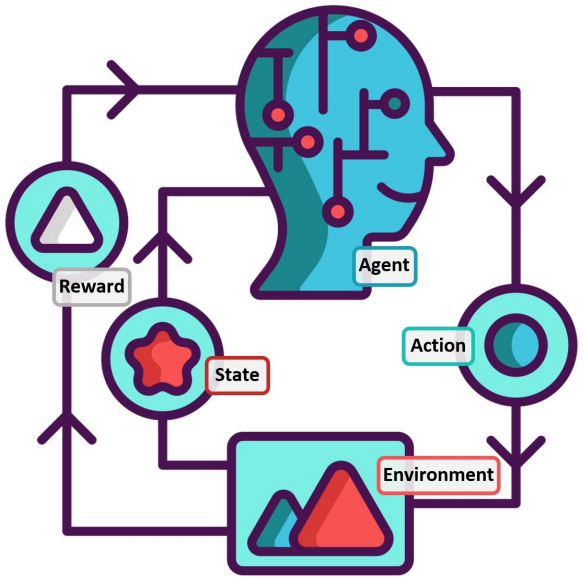
AMCO
ARTIFICIAL INTELLIGENCE, MACHINE LEARNING AND CONTROL RESEARCH GROUP

Reinforcement Learning
2025/2026

RL2
Reinforcement Learning Research Lab @ UniPD

Lecture #19
Policy Gradient Methods: Actor Critic Approaches & Batch Methods

Gian Antonio Susto



Announcements before starting

- The google form is open!
- As stated previously, if you have conflicts (for example with other partial) you are allowed to be tested on the second part of the course material during the January exam

VFA: Chapter 9 & 10 Exam Material

Chapter 9

- 9.1, 9.2, 9.3, 9.4 (no proof of convergence of linear TD), 9.7
- 9.5: 9.5.3, 9.5.4
- 9.5: optional 9.5.1, 9.5.2, 9.5.5
- 9.6: optional
- Not recommend for a first study: 9.8, 9.9, 9.10, 9.11

Chapter 10

- 10.1, 10.2
- Optional: 10.3, 10.4

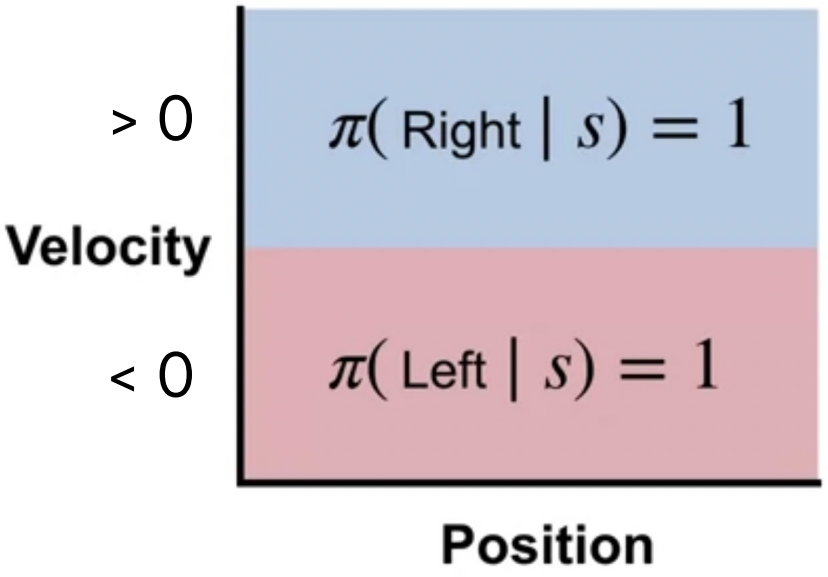
Btw, Chapter 11 (off-policy methods with approximation) is a nice reading, but not exam material!

Recap – The Mountain car problem

Example: Mountain Car problem

Was the usage of V truly necessary in this case?

the



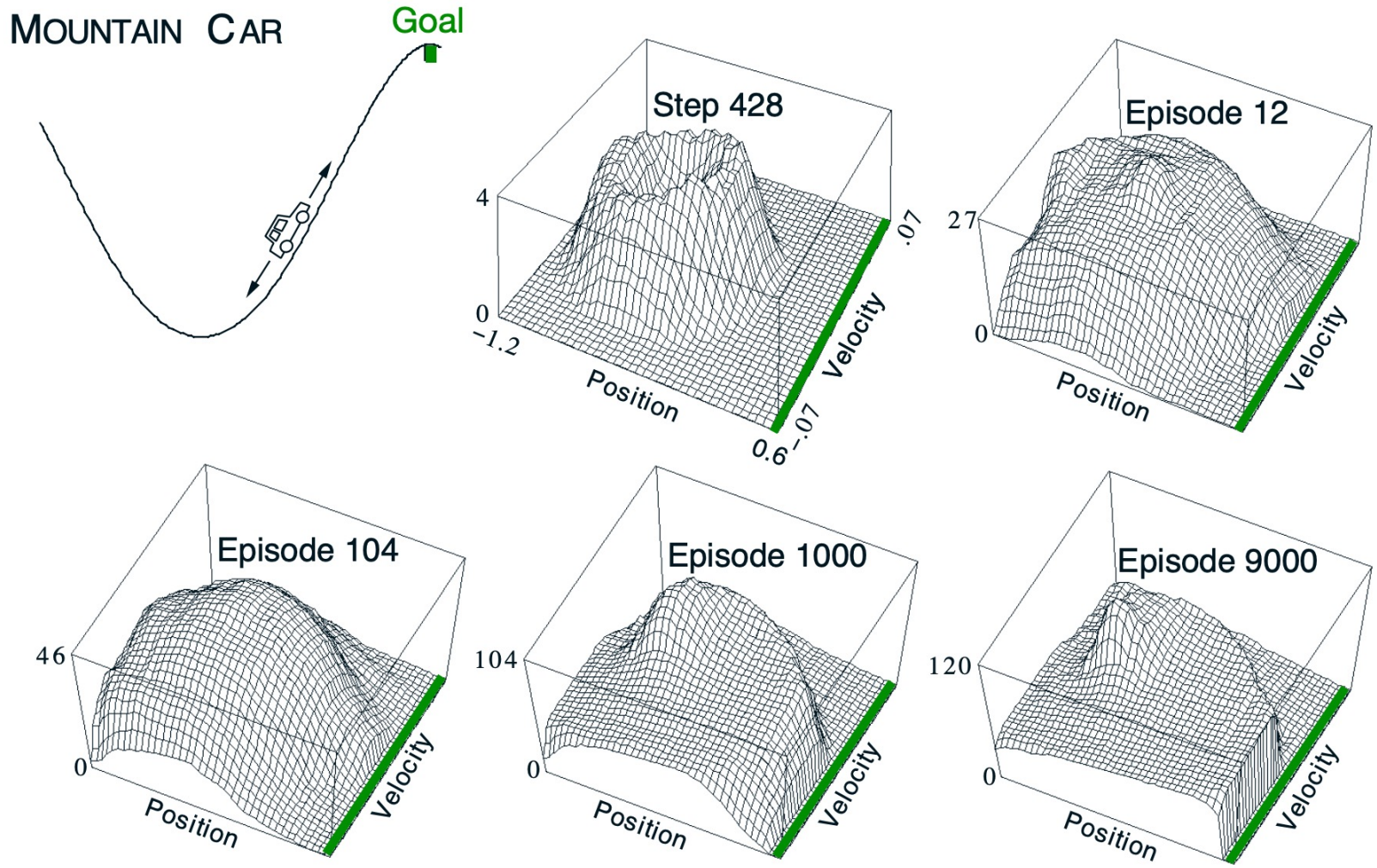
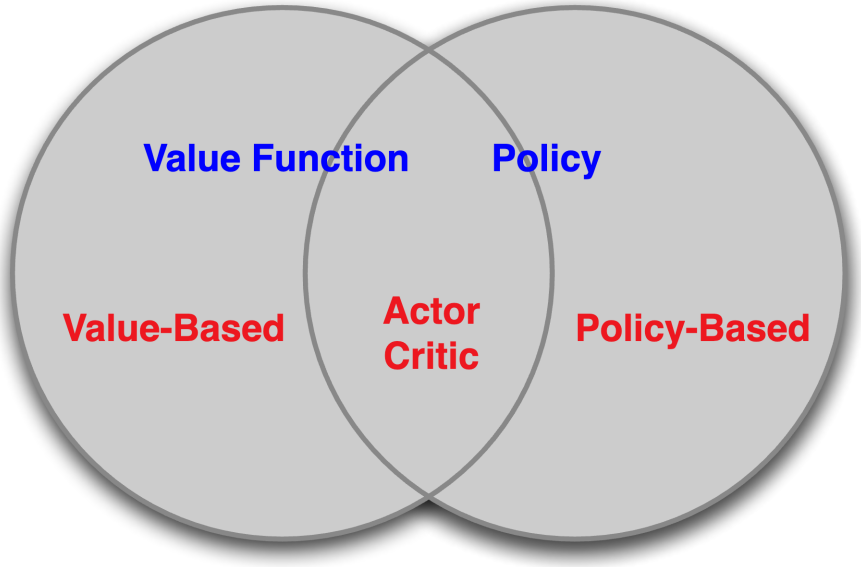


Figure 10.1: The Mountain Car task (upper left panel) and the cost-to-go function ($-\max_a \hat{q}(s, a, \mathbf{w})$) learned during one run.

Recap – There are RL approaches that do not need a value function!



Up until now we have been dealing with Value-Based approaches!

That are approaches that do not need the definition of a Value function (V and/or Q) and directly act on the policy!

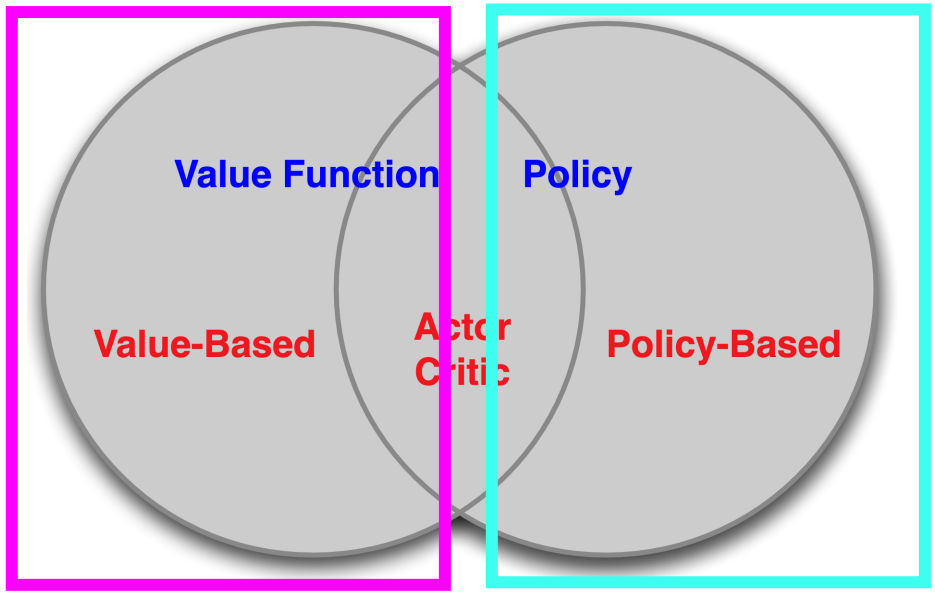
Crude taxonomy of model-free RL approaches:

Value Based
- Learnt Value Function
- Implicit policy(e.g. epsilon-greedy)

Policy Based
- No Value Function
- Learnt Policy

Actor-Critic
- Learnt Value Function
- Learnt Policy

Recap – There are RL approaches that do not need a value function!

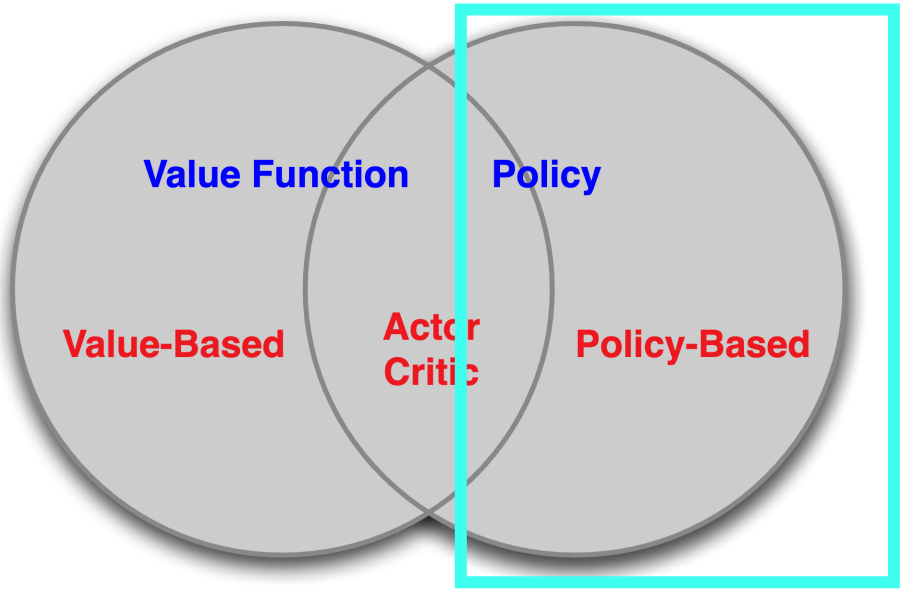


$$V_\theta(s) \approx V^\pi(s)$$
$$Q_\theta(s,a) \approx Q^\pi(s,a)$$

We will directly parametrize the policy and use data to find 'better parameters' (remember that π still need to be a distribution!)

$$\pi_\theta(s,a) = \mathbb{P}[a \mid s,\theta]$$

Recap – Policy-based approaches



Pros of policy-based approaches:

- Better convergence properties
- Effective in high-dimensional or continuous action spaces
- Can learn stochastic policies (remember, in value-based there is always a ‘max’ operation involved)

Cons:

- Typically converge to a local rather than global optimum
- Evaluating how good a policy is, it is typically inefficient and high variance

Recap – A typical Policy parametrization

- We consider that the policy is differentiable with respect to its parameters θ
- The derivative $\nabla_{\boldsymbol{\theta}} \pi(a|s, \boldsymbol{\theta})$ exists and it is always finite
- An example of policy function, the exponential softmax distribution ->

$$\pi_\theta(s, a) = \mathbb{P}[a \mid s, \theta]$$

$$\pi(a|s, \boldsymbol{\theta}) = \frac{\exp(h(s, a, \boldsymbol{\theta}))}{\sum_b \exp(h(s, b, \boldsymbol{\theta}))}$$

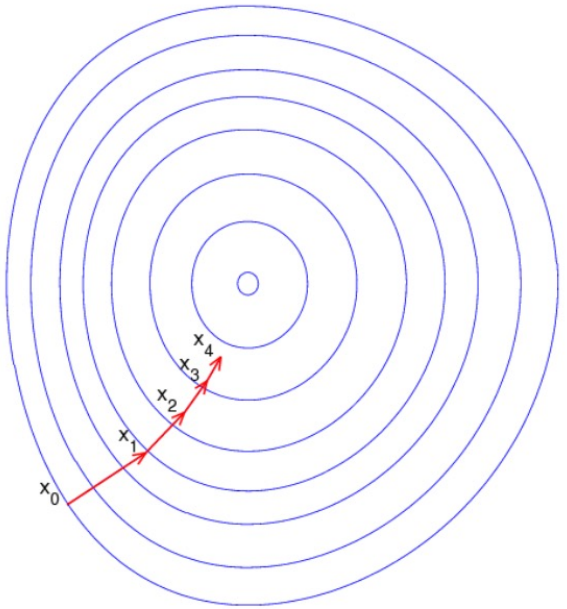
where h() are parametrized numerical preferences

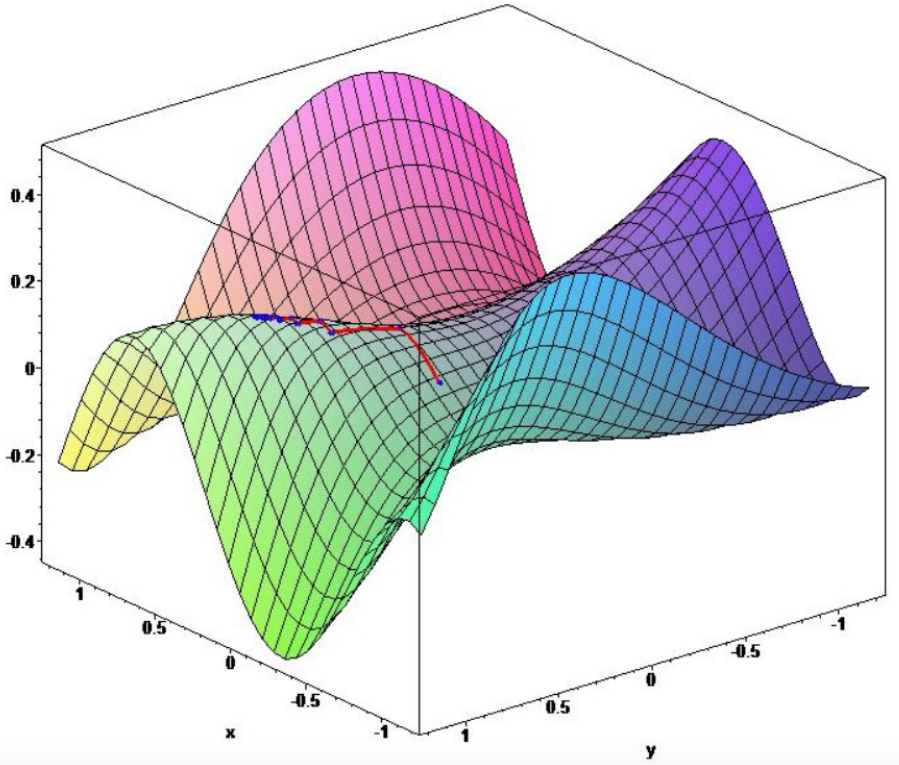
(similarly to what we have seen for example in Gradient Bandits)

Recap – Policy Gradient

- Let $J(\theta)$ be any policy objective function
- Policy gradient algorithms search for a local maximum in $J(\theta)$ by ascending the gradient of the policy w.r.t. parameters θ

$$\Delta\theta = \alpha \nabla_\theta J(\theta)$$





$$\theta_{t+1} = \theta_t + \alpha\Delta\theta$$

Step-size

$$\nabla_\theta J(\theta) = \begin{pmatrix} \frac{\partial J(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{pmatrix}$$

Policy gradient

Recap – The Policy Gradient Theorem (sec. 13.2)

It can be shown – in the episodic case – that (see if curious the proof in the book)

$$\nabla J(\boldsymbol{\theta}) \propto \sum_{s} \mu(s) \sum_{a} q_\pi(s,a) \nabla \pi(a|s,\boldsymbol{\theta})$$

Unfortunately, in stochastic gradient ascent this is not directly usable: we need a way to obtain samples (something that I can get from experience) such that the expectation of the sample gradient is proportional to the actual gradient of the performance measure as a function of the parameter

$J(\theta)$: performance measure

$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \alpha \widehat{\nabla J(\boldsymbol{\theta}_t)}$ gradient ascent

Recap – Towards REINFORCE: Monte Carlo Policy Gradient

We got rid of q (and mu) and we replace it with data coming from experience!

$$\nabla J(\boldsymbol{\theta}) \propto \sum_s \mu(s) \sum_a q_\pi(s,a) \nabla_{\boldsymbol{\theta}} \pi(a|s,\boldsymbol{\theta}),$$

$$= \mathbb{E}_\pi \left[\sum_a q_\pi(S_t,a) \nabla_{\boldsymbol{\theta}} \pi(a|S_t,\boldsymbol{\theta}) \right]$$

$$= \mathbb{E}_\pi \left[\sum_a \pi(a|S_t,\boldsymbol{\theta}) q_\pi(S_t,a) \frac{\nabla_{\boldsymbol{\theta}} \pi(a|S_t,\boldsymbol{\theta})}{\pi(a|S_t,\boldsymbol{\theta})} \right]$$

$$= \mathbb{E}_\pi \left[q_\pi(S_t,A_t) \frac{\nabla_{\boldsymbol{\theta}} \pi(A_t|S_t,\boldsymbol{\theta})}{\pi(A_t|S_t,\boldsymbol{\theta})} \right]$$

$$= \mathbb{E}_\pi \left[G_t \frac{\nabla_{\boldsymbol{\theta}} \pi(A_t|S_t,\boldsymbol{\theta})}{\pi(A_t|S_t,\boldsymbol{\theta})} \right]$$

$$(\text{because } \mathbb{E}_\pi[G_t|S_t,A_t] = q_\pi(S_t,A_t))$$

Recap – The REINFORCE update (A Monte Carlo approach)

$$\boldsymbol{\theta}_{t+1} = \boldsymbol{\theta}_t + \alpha \widehat{\nabla J(\boldsymbol{\theta}_t)}$$

$$\boldsymbol{\theta}_{t+1} \doteq \boldsymbol{\theta}_t + \alpha G_t \frac{\nabla \pi(A_t|S_t, \boldsymbol{\theta}_t)}{\pi(A_t|S_t, \boldsymbol{\theta}_t)}$$

$$\doteq \boldsymbol{\theta}_t + \alpha G_t \nabla_{\boldsymbol{\theta}} \ln \pi(A_t|S_t, \boldsymbol{\theta}_t)$$

Score function

The update increases the parameter vector in a direction:

(i) proportional to the return

(ii) inversely proportional to the action probability

Recap – The REINFORCE algorithm (A Monte Carlo approach)

REINFORCE: Monte-Carlo Policy-Gradient Control (episodic) for π_*

Input: a differentiable policy parameterization $\pi(a|s, \boldsymbol{\theta})$
Algorithm parameter: step size $\alpha > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot, \boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ (G_t)
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \gamma^t G \nabla \ln \pi(A_t|S_t, \boldsymbol{\theta})$

No V or Q is involved!

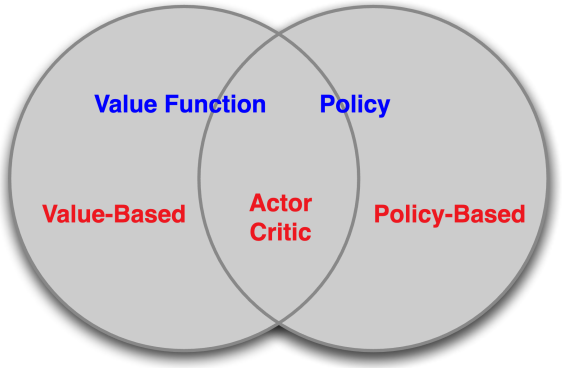
Recap – REINFORCE with baseline

REINFORCE with Baseline (episodic), for estimating $\pi_{\boldsymbol{\theta}} \approx \pi_*$

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Input: a differentiable state-value function parameterization $\hat{v}(s,\mathbf{w})$
Algorithm parameters: step sizes $\alpha^{\boldsymbol{\theta}} > 0$, $\alpha^{\mathbf{w}} > 0$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$ and state-value weights $\mathbf{w} \in \mathbb{R}^{d}$ (e.g., to $\mathbf{0}$)

Loop forever (for each episode):
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot,\boldsymbol{\theta})$
  Loop for each step of the episode $t = 0, 1, \ldots, T-1$:
    $G \leftarrow \sum_{k=t+1}^{T} \gamma^{k-t-1} R_k$ (G_t)
    $\delta \leftarrow G - \hat{v}(S_t,\mathbf{w})$
    $\mathbf{w} \leftarrow \mathbf{w} + \alpha^{\mathbf{w}} \delta \nabla \hat{v}(S_t,\mathbf{w})$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha^{\boldsymbol{\theta}} \gamma^t \delta \nabla \ln \pi(A_t|S_t,\boldsymbol{\theta})$

Actor-Critic Approaches



Actor:

- Improve the policy
- Take care of the control task

Critic:

- Evaluate the policy
- Take care of the prediction task

Actor-Critic vs REINFORCE with baseline

Although the REINFORCE with baseline method learns both a policy and a state value function, we do not consider it to be an actor–critic method

- Because its state-value function is used only as a baseline, not as a 'true' critic.
- That is, it is not used for bootstrapping (updating the value estimate for a state from the estimated values of subsequent states), but only as a baseline for the state whose estimate is being updated.
- Bootstrapping introduces bias and an asymptotic dependence on the quality of the function approximation
- The bias introduced through bootstrapping and reliance on the state representation is often beneficial because it reduces variance and accelerates learning
- REINFORCE with baseline is unbiased and will converge asymptotically to a local minimum, but like all Monte Carlo methods it tends to learn slowly (produce estimates of high variance) and to be inconvenient to implement online or for continuing problems.

Actor-Critic

From the policy gradient theorem:

$$\nabla J(\theta) = \mathbb{E}_\pi \left[\nabla \ln \pi(a|s,\theta) q_\pi(s,a) \right]$$

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What about approximating it ? $\hat{q}_\pi(s,a,\mathbf{w}) \approx q_\pi(s,a)$

Actor-Critic

From the policy gradient theorem:

$$\nabla J(\theta) = \mathbb{E}_\pi \left[\nabla \ln \pi(a|s,\theta) q_\pi(s,a) \right]$$

What about approximating it ? $\hat{q}_\pi(s,a,\mathbf{w}) \approx q_\pi(s,a)$

- We use a critic to estimate the action-value function
- Actor-critic algorithm maintain two sets of parameters:

➔ Critic update action-value function parameters **w**

➔ Actor updates policy parameters θ in the direction suggested by the critic

- The critic is solving the prediction problem, and we can re-use what we already know (Monte Carlo, TD learning, TD(lambda))

QAC: (q) Actor-Critic

- Linear function approximation

$$\hat{q}_\pi(s, a, \mathbf{w}) = \mathbf{x}(s, a)^T \mathbf{w}$$

- Critic: TD(0)

Algorithm QAC

Initialize policy parameter θ and w

Algorithm parameter : step-size $\alpha, \beta > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s, \theta)$

Loop for each step of the episode $t = 0, \ldots, T-1$

Observe reward r and new state s'

Sample action $a' \sim \pi(a'|s', \theta)$

$$\delta = r + \gamma \hat{q}_\pi(s', a', \mathbf{w}) - \hat{q}_\pi(s, a, \mathbf{w})$$

$$\theta = \theta + \alpha \nabla_\theta \ln \pi(a|s, \theta)\, \hat{q}_\pi(s, a, \mathbf{w})$$

$$\mathbf{w} = \mathbf{w} + \beta \delta \mathbf{x}(s, a)$$

$$a \leftarrow a', s \leftarrow s'$$

QAC: (q) Actor-Critic

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$$\mathbf{w} = \mathbf{w} + \beta \delta \mathbf{x}(s, a)$$

$$a \leftarrow a', s \leftarrow s'$$

The book show you directly the TD(lambda) version of the algorithm

QAC: (q) Actor-Critic

- Linear function approximation

$$\hat{q}_\pi(s, a, \mathbf{w}) = \mathbf{x}(s, a)^T \mathbf{w}$$

- Critic: TD(0)

Algorithm QAC

Actor - control

Initialize policy parameter θ and w

Algorithm parameter : step-size $\alpha, \beta > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s, \theta)$

Loop for each step of the episode $t = 0, \ldots, T-1$

Observe reward r and new state s'

Sample action $a' \sim \pi(a'|s', \theta)$

$\delta = r + \gamma \hat{q}_\pi(s', a', \mathbf{w}) - \hat{q}_\pi(s, a, \mathbf{w})$

$\theta = \theta + \alpha \nabla_\theta \ln \pi(a|s, \theta)\, \hat{q}_\pi(s, a, \mathbf{w})$

$\mathbf{w} = \mathbf{w} + \beta \delta \mathbf{x}(s, a)$

$a \leftarrow a', s \leftarrow s'$

QAC: (q) Actor-Critic

- Linear function approximation

$$\hat{q}_\pi(s, a, \mathbf{w}) = \mathbf{x}(s, a)^T \mathbf{w}$$

- Critic: TD(0)

Algorithm QAC

Initialize policy parameter θ and w

Algorithm parameter : step-size α $\beta > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s, \theta)$

Loop for each step of the episode $t = 0, \ldots, T-1$

Observe reward r and new state s'

Sample action $a' \sim \pi(a'|s', \theta)$

$$\delta = r + \gamma \hat{q}_\pi(s', a', \mathbf{w}) - \hat{q}_\pi(s, a, \mathbf{w})$$

$$\theta = \theta + \alpha \nabla_\theta \ln \pi(a|s, \theta)\, \hat{q}_\pi(s, a, \mathbf{w})$$

$$\mathbf{w} = \mathbf{w} + \beta \delta \mathbf{x}(s, a)$$

$a \leftarrow a', s \leftarrow s'$

Critic - prediction

QAC: (q) Actor-Critic

- Linear function approximation

$$\hat{q}_\pi(s, a, \mathbf{w}) = \mathbf{x}(s, a)^T \mathbf{w}$$

- Critic: TD(0)

Algorithm QAC

Initialize policy parameter θ and w

Algorithm parameter : step-size $\alpha, \beta > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s, \theta)$

Loop for each step of the episode $t = 0, \ldots, T-1$

Observe reward r and new state s'

Sample action $a' \sim \pi(a'|s', \theta)$

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$$\mathbf{w} = \mathbf{w} + \beta \delta \mathbf{x}(s, a)$$

$$a \leftarrow a', s \leftarrow s'$$

Updates are done are each step! This version is a TD approach

QAC: (q) Actor-Critic

- Linear function approximation

$$\hat{q}_\pi(s, a, \mathbf{w}) = \mathbf{x}(s, a)^T \mathbf{w}$$

- Critic: TD(0)

Algorithm QAC

Initialize policy parameter θ and w

Algorithm parameter : step-size $\alpha, \beta > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s, \theta)$

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$$\mathbf{w} = \mathbf{w} + \beta \delta \mathbf{x}(s, a)$$

$$a \leftarrow a', s \leftarrow s'$$

Note that at each step, I am changing policy! Is a sort of value iteration...

Actor-Critic with a baseline: advantage function 1/2

- A good baseline is:

$$b(s) = v_{\pi_\theta}(s)$$

- Let us introduce the advantage function:

$$A_{\pi_\theta}(s,a) = q_{\pi_\theta}(s,a) - v_{\pi_\theta}(s)$$

in the case of optimal policy A=0 for optimal action in state s

- We can have a re-write the estimator of the gradient considering also the new baseline:

$$\nabla J(\theta) = \mathbb{E}_\pi \left[\nabla \ln \pi(a|s,\theta)\, A_{\pi_\theta}(s,a) \right]$$

Actor-Critic with a baseline: advantage function 1/2

- A good baseline is:

$$b(s) = v_{\pi_\theta}(s)$$

What is the advantage in the case of *a* already according to the policy?

- Let us introduce the advantage function:

$$A_{\pi_\theta}(s,a) = q_{\pi_\theta}(s,a) - v_{\pi_\theta}(s)$$

in the case of optimal policy A=0 for optimal action in state s

- We can have a re-write the estimator of the gradient considering also the new baseline:

$$\nabla J(\theta) = \mathbb{E}_\pi \left[\nabla \ln \pi(a|s,\theta)\, A_{\pi_\theta}(s,a) \right]$$

Actor-Critic with a baseline: advantage function 2/2

- The advantage function can significantly reduce variance of policy gradient
- The critic should really estimate the advantage function
- For example, by estimating both q and v
- Using two function approximators and two parameter vectors

$$\hat{v}_{\pi_\theta}(s, \mathbf{v}) \approx v_{\pi_\theta}(s)$$

$$\hat{q}_{\pi_\theta}(s, a, \mathbf{w}) \approx q_{\pi_\theta}(s, a)$$

$$A_{\pi_\theta}(s, a) = \hat{q}_{\pi_\theta}(s, a, \mathbf{w}) - \hat{v}_{\pi_\theta}(s, \mathbf{v})$$

- And updating both value functions (for example by TD learning)

Initialize policy parameter θ, w, v

Algorithm parameter : step-size $\alpha, \beta, \xi > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s, \theta)$

Loop for each step of the episode $t = 0, \ldots, T-1$

Observe reward r and new state s'

Sample action $a' \sim \pi(a'|s', \theta)$

$$\delta_q = r + \gamma \hat{q}_\pi(s', a', \mathbf{w}) - \hat{q}_\pi(s, a, \mathbf{w})$$

$$\delta_v = r + \gamma \hat{v}_\pi(s', \mathbf{v}) - \hat{v}_\pi(s, \mathbf{v})$$

$$\theta = \theta + \alpha \left(\hat{q}_\pi(s, a, \mathbf{w}) - \hat{v}_\pi(s, \mathbf{v})\right) \nabla_\theta \ln \pi(a|s, \theta)$$

$$\mathbf{w} = \mathbf{w} + \beta \delta_q \nabla \hat{q}_\pi(s, a, \mathbf{w})$$

$$\mathbf{v} = \mathbf{v} + \xi \delta_v \nabla \hat{v}(s, \mathbf{v})$$

$a \leftarrow a', s \leftarrow s'$

- Linear function approximation
- Critic: TD(0)

Initialize policy parameter θ w, v

Algorithm parameter : step-size $\alpha, \beta, \xi > 0$

Actor is the same...

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s,\theta)$

Loop for each step of the episode $t = 0, \ldots, T-1$

Observe reward r and new state s'

Sample action $a' \sim \pi(a'|s',\theta)$

$$\delta_q = r + \gamma \hat{q}_\pi(s', a', \mathbf{w}) - \hat{q}_\pi(s, a, \mathbf{w})$$

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$$\mathbf{v} = \mathbf{v} + \xi \delta_v \nabla \hat{v}(s, \mathbf{v})$$

$a \leftarrow a'$, $s \leftarrow s'$

Initialize policy parameter θ, w, v

Algorithm parameter : step-size $\alpha, \beta, \xi > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s,\theta)$

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Observe reward r and new state s'

Sample action $a' \sim \pi(a'|s',\theta)$

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$$\theta = \theta + \alpha \left(\hat{q}_\pi(s, a, \mathbf{w}) - \hat{v}_\pi(s, \mathbf{v})\right) \nabla_\theta \ln \pi(a|s,\theta)$$

$$\mathbf{w} = \mathbf{w} + \beta \delta_q \nabla \hat{q}_\pi(s, a, \mathbf{w})$$

$$\mathbf{v} = \mathbf{v} + \xi \delta_v \nabla \hat{v}(s, \mathbf{v})$$

$a \leftarrow a', s \leftarrow s'$

Critic is now composed by both a v and q components

Initialize policy parameter θ, w, v

Algorithm parameter : step-size $\alpha, \beta, \xi > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s, \theta)$

Loop for each step of the episode $t = 0, \ldots, T-1$

Observe reward r and new state s'

Sample action $a' \sim \pi(a'|s', \theta)$

$$\delta_q = r + \gamma \hat{q}_\pi(s', a', \mathbf{w}) - \hat{q}_\pi(s, a, \mathbf{w})$$

$$\delta_v = r + \gamma \hat{v}_\pi(s', \mathbf{v}) - \hat{v}_\pi(s, \mathbf{v})$$

$$\theta = \theta + \alpha \left(\hat{q}_\pi(s, a, \mathbf{w}) - \hat{v}_\pi(s, \mathbf{v})\right) \nabla_\theta \ln \pi(a|s, \theta)$$

$$\mathbf{w} = \mathbf{w} + \beta \delta_q \nabla \hat{q}_\pi(s, a, \mathbf{w})$$

$$\mathbf{v} = \mathbf{v} + \xi \delta_v \nabla \hat{v}(s, \mathbf{v})$$

$a \leftarrow a', s \leftarrow s'$

Let's try to simplify the algorithm...

Estimating the advantage function

- Consider the TD error $\delta_{\pi_\theta} = r + \gamma v_{\pi_\theta}(s') - v_{\pi_\theta}(s)$

- The TD error is an unbiased estimate of the advantage function

$$\begin{aligned}\mathbb{E}\left[\delta_{\pi_\theta}|s,a\right] &= \mathbb{E}\left[r + \gamma v_{\pi_\theta}(s')|s,a\right] - v_{\pi_\theta}(s) \\ &= q_{\pi_\theta}(s,a) - v_{\pi_\theta}(s) \\ &= A_{\pi_\theta}(s,a)\end{aligned}$$

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- Let's exploit this:

$$\nabla J(\theta) = \mathbb{E}_\pi\left[\nabla \ln \pi(a|s,\theta)\, A_{\pi_\theta}(s,a)\right]$$

$$\nabla J(\theta) = \mathbb{E}_\pi\left[\nabla_\theta \ln \pi(a|s,\theta)\, \delta_{\pi_\theta}\right]$$

Estimating the advantage function

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In practice we can use an approximator:

$$\delta_{\mathbf{v}} = r + \gamma \hat{v}(s', \mathbf{v}) - \hat{v}(s, \mathbf{v})$$

Initialize policy parameter θ, v

Algorithm parameter : step-size α, $\beta > 0$

Repeat (for each episode) :

Initialize s and take an action $a \sim \pi(a|s,\theta)$

Loop for each step of the episode $t = 0, \ldots, T-1$

Observe reward r and new state s'

$$\delta_{\mathbf{v}} = r + \gamma \hat{v}(s', \mathbf{v}) - \hat{v}(s, \mathbf{v})$$

$$\theta = \theta + \alpha\, \delta_{\mathbf{v}}\, \nabla_\theta \ln \pi(a|s,\theta)$$

$$\mathbf{v} = \mathbf{v} + \beta \delta_{\mathbf{v}} \nabla \hat{v}(s, \mathbf{v})$$

$s \leftarrow s'$

Now the critic is simplified! We get only 1 ste of parameters

Critics at different time-scales

- Critic can estimate value function $v_{\pi_\theta}(s)$ from many targets at different time-scales. From last lectures :
  - For MC, the target is the return G_t

$$\Delta \mathbf{v} = \alpha(G_t - \hat{v}(s, \mathbf{v}))\mathbf{x}(s)$$

  - For TD(0), the target is the TD target $r + \gamma \hat{v}(s', \mathbf{v})$

$$\Delta \mathbf{v} = \alpha(r + \gamma \hat{v}(s', \mathbf{v}) - \hat{v}(s, \mathbf{v}))\mathbf{x}(s)$$

  - For forward-view TD(λ), the target is the λ - return G_t^λ

$$\Delta \mathbf{v} = \alpha(G_t^\lambda - \hat{v}(s, \mathbf{v}))\mathbf{x}(s)$$

  - For backward-view TD(λ), we use eligibility traces

$$\delta_t = r_{t+1} + \gamma \hat{v}(s_{t+1}, \mathbf{v}_t) - \hat{v}(s_t, \mathbf{v}_t)$$
$$e_t = \gamma \lambda e_{t-1} + \mathbf{x}(s_t)$$
$$\Delta \mathbf{v} = \alpha \delta_t e_t$$

Actors at different time-scales 1/2

- The policy gradient can also be estimated at many time-scales

$$\nabla J(\theta) = \mathbb{E}_\pi \left[\nabla \ln \pi(a|s,\theta)\, A_{\pi_\theta}(s,a) \right]$$

- Monte-Carlo policy gradient uses error from complete error

$$\Delta\theta = \alpha(G_t - \hat{v}(S_t, \mathbf{v}))\nabla_\theta \ln \pi(A_t|S_t,\theta)$$

- Actor-critic policy gradient uses the one-step TD error

$$\Delta\theta = \alpha(r + \gamma\hat{v}(S_{t+1}, \mathbf{v}) - \hat{v}(S_t, \mathbf{v}))\nabla_\theta \ln \pi(A_t|S_t,\theta)$$

Actors at different time-scales 2/2

- Just-like forward-view TD(λ), we can mix over time-scales

$$\Delta\theta = \alpha(G_t^\lambda - \hat{v}(S_t, \mathbf{v}))\nabla_\theta \ln \pi(A_t|S_t, \theta)$$

  where $G_t^\lambda - \hat{v}(S_t, \mathbf{v})$ is a biased estimate of advantage function

- Like backward-view TD(λ), we can also use eligibility traces
  - By equivalence with TD(λ), substituting $\mathbf{x}(s) = \nabla_\theta \ln \pi(A|S, \theta)$

$$\begin{aligned}
\delta_t &= r_{t+1} + \gamma \hat{v}(s_{t+1}, \mathbf{v}_t) - \hat{v}(s_t, \mathbf{v}_t) \\
e_t &= \gamma\lambda e_{t-1} + \nabla_\theta \ln \pi(a_t|s_t, \theta_t) \\
\Delta\theta &= \alpha\delta_t e_t
\end{aligned}$$

- This update can be applied online, to incomplete sequences

Actors at different time-scales 2/2

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Does it make sense to have actor and critic at different time-scales?

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$$
$$\Delta\theta = \alpha\delta_t e_t$$

Does it make sense to have actor and critic at different time-scales?

We can mix approaches, but critic should be 'faster' (TD-0 for example) than the actor, since the actor improves based on the knowledge provided by the critic!

- This update can be applied online, to incomplete sequences

Actor-critic: some considerations 1/2

- Approximating the policy gradient introduces bias
- A biased policy gradient may not find the right solution
- Luckily, if we choose value function approximation carefully, then we can avoid introducing any bias: we can still follow the exact policy gradient
- Book material:
  - 13.1
  - 13.2 (proof is optional)
  - 13.3
  - 13.4
  - 13.5 (TD-lambda version)
  - Optional 13.6, 13.7 -> continuous case (more on next slide)

Actor-critic: some considerations 2/2

- (Optional) Policy gradient for continuous problems: we define performance in terms of the average rate of reward per time step

$$
\begin{aligned}
J(\boldsymbol{\theta}) \doteq r(\pi) \doteq & \lim_{h \to \infty} \frac{1}{h} \sum_{t=1}^{n} \mathbb{E}[R_t \mid A_{0:t-1} \sim \pi] \\
& = \lim_{t \to \infty} \mathbb{E}[R_t \mid A_{0:t-1} \sim \pi] \\
& = \sum_s \mu(s) \sum_a \pi(a|s) \sum_{s',r} p(s', r|s, a) r
\end{aligned}
$$

- (Optional) Policy parametrization can be extended to continuous scenarios, for example

$$
\pi(a|s, \boldsymbol{\theta}) \doteq \frac{1}{\sigma(s, \boldsymbol{\theta})\sqrt{2\pi}} \exp\left(-\frac{(a - \mu(s, \boldsymbol{\theta}))^2}{2\sigma(s, \boldsymbol{\theta})^2} \right)
$$

Policy Gradient: Chapter 13

- 13.1
- 13.2 (Proof of Policy Gradient Theorem Optional)
- 13.3
- 13.4
- 13.5

Batch methods

Incremental vs. Batch approaches

- Up until now we have been dealing with incremental approaches: data and experience is collected ‘on-line’ in an incremental fashion
- Batch approaches instead store data during experience to achieve some benefits
- Batch approaches were a breakthrough in ATARI [1]
- Notes:
  - No dedicated chapter for batch approaches (some references in chapter 16)
  - We briefly talked about ‘batch’ in the past…

[1] Mnih et al., “Human-level control through deep reinforcement learning” (Nature, 2015)

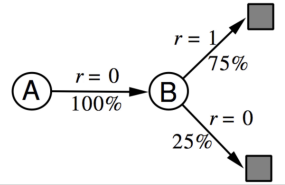
Prediction: (batch) MC and TD – AB Example

Two states, no discounting, 8 episodes of experience:

A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0

What is $V(A)$ and what is $V(B)$?

$V(B) = 3/4$
$V(A) = 0$ is the Monte Carlo solution
-> MC converges to solution with minimun mean-squared error
$V(A) = 3/4$ is the TD(0) solution
-> TD(0) (implicitely) converges to solution of max likelihood Markov model



Batch methods in Reinforcement Learning

- Gradient descent is simple and appealing
- But it is not sample efficient (one of the reason we introduced off-policy)
- Batch methods seek to find the best fitting value function given the agent's experience (training data)

Batch methods in Reinforcement Learning

- Gradient descent is simple and appealing
- But it is not sample efficient (one of the reason we introduced off-policy)
- Batch methods seek to find the best fitting value function given the agent's experience (training data)

Intuition behind batch methods: *in general one of the requirement for SGD optimization is that the training data is indipendent and identically distributed and when an Agent interacts with the environment, the sequence of experience tuples can be highly correlated.*

Learning algorithms that learn from each of these experiences tuples in sequential order runs the risk of getting swayed by the effects of this correlation

Experience Replay

- We can improve this situation using a large buffer of past experience and sample training data from it (a random subset), instead of using latest experience
- This technique is called **replay buffer or experience buffer**. The replay buffer contains a collection of tuples (S, A, R, S'). The tuples are gradually added to the buffer as we are interacting with the environment*
- The act of sampling a small batch of tuples from the replay buffer in order to learn is known as **experience replay**
- In addition to breaking harmful correlations, experience replay allows us to learn more from individual tuples multiple times, recall rare occurrences, and in general make better use of our experience

*The simplest implementation is a buffer of fixed size, with new data added tothe end of the buffer so that it pushes the oldest experience out of it

Buffer Reply: Least Squares Prediction

- Given value function approximation $\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$
- and experience $\mathcal{D}$ consisting of $\langle state, value \rangle$ pairs

$$\mathcal{D} = \{\langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, \ldots, \langle s_T, v_T^\pi \rangle\}$$

- which parameters $\mathbf{w}$ give the *best fitting* value function $\hat{v}(s, \mathbf{w})$

Buffer Reply: Least Squares Prediction

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- which parameters $\mathbf{w}$ give the *best fitting* value function $\hat{v}(s, \mathbf{w})$
- **Least squares** algorithms find parameter vector $\mathbf{w}$ minimising sum-squared error between $\hat{v}(s_t, \mathbf{w})$ and v_t^π

$$\begin{aligned} LS(\mathbf{w}) &= \sum_{t=1}^{T} (v_t^\pi - \hat{v}(s_t, \mathbf{w}))^2 \\ &= \mathbb{E}_{\mathcal{D}} \left[(v^\pi - \hat{v}(s, \mathbf{w}))^2 \right] \end{aligned}$$

SGD with Experience Replay

Given experience $\mathcal{D}$ consisting of $\langle state, value \rangle$ pairs

$$\mathcal{D} = \left\{ \left\langle s_1, v_1^\pi \right\rangle, \left\langle s_2, v_2^\pi \right\rangle, \ldots, \left\langle s_T, v_T^\pi \right\rangle \right\}$$

Repeat :

1. Sample state, value from experience

$$\left\langle s, v^\pi \right\rangle \sim \mathcal{D}$$

2. Apply stochastic gradient descent update

$$\Delta \mathbf{w} = \alpha \left(v^\pi - \hat{v}(s, \mathbf{w}) \right) \nabla_{\mathbf{w}} \hat{v}(s, \mathbf{w})$$

$$\mathbf{w} \leftarrow \mathbf{w} + \Delta \mathbf{w}$$

SGD with Experience Replay

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We are assuming the availability of the oracle...

SGD with Experience Replay in practice

- We do not know true values v_t^π
- In practice, our *training data* must use noisy or biased samples of v_t^π

LSMC Least Squares Monte-Carlo uses return

$$v_t^\pi \approx G_t$$

LSTD Least Squares Temporal-Difference uses TD target

$$v_t^\pi \approx R_{t+1} + \gamma\, \hat{v}(S_{t+1}, \mathbf{w})$$

LSTD(λ) Least Squares TD(λ) uses λ-return

$$v_t^\pi \approx G_t^\lambda$$

Linear Least Squares Prediction Algorithms

AMCO
ARTIFICIAL INTELLIGENCE, MACHINE LEARNING AND CONTROL RESEARCH GROUP

Reinforcement Learning 2025/2026

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Thank you!
Questions?

Lecture #19:
Policy Gradient Methods: Actor Critic Approaches & Batch Methods

Gian Antonio Susto