UNIVERSITÀ DEGLI STUDI DI PADOVA

Local equalization and specification

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Agenda

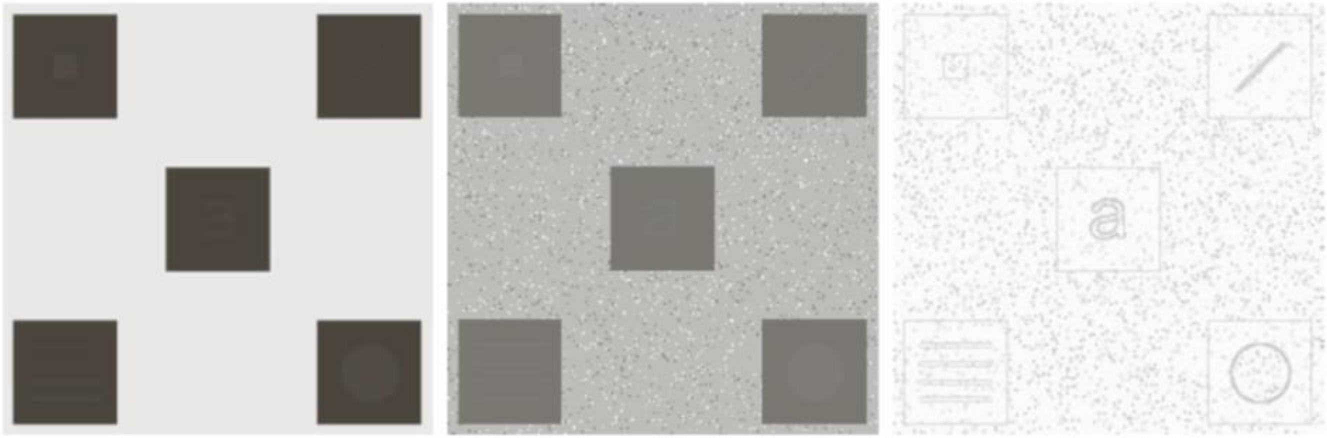
- Using multiple equalization functions
- Specifying a shape for the histogram

Extending the histogram equalization

- The concept of histogram equalization can be extended
  - Local histogram equalization
  - Histogram specification: the output shape is not flat, but *specified* by a given function

Local histogram equalization

- Local histogram equalization can be useful if different regions of the image have very different pixel distribution characteristics



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3.

Histogram specification

- Histogram specification process:
  - Equalize the histogram
  - Specify the desired output shape of the histogram
    - This is done by listing all the points in the histogram
  - Obtain the inverse transformation
  - Apply the two transformations
    - Equalization
    - Transformation to desired shape
  - Map the two together

Histogram specification

Equalization process on the image

$$r = I(x,y) \longrightarrow s = T(r)$$

$$z \longrightarrow s = G(z)$$

"Abstract" equalization process on a histogram distribution defined by us

Histogram specification

Equalization process on the image

$r = I(x, y) \rightarrow s = T(r)$

$z \leftarrow s = G(z)$

Invert the function

Histogram specification

Equalization process on the image

$$\begin{tikzcd}
r = I(x,y) \arrow[r] & s = T(r) \arrow[d] \\
z & s = G(z) \arrow[l]
\end{tikzcd}$$

in blue abstract equalization

Connect the two processes

Invert the function

invert the blue arrow

Histogram specification

Mathematical formulation:

- Equalize the input image

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

- Define the desired output Probability Mass Function (PMF) $p_z(z_i)$ and evaluate the corresponding CDF, representing the **target**

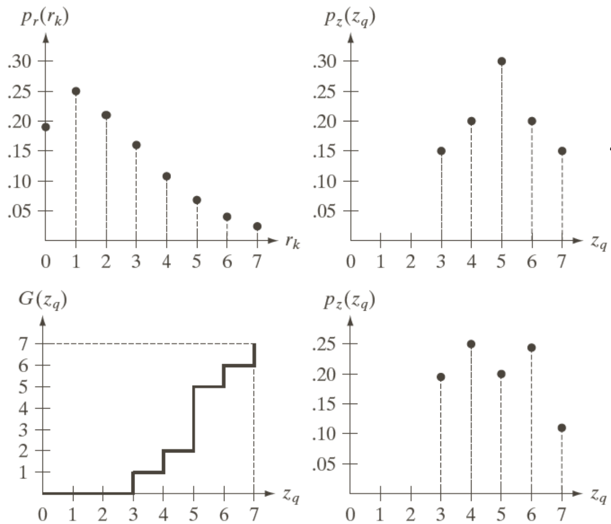
$$s = G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

- Obtain the inverse transformation (mapping from s to z)

$$z_q = G^{-1}(s_k)$$

- Equalize the input image ($r \rightarrow s$) and apply the inverse mapping $z = G^{-1}(s)$

Histogram specification – example



specif.

result

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2 Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).

1

S=T(r)	val	rounded
S_0	1.33	1
S_1	3.08	3
S_2	4.55	5
S_3	5.67	6
S_4	6.23	6
S_5	6.65	7
S_6	6.86	7
S_7	7.00	7

$G(z_q)$	val	rounded
$G(z_0)$	0	0
$G(z_1)$	0	0
$G(z_2)$	0	0
$G(z_3)$	1.05	1
$G(z_4)$	2.45	2
$G(z_5)$	4.55	5
$G(z_6)$	5.95	6
$G(z_7)$	7.00	7

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i)$$

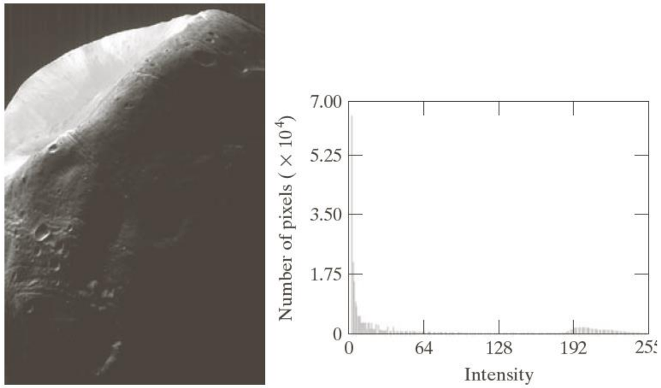
$$z_q = G^{-1}(s_k)$$

Histogram specification

- How could specification be useful?

Equalization vs specification

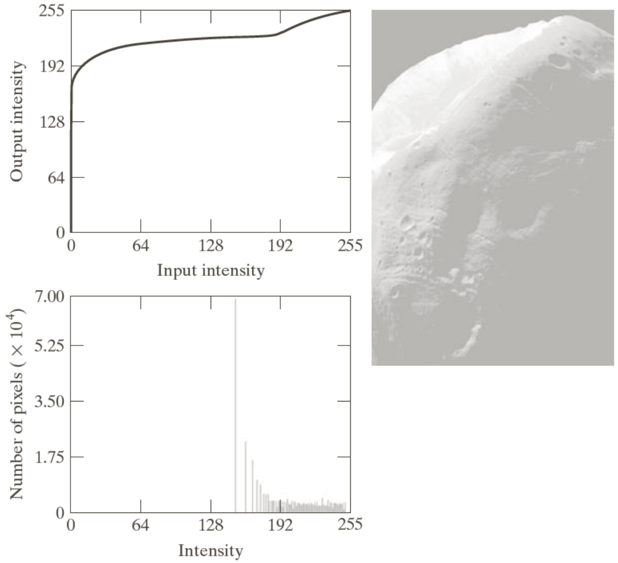
- Equalization can be unsuitable
  - When the number of pixels with low gray levels is very high
- Specification function manually defined



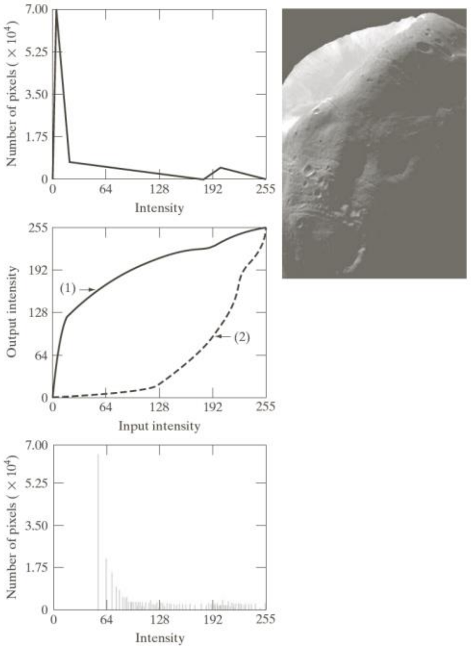
a b

FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

Equalization



Specification



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