3. Abstract Oriented Dynamical System $\longrightarrow$ AODS

giovedì 4 giugno 2020 21:28

$$S = \{T, W, \Sigma\} \;\Big|\; W = U \times Y$$

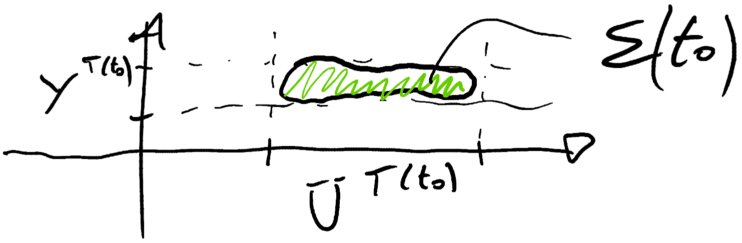
U = input values set
Y = output values set

$$\downarrow$$

$$S = \{T, U \times Y, \Sigma\}$$

$$\Sigma = \{\Sigma(t_0) \subset U^{T(t_0)} \times Y^{T(t_0)},\ t_0 \in T : CRT\}$$

$$CRT: \quad \Sigma(t_0)\big|_{T(t_1)} \subset \Sigma(t_1)$$



PROPERTIES

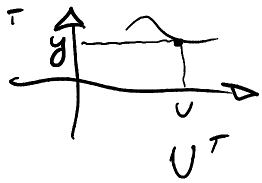
- Uniformity

$$\exists\ \Sigma_{UN} \subset U^T \times Y^T : \ \forall t_0 \ \ \Sigma(t_0) = \Sigma_{UN}\big|_{T(t_0)}$$

- Connection

Σ_{UN} is the graph of a function Y^T

$$y : U^T \to Y^T$$



- Stationariety

$$\Delta_{\bar{t}}\, \Sigma(t_0) = \Sigma(t_0 + \bar{t}) \quad \forall t_0, \bar{t}$$

- Linearity

$\Sigma(t_0)$ is linear $\forall t_0$

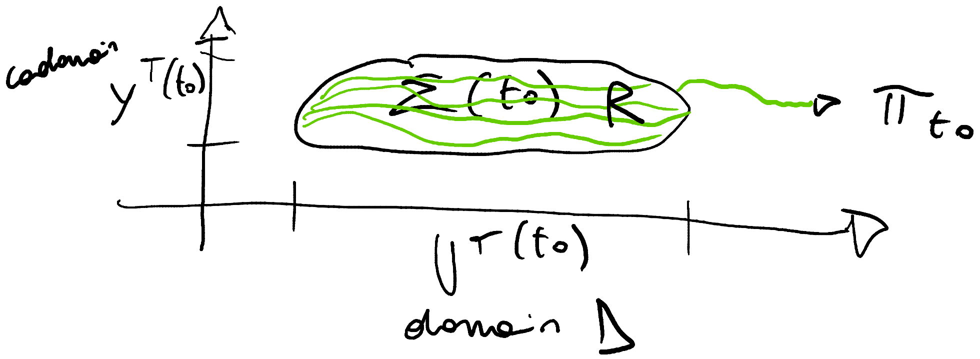
$(u_0^1, y_0^1)\ (u_0^2, y_0^2) \in \Sigma(t_0)$ is linear

$(u_0^1, y_0^1)\ (u_0^2, y_0^2) \in \Sigma(t_0)$ is linear

Remark:

The state x_0 represents the additional information to u_0 to specify y_0.
The state is a parametrization of all $\Sigma(t_0)$ with $t_0 \in T$ which satisfies some properties

Parametric representation of $R \subset U \times Y$



The parametrization enables me to express all the functions in terms of the values of various parameters

Let $\hat{\Pi} = \{(\overset{\text{set}}{P}, \Pi_{t_0}), t_0 \in T\}$ a parametrization of $\Sigma(t_0)$

with $\hat{\Pi}_{t_0} : P \times D(\Sigma(t_0)) \to R(\Sigma(t_0))$

such that

$(u_0, y_0) \in \Sigma(t_0) \Rightarrow \exists\, x_0 : y_0 = \hat{\Pi}_{t_0}(x_0, y_0)$ $\quad x_0 \in P$,

$u_0 \in D(\Sigma(t_0)) \Rightarrow (u_0, \hat{\Pi}_{t_0}(x_0, y_0)) \in \Sigma(t_0)$

- Causality

- causality

S is causal if there exists at least a parametric representation which is causal i.e.

$$\forall t_0 \in T \quad \forall x_0 \in P \quad \forall \bar{t} \in T(t_0)$$

$$u_{[t_0,\bar{t}]} = u'_{[t_0,\bar{t}]} \Rightarrow \pi_{t_0}(x_0,u)(\bar{t}) = \pi_{t_0}(x_0,u')(\bar{t})$$

(if $[t_0,\bar{t})$ → strictly causal)