Università degli Studi di Padova

Image histogram and equalization

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Agenda

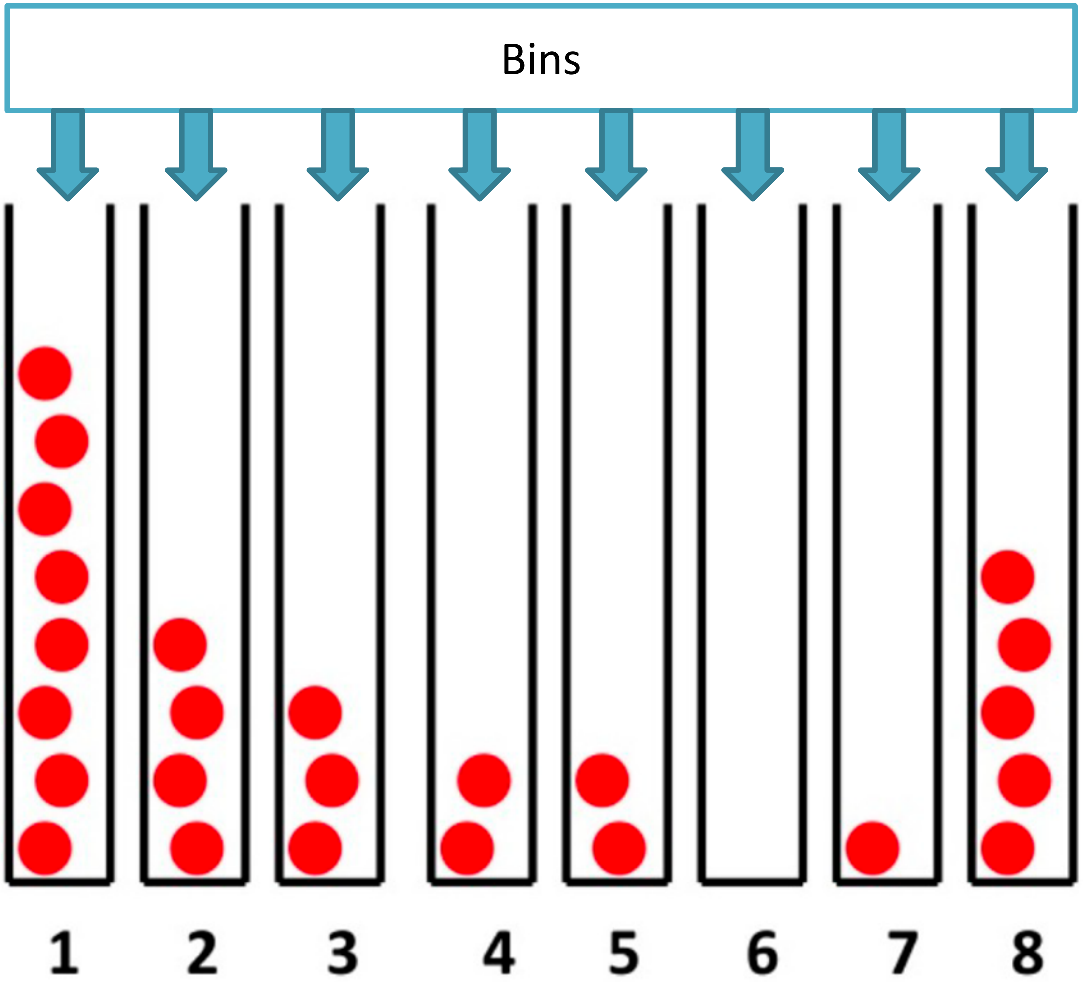
- Image histograms
- Working on the histogram of an image
  - Histogram equalization

Histograms and images

- What is a histogram?
- How can a histogram be evaluated from an image?

Histograms and images

1	8	4	3	4
1	1	1	7	8
8	8	3	3	1
2	2	1	5	2
1	1	8	5	2



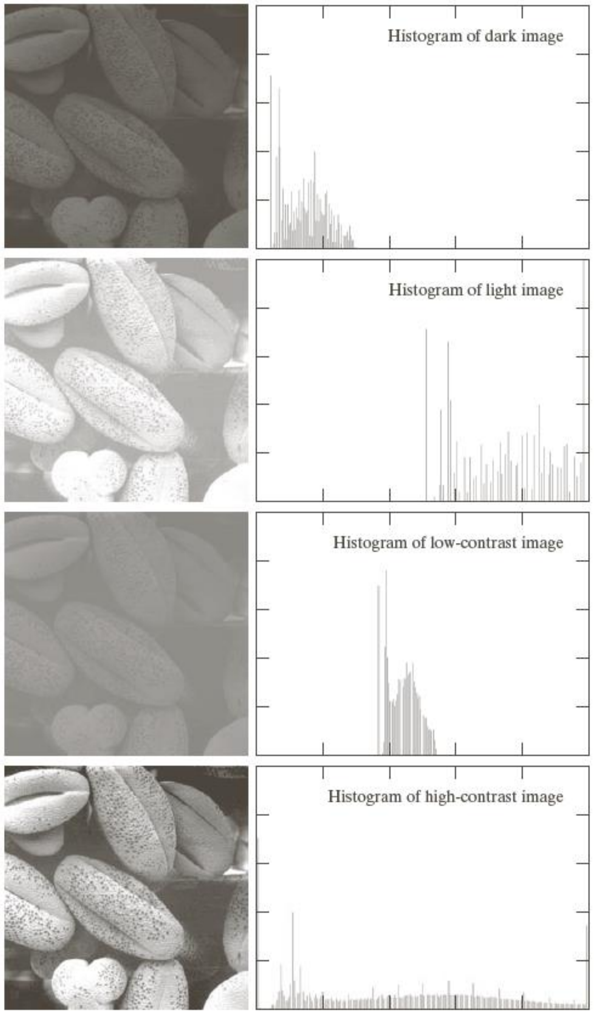
Histograms

- Histograms of the grayscale values

$$p(r_k) = \frac{\boxed{h(r_k)}}{MN} = \frac{n_k}{MN}$$

# of pixels whose intensity is r_k

- Can be treated as a probabilistic density function (PDF)



PDF/CDF – recall

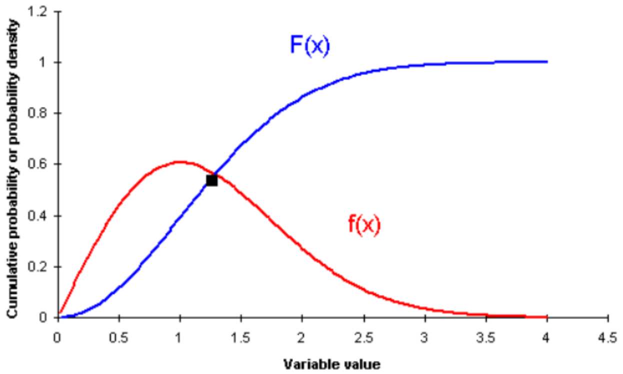
- Cumulative Distribution Function (CDF)

$$F_X(x) = P(X \leq x)$$

- Probability Density Function (PDF)

$$f_X(x) = \frac{d}{dx} F_X(x)$$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt$$



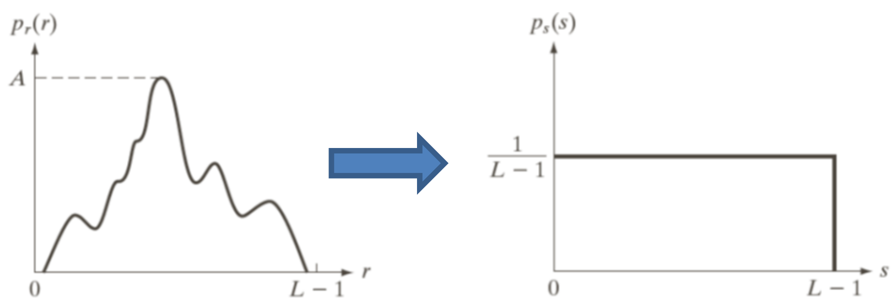
Histograms

- Histograms are widely used for:
  - Evaluating image statistics
  - Compression
  - Segmentation
  - Image enhancement
- Is it meaningful to "modify the histogram"?
  - Which operations could we apply?

- No spoiler ☺

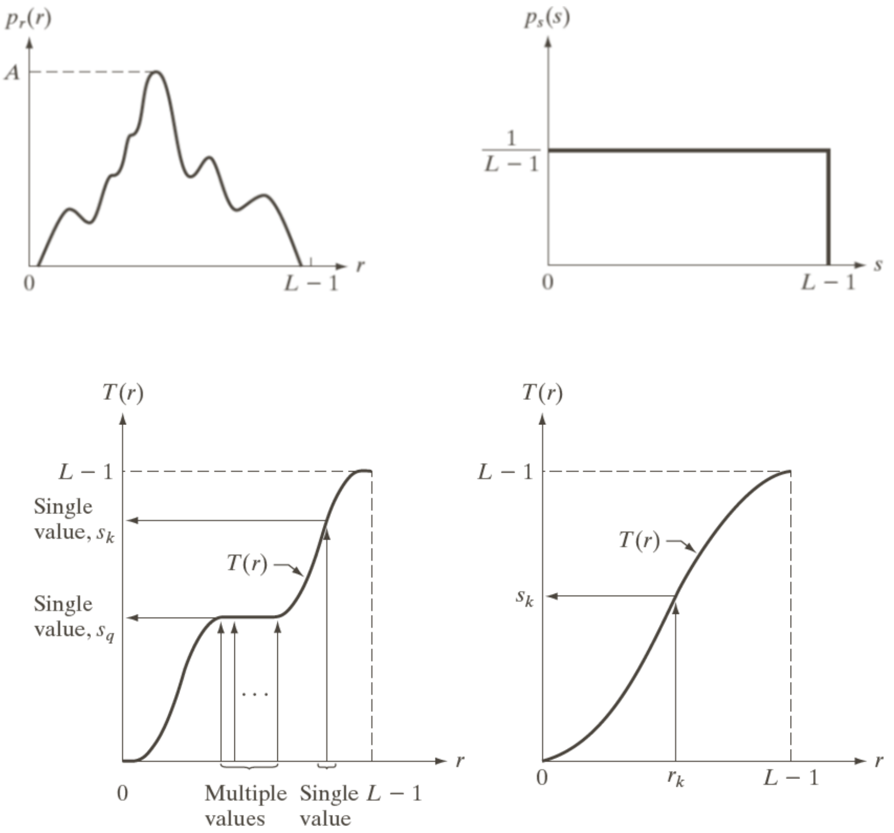
Histogram equalization

- Histogram equalization is a process that flattens the histogram



Histogram equalization

- Histogram equalization
- Based on a equalization function
  - How to find this function?



Equalization function

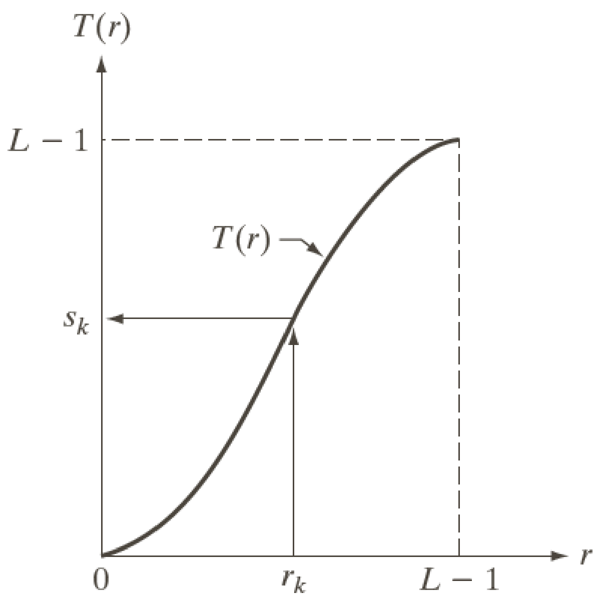
- The function $T(r)$ capable of equalizing the histogram is the CDF

spiegazione intuitiva, i valori di grigio con frequenza piu alta hanno una pmf maggiore che porta ad aumentare la derivata della cdf, che quindi e' piu ripida, questo porta ad avere uno "stretch" maggiore allontanando i pixel con colori simili portandoli ad avere colori piu diversi, viceversa dove la frequenza e' bassa i colori vengono resi piu vicini -> stiamo appiattendo la derivata ovvero la pmf

$$s = T(r) = \boxed{(L-1)\int_0^r p_r(w)dw}$$

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

L-1 e' per avere i valori non tra 0 e 1 come una normale CDF ma tra 0 e L-1 come un immagine di int



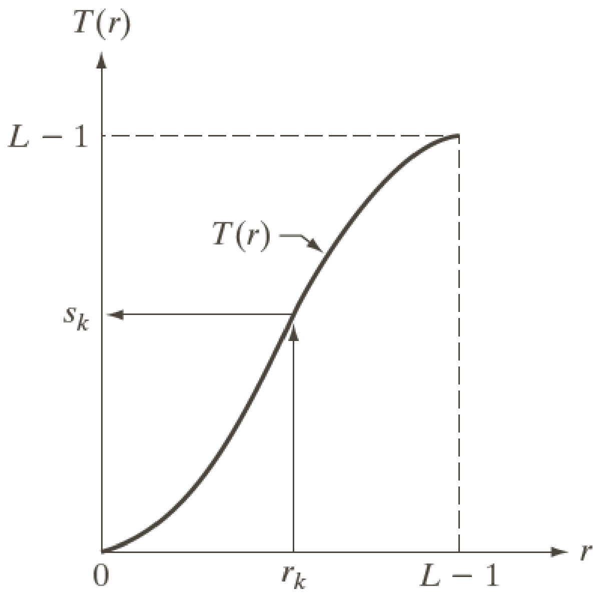
Cumulative Distribution Function (CDF) of the RV r

Equalization function

- $T(r)$ is monotonically non-decreasing
  - The inverse function is available
- The function is bounded

$$0 \le T(r) \le L - 1$$
$$0 \le r \le L - 1$$

- $T(r)$ continuous and differentiable



Equalization: a 3-bit example

- Now consider an example
  - An image with 8 gray levels
  - Gray level distribution is given by

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Equalization: a 3-bit example

- Now consider an example
  - The CDF is given by:

$$s_i = 7\sum_{j=0}^{i} p_r(r_j)$$

- How can we apply this formula to equalize the image?

Equalization: a 3-bit example

- Using the info about pixels and the CDF formula we can fill this table

r_k	n_k	$p_r(r_k) = n_k/MN$
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$r_4 = 4$	329	0.08
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$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_i = 7\sum_{j=0}^{i} p_r(r_j)$$

r		s_i	round
0	S_0	1.33	1
1	S_1	3.08	3
2	S_2	4.55	5
3	S_3	5.67	6
4	S_4	6.23	6
5	S_5	6.65	7
6	S_6	6.86	7
7	S_7	7.00	7

Input values

Output values

Equalization: a 3-bit example

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
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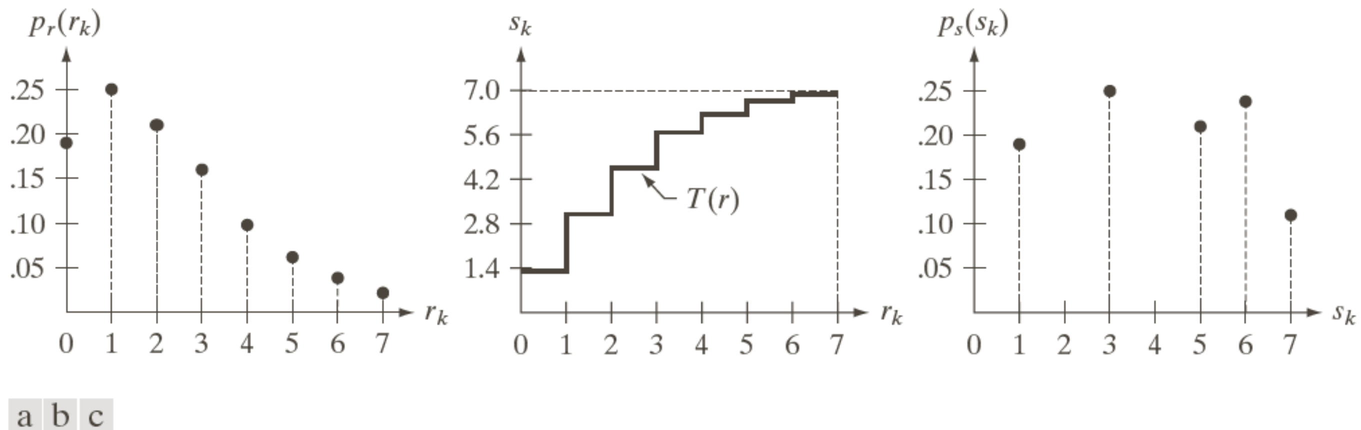
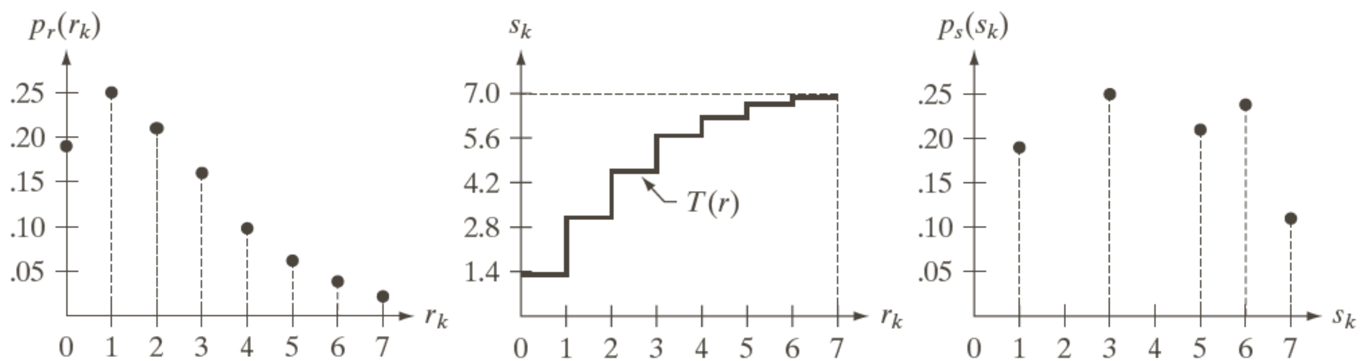


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Equalization: a 3-bit example

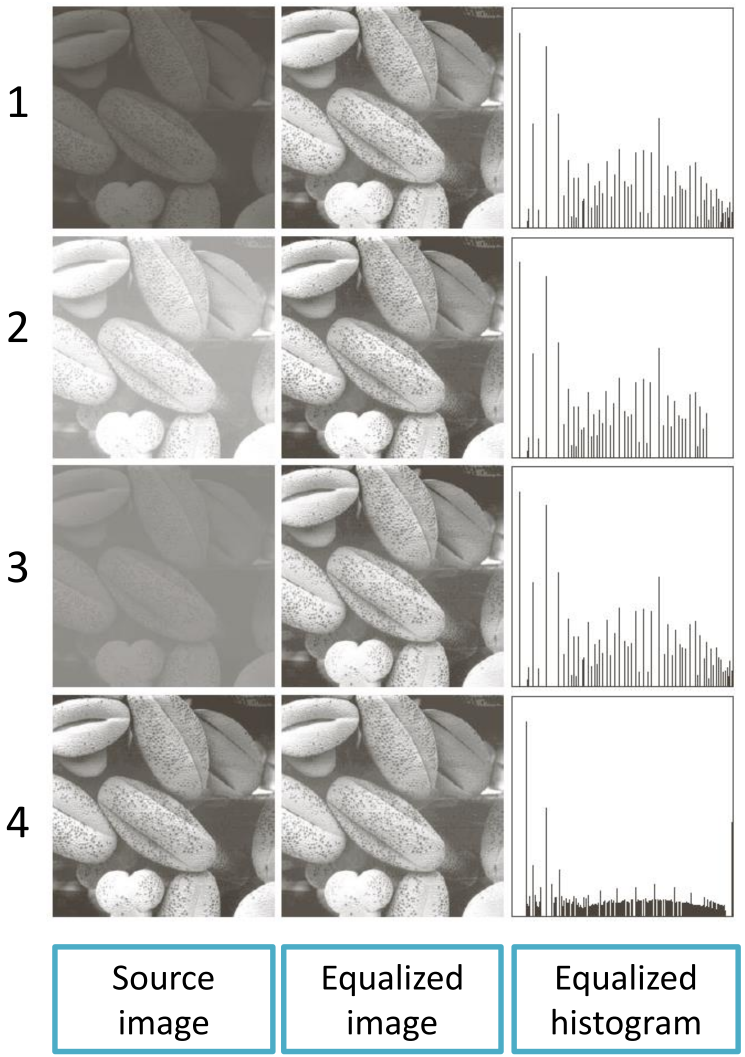
- The output is not perfectly flat
  - Caused by the discrete nature of data



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

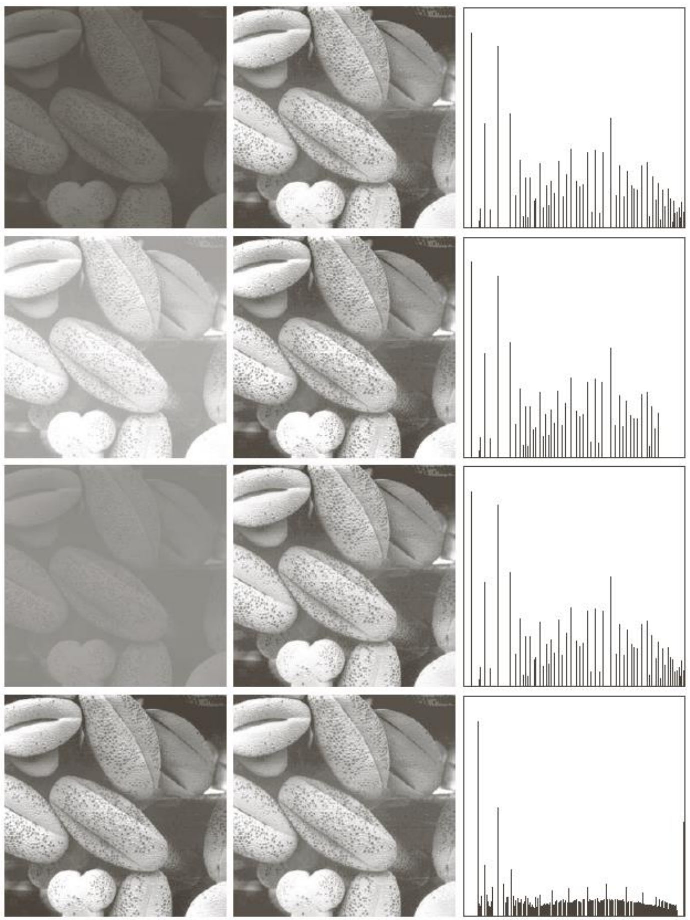
Equalization: example



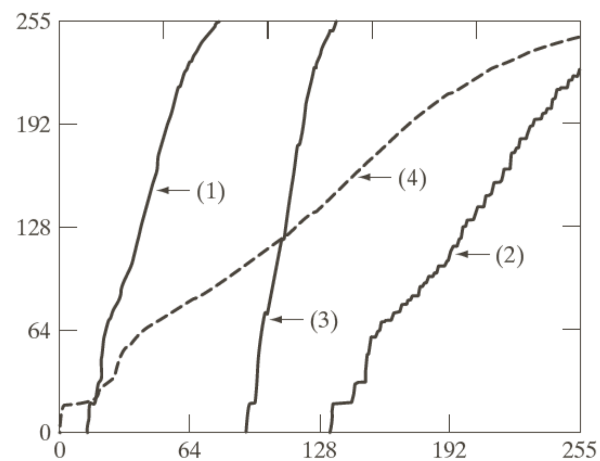
- Here you can see the corresponding input and output images
- Compare the source images
- Compare the equalized images
- What do you observe?

Equalization: example

1
2
3
4



Source image	Equalized image	Equalized histogram



Equalization functions

Histograms in OpenCV

Histograms in OpenCV

- Data structure: array
  - `cv::Mat` and `vector<>`
- cv::calcHist function

```
void cv::calcHist(
  cv::InputArrayOfArrays  images,              // vector of 8U or 32F images
  const vector<int>&      channels,            // lists channels to use
  cv::InputArray          mask,                // in 'images' count, iff 'mask'
                                               // nonzero
  cv::OutputArray         hist,                // output histogram array
  const vector<int>       histSize,            // hist sizes in each dimension
  const vector<float>&    ranges,              // pairs give bin sizes in a
                                               // flat list
  bool                    accumulate = false   // if true, add to 'hist', else
                                               // replace
);
```

Histograms in OpenCV

- Your duty: check the calcHist function and tutorial

§ calcHist() [1/3]

```
void cv::calcHist ( const Mat *      images,
                    int              nimages,
                    const int *      channels,
                    InputArray       mask,
                    OutputArray      hist,
                    int              dims,
                    const int *      histSize,
                    const float **   ranges,
                    bool             uniform = true,
                    bool             accumulate = false
                  )
```

Python:

```
hist = cv.calcHist( images, channels, mask, histSize, ranges[, hist[, accumulate]] )
```

calcHist params

Parameters

images Source arrays. They all should have the same depth, CV_8U, CV_16U or CV_32F , and the same size. Each of them can have an arbitrary number of channels.

nimages Number of source images.

channels List of the dims channels used to compute the histogram. The first array channels are numerated from 0 to images[0].channels()-1 , the second array channels are counted from images[0].channels() to images[0].channels() + images[1].channels()-1, and so on.

mask Optional mask. If the matrix is not empty, it must be an 8-bit array of the same size as images[i] . The non-zero mask elements mark the array elements counted in the histogram.

hist Output histogram, which is a dense or sparse dims -dimensional array.

dims Histogram dimensionality that must be positive and not greater than CV_MAX_DIMS (equal to 32 in the current OpenCV version).

histSize Array of histogram sizes in each dimension.

ranges Array of the dims arrays of the histogram bin boundaries in each dimension. When the histogram is uniform (uniform =true), then for each dimension i it is enough to specify the lower (inclusive) boundary L_0 of the 0-th histogram bin and the upper (exclusive) boundary $U_{\texttt{histSize}[i]-1}$ for the last histogram bin histSize[i]-1 . That is, in case of a uniform histogram each of ranges[i] is an array of 2 elements. When the histogram is not uniform (uniform=false), then each of ranges[i] contains histSize[i]+1 elements: $L_0, U_0 = L_1, U_1 = L_2, \ldots, U_{\texttt{histSize[i]}-2} = L_{\texttt{histSize[i]}-1}, U_{\texttt{histSize[i]}-1}$. The array elements, that are not between L_0 and $U_{\texttt{histSize[i]}-1}$, are not counted in the histogram.

uniform Flag indicating whether the histogram is uniform or not (see above).

accumulate Accumulation flag. If it is set, the histogram is not cleared in the beginning when it is allocated. This feature enables you to compute a single histogram from several sets of arrays, or to update the histogram in time.

calcHist – example

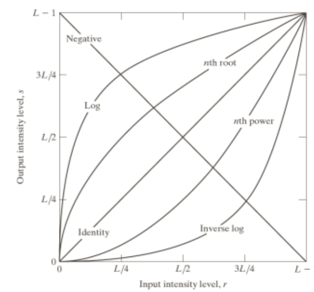

```
#include <opencv2/imgproc.hpp>
#include <opencv2/highgui.hpp>

using namespace cv;

int main( int argc, char** argv )
{
 Mat src, hsv;
 if( argc != 2 || !(src=imread(argv[1], 1)).data )
 return -1;

 cvtColor(src, hsv, COLOR_BGR2HSV);

 // Quantize the hue to 30 levels
 // and the saturation to 32 levels
 int hbins = 30, sbins = 32;
 int histSize[] = {hbins, sbins};
 // hue varies from 0 to 179, see cvtColor
 float hranges[] = { 0, 180 };
 // saturation varies from 0 (black-gray-white) to
 // 255 (pure spectrum color)
 float sranges[] = { 0, 256 };
 const float* ranges[] = { hranges, sranges };
    MatND hist;
 // we compute the histogram from the 0-th and 1-st channels
 int channels[] = {0, 1};

 calcHist( &hsv, 1, channels, Mat(), // do not use mask
             hist, 2, histSize, ranges,
 true, // the histogram is uniform
 false );
 double maxVal=0;
 minMaxLoc(hist, 0, &maxVal, 0, 0);
```

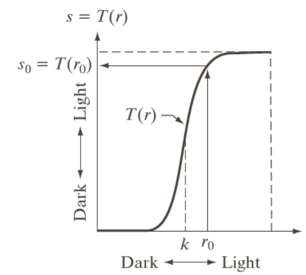
calcHist – example

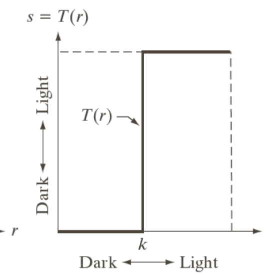

```
int scale = 10;
Mat histImg = Mat::zeros(sbins*scale, hbins*10, CV_8UC3);

for( int h = 0; h < hbins; h++ )
for( int s = 0; s < sbins; s++ )
       {
float binVal = hist.at<float>(h, s);
int intensity = cvRound(binVal*255/maxVal);
rectangle( histImg, Point(h*scale, s*scale),
Point( (h+1)*scale - 1, (s+1)*scale - 1),
Scalar::all(intensity),
CV_FILLED );
       }

namedWindow( "Source", 1 );
imshow( "Source", src );

namedWindow( "H-S Histogram", 1 );
imshow( "H-S Histogram", histImg );
waitKey();
}
```

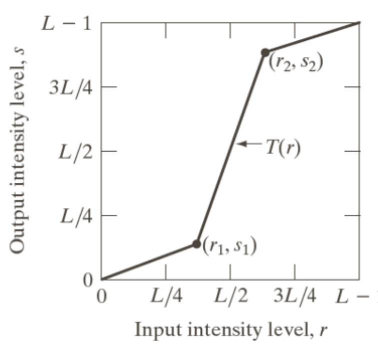
Single-pixel operations

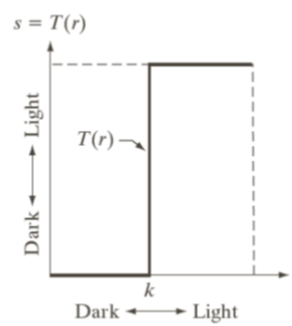
- Recap of the transformations analyzed so far
  - Negative
  - Logarithm
  - Gamma
  - Contrast stretching
  - Thresholing
  - Histogram equalization

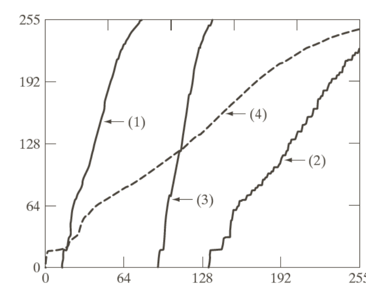












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