UNIVERSITÀ DEGLI STUDI DI PADOVA

Segmentation by thresholding & the Otsu's method

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Agenda

- Segmentation by thresholding
- Critical noise factors with thresholding
- Otsu's method

Segmentation techniques

- Segmentation by thresholding (histogram-based)
- Region growing methods
- Watershed transformation
- Clustering-based methods
- Model-based segmentation
- Edge-based methods
- Graph partitioning methods
- Multi-scale segmentation
- Many others…

Segmentation by thresholding

- Can a simple thresholding process be a segmentation technique?

Segmentation by thresholding

- Segmentation needs one or more criteria
- Criteria may be defined on the histogram
  – A simple example: applying a threshold and selecting the two resulting segments
    - In such case, would the segments be connected?
  – This approach can be extended to multiple thresholds/ranges
    - One segment per range

Segmentation by thresholding

- Thresholding becomes an important task!
- We shall further work on it… for example:
  - Global threshold vs local threshold
  - Threshold selection process

Segmentation: principles – recall

- Subdivide an image into n regions $R_1, R_2, \dots, R_n$ such that
  - $\bigcup_{i=1}^{n} R_i = R$
  - $R_i \cap R_j = \emptyset \quad \forall i, j \ (i \neq j)$
  - Optionally: each region shall be connected
- Two main criteria:
  - Similarity (between pixels in the same region)
  - Discontinuity (between pixels in different regions)
- Segmentation by thresholding: what is the driving criterion?

Threshold selection

- Selecting the right threshold may be trivial or tricky
- The threshold is the **only parameter** in this kind of segmentation
- Let's see some examples…

Global thresholding

- Where would you place a threshold?

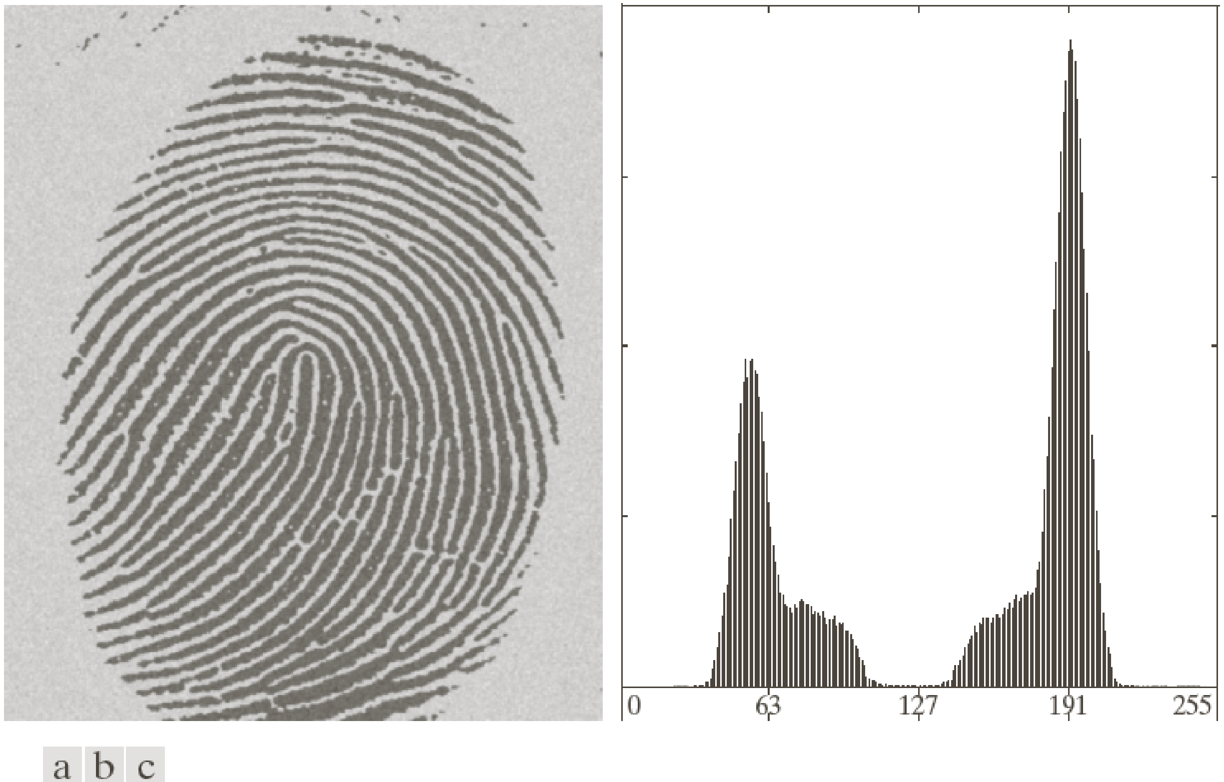


FIGURE 10.38 (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (the border was added for clarity). (Original courtesy of the National Institute of Standards and Technology.)

Global thresholding

- Where would you place a threshold?

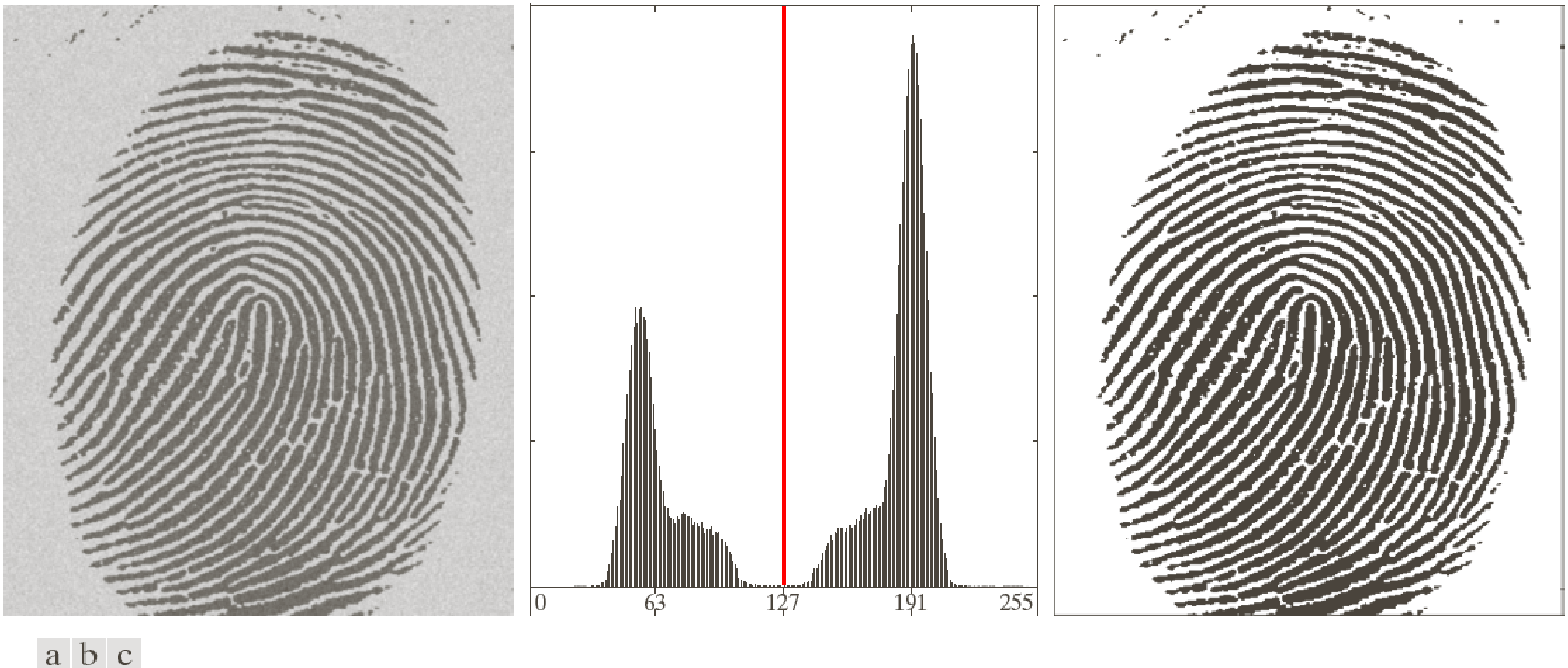


FIGURE 10.38 (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (the border was added for clarity). (Original courtesy of the National Institute of Standards and Technology.)

Thresholds and noise

- Problems arise in noisy images
  - Where would you place a threshold?

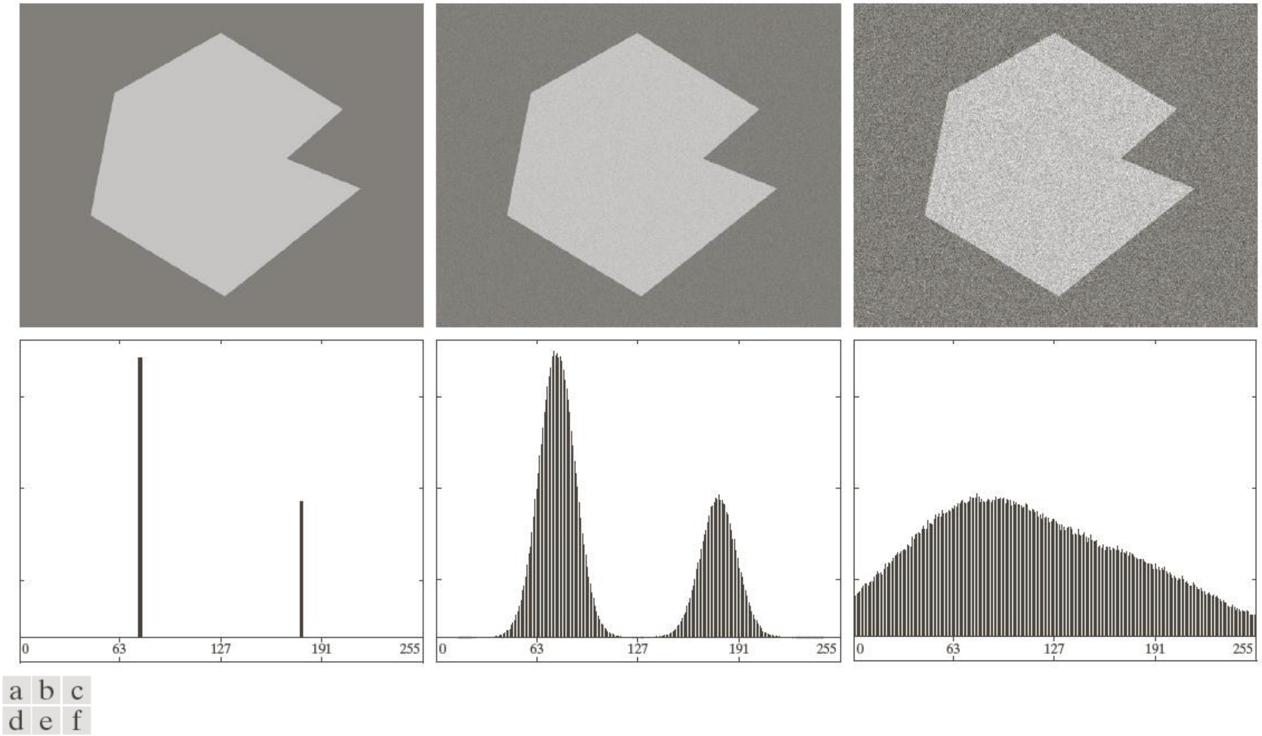


FIGURE 10.36 (a) Noiseless 8-bit image. (b) Image with additive Gaussian noise of mean 0 and standard deviation of 10 intensity levels. (c) Image with additive Gaussian noise of mean 0 and standard deviation of 50 intensity levels. (d)–(f) Corresponding histograms.

Thresholds and illumination

- Problems arise with illumination changes
  – Where would you place a threshold?

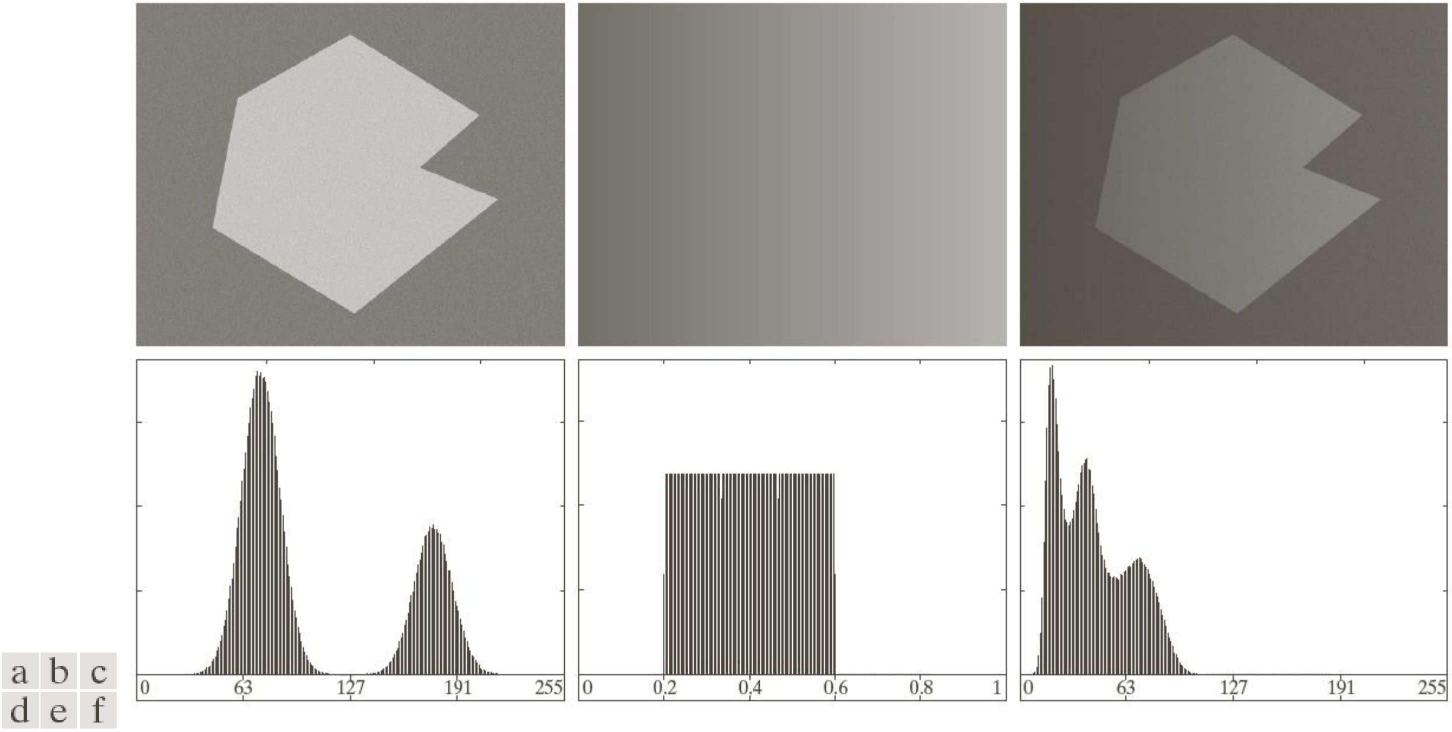
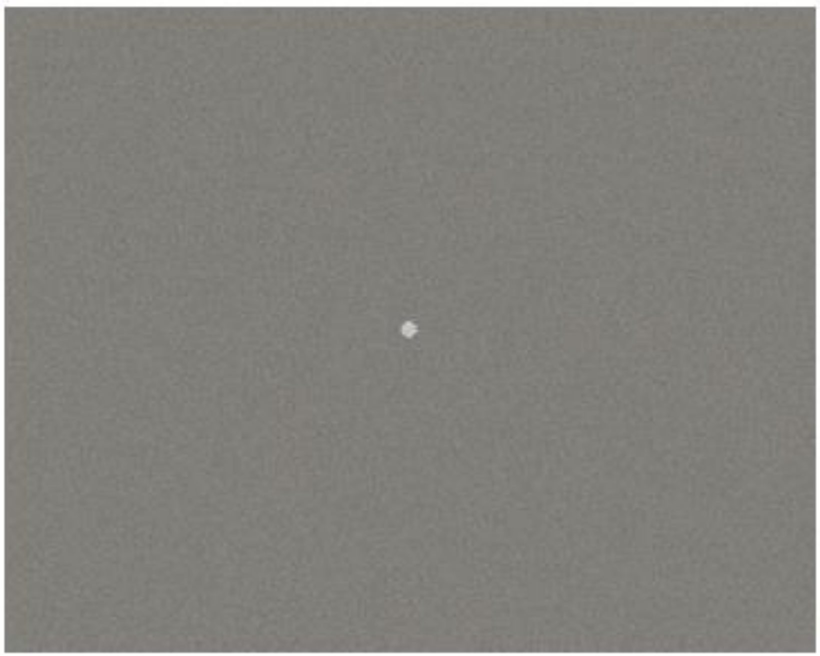
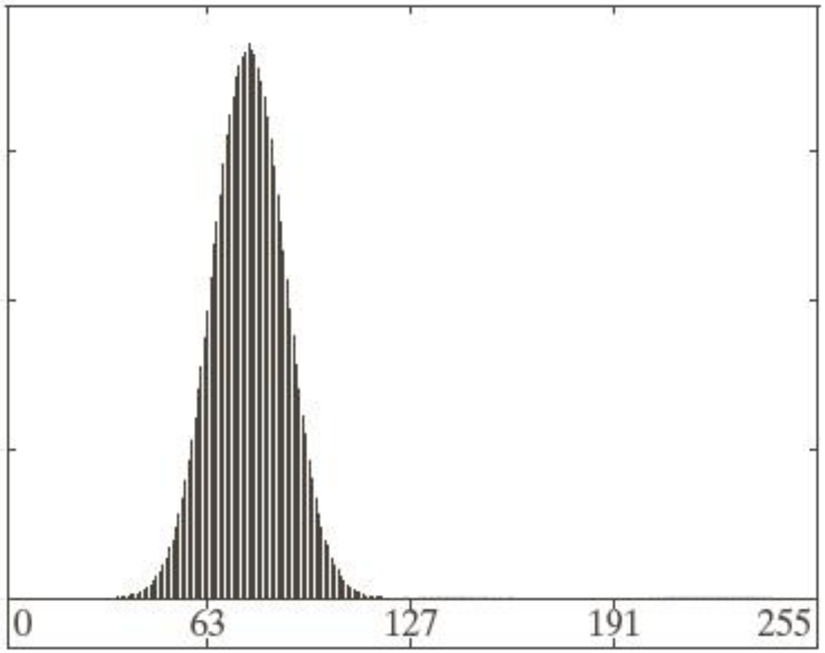


FIGURE 10.37 (a) Noisy image. (b) Intensity ramp in the range [0.2, 0.6]. (c) Product of (a) and (b). (d)–(f) Corresponding histograms.

Thresholds and size

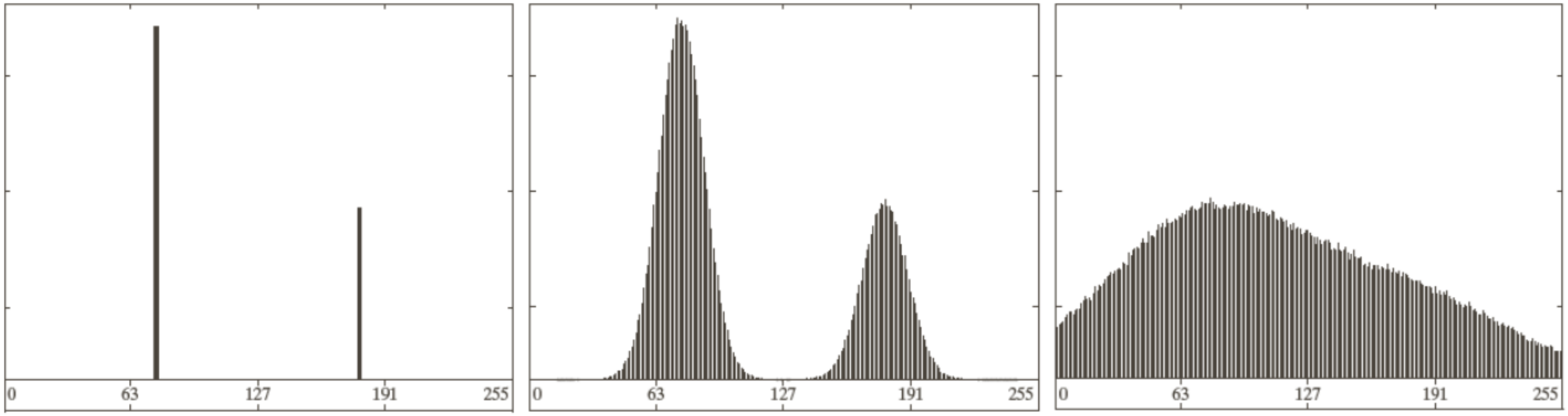
- Small regions have very limited impact on the histogram





Locating the threshold

- Thresholding is effective depending on:
  - Distance between peaks
  - Image noise
  - Relative size of the regions
  - Illumination properties
    - Similar effect when the reflectance properties of the objects is not uniform



Locating the threshold

- So far: we always solved the problem inspecting the histogram and choosing an appropriate threshold value
- Can we make this process automatic?

Otsu's optimal threshold

Otsu's method

- A global thresholding method based on the histogram
- Assumes that two classes are created by thresholding
- Finds the optimal threshold
  - Maximizes inter-class (between-class) variance
    - A measure of the difference between the two classes
  - Minimize intra-class variance

N. Otsu, “A threshold selection method from gray-level histograms”, IEEE Transactions on Systems, Man and Cybernetics, vol. 9, no. 1, January 1979.

Otsu's method: algorithm/1

- Compute the normalized histogram
  - Recall – normalized means that:

$$\sum_{i=0}^{L-1} p_i = 1$$

- Set a threshold $T(k) = k$
  - It divides the image into two classes:
    - Below threshold – class C_1
    - Above threshold – class C_2

Otsu's method: algorithm/2

- Compute probability: $P_1(k)$ (**below threshold**) for $k = 0, 1, \dots, L-1$:

$$P_1(k) = \sum_{i=0}^{k} p_i$$

  – This is the probability that a pixel belongs to C_1

- Compute the complementary probability (**above threshold**): $P_2(k) = 1 - P_1(k)$

Otsu's method: algorithm/3

- Compute the image global mean, i.e. the average (whole) image intensity m_g:

$$m_G = \sum_{i=0}^{L-1} i p_i$$

- Compute the cumulative mean up to level k:

$$m(k) = \sum_{i=0}^{k} i p_i$$

Observation/1

- Observe that:

$$m(k) = \sum_{i=0}^{k} ip_i$$

is the cumulative mean up to level k normalized over the *whole* image

- What is we want to refer such cumulative mean to C_1?
  - We can further normalize to the number of pixels in C_1 i.e. $P_1(k)$

Otsu's method: algorithm/4

- Compute pixel mean intensity value referred to classes 1 and 2: $m_1(k)$, $m_2(k)$ for $k = 0, 1, \dots, L-1$:

$$m_1(k) = \frac{1}{P_1(k)} \sum_{i=0}^{k} i p_i$$

$$m_2(k) = \frac{1}{P_2(k)} \sum_{i=k+1}^{L-1} i p_i$$

Observation/2

- What's the difference between:
  - Mean intensity value in class 1:

$$m_1(k) = \frac{1}{P_1(k)} \sum_{i=0}^{k} i p_i$$

  - And the cumulative mean up to level k:

$$m(k) = \sum_{i=0}^{k} i p_i$$

Observation/2

- Consider a toy example – an image having:
  - 50% of the pixels having gray level 100
  - 50% of the pixels having gray level 200
- Set the threshold k=150
- Calculate the mean intensity value and the cumulative mean for class 1 (below threshold)

100
50

- Anti-spoiler ☺

Observation/2

- The difference is linked to these concepts:
  - The mean intensity values is referred to the pixels in C_1
  - The cumulative mean is referred to all the pixels in the image

Observation/2

- Before proceeding, observe that:

$$P_1 + P_2 = 1$$

and

$$P_1 m_1 + P_2 m_2 = m_g$$

Otsu's method: algorithm/5

- Consider σ_G^2 that is the global variance (related to all pixels in the image)

$$\sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i$$

- Define the inter-class (between-class) variance σ_B^2 (omitting dependency on k) as:

$$\sigma_B^2(k) = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2$$

- The "quality" of the threshold is defined by:

$$\eta = \frac{\sigma_B^2(k)}{\sigma_G^2}$$

Otsu's method: algorithm/6

- The expression of:

$$\sigma_B^2(k) = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2$$

can be rewritten (omitting dependency on k) as*:

$$\sigma_B^2(k) = P_1 P_2(m_1 - m_2)^2$$

for $k = 0, 1, \ldots, L-1$

*homework: check calculations! You are required to know them

Otsu's method: algorithm/6

- A different formulation can be derived:

$$\sigma_B^2(k) = \frac{(m_G P_1 - m)^2}{P_1(1 - P_1)}$$

for $k = 0, 1, \ldots, L - 1$

- Recall: P_1, P_2, m are functions of k
- This formula is computationally more efficient
  - Only m and P_1 need to be computed for every value of k

Otsu's method: algorithm/7

- The **optimal threshold** can be found by maximixing η, that is finding:

$$k^* \text{ s.t. } \sigma_B^2(k^*) = \max_k(\sigma_B^2(k))$$

- Segment the image using k^* as a threshold

Intra- vs inter-class variance

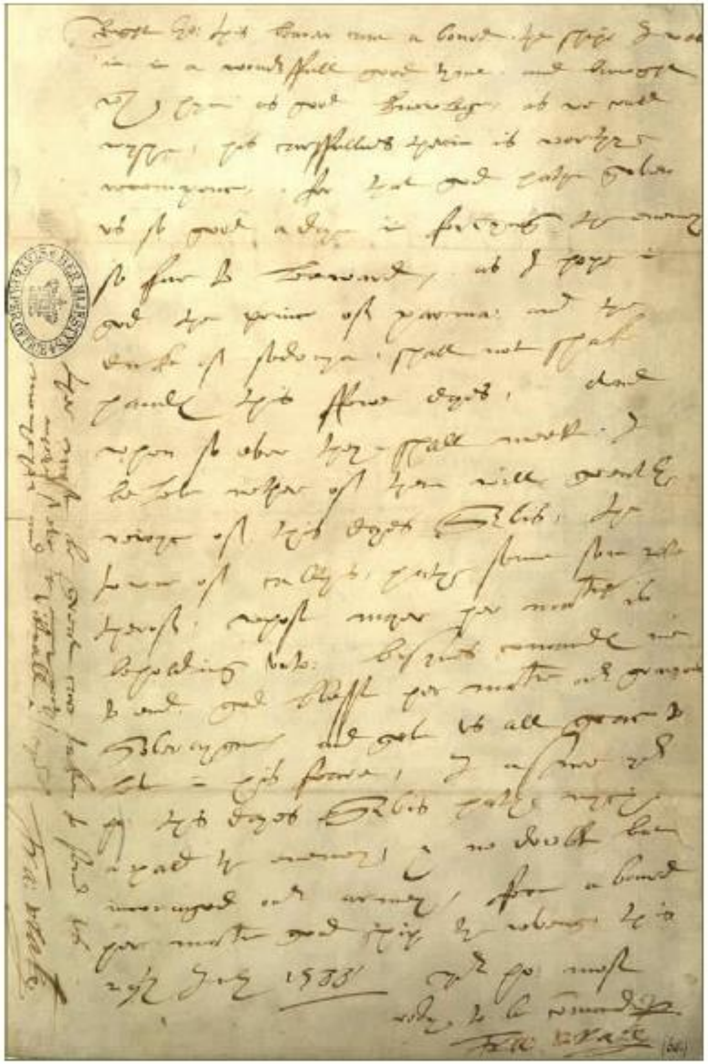
- We have previously used the inter-class variance
- The intra-class variance can be also calculated
  - It is defined as

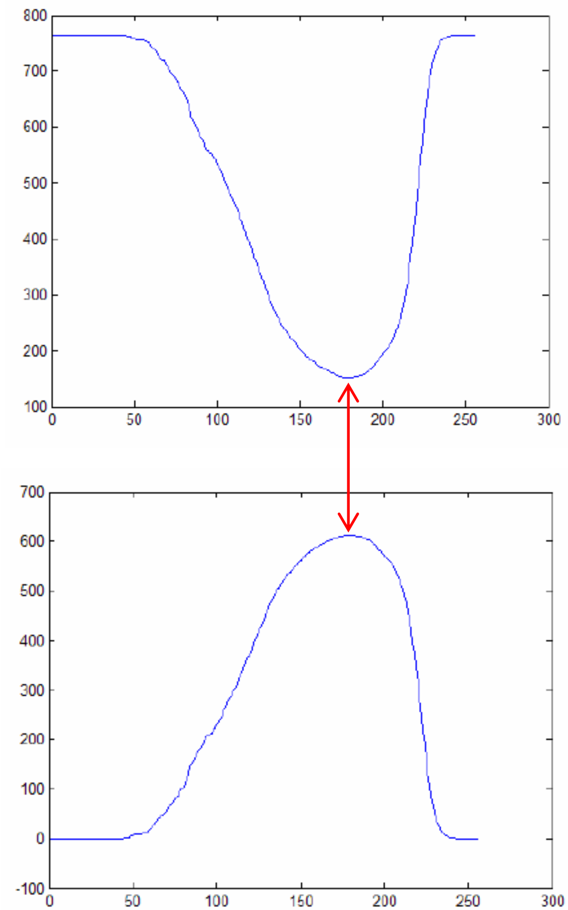
$$\sigma_{in}^2 = P_1\sigma_1^2 + P_2\sigma_2^2$$

- The global variance can be expressed as:

$$\sigma_G^2 = \sigma_{in}^2 + \sigma_B^2$$

Otsu – example

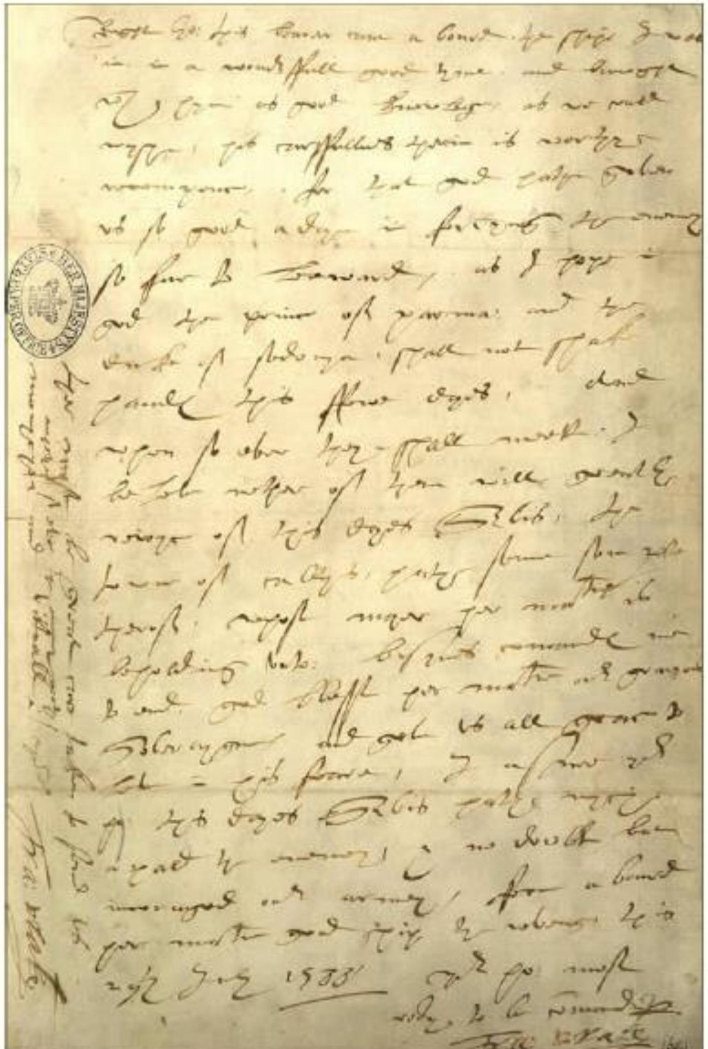


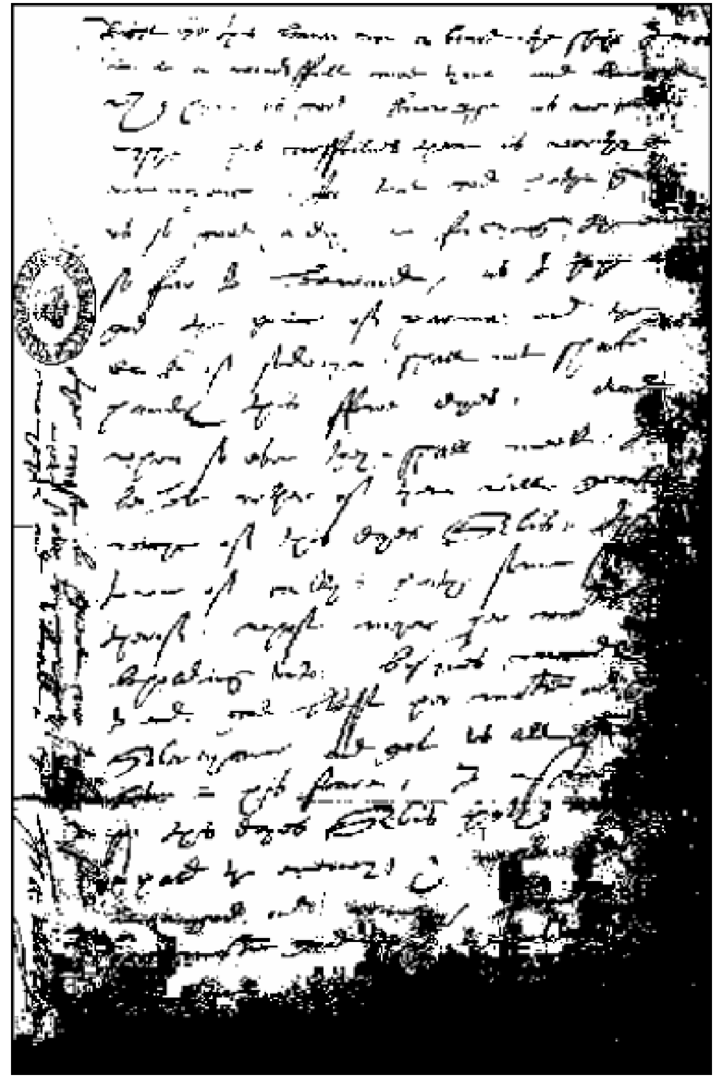


σ^2_{intra}

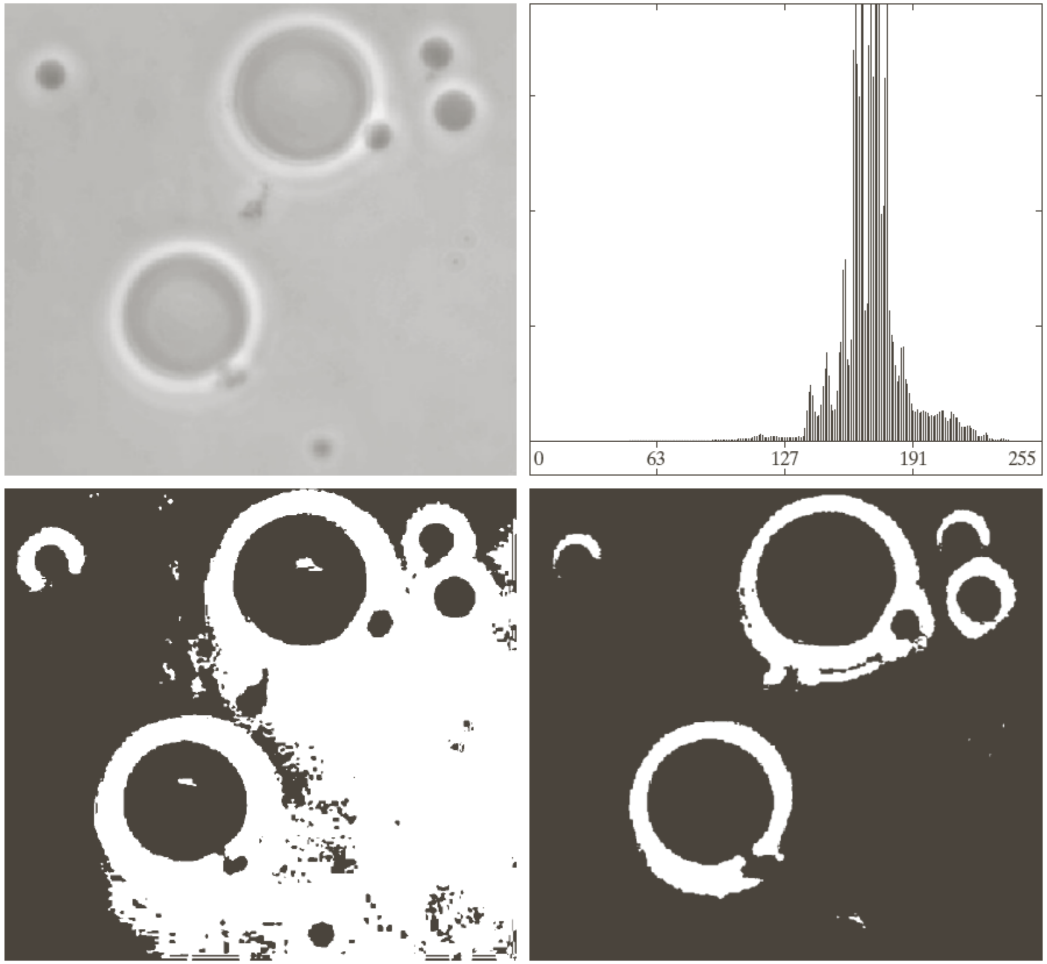
$\sigma^2_B = \sigma^2_{inter}$

Otsu – example





Otsu – example



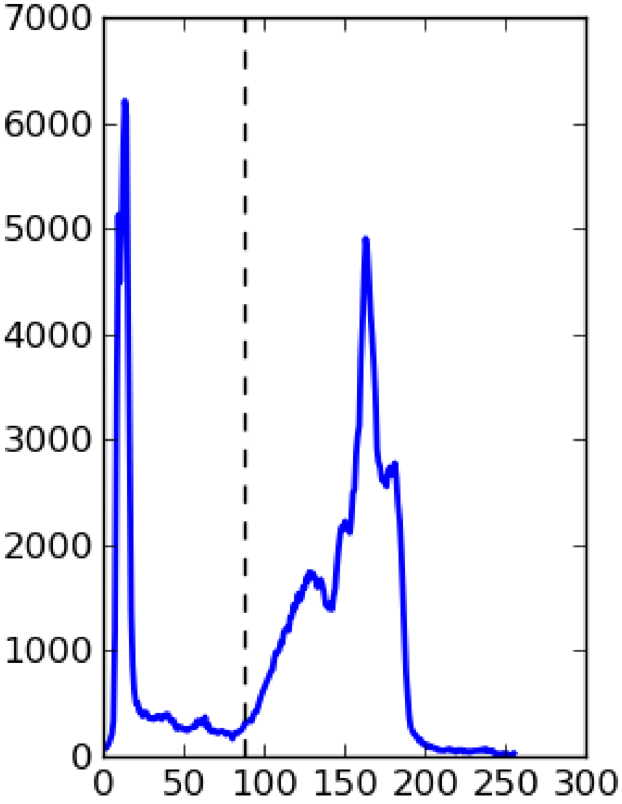
K-means: T=169

Otsu: T=181

a b
c d

FIGURE 10.39 (a) Original image. (b) Histogram (high peaks were clipped to highlight details in the lower values). (c) Segmentation result using the basic global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (Original image courtesy of Professor Daniel A. Hammer, the University of Pennsylvania.)

Otsu – example



Smoothing

- Smoothing can help Otsu's method

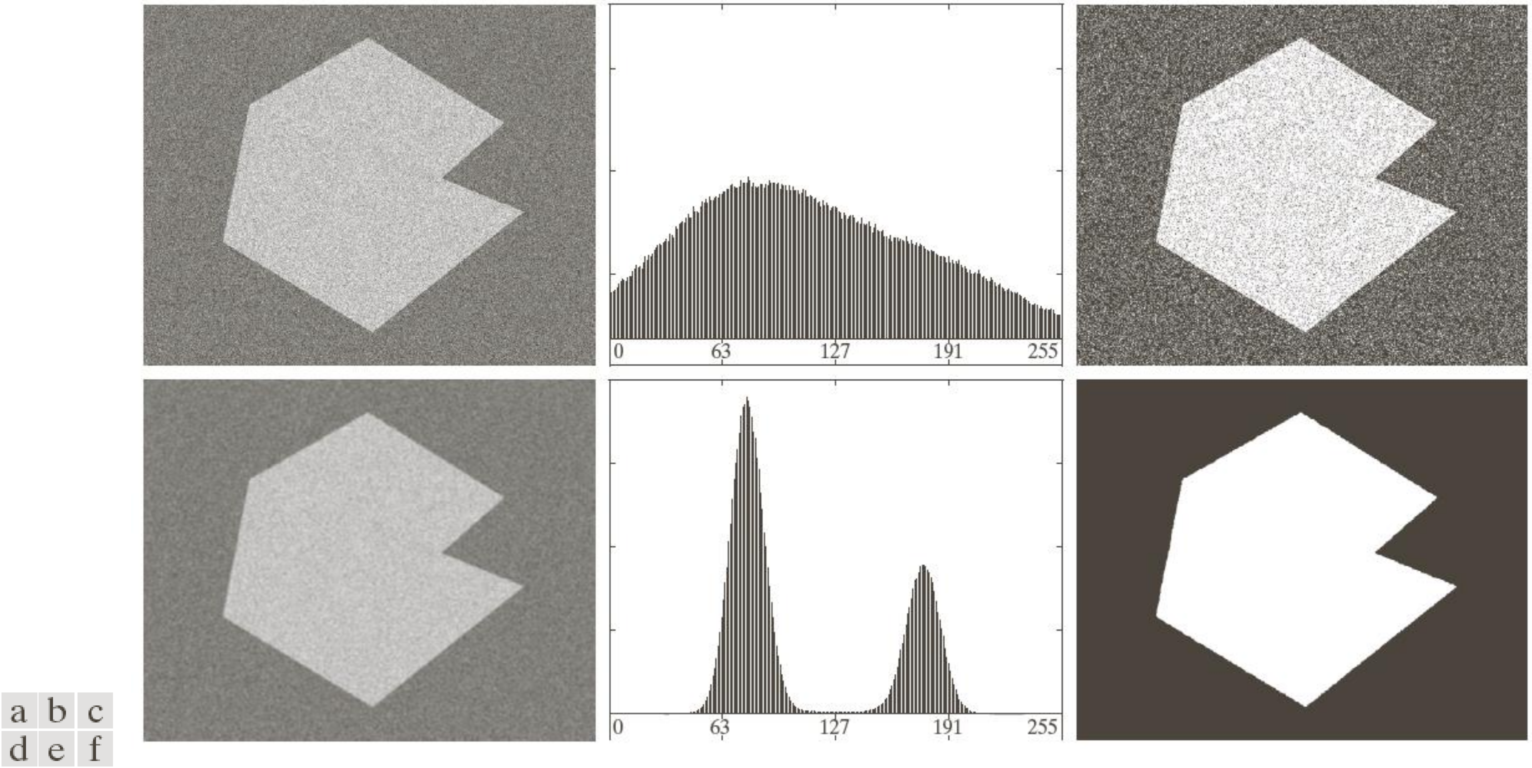
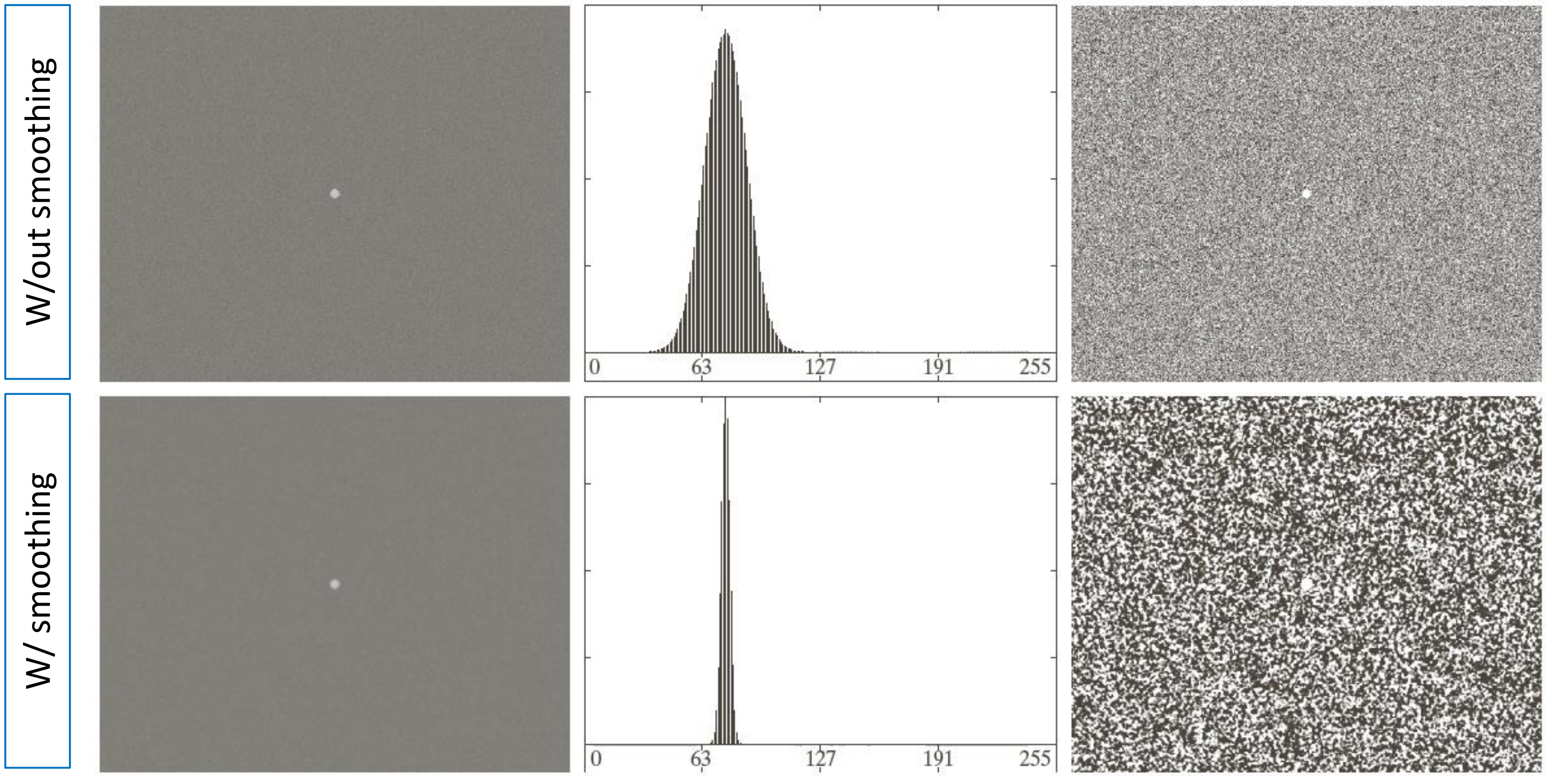


FIGURE 10.40 (a) Noisy image from Fig. 10.36 and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method.

Otsu – issues



a	b	c
d	e	f

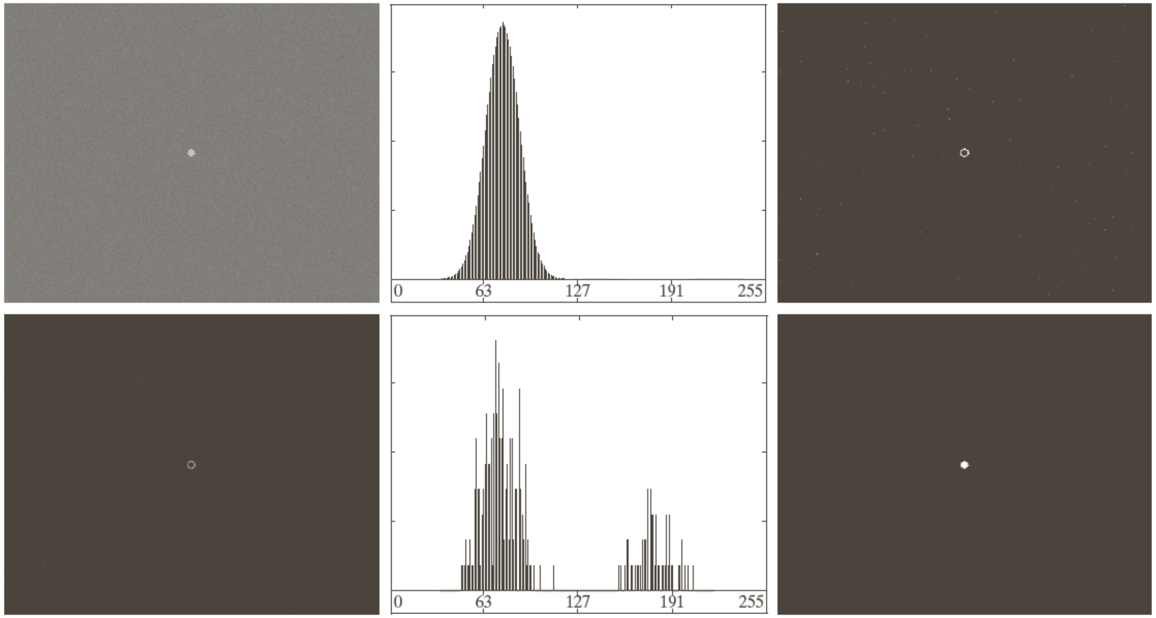
FIGURE 10.41 (a) Noisy image and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method. Thresholding failed in both cases.

Extending Otsu's method

- The Otsu's method can be combined with other techniques
  - E.g.: edge detection
- The Otsu's method can be generalized to:
  - Non-global thresholding
  - Multiple categories

Extension: edge detection

- Compute the histogram and the threshold on the edge image (or combination of edge with the original image)
- Apply the threshold to the original image



a b c
d e f

FIGURE 10.42 (a) Noisy image from Fig. 10.41(a) and (b) its histogram. (c) Gradient magnitude image thresholded at the 99.7 percentile. (d) Image formed as the product of (a) and (c). (e) Histogram of the nonzero pixels in the image in (d). (f) Result of segmenting image (a) with the Otsu threshold based on the histogram in (e). The threshold was 134, which is approximately midway between the peaks in this histogram.

Extension: variable thresholding

- Process different regions of the image using different thresholds

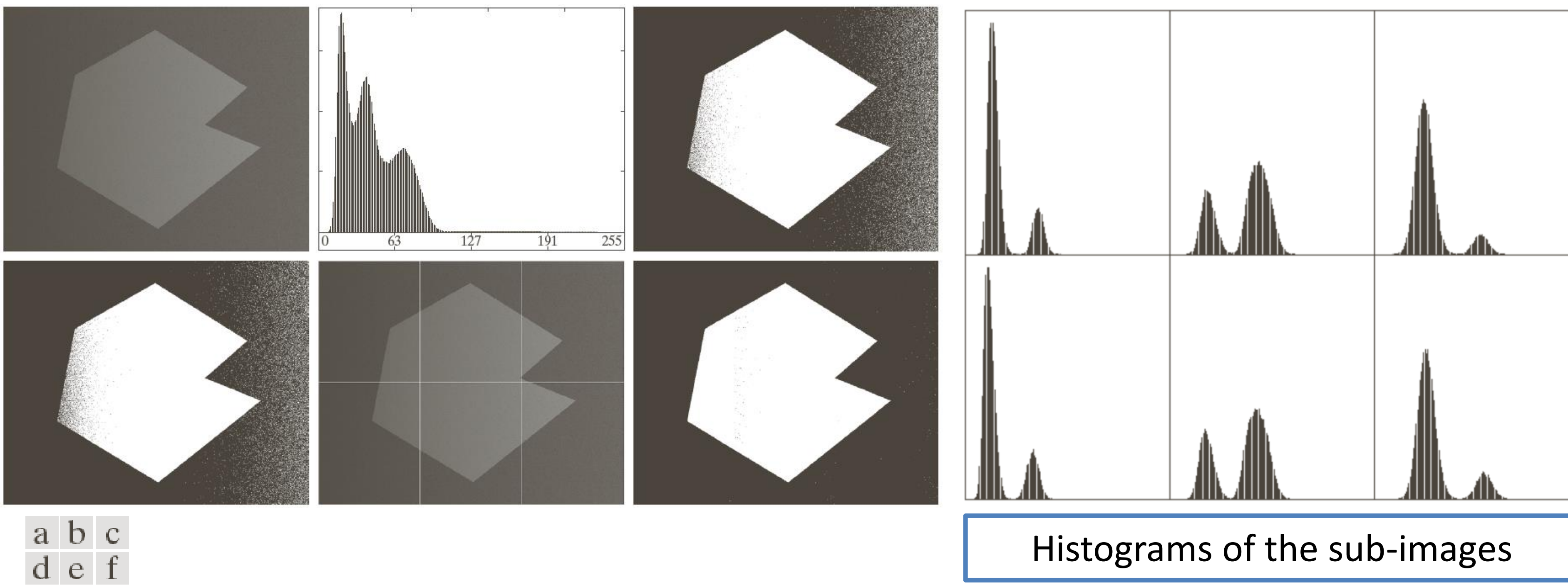


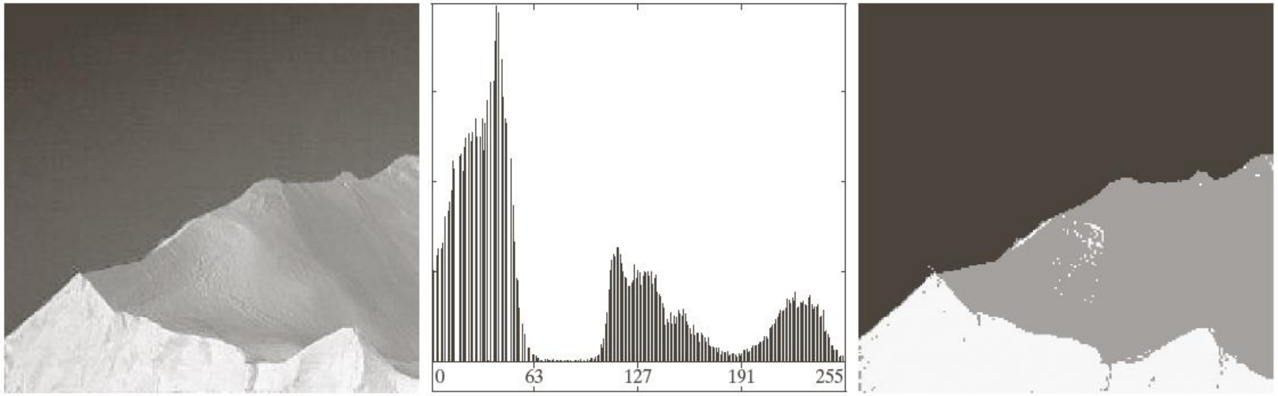
FIGURE 10.46 (a) Noisy, shaded image and (b) its histogram. (c) Segmentation of (a) using the iterative global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (e) Image subdivided into six subimages. (f) Result of applying Otsu's method to each subimage individually.

Extension: multiple thresholds

- Subdivision into multiple regions
- Maximize inter-class variance:

$$\sigma_B^2 = \sum_{j=1}^{N} P_j \left(m_j - m_G\right)^2$$

- Extension of the Otsu's method to N regions



a b c

FIGURE 10.45 (a) Image of iceberg. (b) Histogram. (c) Image segmented into three regions using dual Otsu thresholds. (Original image courtesy of NOAA.)

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Segmentation, thresholding & clustering

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