Humanoids exercises

① Given an humanoid that must go from (x_s, y_s) to (x_g, y_g) in an environment that contains obstacles, explain the RRT planner and:

1. definition of configuration and the associated configuration space
2. Choice of the configuration space distance
3. expansion mechanism
4. discuss completeness properties of the RRT planner

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1. The configuration space of legged robots combines the configuration $\hat{q} \in \mathbb{R}^N$ of their N joints with a global position $x_0 \in \mathbb{R}^3$ and orientation $\theta_0 \in \mathbb{R}^3$ $(SO(3))$

$$q = \begin{pmatrix} \hat{q} \\ x_0 \\ \theta_0 \end{pmatrix}$$

2. Given $p(q)$, a control point representing a point of the robot in W when the configuration is q

distance :

$$d(q_A, q_B) = \max_{\substack{p \in B \\ \hookrightarrow \text{robot}}} \| p(q_A) - p(q_B) \|$$

maximum displacement between 2 configurations

3 a. RRT is a probabilistic method in which samples of C are randomly extracted.
A tree Ts rooted at q_s is built.
Then the following algorithm is applied:
- Generate qrand in C with uniform probability distribution
- Search the tree for the nearest configuration qnear
- choose qnew at a distance δ from qnear in the direction of qrand
- Check for collision qnew and the segment from qnear to qnew
- if check is negative add qnew to Ts (expansion)

4. RRT as a probabilistic method builds an extremely approximated version of Cfree, and doesn't build CO, in fact it computes B(q) volum to check the collisions
It requires an a priori knowledge of the geometry and poses of the obstacles.
By the way this method covers rapidly Cfree because the expansion is biased towards unexplored areas, this improves efficency.
A problem is that there is no bias that drives the expansion towards the goal.
To solve it, the most used approach is to make the search bidirectional by growing another tree rooted at qg.

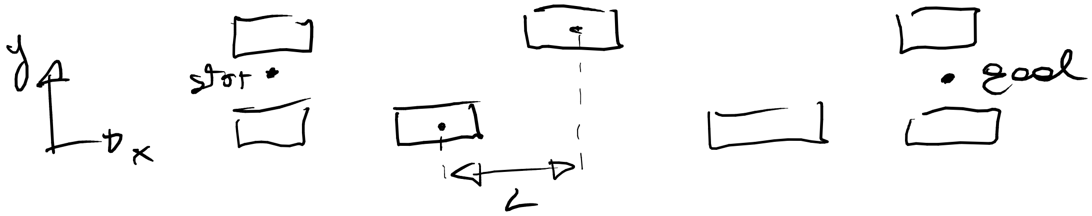
or equivalently, setting $q_{rand} = q_{\varepsilon}$ so that the expansion step is greedy and aimed towards the goal with operations of exploration and exploitation. A popular choise is ε-greedy.

Then it is a single-query algorithm because the tree is rooted at the starting configuration, so every new problem demands the construction of a new tree.

② Regarding humanoid robots, starting from their dynamic model, tell how to conclude to a LIP model.
Then explain how to use it in gait generation

The gait generation algorithm consists on:

1 Plan the footsteps (offline)
- timing and lengths (desired speed)



$$\frac{L}{T} = v \text{ (velocity)}$$

- obstacles and other tasks

2 Plan ZMP trajectory (interpolation)

3 Compute a CoM trajectory consistent with the planned ZMP trajectory:
- Flat ground
- constant height of CoM
- Internal angular momentum neglected

LIP model: $\ddot{c}^x = \frac{g}{c^z}(c^x - z^x)$ ↳ planned ZMP trajectory
along sagittal direction

4 Track the desired CoM trajectory
A. define a swinging foot trajectory
B. use kinematic control to obtain reference joint trajectories
C. send the reference joint profiles to the actuators

We can use the MPC as a predictor.
Prediction model using the LIP:

$$\frac{d}{dt}\begin{pmatrix} c \\ \dot{c} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\eta^2 & 0 \end{pmatrix}\begin{pmatrix} c \\ \dot{c} \end{pmatrix} + \begin{pmatrix} 0 \\ \eta^2 \end{pmatrix} z \qquad \eta^2 = \frac{g}{c^z}$$

where the ZMP is the input

$$ZMP \to \boxed{LIP} \to COM$$

The cost function is

$$J = \sum_{i=k}^{k+N-1} \left((\dddot{c}_i^{\,x})^2 + (\dddot{c}_i^{\,y})^2 \right)$$

Decomposing the LIP there is a stable and an unstable dynamics, therefore we have to impose the condition at every MPC iteration

$$x_u^k = \eta \int_{t_k}^{\infty} e^{-\eta(\tau - t_k)} z(\tau)\, d\tau \to \text{stability constraint at the initial state}$$

③ Prove that a humanoid robot can be modelized as an inverted pendulum

Starting from the Lagrangian dynamic model

$$M(q)\left[\begin{pmatrix}\ddot{\hat{q}}\\ \ddot{x}_0\\ \ddot{\theta}_0\end{pmatrix}+\begin{pmatrix}0\\ g\\ 0\end{pmatrix}\right]+n(q,\dot{q})=\begin{pmatrix}u\\ 0\\ 0\end{pmatrix}+\sum_i C_i(q)^T f_i$$

Since the vector u has the same size of the vector $\hat{q}$ of joint positions, the whole dynamics including the global position x_0 and orientation θ_0 is underactuated if no extend forces f_i are exerted.
The part which is not actuated involves the Newton and Euler equations of motion of the robot taken as a whole.

While standing still, walking or running on a flat ground (reference frame oriented along the ground with z-axis orthogonal to it, contact points p_i such that $p_i^z = 0$, $\forall i$):

Newton equation $\rightarrow$ $m(\ddot{c}+g)=\sum_i f_i$

m: mass of the robot
c: position of the CoM

Euler equation $\rightarrow$ $\dot{L}=\sum_i (p_i - c)\times f_i$
p_i: points of applications of the forces f_i
and

The newton equation shows that the robots needs external forces f_i in order to move the CoM in a direction different from the gravity

The Euler equation shows that the positions of the points p_i wrt the CoM c is important to keep the angular momentum L under control

Consider now the sum of the Euler equation and the cross product of the CoM c with the Newton equation:

$$m c \times (\ddot{c} + g) + \dot{L} = \sum_i p_i \times f_i$$

and divide the result by the z-coordinate of the Newton equation to obtain

$$\frac{m c \times (\ddot{c} + g) + \dot{L}}{m(\ddot{c}^z + g^z)} = \frac{\sum_i p_i \times f_i}{\sum_i f_i^z}$$

since $p_i^z = 0$, the x and y coordinates of this equation can be simplified in this way:

$$c^{x,y} - \frac{c^z}{\ddot{c}^z + g^z}(\ddot{c}^{x,y} + g^{x,y}) + \frac{1}{m(\ddot{c}^z + g^z)} S \dot{L}^{x,y} = \underbrace{\frac{\sum_i f_i^z p_i^{x,y}}{\sum_i f_i^z}}_{\substack{\text{CoP} \\ z^{x,y}}}$$

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Rewriting the latter isolating $\ddot{c}^{x,y}$:

$$\frac{c^z}{\ddot{c}^z + g^z} \ddot{c}^{x,y} = -\frac{c^z}{\ddot{c}^z + g^z} g^{x,y} + (c^{x,y} - z^{x,y}) + \frac{S \dot{L}^{x,y}}{m(\ddot{c}^z + g^z)}$$

In case the robot is walking on an horizontal ground, the z-axis is aligned with the gravity, so $g^{x,y} = 0$.

Assuming then that the CoM moves strictly horizontally above the ground, c^z is costant and $\ddot{c}^z = 0$, the previous equation becomes

$$\frac{c^z}{g^z} \ddot{c}^{x,y} = (c^{x,y} - z^{x,y}) + \frac{S\dot{L}^{x,y}}{mg^z}$$

Variations of the angular momentum L can also be bounded in the x and y directions, and for sake of simplicity, by considering these variation equal to 0, we obtain the linear inverted pendulum equation

$$c^{x,y} - \frac{c^z}{g^z} \ddot{c}^{x,y} = z^{x,y}$$