20. Asymptotic output zeroing problem (AOZP)

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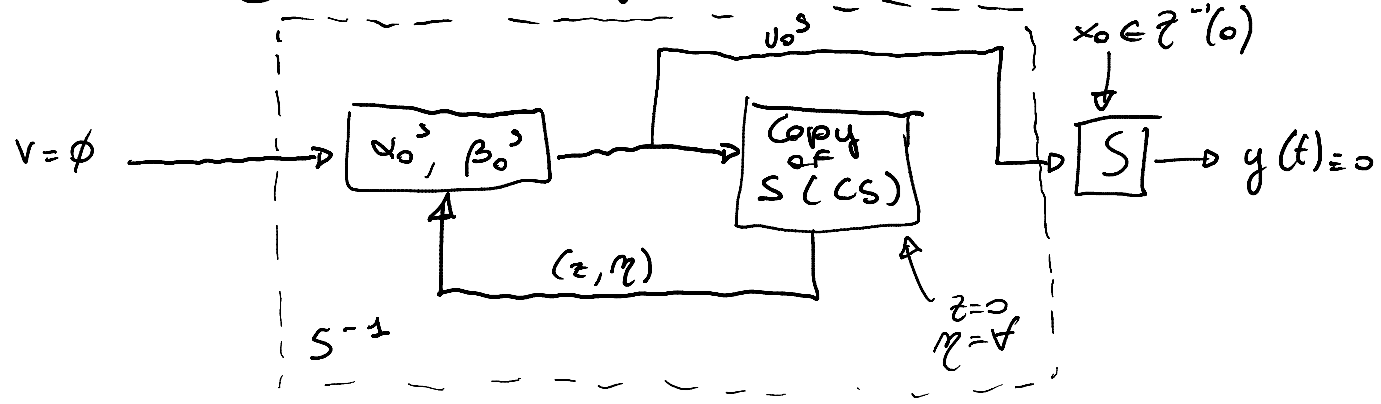
The output zeroing problem it's related to the possibility of making $y(t) \equiv 0$ by means of a feedback (α_0^s, β_0^s).



The full block S^{-1} is the "right inverse", a system which produces the input to the system S for tracking the given reference, if feeded with this last one.

A SISO NL system is locally right invertible around any point at which the relative degree is defined.

$$\dot{\eta} = q(z, \eta)$$

$$v_0^s = -\frac{b(z,\eta)}{a(z,\eta)} + \frac{1}{a(z,\eta)} \left(\sum_{i=1}^{r} \alpha_{i-1}^* z_i \right)$$

if (1) $\alpha = (\alpha_0^*, \dots, \alpha_{r-1}^*)$ is Hurwitz.

(2) $\dot{\eta} = q(0, \eta)$ is AS

then (z, η) goes to zero and $y(t) = 0$

the inverse dynamics is $\dot{\eta} = q(z, \eta)$ which is AS if $\dot{\eta} = q(0, \eta)$ is AS

Prop: For what concerns the boundedness of the evolutions in presence of external input $v \neq 0$

$$\begin{cases} \dot{z} = Az + Bv \\ \dot{\eta} = Rz + Q\eta \end{cases}$$

if the system is linear, when A and Q have Hurwitz eigenvalues, then

$$\forall |v| < M \text{ (bounded)} \Rightarrow \begin{matrix} |z| < k_1 \\ |\eta| < k_2 \end{matrix} \text{ (bounded)}$$

$\forall |v| < M$ (bounded) $\Rightarrow$ $|z| < K_1$, $|\eta| < K_2$ (bounded)

In NL systems this is not true, in fact

$\dot{\eta} = q(z, \eta)$

$|z|$ bounded $\not\Rightarrow$ $|\eta|$ bounded

Prop: $\begin{cases} \dot{z} = Az + Bv \\ \dot{\eta} = q(z, \eta) \end{cases}$

if $\sigma(A) \in \mathbb{C}^-$ & $\dot{\eta} = q(0, \eta)$ LAS in $\eta = 0$

then $\forall \varepsilon \; \exists \delta, K : \|x_0\| < \delta$ & $|v(t)| < \varepsilon \; \forall t$

$\Rightarrow \|\eta(t)\| < \varepsilon \quad \forall t \geq 0$

i.e., at least locally, for small values of v the evolutions remain bounded.

In conclusion when a linearizing feedback is applied, the stability of the zero dynamics is a necessary condition for the stability of the system and for the boundedness of evolutions for bounded input.

But it is not a sufficient condition, therefore ad hoc study and investigation on the whole control system are needed to assure stability.

What is really needed is the stability of the inverse dynamics.