CONSTRAINT SATISFACTION PROBLEMS – PART II

Chapter 6

Outline

- Constraint Satisfaction Problems (CSP)
- Backtracking search for CSPs
- Local search for CSPs

Review: Constraint satisfaction problem

- **Set of variables X** = $\{X_1, X_2, \ldots, X_n\}$
- **Set of Domains D** = $\{D_1, D_2, \ldots, D_n\}$
  - Each domain **Di** consists of a set of **allowable values** for variable X_i.
- **Set of constraints C** = $\{\ c_i = (scope_i, rel_i) \mid i = 1, \ldots, h\}$
  - **$scope_i$: subset of X**, the **variables** that are **constrained** by c_i
  - **rel_i**: is a **relation** and tells us which **simultaneous assignments of values** to variables in **$scope_i$** are **allowed**

Review: Constraint satisfaction problem

- **State:** defined by an **assignment** of values **to some or all** of the **variables**, $\{X_i = v_i, X_j = v_j, \ldots\}$
- **Assignment** can be:
  - **Consistent:** it does **not violate** any constraints
  - **Complete:** **every variable** is assigned
  - **Partial:** **only some of the variables** are assigned
- **Solution:** a **consistent and complete** assignment

3-SAT example

$$(x_1 \vee x_2 \vee x_6) \wedge (\neg x_1 \vee x_3 \vee x_4) \wedge$$
$$(\neg x_4 \vee \neg x_5 \vee x_6) \wedge (x_2 \vee x_5 \vee \neg x_6)$$

- **Non-binary CSP:**
  - Boolean variables: $x_1, \ldots, x_6$
  - Constraints: one for each clause

$$C_1(x_1, x_2, x_6) = \{(0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$$

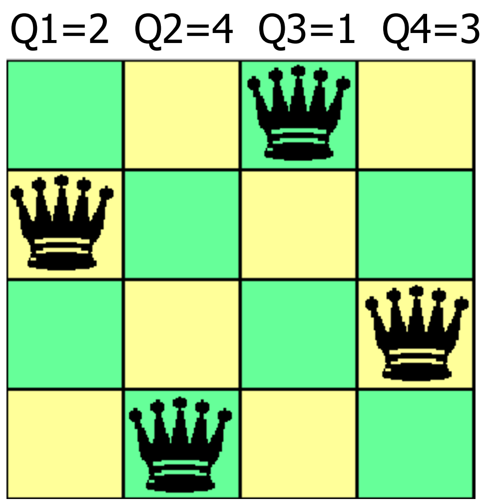
$$C_2(x_1, x_3, x_4) = \ldots$$

Example of CSP: 4-Queens Problem

- Place **one queen** in each column such that they do not attack each other
- Variables: Q1, Q2, Q3, Q4 (one per column)
- Domains: Di = [1, 2, 3, 4] (row position of a queen)
- Constraints:
  - $Qi \neq Qj$ for all i,j (cannot be in the same row)
  - $|Qi-Qj| \neq |i-j|$ (cannot be in the same diagonal)

Translate each constraint into a set of allowable values for its variables

- E.g., values for (Q1,Q2) are
  (1,3) (1,4) (2,4) (3,1) (4,1) (4,2)



Example: Cryptarithmetic

```
  T W O
+ T W O
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F O U R
```

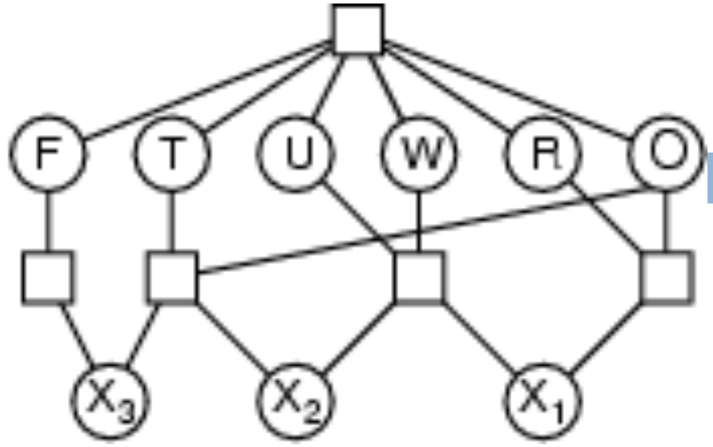

- Each letter represents a different digit
- The aim is to find a substitution of digits for letters such that the resulting sum is arithmetically correct

- Variables: $F, T, U, W, R, O, X_1, X_2, X_3$
- Domains: $\{0,1,2,3,4,5,6,7,8,9\}$

- Constraints:
  - *Alldiff* (F,T,U,W,R,O) non i riporti
  - $O + O = R + 10 \cdot X_1$
  - $X_1 + W + W = U + 10 \cdot X_2$
  - $X_2 + T + T = O + 10 \cdot X_3$
  - $X_3 = F$

Some real-world CSPs

- Assignment problems
  - e.g., who teaches what class
- Timetabling problems
  - e.g., which class is offered when and where?
- Transportation scheduling
- Factory scheduling
- Many real-world problems involve real-valued variables