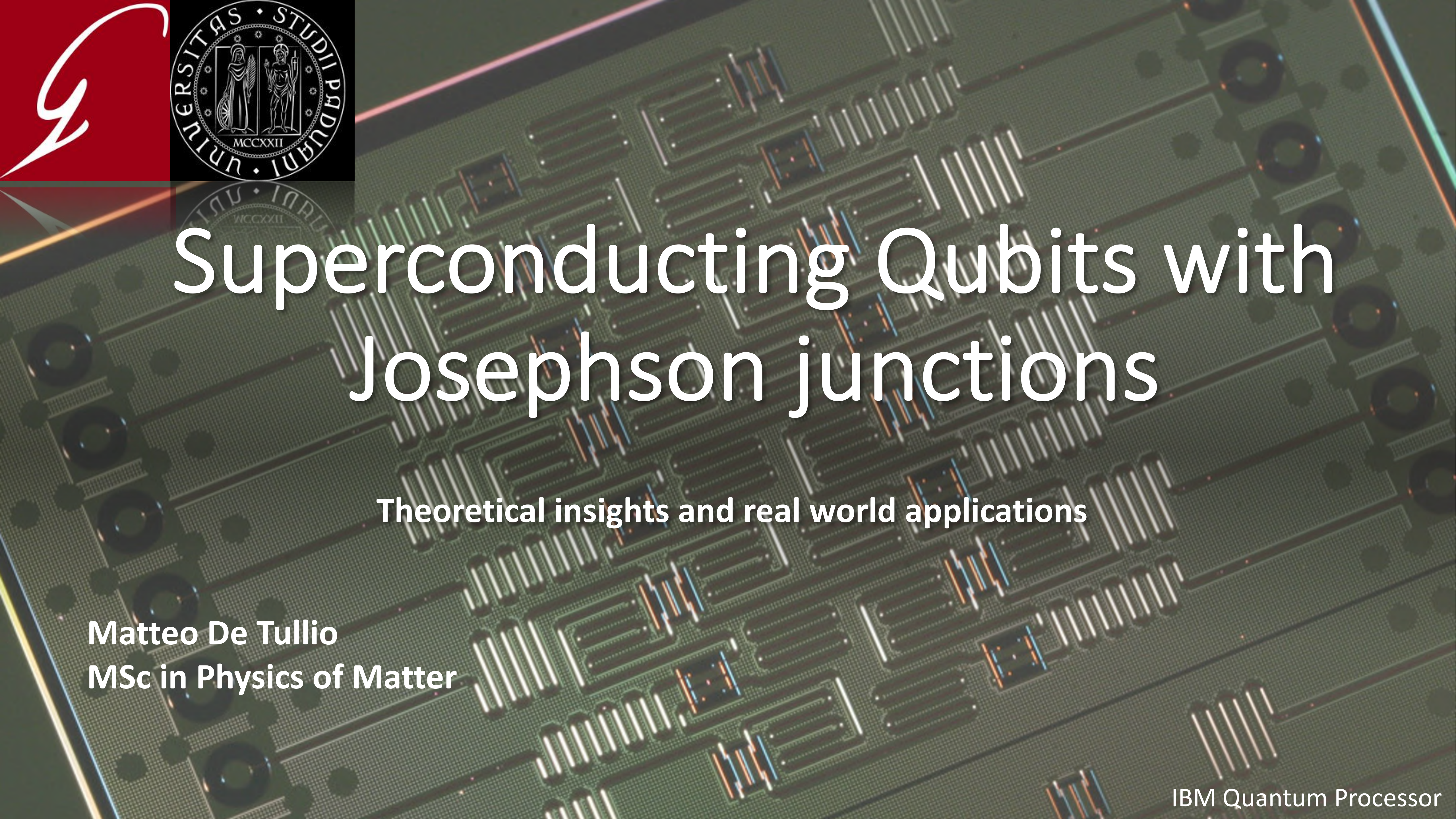
Superconducting Qubits with Josephson junctions

Theoretical insights and real world applications

Matteo De Tullio
MSc in Physics of Matter



IBM Quantum Processor

this talk will last ~ 25 minutes

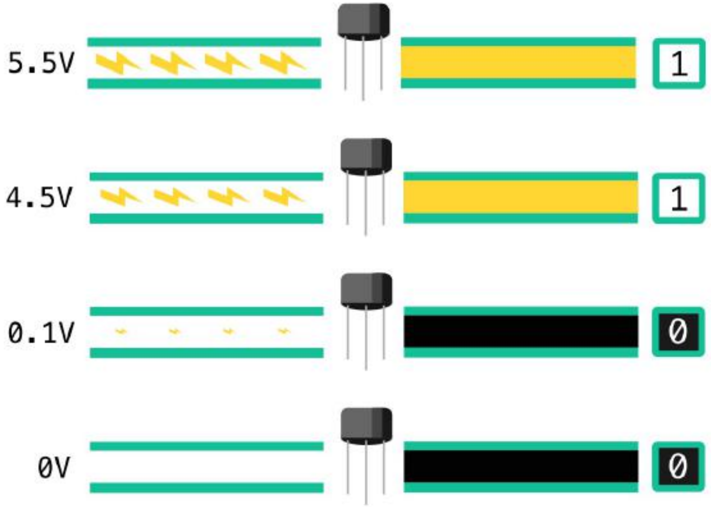
Outline

Artificial atoms	Why superconductors?	Making a non-linear resonator: the Josephson junction	The SC qubits family tree
The transmon qubit	Qubits gates: single and coupled	Qubit readout	Problems and applications

Bit vs qubit

Classic bit (binary digit)

- Two levels system (0 or 1 logic state)
- Physically implementations:
  - electrical switches (*transistors*)
  - distinct levels of voltage/currents in a circuit
  - magnetization directions…
- A measurement does not affect the state

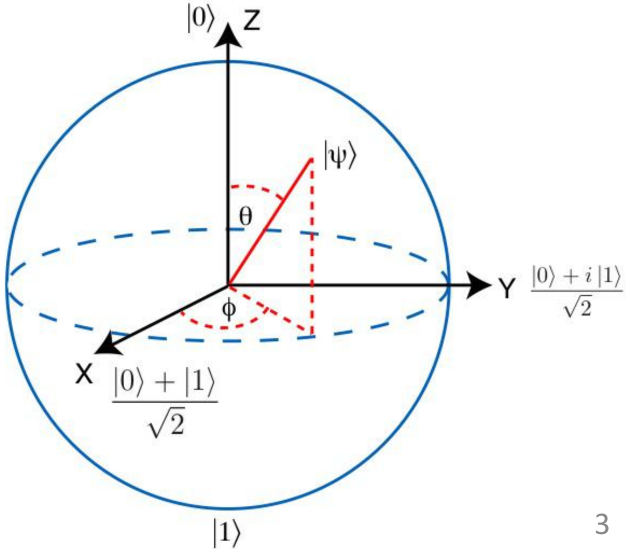


Qubit (quantum bit)

- Coherent superposition of two quantum states
- Wave vector pointing towards a position in the Bloch sphere:

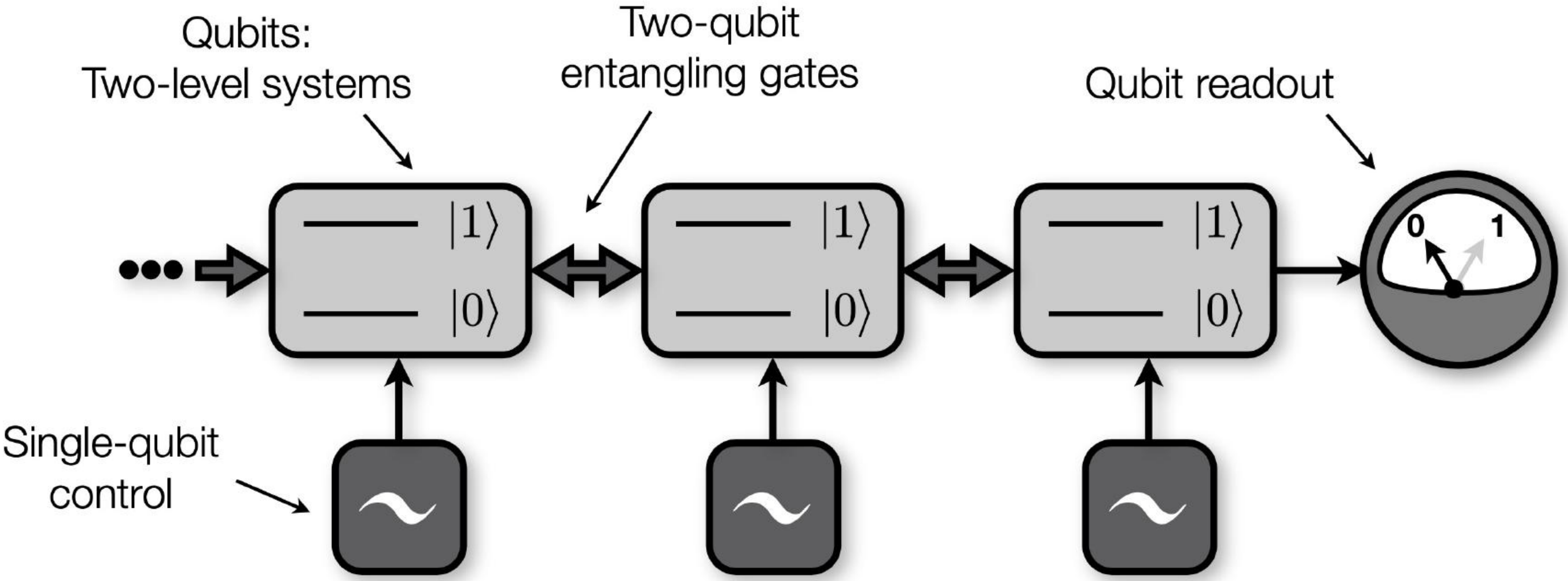
$$\psi = \alpha|0\rangle + \beta|1\rangle$$

- A measurement destroys coherence



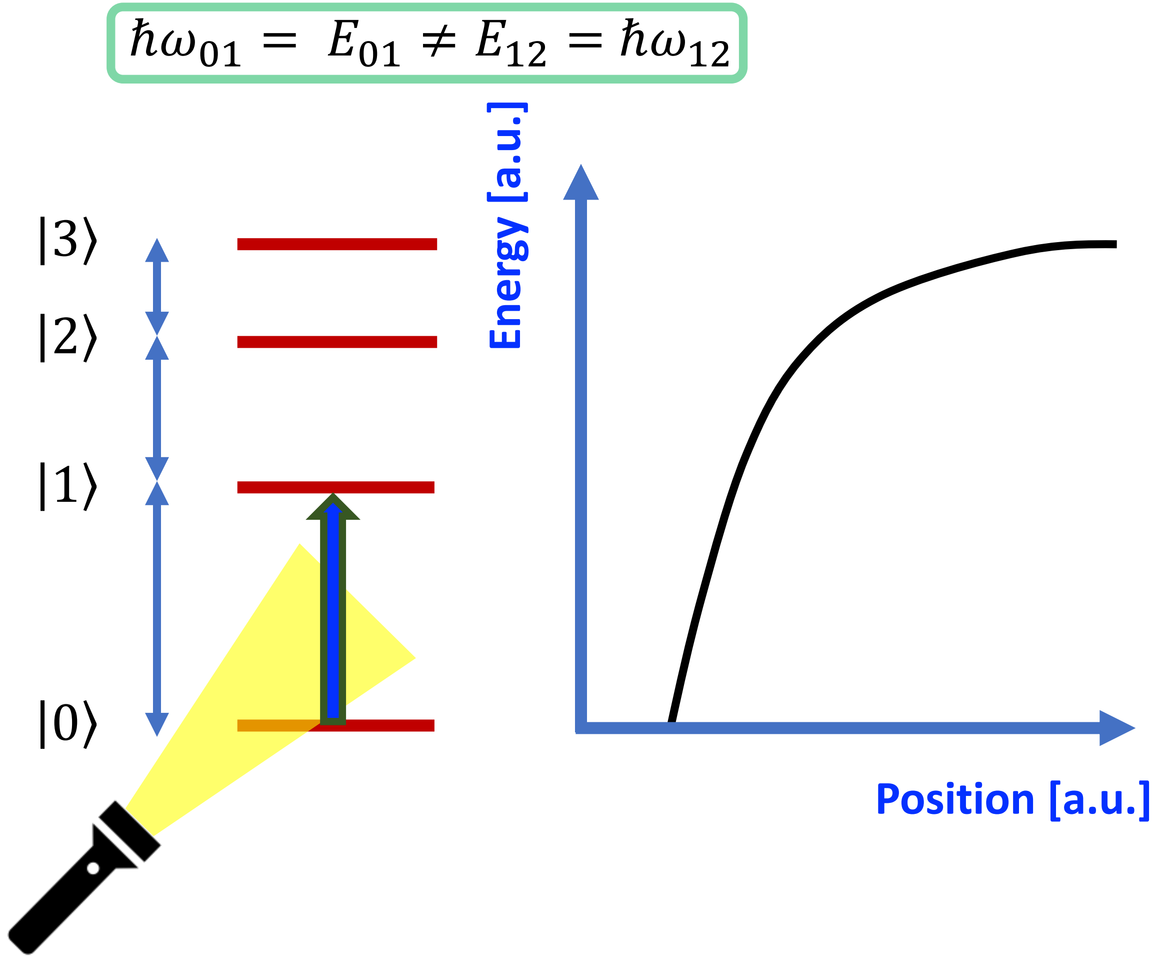
The challenge

Dipartimento di Fisica e Astronomia Galileo Galilei



D. DiVincenzo, Fortschritte der Physik **48**, 771 (2000)

Nature "atomic" atoms



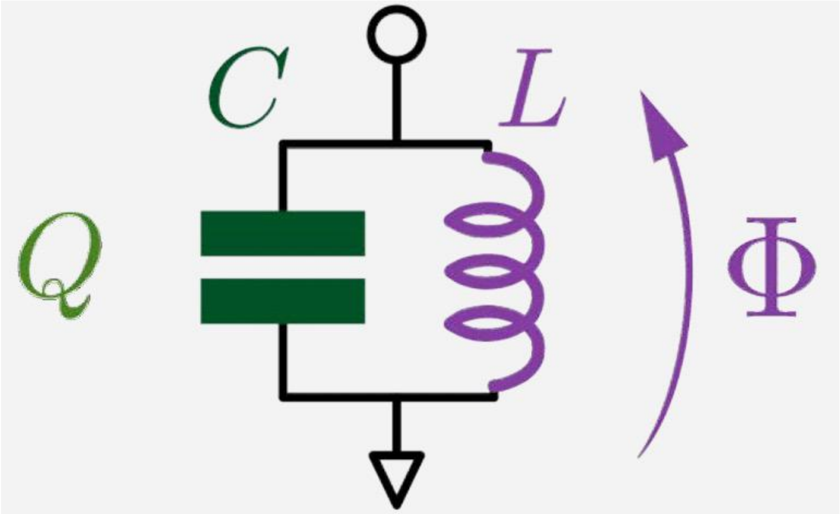
- Laser tuned at the right transition frequency ω_{01}

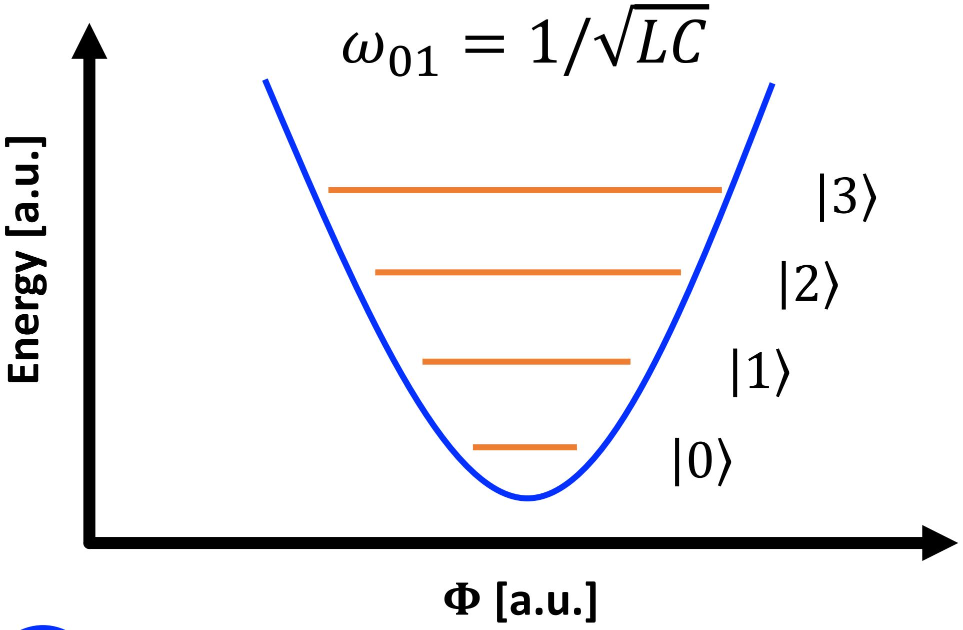
- $T_{decay} \sim years$
- $T_{decoh} \geq 10\ seconds$
- $T_{pulse} \sim 5\ \mu s$

Low error per gate $\sim 0.48\ \%$

"*Artificial*" atoms – LC harmonic oscillator

Dipartimento di Fisica e Astronomia Galileo Galilei





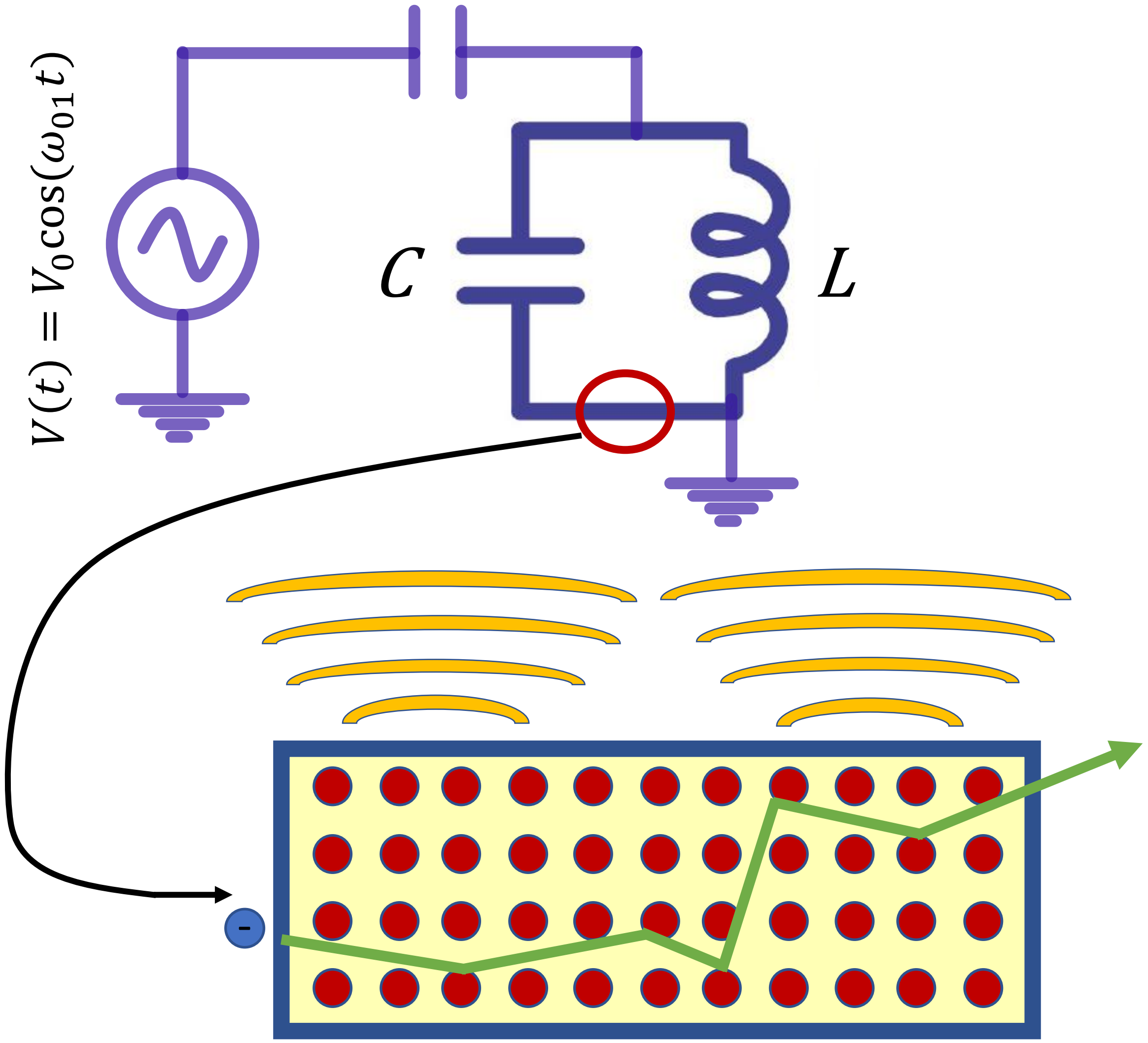
$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

Capacitive term

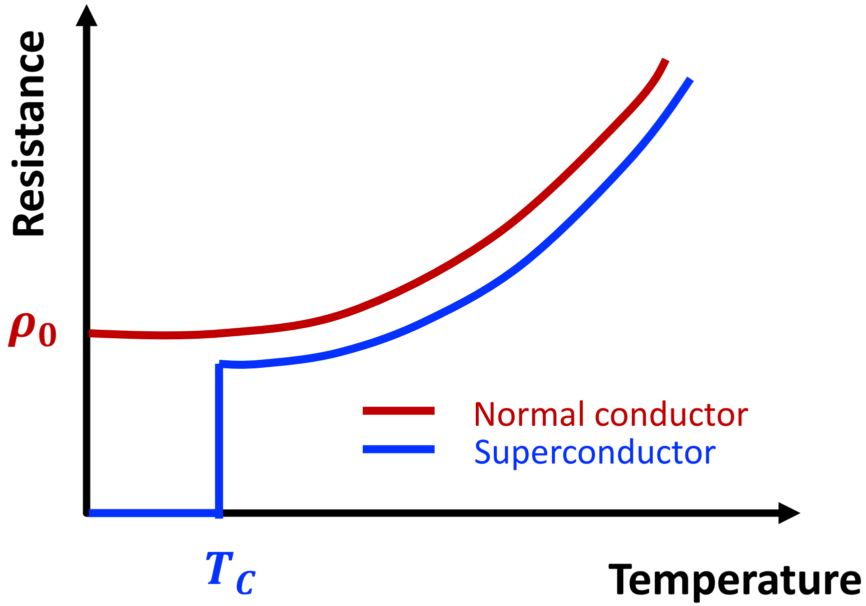
Inductive term

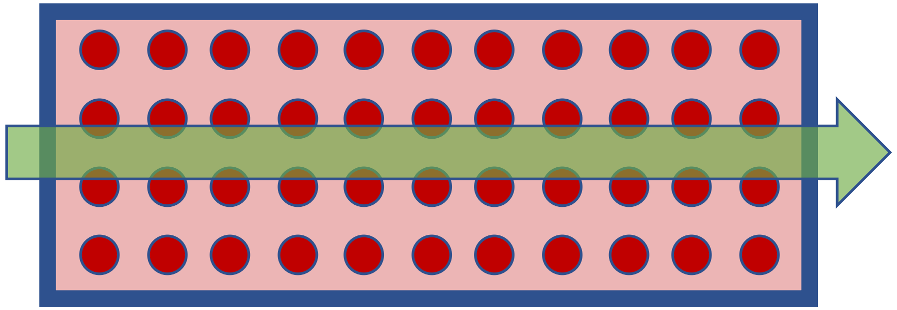
How to make a quantum circuit

Dipartimento di Fisica e Astronomia Galileo Galilei



Normal conductor ➡ Joule heating, leakage of information





Superconducting wire ➡ quantum mechanical behavior is preserved!

The Helium dilution refrigerator

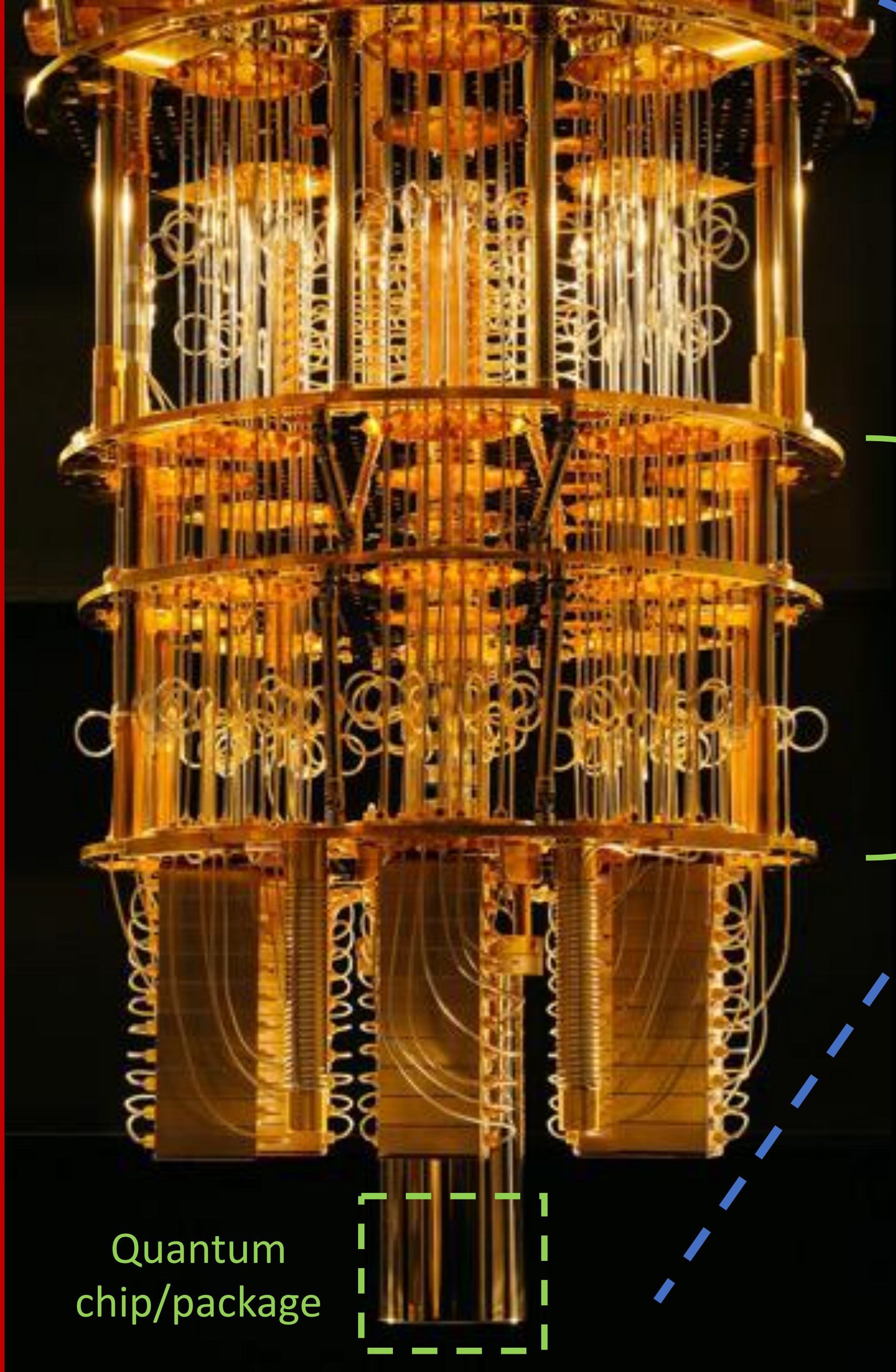
$$\omega_{01} = \frac{1}{\sqrt{LC}} \sim 10\ GHz$$

Quantum condition:

$$T \ll \frac{\hbar\omega_{01}}{K_B} \sim 0.3\ K$$

Commercial He dilution refrigerators:

$$T = 20\ mK$$



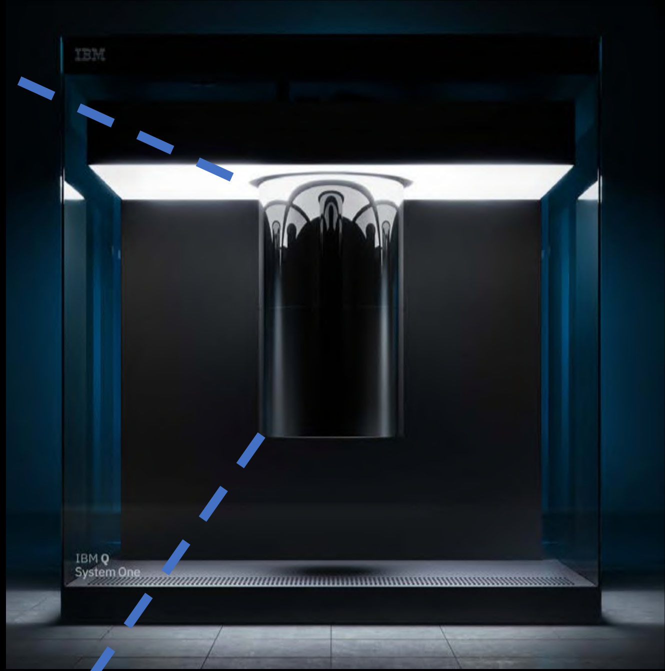
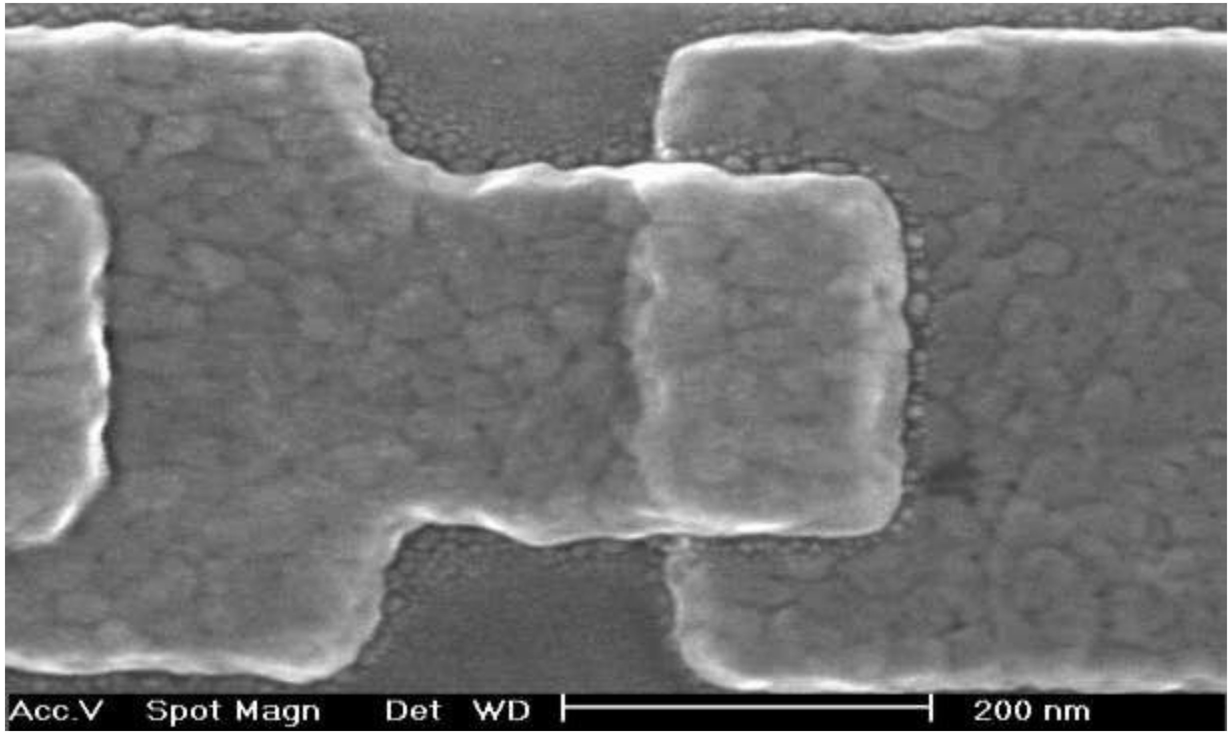
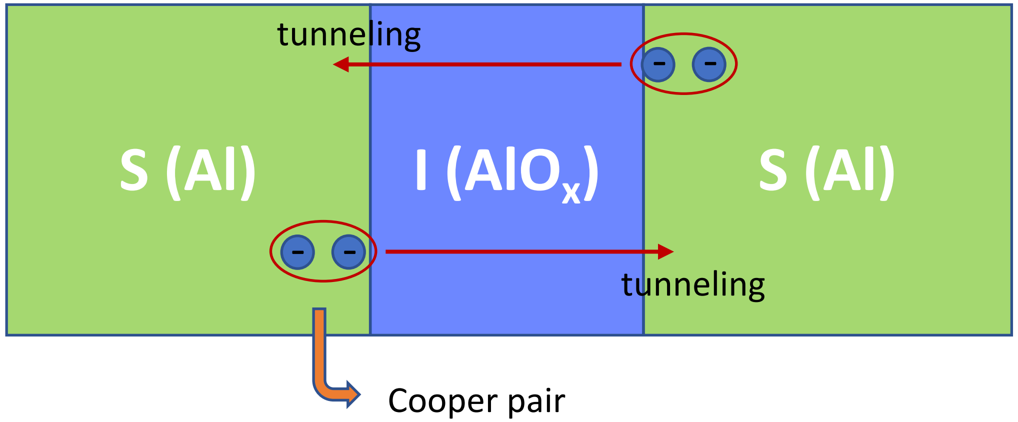


Illustration: IBM, Qiskit Textbook

The Josephson junction – *anharmonic* oscillator (I)

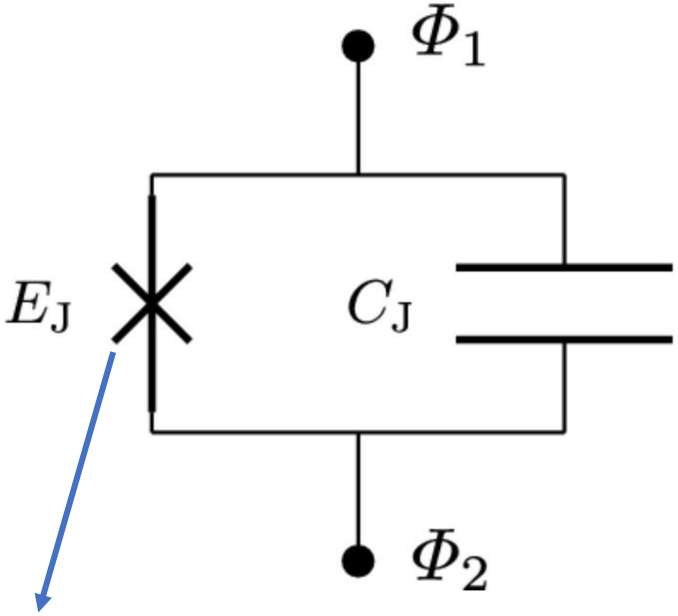




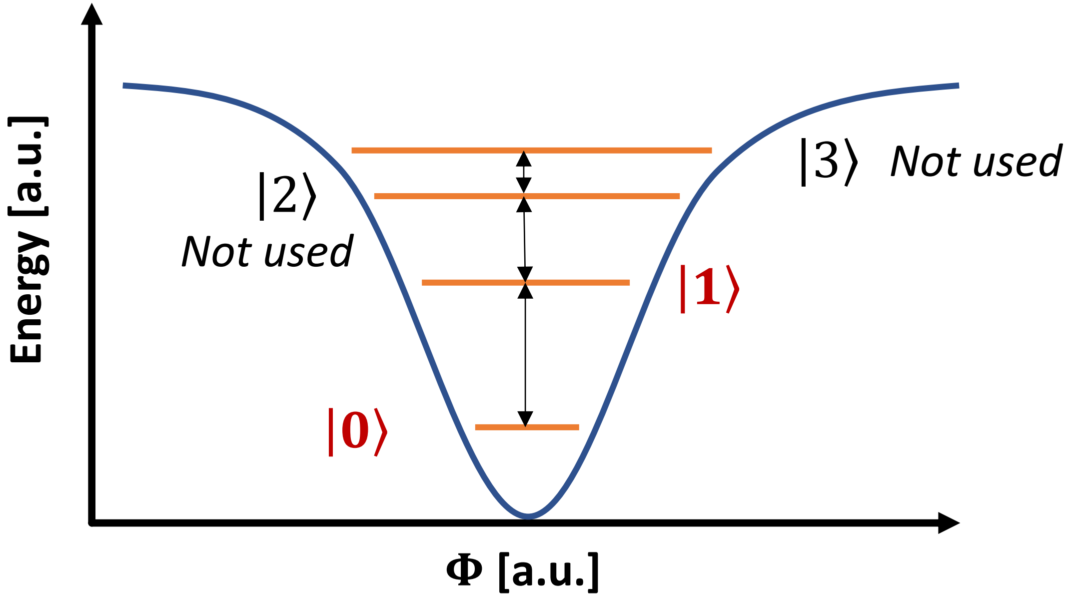
$$I = I_0 \sin(2\pi\Phi/\Phi_0) \quad \Rightarrow \quad L_J = \left(\frac{\partial I}{\partial \Phi}\right)^{-1} = \frac{\Phi_0}{2\pi I_0}\frac{1}{\cos(2\pi\Phi/\Phi_0)}$$

Niemczyk,… . (2009). Fabrication Technology of and Symmetry Breaking in Superconducting Quantum Circuits. 22. 10.1088/0953-2048/22/3/034009.

The Josephson junction – *anharmonic* oscillator (II)



Josephson junction symbol



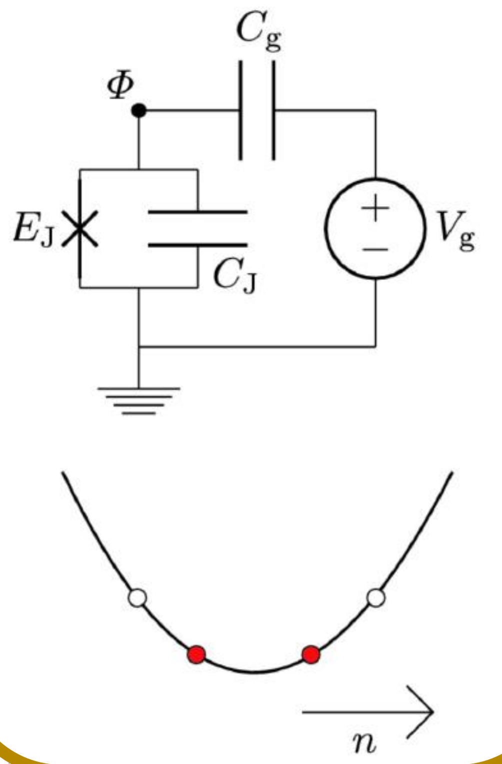
$$H = \frac{Q^2}{2C} - E_J \cos(2\pi\Phi/\Phi_0)$$

A.F. Kockum, Quantum Optics with Artificial Atoms. Ph.D. thesis, Chalmers University of Technology, 2014.

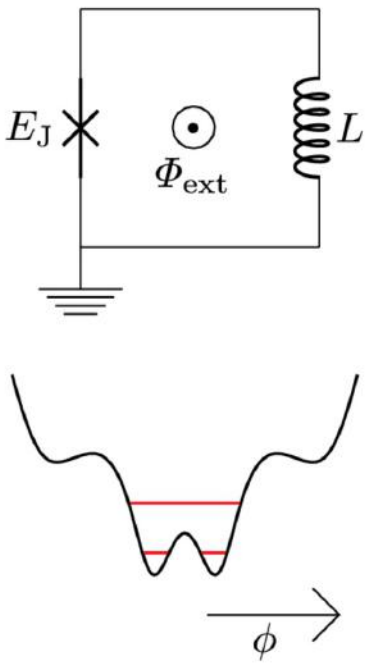
Josephson-junction based qubits

$[\phi, n] = i \Rightarrow \Delta\phi\Delta n \geq 1$ *Heisenberg uncertainty*

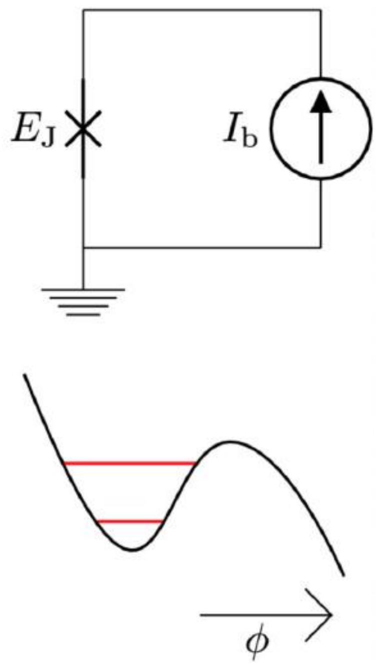
Charge qubit



Flux qubit



Phase qubit



$\frac{E_J}{E_C} \ll 1$

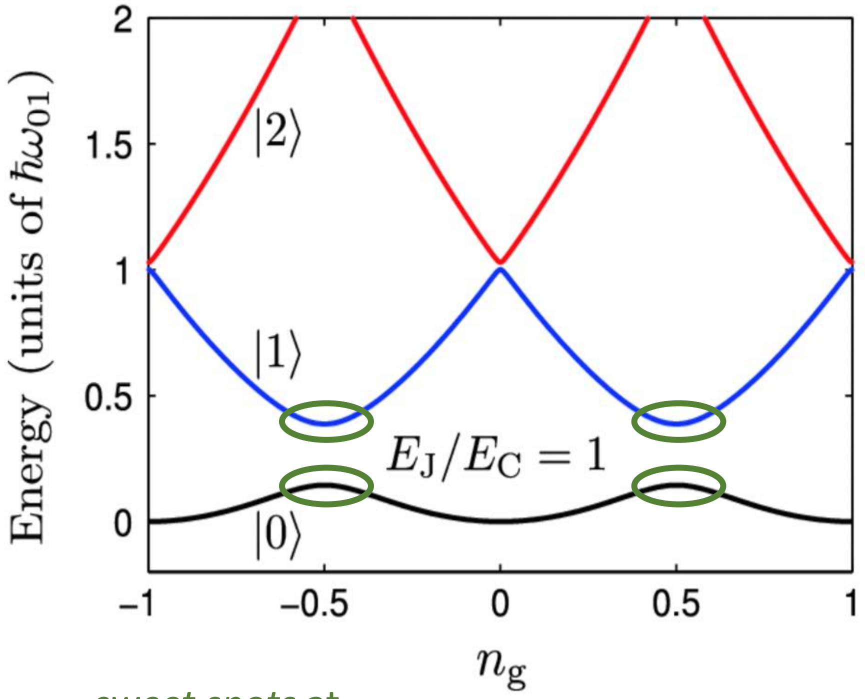
- large ϕ quantum fluctuations
- well defined charge number *n*

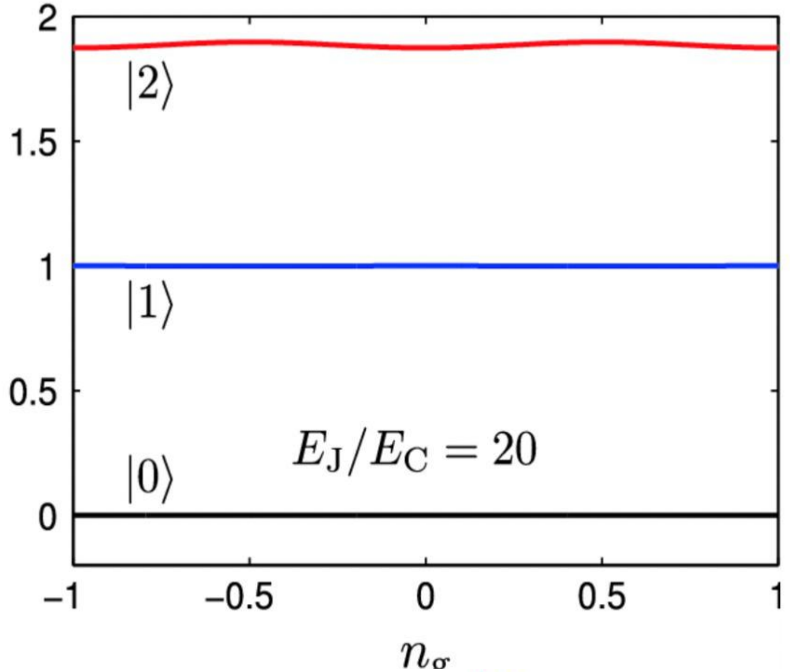
$\frac{E_J}{E_C} \gg 1$

Opposite behavior w.r.t. Charge qubit

X. Gu..., Microwave photonics with superconducting quantum circuits. Phys. Rep. **718–719**, 1–102 (2017).

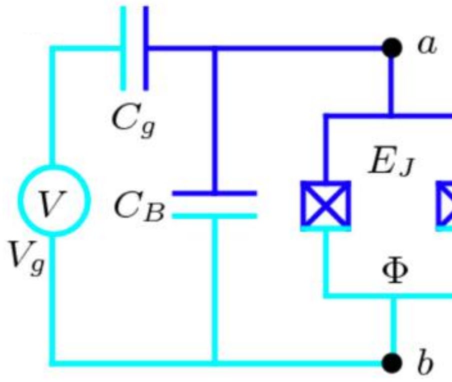
The transmon qubit





sweet spots at
$n_g = m + \frac{1}{2}$,
$m \in \mathbb{Z}$

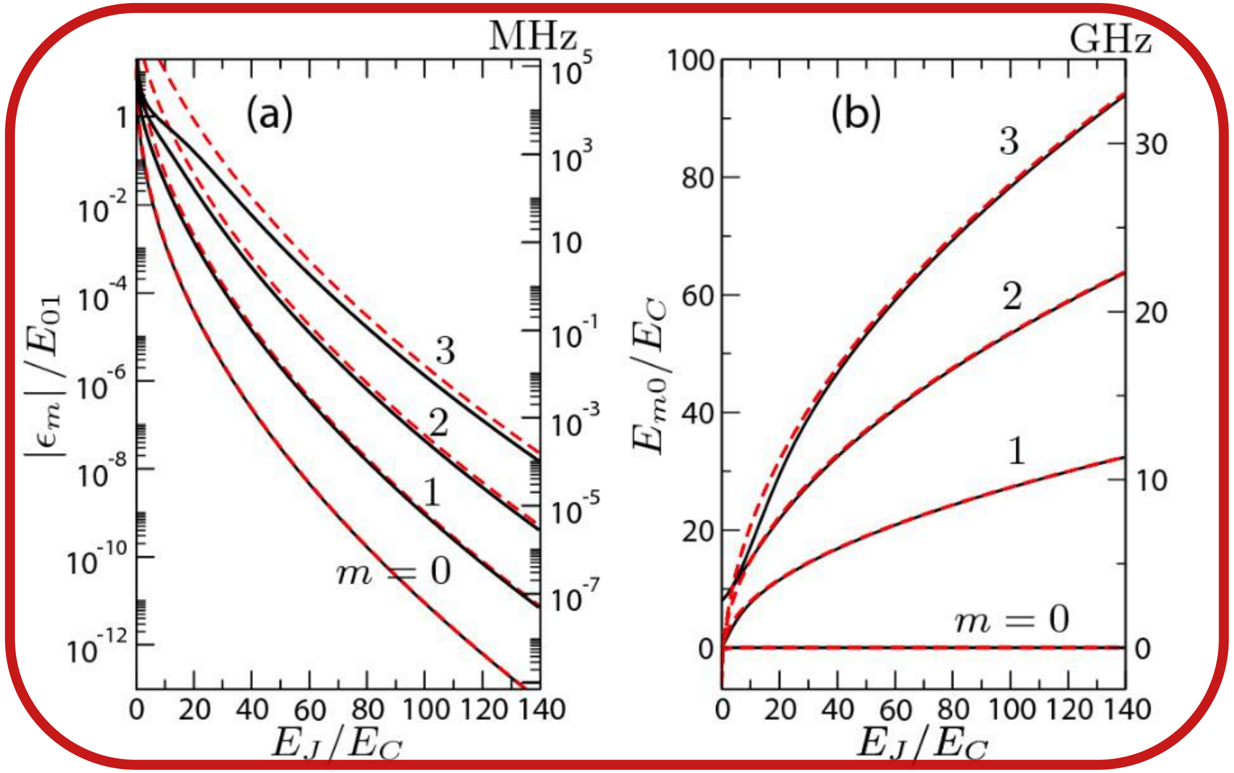
$$H = 4\,E_C\,(n - n_g)^2 + E_J\cos(\Phi)$$



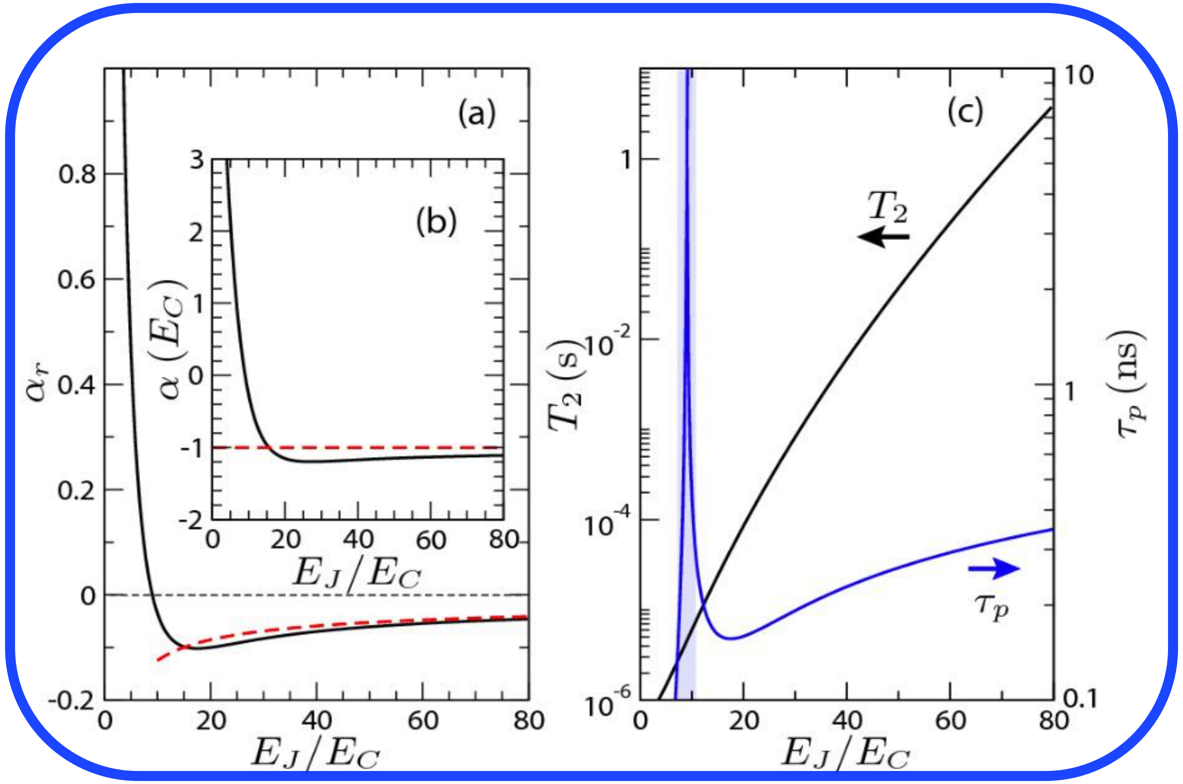
Koch, Jens & Yu, .. (2007). Physical Review A. 76. 10.1103/PhysRevA.76.042319

Charge dispersion vs anharmonicity at degeneracy points

Charge dispersion



Anharmonicity



$\epsilon_m \propto e^{-\sqrt{8E_J/E_C}}$ for $E_J/E_C \gg 1$

$\alpha_r \propto -\sqrt{8E_J/E_C}$ for $E_J/E_C \gg 1$

Koch, Jens & Yu, .. (2007). Physical Review A. 76. 10.1103/PhysRevA.76.042319

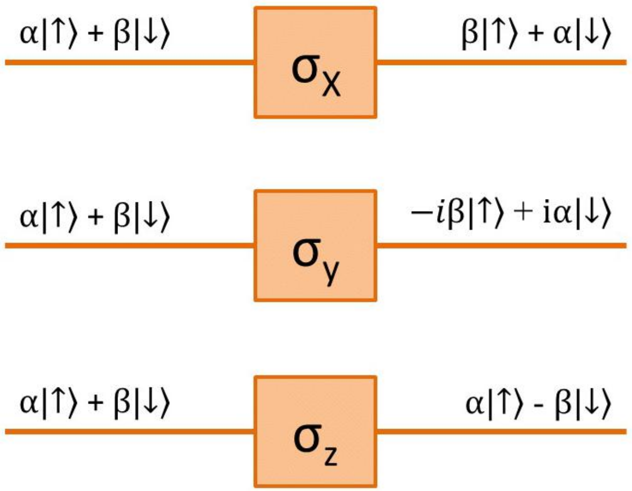
The DiVincenzo criteria

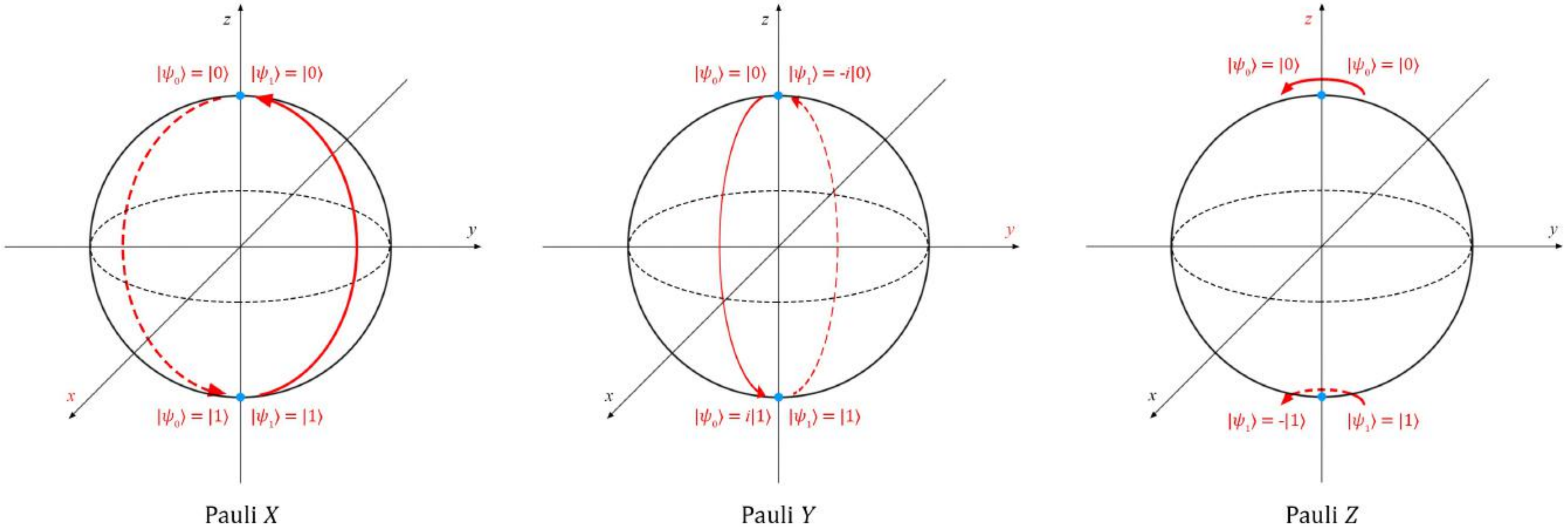
1. **Qubits**: The state must be described as a normalized superposition of two states and the internal Hamiltonian of the system must be known
2. **Initialization**: it must be possible to initialize these qubits to a simple and known state
3. **Gates**: it must be possible to perform both single and two qubit gates on the qubits with high fidelity
4. **Readout**: it must be possible to measure the states of the qubits.
5. **Coherence**: the coherence times of the qubits must be long enough to allow a large number of gates to be performed in sequence before a significant loss of quantum coherence occurs

D.P. DiVincenzo, The physical implementation of quantum computation. Fortschritte der Phys. **48**, 771 (2000)

Qubit single gates

Dipartimento di Fisica e Astronomia Galileo Galilei





Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Microsoft Azure Quantum documentation

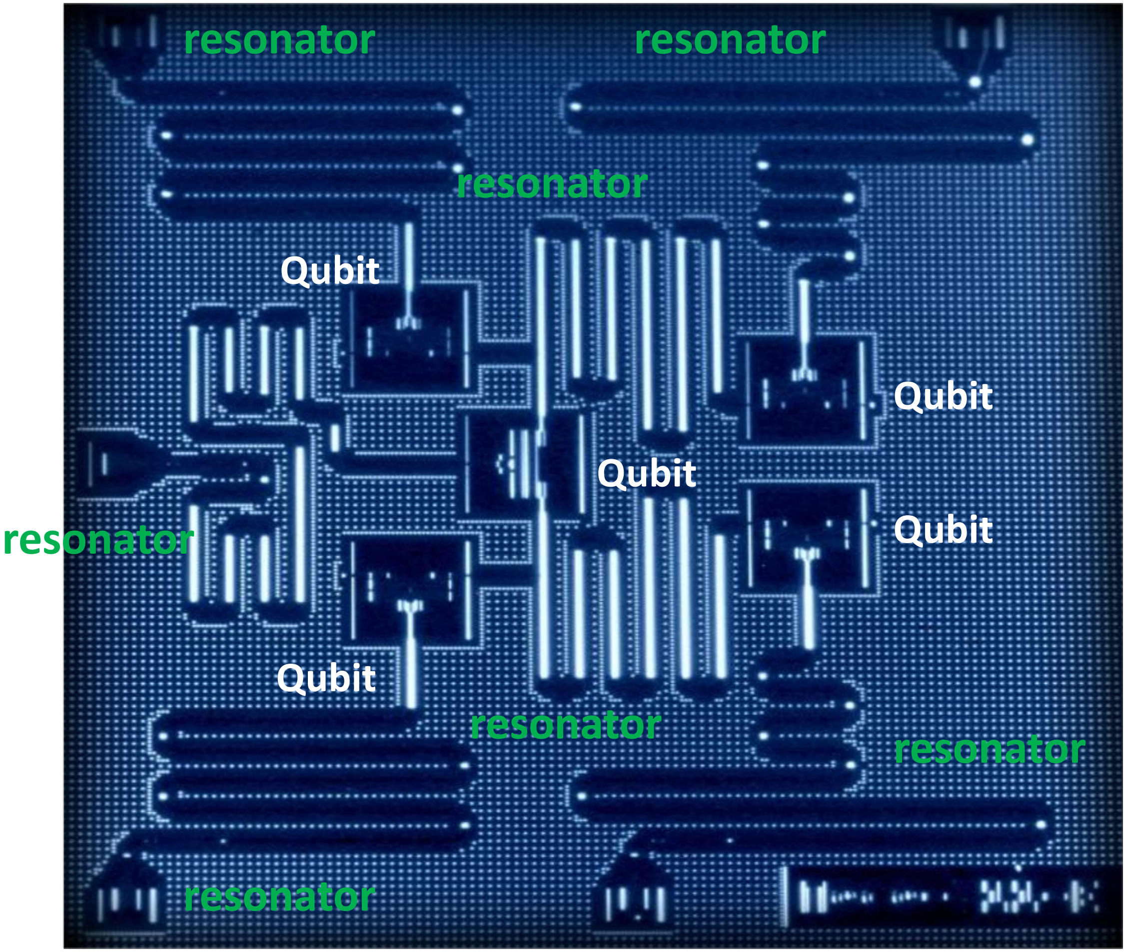
Coupling J-J qubits

Josephson junction coupling

Direct connection:
- capacitively
- inductively

Connection by intermediate electrical coupling circuit:
- LC resonator
- Another J-J qubit
- DC-Squid

Intermediate quantum bus



Turn off qubit coupling: <u>tuning transition frequencies far from resonance with each other</u>

Qubit readout

Qubit + **Resonator** + **Interaction**

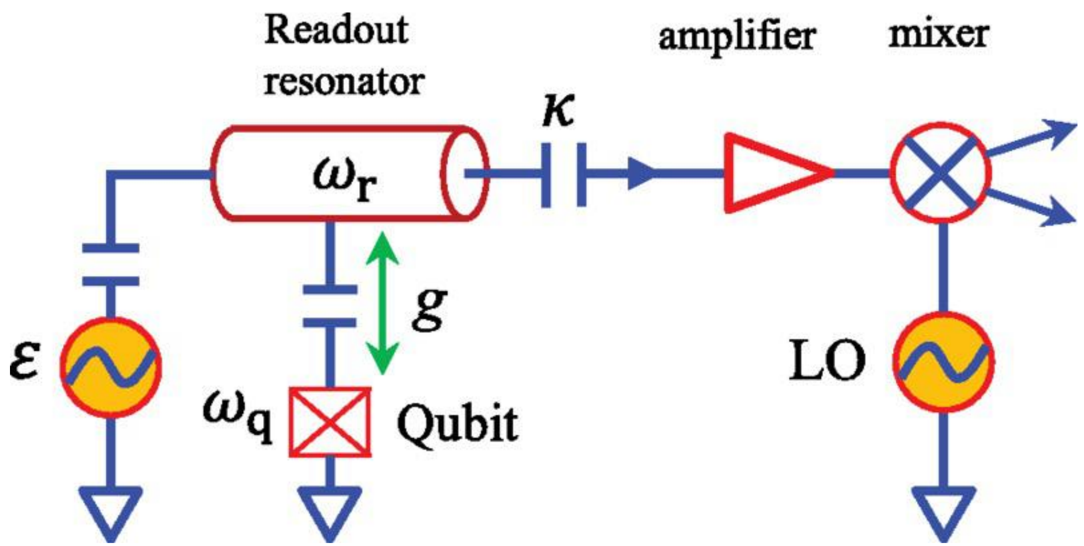
Frequency ω_q

Frequency ω_r

Interaction strength g

$$\boldsymbol{H = H_{Qub} + H_{Res} + H_{Int}}$$

Jaynes-Cummings Hamiltonian



Dispersive interaction condition: $\boldsymbol{\omega_r - \omega_q \gg g}$

$$H = \hbar \frac{g^2}{\omega_r - \omega_q}\left(|1\rangle\langle 1| + a^\dagger a\, \sigma_z\right)$$

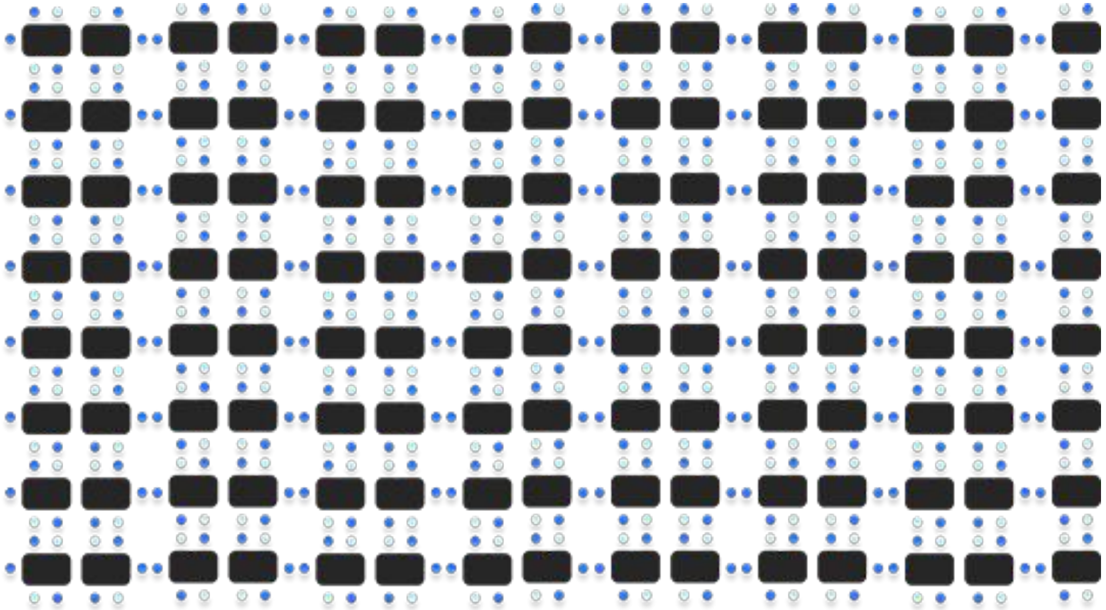
Time evolution:

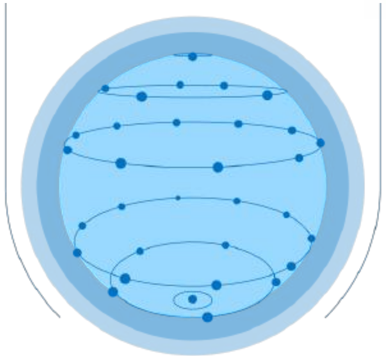
$$\boldsymbol{e^{-iHt/\hbar}}|0\rangle|\alpha\rangle = |0\rangle\left|\alpha\, \boldsymbol{e^{-ig^2t/(\omega_r-\omega_q)}}\right\rangle$$

$$\boldsymbol{e^{-iHt/\hbar}}|1\rangle|\alpha\rangle = |1\rangle\left|\alpha\, \boldsymbol{e^{ig^2t/(\omega_r-\omega_q)}}\right\rangle$$

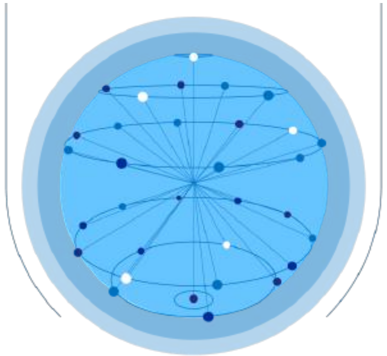
Physical Review A. 92. 10.1103/PhysRevA.92.012325.

Supercomputer vs quantum computer

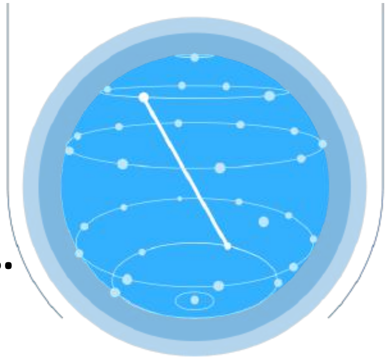




1. **Activation: 2^n states**



2. **Encoding: applying gates**



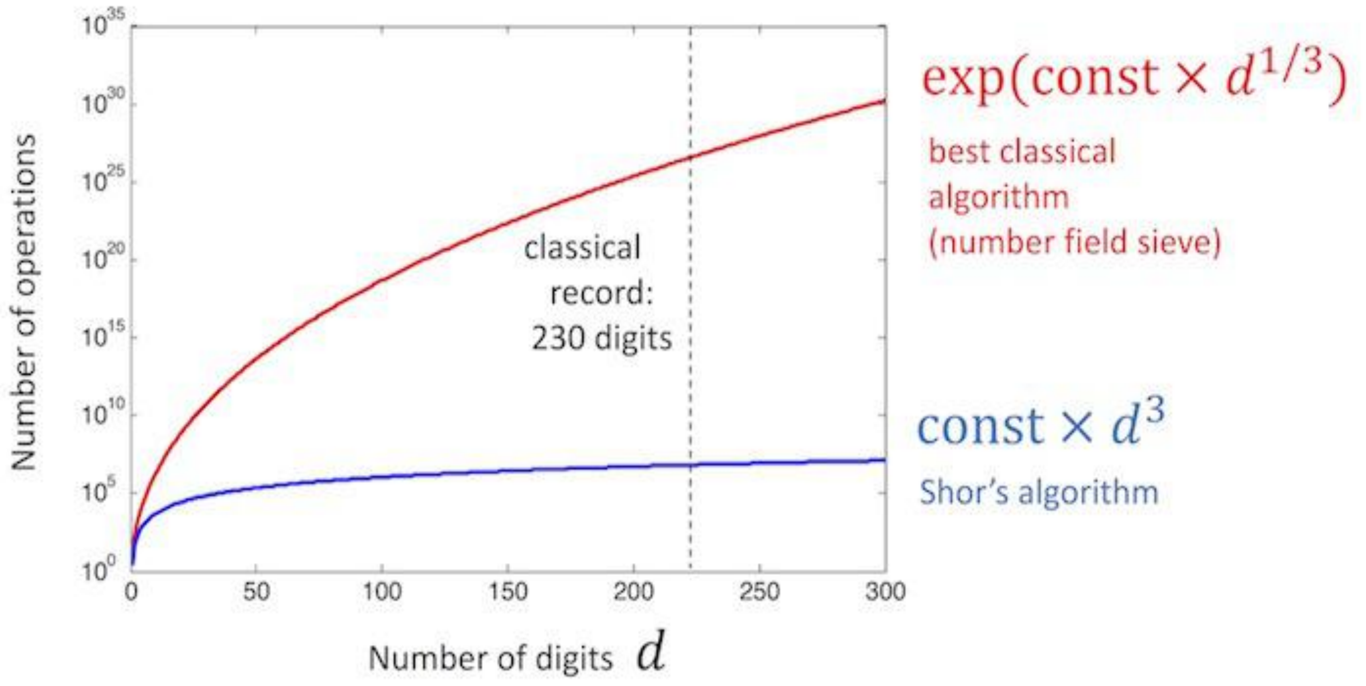
3. **Solution: using intereference and translating to 010011…**

- 5 people – 120 combinations
- 10 people – 3.628.800 combinations!

Primes factorization and searching algorithms

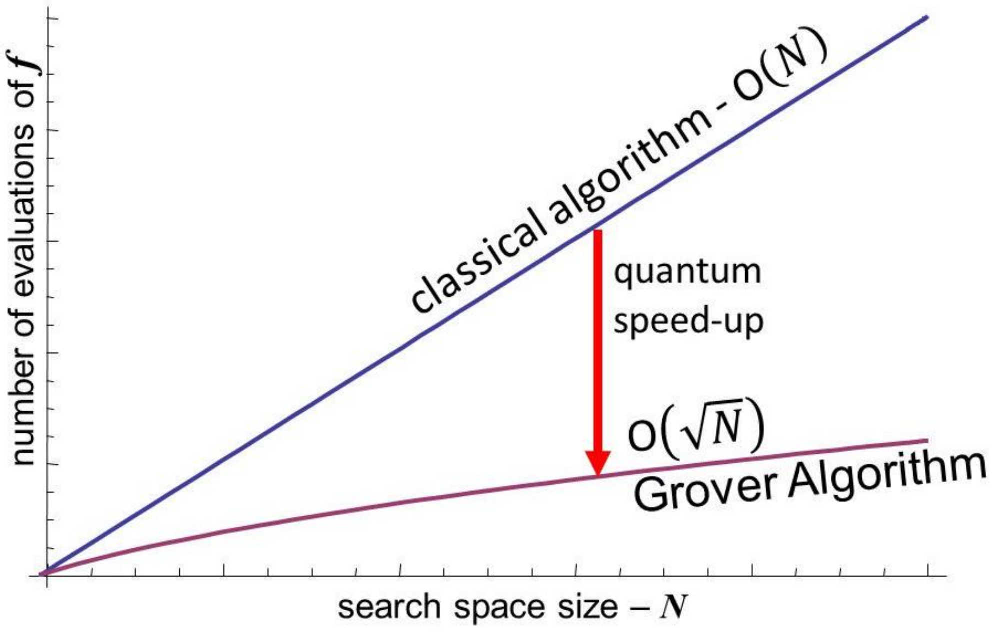
Dipartimento di Fisica e Astronomia Galileo Galilei

Primes factorization



IBM Quantum

Searching from a list



$M = p \cdot q$

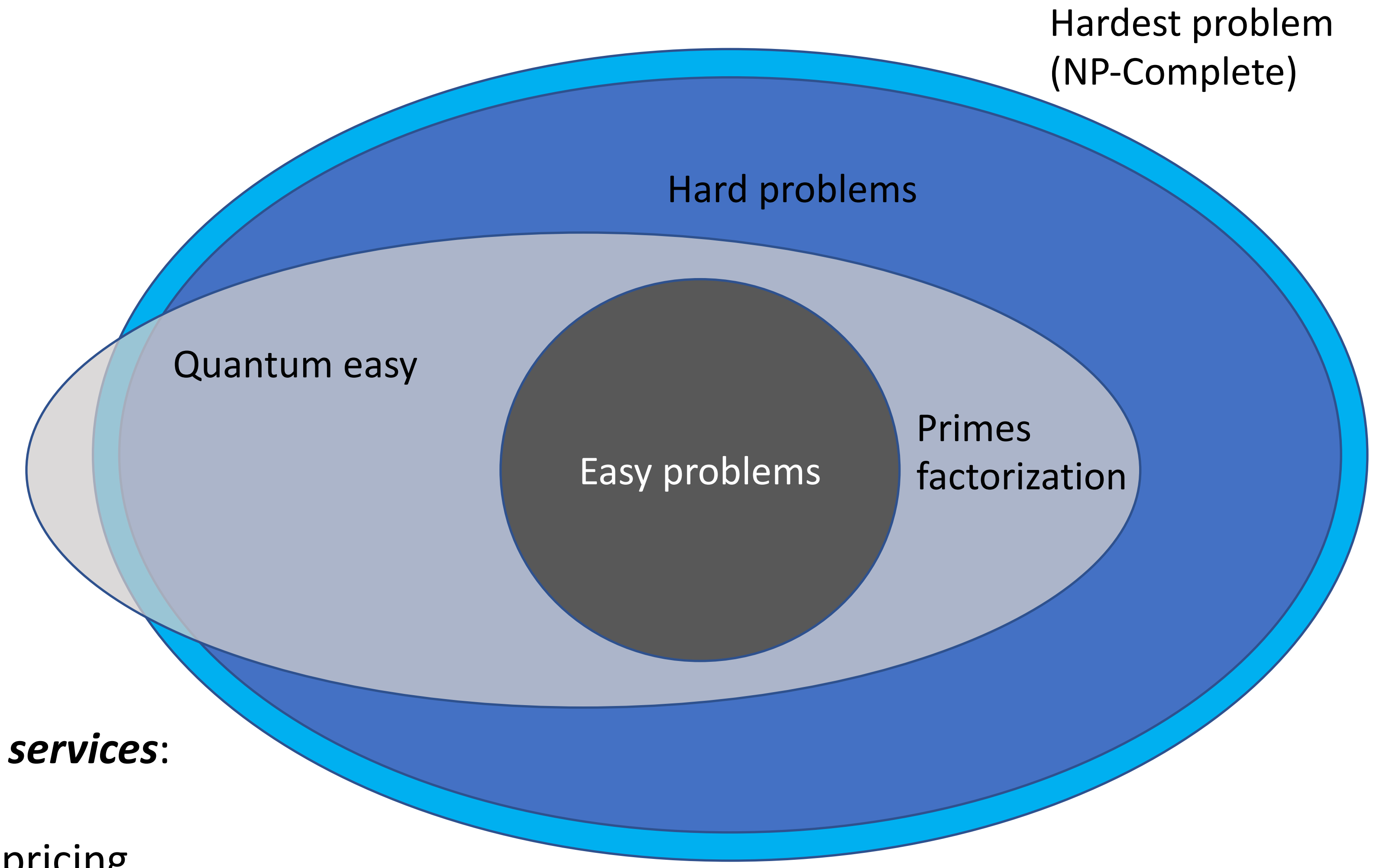
p, q prime numbers

Classical	$t \sim exp(O(d^3))$	$2.8 \cdot 10^{22}$ years
Quantum	$t \sim O(d^3)$	$100\ s$

Applications

Combinatorial optimization problems:

- Assignment problem
- Closure problem
- Constraint satisfaction problem
- Cutting stock problem
- Integer programming
- Knapsack problem
- Minimum spanning tree
- Scheduling problem
- Traveling salesman problem
- Vehicle routing and rescheduling

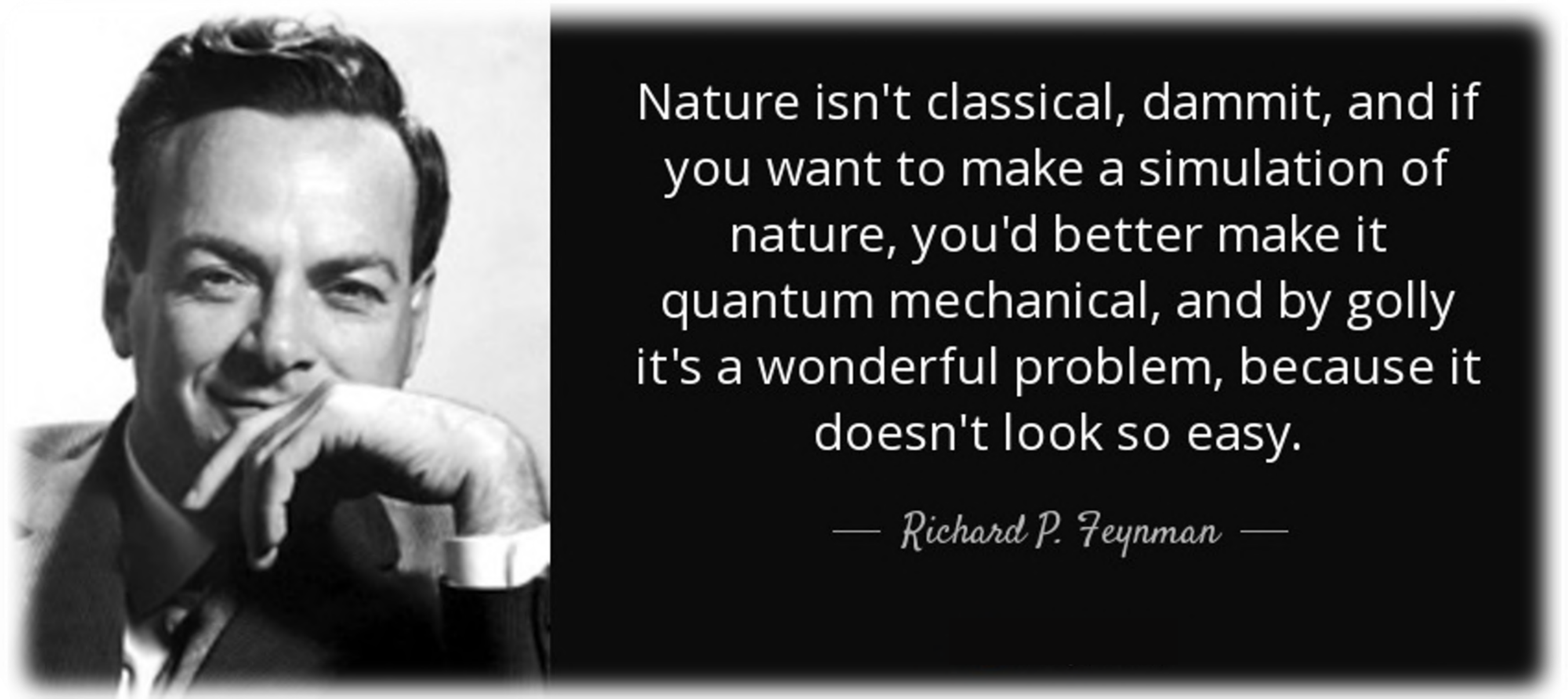


Chemistry models:

- Material design
- Drug discovery

Financial services:

- Asset pricing
- Risk analysis
- Rare events simulation


Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy.
— Richard P. Feynman —