



# EXERCISES - 3

# Weighted CSP

Consider a **weighted CSP**

- ▣ with three **variables**  $X, Y, Z$
- ▣ with domains  $D_X=\{a,b\}$ ,  $D_Y=\{c,d\}$ , and  $D_Z=\{f,g\}$
- ▣ with five **weighted constraints**
  - $c1 = \langle f_1, \{X\} \rangle$
  - $c2 = \langle f_2, \{Y\} \rangle$
  - $c3 = \langle f_3, \{Z\} \rangle$
  - $c4 = \langle f_4, \{X, Y\} \rangle$
  - $c5 = \langle f_5, \{Y, Z\} \rangle$

where  $f_i, i=1,..5$  are shown in the next slide

# Weighted CSP

## weighted constraints

- $c1 = \langle f_1, \{X\} \rangle$
- $c2 = \langle f_2, \{Y\} \rangle$
- $c3 = \langle f_3, \{Z\} \rangle$
- $c4 = \langle f_4, \{X, Y\} \rangle$
- $c5 = \langle f_5, \{Y, Z\} \rangle$

$$f_1(a) = 15$$
$$f_1(b) = 10$$

$$f_2(c) = 20$$
$$f_2(d) = 25$$

$$f_3(f) = 45$$
$$f_3(g) = 20$$



$$f_4(a, c) = 10$$
$$f_4(a, d) = 15$$
$$f_4(b, c) = 25$$
$$f_4(b, d) = 20$$

$$f_5(c, f) = 30$$
$$f_5(c, g) = 40$$
$$f_5(d, f) = 45$$
$$f_5(d, g) = 60$$

# Weighted CSP

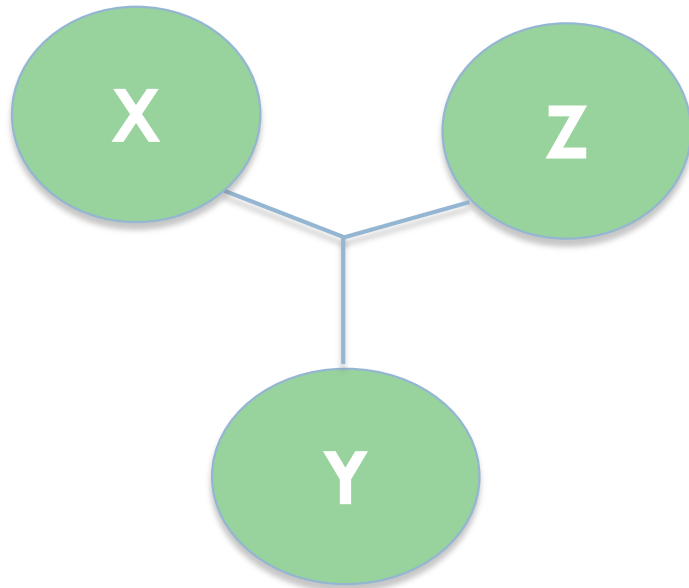


- Define the **constraint obtained by combining** the weighted constraints  $c_1, c_2, c_3, c_4, c_5$
- What is the **optimal solution** of the weighted CSP?
- What is the **preference value** of the optimal solution?

1. Define the constraint obtained by combining the weighted constraints  $c_1, c_2, c_3, c_4, c_5$

### Weighted c-semiring

$$S_{FCSP} = \langle R^+ \cup \{+\infty\}, \text{min}, +, +\infty, 0 \rangle$$



$f_1(a) = 15$	$f_2(c) = 20$	$f_3(f) = 45$
$f_1(b) = 10$	$f_2(d) = 25$	$f_3(g) = 20$
<div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <div style="background-color: #90EE90; border-radius: 50%; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center; margin: 0 auto;">X</div> </div> <div style="text-align: center;"> <div style="background-color: #90EE90; border-radius: 50%; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center; margin: 0 auto;">Y</div> </div> <div style="text-align: center;"> <div style="background-color: #90EE90; border-radius: 50%; width: 40px; height: 40px; display: flex; align-items: center; justify-content: center; margin: 0 auto;">Z</div> </div> </div>		
$f_4(a, c) = 10$	$f_5(c, f) = 30$	
$f_4(a, d) = 15$	$f_5(c, g) = 40$	
$f_4(b, c) = 25$	$f_5(d, f) = 45$	
$f_4(b, d) = 20$	$f_5(d, g) = 60$	

The combined constraint is

$c_6 = \langle f_6, \{X, Y, Z\} \rangle$ , where

$$\begin{array}{c}
 X \ Y \ Z \\
 \downarrow \downarrow \downarrow \\
 f_6(a, c, f) = f_1(a) + f_2(c) + f_3(f) + f_4(a, c) + f_5(c, f) \\
 = 15 + 20 + 45 + 10 + 30 = 120
 \end{array}$$

$$f_6(a, c, g) = 15 + 20 + 20 + 10 + 40 = 105$$

$$f_6(a, d, f) = 15 + 25 + 45 + 15 + 45 = 145$$

$$f_6(a, d, g) = 15 + 25 + 20 + 15 + 60 = 135$$

$$f_6(b, c, f) = 10 + 20 + 45 + 25 + 30 = 130$$

$$f_6(b, c, g) = 10 + 20 + 20 + 25 + 40 = 115$$

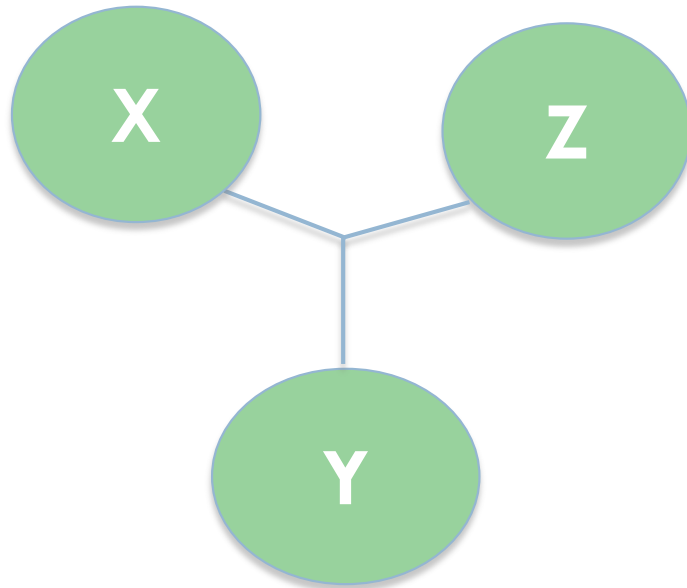
$$f_6(b, d, f) = 10 + 25 + 45 + 20 + 45 = 145$$

$$f_6(b, d, g) = 10 + 25 + 20 + 20 + 60 = 135$$

2. What are the **optimal solutions** of the weighted CSP?
3. What is the **preference value** of the optimal solutions?

### Weighted c-semiring

$$S_{FCSP} = \langle R^+ \cup \{+\infty\}, \min, +, +\infty, 0 \rangle$$



Optimal solution: **s = (a, c, g)**

Preference value of the optimal solution is **105**

$f_1(a) = 15$	$f_2(c) = 20$	$f_3(f) = 45$
$f_1(b) = 10$	$f_2(d) = 25$	$f_3(g) = 20$
$f_4(a, c) = 10$	$f_5(c, f) = 30$	
$f_4(a, d) = 15$	$f_5(c, g) = 40$	
$f_4(b, c) = 25$	$f_5(d, f) = 45$	
$f_4(b, d) = 20$	$f_5(d, g) = 60$	

The **combined constraint** is

$c6 = \langle f6, \{X, Y, Z\} \rangle$ , where

$$\begin{aligned}
 &\begin{matrix} X & Y & Z \\ \downarrow & \downarrow & \downarrow \end{matrix} \\
 f6(a, c, f) &= f1(a) + f2(c) + f3(f) + f4(a, c) + f5(c, f) \\
 &= 15 + 20 + 45 + 10 + 30 = 120
 \end{aligned}$$

$$f6(a, c, g) = 15 + 20 + 20 + 10 + 40 = 105$$

$$f6(a, d, f) = 15 + 25 + 45 + 15 + 45 = 145$$

$$f6(a, d, g) = 15 + 25 + 20 + 15 + 60 = 135$$

$$f6(b, c, f) = 10 + 20 + 45 + 25 + 30 = 130$$

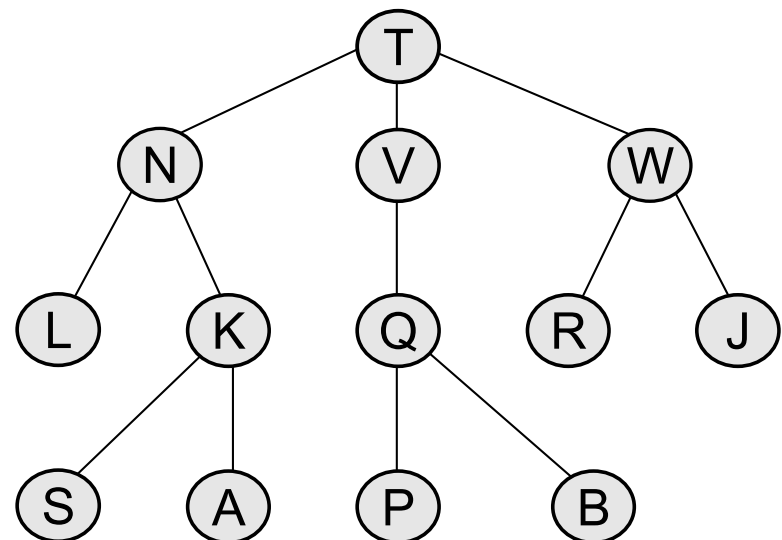
$$f6(b, c, g) = 10 + 20 + 20 + 25 + 40 = 115$$

$$f6(b, d, f) = 10 + 25 + 45 + 20 + 45 = 145$$

$$f6(b, d, g) = 10 + 25 + 20 + 20 + 60 = 135$$

# Search strategies

- Enumerate the **order** in which **all nodes** (of the tree below) are chosen **for expansion** by the **following search strategies**:
  - Depth First Search
  - Breadth First Search
  - Iterative Deepening Search



# Review: Search strategies

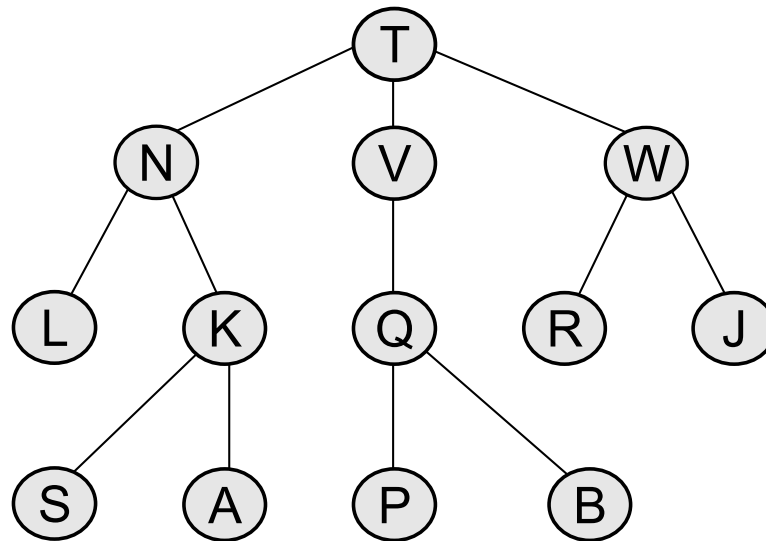


- A search strategy is defined by picking the order of node expansion
- Breadth-first search expands the shallowest unexpanded node first
- Depth-first search expands the deepest unexpanded node first
- Iterative deepening search calls depth-first search  
with increasing depth limits until a goal is found



# Search strategies

Depth-first search expands the deepest unexpanded node first

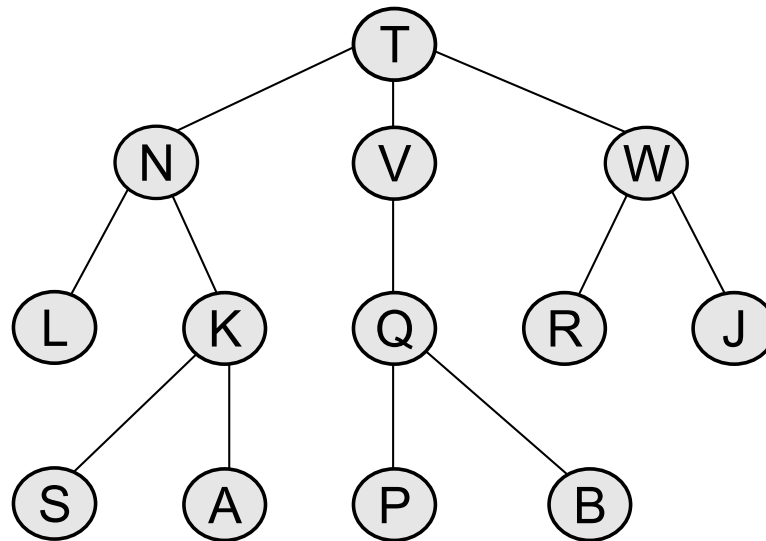


## ■ Depth First Search

T N L K S A V Q P B W R J

# Search strategies

Breadth-first search expands the shallowest unexpanded node first

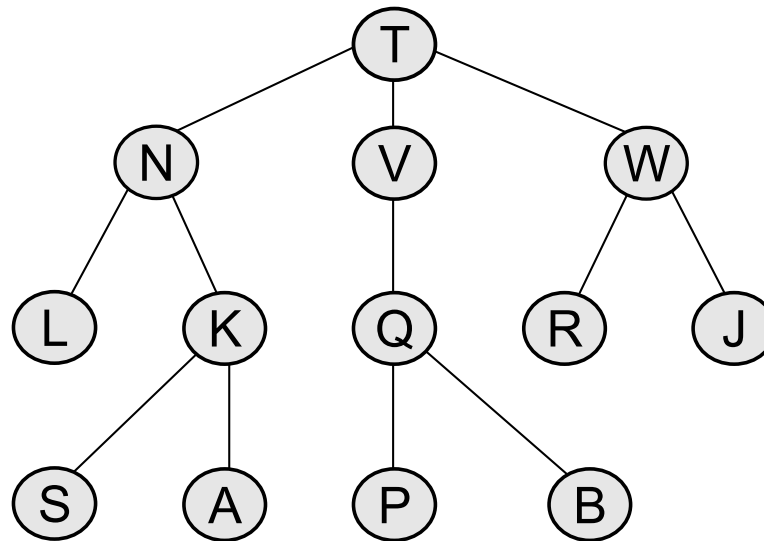


## ■ Breath First Search

T N V W L K Q R J S A P B

# Search strategies

Iterative deepening search  
calls depth-first search  
with increasing depth limits  
until a goal is found



## ■ Iterative Deepening Search

T; T N V W; T N L K V Q W R J; T N L K S A V Q P B W R J