INFORMED SEARCH ALGORITHMS

Chapter 3

Best-first search

- Informed search algorithms

  use problem-specific knowledge to speed up the search process

Outline

- Best-first search
  - **Greedy** best-first search
  - **A*** search
- Heuristics

Review: Tree search algorithms

function TREE-SEARCH(problem) **returns** a solution, or failure

initialize the frontier using the initial state of problem

loop do

if the frontier is empty **then return** failure

choose a leaf node and remove it from the frontier

if the node contains a goal state **then return** the corresponding solution

expand the chosen node, adding the resulting nodes to the frontier

A search strategy is defined by picking the order of node expansion

Best-first search

- Informed search algorithms

  use problem-specific knowledge to speed up the search process

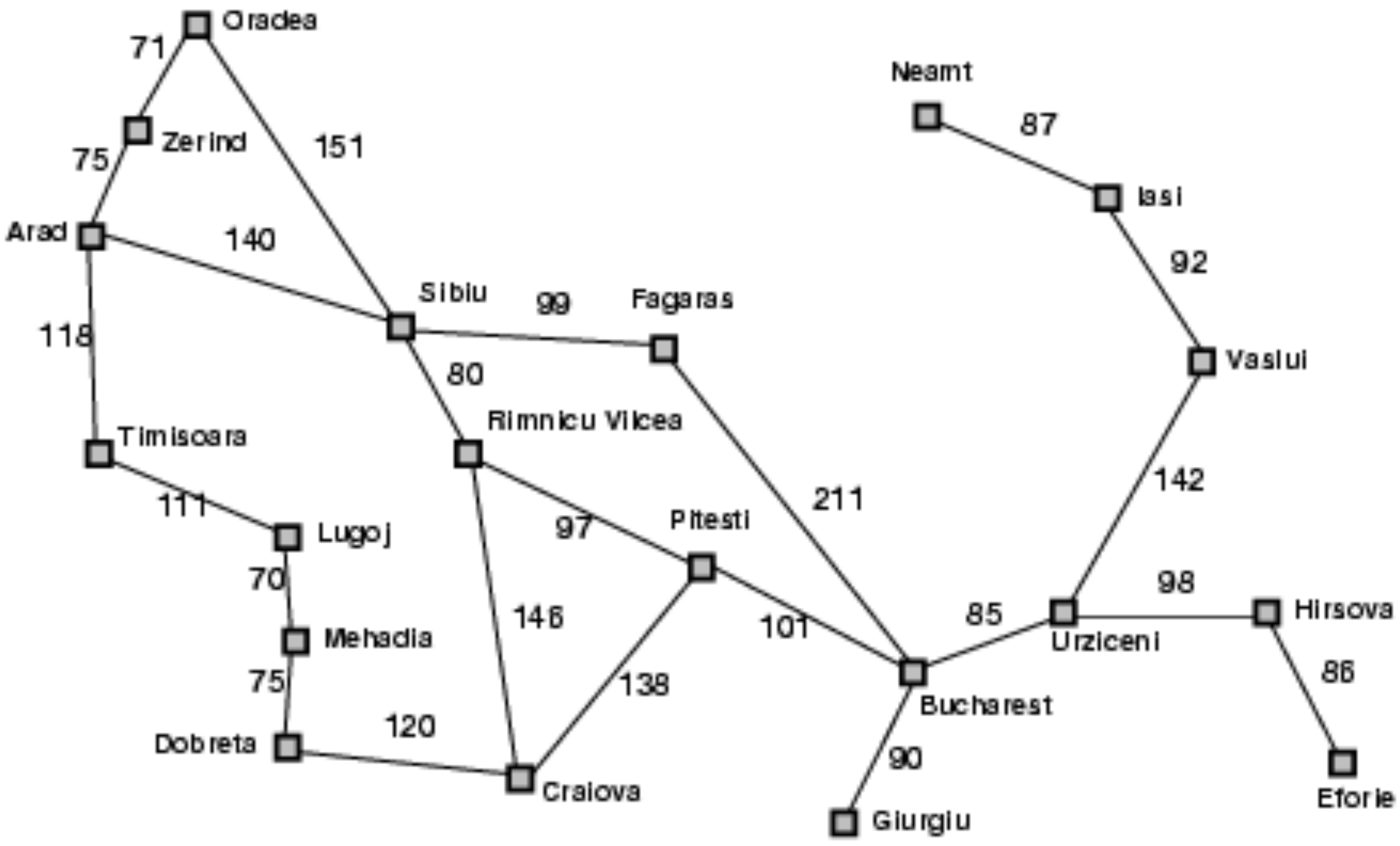
- Idea: use an **evaluation function f(n)** for each node
  - Estimate of "desiderability"
  - to choose the node selected for expansion
  - is construed as a cost estimate
  - Expand node with the lowest f(n) first

- Special cases
  - **Greedy** best-first search
  - $\mathbf{A}^*$ search

Best-first search

- Most algorithms use a **heuristic function** h(n) as a component of f
  - **h(n) =** __**estimated cost**__ of the __**cheapest path**__ from the state of the node n to a goal state
  - depends only on the state associated with n
  - Assumptions:
    - h(n) is non negative
    - h(n)=0 at every goal state
  - **Example:** in Romania, we may __**estimate**__
    - the **cost of the cheapest path** from Arad to Bucharest
    - **via** the __**straight-line distance**__ from Arad to Bucharest
    - $h_{SLD}(n)$ = straight-line distance from n to Bucarest

Romania with step costs in km



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
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Greedy best-first search

- Evaluation function $\boxed{f(n) = h(n)}$ (heuristic)

  ***h(n)*:** estimated cost of the cheapest path from **n** to **goal**

- Example: $h_{SLD}(n)$ = *straight-line distance* **from *n* to Bucharest**

- Greedy best-first search expands the node that appears to be closest to goal

We expand **first** the node with the lowest value of h(n)

Greedy best-first search example

h(n)

Straight–line distance
to Bucharest

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Greedy best-first search example

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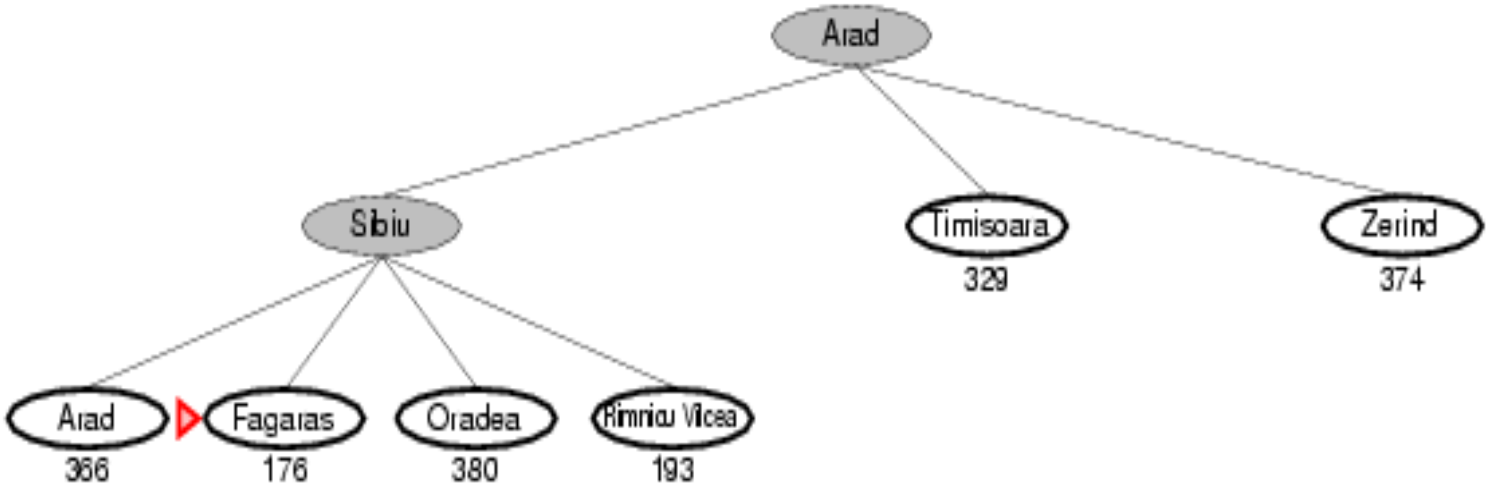
Greedy best-first search example



h(n)

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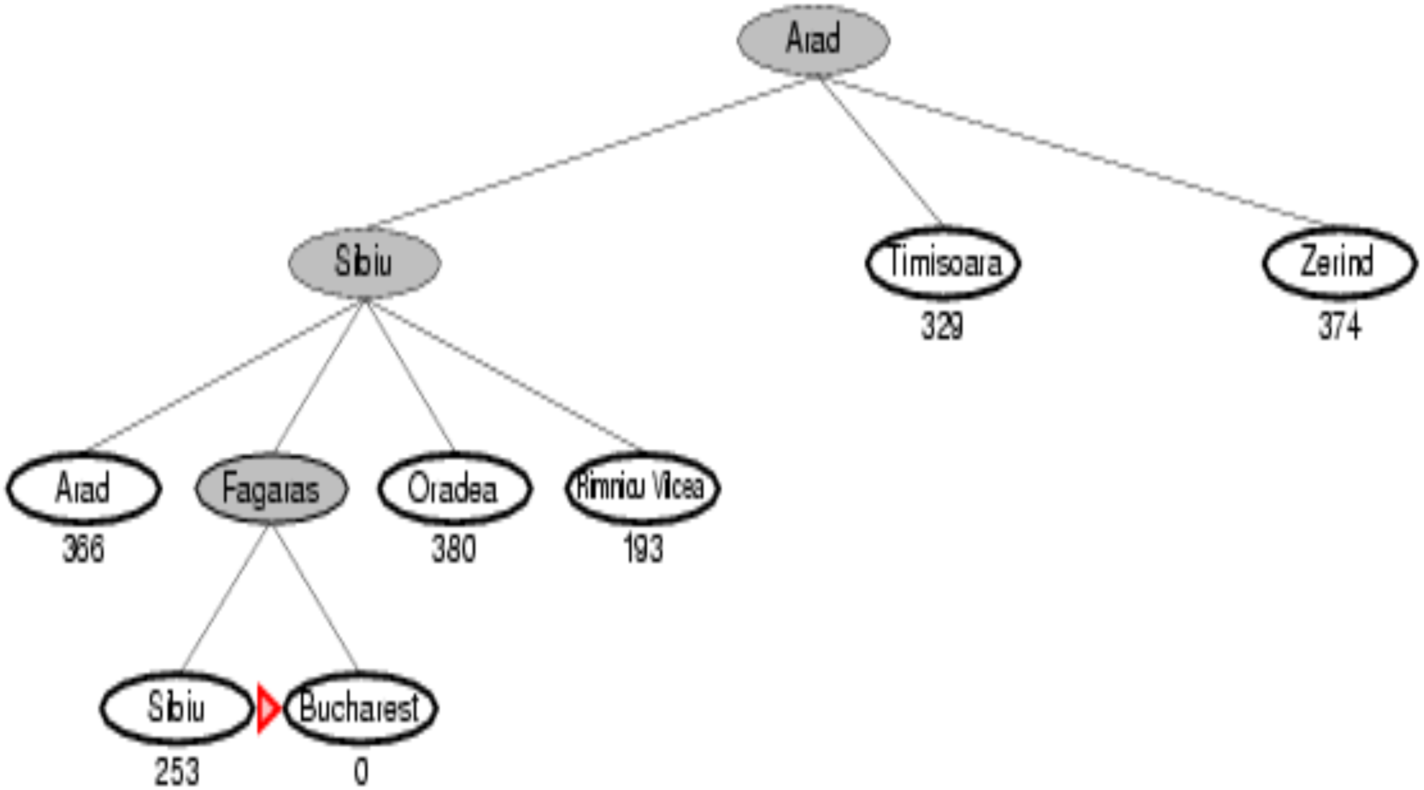
Greedy best-first search example



h(n)

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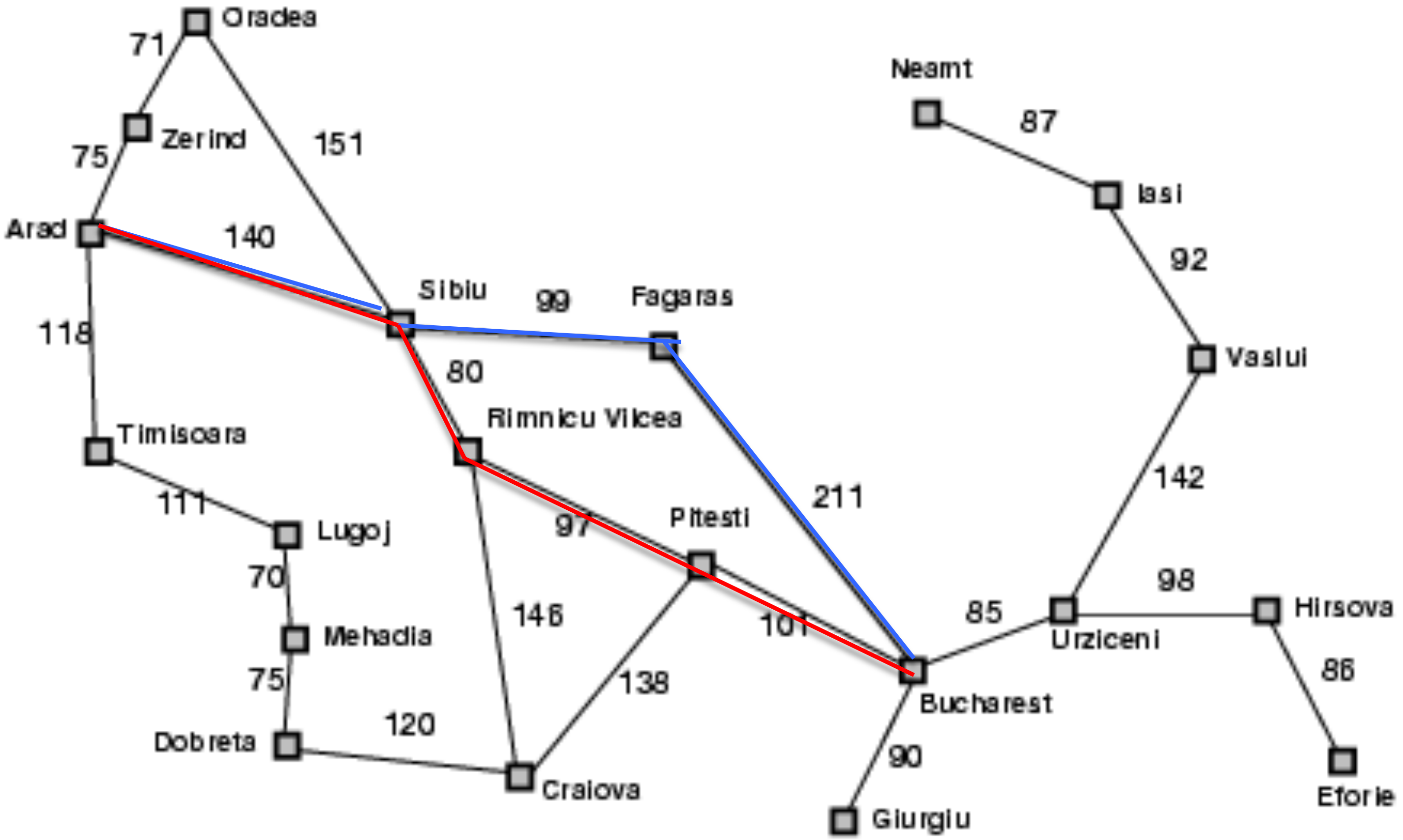
returned by **greedy BFS**

optimal



h(n)

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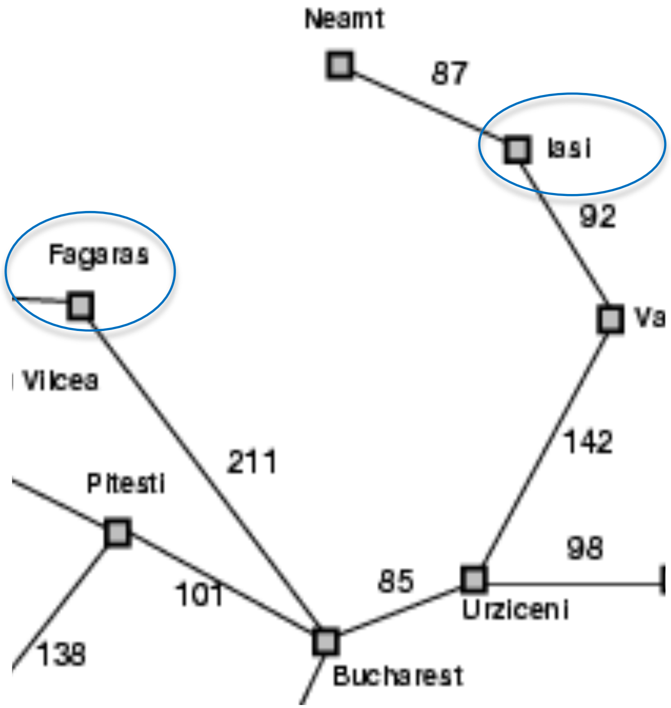
Properties of greedy best-first search

b: branching factor
m: the maximum depth of the search space

- Complete? No (tree-search)
  - NO, consider the problem of going from Iasi to Fagars: Iasi→ Neamt →Iasi→ Neamt

    The **heuristic** suggests **Neamt** be expanded **first** because it is closest to **Fagaras**,

    but **it is a dead end**
  - Complete in finite state with repeated-state checking



- Time? $O(b^m)$, but a good heuristic can give dramatic improvement
- Space? $O(b^m)$ -- keeps all nodes in memory
- Optimal? No

Outline

- Best-first search
  - **Greedy** best-first search
  - **A*** search
- Heuristics

A^* search

- **Idea:** avoid expanding **paths** that are **already expensive**
- Evaluation function $\boxed{f(n) = g(n) + h(n)}$
  - **g(n)** = **path cost** from the **start node** to **node n**
  - **h(n)** = estimated cost of the cheapest path from **n** to **goal**
  - **f(n)** = estimated cost of the cheapest **solution through n**
- Like **uniform-cost search** with **f=g+h** instead of **g**

A* search example

We expand **first** the node with the **lowest value** of **g(n) + h(n)**

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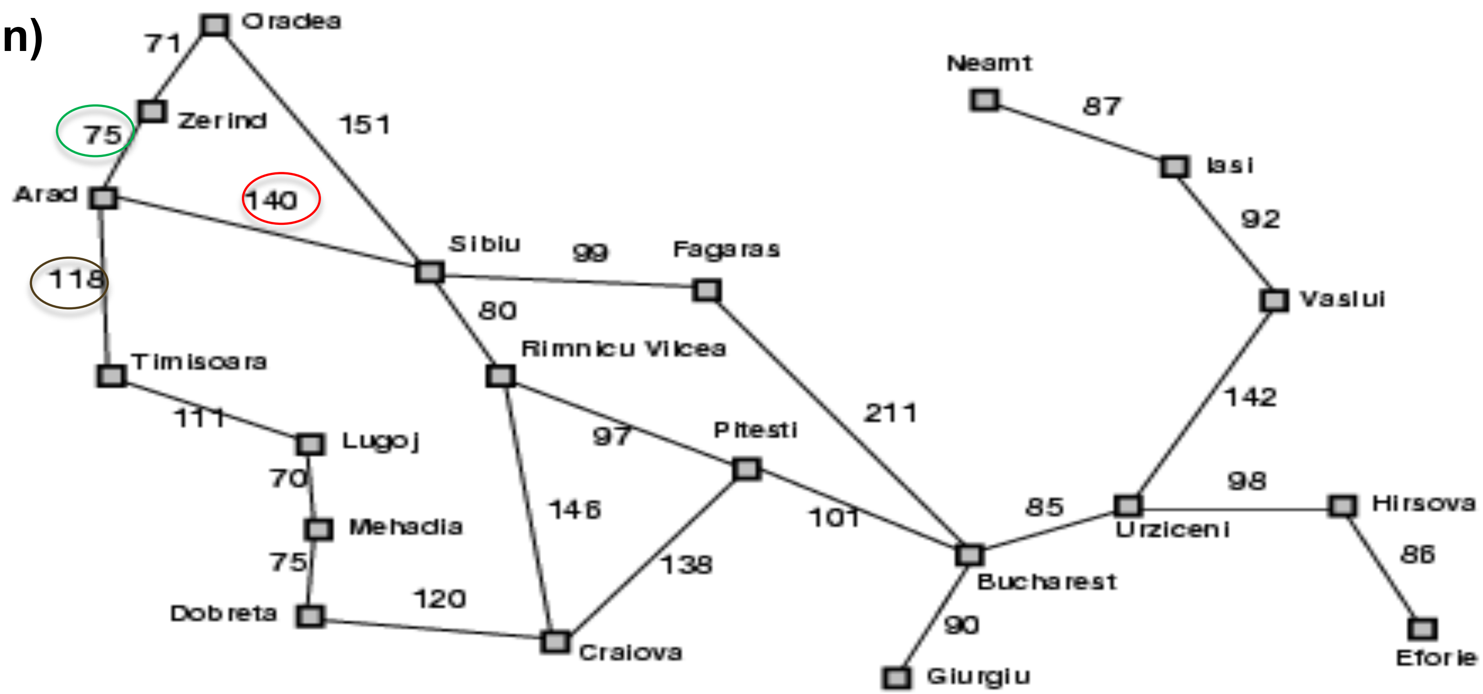
A* search example

We expand first the node with the **lowest value** of **g(n)** + **h(n)**

g(n)



h(n)

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A* search example

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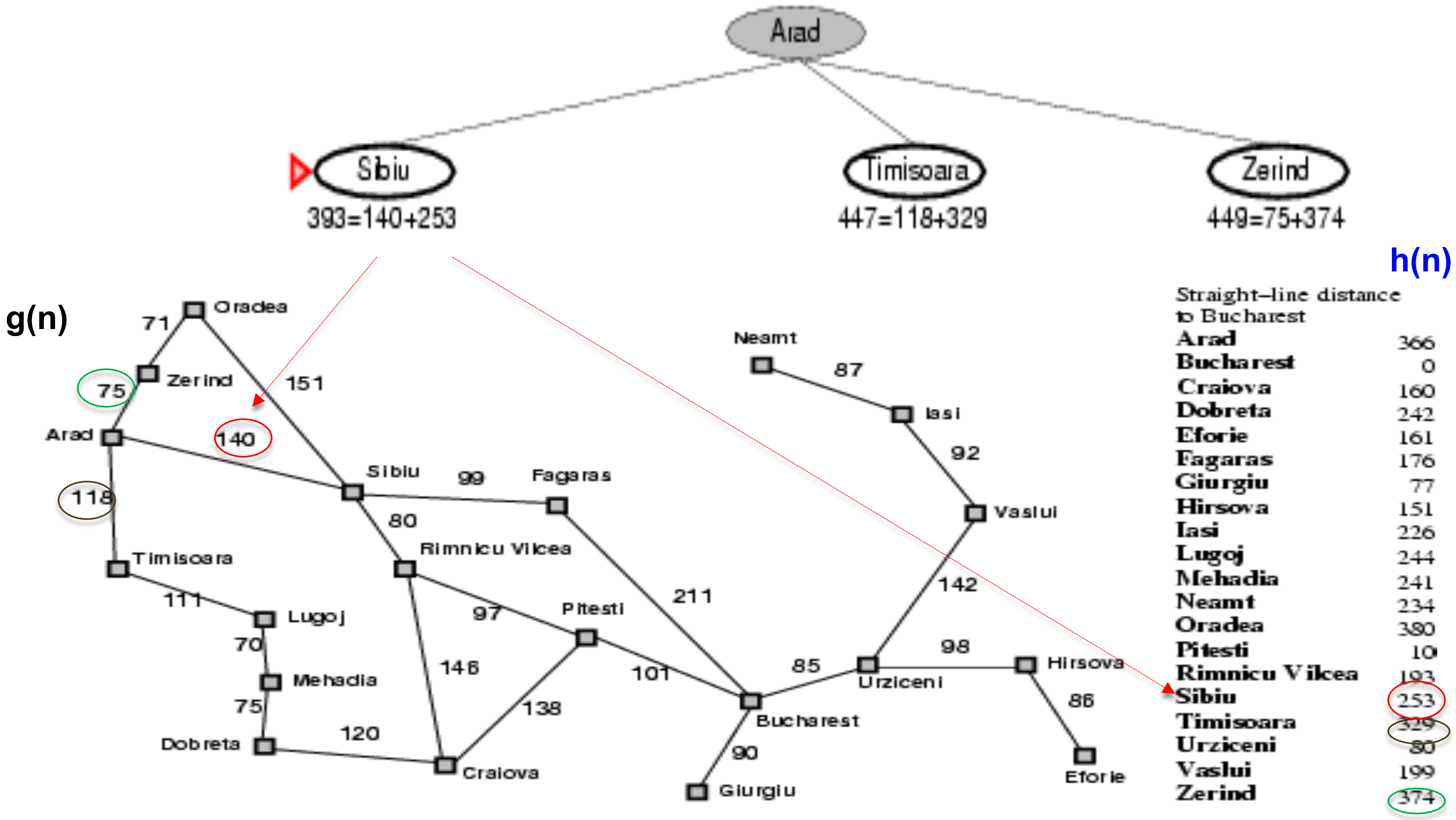

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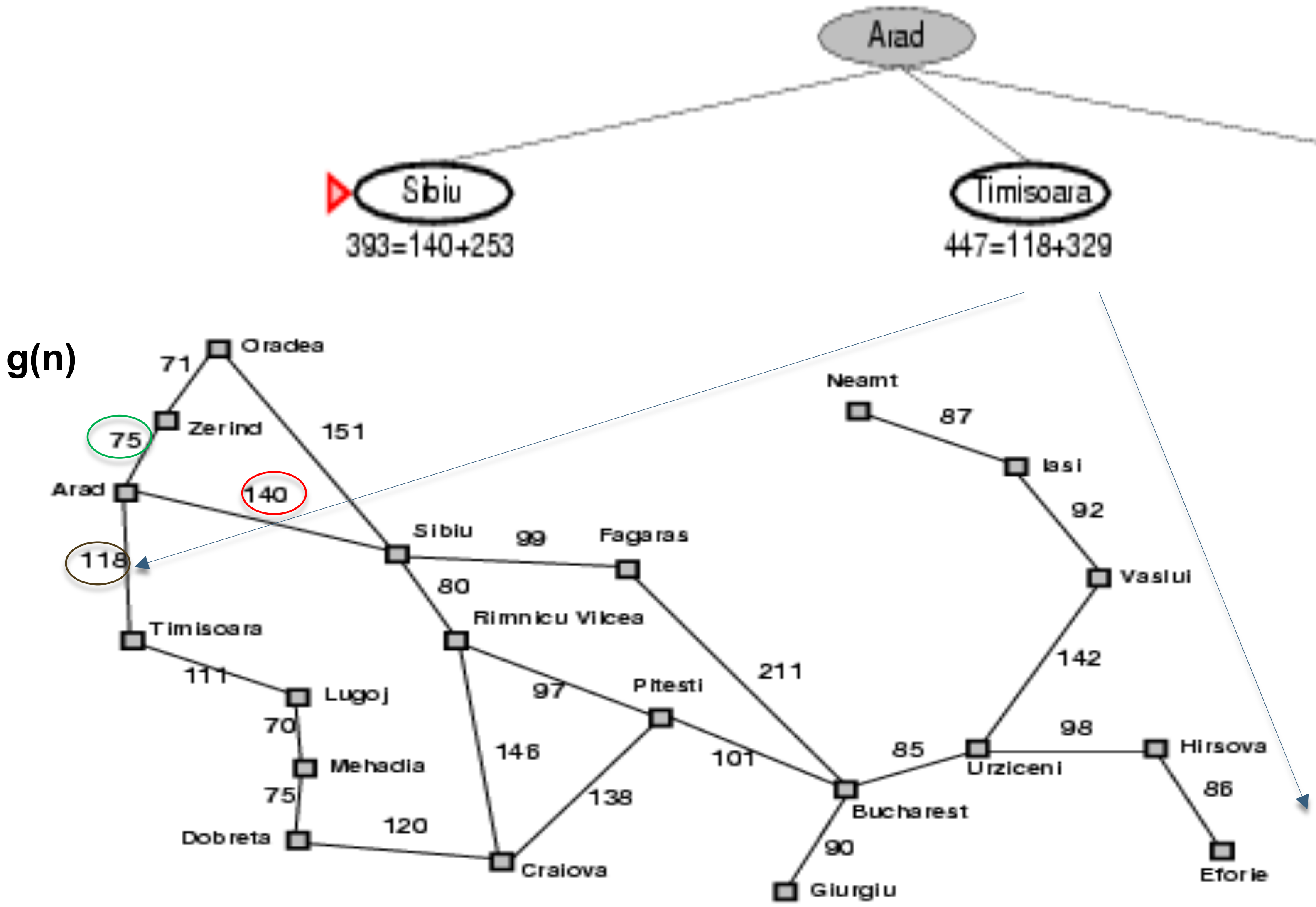

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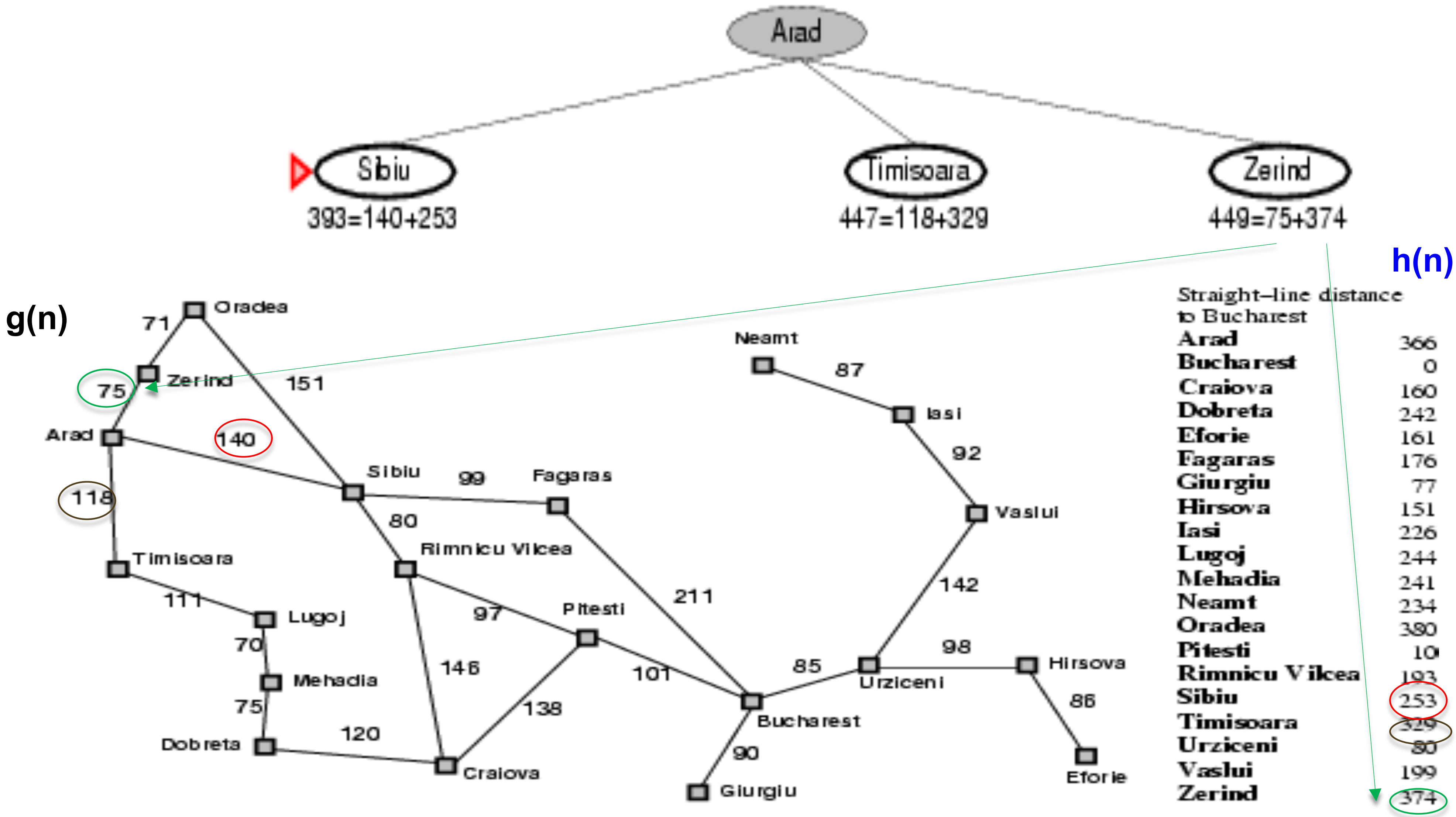

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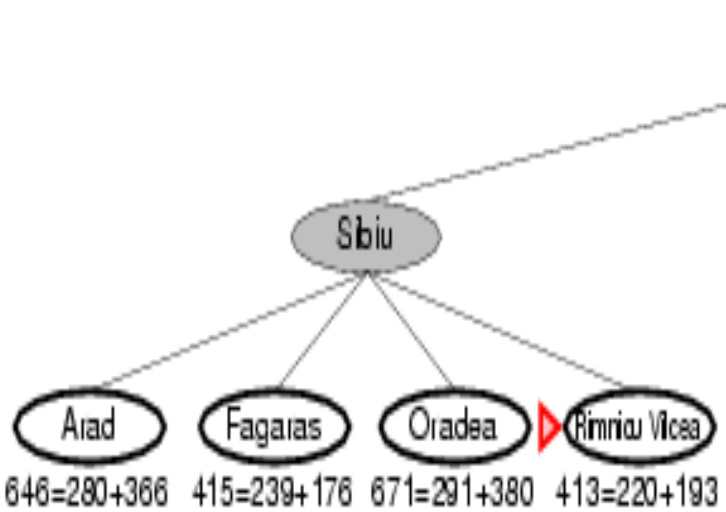

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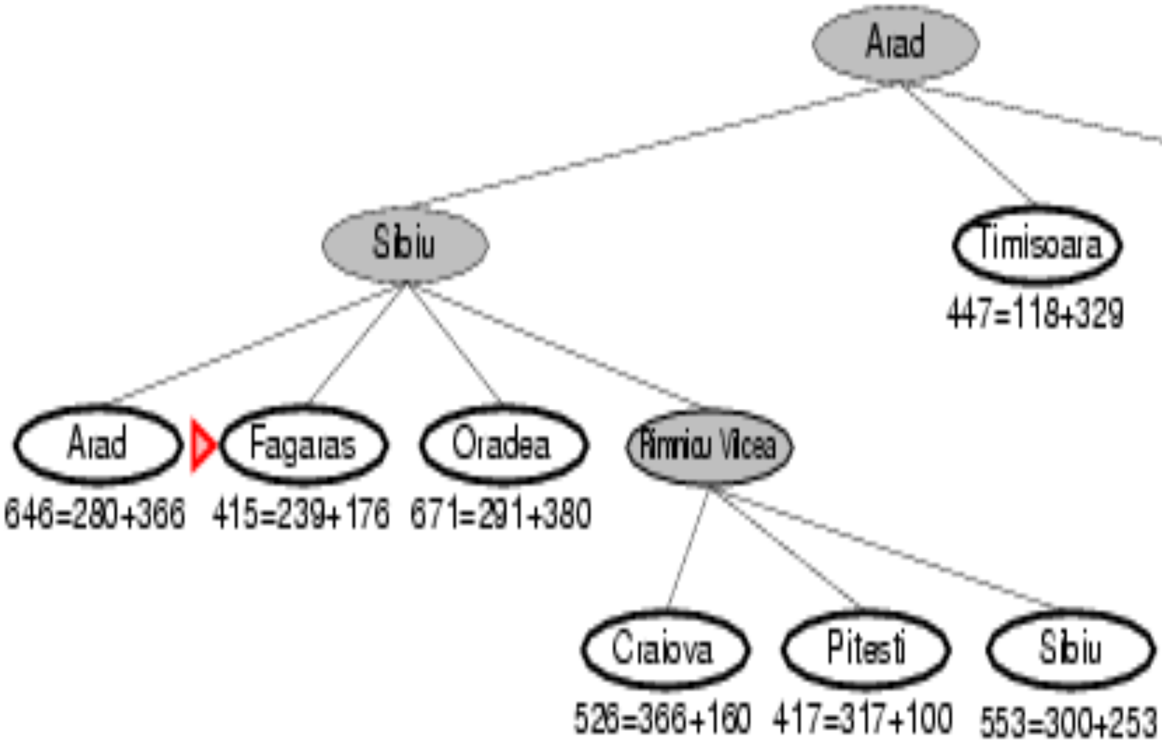

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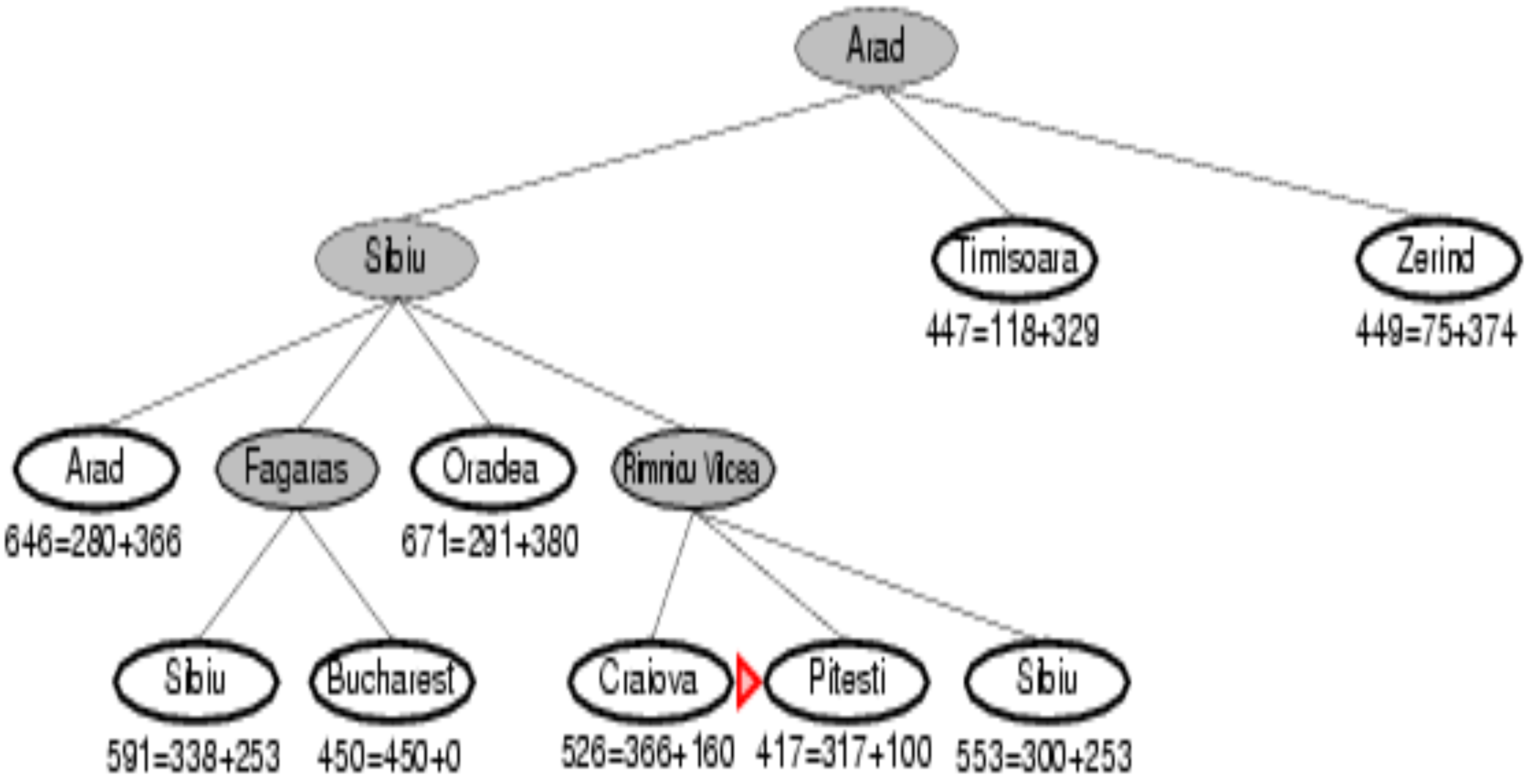

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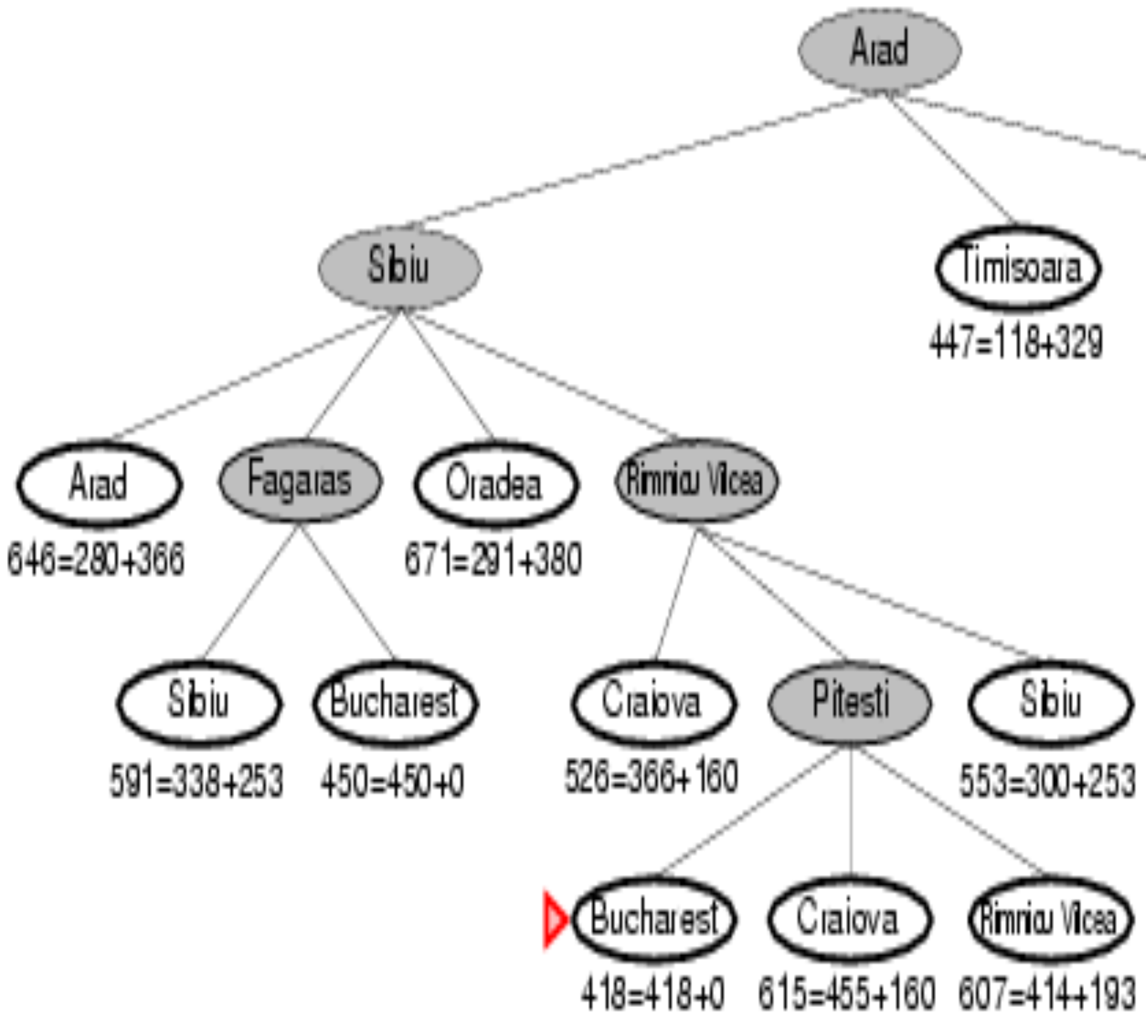

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Admissible heuristics

- A heuristic h is **admissible** if

  for every node *n*, $h(n) \leq h^*(n)$

  where $h^*(n)$ is the **true cost** from *n* to ***goal***

- An admissible heuristic never overestimates the cost to reach the goal

- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)

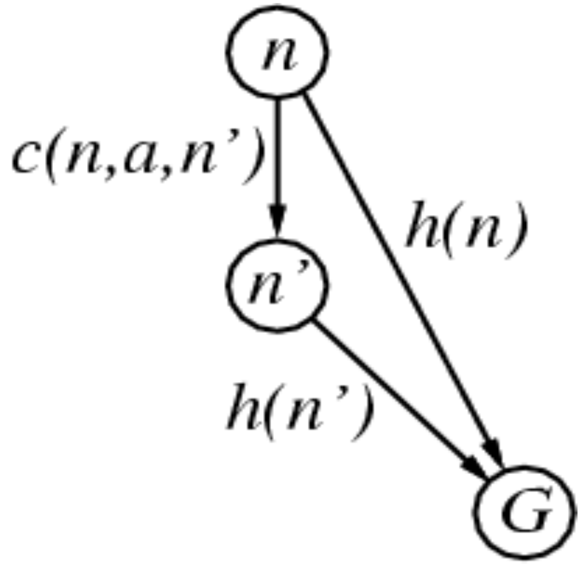
- **Theorem**: If *h* is admissible,

  A^* using TREE-SEARCH is optimal

Consistent heuristics

- A heuristic h is **consistent**
  if **for every** node ***n***,
  for every successor *n'* of *n* generated **by an action *a***,

$$h(n) \leq c(n, a, n') + h(n')$$



- Instance of triangular inequality

- Consistency → admissibility

h consistent → A* graph-search optimal

- **Theorem**: If *h* is consistent
  A* using `GRAPH-SEARCH` is optimal

Properties of A*

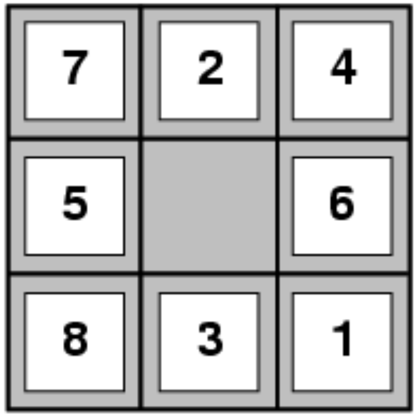
- Complete? Yes (unless there are infinitely many nodes with f $\leq f(G)$)
- Time? Exponential
- Space? Keeps all nodes in memory
- Optimal? Yes (with `TREE-SEARCH`, admissible heuristic;
  `GRAPH-SEARCH`, consistent heuristic)

Admissible heuristics

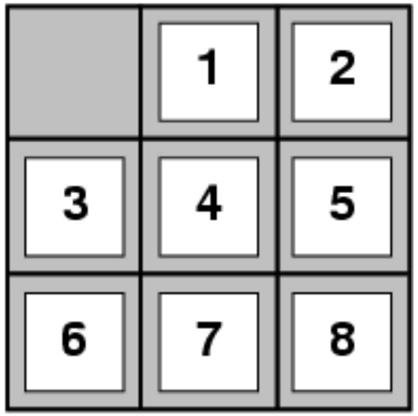
Review: **8-puzzle**
goal: to slide the tiles **horizontally or vertically** into the empty space **until** the configuration matches the goal configuration

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance
  (i.e, the sum of the horizontal and vertical distances of the tiles from their goal positions)



Start State



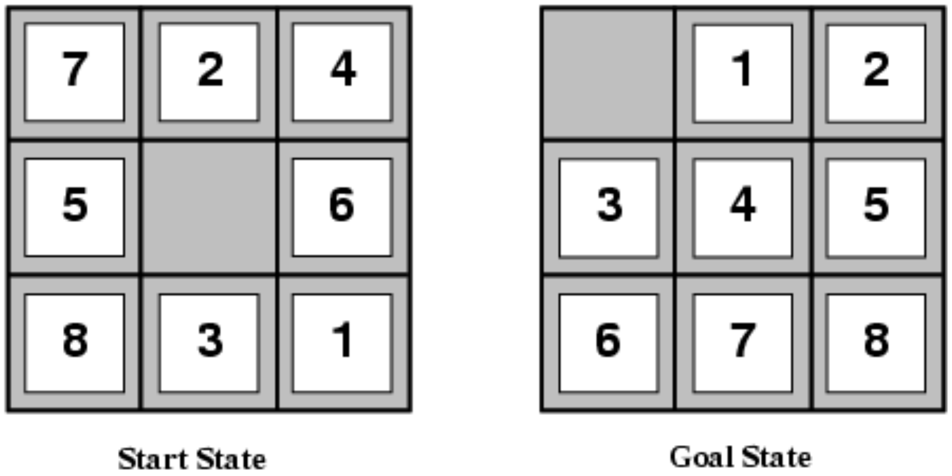
Goal State

- $h_1(S)$ = ?
- $h_2(S)$ = ?

Admissible heuristics

E.g., for the 8-puzzle:

- ***$h_1(n)$*** = number of misplaced tiles



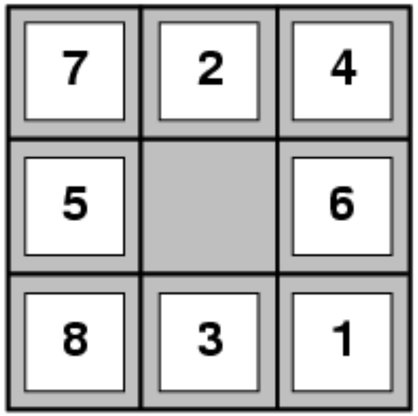
- $h_1(S)$?
- $h_1(S)$ = **8**
- Admissible: any tile out of place must be moved at least once

Admissible heuristics

E.g., for the 8-puzzle:

- $\boldsymbol{h_2(n)}$ = Manhattan distance

(i.e., the sum of the horizontal and vertical distances of the tiles from their goal positions)



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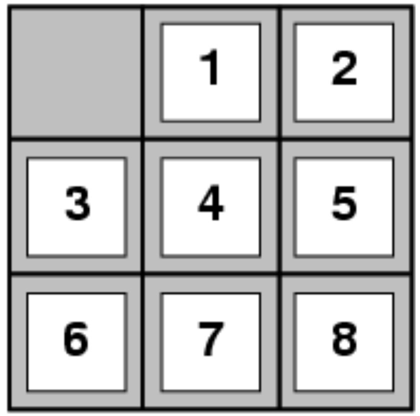

Goal State

- $h_2(S)$ =?
- $h_2(S)$ = 3+1+2+2+2+3+3+2 = **18**
- Admissible: any move can move at most one tile one step closer to the goal

Dominance

8-puzzle:
h_1 number of misplaced tiles
h_2 Manhattan distance

- If $h_2(n) \geq h_1(n)$ **for all node *n*** (both admissible)

  then h_2 **dominates** h_1

- h_2 is **better than** h_1 **for search**

  h_1 will **<u>expand</u> all the nodes** h_2 expands and **<u>possibly more</u>**

- **Comparison** of the **search costs** (average number of generated nodes) for **A* with** h_1, **A* with** h_2 and **IDS** (Iterative Deepening Search)
  - ***d* =12** **IDS** = 3,644,035 nodes
    $A^*(h_1)$ = 227 nodes
    $A^*(h_2)$ = **73 nodes**
  - ***d*=24** **IDS** = too many nodes
    $A^*(h_1)$ = 39,135 nodes
    $A^*(h_2)$ = **1,641 nodes**

d = solution length

Generating admissible heuristics from relaxed problems

- A **problem** with **fewer restrictions on the actions** is called a **relaxed problem**
- A **superset of actions** is available from **each state**
- The **graph** of the relaxed problem is a **supergraph** of the original
- Any optimal solution of the original problem is also
  a solution of the relaxed problem
- The cost of an optimal solution of the relaxed problem
  may be the same **or** lower than
  the cost of an optimal solution of the original problem

→

The cost of an optimal solution to a relaxed problem
is an **admissible heuristic** for the original problem

Generating admissible heuristics from relaxed problems

Admissible **heuristics** for the 8-puzzle:

h_1 number of misplaced tiles

h_2 Manhattan distance

- **Admissible heuristics** can be derived from the exact solution cost of a relaxed version of the problem

- If the rules of the **8-puzzle** are relaxed so that **a tile can move** anywhere (**instead** of just to the adjacent **empty square**) then $h_1(n)$ gives the **exact** number of steps of the optimal solution

- If the rules of the **8-puzzle** are relaxed so that **a tile can move** one square in any direction, even onto an occupied square, then $h_2(n)$ gives the **exact** number of steps of the optimal solution