23. Internal and global stability

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$\dot{x} = f(x,u)$

Suppose the system in free evolution ($u=0$) starting from an initial state x_e.

With no input the system remains in x_e.

$x(t) = x_e \quad \forall t \geq 0$

$\Rightarrow \dot{x} = f(x_e, 0) = 0$

Evaluating the evolutions which start from x_0 near x_e:

- **Stability**

  The evolution remains in the neighborhood of x_e

  $\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0$:

  $\| x_0 - x_e \| < \delta_\varepsilon \Rightarrow \| x(t) - x_e \| < \varepsilon \quad \forall t \geq 0$

- **Attractiveness**

  x_e is attractive if and only if:

  $\exists \delta_a : \quad \| x_0 - x_e \| < \delta_a \Rightarrow \lim_{t \to +\infty} \| x(t) - x_e \| = 0$

  <u>note</u>: attractiveness does not imply stability

- **Asymptotic stability** $\Rightarrow$ it is a local concept

  x_e is asymptotically stable if it is stable and attractive

- **Exponential stability**

  x_e is ES if there exists $\lambda, \alpha, \delta > 0$:

  $\| x(t) - x_e \| < \alpha e^{-\lambda t} \| x_0 - x_e \|$

  $\forall x_0 : \| x_0 - x_e \| < \delta$

\# For local there can be more equilibria
For Global only one!

- **Global asymptotic stability**

  $\delta_a = \infty$ and the state converges to $x_e \quad \forall x_0$ (x_e unique equilibrium)

- **Global exponential stability**

  $\delta = \infty$ and the state converges to $x_e \quad \forall x_0$