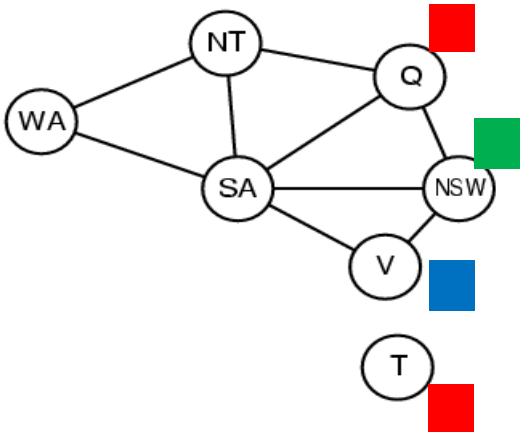
# CONSTRAINT SATISFACTION PROBLEMS – PART IV

Chapter 6

# Outline

- Constraint Satisfaction Problems (CSP)
- Backtracking search
- Backjumping
- No-good
- Forward checking
- Constraint propagation
- Local search for CSPs

# Chronological backtracking



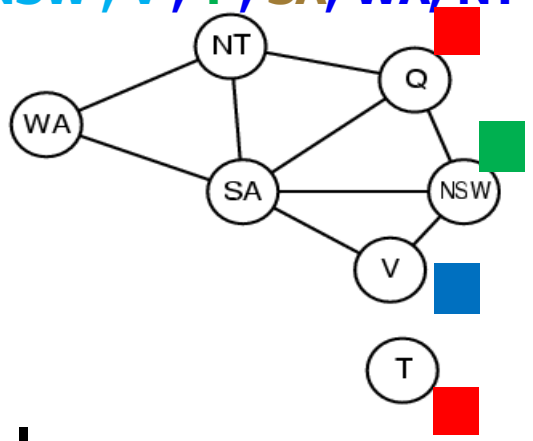

## Chronological backtracking

- Backtrack to the previous variable and try another value

- **Example:** Assume a **fixed variable ordering** Q, NSW , V , **T** , **SA**, WA, NT
  - Suppose the **partial assignment** {Q=red, NSW =green, V =blue, **T** =red}
  - The next variable is **SA**, but **every value violates** a constraint
  - We **back up to T** and try a new color for Tasmania. This is **not useful** ! recoloring Tasmania cannot possibly resolve the problem with **SA**

Assume variable ordering **Q, NSW , V , T , SA, WA, NT**

# Backjumping

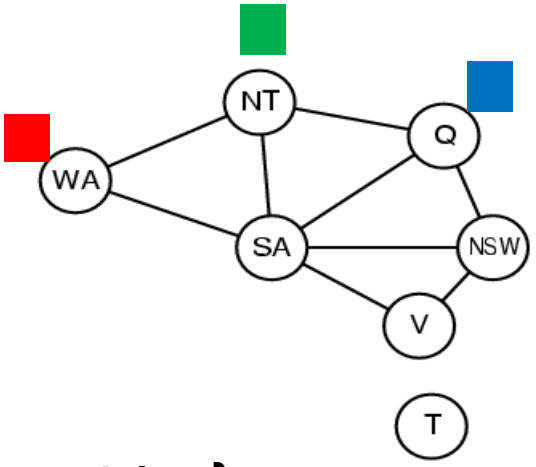



- Backtrack to **a variable** that might **fix the problem**
- The **set of these variables** is called the **conflict set**
  - **Example:** The conflict set for **SA** is {Q, NSW, V}
- Backjumping: Backtrack to the ***most recent*** **variable** in the **conflict set**
  - **Example:** We jump over **T** and try a new value for V

# No-good

- To avoid redundant work
  - **To avoid** running into the **same problem again**
  - **Constraint learning**: finding a minimum set of variables from the conflict set that cause the problem. This set of variables with their corresponding values is called **no-good**
  - **Record** the **no-good** by adding a **new constraint** to the CSP

# No-good



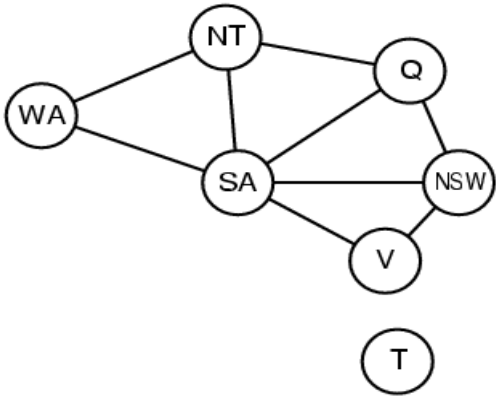

## Example

- Consider the state {WA = red, NT = green, Q = blue}
- This state is a no-good, because there is no valid assignment to SA

- If the **search tree** starts by assigning values for WA, NT, Q →
  **recording this no-good would not help**
  because once we prune this branch from the search tree,
  we will **never encounter this combination** again

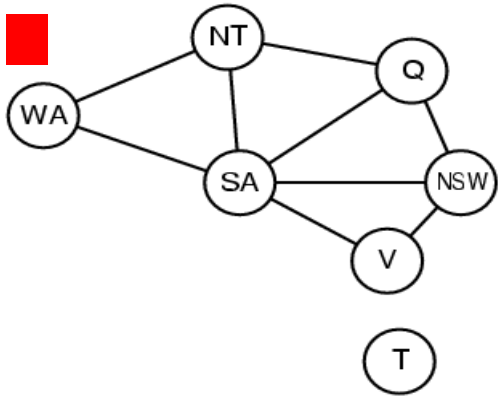
- If the **search tree** starts by assigning values for V, T →
  **Useful to record** {WA = red, NT = green, Q = blue}
  as **a no-good** because we will run into the **same problem**
  again **for each possible set of assignments** to V and T

# Forward checking




- Idea:
  - **Keep track** of **remaining legal values** for **unassigned variables**
  - **Terminate** search when any **variable** has **no legal values**

# Forward checking




- Idea:
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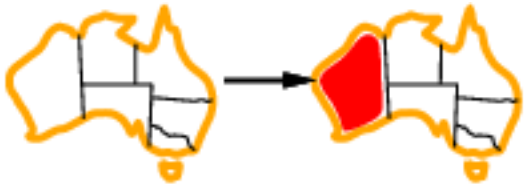
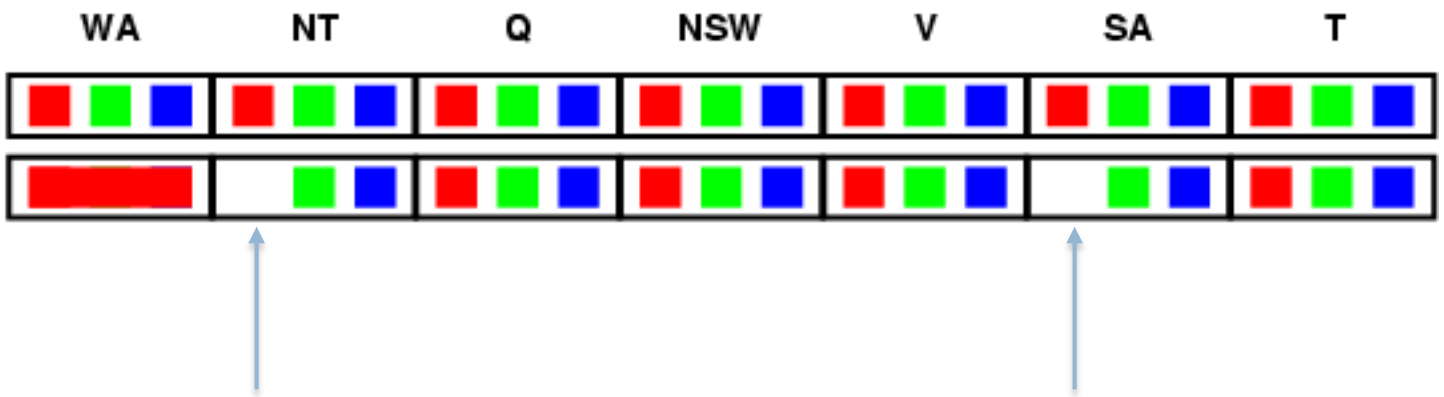
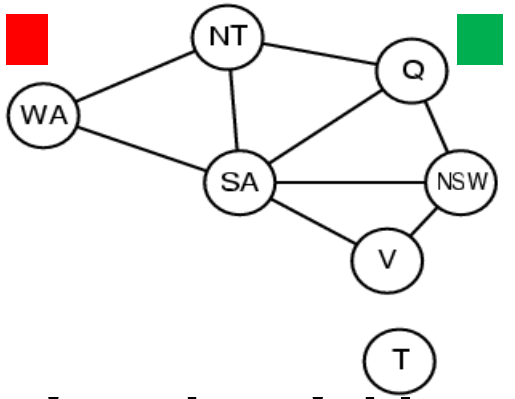
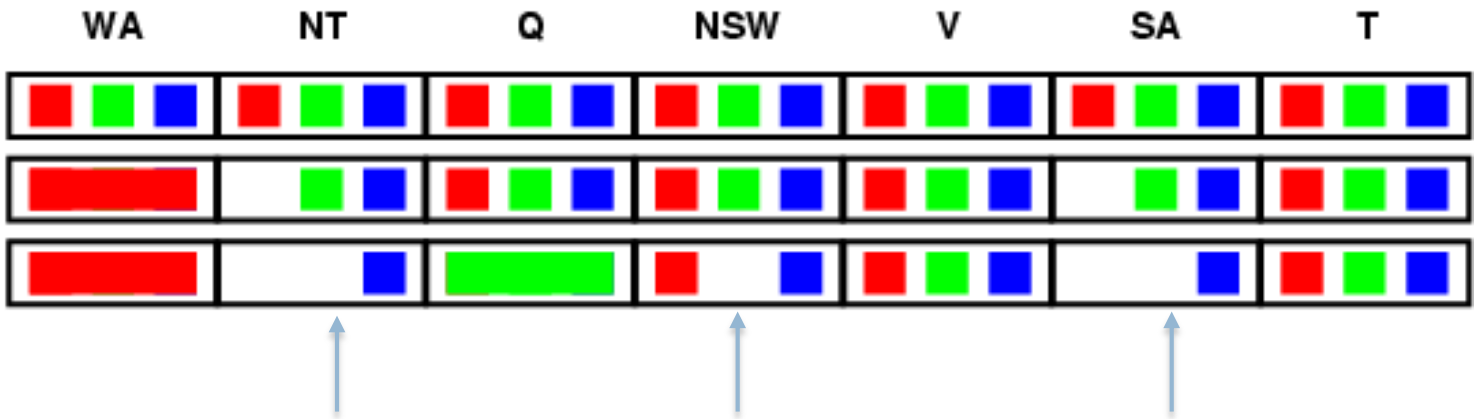




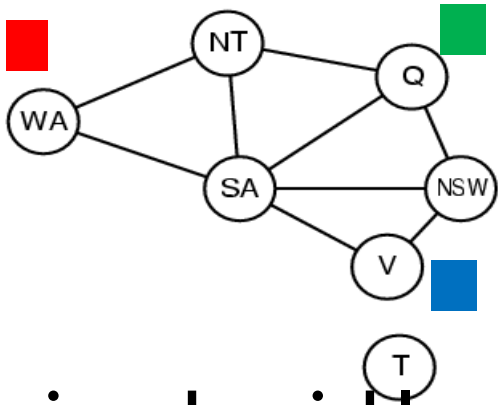
# Forward checking



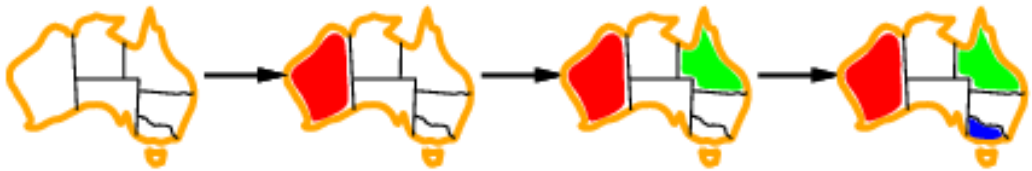

- Idea:
  - **Keep track** of remaining **legal values** for **unassigned variables**
  - **Terminate** search when any **variable** has **no legal values**

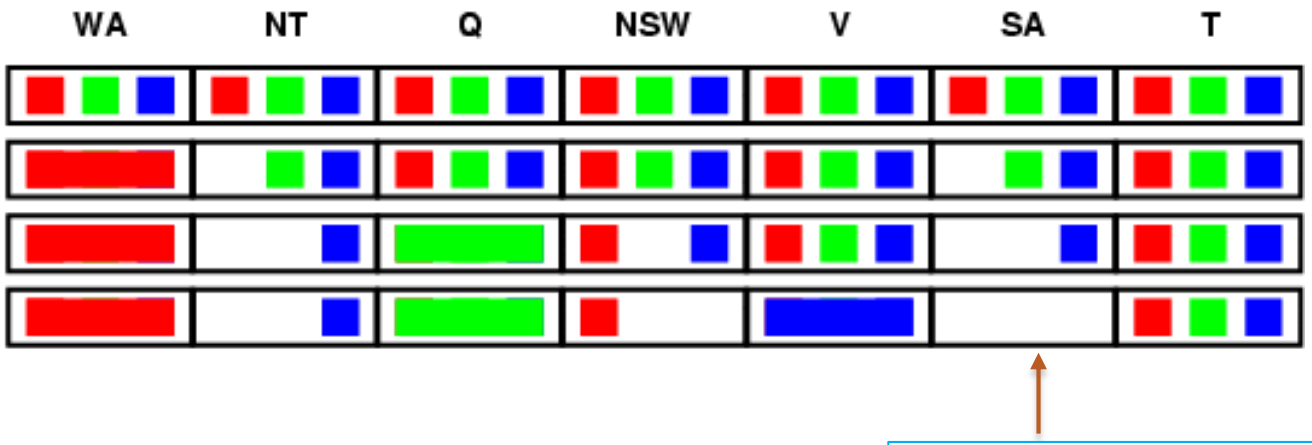


# Forward checking

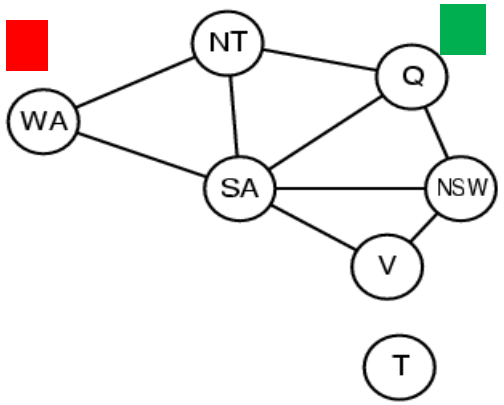



- Idea:
  - **Keep track** of remaining **legal values** for **unassigned variables**
  - **Terminate** search when any **variable** has **no legal values**





# Constraint propagation




- **Forward checking** propagates information
  from **assigned** to **unassigned** variables,
  but **doesn't provide** early detection **for all failures**:


- **Constraint propagation** repeatedly enforces **constraints locally**

# CONSTRAINT SATISFACTION PROBLEMS – PART V

Chapter 6

# Outline

- Constraint Satisfaction Problems (CSP)
- Backtracking search
- Forward checking
- Constraint propagation
- Local search for CSPs
- Structure of the problem

# Constraint propagation

- **Constraint propagation**
  - **Using constraints to reduce the number of legal values for a variable** → this can **reduce** the legal values **for another variable …**
  - May be **interleaved with search**
  - May be done **as a preprocessing step**, before search starts
  - **Sometimes** it **can solve the whole problem**, so **no search is required**
  - The key idea is local consistency
    - By **enforcing local consistency** in each part of the constraint graph → **inconsistent values** are **eliminated** in graph
    - There are **different types** of local consistency

# Node consistency

- A **variable** (corresponding to a node in the CSP network) is **node-consistent** if **all the values** in the **variable's domain satisfy** the variable's **unary constraints**
- **Example** (a variant of map coloring)
  - Assume South Australians **dislike green**
  - Variable SA starts with domain {red,green,blue}
  - We can **make SA node consistent** by **eliminating green**, leaving SA with the reduced domain {red,blue}
- A **CSP network** is **node-consistent** if **every variable** in the network is **node-consistent**

# CSPs with binary constraints

- **Unary constraints** can be eliminated by running **node consistency**
- **All n-ary constraints** can be transformed **into binary ones**

We will consider **CSPs** with only **binary constraints**

# Arc consistency

- **Simplest form** of propagation **makes** each arc consistent

- A **variable** is **arc-consistent** if **every value** in its domain satisfies the variable's **binary constraints**

  **Formally:**

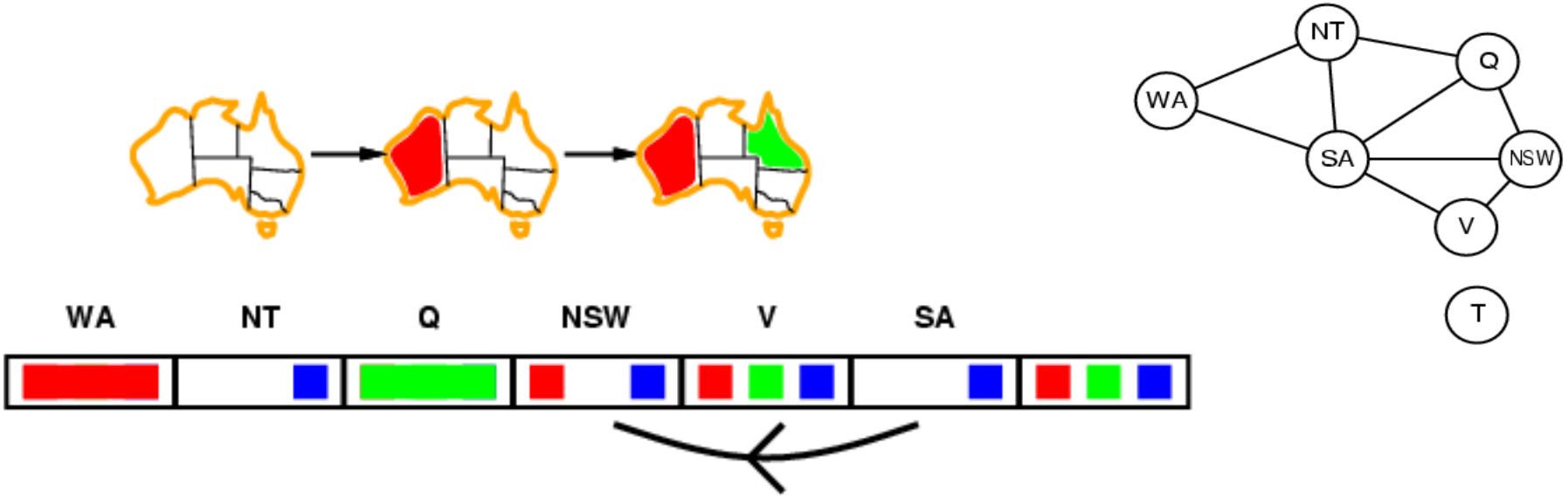
  Assume there is a binary **constraint** between $\mathbf{X_i}$ and $\mathbf{X_{i'}}$

  $\mathbf{X_i}$ is **arc consistent** with respect to $\mathbf{X_i}$ iff

  **for every value** *x* for $\mathbf{X_i}$, there is **some allowed** *y* for $\mathbf{X_i}$ that **satisfies** the binary constraint between $\mathbf{X_i}$ and $\mathbf{X_i}$

# Arc consistency

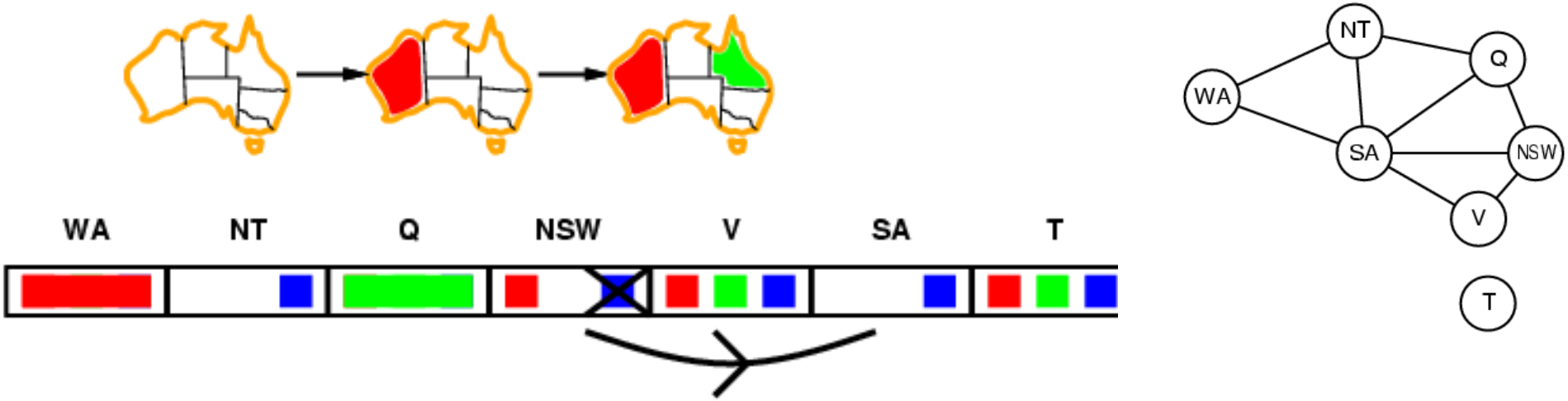
- $X_i$ is **arc consistent** with respect to $X_j$ iff
  for every value *x* for $X_i$, there is some allowed *y* for $X_j$




**SA** is **arc-consistent** with respect to **NSW**

# Arc consistency

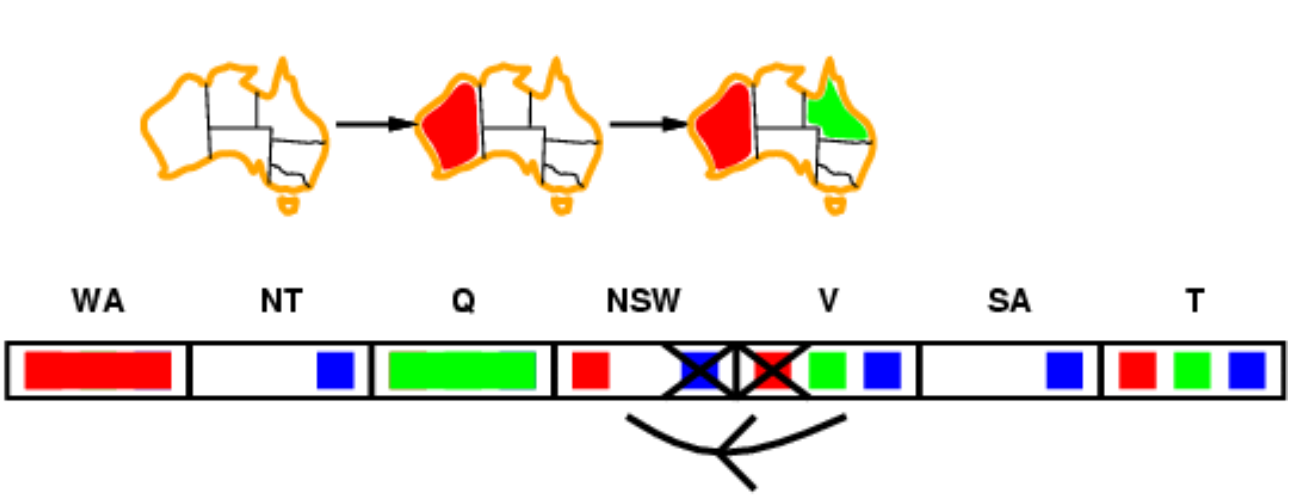
- $X_i$ is **arc consistent** with respect to $X_j$ iff
  for every value *x* for $X_i$, there is some allowed *y* for $X_j$

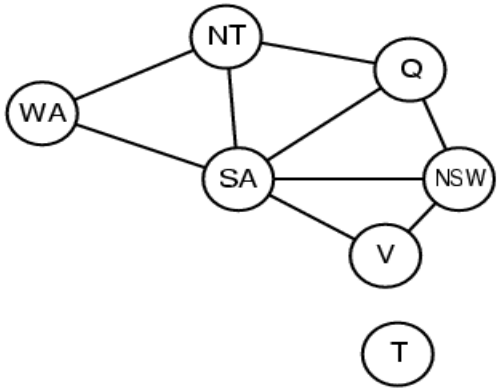



- **NSW** is **not** arc-consistent with respect to **SA**
- **To make NSW arc-consistent** with **SA**, it is sufficient to **remove** the value ***blue*** from the domain of **NSW**

# Arc consistency

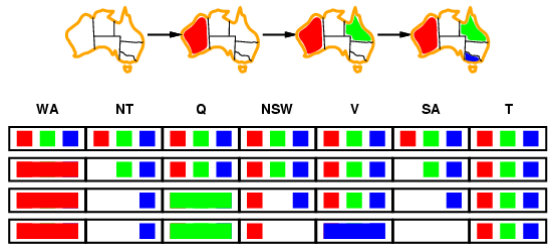
- $X_i$ is **arc consistent** with respect to $X_j$ iff
  for every value *x* for $X_i$, there is some allowed *y* for $X_j$



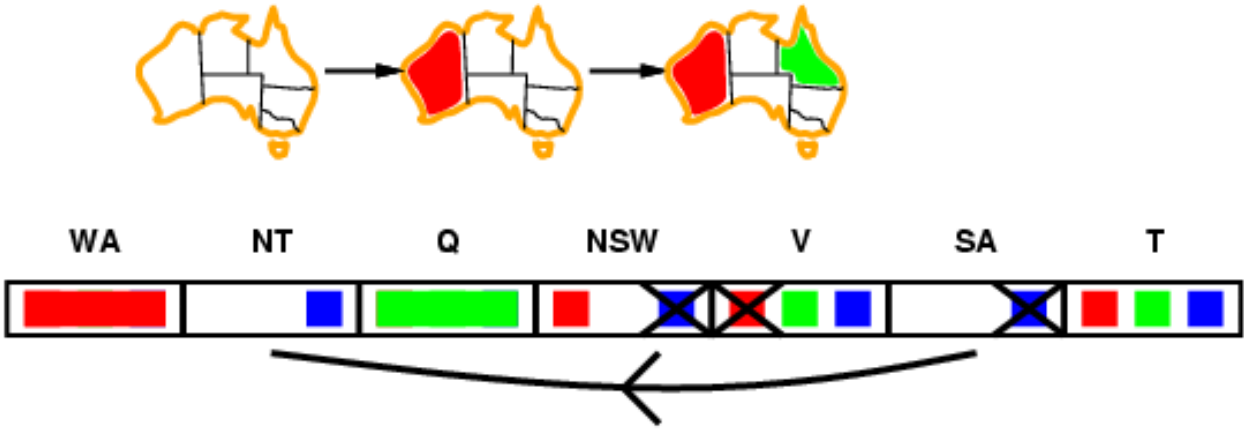



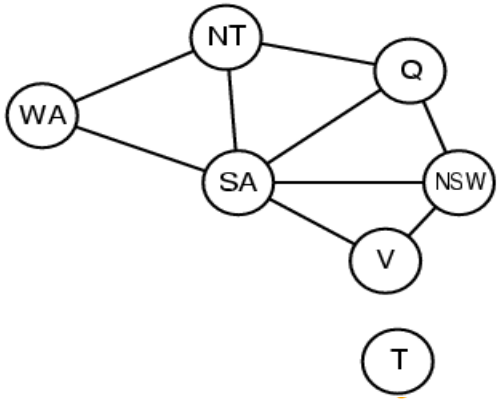

If a **variable *X* loses a value,**
**neighbors of *X*** need to be **rechecked**

# Arc consistency




- $X_i$ is **arc consistent** with respect to $X_j$ iff
  for every value $x$ for $X_i$, there is some allowed $y$ for $X_j$




- **Arc consistency detects failure earlier** than **forward checking**
- Can be run as a preprocessor or after each assignment

# Arc consistency algorithm AC-3 (Mackworth 1977)

**function** AC-3( *csp*) **returns** the CSP, possibly with reduced domains

**inputs**: *csp*, a binary CSP with variables $\{X_1, X_2, \ldots, X_n\}$

**local variables**: *queue*, a queue of arcs, initially all the arcs in *csp*

**while** *queue* is not empty **do**
  $(X_i, X_j) \leftarrow$ REMOVE-FIRST(*queue*)
  **if** RM-INCONSISTENT-VALUES($X_i, X_j$) **then**
    **for each** $X_k$ **in** NEIGHBORS[$X_i$] **do**
      add $(X_k, X_i)$ to *queue*

- If **Di** unchanged → the algorithm just moves on to the next arc
- If **Di** reduced → we add to the queue all **arcs (Xk,Xi)** where Xk is a neighbor of Xi

---

**function** RM-INCONSISTENT-VALUES( $X_i, X_j$) **returns** true iff remove a value

*removed* ← *false*
**for each** $x$ **in** DOMAIN[$X_i$] **do**
  **if** no value $y$ in DOMAIN[$X_j$] allows $(x,y)$ to satisfy constraint($X_i, X_j$)
    **then** delete $x$ from DOMAIN[$X_i$]; *removed* ← *true*
**return** *removed*

# Complexity of AC-3

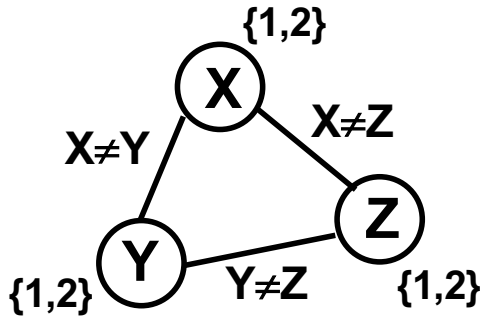
- For binary CSPs
  - n: number of variables
  - c: number of binary constraints (arcs)
  - d: domain size

- Time: $O(cd^3)$
  - **Each arc** $(X_k, X_i)$ **inserted** in the queue **only d times**
    because $X_i$ has **at most d values** to delete
  - **Checking consistency of an arc** can be done in $O(d^2)$ time
  - Thus, $O(cd^3)$ total worst-case time

# Is arc consistency enough?

- By **using AC** we can **remove many incompatible values**
  - Do we **get a solution?**
  - Do we know **if there exists a solution?**
- Unfortunately, the answer to **both** above questions is NO!

- ***Example:***




**CSP is arc consistent**
**but there is no solution**

# Is arc consistency enough?

- **So what are the benefits of AC?**
  - **Sometimes** we have a **solution/failure after AC**
    - **a domain is empty → no solution exists**
    - **all the domains are singleton → we have a solution**

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  - **In general, AC prunes the search space → equivalent easier problem**

# Path consistency (PC)

- **How to strengthen** the consistency level?
- Require **consistency** over **more than one constraint**

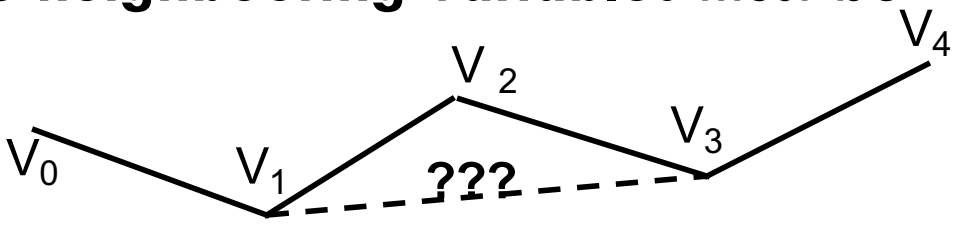
- A **two-variable set {Xi, Xj}** is **path-consistent** with respect to a third **variable Xm** if

  ∀assignment **{Xi = a, Xj = b} consistent** with the constraints on **{Xi , Xj}**
  ∃assignment to **Xm** that satisfies
  - the **constraints on {Xi , Xm}** and
  - the **constraints on{Xm , Xj }**

- This is called path consistency since it is like to consider a **path** from **Xi** to **Xj** with **Xm** in the middle

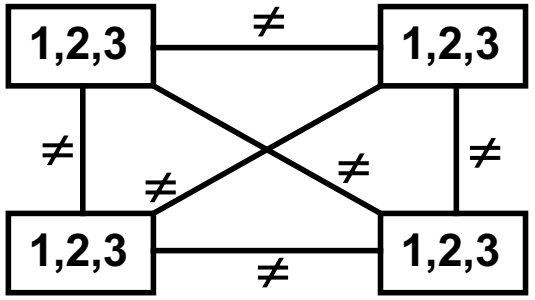
# Path consistency (PC)

- Path consistency
  - does **not guarantee** that **all the constraints** among the **variables on the path** are satisfied
  - **only the constraints** between the **neighbouring variables** must be satisfied




- PC is still **not a complete** technique

A,B,C,D in {1,2,3}
A≠B, A≠C, A≠D, B≠C, B≠D, C≠D
is PC but has not solution




**Other stronger consistency notions…**

# Review: Constraint propagation

- **Constraint propagation**
  - May be **interleaved with search**
  - May be done **as a preprocessing step**, before search starts
  - **Sometimes** it **can solve the whole problem**, so **no search is required**

  - By **enforcing** **local consistency** (arc consistent, or path consistency,…) in each part of the constraint graph → **inconsistent** values are **eliminated** in graph