3. EAP (MIMO Systems)

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Assuming $p = 2$

(A_c, B_c). the controllable canonical form is

$$A_c = \left(\begin{array}{cccc|cccc} 0 & 1 & & 0 & & & & \\ & & \ddots & \vdots & & 0 & & \\ & & & 1 & & & & \\ * & * & \cdots & \underline{a}_{n_1} & \cdots & & & * \\ \hline & & & & 0 & 1 & \cdots & 0 \\ & 0 & & & & & \ddots & \vdots \\ & & & & & & & 1 \\ * & \cdots & & \underline{a}_n & \cdots & & & * \end{array}\right) \qquad B_c = \left(\begin{array}{c|c} 0 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ \hline 0 & 0 \\ \vdots & \vdots \\ 0 & 1 \end{array}\right)$$

$(n_1 \times n_2)$ $\quad$ $(n_2 \times n_2)$

$n = n_1 + n_2$ $\quad$ $\underline{a}_{n_1}, \underline{a}_n \in \mathbb{R} \times \mathbb{R}^n$

$\Lambda = \{ \lambda_1, \dots, \lambda_n \}$ eigenvalues

$= \{ \lambda_1^*, \dots, \lambda^1{}_{n_1}^*, \lambda_2^*, \dots, \lambda^2{}_{n_2}^* \}$

$$p_1^*(\lambda) = (\lambda - \lambda_1^{1*}) \cdots (\lambda - \lambda_{n_1}^{1*}) = a_0^{1*} + a_1^{1*}\lambda + \dots + a_{n_1-1}^{1*}\lambda^{n_1-1} + \lambda^{n_1}$$

$$p_2^*(\lambda) = (\lambda - \lambda_1^{2*}) \cdots (\lambda - \lambda_{n_2}^{2*}) = a_0^{2*} + a_1^{2*}\lambda + \dots + a_{n_2-1}^{2*}\lambda^{n_2-1} + \lambda^{n_2}$$

$$F_c = -\begin{pmatrix} \underline{a}_{n_1} \\ \underline{a}_n \end{pmatrix} - \begin{pmatrix} a_0^{1*} & \cdots & a_{n_1-1}^{1*} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & a_0^{2*} & \cdots & a_{n_2-1}^{2*} \end{pmatrix} = \begin{pmatrix} f_{11} & \cdots & f_{1n} \\ f_{21} & \cdots & f_{2n} \end{pmatrix}$$

is such that $\qquad f = a - a^*$

$\sigma(A_c + B_c F_c) = \Lambda$

MIMO reachability

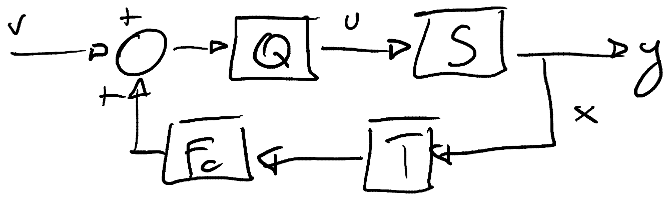
In general: given (A, B) reachable $\qquad \rho(B) = p$

$\exists\, T : |T| \neq 0 \quad n \times n$

$\exists\, Q : |Q| \neq 0 \quad m \times m$

such that:

$$\begin{cases} T A T^{-1} = A_c \\ T B Q = B_c \end{cases}$$



The feedback matrix can be rewritten as

$$(A_c + B_c F_c) = (T A T^{-1} + T B Q F_c) = T(A + BQ F_c T)$$

The feedback matrix can be rewritten as

$$(A_c + B_c F_c) = (TAT^{-1} + TBQF_c) = T(A + BQF_cT)$$

F_c assigns eigenvalues for (A_c, B_c)

$F = QF_cT$ assigns eigenvalues for (A, B)

Consider the reachability matrix

$$\mathcal{R} = \left(B_1 \; AB_1 \; \cdots \; A^{n_1-1}B_1 \;\vdots\; B_2 \; AB_2 \; \cdots \; A^{n_2-1}B_2 \right)$$

and γ_1 the n_1-th row of $\mathcal{R}^{-1}$, γ_2 the n-th row of $\mathcal{R}^{-1}$

$T \cdot \mathcal{R} \to |T| \neq 0$

$$T = \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_1 A^{n_1-1} \\ \gamma_2 \\ \vdots \\ \gamma_2 A^{n_2-1} \end{pmatrix} \qquad Q = \Bigg[\underbrace{\begin{pmatrix} \gamma_1 A^{n_1-1} \\ \gamma_2 A^{n_2-1} \end{pmatrix} B}_{\begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix} \neq 0} \Bigg]^{-1}$$

$$TAT^{-1} = A_c \to TA = A_cT \to A_c$$

$$TBQ = \begin{pmatrix} \gamma_1 B \\ \vdots \\ \gamma_1 A^{n_1-1}B \\ \gamma_2 B \\ \vdots \\ \gamma_2 A^{n_2-1}B \end{pmatrix} Q = \begin{pmatrix} 0 & 0 \\ \vdots & \vdots \\ \gamma_1 A^{n_1-1} & \\ 0 & 0 \\ \vdots & \vdots \\ \gamma_2 A^{n_2-1} & \end{pmatrix} Q = B_c$$

$$F = QF_cT = -Q\begin{pmatrix} a_{n_1} \\ a_n \end{pmatrix} T - Q\left(\begin{array}{ccc|ccc} a_0^{1*} & \cdots & a_{n_1-1}^{1*} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & a_0^{2*} & \cdots & a_{n_2-1}^{2*} \end{array} \right) T$$

$$= \underbrace{-Q\begin{pmatrix} \gamma_1 A^{n_1} \\ \gamma_2 A^{n_2} \end{pmatrix}}_{F^0} + \underbrace{Q\left(\begin{pmatrix} \gamma_1 A^{n_1} \\ \gamma_2 A^{n_2} \end{pmatrix} - \begin{pmatrix} \gamma_1 P_1^*(A) \\ \gamma_2 P_2^*(A) \end{pmatrix} \right)}_{F^A}$$

$$= -Q\begin{pmatrix} \gamma_1 P_1^*(A) \\ \gamma_2 P_2^*(A) \end{pmatrix} \qquad \begin{aligned} Q &= 2 \times 2 \\ \gamma_i &= 1 \times n_i \\ P^* &= n \times n \end{aligned}$$

 # Remark: Reachability is not modified by state feedback

Observability can be changed by state feed.