Normal form

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$$\begin{cases} \dot{x} = -x + y + f_1(x,y) \\ \dot{y} = -y + f_2(x,y) \end{cases} \Rightarrow A = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} \quad f(x,y) = \begin{pmatrix} -x+y \\ -y \end{pmatrix}$$

since the matrix A is already in Jordan form we can apply directly the Poincaré normal form

First we need to define a base in $\mathbb{R}^2$ for 2^{nd} order nonlinearities:

$$H_2 = \operatorname{span}\left\{ \begin{pmatrix} x^2 \\ 0 \end{pmatrix} \begin{pmatrix} xy \\ 0 \end{pmatrix} \begin{pmatrix} y^2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ x^2 \end{pmatrix} \begin{pmatrix} 0 \\ xy \end{pmatrix} \begin{pmatrix} 0 \\ y^2 \end{pmatrix} \right\}$$

Then we need to verify $T_J(H_2) = [h_2(y), J(y)]$

$$T_J\left(\begin{bmatrix} x^2 \\ 0 \end{bmatrix}\right) = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} x^2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2x & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -x+y \\ -y \end{pmatrix} = \begin{pmatrix} -x^2 \\ 0 \end{pmatrix} - \begin{pmatrix} -2x^2+2xy \\ 0 \end{pmatrix} = \overset{V_1}{\begin{pmatrix} x^2-2xy \\ 0 \end{pmatrix}}$$

$$T_J\left(\begin{bmatrix} xy \\ 0 \end{bmatrix}\right) = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} xy \\ 0 \end{pmatrix} - \begin{pmatrix} y & x \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -x+y \\ -y \end{pmatrix} = \begin{pmatrix} -xy \\ 0 \end{pmatrix} - \begin{pmatrix} y^2-2xy \\ 0 \end{pmatrix} = \overset{V_2}{\begin{pmatrix} y^2+xy \\ 0 \end{pmatrix}}$$

$$T_J\left(\begin{bmatrix} y^2 \\ 0 \end{bmatrix}\right) = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} y^2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 & 2y \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -x+y \\ -y \end{pmatrix} = \begin{pmatrix} -y^2 \\ 0 \end{pmatrix} - \begin{pmatrix} -2y^2 \\ 0 \end{pmatrix} = \begin{pmatrix} y^2 \\ 0 \end{pmatrix} \; V_3$$

$$T_J\left(\begin{bmatrix} 0 \\ x^2 \end{bmatrix}\right) = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ x^2 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 2x & 0 \end{pmatrix}\begin{pmatrix} -x+y \\ -y \end{pmatrix} = \begin{pmatrix} x^2 \\ -x^2 \end{pmatrix} - \begin{pmatrix} 0 \\ -2x^2+2xy \end{pmatrix} = \begin{pmatrix} x^2 \\ x^2-2xy \end{pmatrix} \; V_4$$

$$T_J\left(\begin{bmatrix} 0 \\ xy \end{bmatrix}\right) = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ xy \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ y & x \end{pmatrix}\begin{pmatrix} -x+y \\ -y \end{pmatrix} = \begin{pmatrix} xy \\ -xy \end{pmatrix} - \begin{pmatrix} 0 \\ y^2-2xy \end{pmatrix} = \begin{pmatrix} xy \\ -y^2+xy \end{pmatrix} \; V_5$$

$$T_J\left(\begin{bmatrix} 0 \\ y^2 \end{bmatrix}\right) = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ y^2 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 2y \end{pmatrix}\begin{pmatrix} -x+y \\ -y \end{pmatrix} = \begin{pmatrix} y^2 \\ -y^2 \end{pmatrix} - \begin{pmatrix} 0 \\ -2y^2 \end{pmatrix} = \begin{pmatrix} y^2 \\ y^2 \end{pmatrix} \cdot V_6$$

$$\begin{aligned} T_J(H_2) &= \operatorname{span}\{V_1, V_2, V_3, V_4, V_5, V_6\} \\ &= \operatorname{span}\left\{ \begin{pmatrix} x^2-2xy \\ 0 \end{pmatrix} \begin{pmatrix} xy-y^2 \\ 0 \end{pmatrix} \begin{pmatrix} y^2 \\ 0 \end{pmatrix} \begin{pmatrix} x^2 \\ x^2-xy \end{pmatrix} \begin{pmatrix} xy \\ xy-y^2 \end{pmatrix} \begin{pmatrix} y^2 \\ y^2 \end{pmatrix} \right\} \end{aligned}$$

We can remove all 2^{nd} order nonlinearities belonging to $T_J(H_2)$

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

to see the second order nonlinearities that we can delete we need to use the poincaré normal form.

The matrix is already in Jordan form $\lambda_i = -1$

Define a basis for $\mathbb{R}^3$, since we want to remove the 2^{nd} order

The matrix is already in Jordan form $\lambda_1 = 1$

Define a basis for $\mathbb{R}^3$, since we want to remove the 2nd order nonlinearities we use the quadratic terms

$$H_2 = \text{span}\left\{ \begin{pmatrix} x^2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} xy \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} y^2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} yz \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} z^2 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} xz \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ x^2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ y^2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ z^2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ xy \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ yz \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ zx \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ x^2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ y^2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ z^2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ xy \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ yz \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ zx \end{pmatrix} \right\}$$

Now check $J_S(H_2) = [H_2, J] = \frac{\partial J}{\partial v} H_2(v) - \frac{\partial H_2}{\partial v} J$

since $A = \frac{\partial J}{\partial V} \Rightarrow f(x,y,z) = \begin{pmatrix} -x \\ -y+z \\ -z \end{pmatrix}$

brackets :