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DEGLI STUDI
DI PADOVA

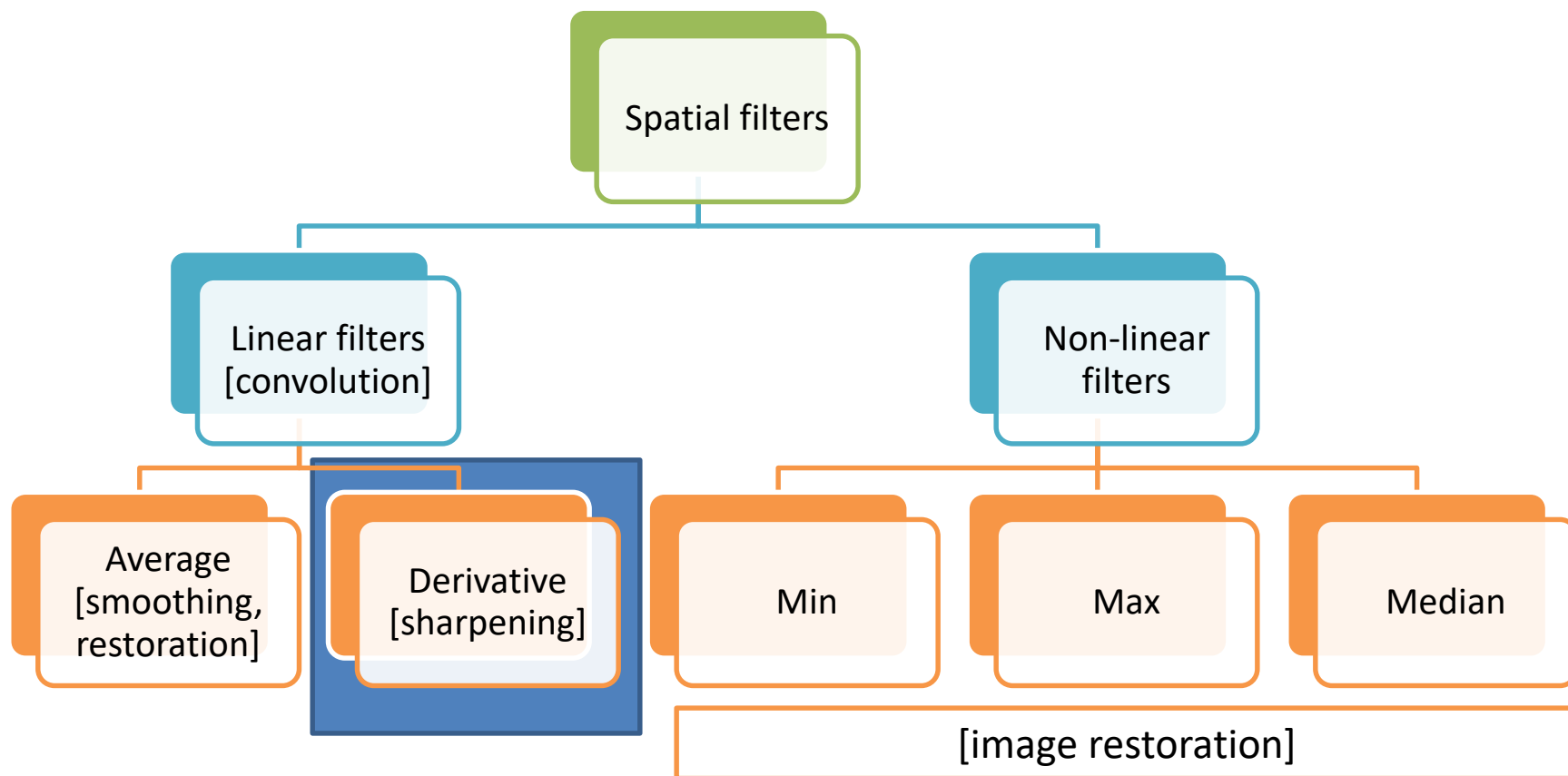
Spatial filtering – non-linear filters
Image restoration

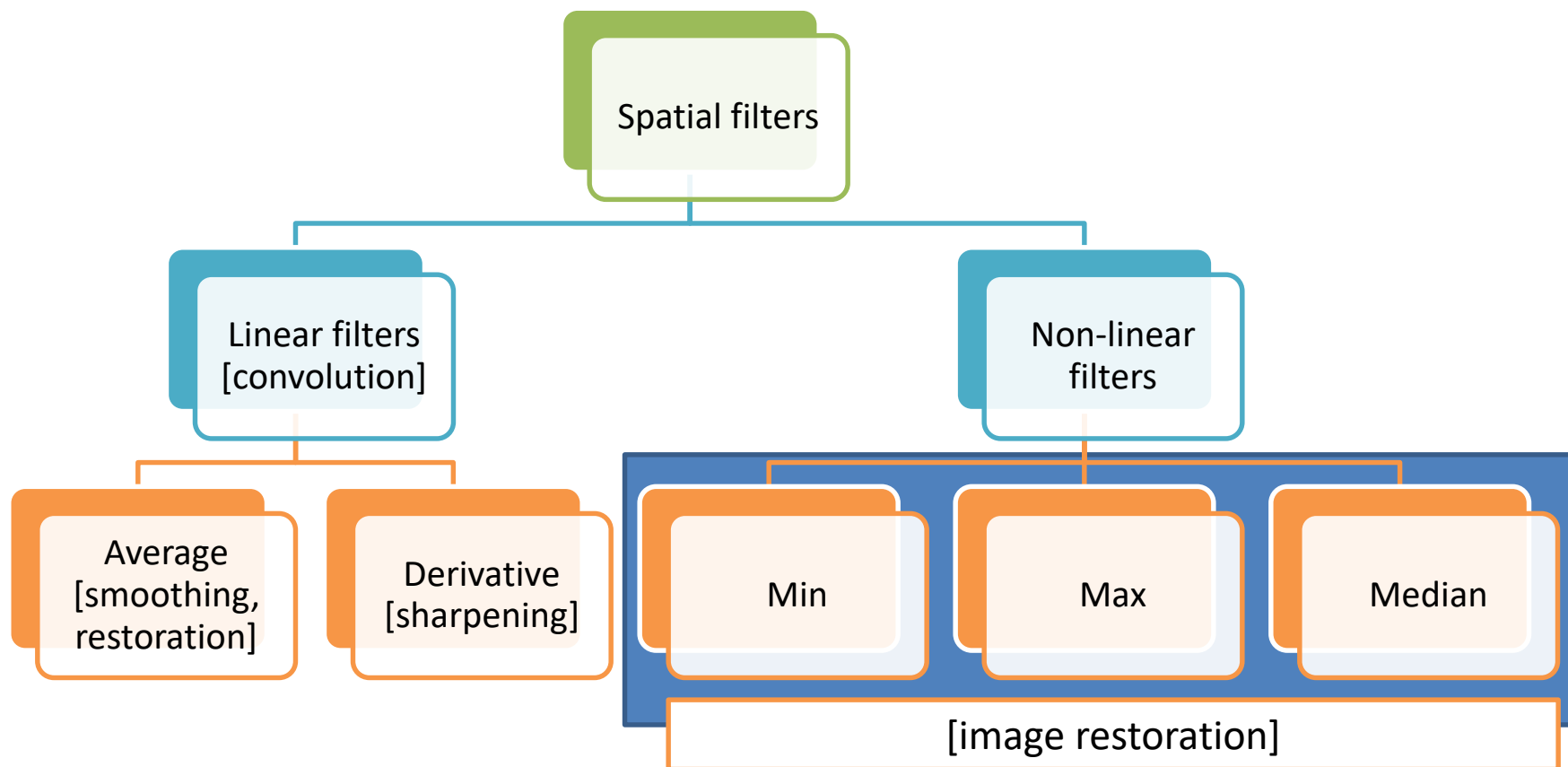
Stefano Ghidoni





- Linear and non-linear filters
- Images and noise, noise models
- Image restoration







- Non-linear filters implement non-linear operations
- What is a non-linear operation?



- No spoiler 😊



- Non-linear filters implement non-linear operations
- What is a non-linear operation?
- Examples of non-linear operations:
 - Min
 - Max
 - Median
- Can you think of other non-linear operations?

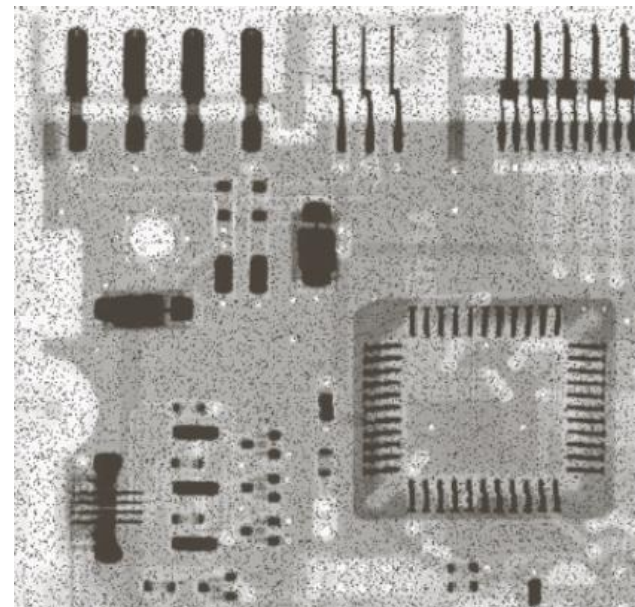


- Besides their mathematical formulation, what can be the advantages of a non-linear filter over a linear one?

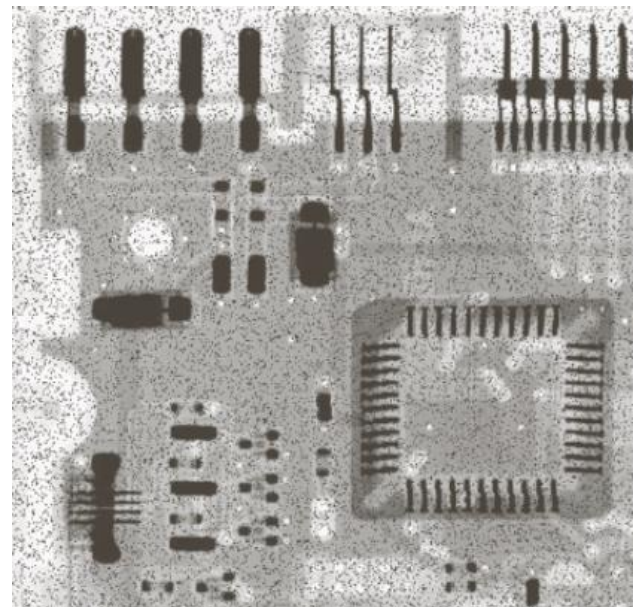


- Besides their mathematical formulation, what can be the advantages of a non-linear filter over a linear one?
- Non-linear filters can *suppress* components
 - Ex: a median filter can remove a spike (a single element that is strongly different from the others)
 - In our context: an element is a pixel

- How do spikes appear in an image?
- An example: microchip microscopic image
- What can you say about this image?



- How do spikes appear in an image?
- Image corrupted by noise
- The task of removing the noise in an image is often referred to as **image restoration**



- We can model the acquired image as:

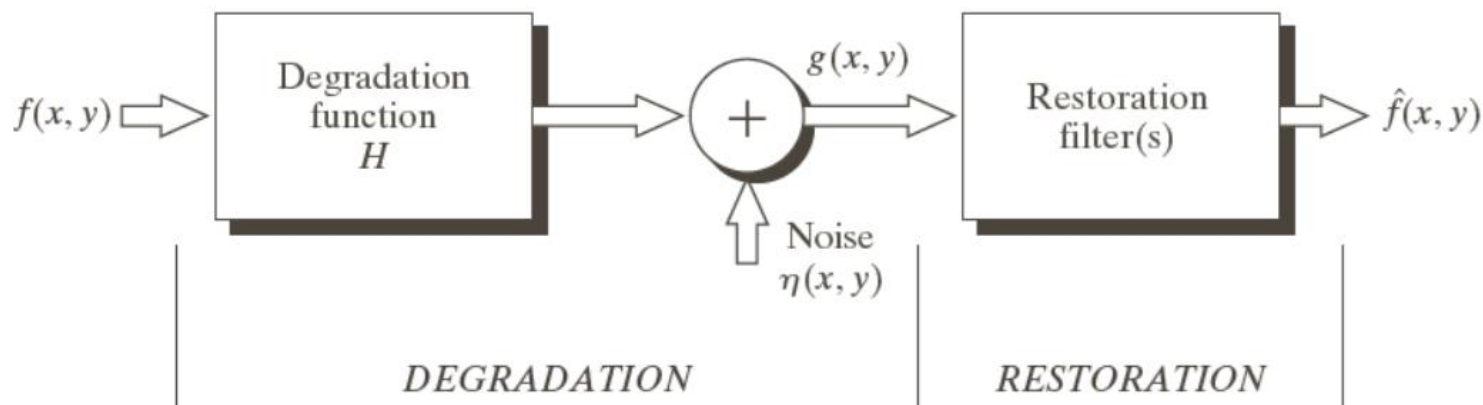
$$f_{\text{acq}}(x, y) = f(x, y) + \eta(x, y)$$

- Ideal image + noise

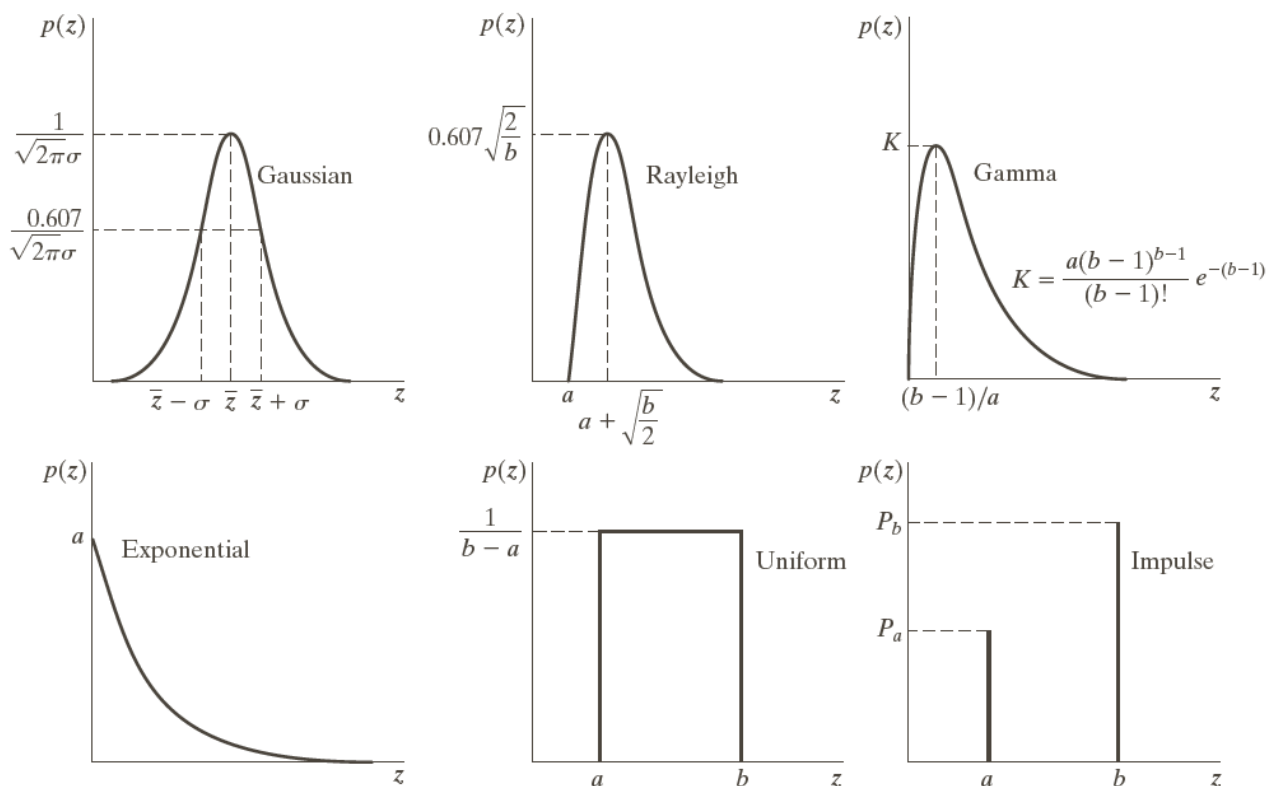
- Image restoration

- Estimate noise properties
 - Remove noise

Many types
of noise



- Several noise models are available



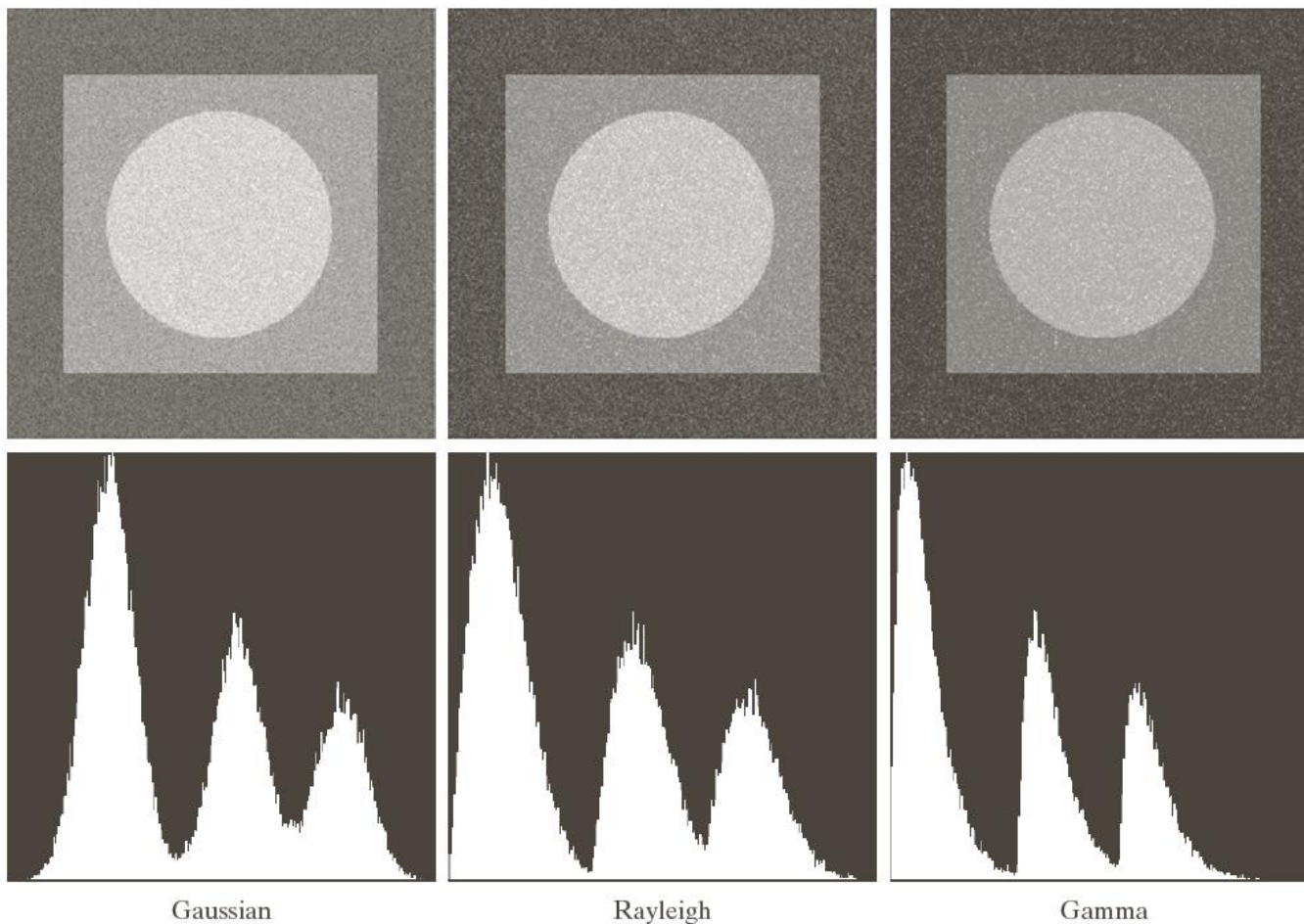
a	b	c
d	e	f

FIGURE 5.2 Some important probability density functions.

- Let's analyze how the different noise models affect a test pattern

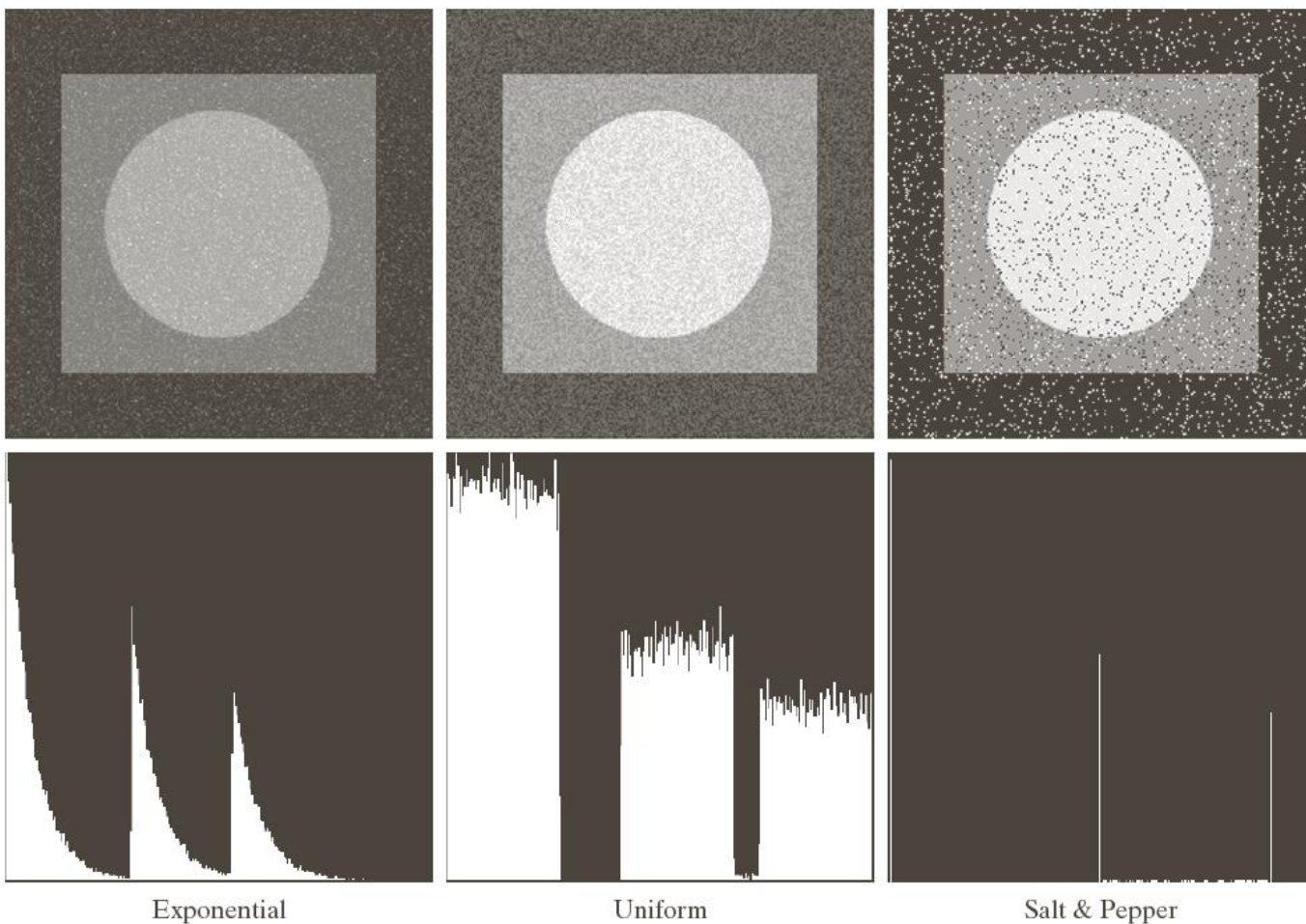


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.



a	b	c
d	e	f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.



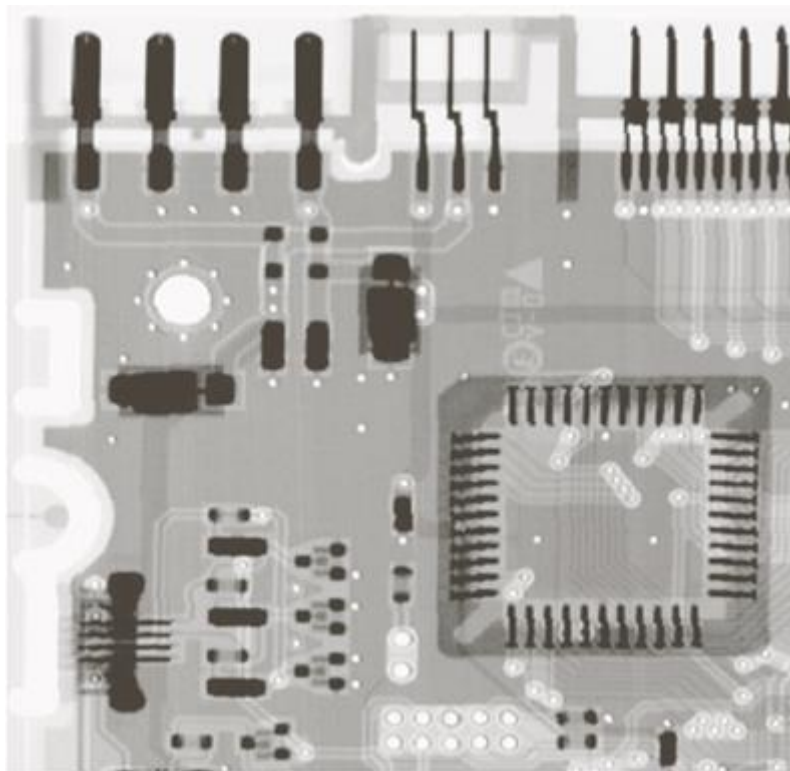
g	h	i
j	k	l

FIGURE 5.4 (Continued) Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

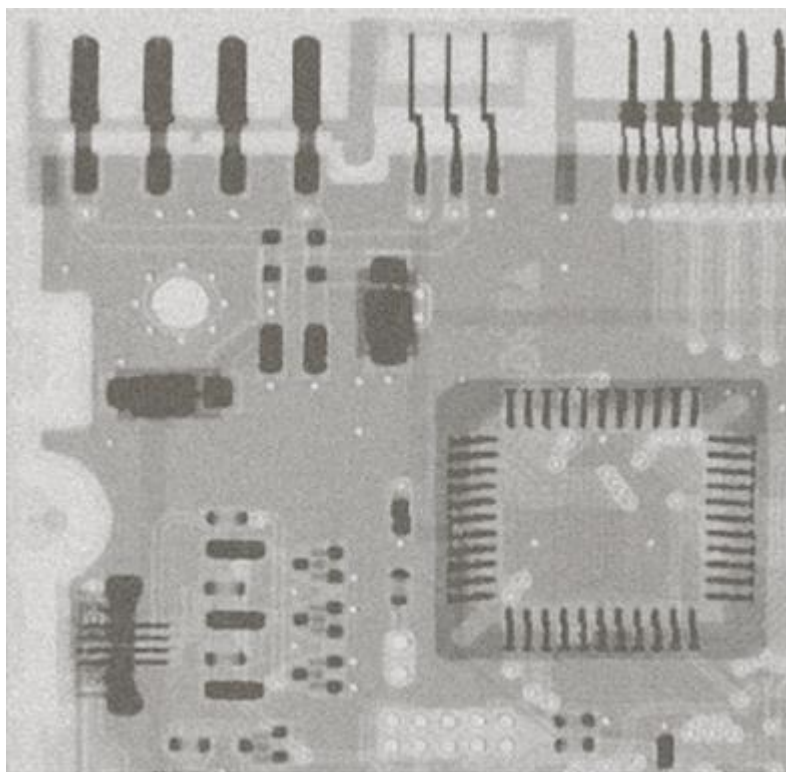


- A corrupted image may be restored using:
 - Smoothing filters
 - Averaging
 - Gaussian filter
 - Non-linear filters
 - Median filter
 - Min and max filters
- Example: image corrupted with gaussian noise
 - Restored using an average filter

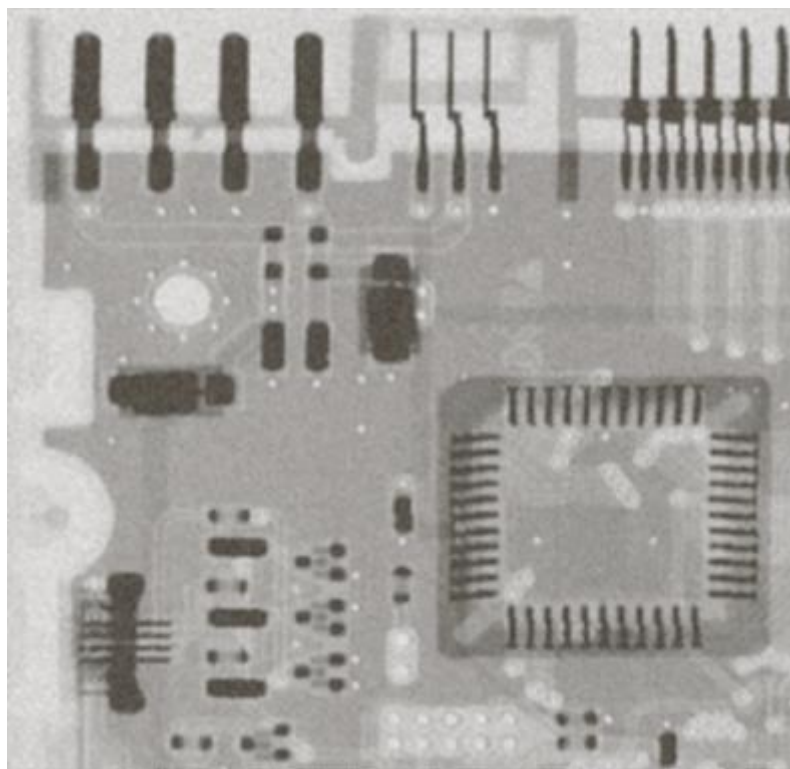
- Original image



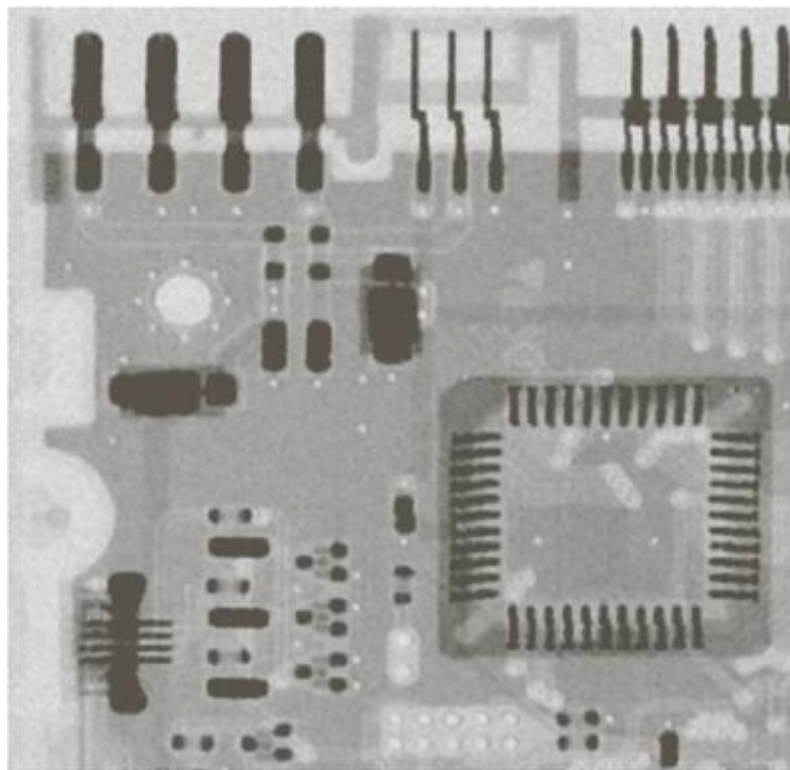
- Corrupted image (gaussian noise)



- Restored image (arithmetic mean)



- Restored image (geometric mean)

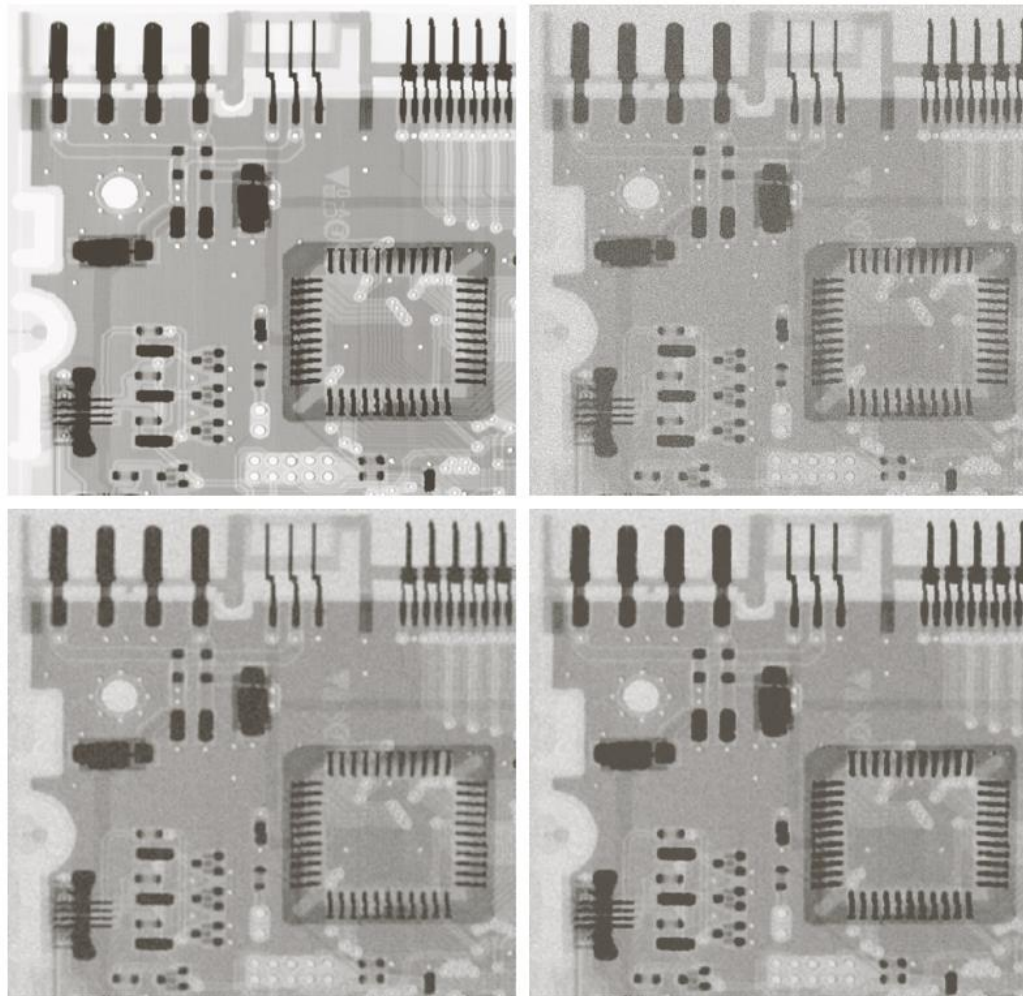


a b
c d

FIGURE 5.7

(a) X-ray image.
(b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3 . (d) Result of filtering with a geometric mean filter of the same size.

(Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



What is the visible difference between arithmetic and geometric mean?



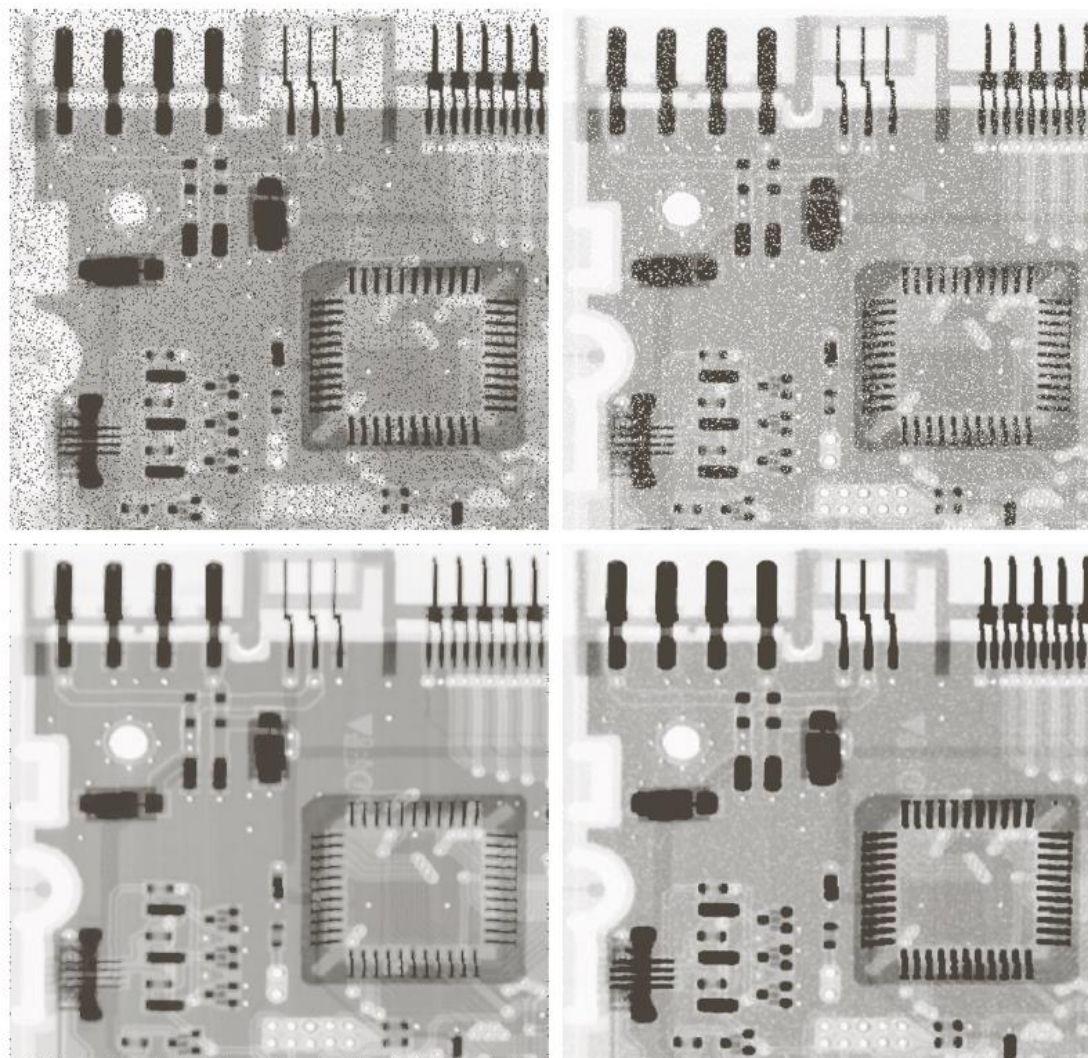
- A corrupted image may be restored using:
 - Smoothing filters
 - Averaging
 - Gaussian filter
 - Non-linear filters
 - Median filter
 - Min and max filters
- Example: image corrupted with salt noise **or** pepper noise
 - Restored using other mean filters



- Several averaging filters are available:
 - Arithmetic mean
 - **Alpha-trimmed mean**
 - Geometric mean
 - Harmonic mean
 - Dominated by the minimum of its arguments
 - In the general case:
 $\text{harmonic} < \text{geometric} < \text{arithmetic mean}$
 - Contraharmonic mean



<i>Name</i>	<i>Equation</i>	<i>Removes Gaussian Noise</i>	<i>Removes Salt Noise</i>	<i>Removes Pepper Noise</i>
Arithmetic mean	$g(x, y) = \frac{1}{mn} \sum_{s,t \in R} f(s, t)$	Yes	No	No
Geometric mean	$g(x, y) = \left[\prod_{s,t \in R} f(s, t) \right]^{\frac{1}{mn}}$	Yes	No	No
Harmonic mean	$g(x, y) = \frac{mn}{\sum_{s,t \in R} \frac{1}{f(s, t)}}$	Yes	Yes	No
Contraharmonic mean	$g(x, y) = \frac{\sum_{s,t \in R} f(s, t)^{Q+1}}{\sum_{s,t \in R} f(s, t)^Q}$	Yes ($Q=0 \rightarrow \text{mean}$)	$Q < 0$ ($Q=-1 \rightarrow \text{harmonic}$)	$Q > 0$ (not both)

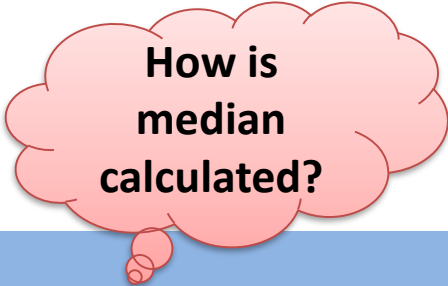


a	b
c	d

FIGURE 5.8

(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

- A corrupted image may be restored using:
 - Smoothing filters
 - Averaging
 - Gaussian filter
 - Non-linear filters
 - Median filter
 - Min and max filters
- Example: image corrupted with gaussalt&pepper noise
 - Restored using a median filter and an alpha-trimmed mean

A pink thought bubble with a small tail pointing towards the bottom left.

How is
median
calculated?

a b
c d

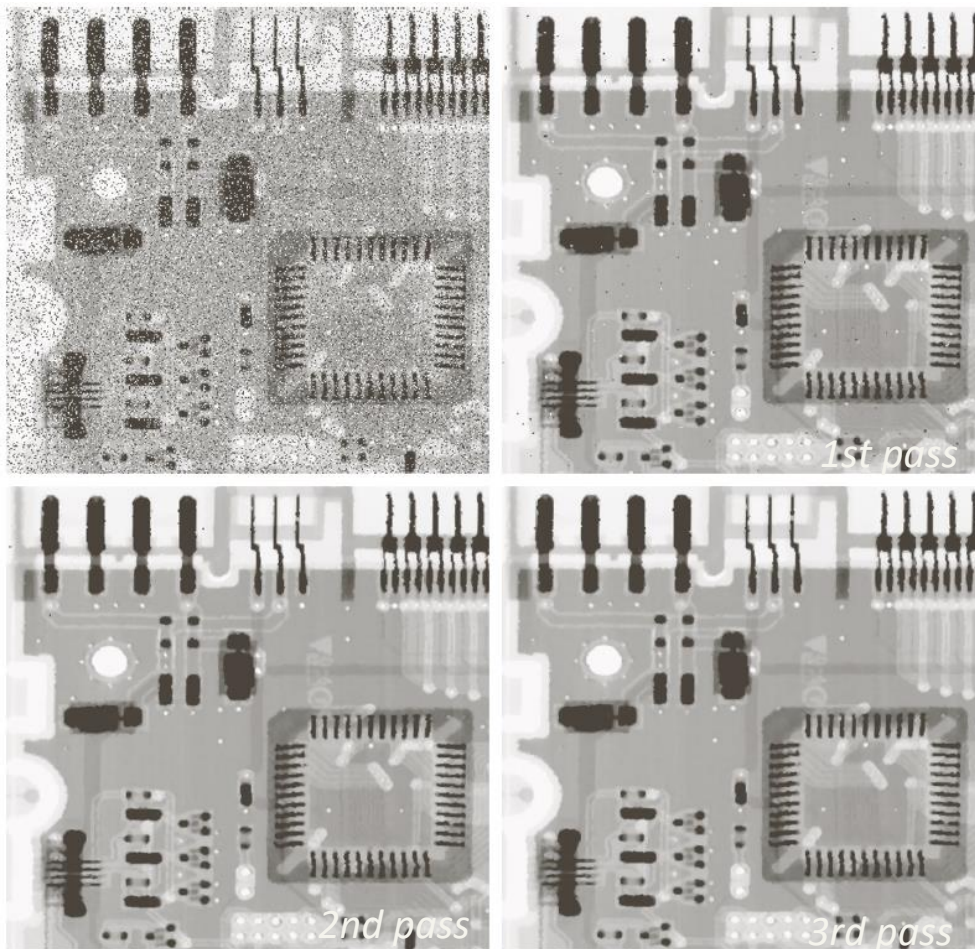
FIGURE 5.10

(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.

(b) Result of one pass with a median filter of size 3×3 .

(c) Result of processing (b) with this filter.

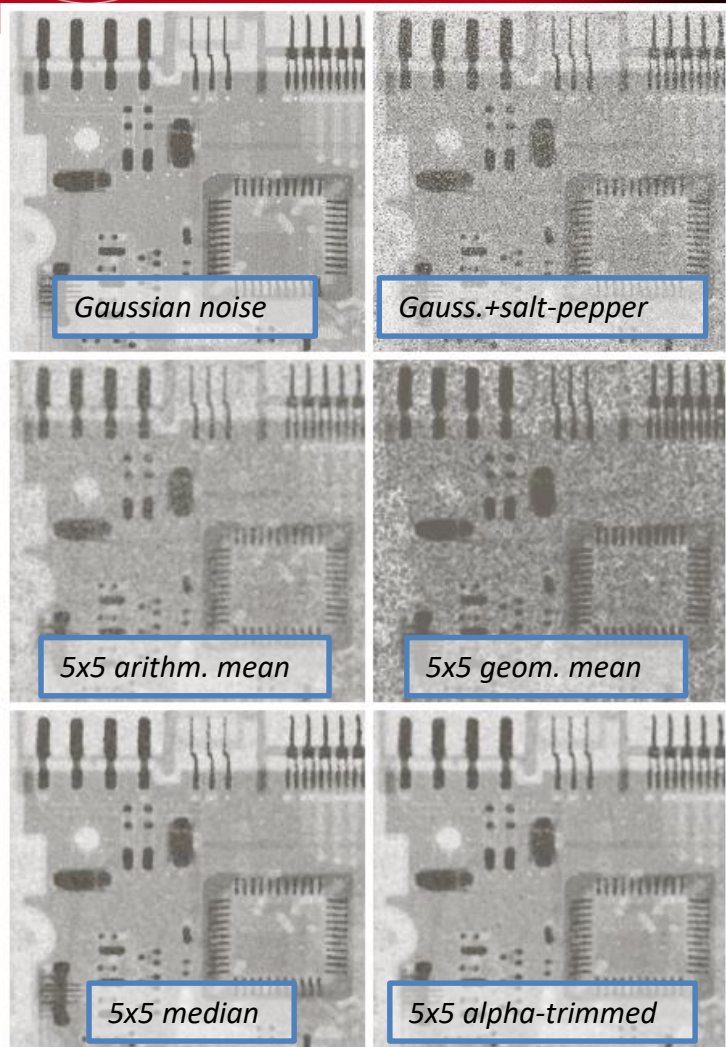
(d) Result of processing (c) with the same filter.



3	7	4
8	2	92
10	10	5



2	3	4	5	7	8	10	10	92
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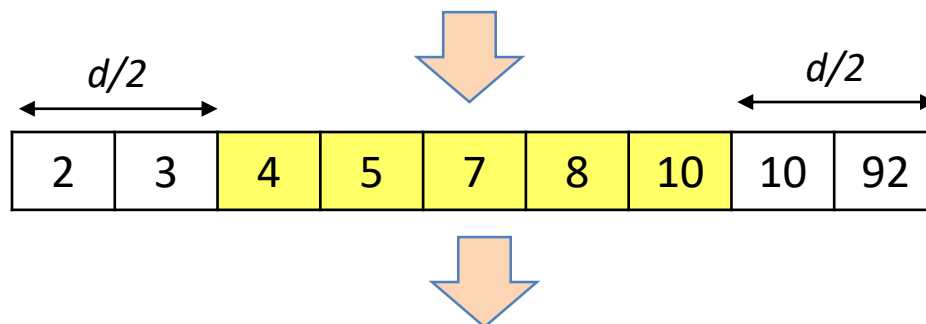
a b
c d
e f

FIGURE 5.12

(a) Image corrupted by additive uniform noise.
(b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5 ;
(c) arithmetic mean filter;
(d) geometric mean filter;
(e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

$$g(x, y) = \frac{1}{mn - d} \sum_{s, t \in R_r} f(s, t)$$

3	7	4
8	2	92
10	10	5



Hybrid between mean and median filter

$D=0$: mean filter

$D=mn-1$: median filter



- A corrupted image may be restored using:
 - Smoothing filters
 - Averaging
 - Gaussian filter
 - Non-linear filters
 - Median filter
 - Min and max filters

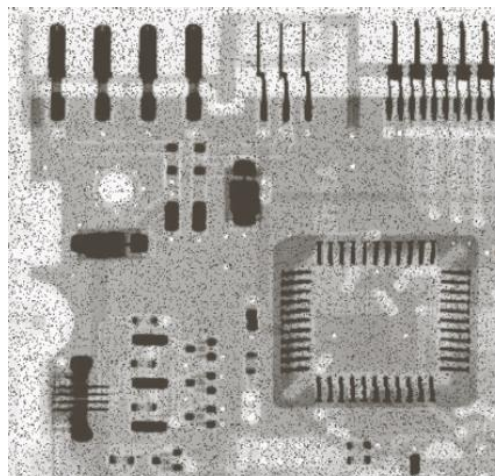


- A corrupted image may be restored using:
 - Smoothing filters
 - Averaging
 - Gaussian filter
 - Non-linear filters
 - Median filter
 - Min and max filters



- A corrupted image may be restored using:
 - Smoothing filters
 - Averaging
 - Gaussian filter
 - Non-linear filters
 - Median filter
 - Min and max filters
- Example: image corrupted with gaussalt&pepper noise
 - Restored using a max/min filter

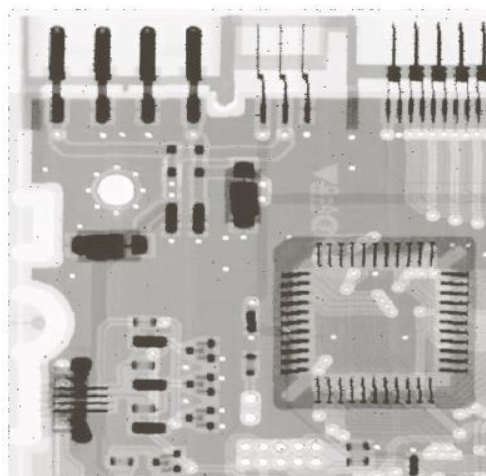
- Max filter: highlights salt noise, removes pepper noise
- Min filter: highlights pepper noise, removes salt noise



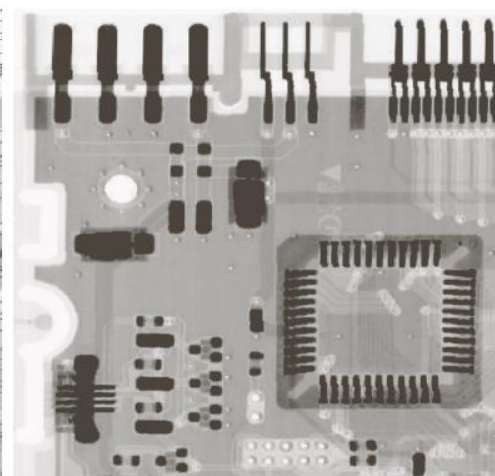
Original

a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3 . (b) Result of filtering 5.8(b) with a min filter of the same size.



Max filter



Min filter



- The filters discussed so far operate in the same way on
 - Every image
 - Every part of the image
- Smarter behaviors can be designed...



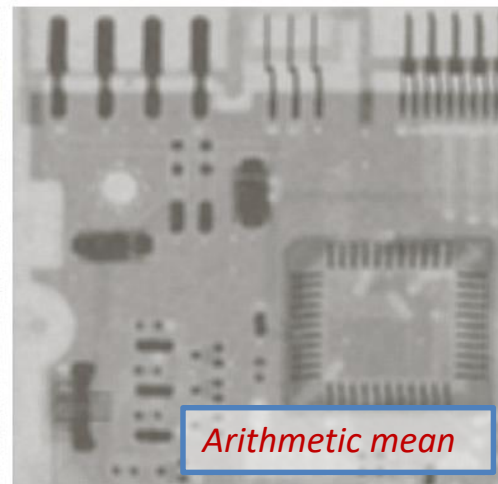
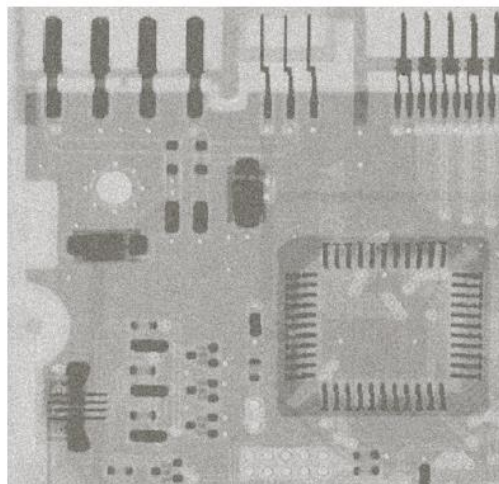
- Adaptive filters tune their effect comparing
 - Local image variance - σ_L^2
 - Noise variance - σ_η^2
- When $\sigma_\eta^2 \ll \sigma_L^2$ the filter should be weak
 - Edges / other image elements
- When $\sigma_\eta^2 \approx \sigma_L^2$ the filter should be strong
- Skip the mathematical details here, just see what is the effect
 - Several algorithms and approaches
 - We will see an example in detail in the next lectures

- Adaptive mean

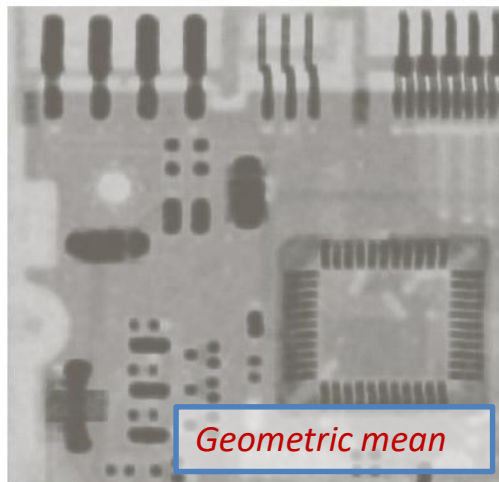
a	b
c	d

FIGURE 5.13

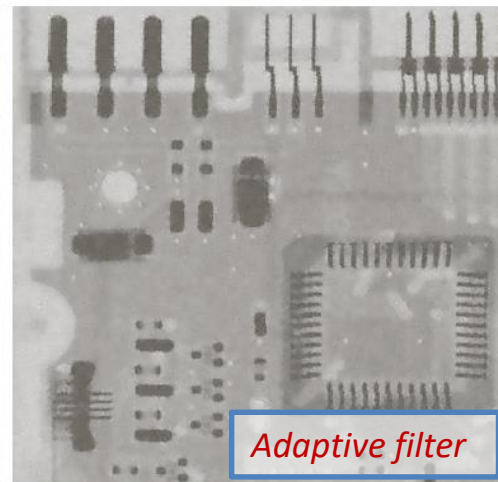
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000.
(b) Result of arithmetic mean filtering.
(c) Result of geometric mean filtering.
(d) Result of adaptive noise reduction filtering. All filters were of size 7×7 .



Arithmetic mean



Geometric mean



Adaptive filter

- Adaptive median

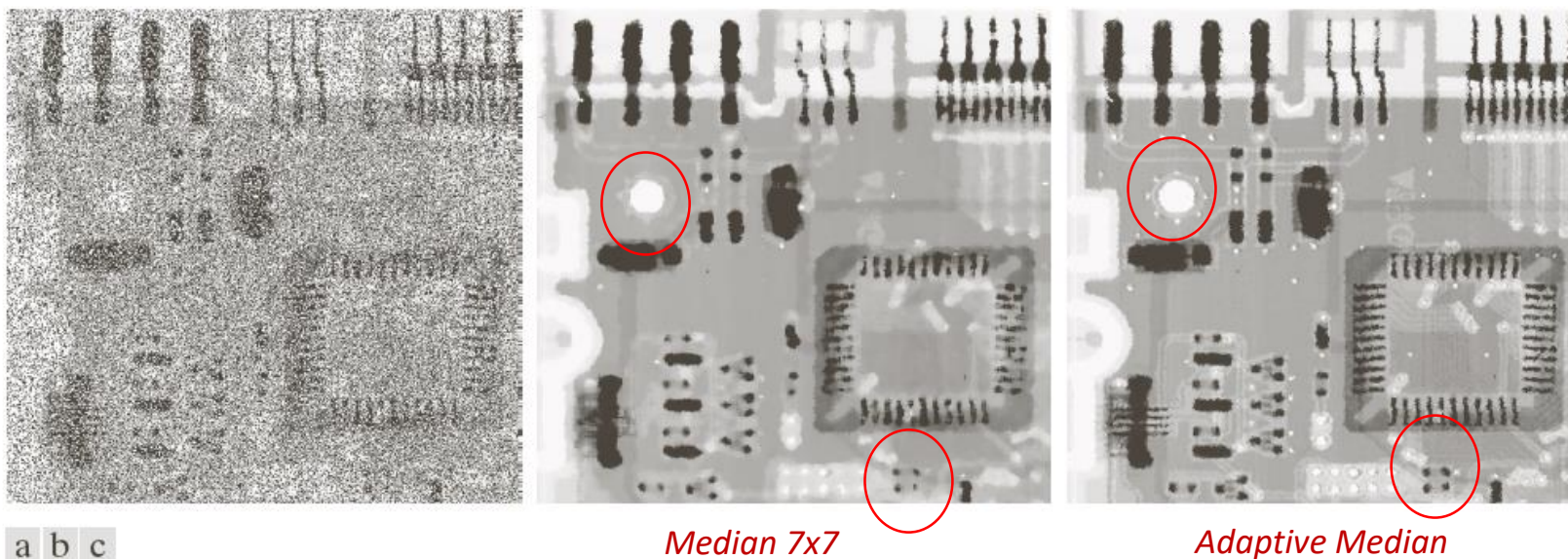


FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.



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Spatial filtering – non-linear filters
Image restoration

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