

Mining Unstructured Data

3. POS tagging



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Outline

POS tagging

POS Taggers

- 1 POS tagging
 - Goal and motivation
 - Part of Speech categories

- 2 POS Taggers
 - Stochastic taggers
 - Hidden Markov Model
 - Viterbi algorithm

Outline

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Goal and motivation

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Goal

- Morphological analysis provides lexical information related to forms (POS, num, gen, tense, ...)
- Multiple analyses can result (POS tags from Penn Treebank tagset)

form	analyses	example of use
fish	NNS	'Cats eat fish'
	VB	'I want to fish'
bass	NN	'I saw you play the bass'
	JJ	'Bass clarinets sound good'

- **Goal:** disambiguate POS of word forms occurring in text

Motivation

Examples of applications of POS tagging:

- Syntactic parsing: words with the same POS tag play a similar syntactic role

Ex: a determiner followed by a common noun is a noun phrase

- Machine translation

Ex: (POS tags from Penn Treebank tagset)

'El hombre	bajo	toca el	bajo	bajo	el puente'
POS	NN		NN	NN	
tagging	JJ		JJ	JJ	
	IN		IN	IN	
	VB		VB	VB	
possible	low		bass	under	
English	small			below	
words	short				
	poor				
'The	small	man plays the	bass	under	the bridge'

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Part of Speech
categories

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Open class vs. Closed class

- General classes:
 - **Closed class:** never invent new closed items (functional words)
Usual subclasses for indo-european languages:
prepositions, conjunctions, determiners, pronouns, auxiliary verbs or particles (prepositions or adverbs in phrasal verbs)
 - **Open class:** new open items can be invented
Usual subclasses for indo-european languages:
nouns, non-auxiliary verbs, adjectives and adverbs
- Each language defines its particular set of subclasses
- Subclasses can be represented with a particular granularity by a set of categories
 - Ex: **Brown corpus:** annotated with 87 different POS tags
 - Ex: **Penn Treebank corpus:** with 48 different POS tags

POS tagging

Part of Speech
categories

POS Taggers

Penn Treebank tagset

POS tagging

Part of Speech
categories

POS Taggers

CC	Coordinating conjunction	PP	Possessive pronoun
CD	Cardinal number	RB	Adverb
DT	Determiner	RBR	Adverb, comparative
EX	Existential there	RBS	Adverb, superlative
FW	Foreign word	RP	Particle
IN	Preposition	SYM	Symbol
JJ	Adjective	TO	to
JJR	Adjective, comparative	UH	Interjection
JJS	Adjective, superlative	VB	Verb, base form
LS	List item marker	VBD	Verb, past tense
MD	Modal	VBG	Verb, gerund
NN	Noun, singular	VCN	Verb, past participle
NNP	Proper noun, singular	VBP	Verb, non-3rd ps. sing. present
NNS	Noun, plural	VBZ	Verb, 3rd ps. sing. present
NNPS	Proper noun, plural	WDT	wh-determiner
PDT	Predeterminer	WP	wh-pronoun
POS	Possessive ending	WP\$	Possessive wh-pronoun
PRP	Personal pronoun	WRB	wh-adverb

12 categories more related to punctuation marks

Ex: to/TO give/VB priority/NN to/IN teacher/NN pay/NN rises/NNS

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POS tagging methods

POS tagging

POS Taggers

Frequently used methods:

- Rule-based methods:

- Rules built manually are not frequently used. High production cost
- Rules learnt automatically from training corpus.

Ex: Brill's tagger.

- Stochastic methods:

- Based on Hidden Markov Models learnt automatically from training corpus.

Outline

POS tagging

POS Taggers

Stochastic taggers

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Stochastic taggers

Goal: Assign the most likely POS-tag sequence to a word sequence.

POS tagging

POS Taggers

Stochastic taggers

$W = w_1 \dots w_n$ (a word sequence)

$T = t_1 \dots t_n$ (a POS-tag sequence)

Tagger result: $\hat{T} = \underset{T}{\operatorname{argmax}} P(T|W)$

1 How is $P(T|W)$ computed?

Apply a [Hidden Markov Model](#)

2 How is \hat{T} found?

Apply [Viterbi algorithm](#)

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POS Taggers

Hidden Markov
Model

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Preliminaries: Markov model

- $X = (X_1, \dots, X_T)$ sequence of random variables taking values in observed states $S = \{s_1, \dots, s_N\}$

- Inference: Sequence probability $P(X)$?

- Markov Properties

- Limited Horizon:

$$P(X_{t+1} = s_k \mid X_1, \dots, X_t) = P(X_{t+1} = s_k \mid X_t)$$

- Time Invariant (Stationary):

$$P(X_{t+1} = s_i \mid X_t = s_j) = P(X_2 = s_i \mid X_1 = s_j)$$

- Transition matrix:

$$a_{ij} = P(X_{t+1} = s_j \mid X_t = s_i); \quad \forall i, j \ a_{ij} \geq 0; \quad \forall i \ \sum_{j=1}^N a_{ij} = 1$$

- Initial probabilities (or extra state s_0):

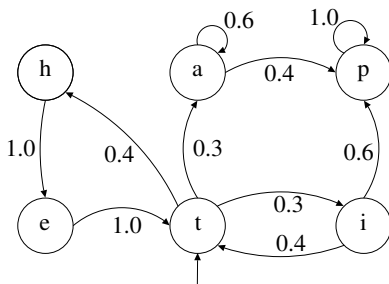
$$\pi_i = P(X_1 = s_i); \quad \sum_{i=1}^N \pi_i = 1$$

Preliminaries: Markov model

Sequence probability: (Bayesian rule+limited horizon)

$$\begin{aligned} P(X_1, \dots, X_T) &= \\ &= P(X_1)P(X_2 | X_1)P(X_3 | X_1X_2) \dots P(X_T | X_1 \dots X_{T-1}) \\ &= P(X_1)P(X_2 | X_1)P(X_3 | X_2) \dots P(X_T | X_{T-1}) \\ &= \pi_{x_1} \prod_{t=2}^T a_{x_{t-1}x_t} \end{aligned}$$

Example:



$$P(t, h, e, t, i, p, p) = 1 \cdot (0.4 \cdot 1 \cdot 1 \cdot 0.3 \cdot 0.6 \cdot 1) = 0.072$$

POS tagging

POS Taggers

Hidden Markov
Model

Hidden Markov model

- $X = (X_1, \dots, X_T)$ sequence of random variables taking values in **unobserved [hidden] states** $S = \{s_1, \dots, s_N\}$ given a sequence of observations $O = (O_1, \dots, O_T)$
- Inference: Probability of ...
 - a process: $P(O)$?
 - the state of a process at the end: $P(X_T | O)$?
 - **the explanation of a process**: $P(X_1, \dots, X_T | O)$?
POS tagging: $X = \text{POS tags}$; $O = \text{words}$
- Transition matrix:
$$a_{ij} = P(X_{t+1} = s_j | X_t = s_i); \quad \forall i, j \ a_{ij} \geq 0; \quad \forall i \ \sum_{j=1}^N a_{ij} = 1$$
- Initial probabilities (or extra state s_0):
$$\pi_i = P(X_1 = s_i); \quad \sum_{i=1}^N \pi_i = 1$$
- Emission Probability:
$$b_{ik} = P(O_t = k | X_t = s_i) \quad \forall i, k \ b_{ik} \geq 0; \quad \forall i \ \sum_{k=1}^N b_{ik} = 1$$

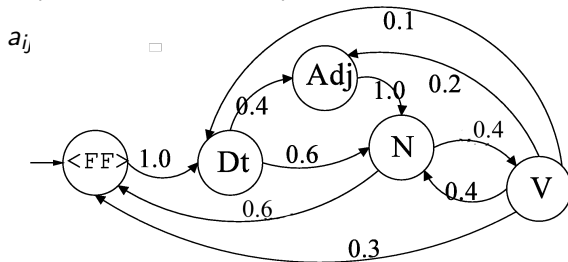
POS tagging

POS Taggers

Hidden Markov
Model

Hidden Markov model

Example (horizon=1; bigrams)



b_{ik}	.	the	this	cat	kid	eats	runs	fish	fresh	little	big
<FF>	1.0										
Dt		0.6	0.4								
N				0.3	0.1		0.1	0.3	0.2		
V				0.1	0.5	0.3	0.1				
Adj				0.1					0.2	0.3	0.4

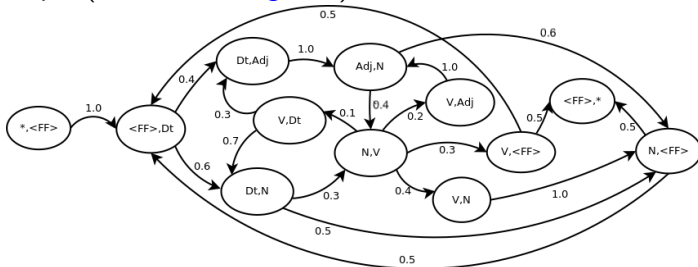
POS tagging

POS Taggers

Hidden Markov
Model

Hidden Markov model

Example (horizon=2; **trigrams**)



b_{ik}	.	the	this	cat	kid	eats	runs	fish	fresh	little	big
?,<FF>	1.0										
?,Dt		0.6	0.4								
?,N				0.3	0.1		0.1	0.3	0.2		
?,V					0.1	0.5	0.3	0.1			
?,Adj					0.1				0.2	0.3	0.4

Learning of parameters

POS tagging

POS Taggers

Hidden Markov
Model

- Parameters a_{ij} , b_{ik} and π_i can be estimated over a training corpus C
- Use smoothing techniques
- Use Baum-Welch algorithm
- **learning of parameters is out of this course**

Learning of parameters

Example: MLE estimator; assume u, v, w are different POS tags in the training corpus

- bigram-based HMM

$$a(u, v) \approx P_{MLE}(v | u) = \frac{c(u, v)}{c(u)}$$

$$b(O_i, u) \approx P_{MLE}(O_i | u) = \frac{c(u, O_i)}{c(u)}$$

$$\pi(u) \approx P_{MLE}(u | *) = \frac{c(*, u)}{c(*)}$$

- trigram-based HMM

$$a(uv, vw) \approx P_{MLE}(vw | uv) = \frac{c(u, v, w)}{c(u, v)}$$

$$b(O_i, uv) = b(O_i, v) \approx P_{MLE}(O_i | v) = \frac{c(v, O_i)}{c(v)}$$

$$\pi(*u) \approx P_{MLE}(*u | **) = \frac{c(*, *, u)}{c(**)} \quad \pi(uv) \approx P_{MLE}(uv | *u) = \frac{c(*, u, v)}{c(*u)}$$

Exercise

Given the following corpus,

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
eat/VB breakfast/NN at/IN morning/NN time/NN ./.
take/VB time/NN with/IN arrow/NN projects/NNS ./.
dinner/NN time/NN goes/VBZ before/IN sleep/NN ./.
flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

apply MLE to estimate the non-zero parameters for the POS-tags involved in the sentence using bigrams:

"time flies like horse flies ."

POS tagging

POS Taggers

Hidden Markov
Model

Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
eat/VB breakfast/NN at/IN morning/NN time/NN ./.
take/VB time/NN with/IN arrow/NN projects/NNS ./.
dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$		$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*		*						
NN		NN						
NNS		NNS						
VBP		VBP						
VBZ		VBZ						
IN		IN						
.								

Exercise

- * horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
- * eat/VB breakfast/NN at/IN morning/NN time/NN ./.
- * take/VB time/NN with/IN arrow/NN projects/NNS ./.
- * dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
- * flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
- * bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$		$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	6	*						
NN		NN						
NNS		NNS						
VBP		VBP						
VBZ		VBZ						
IN		IN						
.								

Exercise

* horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 * dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$		$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	6	*	2					
NN		NN						
NNS		NNS						
VBP		VBP						
VBZ		VBZ						
IN		IN						
.								

Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."


POS tagging

POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$		$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	6	*	2					
NN		NN						
NNS		NNS						
VBP		VBP						
VBZ		VBZ						
IN		IN						
.								

$\frac{1}{3}$  **NN**

Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 * flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 * bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

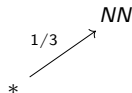
POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$	
*	6
NN	
NNS	
VBP	
VBZ	
IN	
.	

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN						
NNS						
VBP						
VBZ						
IN						



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

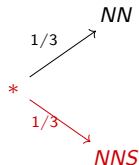
POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$	
*	6
NN	
NNS	
VBP	
VBZ	
IN	
.	

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN						
NNS						
VBP						
VBZ						
IN						



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

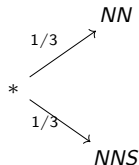
POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$	
*	6
NN	12
NNS	
VBP	
VBZ	
IN	
.	

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN						
NNS						
VBP						
VBZ						
IN						



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

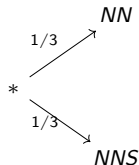
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$c(u)$	
*	6
NN	12
NNS	
VBP	
VBZ	
IN	
.	

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN	3					
NNS						
VBP						
VBZ						
IN						



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

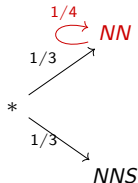
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$c(u)$	
*	6
NN	12
NNS	
VBP	
VBZ	
IN	
.	

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN	3					
NNS						
VBP						
VBZ						
IN						



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

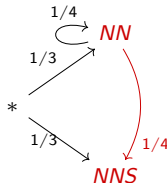
POS tagging

POS Taggers

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Joint transition matrix **A** and initial probabilities π :

$c(u)$		$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	6	*	2	2				
NN	12	NN	3	3				
NNS		NNS						
VBP		VBP						
VBZ		VBZ						
IN		IN						
.								



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN **time/NN flies/VBZ** before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

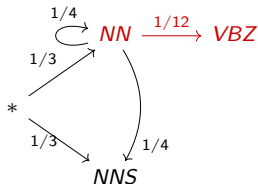
POS Taggers

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Joint transition matrix **A** and initial probabilities π :

$c(u)$	
*	6
NN	12
NNS	
VBP	
VBZ	
IN	
.	

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN	3	3		1		
NNS						
VBP						
VBZ						
IN						



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
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"time flies like horse flies ."

POS tagging

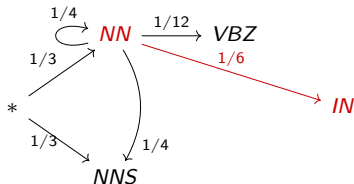
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*	6
NN	12
NNS	
VBP	
VBZ	
IN	
.	

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN	3	3		1	2	
NNS						
VBP						
VBZ						
IN						



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN **time/NN** ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN **sleep/NN** ./.
 flies/NNS smell/VBP an/DT arrow/NN **drink/NN** ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

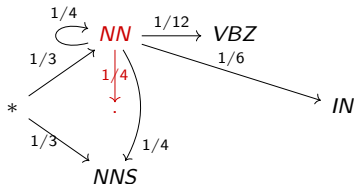
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$c(u)$	
*	6
NN	12
NNS	
VBP	
VBZ	
IN	
.	

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN	3	3		1	2	3
NNS						
VBP						
VBZ						
IN						



Exercise

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 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
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 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

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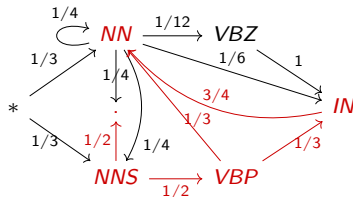
POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$	
*	6
NN	12
NNS	6
VBP	3
VBZ	1
IN	4
.	6

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN	3	3		1	2	3
NNS			3			3
VBP	1				1	
VBZ					1	
IN	3					



Exercise

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
 eat/VB breakfast/NN at/IN morning/NN time/NN ./.
 take/VB time/NN with/IN arrow/NN projects/NNS ./.
 dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
 flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
 bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies."

POS tagging

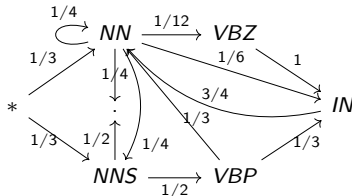
POS Taggers

Hidden Markov
Model

Joint transition matrix **A** and initial probabilities π :

$c(u)$	
*	6
NN	12
NNS	6
VBP	3
VBZ	1
IN	4
.	6

$c(u,v)$	NN	NNS	VBP	VBZ	IN	.
*	2	2				
NN	3	3		1	2	3
NNS			3			3
VBP	1				1	
VBZ					1	
IN	3					



Emission matrix **B**:

$c(u)$	
*	6
NN	12
NNS	6
VBP	3
VBZ	1
IN	4
.	6

$c(u, O_i)$	time	flies	like	horse	.
NN	3			1	
NNS		3			
VBP	1				
VBZ		1			
IN			1		
.					6

$b_{i,k}$	time	flies	like	horse.	.
NN	1/4			1/12	
NNS		1/2			
VBP	1/3				
VBZ		1			
IN			1/4		
.					1

How is the prob. of a POS-tag sequence computed?

Explanation probability:

Generative model (joint probabilities) instead of conditional model

$$P(X | O) = \frac{P(X, O)}{P(O)} \approx P(X, O)$$

$$P(X_1, \dots, X_T, O) = P(X_1, \dots, X_T) \cdot P(O | X_1 \dots X_T)$$

$$P(X_1, \dots, X_T) = \pi_{X_1} \prod_{t=2}^T a_{X_{t-1}X_t}$$

$$P(O | X_1 \dots X_T) = \prod_{t=1}^T b_{O_t X_t}$$

$$P(X_1, \dots, X_T, O) = \pi_{X_1} \cdot b_{O_1 X_1} \cdot \prod_{t=2}^T a_{X_{t-1} X_t} \cdot b_{O_t X_t}$$

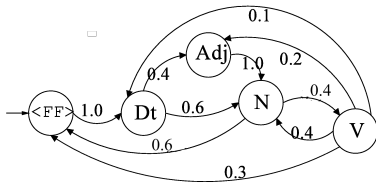
POS tagging

POS Taggers

Hidden Markov
Model

How is the prob. of a POS-tag sequence computed?

Following the previous example



b_{ik}	.	this	cat	eats	fish	...
<FF>	1.0					
Dt		0.4				
N			0.3		0.3	
V				0.5	0.1	
Adj						

$P(X, O) = P(X, ., this, cat, eats, fish) ?$ 7 possible X sequences

$X = \langle FF \rangle, Dt, Adj, N, \langle FF \rangle$

$X = \langle FF \rangle, Dt, Adj, N, V$

$X = \langle FF \rangle, Dt, N, \langle FF \rangle, Dt$

$X = \langle FF \rangle, Dt, N, V, \langle FF \rangle$

$X = \langle FF \rangle, Dt, N, V, N$

$$P(X, O) = (1 \cdot 1) \cdot (1 \cdot 0.4) \cdot (0.6 \cdot 0.3) \cdot (0.4 \cdot 0.5) \cdot (0.4 \cdot 0.3) = 0.001728$$

$X = \langle FF \rangle, Dt, N, V, Adj$

$X = \langle FF \rangle, Dt, N, V, Dt$

POS tagging

POS Taggers

Hidden Markov
Model

How is the best POS-tag sequence found?

POS tagging

POS Taggers

Hidden Markov
Model

We want to find

$$\hat{X} = \operatorname{argmax}_X P(X \mid O) \approx \operatorname{argmax}_X P(X, O)$$

- Brute force, $O(N^T)$

N states (POS tags) and T observations (word sequence length)

- Viterbi algorithm, dynamic programming, $O(T * N^2)$

Outline

POS tagging

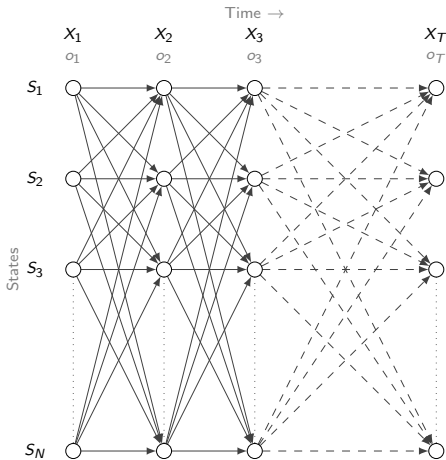
POS Taggers

Viterbi algorithm

- 1 POS tagging
 - Goal and motivation
 - Part of Speech categories

- 2 POS Taggers
 - Stochastic taggers
 - Hidden Markov Model
 - Viterbi algorithm

Viterbi algorithm



Auxiliary structure:

TRELLIS of a fully connected HMM

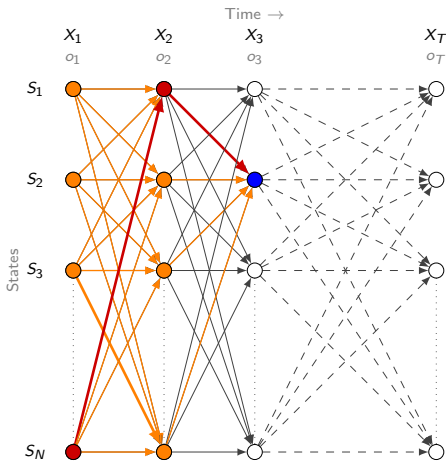
Node (S_i, X_t) stores information related to state S_i about the partial sequence from X_1 to X_t : $\delta_t(i)$ and $\varphi_t(i)$

POS tagging

POS Taggers

Viterbi algorithm

Viterbi algorithm



Auxiliary structure:

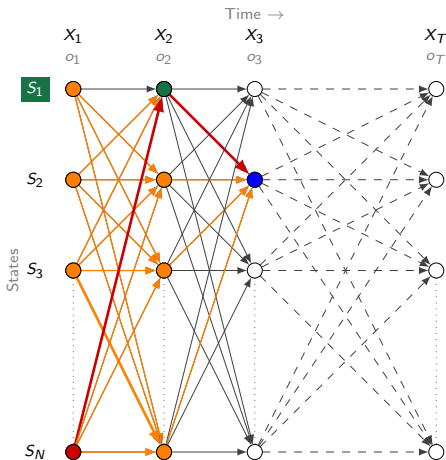
TRELLIS of a fully connected HMM

Node (S_i, X_t) stores information related to state S_i about the partial sequence from X_1 to X_t : $\delta_t(i)$ and $\varphi_t(i)$

$$\delta_t(i) = \max_{X_1, \dots, X_{t-1}} P(X_1, \dots, X_{t-1}, X_t = S_i, O)$$

POS tagging
POS Taggers
Viterbi algorithm

Viterbi algorithm



Auxiliary structure:

TRELLIS of a fully connected HMM

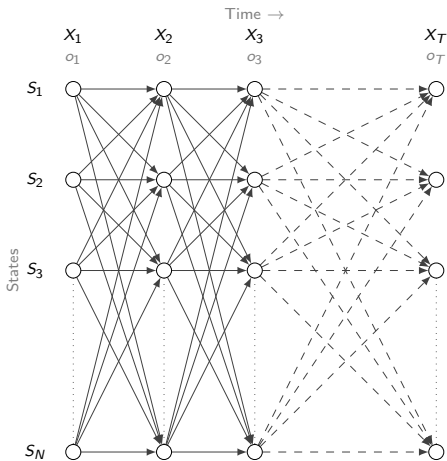
Node (S_i, X_t) stores information related to state S_i about the partial sequence from X_1 to X_t : $\delta_t(i)$ and $\varphi_t(i)$

$$\delta_t(i) = \max_{X_1, \dots, X_{t-1}} P(X_1, \dots, X_{t-1}, X_t = S_i, O)$$

$$\varphi_t(i) = \text{last}(\arg\max_{X_1, \dots, X_{t-1}} P(X_1, \dots, X_{t-1}, X_t = S_i, O))$$

POS tagging
POS Taggers
Viterbi algorithm

Viterbi algorithm



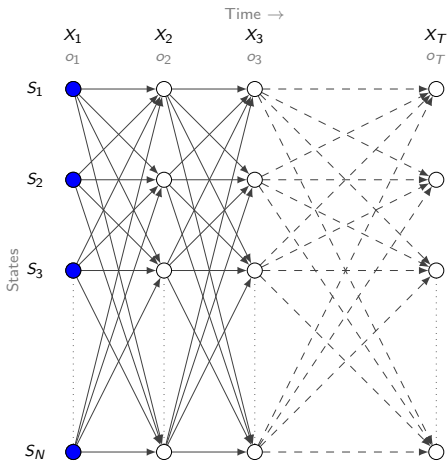
Algorithm:

POS tagging

POS Taggers

Viterbi algorithm

Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

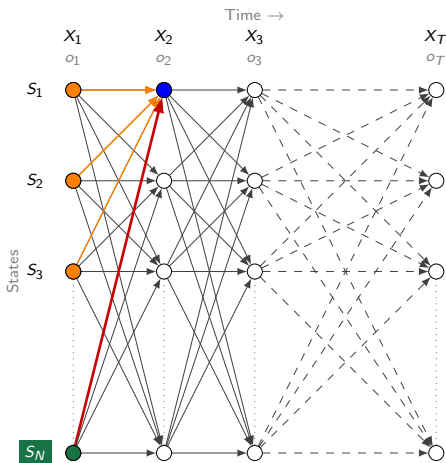
$$\delta_1(j) = \pi_j * b_{j,o_1}$$

POS tagging

POS Taggers

Viterbi algorithm

Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

$$\delta_1(j) = \pi_j * b_{j,o_1}$$

2. Inference step:

$$\forall t = 2 \dots T : \quad \forall j = 1 \dots N :$$

$$\delta_t(j) = \max_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

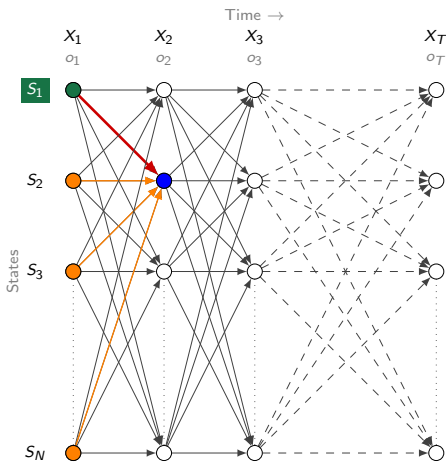
$$\varphi_t(j) = \operatorname{argmax}_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j})$$

POS tagging

POS Taggers

Viterbi algorithm

Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

$$\delta_1(j) = \pi_j * b_{j,o_1}$$

2. Inference step:

$$\forall t = 2 \dots T : \quad \forall j = 1 \dots N :$$

$$\delta_t(j) = \max_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

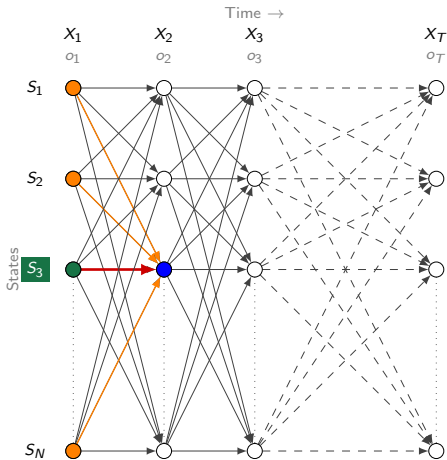
$$\varphi_t(j) = \operatorname{argmax}_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j})$$

POS tagging

POS Taggers

Viterbi algorithm

Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

$$\delta_1(j) = \pi_j * b_{j,o_1}$$

2. Inference step:

$$\forall t = 2 \dots T : \quad \forall j = 1 \dots N :$$

$$\delta_t(j) = \max_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

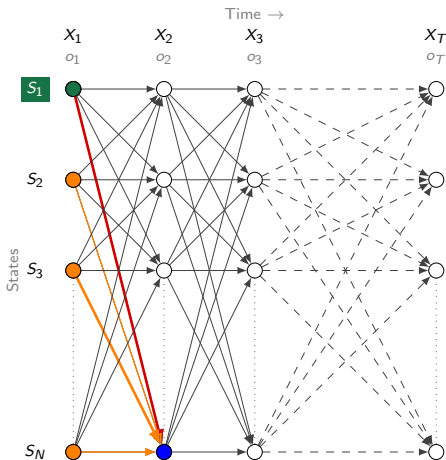
$$\varphi_t(j) = \operatorname{argmax}_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j})$$

POS tagging

POS Taggers

Viterbi algorithm

Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

$$\delta_1(j) = \pi_j * b_{j,o_1}$$

2. Inference step:

$$\forall t = 2 \dots T : \quad \forall j = 1 \dots N :$$

$$\delta_t(j) = \max_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

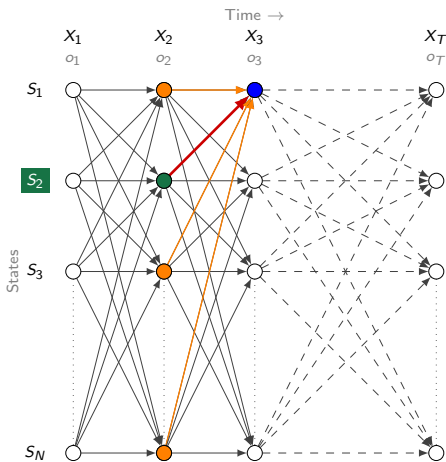
$$\varphi_t(j) = \operatorname{argmax}_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j})$$

POS tagging

POS Taggers

Viterbi algorithm

Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

$$\delta_1(j) = \pi_j * b_{j,o_1}$$

2. Inference step:

$$\forall t = 2 \dots T : \quad \forall j = 1 \dots N :$$

$$\delta_t(j) = \max_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

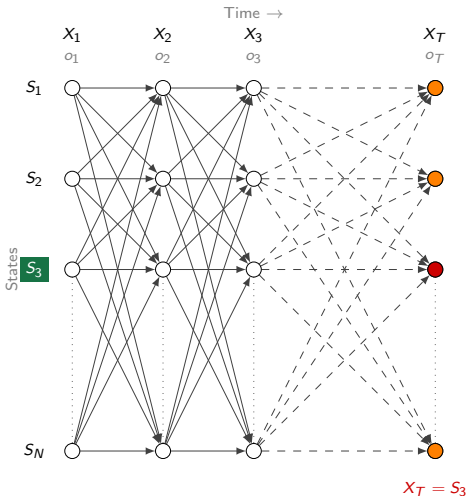
$$\varphi_t(j) = \operatorname{argmax}_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j})$$

POS tagging

POS Taggers

Viterbi algorithm

Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N : \\ \delta_1(j) = \pi_j * b_{j,o_1}$$

2. Inference step:

$$\forall t = 2 \dots T : \quad \forall j = 1 \dots N :$$

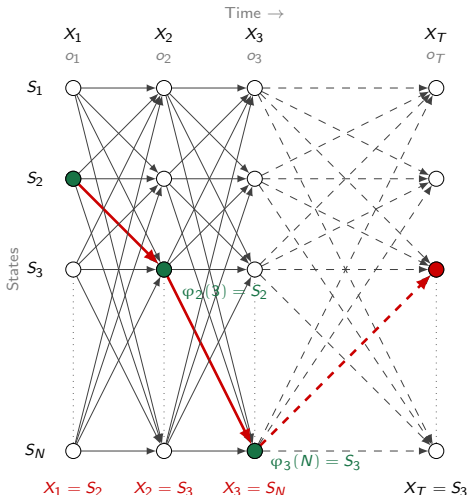
$$\delta_t(j) = \max_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

$$\varphi_t(j) = \operatorname{argmax}_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j})$$

3. Termination step:

$$X_T = \operatorname{argmax}_i (\delta_T(i))$$

Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

$$\delta_1(j) = \pi_j * b_{j,o_1}$$

2. Inference step:

$$\forall t = 2 \dots T : \quad \forall j = 1 \dots N :$$

$$\delta_t(j) = \max_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

$$\varphi_t(j) = \operatorname{argmax}_{1 \leq i \leq N} (\delta_{t-1}(i) * a_{i,j})$$

3. Termination step:

$$X_T = \operatorname{argmax}_i (\delta_T(i))$$

4. Backward path readout step:

$$\forall t = 1 \dots T - 1 :$$

$$X_t = \varphi_{t+1}(X_{t+1})$$

POS tagging

POS Taggers

Viterbi algorithm

Exercise

Apply Viterbi algorithm using the following HMM to

The	kid	fishes	fish
DT	NN	NNS	NN
			NNS
	JJ	VBZ	VBP

POS tagging

POS Taggers

Viterbi algorithm

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

B	the	big	kid	fish	time	fishes	times
DT	1						
JJ		0.8	0.2				
NN			0.3	0.4	0.3		
NNS				0.3		0.4	0.3
VBZ						0.6	0.4
VBP				0.7	0.3		

Viterbi algorithm

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

$$O_4 = \text{fish}$$
$$\begin{aligned} \bar{S}_1 &= \bar{D}\bar{T} \\ \bar{S}_2 &= \bar{J}\bar{J} \\ \bar{S}_3 &= \bar{N}\bar{N} \\ \bar{S}_4 &= \bar{N}\bar{N}\bar{S} \\ \bar{S}_5 &= \bar{V}\bar{B}\bar{Z} \\ \bar{S}_6 &= \bar{V}\bar{B}\bar{P} \end{aligned}$$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

$O_1 = \text{the } X_1$

$$O_2 = \text{kid}$$
$$O_3 = \text{fishes}$$
$$O_4 = \text{fish}$$
[illegible]

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ	0	0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$			
	$\delta = 0.4$			
$\bar{s}_2 = \text{JJ}$	$\bar{\pi} = 0.2$ $\bar{b} = 0$			
	$\delta = 0$			
$\bar{s}_3 = \text{NN}$				
$\bar{s}_4 = \text{NNS}$				
$\bar{s}_5 = \text{VBZ}$				
$\bar{s}_6 = \text{VBP}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	0
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN	0	0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$ $\delta = 0.4$			
$\bar{s}_2 = \text{JJ}$				
$\bar{s}_3 = \text{NN}$	$\bar{\pi} = 0$ $\bar{b} = 0$ $\delta = 0$			
$\bar{s}_4 = \text{NNS}$				
$\bar{s}_5 = \text{VBZ}$				
$\bar{s}_6 = \text{VBP}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS	0		0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$	$\bar{\pi} = 0.4$	$\bar{b} = 1$		
	$\delta = 0.4$			
$\bar{s}_2 = \text{JJ}$				
$\bar{s}_3 = \text{NN}$				
$\bar{s}_4 = \text{NNS}$	$\bar{\pi} = 0.3$	$\bar{b} = 0$		
	$\delta = 0$			
$\bar{s}_5 = \text{VBZ}$				
$\bar{s}_6 = \text{VBP}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	0
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ	0			0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$			
$\bar{s}_2 = \text{JJ}$	$\delta = 0.4$			
$\bar{s}_3 = \text{NN}$				
$\bar{s}_4 = \text{NNS}$				
$\bar{s}_5 = \text{VBZ}$	$\bar{\pi} = 0$ $\bar{b} = 0$			
$\bar{s}_6 = \text{VBP}$	$\delta = 0$			

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP	0		0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$			
	$\delta = 0.4$			
$\bar{s}_2 = \text{JJ}$				
$\bar{s}_3 = \text{NN}$				
$\bar{s}_4 = \text{NNS}$				
$\bar{s}_5 = \text{VBZ}$				
$\bar{s}_6 = \text{VBP}$	$\bar{\pi} = 0.1$ $\bar{b} = 0$			
	$\delta = 0$			

Viterbi algorithm

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

$$O_4 = \text{fish}$$
[illegible]

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT	0	0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

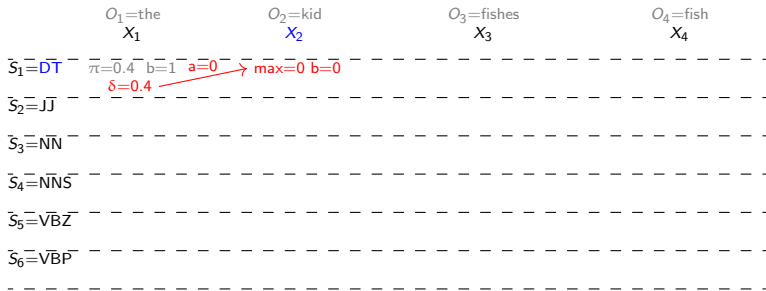
B	the	kid	fish	fishes
DT	1	0		
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$



Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ	0		0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

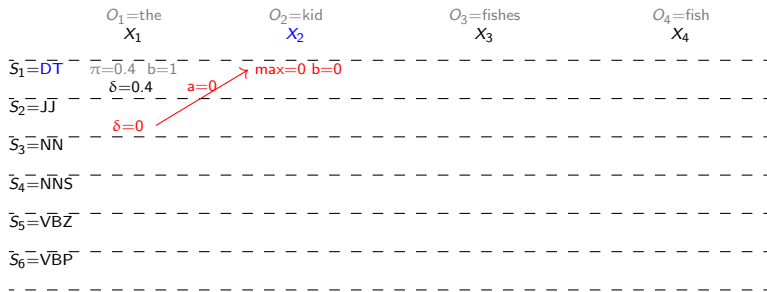
B	the	kid	fish	fishes
DT	1	0		
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$



Exercise

POS tagging

POS Taggers

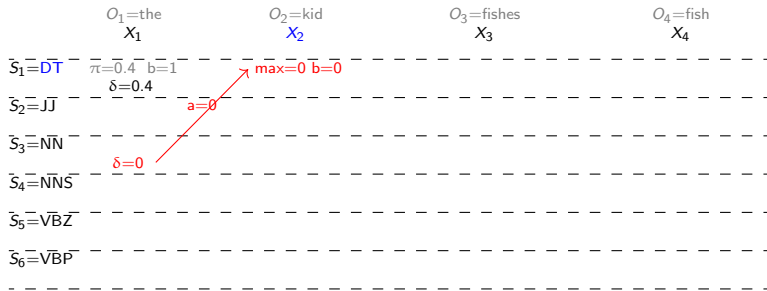
Viterbi algorithm

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN	0			0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1	0		
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$



- POS tagging
- POS Taggers
 - Viterbi algorithm

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS	0					1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1	0		
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

Diagram illustrating a 2D lattice structure with 6 rows and 4 columns. The columns are labeled $O_1 = \text{the } X_1$, $O_2 = \text{kid } X_2$, $O_3 = \text{fishes } X_3$, and $O_4 = \text{fish } X_4$. The rows are labeled $S_1 = \text{D } T$, $S_2 = \text{J } J$, $S_3 = \text{N } N$, $S_4 = \text{N } N \text{ S}$, $S_5 = \text{V } B \text{ Z}$, and $S_6 = \text{V } B \text{ P}$. A red arrow points from the intersection of S_4 and X_1 to the intersection of S_1 and X_2 . Labels along the arrow include $a=0$, $\delta=0$, and $\text{max}=0 \text{ b}=0$. Other labels include $\pi=0.4 \text{ b}=1$ and $\delta=0.4$ near the top left.

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

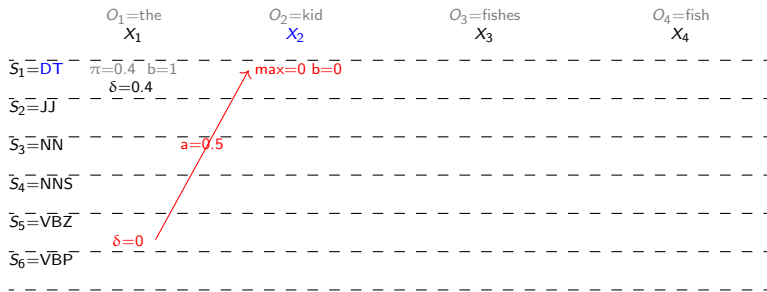
B	the	kid	fish	fishes
DT	1	0		
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{ij}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{ij})$



Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

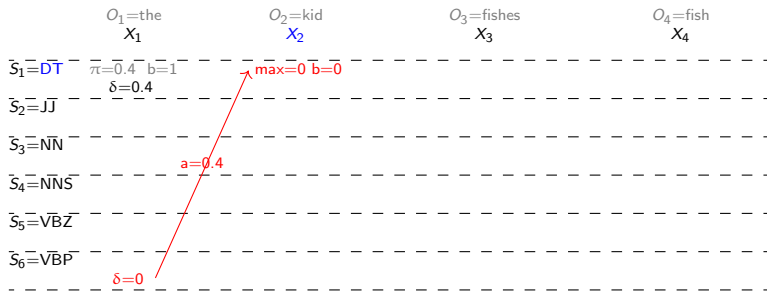
B	the	kid	fish	fishes
DT	1	0		
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$



Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_2 = \text{JJ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_3 = \text{NN}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_4 = \text{NNS}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$
$\bar{s}_5 = \text{VBZ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_6 = \text{VBP}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_7 = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_8 = \text{JJ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$
$\bar{s}_9 = \text{NN}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{10} = \text{NNS}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{11} = \text{VBZ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{12} = \text{VBP}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$
$\bar{s}_{13} = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{14} = \text{JJ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{15} = \text{NN}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{16} = \text{NNS}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$
$\bar{s}_{17} = \text{VBZ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{18} = \text{VBP}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{19} = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{20} = \text{JJ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$
$\bar{s}_{21} = \text{NN}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{22} = \text{NNS}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{23} = \text{VBZ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{24} = \text{VBP}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$
$\bar{s}_{25} = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{26} = \text{JJ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{27} = \text{NN}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{28} = \text{NNS}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$
$\bar{s}_{29} = \text{VBZ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{30} = \text{VBP}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{31} = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{32} = \text{JJ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$
$\bar{s}_{33} = \text{NN}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{34} = \text{NNS}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{35} = \text{VBZ}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{s}_{36} = \text{VBP}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$		
$\bar{S}_2 = \text{JJ}$	$\delta = 0.4$ $a = 0.2$	$\max = 0.08$ $b = 0.2$	
$\bar{S}_3 = \text{NN}$			
$\bar{S}_4 = \text{NNS}$			
$\bar{S}_5 = \text{VBZ}$			
$\bar{S}_6 = \text{VBP}$			

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ		0	0.8	0.2		
NN		0		0.1	0.9	
NNS		0				1
VBZ	0.5	0	0.2	0.3		
VBP	0.4	0	0.4	0.2		

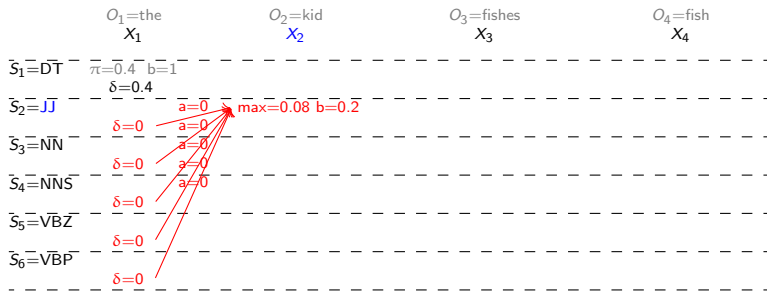
B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$



Viterbi algorithm

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

$$O_4 = \text{fish}$$

$\bar{S}_1=\overline{DT}$	$\pi=0.4$	$b=1$
$\bar{S}_2=\overline{JJ}$	$\delta=0.4$	
	$\max=0.08$	$b=0.2$
$\bar{S}_3=\overline{NN}$	$\delta=0.016,$	$\varphi=D\overline{T}$
$\bar{S}_4=\overline{NNS}$		
$\bar{S}_5=\overline{VBZ}$		
$\bar{S}_6=\overline{VBP}$		

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{i,j})$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$			
$\bar{S}_2 = \text{JJ}$	$\delta = 0.4$	$\text{max} = 0.08$ $\bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$	$a = 0.5$	$\delta = 0.016$ $\varphi = \text{DT}$	$\text{max} = 0.2$ $\bar{b} = 0.3$	
$\bar{S}_4 = \text{NNS}$				
$\bar{S}_5 = \text{VBZ}$				
$\bar{S}_6 = \text{VBP}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{ij}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{ij})$

	$O_1=\text{the}$ X_1	$O_2=\text{kid}$ X_2	$O_3=\text{fishes}$ X_3	$O_4=\text{fish}$ X_4
$\bar{S}_1=\bar{\text{DT}}$	$\bar{\pi}=0.4$ $\bar{b}=1$			
$\bar{S}_2=\bar{\text{JJ}}$	$\delta=0.4$	$\bar{\max}=0.08$ $\bar{b}=0.2$		
$\bar{S}_3=\bar{\text{NN}}$		$\delta=0.016$ $\varphi=\text{DT}$	$\bar{\max}=0.2$ $\bar{b}=0.3$	
$\bar{S}_4=\bar{\text{NNS}}$		$\delta=0.06$ $\varphi=\text{DT}$		
$\bar{S}_5=\bar{\text{VBZ}}$				
$\bar{S}_6=\bar{\text{VBP}}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

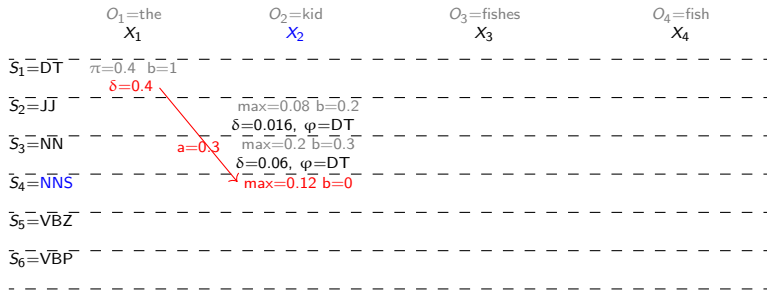
B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS		0	0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{ij}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{ij})$



Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

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Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$			
	$\delta = 0.4$		
$\bar{s}_2 = \text{JJ}$	$\bar{\max} = 0.08$ $\bar{b} = 0.2$		
	$\delta = 0.016$, $\varphi = \text{DT}$		
$\bar{s}_3 = \text{NN}$	$\bar{\max} = 0.2$ $\bar{b} = 0.3$		
	$\delta = 0.06$, $\varphi = \text{DT}$		
$\bar{s}_4 = \text{NNS}$	$\bar{\max} = 0.12$ $\bar{b} = 0$		
	$\delta = 0$, $\varphi = \text{DT}$		
$\bar{s}_5 = \text{VBZ}$			
$\bar{s}_6 = \text{VBP}$			

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3	0	
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

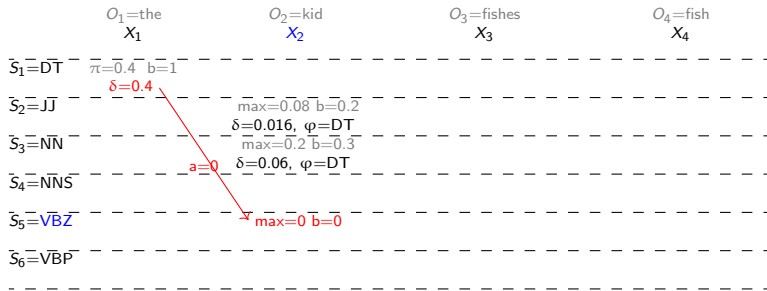
B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ		0		0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{ij}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{ij})$



Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$		
$\bar{S}_2 = \text{JJ}$	$\delta = 0.4$	$\bar{\max} = 0.08$ $\bar{b} = 0.2$	
$\bar{S}_3 = \text{NN}$	$\delta = 0.016$, $\varphi = \text{DT}$	$\bar{\max} = 0.2$ $\bar{b} = 0.3$	
$\bar{S}_4 = \text{NNS}$	$\delta = 0.06$, $\varphi = \text{DT}$		
$\bar{S}_5 = \text{VBZ}$	$\bar{\max} = 0$ $\bar{b} = 0$		
$\bar{S}_6 = \text{VBP}$	$\delta = 0$, $\varphi = \text{DT}$		

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		0
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

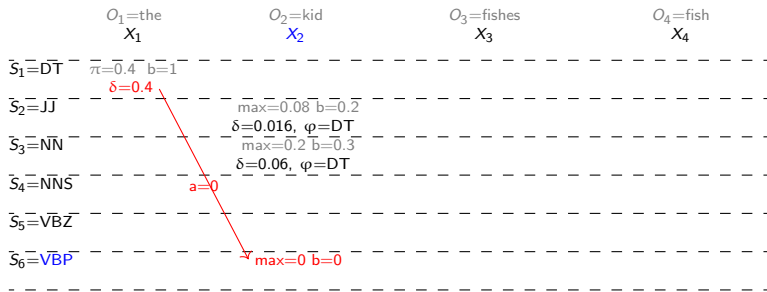
B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP		0	0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{i,j})$



Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{ij}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{ij})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$		
	$\delta = 0.4$		
$\bar{S}_2 = \text{JJ}$		$\bar{\max} = 0.08$ $\bar{b} = 0.2$	
		$\delta = 0.016$, $\varphi = \text{DT}$	
$\bar{S}_3 = \text{NN}$		$\bar{\max} = 0.2$ $\bar{b} = 0.3$	
		$\delta = 0.06$, $\varphi = \text{DT}$	
$\bar{S}_4 = \text{NNS}$			
$\bar{S}_5 = \text{VBZ}$			
$\bar{S}_6 = \text{VBP}$		$\bar{\max} = 0$ $\bar{b} = 0$	
		$\delta = 0$, $\varphi = \text{DT}$	

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			0
JJ		0.2		0
NN		0.3	0.4	0
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	0

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{i,j})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$	$\bar{b} = 0$	
$\bar{s}_2 = \text{JJ}$	$\delta = 0.4$	$\delta = 0, \varphi = \text{DT}$	
$\bar{s}_3 = \text{NN}$	$\max = 0.08$ $\bar{b} = 0.2$	$\bar{b} = 0$	
$\bar{s}_4 = \text{NNS}$	$\delta = 0.016, \varphi = \text{DT}$	$\delta = 0, \varphi = \text{DT}$	
	$\max = 0.2$ $\bar{b} = 0.3$	$\bar{b} = 0$	
	$\delta = 0.06, \varphi = \text{DT}$	$\delta = 0, \varphi = \text{DT}$	
$\bar{s}_5 = \text{VBZ}$			
$\bar{s}_6 = \text{VBP}$		$\bar{b} = 0$	
		$\delta = 0, \varphi = \text{DT}$	

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{ij}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{ij})$

	$O_1=\text{the}$ X_1	$O_2=\text{kid}$ X_2	$O_3=\text{fishes}$ X_3	$O_4=\text{fish}$ X_4
$\bar{S}_1=\bar{\text{DT}}$	$\bar{\pi}=0.4$ $\bar{b}=1$			
$\bar{S}_2=\bar{\text{JJ}}$	$\delta=0.4$	$\bar{\max}=0.08$ $\bar{b}=0.2$		
$\bar{S}_3=\bar{\text{NN}}$		$\delta=0.016$, $\varphi=\text{DT}$ $\bar{\max}=0.2$ $\bar{b}=0.3$	$\bar{a}=0.2$	
$\bar{S}_4=\bar{\text{NNS}}$		$\delta=0.06$, $\varphi=\text{DT}$	$\bar{a}=0.1$	$\bar{\max}=0.006$ $\bar{b}=0.4$
$\bar{S}_5=\bar{\text{VBZ}}$				
$\bar{S}_6=\bar{\text{VBP}}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4, \bar{b} = 1$			
$\bar{S}_2 = \text{JJ}$	$\delta = 0.4$	$\bar{\max} = 0.08, \bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$		$\delta = 0.016, \varphi = \text{DT}$		
$\bar{S}_4 = \text{NNS}$		$\bar{\max} = 0.2, \bar{b} = 0.3$		
		$\delta = 0.06, \varphi = \text{DT}$	$\delta = 0.1$	
$\bar{S}_5 = \text{VBZ}$			$\bar{\max} = 0.006, \bar{b} = 0.4$	
			$\delta = 0.0024, \varphi = \text{NN}$	
$\bar{S}_6 = \text{VBP}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2	0	
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$			
$\bar{S}_2 = \text{JJ}$	$\delta = 0.4$	$\max = 0.08$ $\bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$		$\delta = 0.016$, $\varphi = \text{DT}$		
$\bar{S}_4 = \text{NNS}$		$\max = 0.2$ $\bar{b} = 0.3$		
$\bar{S}_5 = \text{VBZ}$		$\delta = 0.06$, $\varphi = \text{DT}$	$\max = 0.006$ $\bar{b} = 0.4$	
$\bar{S}_6 = \text{VBP}$			$\delta = 0.0024$, $\varphi = \text{NN}$	
			$\max = 0.054$ $\bar{b} = 0.6$	

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4, \bar{b} = 1$ $\delta = 0.4$			
$\bar{S}_2 = \text{JJ}$		$\bar{\max} = 0.08, \bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$		$\delta = 0.016, \varphi = \text{DT}$ $\bar{\max} = 0.2, \bar{b} = 0.3$		
$\bar{S}_4 = \text{NNS}$		$\delta = 0.06, \varphi = \text{DT}$		
$\bar{S}_5 = \text{VBZ}$			$\bar{\max} = 0.006, \bar{b} = 0.4$ $\delta = 0.0024, \varphi = \text{NN}$ $\bar{\max} = 0.054, \bar{b} = 0.6$	
$\bar{S}_6 = \text{VBP}$			$\delta = 0.0324, \varphi = \text{NN}$	

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1		0	
JJ		0.2	0	
NN		0.3	0.4	
NNS			0.3	0.4
VBZ			0	0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

	$O_1=\text{the}$ X_1	$O_2=\text{kid}$ X_2	$O_3=\text{fishes}$ X_3	$O_4=\text{fish}$ X_4
$\bar{S}_1=\text{DT}$	$\bar{\pi}=0.4$ $\bar{b}=1$			$\bar{b}=0$
$\bar{S}_2=\text{JJ}$	$\delta=0.4$	$\bar{\max}=0.08$ $\bar{b}=0.2$		$\delta=0, \varphi=\text{DT}$
$\bar{S}_3=\text{NN}$		$\delta=0.016, \varphi=\text{DT}$		$\bar{b}=0$
		$\bar{\max}=0.2$ $\bar{b}=0.3$		$\delta=0, \varphi=\text{DT}$
$\bar{S}_4=\text{NNS}$		$\delta=0.06, \varphi=\text{DT}$		
			$\bar{\max}=0.006$ $\bar{b}=0.4$	
$\bar{S}_5=\text{VBZ}$			$\delta=0.0024, \varphi=\text{NN}$	
			$\bar{\max}=0.054$ $\bar{b}=0.6$	$\bar{b}=0$
$\bar{S}_6=\text{VBP}$			$\delta=0.0324, \varphi=\text{NN}$	$\delta=0, \varphi=\text{DT}$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS			0			1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{i,j})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$			
$\bar{S}_2 = \text{JJ}$ $\delta = 0.4$	$\bar{\max} = 0.08$ $\bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$	$\delta = 0.016$, $\varphi = \text{DT}$ $\bar{\max} = 0.2$ $\bar{b} = 0.3$		
$\bar{S}_4 = \text{NNS}$	$\delta = 0.06$, $\varphi = \text{DT}$	$\bar{\max} = 0.006$ $\bar{b} = 0.4$	
$\bar{S}_5 = \text{VBZ}$		$\delta = 0.0024$, $\varphi = \text{NN}$ $\bar{\max} = 0.054$ $\bar{b} = 0.6$	
$\bar{S}_6 = \text{VBP}$		$\delta = 0.0324$, $\varphi = \text{NN}$	$\bar{\max} = 0.0065$ $\bar{b} = 0.4$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{i,j})$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$ $\delta = 0.4$			
$\bar{S}_2 = \text{JJ}$		$\bar{\max} = 0.08$ $\bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$		$\delta = 0.016$, $\varphi = \text{DT}$ $\bar{\max} = 0.2$ $\bar{b} = 0.3$		
$\bar{S}_4 = \text{NNS}$		$\delta = 0.06$, $\varphi = \text{DT}$		
$\bar{S}_5 = \text{VBZ}$			$\bar{\max} = 0.006$ $\bar{b} = 0.4$ $\delta = 0.0024$, $\varphi = \text{NN}$	
$\bar{S}_6 = \text{VBP}$			$\bar{\max} = 0.054$ $\bar{b} = 0.6$ $\delta = 0.0324$ $\varphi = \text{NN}$	$\bar{\max} = 0.0065$ $\bar{b} = 0.4$ $\delta = 0.0026$, $\varphi = \text{VBZ}$

$a = 0.2$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS				0		1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{i,j})$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4, \bar{b} = 1$ $\delta = 0.4$			
$\bar{S}_2 = \text{JJ}$		$\bar{\max} = 0.08, \bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$		$\delta = 0.016, \varphi = \text{DT}$ $\bar{\max} = 0.2, \bar{b} = 0.3$		
$\bar{S}_4 = \text{NNS}$		$\delta = 0.06, \varphi = \text{DT}$		$\bar{\max} = 0.0065, \bar{b} = 0.4$
$\bar{S}_5 = \text{VBZ}$			$\bar{\max} = 0.006, \bar{b} = 0.4$ $\delta = 0.0024, \varphi = \text{NN}$ $\bar{\max} = 0.054, \bar{b} = 0.6$ $\delta = 0.0324, \varphi = \text{NN}$	$\delta = 0.0026, \varphi = \text{VBZ}$ $\bar{\max} = 0.0097, \bar{b} = 0.3$
$\bar{S}_6 = \text{VBP}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{i,j})$

	$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4, \bar{b} = 1$ $\delta = 0.4$			
$\bar{S}_2 = \text{JJ}$		$\bar{\max} = 0.08, \bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$		$\delta = 0.016, \varphi = \text{DT}$ $\bar{\max} = 0.2, \bar{b} = 0.3$		
$\bar{S}_4 = \text{NNS}$		$\delta = 0.06, \varphi = \text{DT}$		$\bar{\max} = 0.0065, \bar{b} = 0.4$ $\delta = 0.0026, \varphi = \text{VBZ}$
$\bar{S}_5 = \text{VBZ}$			$\bar{\max} = 0.006, \bar{b} = 0.4$ $\delta = 0.0024, \varphi = \text{NN}$ $\bar{\max} = 0.054, \bar{b} = 0.6$ $\delta = 0.0324, \varphi = \text{NN}$	$\bar{\max} = 0.0097, \bar{b} = 0.3$ $\delta = 0.0029, \varphi = \text{VBZ}$
$\bar{S}_6 = \text{VBP}$				

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		0
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \arg\max(\delta_{t-1}(i) * a_{i,j})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$ $\bar{\pi} = 0.4$ $\bar{b} = 1$			
$\bar{S}_2 = \text{JJ}$ $\delta = 0.4$			
	$\bar{\max} = 0.08$ $\bar{b} = 0.2$		
$\bar{S}_3 = \text{NN}$	$\delta = 0.016$, $\varphi = \text{DT}$		
	$\bar{\max} = 0.2$ $\bar{b} = 0.3$		
$\bar{S}_4 = \text{NNS}$	$\delta = 0.06$, $\varphi = \text{DT}$		
		$\bar{\max} = 0.0065$ $\bar{b} = 0.4$	
$\bar{S}_5 = \text{VBZ}$		$\delta = 0.0024$, $\varphi = \text{NN}$	$\delta = 0.0026$, $\varphi = \text{VBZ}$
		$\bar{\max} = 0.054$ $\bar{b} = 0.6$	$\bar{\max} = 0.0097$ $\bar{b} = 0.3$
$\bar{S}_6 = \text{VBP}$		$\delta = 0.0324$ $\varphi = \text{NN}$	$\delta = 0.0029$, $\varphi = \text{VBZ}$
			$\bar{\max} = 0.0024$ $\bar{b} = 0.7$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Inference step: $\delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$ $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$

	$O_1=\text{the}$ X_1	$O_2=\text{kid}$ X_2	$O_3=\text{fishes}$ X_3	$O_4=\text{fish}$ X_4
$\bar{S}_1=\bar{\text{DT}}$	$\bar{\pi}=0.4$ $\bar{b}=1$ $\delta=0.4$			
$\bar{S}_2=\text{JJ}$		$\bar{\max}=0.08$ $\bar{b}=0.2$		
$\bar{S}_3=\bar{\text{NN}}$		$\delta=0.016$, $\varphi=\text{DT}$ $\bar{\max}=0.2$ $\bar{b}=0.3$		
$\bar{S}_4=\text{NNS}$		$\delta=0.06$, $\varphi=\text{DT}$		$\bar{\max}=0.0065$ $\bar{b}=0.4$ $\delta=0.0026$, $\varphi=\text{VBZ}$
$\bar{S}_5=\bar{\text{VBZ}}$			$\bar{\max}=0.006$ $\bar{b}=0.4$ $\delta=0.0024$, $\varphi=\text{NN}$ $\bar{\max}=0.054$ $\bar{b}=0.6$ $\delta=0.0324$ $\varphi=\text{NN}$	$\bar{\max}=0.0097$ $\bar{b}=0.3$ $\delta=0.0029$, $\varphi=\text{VBZ}$
$\bar{S}_6=\bar{\text{VBP}}$				$\bar{\max}=0.0024$ $\bar{b}=0.7$ $\delta=0.0017$, $\varphi=\text{NNS}$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Termination step: $X_4 = \operatorname{argmax}_i(\delta_4(i))$

$O_1=\text{the}$ X_1	$O_2=\text{kid}$ X_2	$O_3=\text{fishes}$ X_3	$O_4=\text{fish}$ X_4
$\bar{s}_1=\bar{\text{DT}}$	$\bar{\pi}=0.4 \quad \bar{b}=1$		
$\bar{s}_2=\bar{\text{JJ}}$	$\delta=0.4$		
$\bar{s}_3=\bar{\text{NN}}$	$\max=0.08 \quad \bar{b}=0.2$		
$\bar{s}_4=\bar{\text{NNS}}$	$\delta=0.016, \varphi=\text{DT}$		
$\bar{s}_5=\bar{\text{VBZ}}$	$\max=0.2 \quad \bar{b}=0.3$		
$\bar{s}_6=\bar{\text{VBP}}$	$\delta=0.06, \varphi=\text{DT}$		
		$\max=0.006 \quad \bar{b}=0.4$	
		$\delta=0.0024, \varphi=\text{NN}$	
		$\max=0.054 \quad \bar{b}=0.6$	
		$\delta=0.0324, \varphi=\text{NN}$	
			$\max=0.0065 \quad \bar{b}=0.4$
			$\delta=0.0026, \varphi=\text{VBZ}$
			$\max=0.0097 \quad \bar{b}=0.3$
			$\delta=0.0029, \varphi=\text{VBZ}$
			$\max=0.0024 \quad \bar{b}=0.7$
			$\delta=0.0017, \varphi=\text{NNS}$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Termination step: $X_4 = \operatorname{argmax}_i(\delta_4(i))$

$O_1=\text{the}$ X_1	$O_2=\text{kid}$ X_2	$O_3=\text{fishes}$ X_3	$O_4=\text{fish}$ X_4
$\bar{s}_1=\bar{\text{DT}}$	$\bar{\pi}=0.4 \quad \bar{b}=1$		
$\bar{s}_2=\bar{\text{JJ}}$	$\delta=0.4$		
$\bar{s}_3=\bar{\text{NN}}$	$\bar{\text{max}}=0.08 \quad \bar{b}=0.2$		
	$\delta=0.016, \varphi=\text{DT}$		
$\bar{s}_4=\bar{\text{NNS}}$	$\bar{\text{max}}=0.2 \quad \bar{b}=0.3$		
	$\delta=0.06, \varphi=\text{DT}$		
$\bar{s}_5=\bar{\text{VBZ}}$		$\bar{\text{max}}=0.006 \quad \bar{b}=0.4$	
		$\delta=0.0024, \varphi=\text{NN}$	
$\bar{s}_6=\bar{\text{VBP}}$		$\bar{\text{max}}=0.054 \quad \bar{b}=0.6$	
		$\delta=0.0324 \quad \varphi=\text{NN}$	
			$\bar{\text{max}}=0.0065 \quad \bar{b}=0.4$
			$\delta=0.0026, \varphi=\text{VBZ}$
			$\bar{\text{max}}=0.0097 \quad \bar{b}=0.3$
			$\delta=0.0029, \varphi=\text{VBZ}$
			$\bar{\text{max}}=0.0024 \quad \bar{b}=0.7$
			$\delta=0.0017, \varphi=\text{NNS}$

$X_4 = \text{NNS}$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Backward path readout step: $X_t = \varphi(X_{t+1})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \text{DT}$	$\bar{\pi} = 0.4 \quad \bar{b} = 1$		
	$\delta = 0.4$		
$\bar{s}_2 = \text{JJ}$	$\bar{\max} = 0.08 \quad \bar{b} = 0.2$		
	$\delta = 0.016, \varphi = \text{DT}$		
$\bar{s}_3 = \text{NN}$	$\bar{\max} = 0.2 \quad \bar{b} = 0.3$		$\bar{\max} = 0.0065 \quad \bar{b} = 0.4$
	$\delta = 0.06, \varphi = \text{DT}$		$\delta = 0.0026, \varphi = \text{VBZ}$
$\bar{s}_4 = \text{NNS}$		$\bar{\max} = 0.006 \quad \bar{b} = 0.4$	$\bar{\max} = 0.0097 \quad \bar{b} = 0.3$
		$\delta = 0.0024, \varphi = \text{NN}$	$\delta = 0.0029, \varphi = \text{VBZ}$
$\bar{s}_5 = \text{VBZ}$		$\bar{\max} = 0.054 \quad \bar{b} = 0.6$	
		$\delta = 0.0324 \quad \varphi = \text{NN}$	
$\bar{s}_6 = \text{VBP}$			$\bar{\max} = 0.0024 \quad \bar{b} = 0.7$
			$\delta = 0.0017, \varphi = \text{NNS}$
		$X_3 = \text{VBZ}$	$X_4 = \text{NNS}$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Backward path readout step: $X_t = \varphi(X_{t+1})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{S}_1 = \text{DT}$	$\bar{\pi} = 0.4$ $\bar{b} = 1$		
	$\delta = 0.4$		
$\bar{S}_2 = \text{JJ}$	$\bar{\max} = 0.08$ $\bar{b} = 0.2$		
	$\delta = 0.016$, $\varphi = \text{DT}$		
$\bar{S}_3 = \text{NN}$	$\bar{\max} = 0.2$ $\bar{b} = 0.3$		
	$\delta = 0.06$, $\varphi = \text{DT}$		
$\bar{S}_4 = \text{NNS}$		$\bar{\max} = 0.006$ $\bar{b} = 0.4$	$\bar{\max} = 0.0065$ $\bar{b} = 0.4$
		$\delta = 0.0024$, $\varphi = \text{NN}$	$\delta = 0.0029$, $\varphi = \text{VBZ}$
$\bar{S}_5 = \text{VBZ}$		$\bar{\max} = 0.054$ $\bar{b} = 0.6$	
		$\delta = 0.0324$ $\varphi = \text{NN}$	
$\bar{S}_6 = \text{VBP}$			$\bar{\max} = 0.0024$ $\bar{b} = 0.7$
			$\delta = 0.0017$, $\varphi = \text{NNS}$
	$X_2 = \text{NN}$	$X_3 = \text{VBZ}$	$X_4 = \text{NNS}$

Exercise

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

A	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

B	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

POS Taggers

Viterbi algorithm

Trellis from Viterbi: Backward path readout step: $X_t = \varphi(X_{t+1})$

$O_1 = \text{the}$ X_1	$O_2 = \text{kid}$ X_2	$O_3 = \text{fishes}$ X_3	$O_4 = \text{fish}$ X_4
$\bar{s}_1 = \bar{DT}$	$\bar{\pi} = 0.4 \quad \bar{b} = 1$		
	$\delta = 0.4$		
$\bar{s}_2 = \bar{JJ}$	$\bar{\max} = 0.08 \quad \bar{b} = 0.2$		
	$\delta = 0.016, \varphi = \text{DT}$		
$\bar{s}_3 = \bar{NN}$	$\bar{\max} = 0.2 \quad \bar{b} = 0.3$		
	$\delta = 0.06, \varphi = \text{DT}$		
$\bar{s}_4 = \bar{NNS}$		$\bar{\max} = 0.006 \quad \bar{b} = 0.4$	
		$\delta = 0.0024, \varphi = \text{NN}$	
$\bar{s}_5 = \bar{VBZ}$		$\bar{\max} = 0.054 \quad \bar{b} = 0.6$	
		$\delta = 0.0324, \varphi = \text{NN}$	
$\bar{s}_6 = \bar{VBP}$			$\bar{\max} = 0.0024 \quad \bar{b} = 0.7$
			$\delta = 0.0017, \varphi = \text{NNS}$
$X_1 = \text{DT}$	$X_2 = \text{NN}$	$X_3 = \text{VBZ}$	$X_4 = \text{NNS}$