UNIVERSITÀ DEGLI STUDI DI PADOVA

Images in the frequency domain

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Agenda

- Fourier transform applied to images
- Phase and spectrum
- Aliasing with images

Fourier transform of images

- Images have some differences WRT commonly used signals
  - 2 dimensions
  - Only positive values for x and y

Fourier transforms in 2D

- Continuous 2D Fourier transform of a signal $f(x, y)$

$$F(u, v) = \iint_{-\infty}^{+\infty} f(x, y) e^{-j2\pi(ux+vy)} dxdy$$

$$f(x, y) = \iint_{-\infty}^{+\infty} F(u, v) e^{j2\pi(ux+vy)} dudv$$

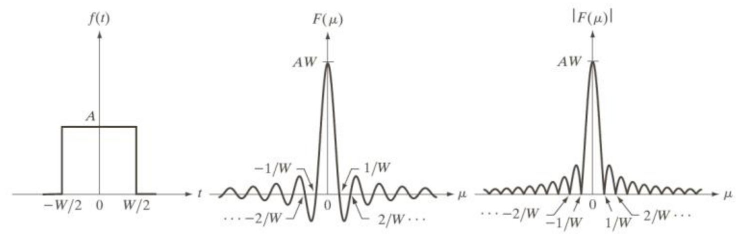
- Discrete 2D Fourier transform (DFT)

$$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$f(x, y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(u, v) e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

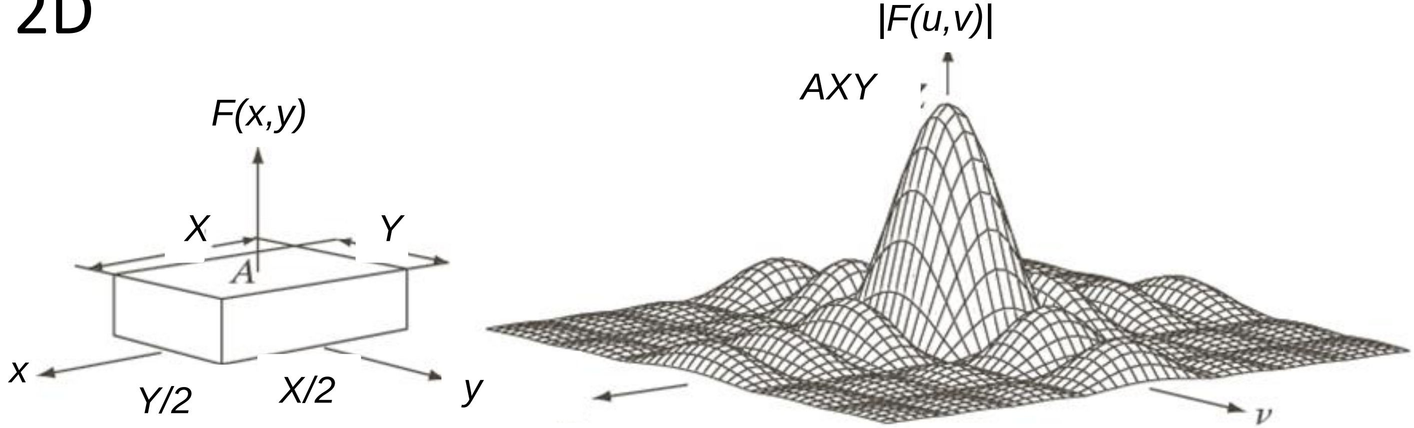
Signal reconstruction

- Recall: signal reconstructed using the rect in frequency
- Rect-sinc transform in 2D



a b c

FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

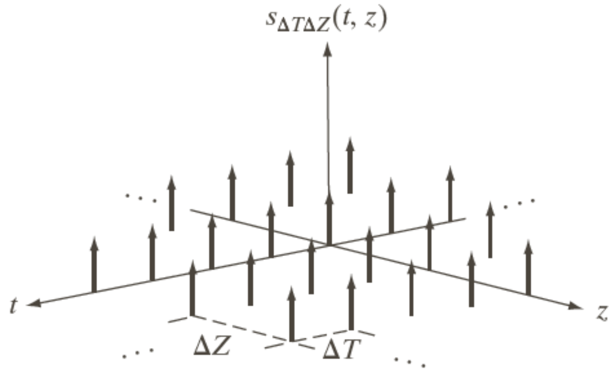


a b

FIGURE 4.13 (a) A 2-D function, and (b) a section of its spectrum (not to scale). The block is longer along the t-axis, so the spectrum is more "contracted" along the μ-axis. Compare with Fig. 4.4.

Sampling in 2D

- Sampling is performed in both directions
- Replicas are generated in both directions



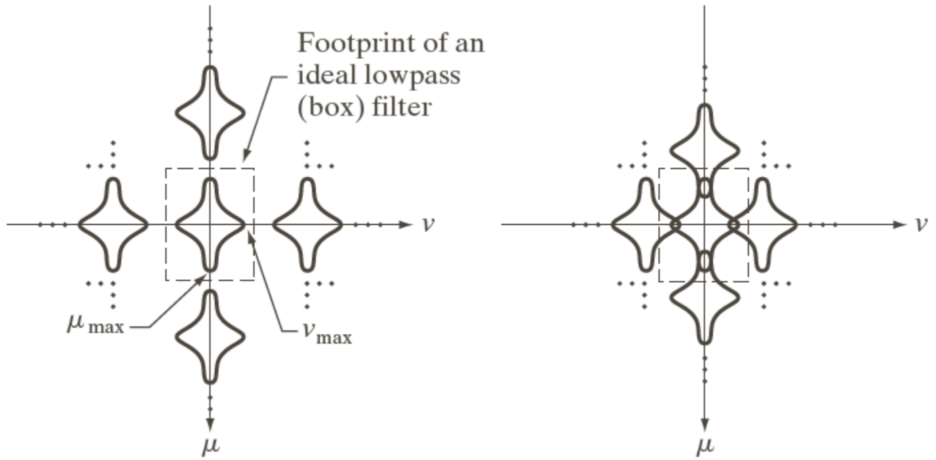


FIGURE 4.15 Two-dimensional Fourier transforms of (a) an over-sampled, and (b) under-sampled band-limited function.

Aliasing

2D sampling theorem

- The image $f(x, y)$ is uniformly sampled over an orthogonal lattice with spacing ΔX and ΔY
- The sampling theorem in 2D becomes

$$F_X = \frac{1}{\Delta X} > 2u_{max} \text{ and } F_Y = \frac{1}{\Delta Y} > 2v_{max}$$

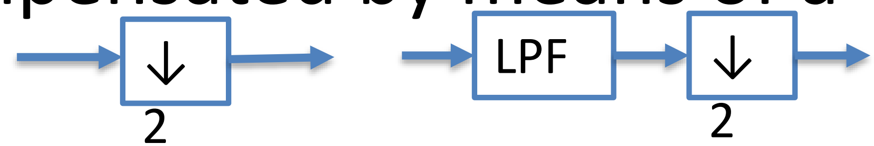
- u_{max} and v_{max} are the maximum spatial frequencies of the signal along X and Y, respectively
  - As in the 1D case: the signal is supposed to be band-limited

Aliasing

- It is interesting to see how aliasing appears in the image
- Example of undersampling (causes aliasing – why?)

2D space aliasing – example

- Aliasing can be compensated by means of a LPF

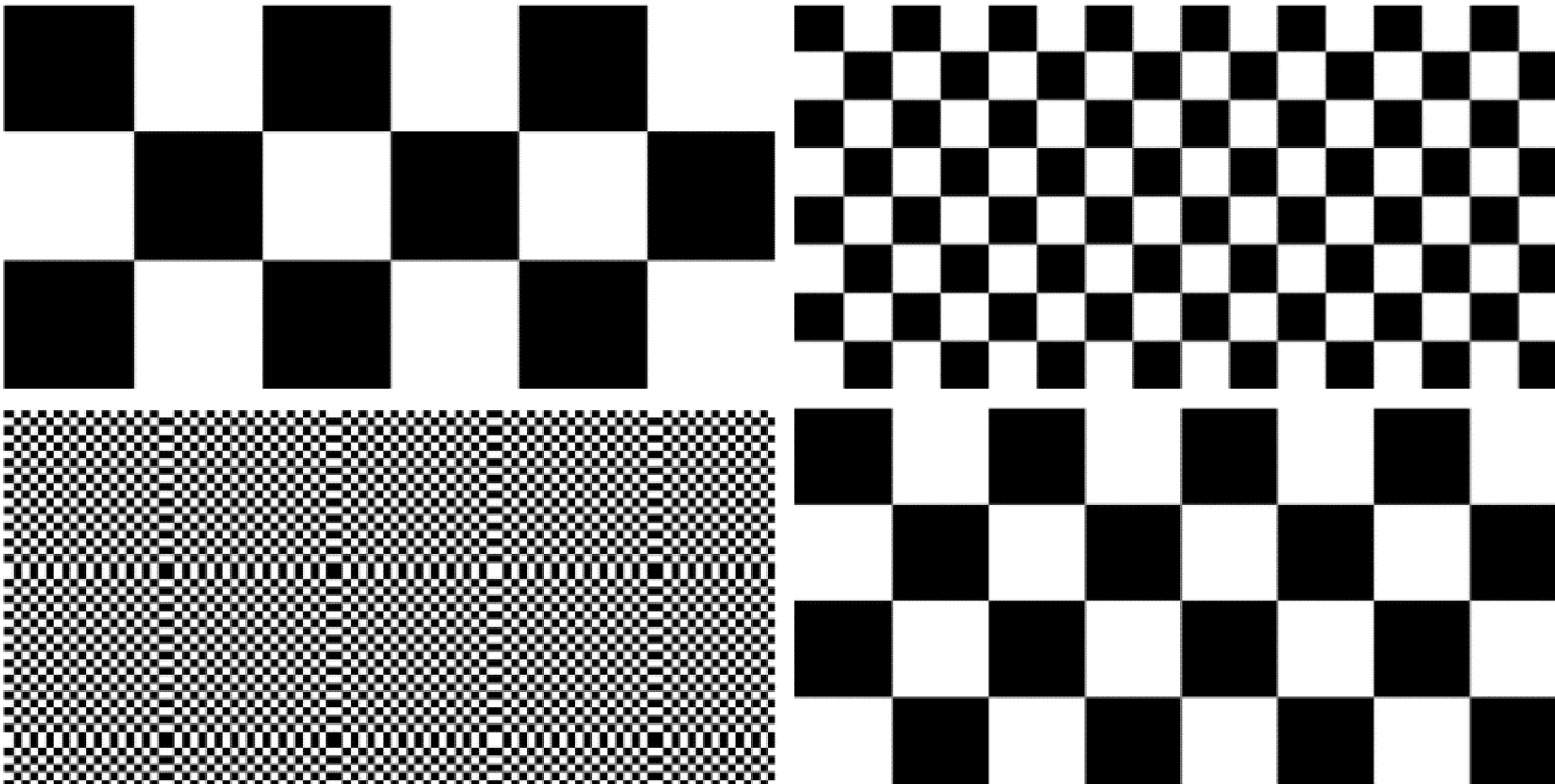


a b c

FIGURE 4.17 Illustration of aliasing on resampled images. (a) A digital image with negligible visual aliasing. (b) Result of resizing the image to 50% of its original size by pixel deletion. Aliasing is clearly visible. (c) Result of blurring the image in (a) with a 3 × 3 averaging filter prior to resizing. The image is slightly more blurred than (b), but aliasing is not longer objectionable. (Original image courtesy of the Signal Compression Laboratory, University of California, Santa Barbara.)

2D space aliasing – example

- Is there aliasing in the images?



2D space aliasing – example

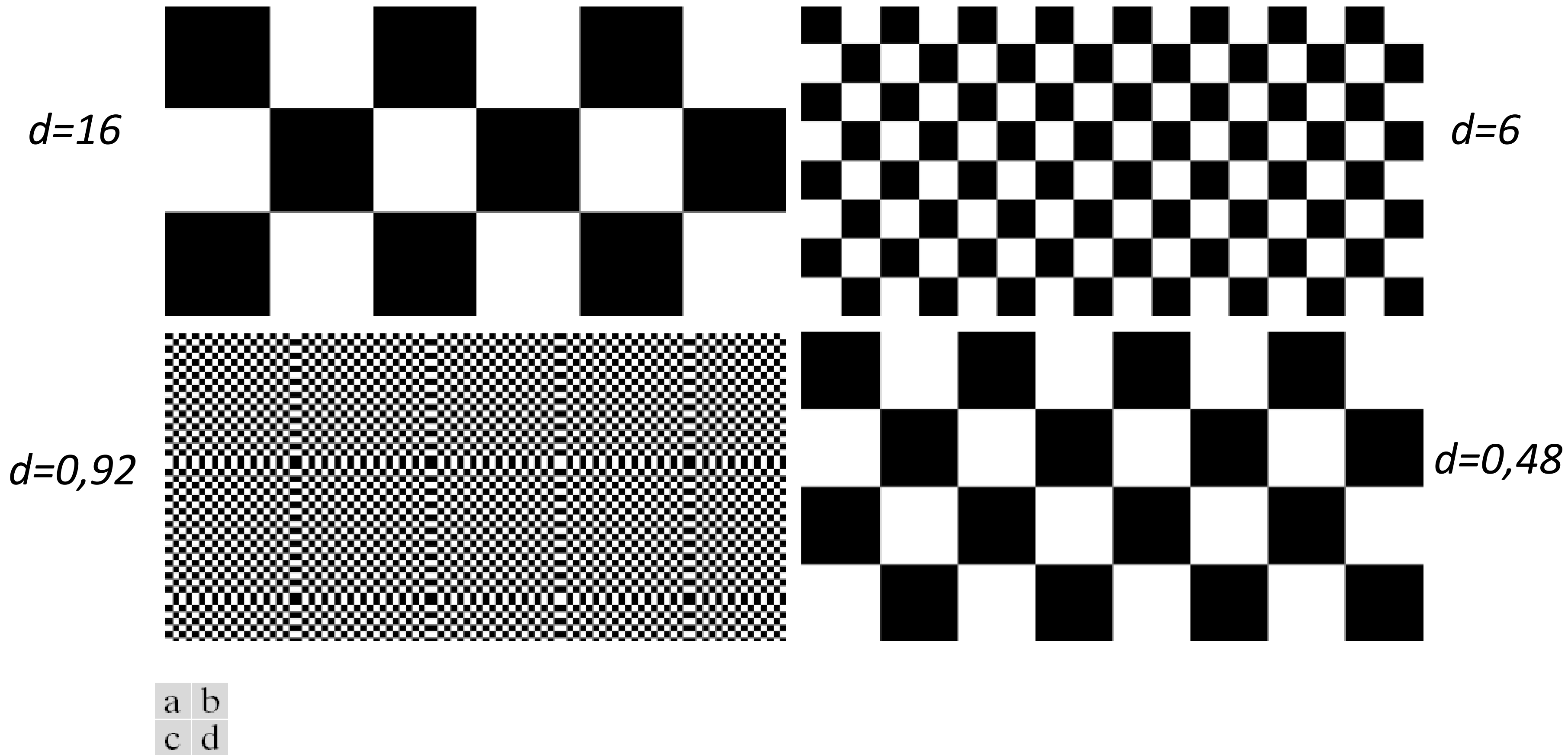


FIGURE 4.16 Aliasing in images. In (a) and (b), the lengths of the sides of the squares are 16 and 6 pixels, respectively, and aliasing is visually negligible. In (c) and (d), the sides of the squares are 0.9174 and 0.4798 pixels, respectively, and the results show significant aliasing. Note that (d) masquerades as a "normal" image.

Temporal aliasing

- Wagon wheel effect

Moiré effect

- The Moiré effect is caused by beating of similar patterns (grating with similar spacing)
  - No aliasing is involved
- It comes into play here because some patterns caused by the Moiré effect are visually similar to aliasing

Moiré effect

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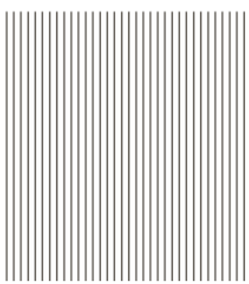

Moiré effect

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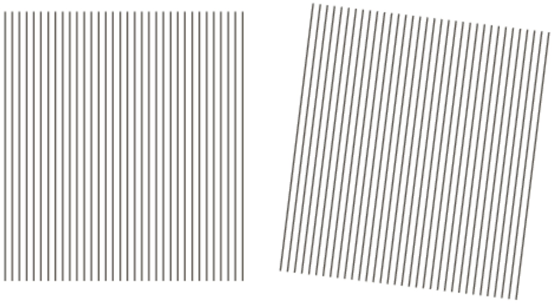

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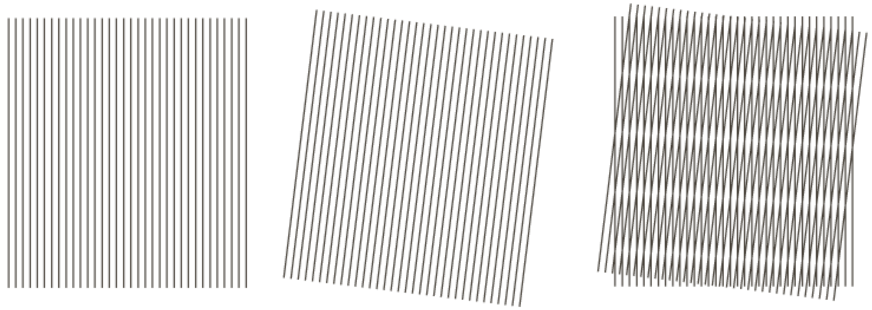

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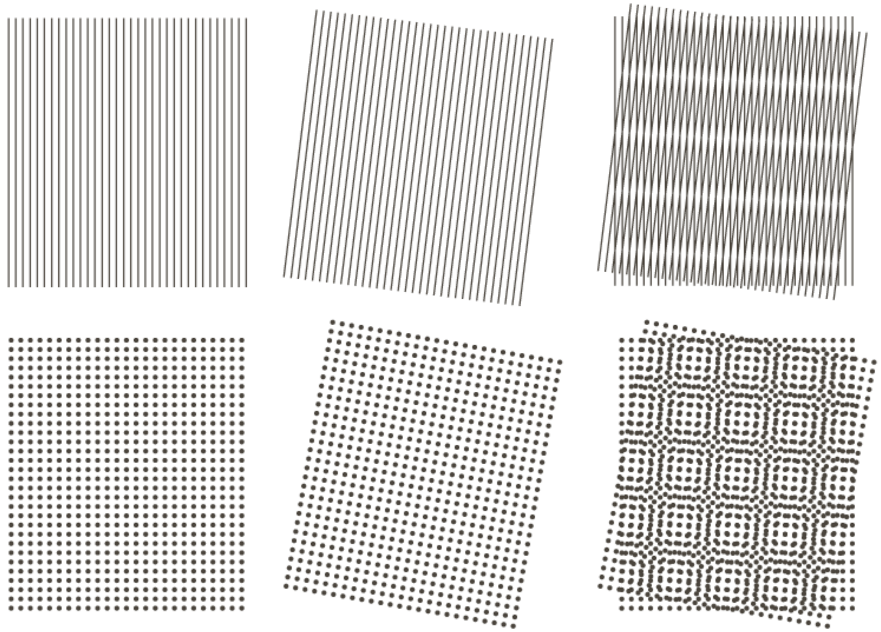

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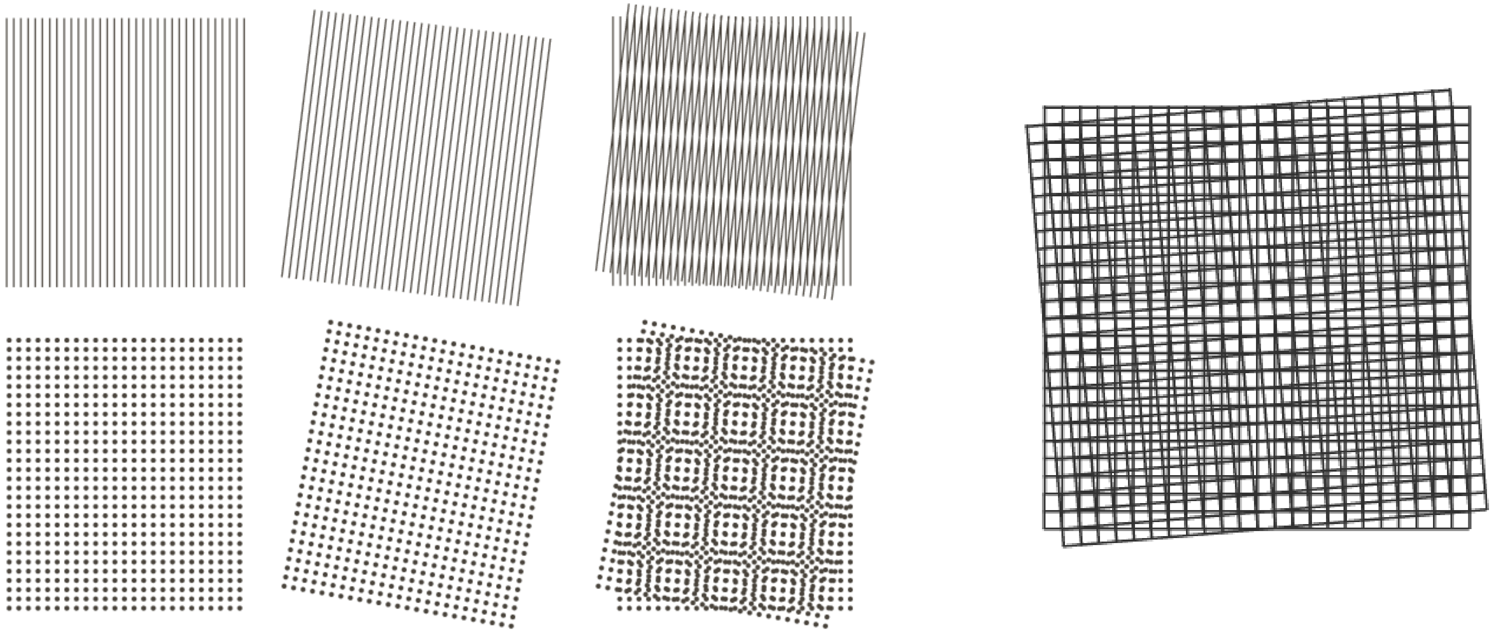


Visual appearance of spectrum and phase

Spectrum and phase

- The DFT of an image can be decomposed into
  - Spectrum
  - Phase

$$F(u,v) = |F(u,v)|e^{j\varphi(u,v)}$$

- Considering the real and imaginary parts of the transform

$$|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$$

$$\varphi(u,v) = \tan^{-1}\frac{I(u,v)}{R(u,v)}$$

Centering the Fourier spectrum

- The Fourier spectrum is usually centered on the 0 value

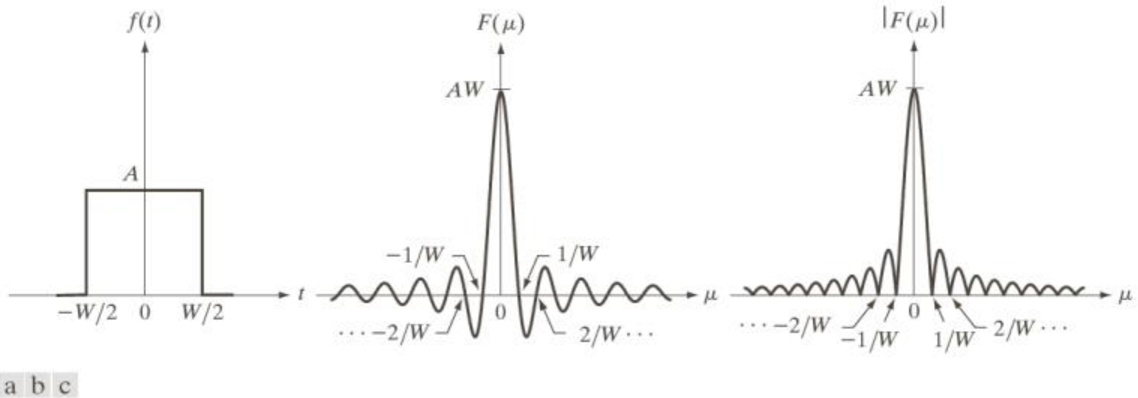
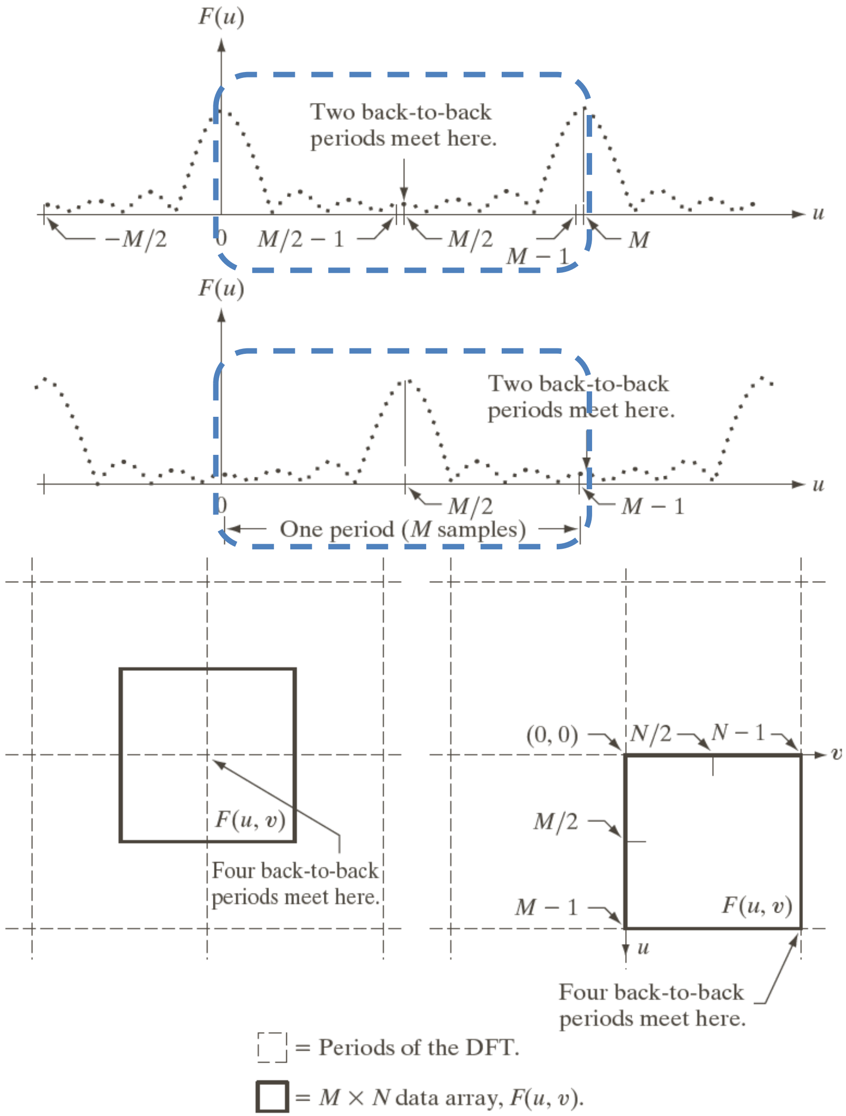


FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

Centering the Fourier spectrum

- The Fourier spectrum is usually centered on the 0 value
- A shift is typically needed to align the Fourier spectrum (expanding towards positive and negative values) with the image representation space
- Thanks to the shift, a single replica is found in the

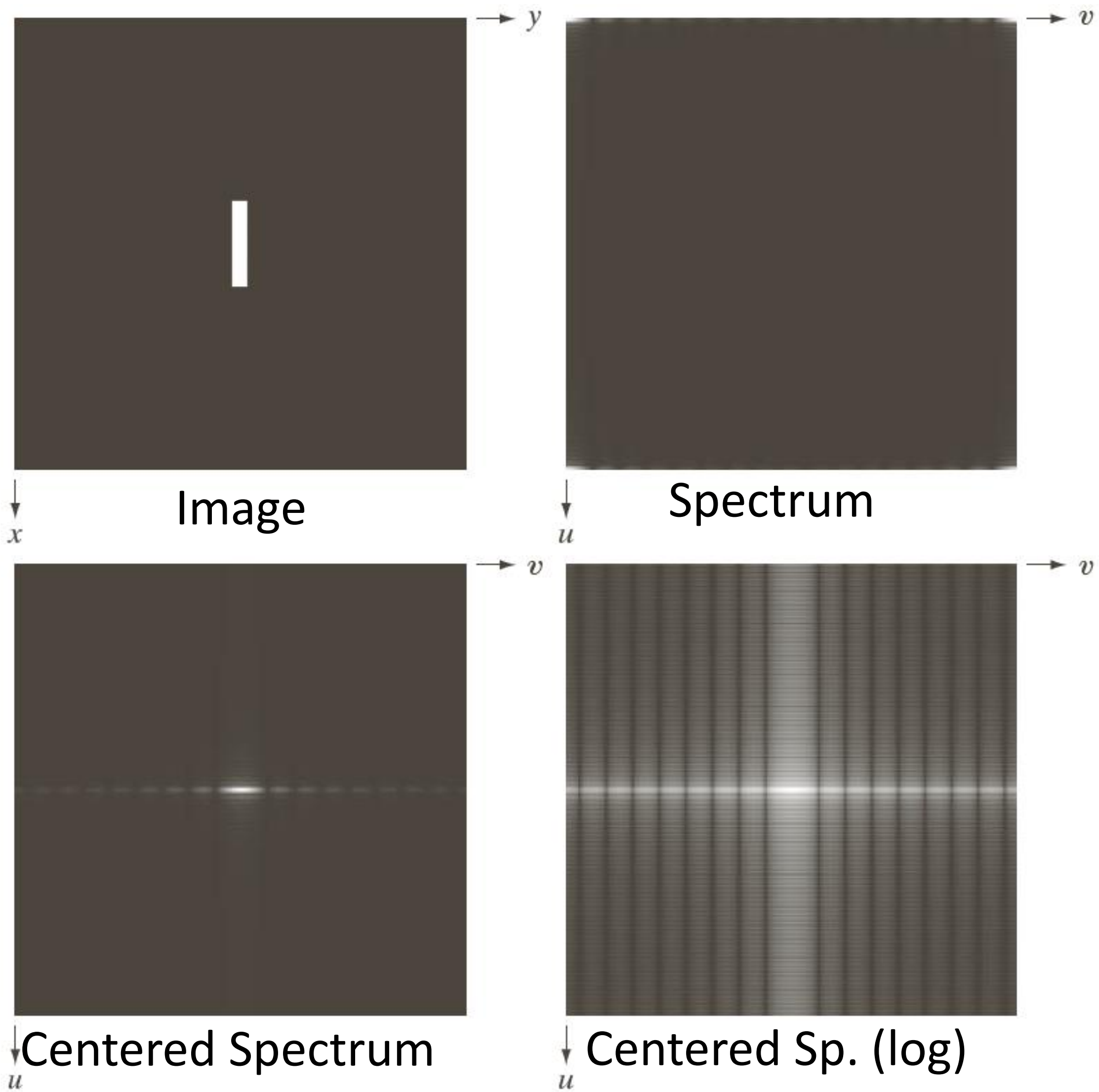
Centering the Fourier spectrum



a
b
c d

FIGURE 4.23 Centering the Fourier transform. (a) A 1-D DFT showing an infinite number of periods. (b) Shifted DFT obtained by multiplying $f(x)$ by $(-1)^x$ before computing $F(u)$. (c) A 2-D DFT showing an infinite number of periods. The solid area is the $M \times N$ data array, $F(u, v)$, obtained with Eq. (4.5-15). This array consists of four quarter periods. (d) A Shifted DFT obtained by multiplying $f(x, y)$ by $(-1)^{x+y}$ before computing $F(u, v)$. The data now contains one complete, centered period, as in (b).

Spectrum centering – example



a b
c d

FIGURE 4.24
(a) Image.
(b) Spectrum showing bright spots in the four corners.
(c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.

Rotation and translation

- Translation does not affect the spectrum
- Rotation affects the spectrum

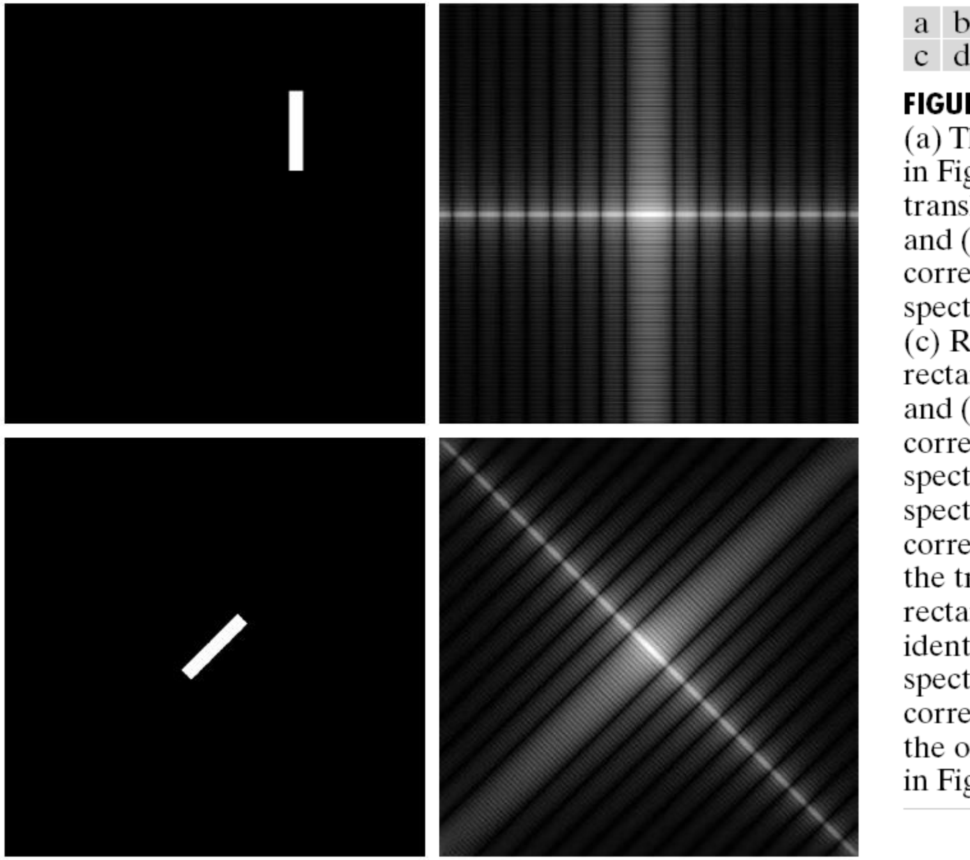


FIGURE 4.25 (a) The rectangle in Fig. 4.24(a) translated, and (b) the corresponding spectrum. (c) Rotated rectangle, and (d) the corresponding spectrum. The spectrum corresponding to the translated rectangle is identical to the spectrum corresponding to the original image in Fig. 4.24(a).

Spectrum and phase

- Both spectrum and phase encode information
- Which one encodes most peculiar info?
- Let's analyze how the image content is encoded

Phase vs spectrum

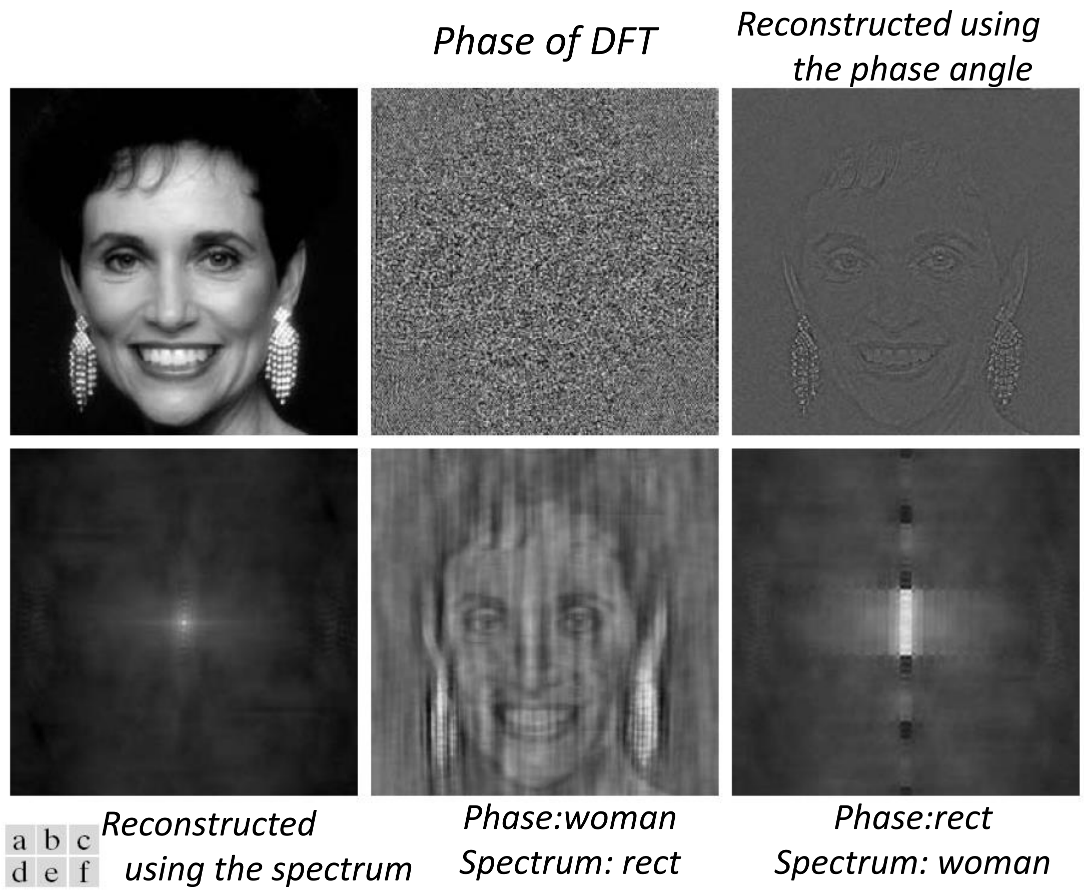


FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

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