UNIVERSITÀ
DEGLI STUDI
DI PADOVA

The Hough transform

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INTELLIGENT AUTONOMOUS SYSTEMS LAB

Agenda

- Finding lines in an image
- The Hough transform
- Generalized Hough transform

Slides by Stefano Ghidoni and Pietro Zanuttigh, figures from Gonzalez & Woods

Finding lines

- Q: How to find lines in an image?

Finding lines

- Q: How to find lines in an image?
- A possible approach:
  - Compute edges
  - Consider all couples of edge points and evaluate the line passing through them
  - Count the number of edge points on such line
  - Complexity: $O(n^2)$ couples – adding comparisons: $O(n^3)$

Hough transform

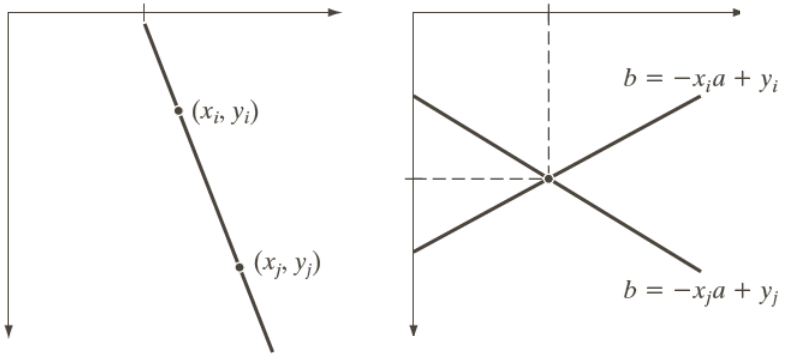
- Alternative approach: using the Hough transform
- A line passing through (x_i, y_i):

$$y_i = ax_i + \mathrm{b}$$

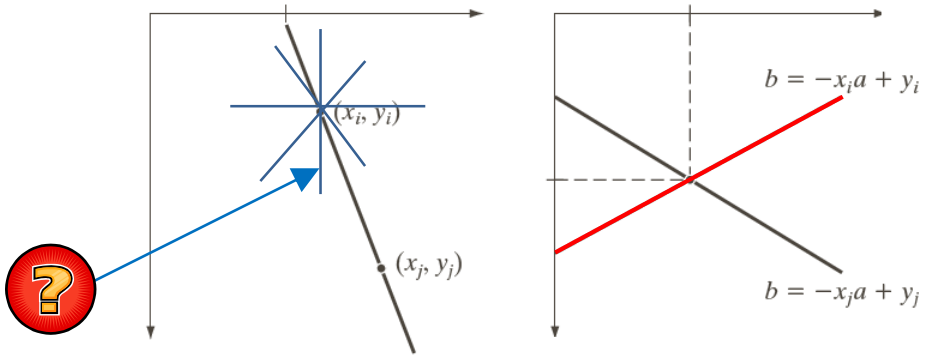
- Now consider the ab-plane and rewrite the equation as

$$b = -x_i a + y_i$$

- The ab-plane is called parameter space



Parameter space



Bundle of lines passing through a point	→	Bundle of lines passing through a point
Line through 2 points	→	Intersection of two lines

What about vertical lines?

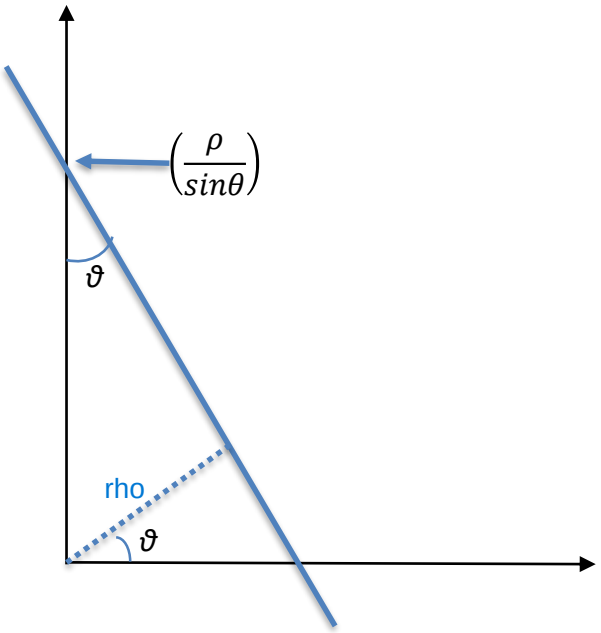
Normal representation

- Solution to the vertical line issue: **normal representation**

$$x\cos\theta + y\sin\theta = \rho$$

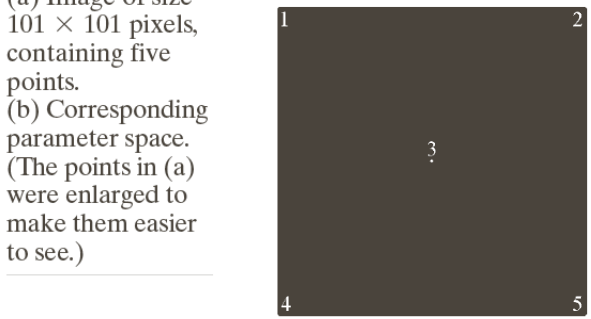
$$y = \left(-\frac{\cos\theta}{\sin\theta}\right)x + \left(\frac{\rho}{\sin\theta}\right)$$

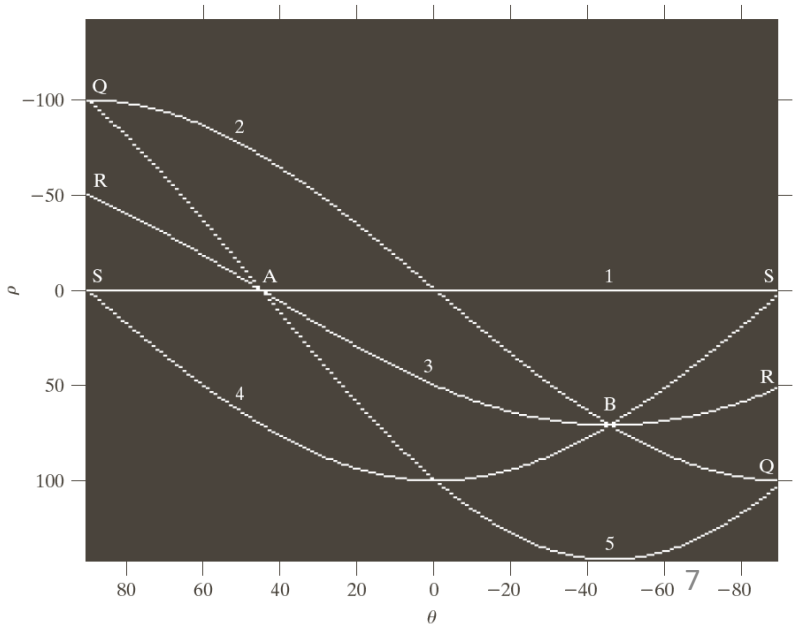
- Parameter space: sinusoidal curve



a
b

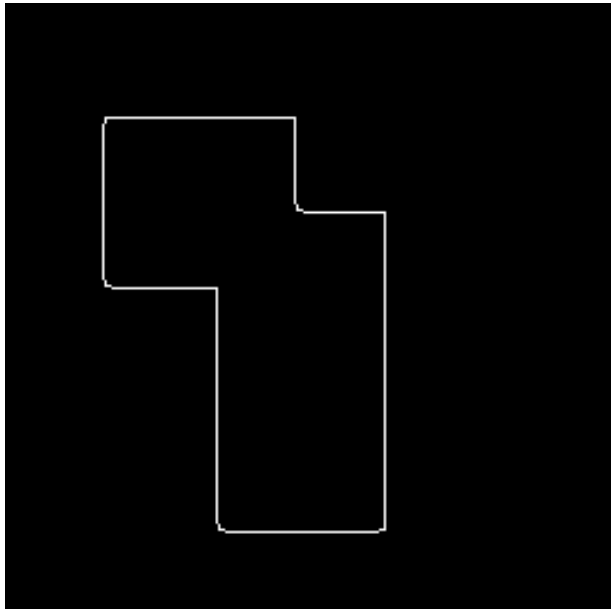
FIGURE 10.33
(a) Image of size 101 × 101 pixels, containing five points.
(b) Corresponding parameter space.
(The points in (a) were enlarged to make them easier to see.)





An example

Image

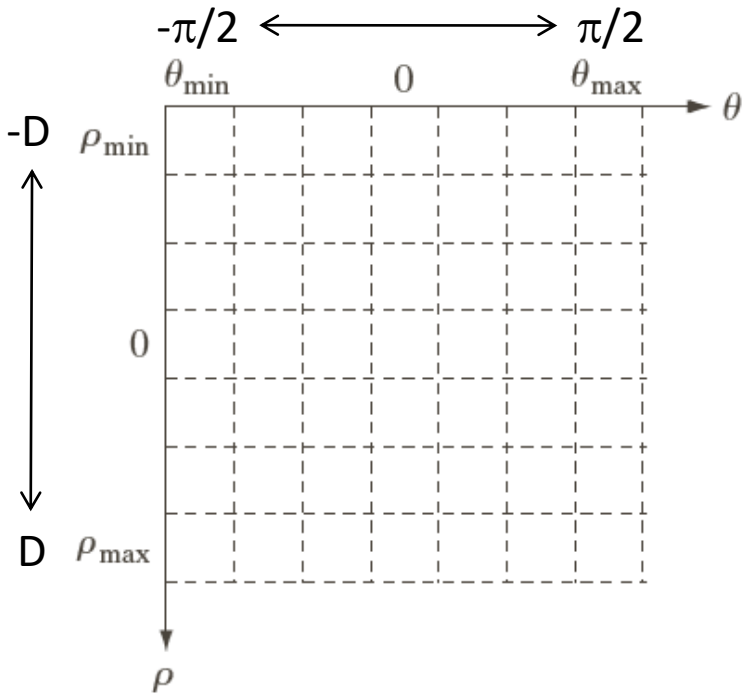


Edges

Hough transform

Accumulation cells

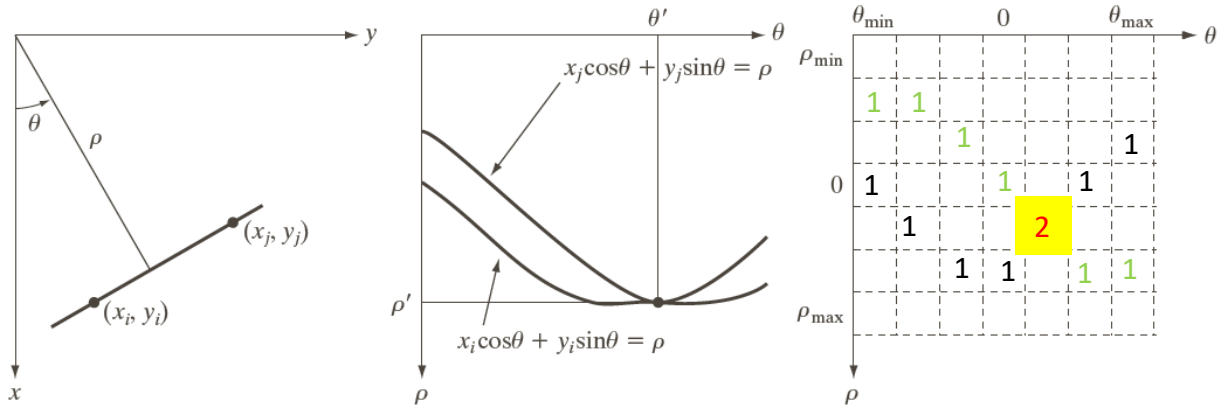
- The parameter space is quantized along ρ and θ



large	small
Few Cells	**Many Cells**
Handle pixels not perfectly aligned	Requires precise alignment
Poor lines localization	Accurate lines localization

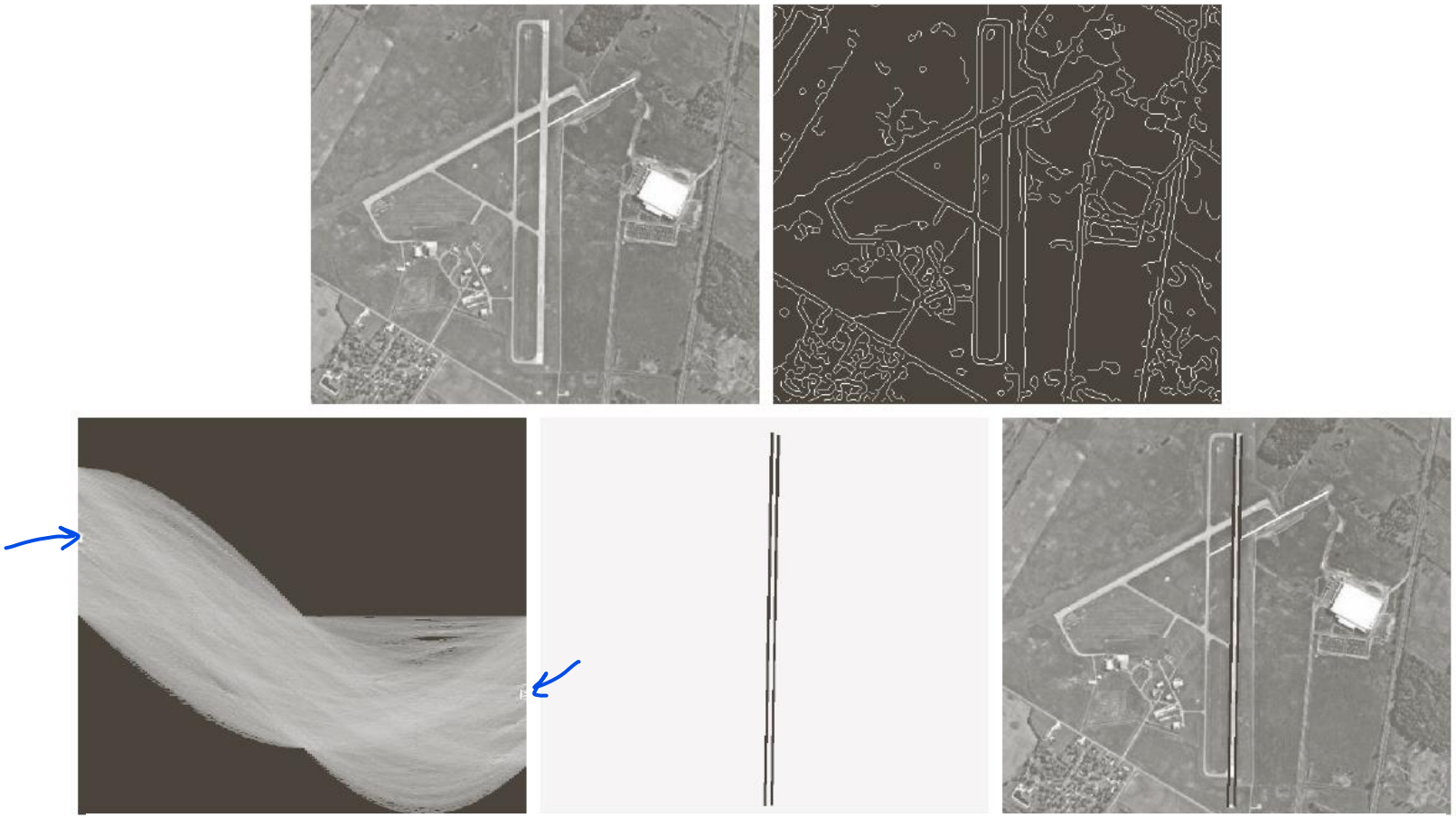
Finding lines using Hough

- For each edge pixel:
  - Compute (ρ, θ) pair values (going along the sinusoidal curve)
  - Crossed cells++
- Counter of each cell: # of pixels on that line



dove si intersecano piu linee nel dominio di hough, ci sono le coordinate rho e theta della linea corrispondente

Hough – example



a b
c d e

FIGURE 10.34 (a) A 502 × 564 aerial image of an airport. (b) Edge image obtained using Canny's algorithm. (c) Hough parameter space (the boxes highlight the points associated with long vertical lines). (d) Lines in the image plane corresponding to the points highlighted by the boxes). (e) Lines superimposed on the original image.

Generalized Hough transform

- The Hough transform works for more complex shapes
- General equation:

$$g(\boldsymbol{v}, \boldsymbol{c}) = 0$$

  - Where $\boldsymbol{v}$ is a vector of coordinates and $\boldsymbol{c}$ a vector of coefficients
- E.g. (circle): $(x - c_1)^2 + (y - c_2)^2 = c_3^2$
- The parameter space might have high dimensionality

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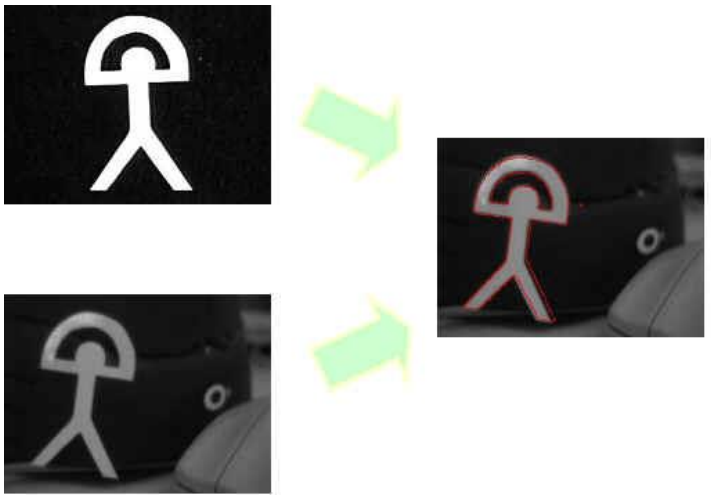
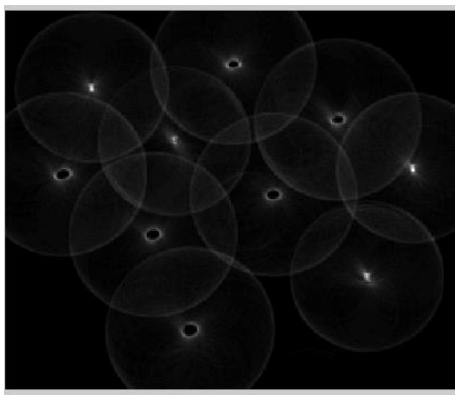
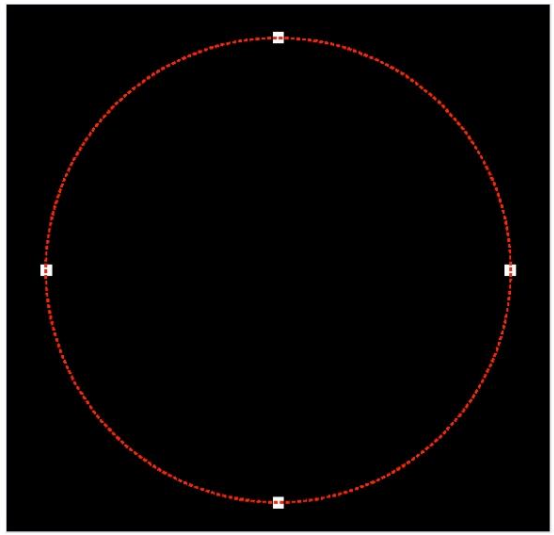
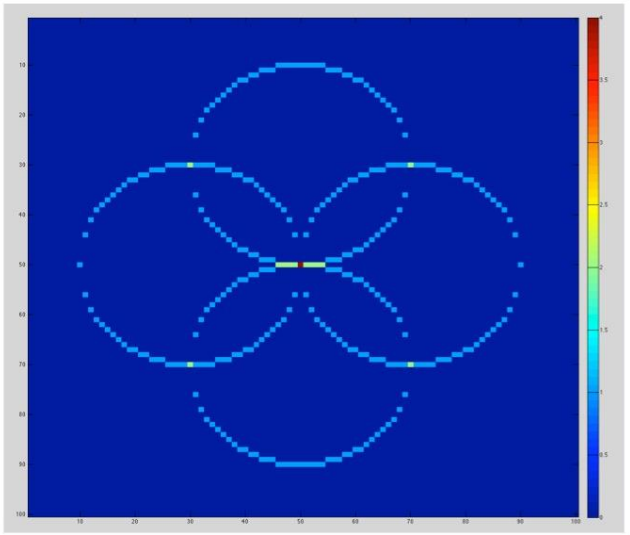
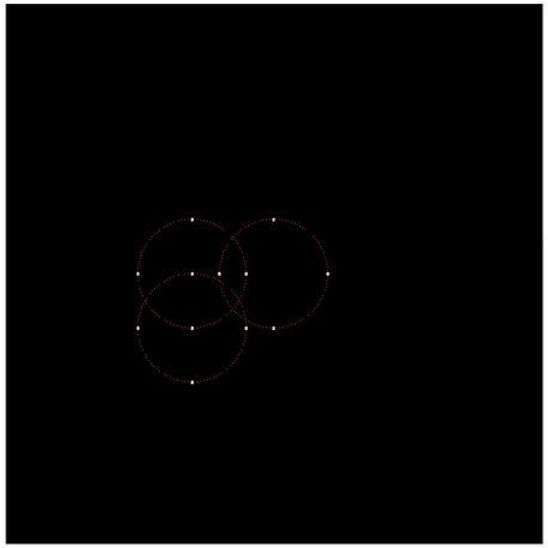


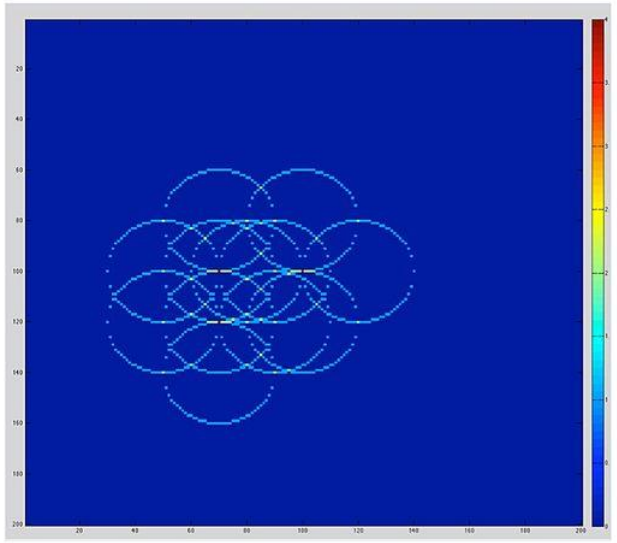


Generalized Hough transform – ex.









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Edge detection and the Hough transform

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