



# UNIVERSITÀ DEGLI STUDI DI PADOVA

## Morphological operators

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- Images represented as sets
- Morphological operators
- Combination of morphological operators



- Low-level, but operate on the **shapes** found in an image
- Generally work on binary images
- Based on set theory
- Set-based description of an image
  - A set is a vector of tuples, each tuple representing the  $(x, y)$  coordinates of a point belonging to the set
  - E.g.: the set of all white pixels in a binary image



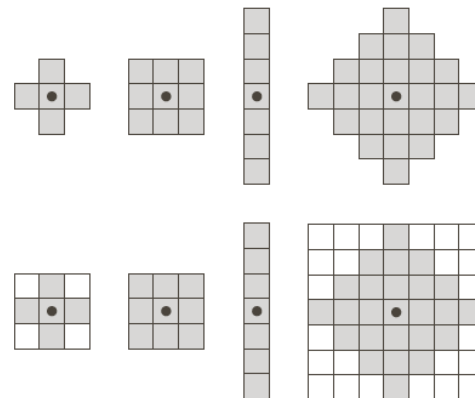
- It is possible to process an image working in this set-based description
- An operator can add or remove pixels to/from a set
- Such operators modify the image **working on the shape**
- Examples: erosion and dilation

# Erosion



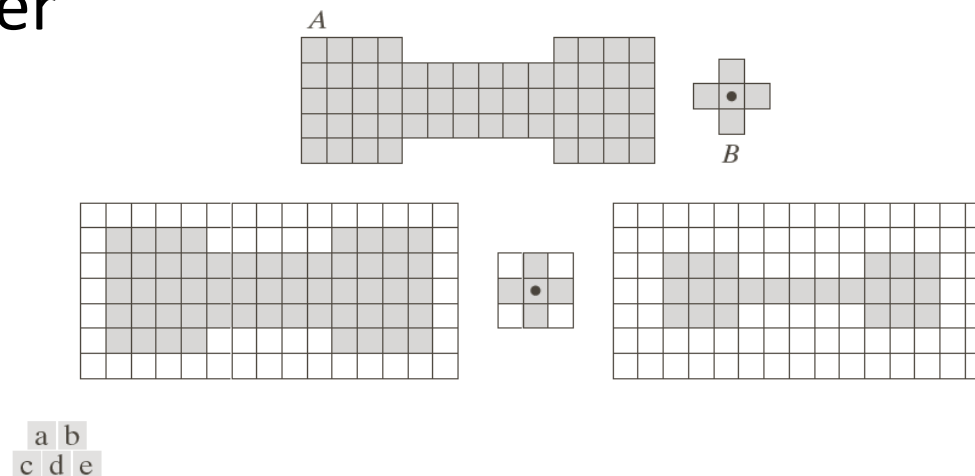
- Consider two sets,  $A$  and  $B$  ( $B$  being the structuring element)
- The erosion of  $A$  by  $B$  is a set defined as:
$$A \ominus B = \{z | (B)_z \subseteq A\}$$
- This means: translate  $B$  to point  $z$ , and keep  $z$  iff the whole structuring element is fully included in  $A$
- Applications:
  - Thinning
  - Separate weakly connected components

- Example of structuring elements



**FIGURE 9.2** First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

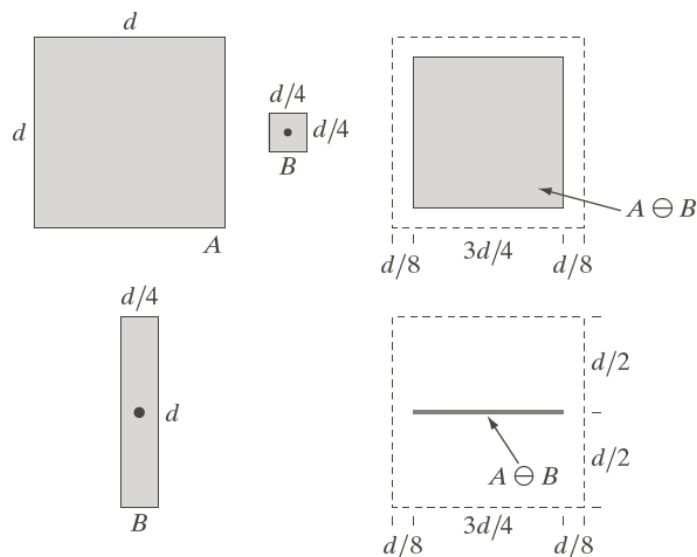
- Results of erosion
  - The final shape is similar to the initial one, but thinner



**FIGURE 9.3** (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

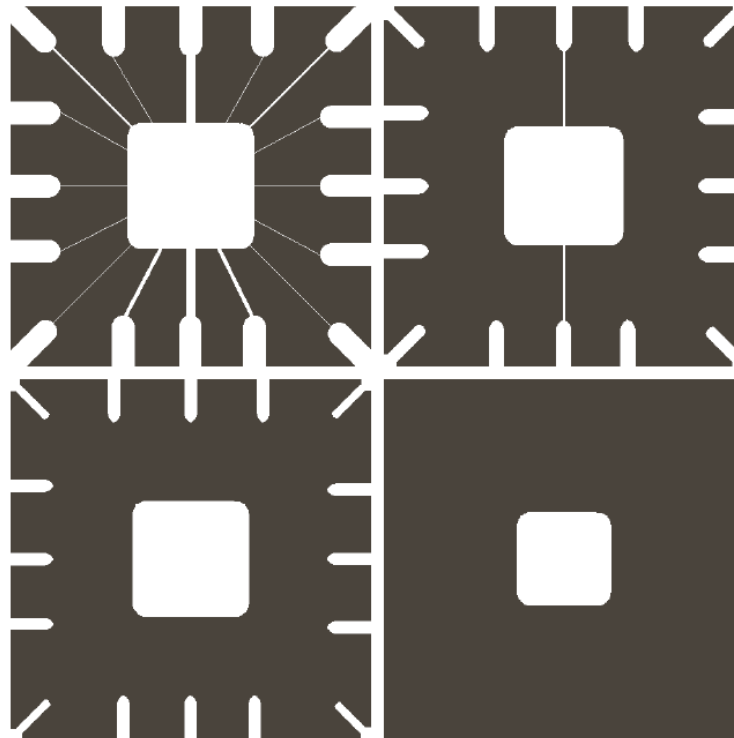


- Results of erosion
  - The shape of the structuring element determines in which direction(s) the erosion operates



**FIGURE 9.4** (a) Set  $A$ . (b) Square structuring element,  $B$ . (c) Erosion of  $A$  by  $B$ , shown shaded. (d) Elongated structuring element. (e) Erosion of  $A$  by  $B$  using this element. The dotted border in (c) and (e) is the boundary of set  $A$ , shown only for reference.

- Erosion separates weakly connected components



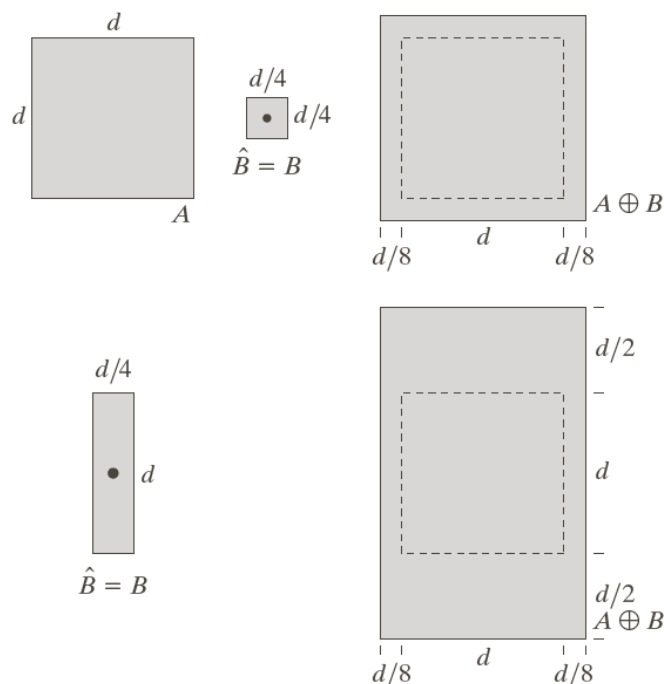
a	b
c	d

**FIGURE 9.5** Using erosion to remove image components. (a) A  $486 \times 486$  binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes  $11 \times 11$ ,  $15 \times 15$ , and  $45 \times 45$ , respectively. The elements of the SEs were all 1s.

# Dilation

- Consider two sets,  $A$  and  $B$  ( $B$  being the structuring element)
- The dilation of  $A$  by  $B$  is a set defined as:
$$A \oplus B = \{z | (B)_z \cap A \neq \emptyset\}$$
- This means: translate  $B$  to point  $z$ , and keep  $z$  iff there is at least one pixel overlapping with  $A$
- Applications:
  - Thickening
  - Merging close, unconnected components

- Results of dilation
  - The shape of the structuring element determines in which direction(s) the erosion operates

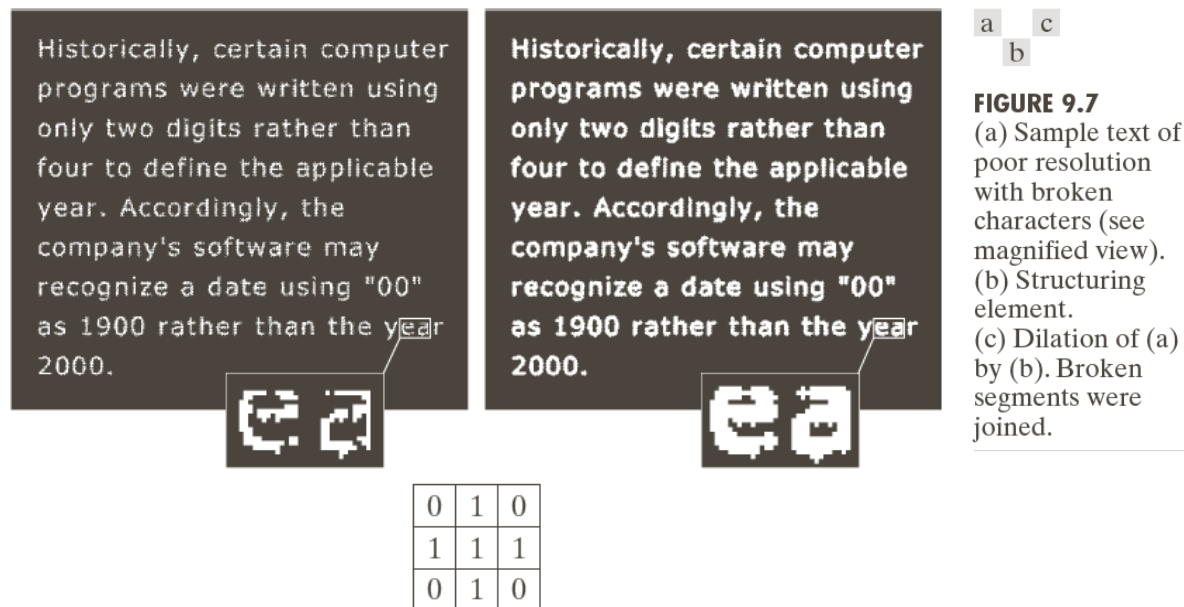


a	b	c
d	e	

**FIGURE 9.6**

(a) Set  $A$ .  
 (b) Square structuring element (the dot denotes the origin).  
 (c) Dilation of  $A$  by  $B$ , shown shaded.  
 (d) Elongated structuring element. (e) Dilation of  $A$  using this element. The dotted border in (c) and (e) is the boundary of set  $A$ , shown only for reference

- Dilation merges close unconnected components



Opening and closing



- Morphological operations can be concatenated
- Opening: erosion + dilation

$$A \circ B = (A \ominus B) \oplus B$$

- Effects
  - Contour smoothing
  - Eliminate thin protrusions without reducing the element size



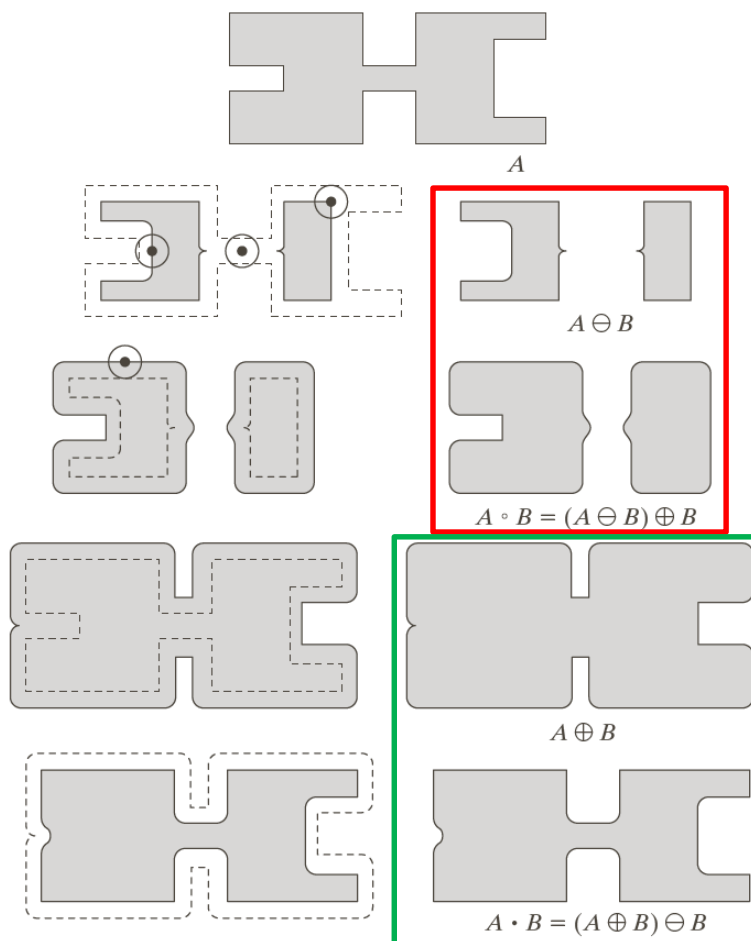


- Morphological operations can be concatenated
- Closing: dilation + erosion

$$A \cdot B = (A \oplus B) \ominus B$$

- Effects
  - Fuse narrow breaks without increasing the element size

- Compare opening vs erosion and closing vs dilation



a
b c
d e
f g
h i

**FIGURE 9.10**  
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.

- Complex combinations may effectively remove noise



a	b
d	c
e	f

**FIGURE 9.11**

(a) Noisy image.  
(b) Structuring element.  
(c) Eroded image.  
(d) Opening of  $A$ .  
(e) Dilation of the opening.  
(f) Closing of the opening.  
(Original image courtesy of the National Institute of Standards and Technology.)



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