Multi-agent decision making: preference reasoning and voting theory

Outline

- Preferences
- Several kinds of preferences
- Preferences in multi-agent decision making
- Voting theory (social choice)
- In multi-agent AI scenarios:
  - Missing and imprecise preferences
  - Computational concerns
  - Large set of candidates
  - Candidate set with a combinatorial structure

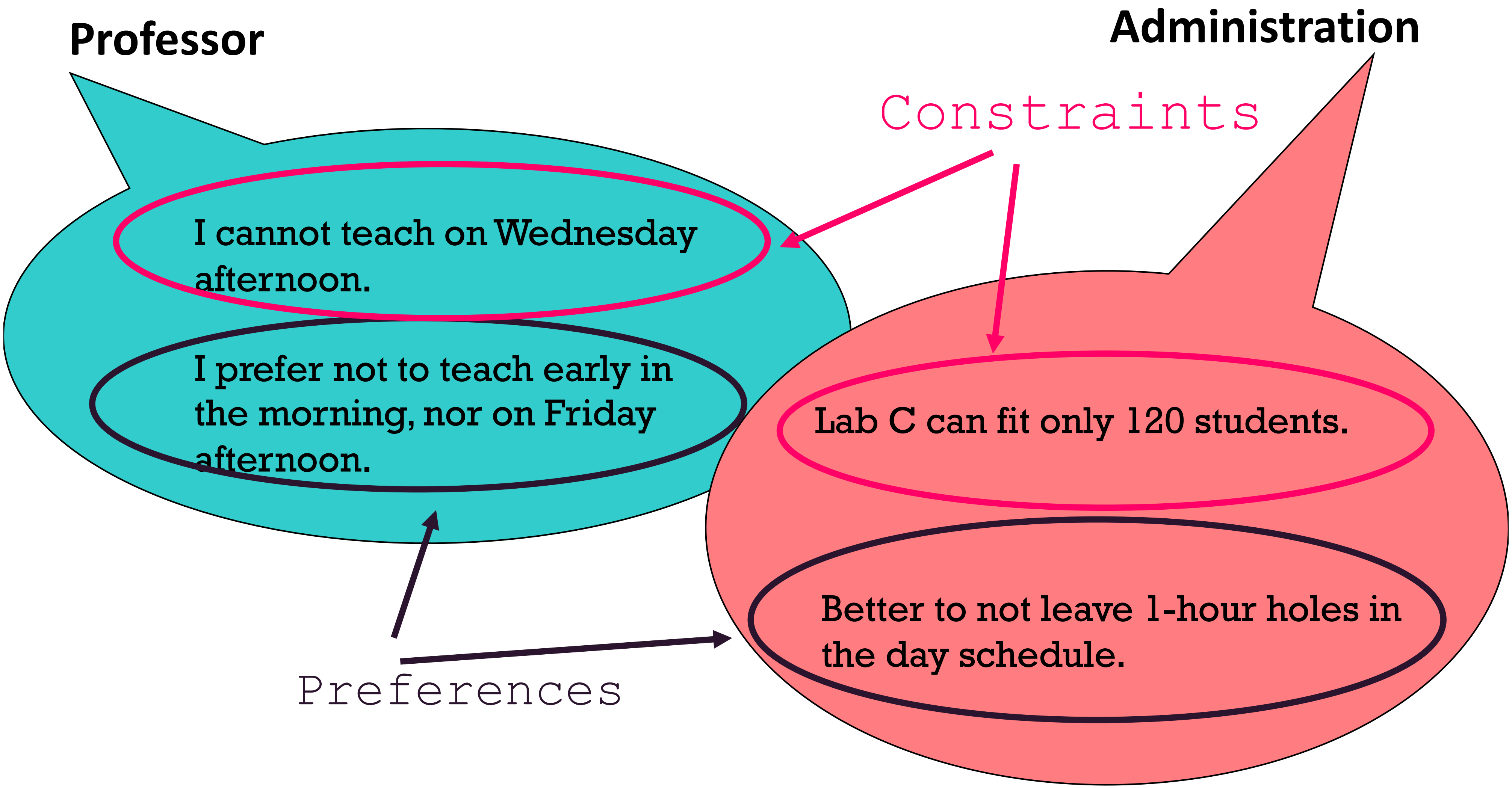
Preferences

- Preferences are **ubiquitous** in **everyday decision making**
  - Essential ingredients in every reasoning tool
- Preferences are **orderings** over possible options
  - Options: candidates, car, computers, books, movies …
- Preferences can model **levels of acceptance**, or **costs**
  - Preferences are <u>tolerant constraints</u>
  - Constraints are strict requirements that must be satisfied

Preferences

- **If all constraints**, possibly
  - no solution, or
  - too many of them, all apparently equally good
- Some problems are **naturally modelled** with preferences
  - I don't like meat, and I prefer fish to cheese
- **Constraints and preferences** may be present in the same problem
  - Ex. Timetabling, ..

University timetable



Several kinds of preferences

- **Unconditional**
  - *I **prefer** taking the bus*
- **Conditional**
  - *I **prefer** taking the bus **if it's raining***
- **Multi-agent**
  - *I like blue, my husband likes green,*

    what color for the new car?

Several kinds of preferences

- **Quantitative**
  - **Numbers**, or ordered set of objects
    - *My preference for ice cream <u>is 0.8</u>, and for cake <u>is 0.6</u>*

- **Qualitative**
  - **Pairwise comparisons**
    - *Ice cream is <u>better than</u> cake*

Two main ways to model compactly preferences

- **Several kinds of preferences**
- **Two compact ways to model preferences**
  - **Soft constraints**
    for modeling **quantitative** and **unconditional** preferences
    - *Ex., My preference for ice cream is 0.8, and for cake is 0.6*
  - **CP-nets**
    for modeling **qualitative** and **conditional** preferences
    - *Ex., Red wine is better than white wine if there is meat*

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II PART

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Preferences for <u>collective decision making</u> in multi-agent systems

- Several **agents**
- **Common** set of **possible decisions**
- **Each agent** has its **<u>preferences</u>** over the possible **decisions**
- **Goal**: to choose <u>one</u> of the decisions, **based on the preferences** of the agents
  - or <u>a set</u> of decisions
  - or <u>a ranking</u> over the decisions

- **AI scenarios** add:
  - Imprecision
  - Uncertainty
  - Complexity concerns
  - Combinatorial structure of the decisions

Applications

- Doodle
  - **Several time slots** under consideration
  - Participants **accept** or **reject** each time slot
    - Very simple way **to express preferences** over time slots
    - Very little information communicated to the system
  - **Collective choice**: a single time slot
    - The one with most acceptance votes from participants
- Other applications
  - Group recommender systems
  - Meta-search engine

How to compute a collective decision?

- Let the **agents** **vote** by expressing their **preferences** over the **possible decisions**
- **Aggregate** the **votes** to get a **single decision**
- Let's look at **voting theory**
  - Agents = Voters
  - Decisions = Candidates
  - Preferences
  - Chosen decision = winner

Voting theory
(Social choice)

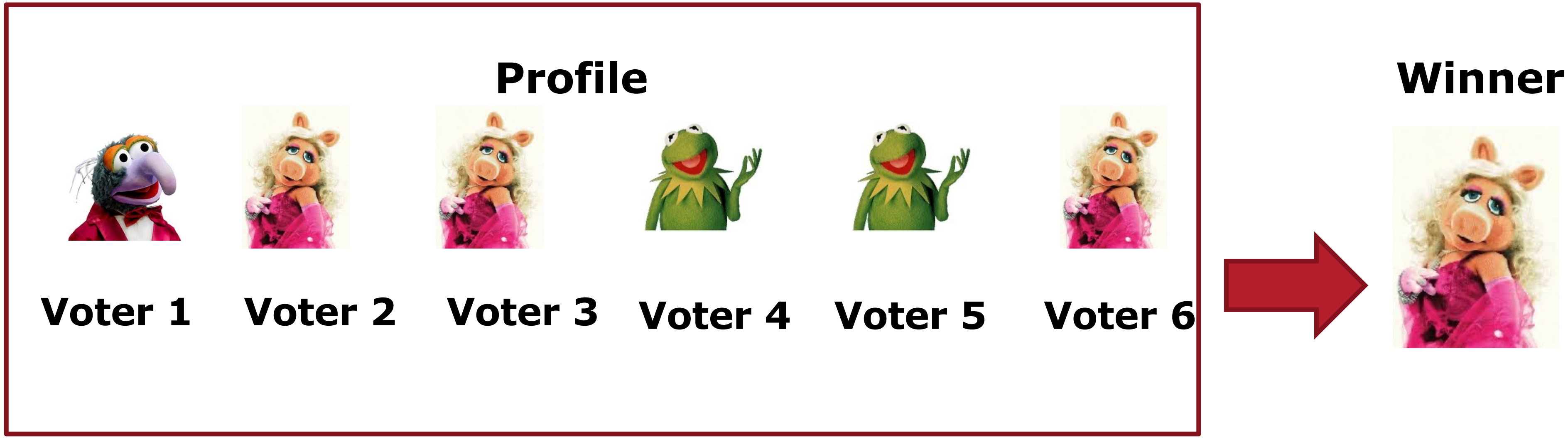
- **Voters**
- **Candidates**
- **Each voter** expresses its **preferences** over the **candidates**
- **Goal**: to choose **one** **candidate** (the winner), based on the voters' preferences
  - Also many candidates, or ranking over candidates
- **Voting Rules** (functions) **to achieve the goal**

Some voting rules

- Plurality
  - **Voting**: each voter provides the most preferred decision
  - **Selection**: the decision preferred by the largest number of voters
- Majority: like plurality, over 2 options

Plurality

- **Voting**: the most preferred decision
- **Selection**: the decision preferred by the largest number of agents
- **Example:**
  - 6 voters
  - 3 candidates:



Another voting rule

- Approval (m options)
  - **Voting:**
    each voter approves any number of options
  - **Selection:** option with most votes

Voting rule used in Doodle

Another voting rule

- Borda
  - **Voting**: each voter provides a ranking over all options
  - **Score of an option** for a voter: number of options that it dominates
  - **Selection**: option with greatest sum of scores

Borda

rank
3
2
1
0
Voter 1
rank
3
2
1
0
Voter 2
rank
3
2
1
0
Voter 3
rank
3
2
1
0
Voter 4
rank
3
2
1
0
Voter 5
Borda Count
9
8
7
6
Winner

Some desirable properties

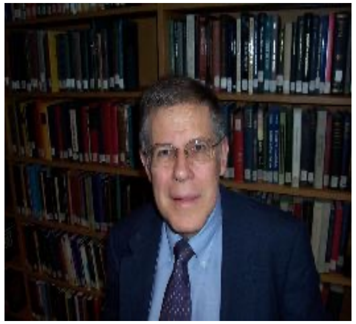
- Unanimity (efficiency)
  - If all voters have the same top choice, it is selected
- Non-dictatorship
  - There is no dictator
  - Dictator: voter such that his top choice always wins, regardless of the votes of other voters
- Non-manipulability
  - There is no incentive for agents to misrepresent the preferences

Two classical impossibility results

- **Arrow's theorem (1951)**
  - Totally ordered preferences
  - It is **impossible** to find a **voting rule** with some desirable properties including
    - unanimity
    - non-dictatoriality

Nobel prize
in Economics 1972

- **Gibbard-Sattherwaite's theorem (1973)**
  - Totally ordered preferences
  - it is **impossible** to have a reasonable **voting rule** that is
    - non-dictatorial
    - non-manipulable



- These impossibility results hold also when we allow partially ordered preferences

Multi-agent decision making: preference reasoning and voting theory

III PART

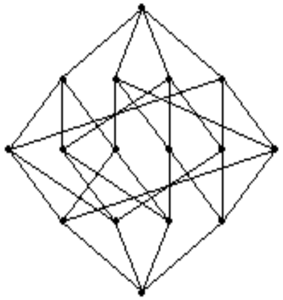
Voting theory and multi-agent systems

- Voting theory (social choice)
  - Voting rules
  - Desirable properties
  - Impossibility results

- In multi-agent AI scenarios, we usually have
  - Incomparability
  - Uncertainty, vagueness, preference elicitation
  - Computational concerns
  - Large set of decisions (candidates) w.r.t. number of agents (voters)
  - Combinatorial structure for the set of decisions (candidates)

Incomparability

- **Preferences** do **not always** induce **a total order** over the options
- Preferences may induce a **partial order** where some options are incomparable



- **Some options** are naturally incomparable
  - Eg., it may be **easy** to compare two apartments
  - but it may be **difficult** to compare an apartment and a house, thus we say they are incomparable
- An agent may have several possibly conflicting preference criteria he wants to follow
  - Eg., a cheap and slow car is incomparable w.r.t. an expensive and fast car
- Many **AI formalisms** to model preferences **allow for partial orders** (eg., soft constraints)

Uncertainty, vagueness

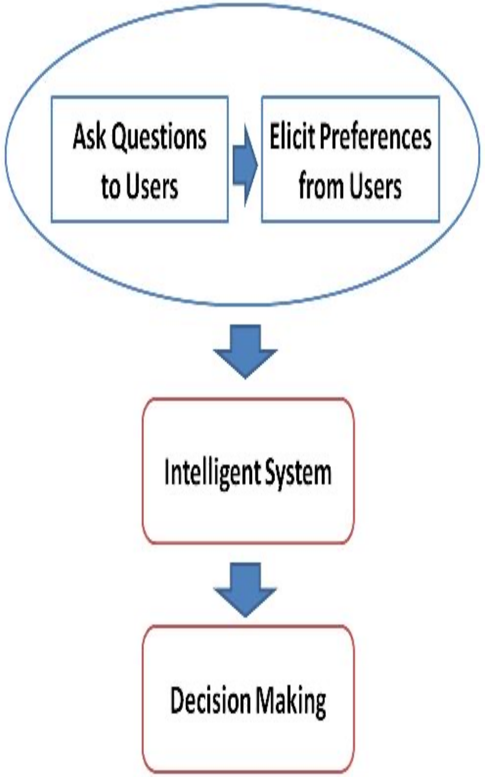
- Missing preferences
  - Too costly to compute them
  - Privacy concerns
  - Ongoing elicitation process

- Imprecise preferences
  - Preferences coming from sensor data
  - Too costly to compute the exact preference
  - Estimates

Aim in AI: Find compact preference formalisms and solving techniques to model and solve problems with missing or imprecise preferences

Preference elicitation

- Some **preferences** may be missing
- Time consuming, costly, difficult, to elicit all preferences
- We want to terminate preferences elicitation as soon as a winner fixed



Computational concerns

- We would like to avoid very costly ways to
  - Model agents' preferences
  - Compute the winner
  - Reason with the agents' preferences

- On the other hand, we need a computational barrier against bad behaviors (such as manipulation)

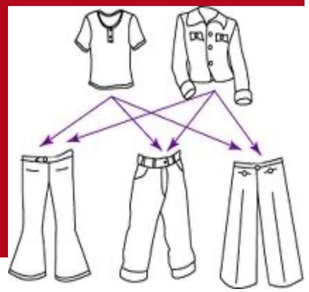
Bartholdi, Tovey, Trick. The computational difficulty of manipulating an election. Social Choice and Welfare 1989

Large set of candidates

- In **AI scenarios**, usually
  the **set of decisions** **(candidates)** is **much larger** than
  the **set of agents** **expressing preferences**
  over the decisions

  - *Eg., many web pages, few search engines*

Combinatorial structure for the set of decisions



- **The set of decisions** may have a **combinatorial structure**
- **Dinner example**:
  - **Three friends** need to decide **what to cook for dinner**
  - **4 items** (pasta, main, dessert, drink)
  - **5 options for each item**
  - 5^4 = **625 possible dinners**
  - It is **unfeasible** providing a **preference ordering over 625** dinners
- In general: Cartesian product of several variable domains
  - **Variables** = **items** of the menu
  - **Domain** of each variable = **5 options**
- **Goal:** Find **compact preference formalism** to express **agents' preferences**

Formalisms to model preferences compactly

- **Preference ordering over a large set of decisions**
  ➔ need to **model them compactly**
  - Otherwise too much space and time to handle such preferences

- Two examples:
  - **Soft constraint formalism**
  - **CP-net formalism**

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IV PART

Soft constraint formalism

- Soft constraint formalism (the c-semiring framework)
  - The agent expresses his preferences over partial assignments of the decision variables
  - From these preferences → the preference ordering over the solution space is generated
  - **Any ordering can be obtained!**

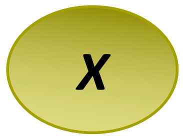
Formalisms to model compactly preferences

- **Soft constraints** model quantitative and unconditional preferences
  - Quantitative ➔ a level of preference for each assignment of the variables in a soft constraint
  - It is difficult **to elicitate** quantitative preferences from user
- Moreover, many problems **need statements** like
  - “I like white wine **if** there is fish” (**conditional**)
  - “white wine is **better than** red wine” (**qualitative**)

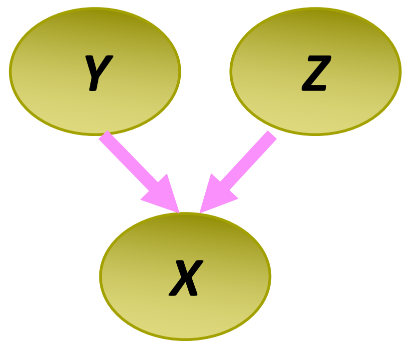
- **CP-net:** formalism to compactly represent qualitative and conditional preferences

CP-net (conditional preference network)

- Variables $\{X_1, \ldots, X_n\}$ with **domains**
- For each variable, a total order over **its values**
- Indipendent variable:
  - $X=v_1 > X=v_2 > \ldots > X=v_k$



- Dependent variable: a total order over **its values** for each combination of values of some other variables
  - $Y=a, Z=b$: $X=v_1 > X=v_2 > \ldots > X=v_k$
  - X depends on Y and Z (parents of X)



- Graphically: **directed graph** over $X_1, \ldots, X_n$
- Possibly **cyclic**

CP-net

- A CP-net over variables V = {X1,...,Xn} is
  - a directed graph G over X1, . . . , Xn
  - whose nodes are annotated with conditional preference tables CPT(Xi) for each Xi ∈ V.
  - Each conditional preference table CPT(Xi) associates a total order ≻ with each instantiation u of Xi's parents Pa(Xi) = U.

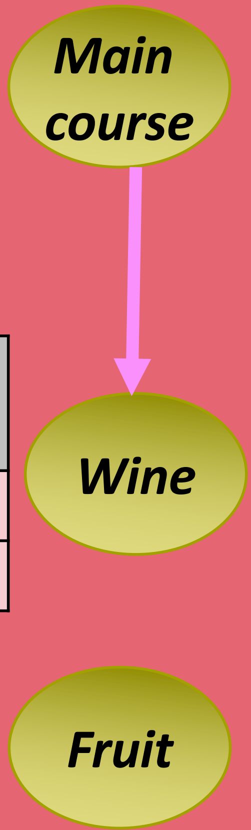
CP-net: an example

CP-table

Main course
fish > meat

Main course	Wine
fish	white > red
meat	red > white

Fruit
peaches > strawberries



Variables

- **MainCourse**
- **Wine**
- **Fruit**

Domains

- $D_{MainCourse}$ = {meat,fish}
- D_{Wine} = {white, red}
- D_{Fruit} ={peaches,strawberries}

Independent variables

- **MainCourse**
- **Fruit**

Dependent variable

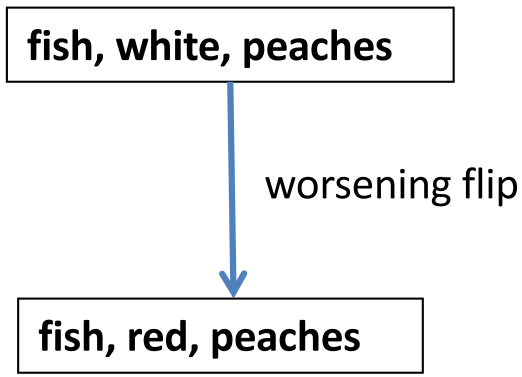
- **Wine**

➢ Boutilier, Brafman, Domshlak,Hoos, Poole. CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements. *J. Artif. Intell. Res. (JAIR)* 21: 135-191 (2004)

CP-net semantics

- Worsening flip: changing <u>the value</u> of <u>a variable</u> in a way that is <u>less preferred</u> in some statement

- Example:



- An outcome O_1 is preferred to O_2 iff
  there is a <u>sequence</u> of <u>worsening flips</u> from O_1 to O_2

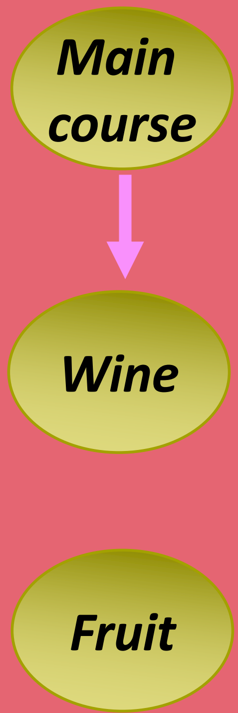
- Optimal outcome: if no other outcome is preferred

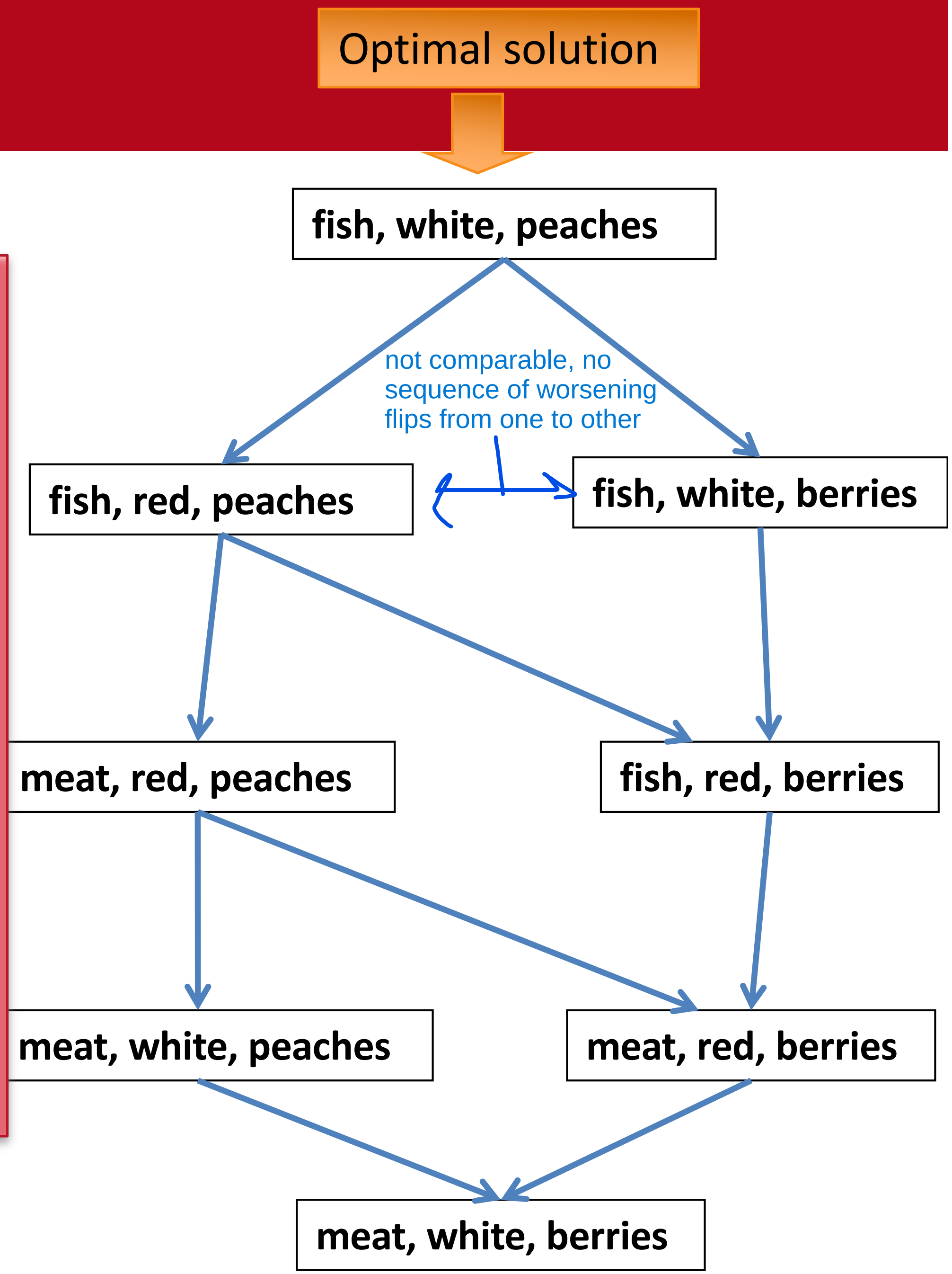
A CP-net induces an ordering over solutions

Main course
fish > meat

Main course	Wine
fish	white > red
meat	red > white

Fruit
peaches > strawberries





Finding an optimal solution in acyclic CP-nets is easy!

Forward sweep algorithm

- First consider independent variables

  Assign them their most preferred values

- Then consider dependent variables, that directly depend on the assigned variables

  Assign them their most preferred values that are consistent with the values previously assigned to their parents

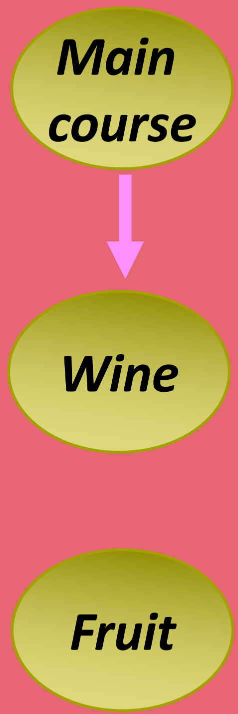
- And so on until we assign a value to all the variables

Finding an optimal solution in acyclic CP-nets is easy!

Main course
fish > meat

Main course	Wine
fish	white > red
meat	red > white

Fruit
peaches > strawberries



Optimal solution

fish, white, peaches

Soft constraints vs. CP-nets

- Different expressive power
- Different computational complexity for reasoning with them

	Soft CSPs	Tree-like soft CSPs	CP-nets	Acyclic CP-nets
Preference orderings	all	all	some	some
Find an optimal decision	difficult	easy	difficult	easy
Compare two decisions	easy	easy	difficult	difficult
Check if a decision is optimal	difficult	easy	easy	easy

Multi-agent decision making: preference reasoning and voting theory

V PART

Sequential voting

- **Several voters**
- **Decisions** are made by **several issues**
- Main idea: **vote separately on each issue**, but do so **sequentially**
- This gives **voters** the opportunity to make **their vote <u>for one issue</u>** depending on the **decisions on previous issues**

- In AI: sequential voting when agents express their preferences
  - via soft constraints
  - via CP-nets

Sequential voting with soft constraints

- How to apply **sequential voting** when **agents** express their **preferences** via **soft constraint problems**?

- Assume the agents have
  - the **same constraint graph**
  - but **different preference values**

Dinner example, three agents, fuzzy constraints

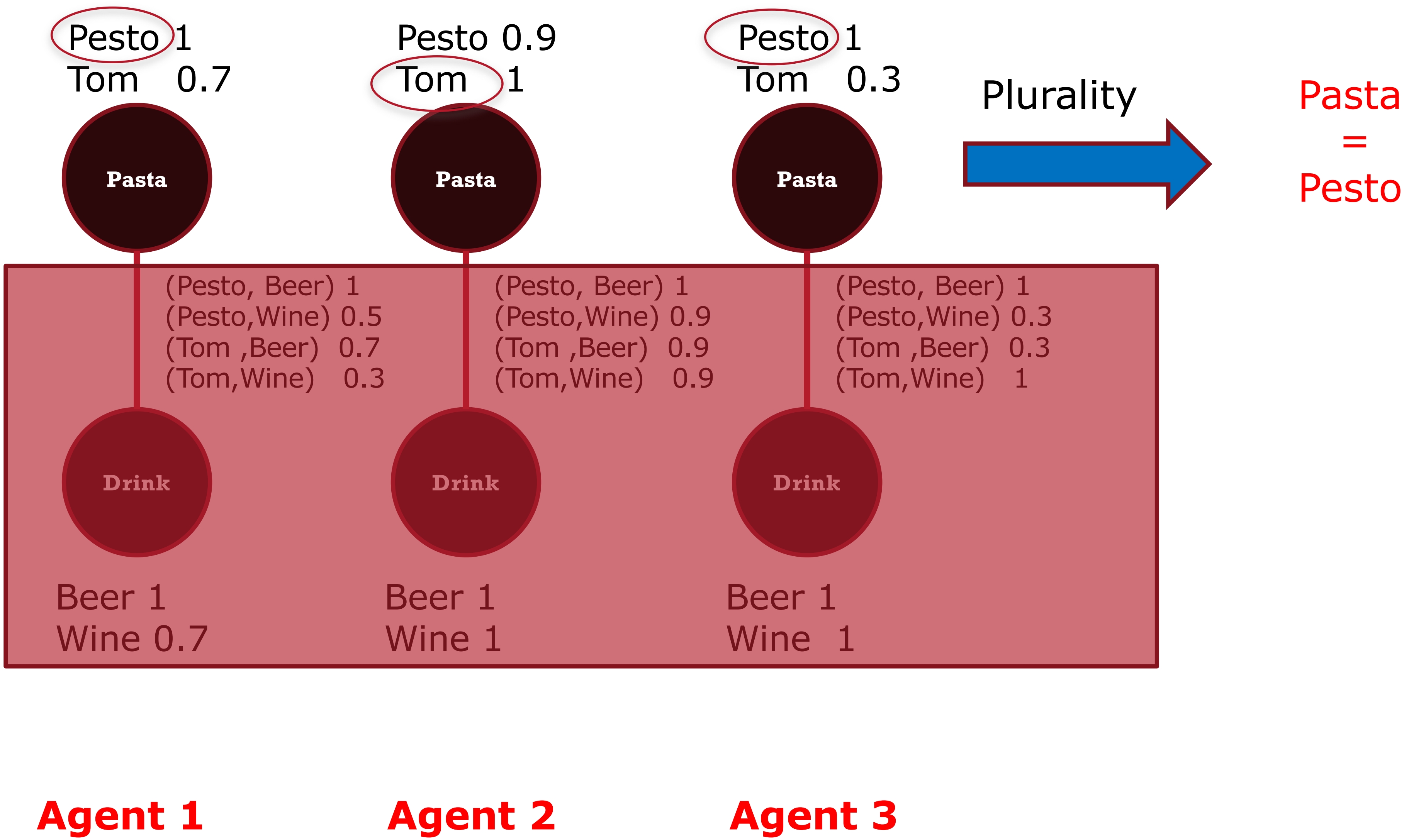
The sequential voting approach

voti del agente

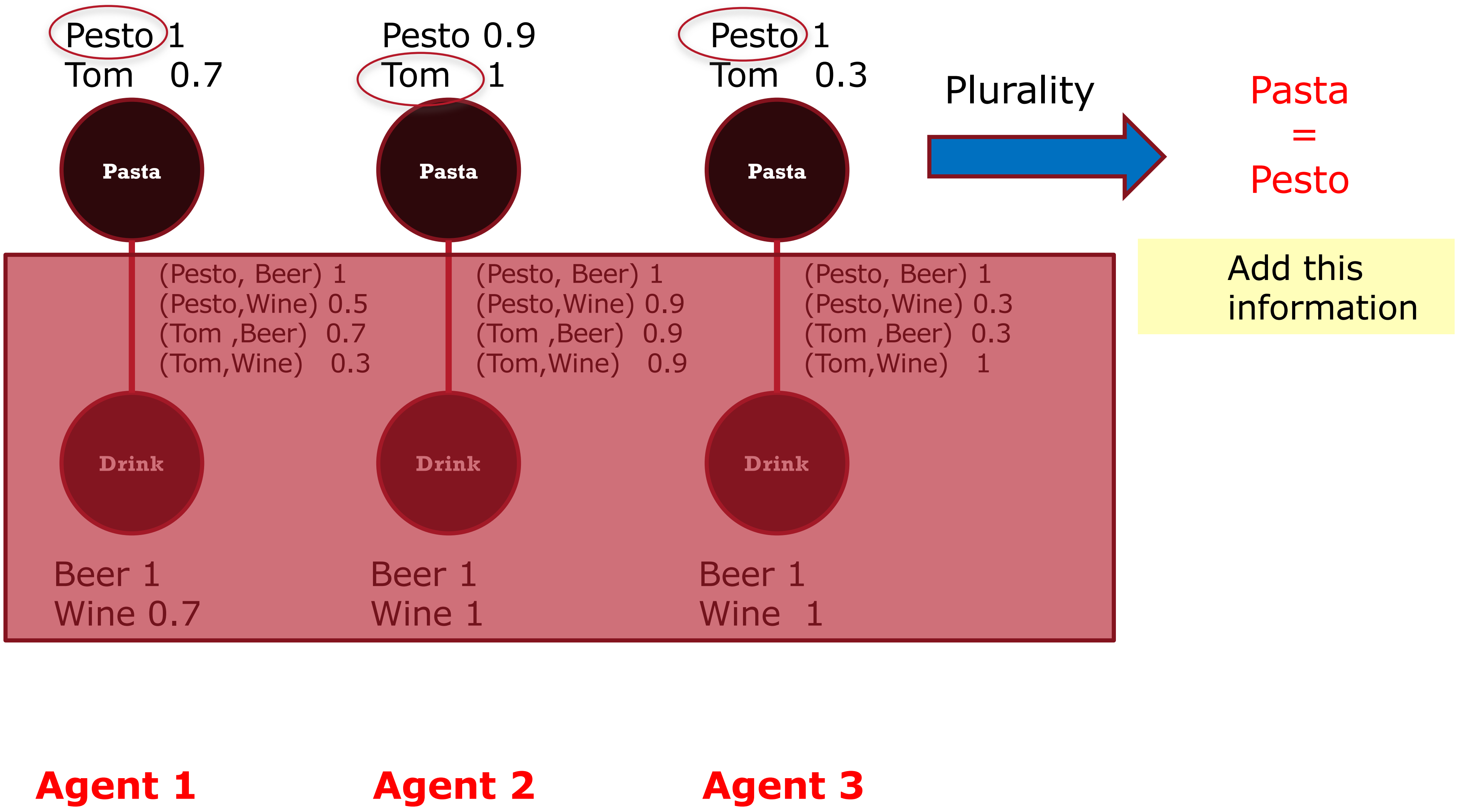
- **For each variable**
  1. **Compute** an **explicit profile** over the **variable domain**
  2. **Apply a voting rule** to this explicit profile →
     the rule will return a **specific value**
     that will be assigned **to this variable**
  3. **Add the information** about the selected variable value

Similar approach used for CP-nets in [Lang, Xia, 2009]

Dinner example using plurality



Dinner example using plurality



Dinner example using plurality

tutte le scelte con tomato dentro sono messe a 0

Dinner example using plurality