

EXERCISE AND SOLUTION

(ARC CONSISTENCY)



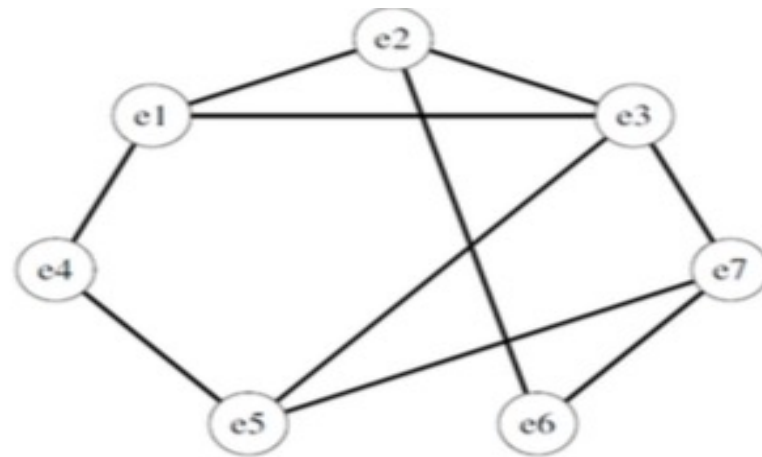
CSP: arc-consistency

Consider a **map coloring problem** that can be modelled via a CSP with

- **Variables:** $e1, e2, e3, e4, e5, e6, e7$
- **Domains:**
 - Domain of $e1, e3, e4, e5, e6$ is $\{ \text{R}, \text{G}, \text{B} \}$
 - Domain of $e2 = \{ \text{R} \}$
 - Domain of $e7 = \{ \text{B} \}$
- **Constraints:** specified by the following constraint graph

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- **Constraints:** specified by this constraint graph.
 - ▣ There is an arc between two variables if they must have different colors



- **Apply arc-consistency** and **show how the domains change**

Review: Arc consistency

- A **variable** is **arc-consistent** if **every value** in its domain satisfies the variable's **binary constraints**

Formally:

Assume there is a **binary constraint** between X_i and X_j ,

X_i is **arc consistent** with respect to X_j iff

for every value x for X_i , there is **some allowed** y for X_j that **satisfies** the binary constraint between X_i and X_j

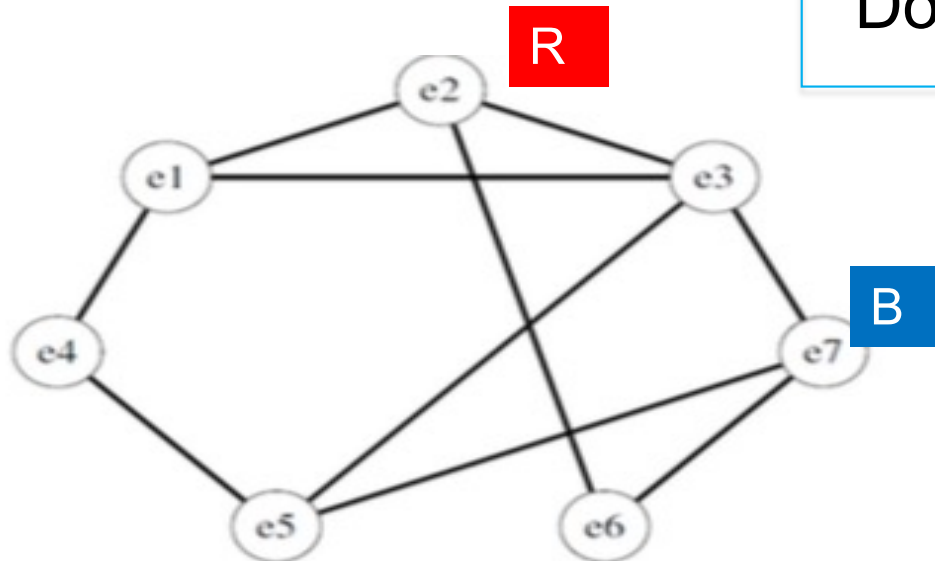
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Domain of e1,e3,e4,e5,e6

$D = \{ \text{R}, \text{G}, \text{B} \}$

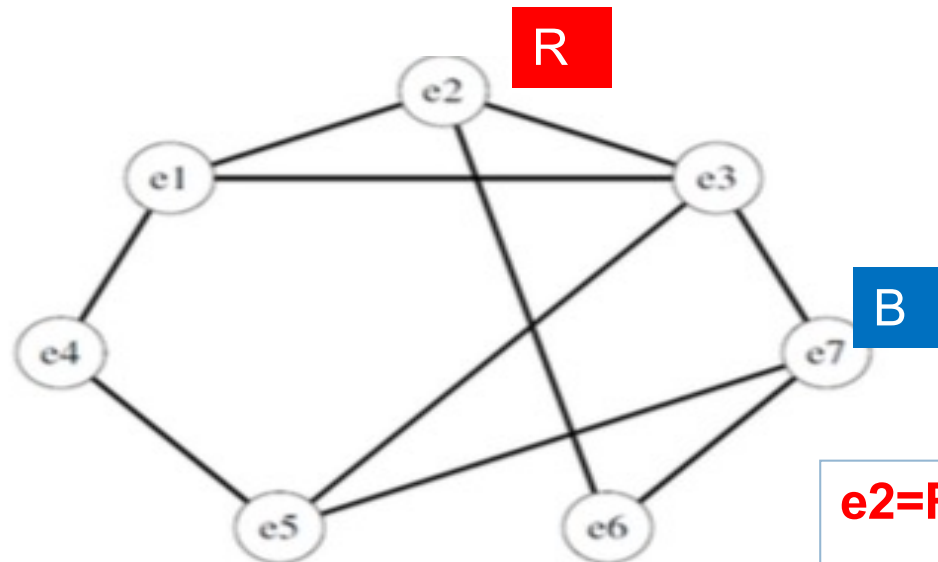
Domain of e2 = $\{ \text{R} \}$

Domain of e7 = $\{ \text{B} \}$



e1	e2	e3	e4	e5	e6	e7
R G B	R	R G B	R G B	R G B	R G B	B

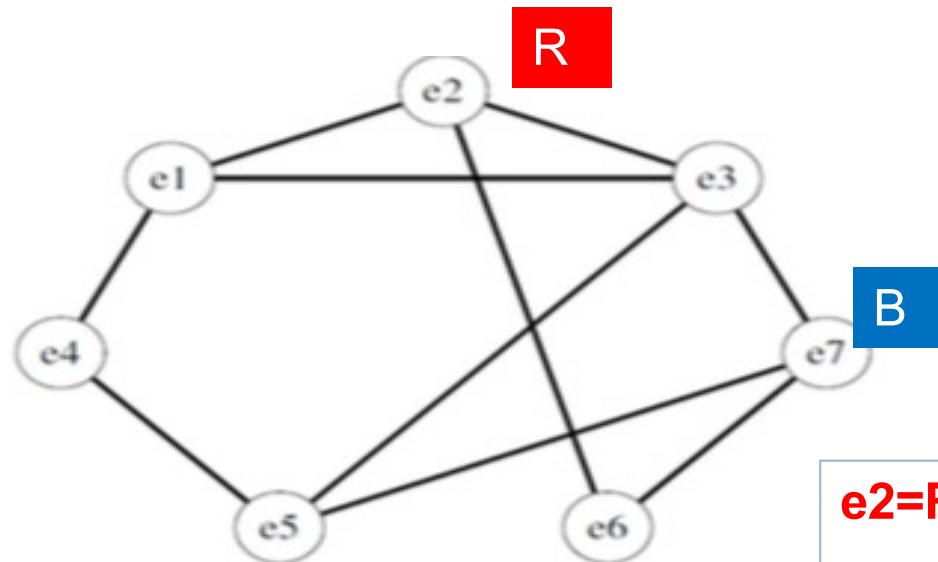
CSP: arc-consistency



e2=R → Remove **R** from the domain of **e1, e3, e6**

e1	e2	e3	e4	e5	e6	e7
R G B	R	R G B	R G B	R G B	R G B	B

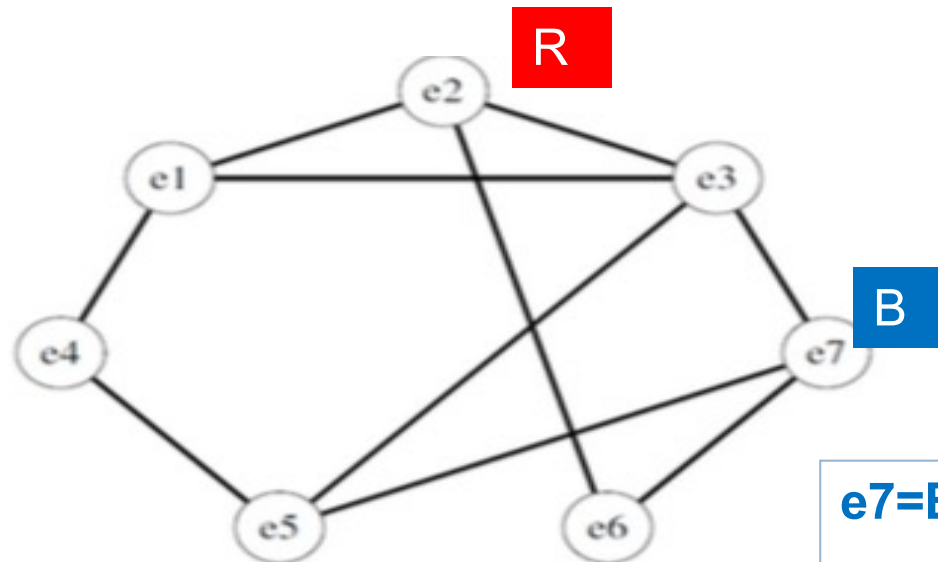
CSP: arc-consistency



e2=R → Remove **R** from the domain of **e1, e3, e6**

e1	e2	e3	e4	e5	e6	e7
<div>G B</div>	<div>R</div>	<div>G B</div>	<div>R G B</div>	<div>R G B</div>	<div>G B</div>	<div>B</div>

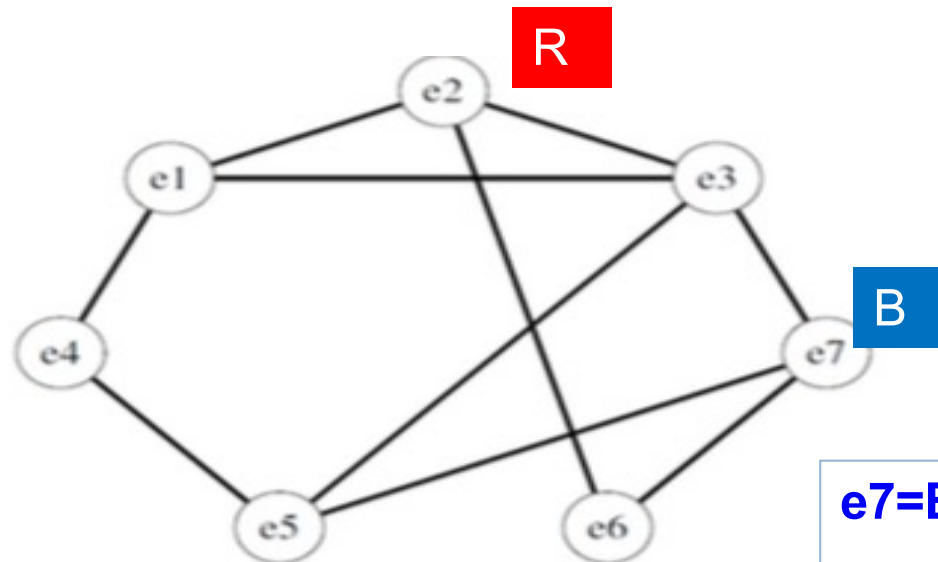
CSP: arc-consistency



e7=B → Remove **B** from the domain of **e3, e5, e6**

e1	e2	e3	e4	e5	e6	e7
G B	R	G B	R G B	R G B	G B	B

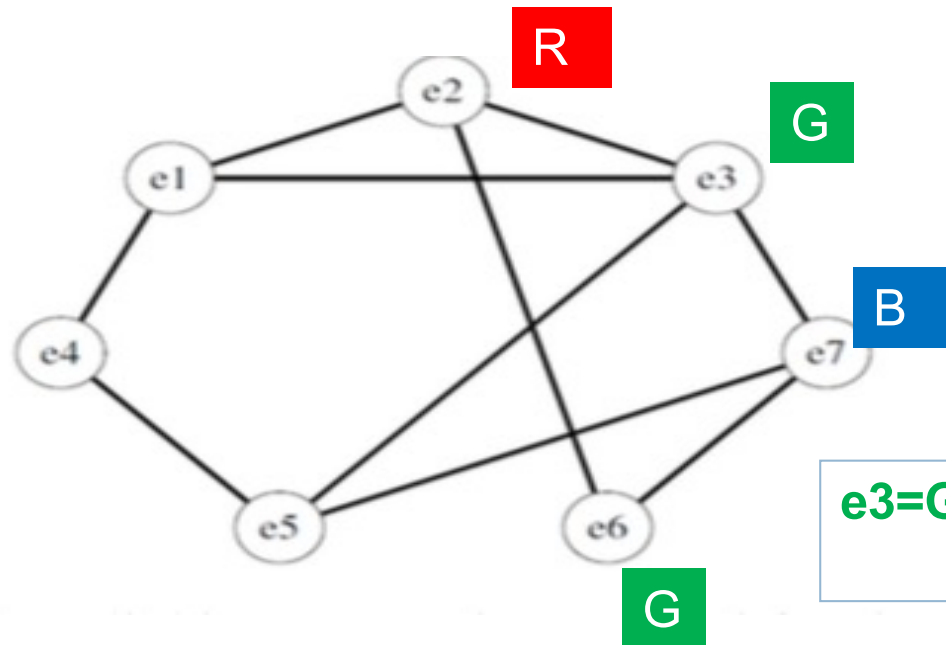
CSP: arc-consistency



e7=B → Remove **B** from the domain of **e3, e5, e6**

e1	e2	e3	e4	e5	e6	e7
<div>G B</div>	<div>R</div>	<div>G</div>	<div>R G B</div>	<div>R G</div>	<div>G</div>	<div>B</div>

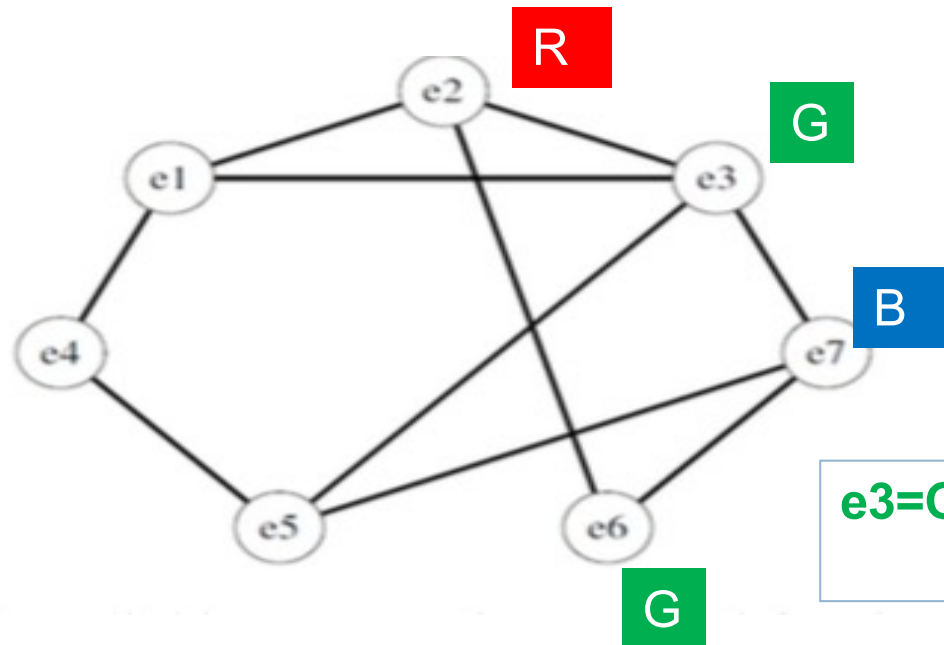
CSP: arc-consistency



e3=G → Remove **G** from the domain of **e1**, **e5**

e1	e2	e3	e4	e5	e6	e7
G B	R	G	R G B	R G	G	B

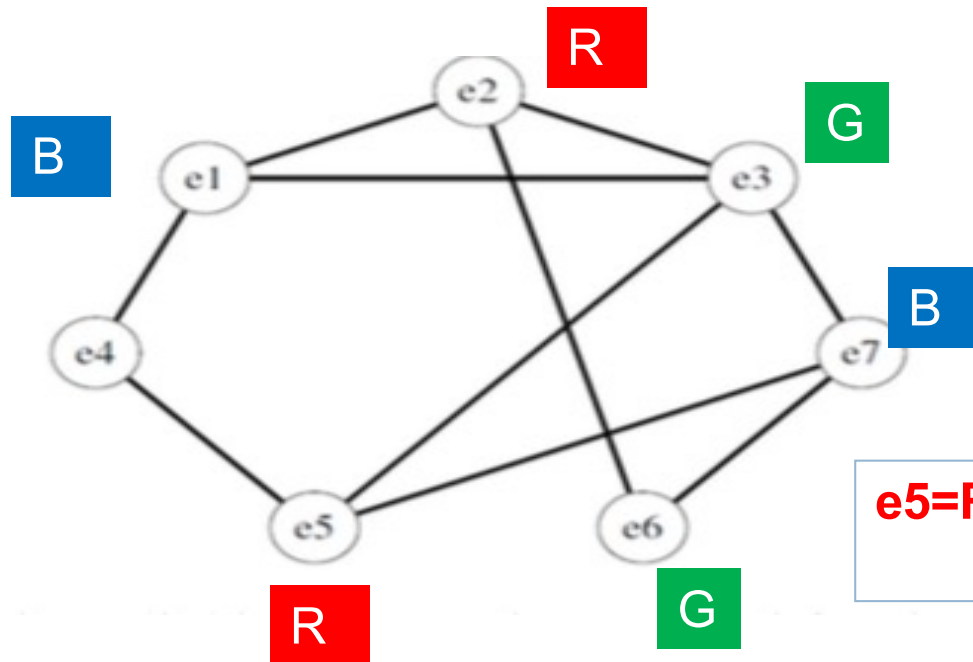
CSP: arc-consistency



e3=G → Remove **G** from the domain of **e1, e5**

e1	e2	e3	e4	e5	e6	e7
B	R	G	R G B	R	G	B

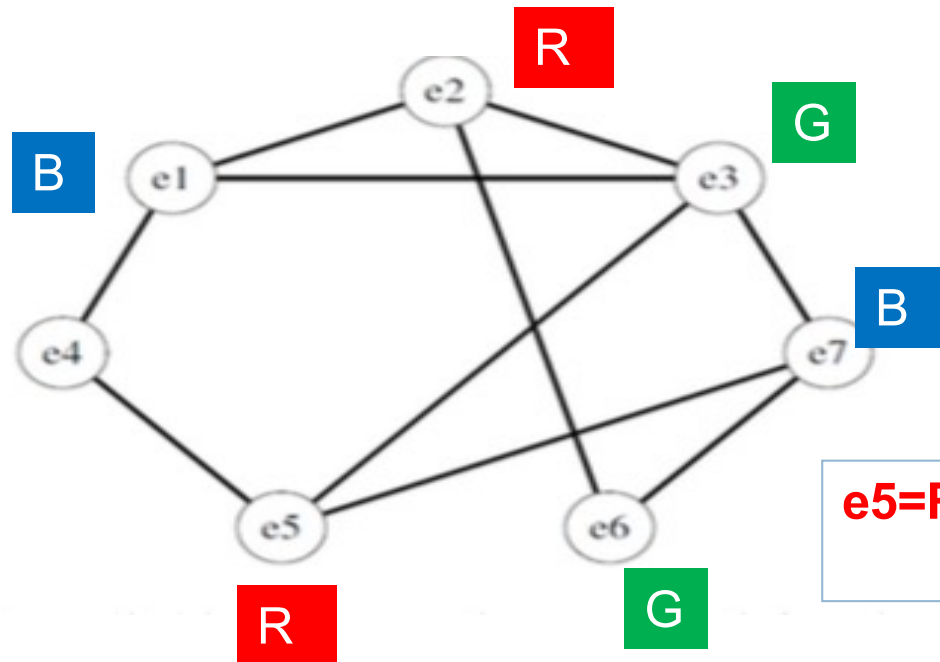
CSP: arc-consistency



e5=R → Remove **R** from the domain of **e4**

e1	e2	e3	e4	e5	e6	e7
B	R	G	R G B	R	G	B

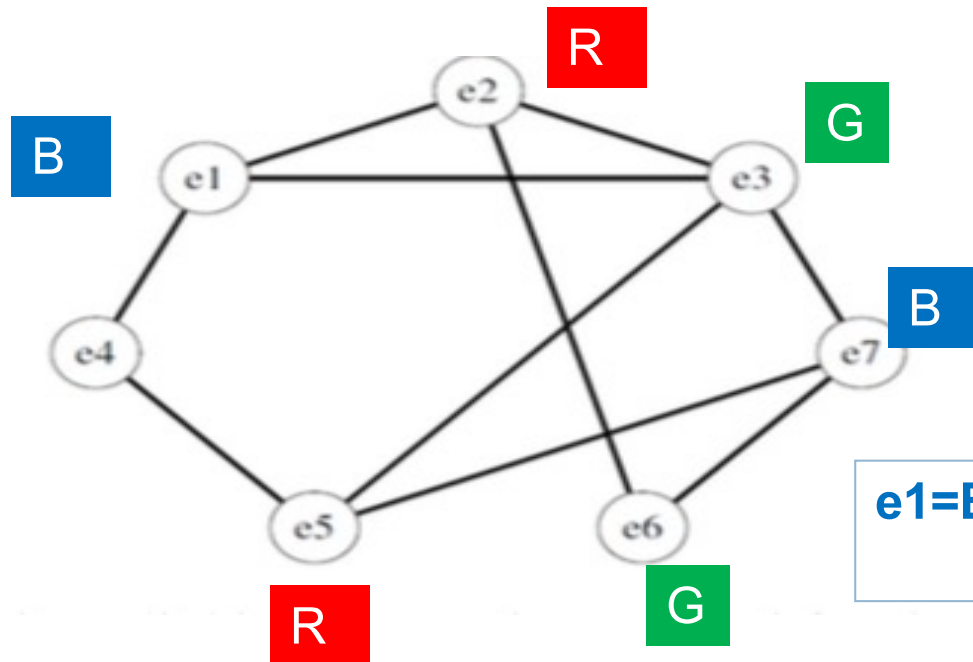
CSP: arc-consistency



e5=R → Remove **R** from the domain of **e4**

e1	e2	e3	e4	e5	e6	e7
B	R	G	G B	R	G	B

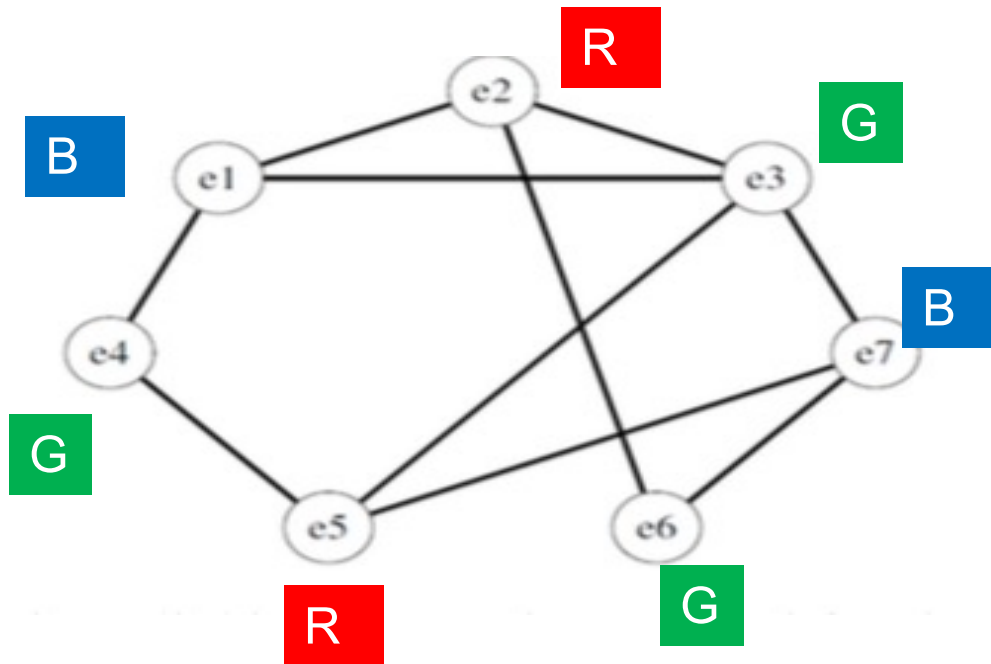
CSP: arc-consistency



e1=B → Remove **B** from the domain of **e4**

e1	e2	e3	e4	e5	e6	e7
B	R	G	G B	R	G	B

CSP: arc-consistency



e1	e2	e3	e4	e5	e6	e7
B	R	G	G	R	G	B