only if task i ends before task j starts.

$$u_j - u_i \leq tottime * y1_{i,j} \forall i \in T \ \forall j \in T \text{ if } P_i = P_j$$

$$u_i - u_j \leq tottime * (1 - y1_{i,j}) \forall i \in T \ \forall j \in T \text{ if } P_i = P_j$$

$$u_j - v_i \leq tottime * y2_{i,j} \forall i \in T \ \forall j \in T \text{ if } P_i = P_j$$

$$v_i - u_j \leq tottime * (1 - y2_{i,j}) \forall i \in T \ \forall j \in T \text{ if } P_i = P_j$$

Constraints

- The total number of people that are already in place P when task j is started can be obtained by the sum of $y1_{i,j} - y2_{i,j}$ between the tasks i that are done in the same place. The following constraint imposes that the number of people in the same place at the same time doesn't go over the maximum.

$$\sum_{i|P_i=P_j} y1_{i,j} - y2_{i,j} \leq N_{P_j} - 1 \; \forall j \in T$$

Constraints

- The final constraint deals with the budget for each person and imposes that, considering the cost of biking and the expense for each completed task, the maximum budget is not surpassed.

$$\sum_{t \in T} c_t z_{n,t} + C_{bike}\left(\sum_{i \in B_{in}} t_{n,i} - \sum_{i \in B_{out}} t_{n,i}\right) \leq W_n \;\forall n \in N$$

Objective function

The model's objective function is:

$$min \quad R\sum_{n\in H}(u_n - v_n) + P\sum_{t\in T}Q_t(1-z_t) - F\sum_{n\in N}\sum_{i\in V}\sum_{j\in V}(f_{i,j}^{W}w_{n,i,j} + f_{i,j}^{C}c_{n,i,j})$$

Parameters R, P and F regulate the relative importance given to the three terms and, unless otherwise specified, are considered fixed to 1.

Gurobi implementation

The implemented code contains a function that import the data from the `txt` file instances, extracting information about the vertices of the multigraph, their respective features and the traveling times and related fitness between them.

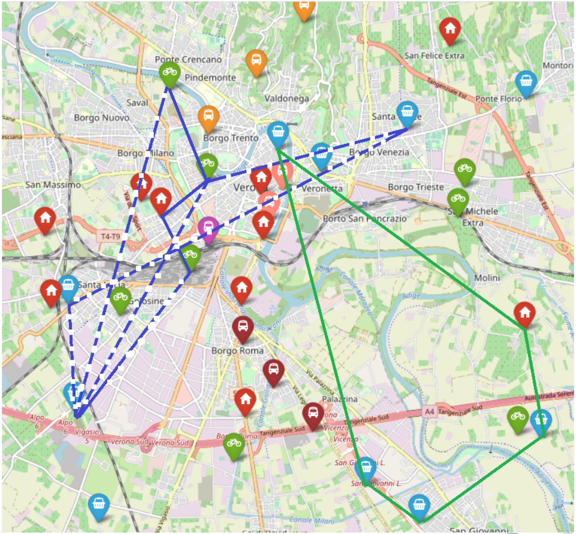
The mathematical model was then implemented using Gurobi, introducing variables and constraints as previously described.

Branch-and-cut inequalities

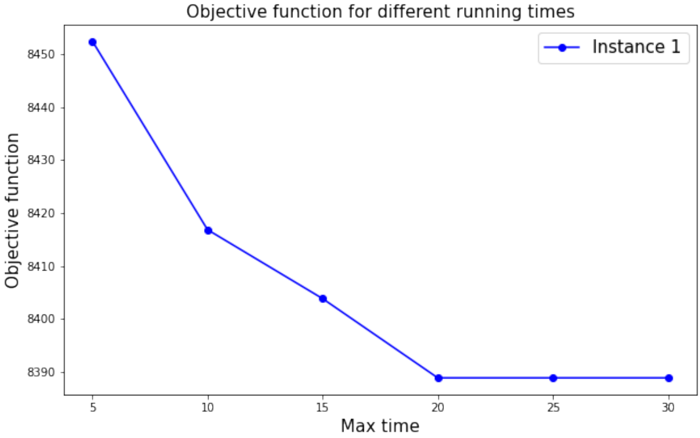
The implementation of a Branch-and-Cut was achieved by relaxing the constraints related to the number of people simultaneously in a same place.

The problem is solved without these constraints: when a MIP solution is found, we check if it violates any of these constraints and, in case it does, the model is solved again after their introduction.

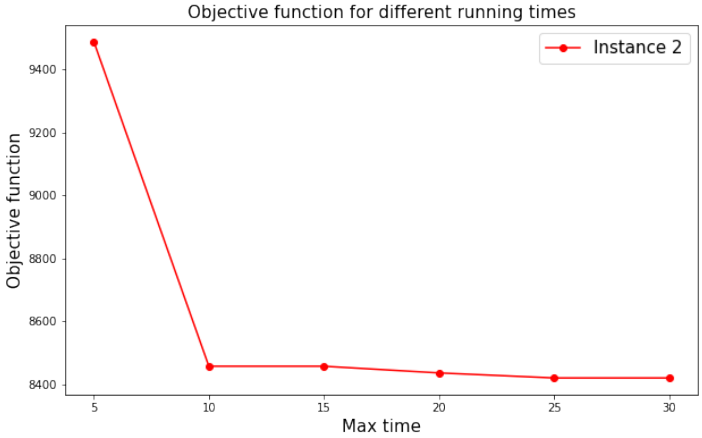
Results


Ponte Crencano
Pindemonte
Saval
Borgo Nuovo
Valdonega
San Felice Extra
Ponte Florio
Borgo Trento
Borgo Venezia
San Massimo
Veronetta
Borgo Trieste
T4-T9
Molini
Borgo Roma
Palazzina
Tangenziale Sud
Verona Sud
Vigasio
Canale Milani

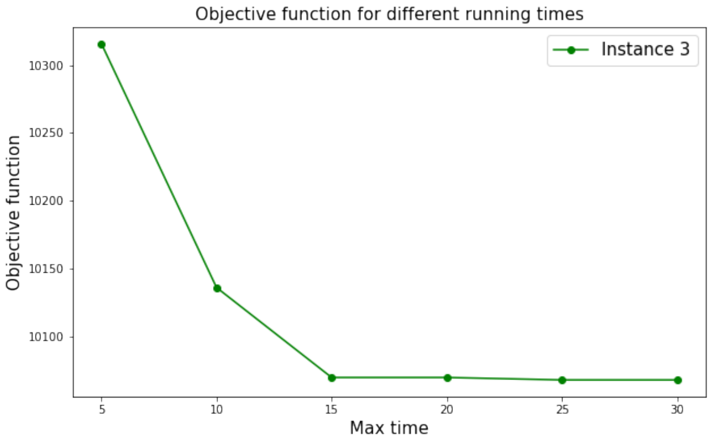
Results


Objective function for different running times
Instance 1
Objective function
8450
8440
8430
8420
8410
8400
8390
5
10
15
20
25
30
Max time

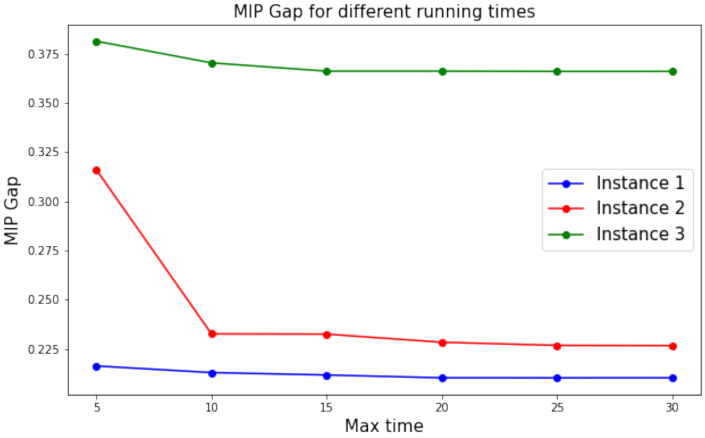
Results


Objective function for different running times
Instance 2
Objective function
9400
9200
9000
8800
8600
8400
5
10
15
20
25
30
Max time

Results


Objective function for different running times
Instance 3
Objective function
10300
10250
10200
10150
10100
5
10
15
20
25
30
Max time

Results


MIP Gap for different running times
MIP Gap
0.375
0.350
0.325
0.300
0.275
0.250
0.225
5
10
15
20
25
30
Max time
Instance 1
Instance 2
Instance 3

Questions

- **How do the solutions change if the maximum number of mode of transportation changes K is set to 2 for every person $n \in N$?**
  For most people the best route would be to take a bike at the beginning of the day, travel with it and leave it at the end of the day. Other people would only walk around or take the bus just once.

Questions

- **How do the solutions change when this K increases?**

K	Objective Function
0	9100.83
2	8130.62
4	8203.86
6	8333.38

Questions

- **How do the solutions change when soft constraints are not allowed anymore?**
  In the absence of soft constraints, all tasks in a task list would have to be completed: this could create an infeasible problem in case the time windows of different tasks didn't match or the budget couldn't allow for all of them to be completed.

Questions

- **How do the solutions change if the goal becomes to only maximize the overall fitness score?**
  In this case, the penalties associated with not doing tasks would be ignored. Total time would also not be a factor. If for a person there exists a route starting and terminating at their home that produces positive fitness, this would be repeated until reaching the maximum time. Otherwise, the person wouldn't move at all.

Thank you for your attention!