UNIVERSITÀ DEGLI STUDI DI PADOVA

Spatial filtering – non-linear filters
Image restoration

Stefano Ghidoni

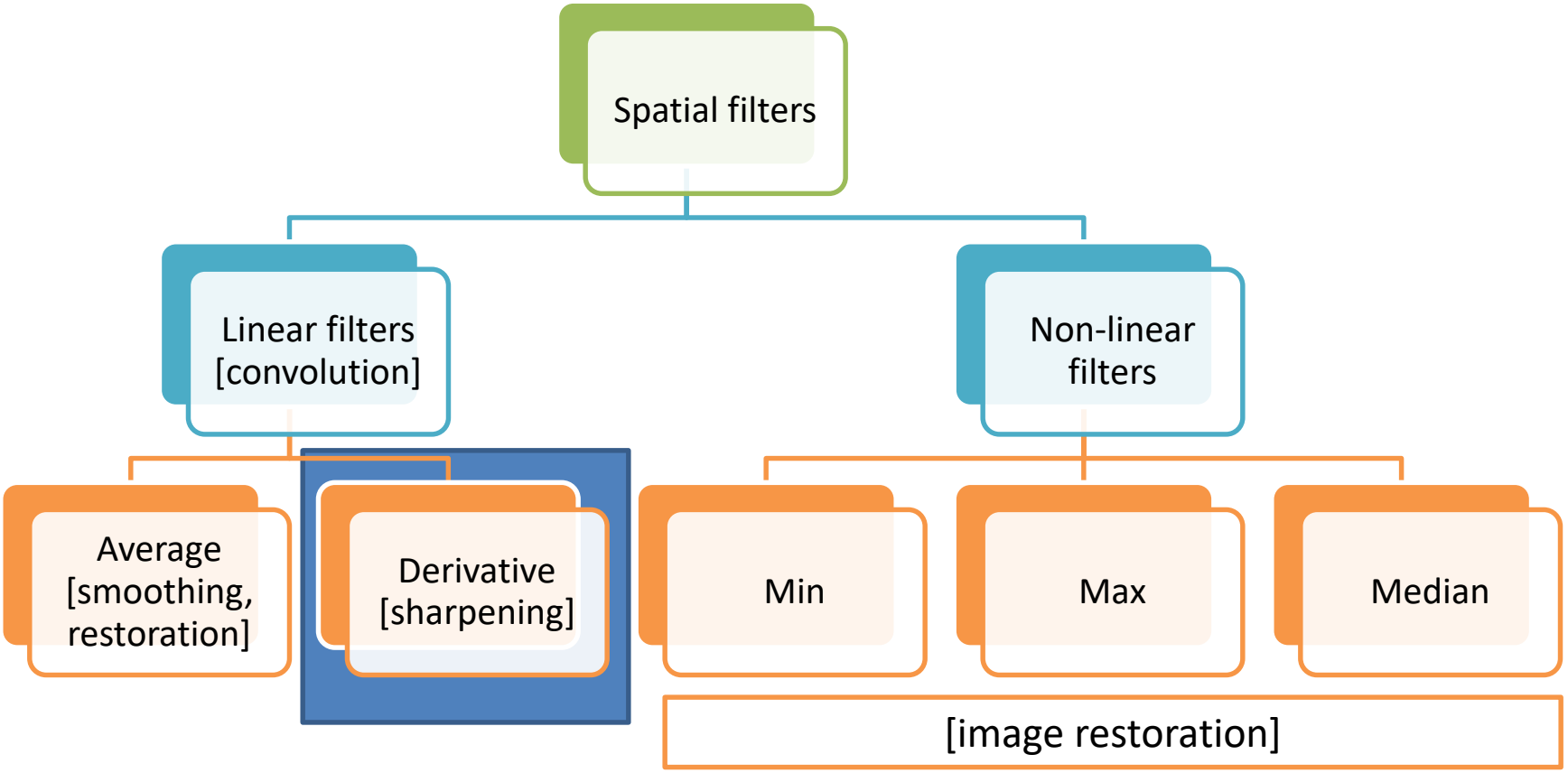
DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

INTELLIGENT AUTONOMOUS SYSTEMS LAB

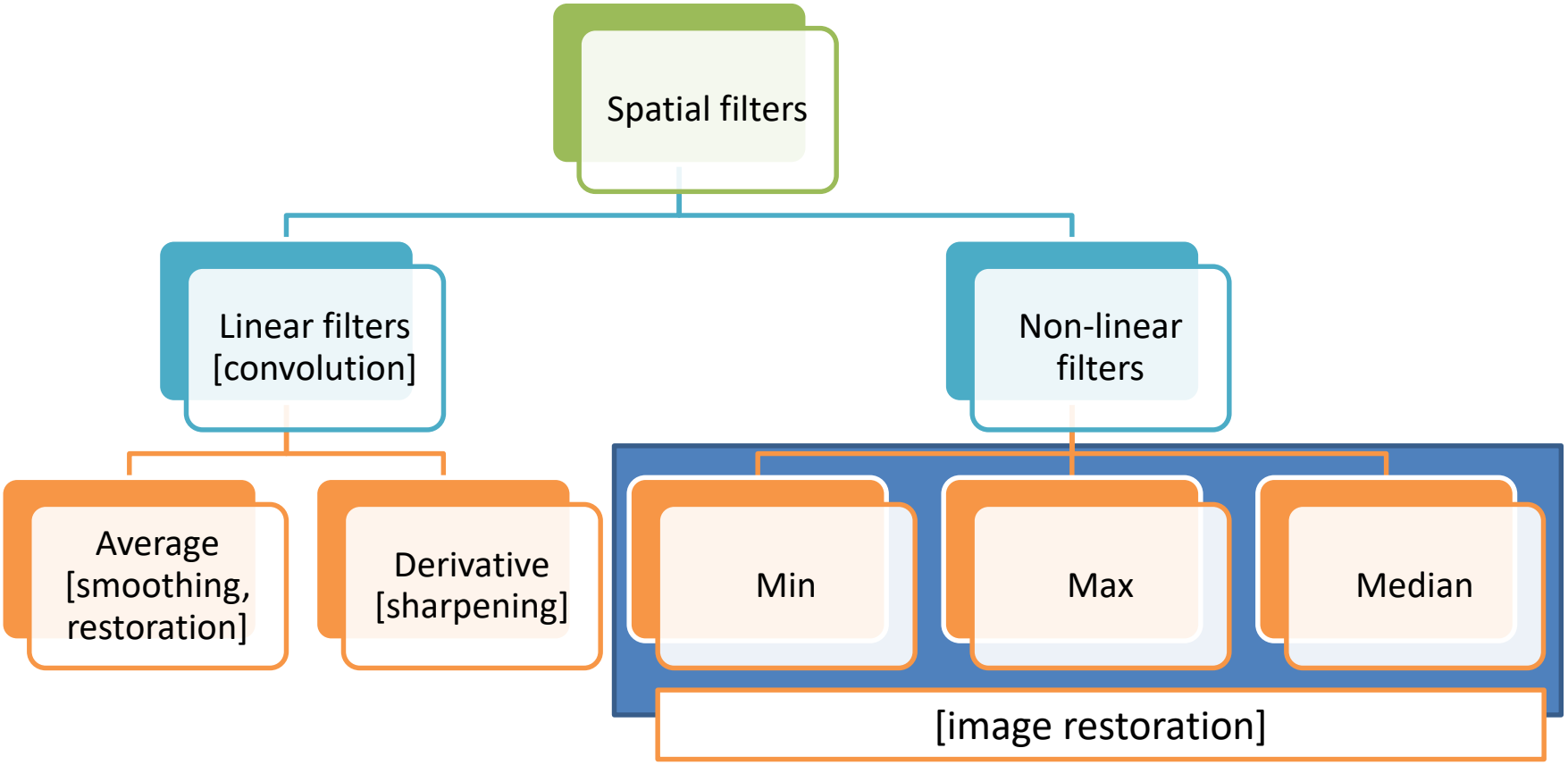
Agenda

- Linear and non-linear filters
- Images and noise, noise models
- Image restoration

Spatial filters overview



Spatial filters overview



Non-linear filters

- Non-linear filters implement non-linear operations
- What is a non-linear operation?

- No spoiler ☺

Non-linear filters

- Non-linear filters implement non-linear operations
- What is a non-linear operation?
- Examples of non-linear operations:
  - Min
  - Max
  - Median
- Can you think of other non-linear operations?

Non-linear filters

- Besides their mathematical formulation, what can be the advantages of a non-linear filter over a linear one?

Non-linear filters

- Besides their mathematical formulation, what can be the advantages of a non-linear filter over a linear one?
- Non-linear filters can *suppress* components
  - Ex: a median filter can remove a spike (a single element that is strongly different from the others)
  - In our context: an element is a pixel

Image restoration

- How do spikes appear in an image?
- An example: microchip microscopic image
- What can you say about this image?

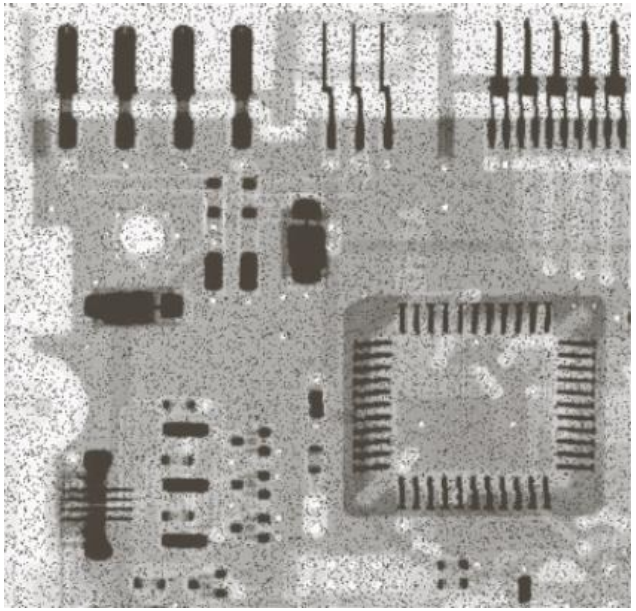


Image restoration

- How do spikes appear in an image?
- Image corrupted by noise
- The task of removing the noise in an image is often referred to as **image restoration**

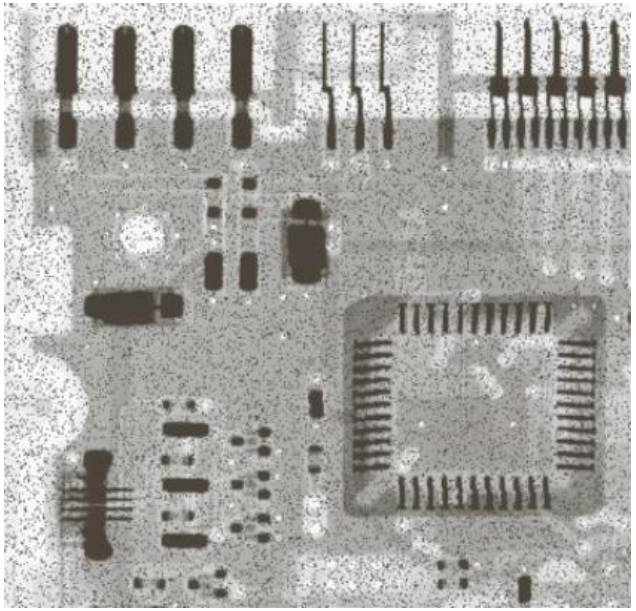
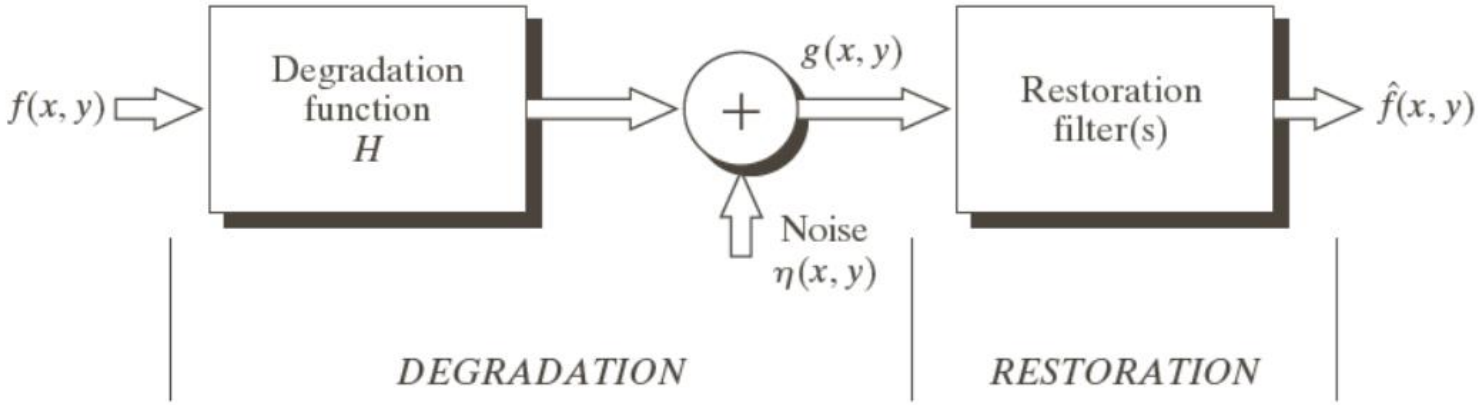


Image restoration

- We can model the acquired image as:

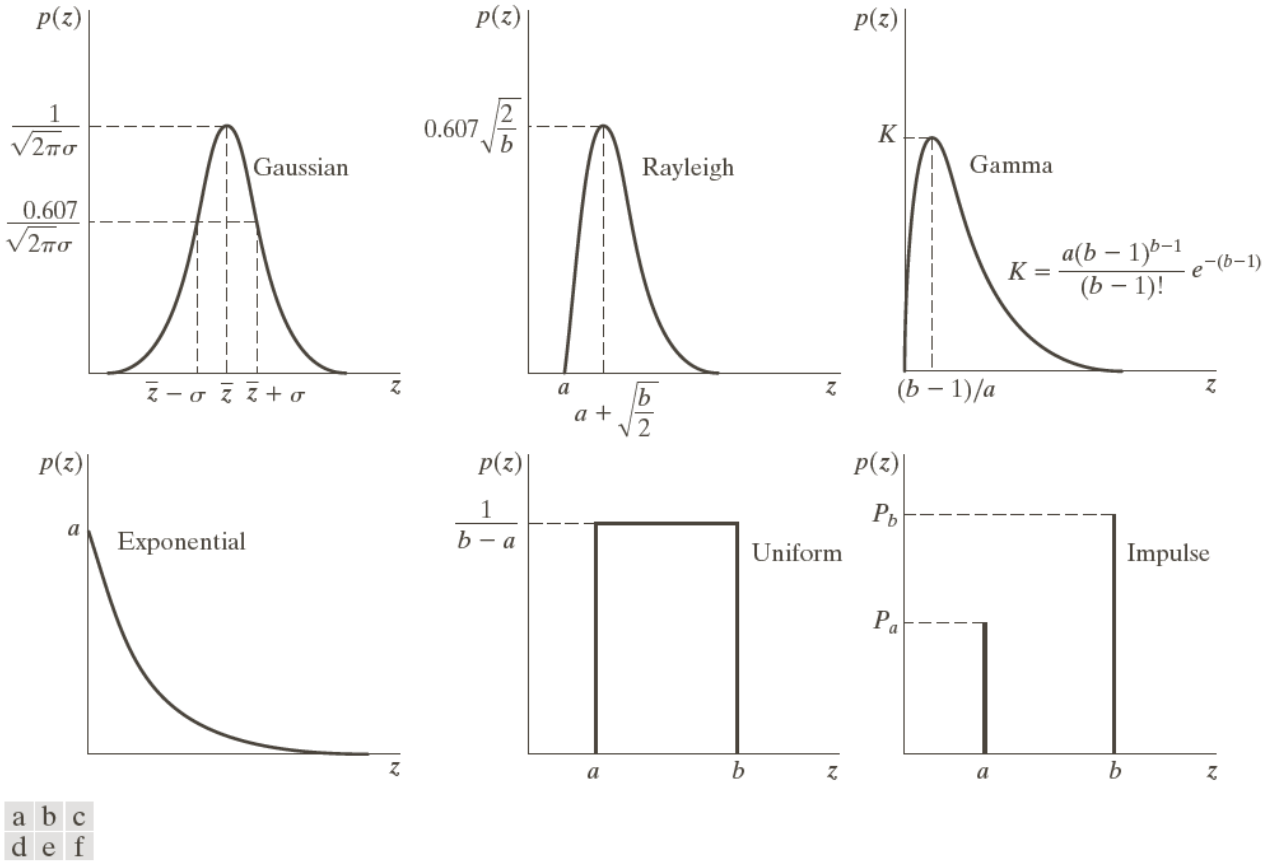
$$f_{\text{acq}}(x, y) = f(x, y) + \eta(x, y)$$

  - Ideal image + noise
- Image restoration
  - Estimate noise properties
  - Remove noise



Noise models

- Several noise models are available



a b c
d e f

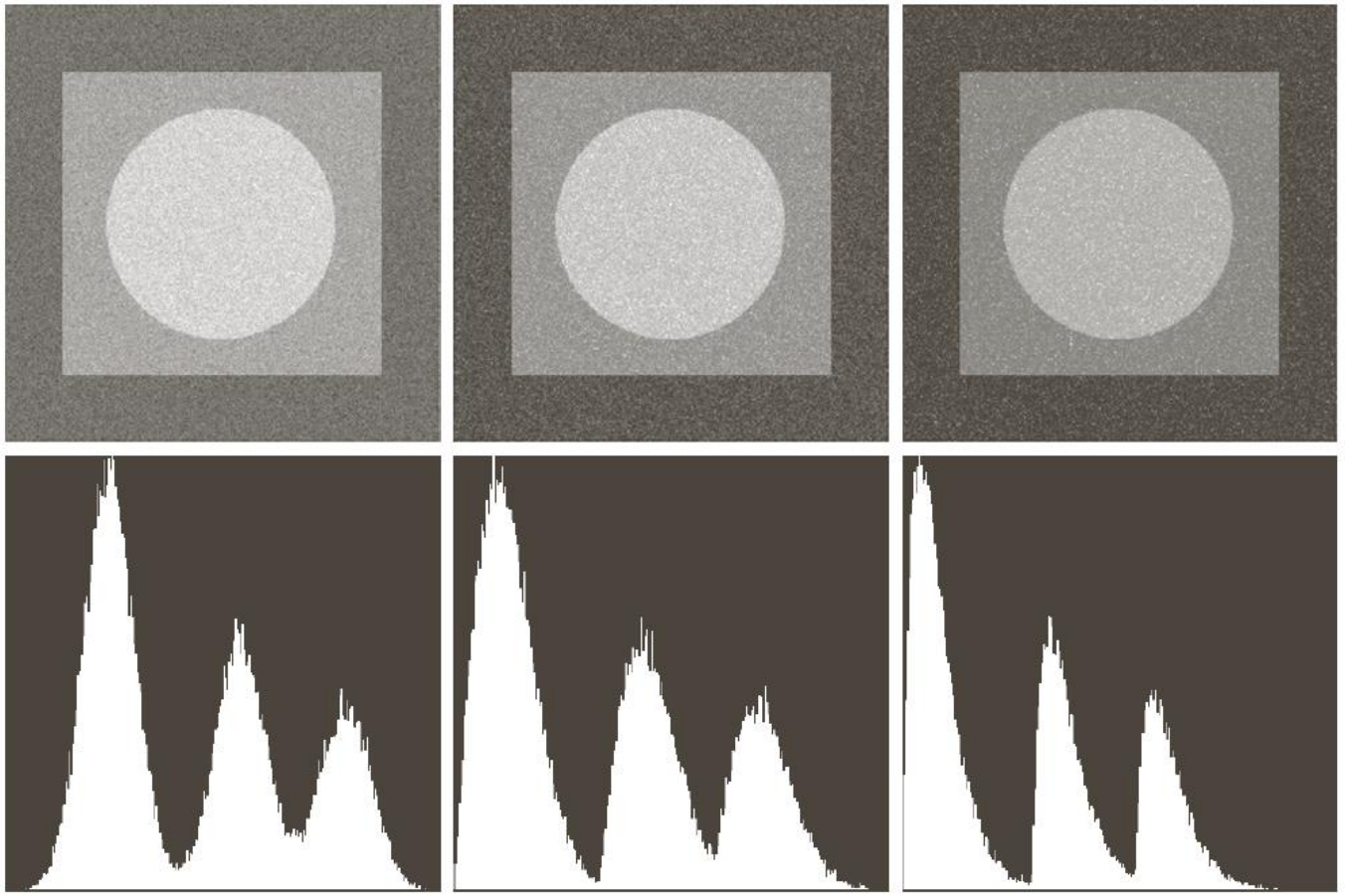
FIGURE 5.2 Some important probability density functions.

Noise models

- Let's analyze how the different noise models affect a test pattern

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

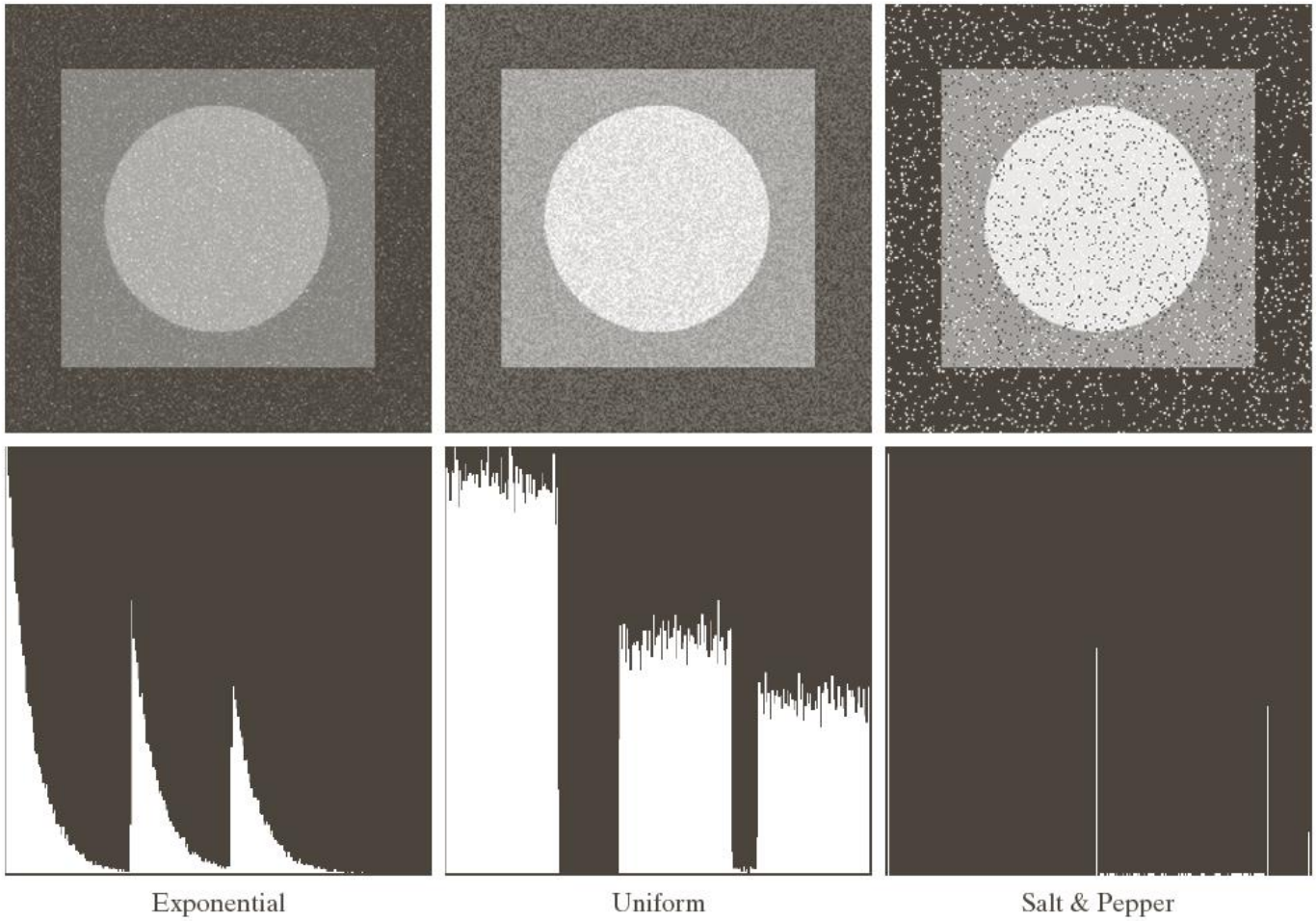
Noise models – example



a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Noise models – example



g h i
j k l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Image restoration

- A corrupted image may be restored using:
  - Smoothing filters
    - Averaging
    - Gaussian filter
  - Non-linear filters
    - Median filter
    - Min and max filters
- Example: image corrupted with gaussian noise
  - Restored using an average filter

Image restoration – example

- Original image

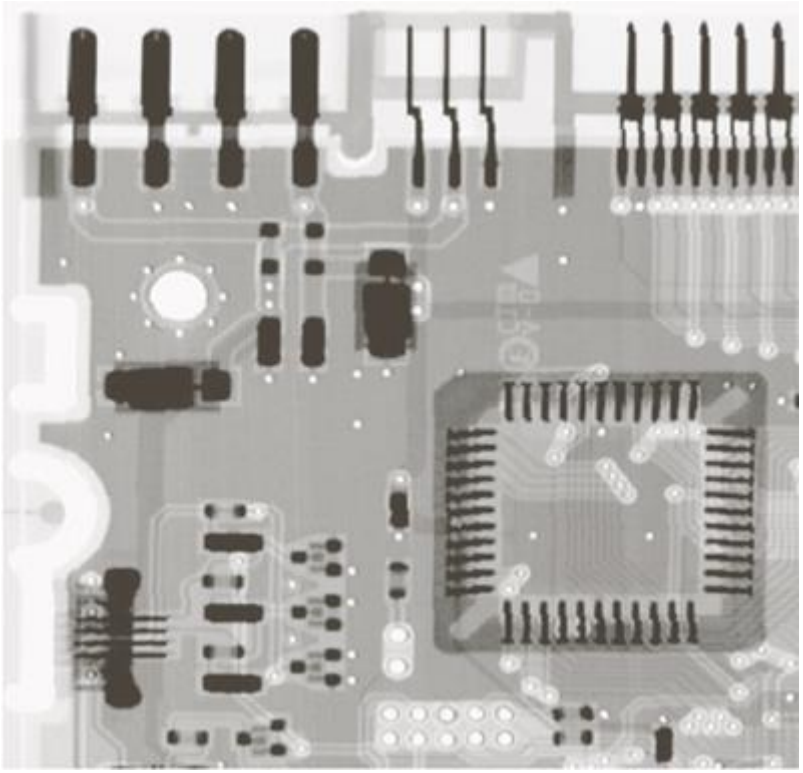


Image restoration – example

- Corrupted image (gaussian noise)

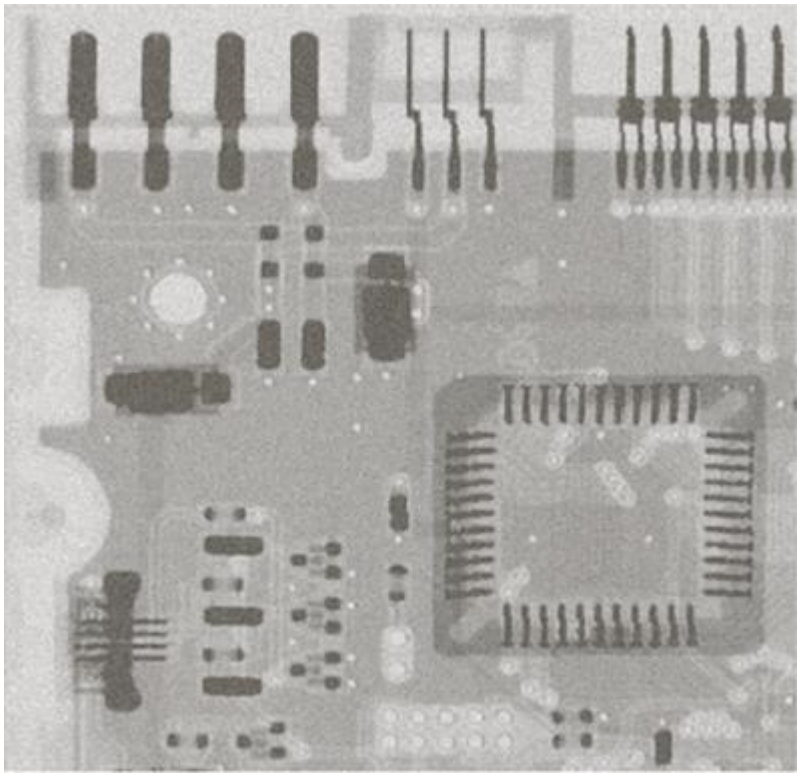


Image restoration – example

- Restored image (arithmetic mean)

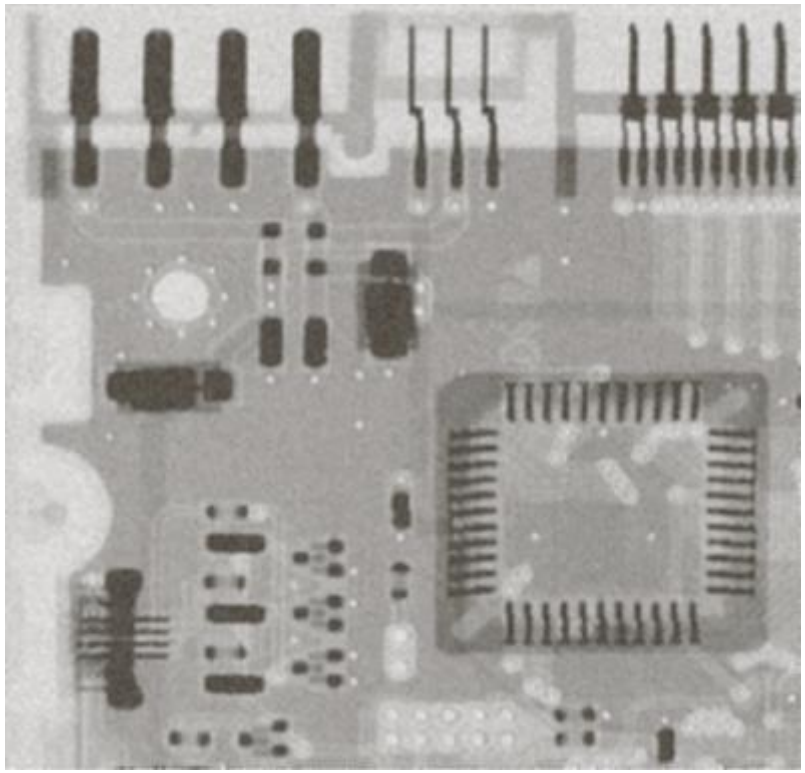


Image restoration – example

- Restored image (geometric mean)

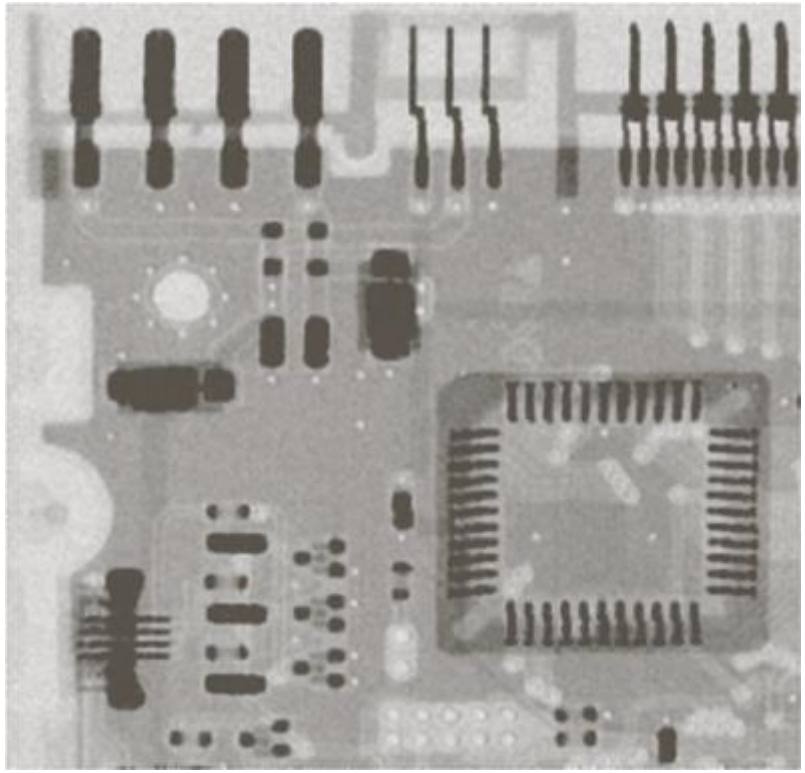
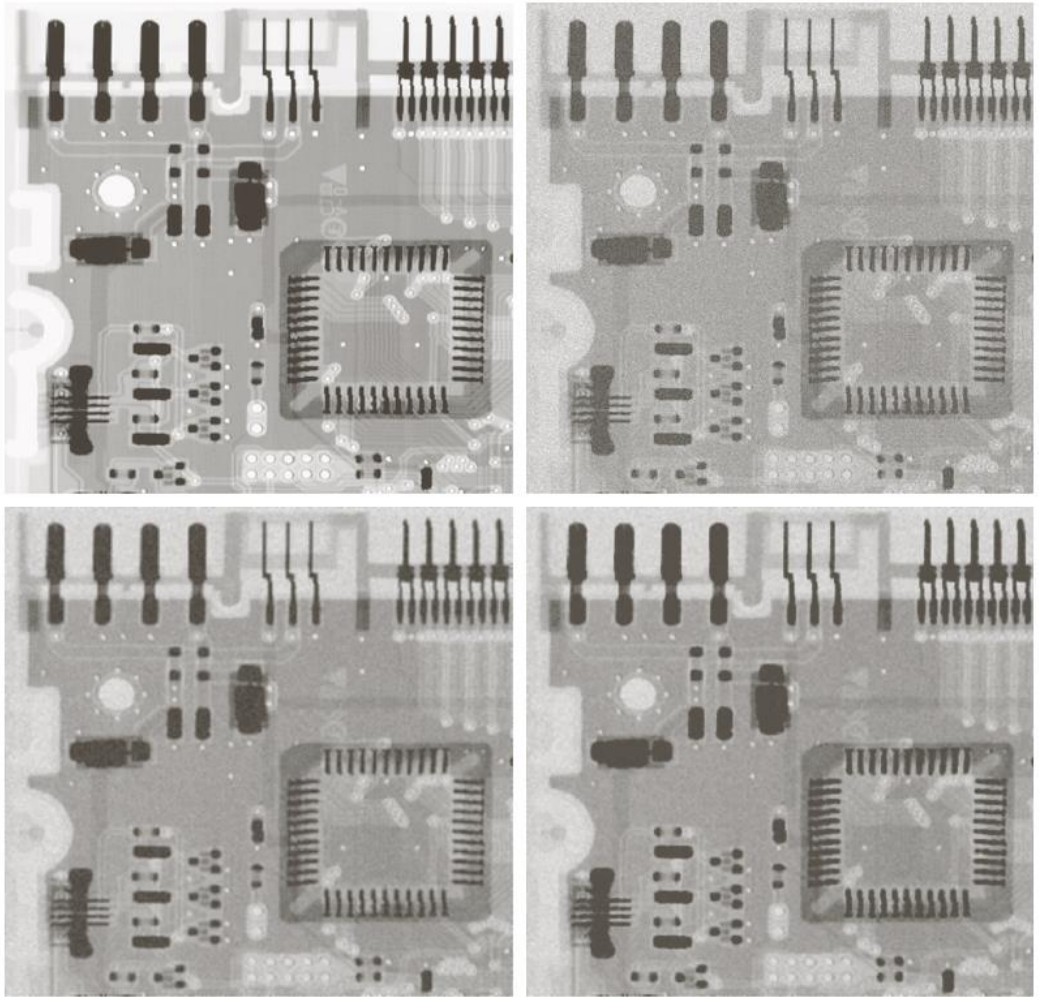


Image restoration – example

a b
c d

FIGURE 5.7
(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



What is the visible difference between arithmetic and geometric mean?

Image restoration

- A corrupted image may be restored using:
  - Smoothing filters
    - Averaging
    - Gaussian filter
  - Non-linear filters
    - Median filter
    - Min and max filters
- Example: image corrupted with salt noise **or** pepper noise
  - Restored using other mean filters

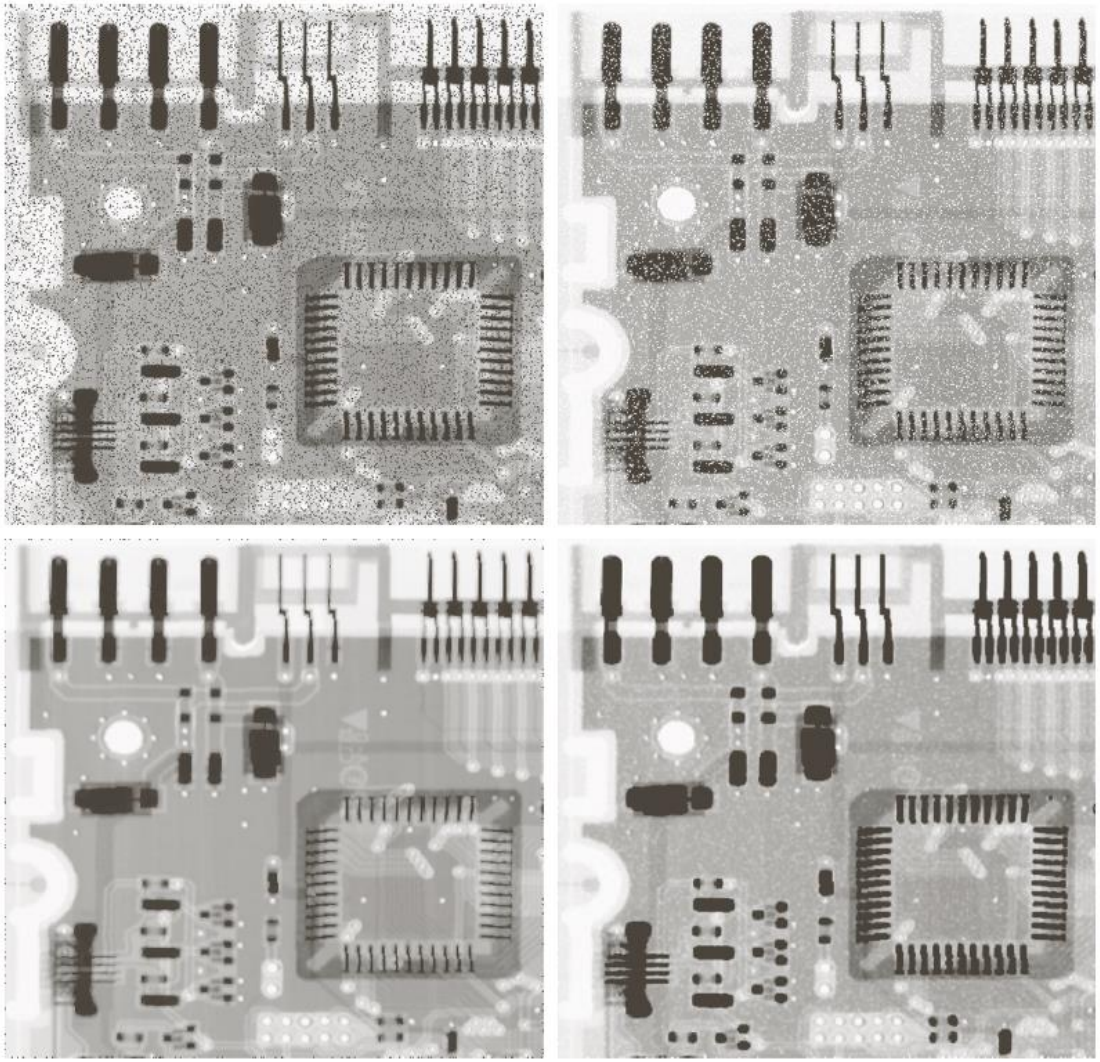
Averaging filters overview

- Several averaging filters are available:
  - Arithmetic mean
    - **Alpha-trimmed mean**
  - Geometric mean
  - Harmonic mean
    - Dominated by the minimum of its arguments
    - In the general case:
      harmonic < geometric < arithmetic mean
  - Contraharmonic mean

Averaging filters overview

Name	Equation	Removes Gaussian Noise	Removes Salt Noise	Removes Pepper Noise
Arithmetic mean	$g(x,y)=\frac{1}{mn}\sum_{s,t\in R} f(s,t)$	Yes	No	No
Geometric mean	$g(x,y)=\left[\prod_{s,t\in R} f(s,t)\right]^{\frac{1}{mn}}$	Yes	No	No
Harmonic mean	$g(x,y)=\frac{mn}{\sum_{s,t\in R}\frac{1}{f(s,t)}}$	Yes	Yes	No
Contraharmonic mean	$g(x,y)=\frac{\sum_{s,t\in R} f(s,t)^{Q+1}}{\sum_{s,t\in R} f(s,t)^{Q}}$	Yes (Q=0→mean)	Q<0 (Q=-1→harmonic)	Q>0 (not both)

Salt noise and pepper noise

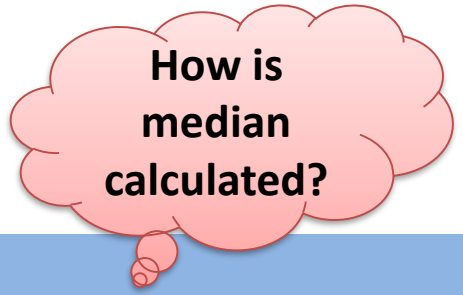


a b
c d

FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

Image restoration

- A corrupted image may be restored using:
  - Smoothing filters
    - Averaging
    - Gaussian filter
  - Non-linear filters
    - Median filter
    - Min and max filters

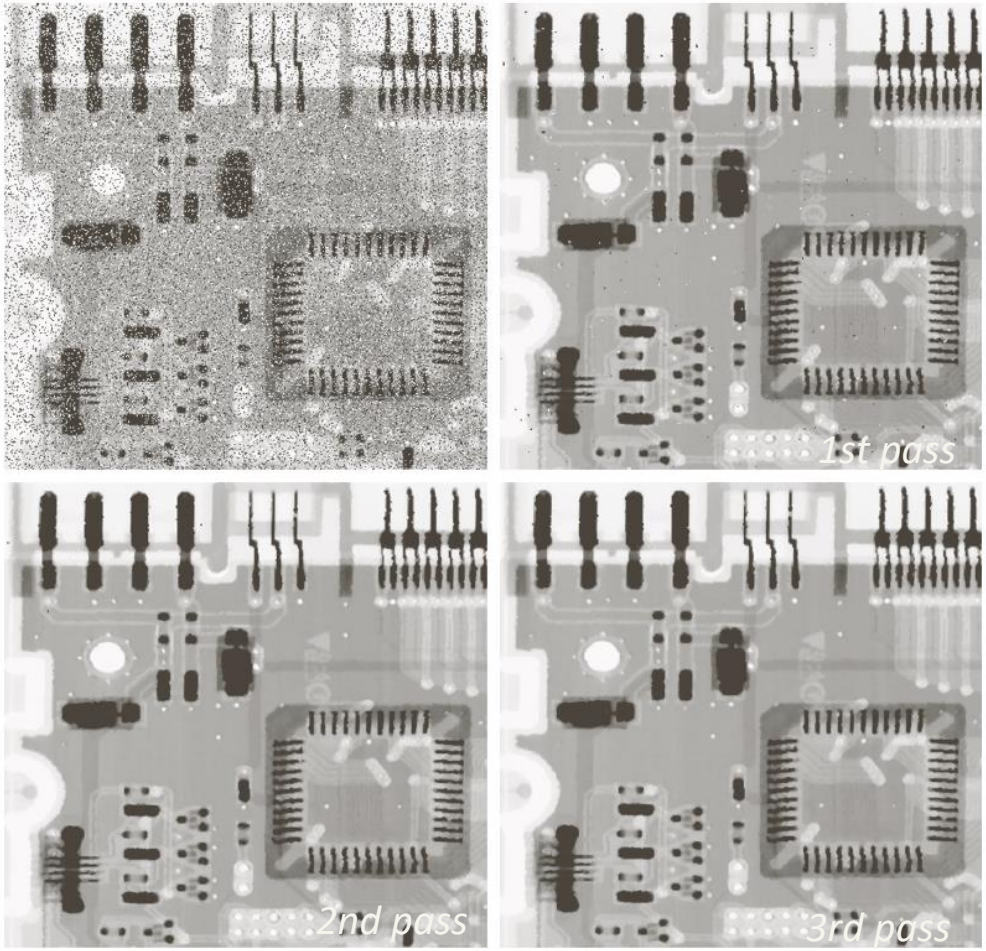


- Example: image corrupted with gausalt&pepper noise
  - Restored using a median filter and an alpha-trimmed mean

Median filter

a b
c d

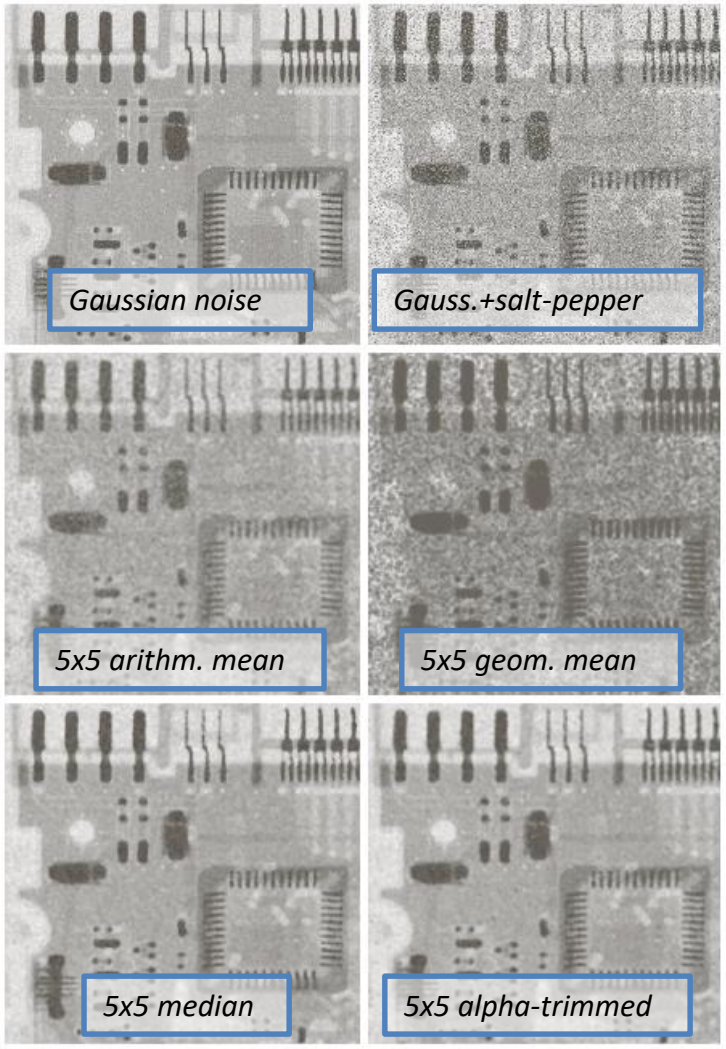
FIGURE 5.10
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size 3×3.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



3	7	4
8	2	92
10	10	5

2	3	4	5	7	8	10	10	92

Alpha-trimmed mean filter

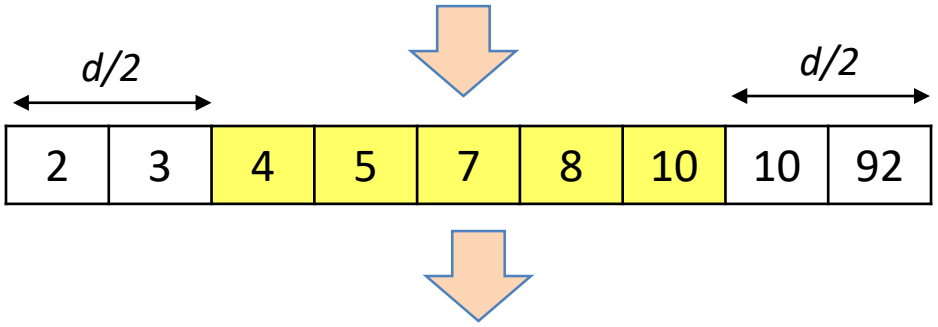


a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.

$$g(x,y) = \frac{1}{mn-d} \sum_{s,t \in R_r} f(s,t)$$

3	7	4
8	2	92
10	10	5



(4+5+7+8+10)/5=6,8

Hybrid between mean and median filter
D=0 : mean filter
D=mn-1 : median filter

Image restoration

- A corrupted image may be restored using:
  - Smoothing filters
    - Averaging
    - Gaussian filter
  - Non-linear filters
    - Median filter
    - Min and max filters

Image restoration

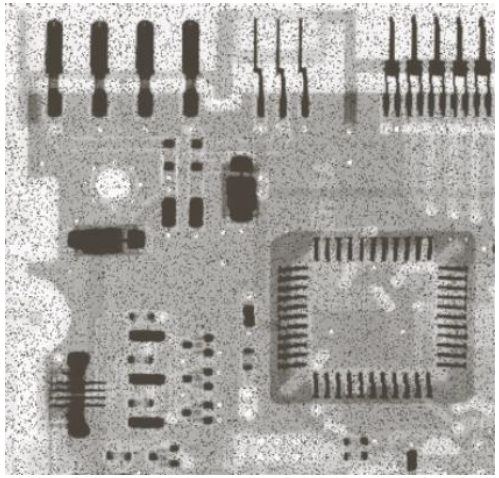
- A corrupted image may be restored using:
  - Smoothing filters
    - Averaging
    - Gaussian filter
  - Non-linear filters
    - Median filter
    - Min and max filters

Image restoration

- A corrupted image may be restored using:
  - Smoothing filters
    - Averaging
    - Gaussian filter
  - Non-linear filters
    - Median filter
    - Min and max filters
- Example: image corrupted with gausalt&pepper noise
  - Restored using a max/min filter

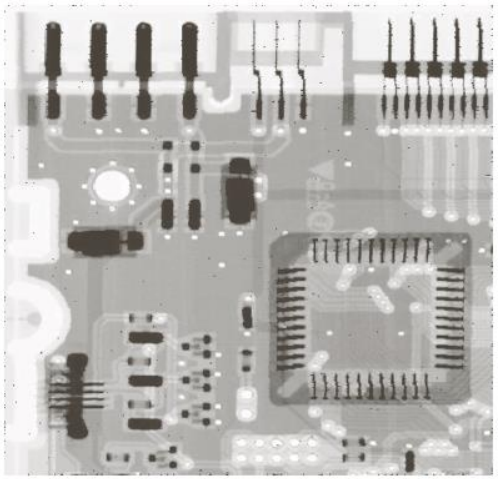
Max and min filters

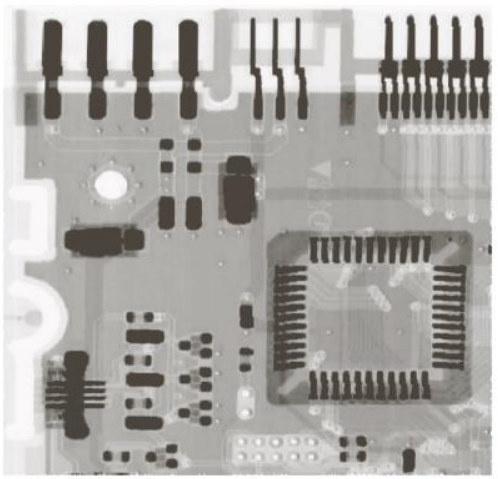
- Max filter: highlights salt noise, removes pepper noise
- Min filter: highlights pepper noise, removes salt noise



a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.





Original

Max filter

Min filter

Adaptive filters

- The filters discussed so far operate in the same way on
  - Every image
  - Every part of the image
- Smarter behaviors can be designed…

Adaptive filters

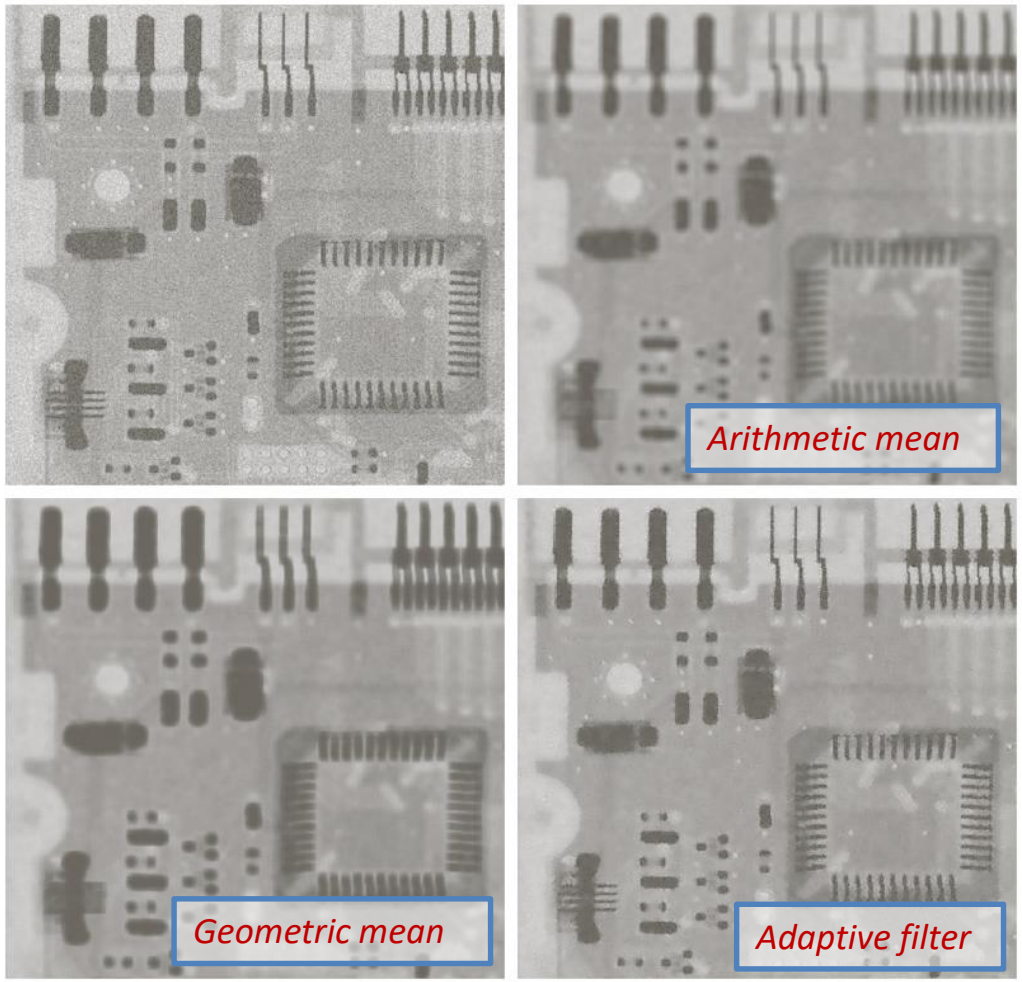
- Adaptive filters tune their effect comparing
  - Local image variance - σ_L^2
  - Noise variance - σ_η^2
- When $\sigma_\eta^2 \ll \sigma_L^2$ the filter should be weak
  - Edges / other image elements
- When $\sigma_\eta^2 \approx \sigma_L^2$ the filter should be strong
- Skip the mathematical details here, just see what is the effect
  - Several algorithms and approaches
  - We will see an example in detail in the next lectures

Adaptive filters

- Adaptive mean

a b
c d

FIGURE 5.13
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



Adaptive filters

- Adaptive median

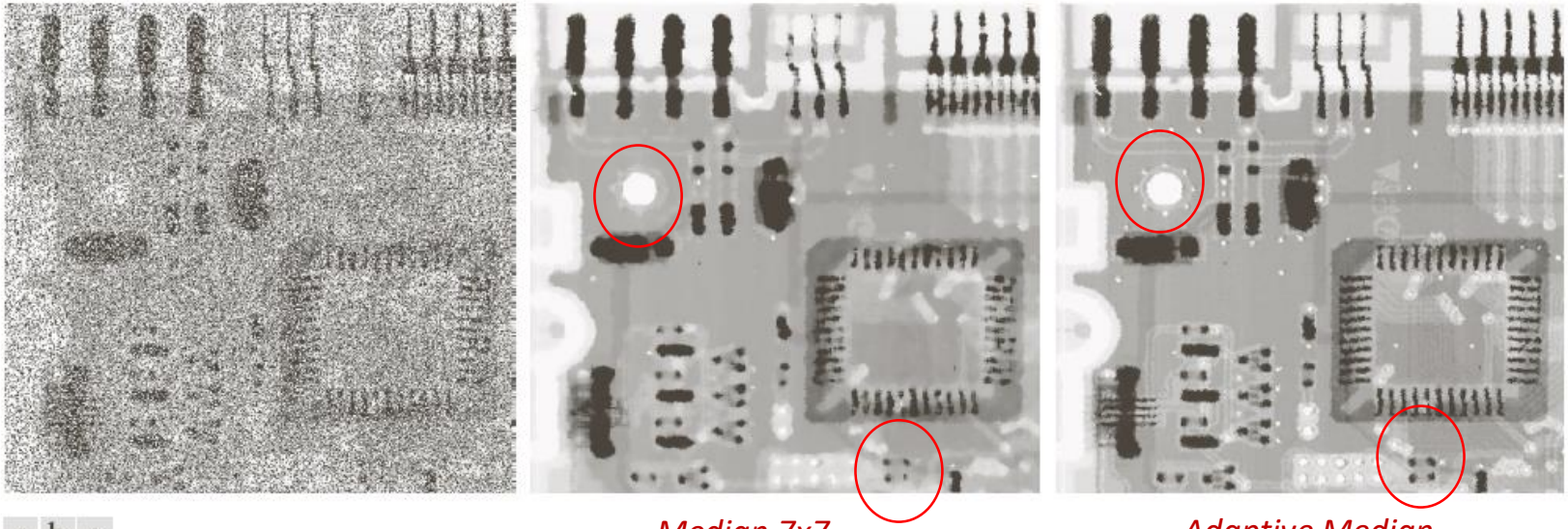


FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

UNIVERSITÀ DEGLI STUDI DI PADOVA

Spatial filtering – non-linear filters
Image restoration

Stefano Ghidoni

DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE

INTELLIGENT AUTONOMOUS SYSTEMS LAB