UNIVERSITÀ DEGLI STUDI DI PADOVA

Density estimation

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Agenda

- Overcoming the limitations of k-means
- Creating density functions
  - Kernels
- Processing density functions

Segmentation techniques

- Segmentation by thresholding (histogram-based)
- Region growing methods
- Watershed transformation
- Clustering-based methods
- Model-based segmentation
- Edge-based methods
- Graph partitioning methods
- Multi-scale segmentation
- Many others…

Recall: segmentation by clustering

- Recall: image segmentation based on a clustering technique
- Recall: image description by means of feature vector
- In such context we analyzed how k-means leads to a segmentation

Limitations of k-means

- K-means is a good choice, but:
  - A parametric method: k needs to be provided
    - Advantage/disadvantage
  - Non-optimal method
  - Forces spherical symmetry (in the N-dimensional space)

Beyond k-means

- How could we derive a deeper analysis on the dataset?
  - You probably know several techniques from previous courses:
    - Recall
    - Discuss with your classmates

- Anti-spoiler ☺

Beyond k-means

- Many possible options
- An idea:
  - Create a density function
  - Look for the modes of the density function

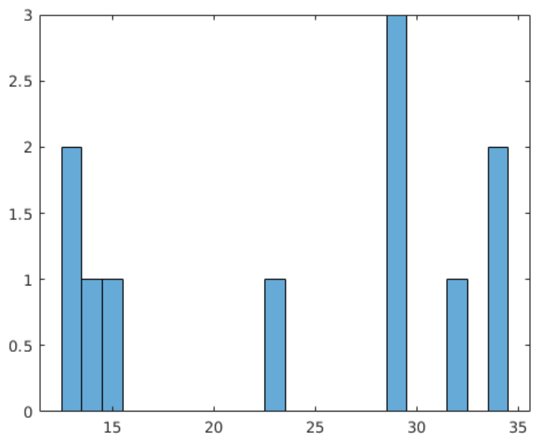
Creating a density function

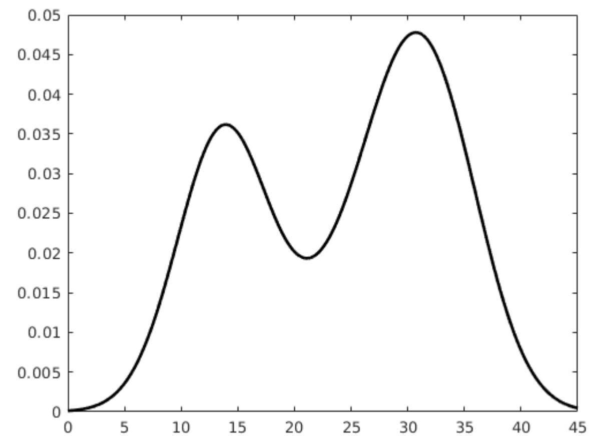
- In computer vision we have pixels, not density functions
- What density function related to an image can we define?

- Anti-spoiler ☺

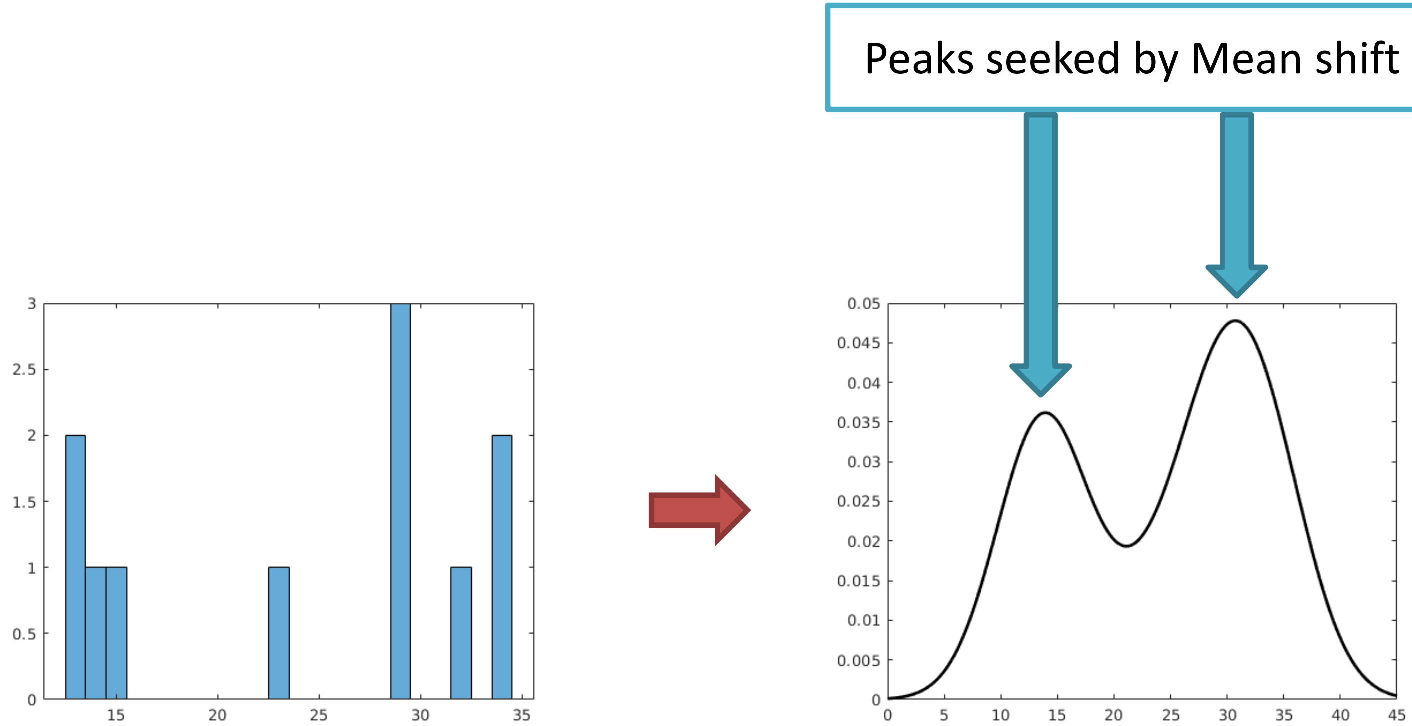
Creating a density function

- Starting point: set of samples
- Desired output: density function (PDF)
- Simple approach: kernel density estimation (AKA Parzen window technique)
  - Convolution with a given kernel of radius r



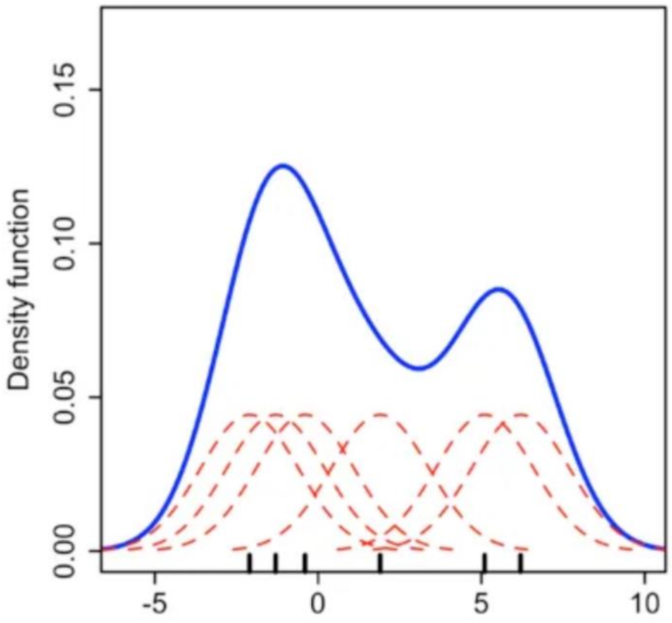


Creating a density function



Creating a density function

- We can create a density function by:
  - Choosing a kernel (e.g. a gaussian function)
  - Centering a kernel on each sample
  - Summing up all the contributions



Kernel

- The kernel is defined as:

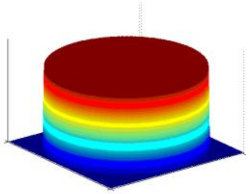
$$K(\boldsymbol{x}) = c_k k\left(\left\|\frac{\boldsymbol{x}}{r}\right\|^2\right) = c_k k\left(\frac{\|\boldsymbol{x}^2\|}{r^2}\right)$$

- For point $\boldsymbol{x}$ in the n-dimensional feature space
- The kernel $K(\cdot)$ integrates to 1
- The kernel has radius r
- $k(\cdot)$ is a 1-dimensional profile generating the kernel
  - Applied to all dimensions of the feature space

Kernels

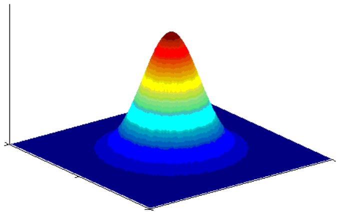
- Several kernels may be used:
  - Uniform kernel

$$k_U(x) = \begin{cases} c_u & \|x\| < 1 \\ 0 & \text{otherwise} \end{cases}$$



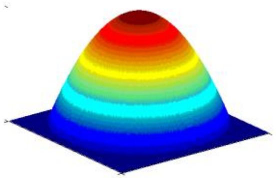
  - Normal kernel

$$k_N = c_n \exp\left(-\frac{1}{2}\|x\|^2\right)$$



  - Epanechnikov kernel

$$k_E(x) = \begin{cases} c_e\left(1 - \|x\|^2\right) & \|x\| < 1 \\ 0 & \text{otherwise} \end{cases}$$



Creating a density function

- The convolution with a given kernel of width r is expressed as the sum of a translated kernel for each data point
- A function of the N data points

$$f(\boldsymbol{x}) = \frac{1}{Nr^n} \sum_{i=1}^{N} K(\boldsymbol{x} - \boldsymbol{x}_i) = \frac{c_k}{Nr^n} \sum_{i=1}^{N} k\left(\frac{\|\boldsymbol{x} - \boldsymbol{x}_i\|^2}{r^2}\right)$$

Where $\boldsymbol{x}_i$ are the input samples and $k(\cdot)$ is the kernel function

- The factor r^n normalizes by the number of dimensions
  - Vector $\boldsymbol{x} \in \mathbb{R}^{\boldsymbol{n}}$

Density function processing

- We described how to derive a density function
- We can then
  - Find the major peaks (modes)
  - Identify regions of the input space that climb to the same peak
    - Such regions belong to the same region/cluster
- Intuition about how the density function could be used for segmentation

Density function processing

- We described how to derive a density function, but…
- Consider the complexity in an n-dimensional space!
- Creating a density function is computationally complex
  - Possible but inefficient
  - Really needed or unnecessary?
  - Alternatives?

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Density estimation

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