31. Achieving relative degree via Dynamic Extension

lunedì 13 luglio 2020 23:40

A nonlinear MIMO system which has strong vector relative degree allows:

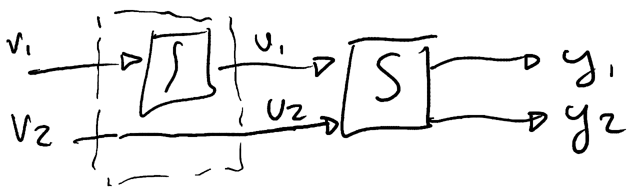
- fully linearization and readability by mean of a feedback and coordinate transformation
- to solve the non-interactive control problem.

It's interesting to note, by means of a feedback, a system which does not have strong vector relative degree into one having it.

More precisely, it is wanted $r_1 + r_2 + \dots + r_m = n$.
If this is the case a trivial zero dynamics is obtained (z^* degenerates into a single point x_0).

This result cannot be achieved by means of a static state feedback because it does not change the relative degree (relative degree is invariant under static feedback).

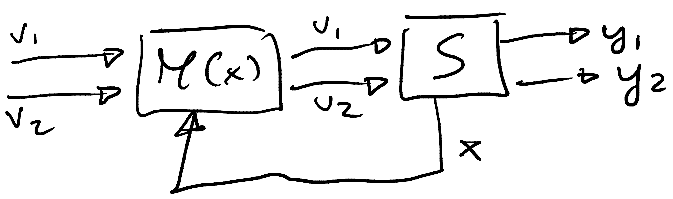
A useful idea to obtain higher relative degree is based on the usage of integrators on input channels



If $\rho(A(x)) = k < m$ (i.e. $r_1 + \dots + r_m < n$) then

$$\exists\, M(x) \ni A(x)\cdot M(x) = \left(\begin{array}{c|c} I_{k\times k} & 0 \\ \hline \# & 0 \end{array}\right) \qquad M \in \mathbb{R}^{m\times m}$$

describing the structure for the implementation of delays



Dynamic Extension Algorithm

Main algorithm ... considers an iterative procedure for

Dynamic extension algorithm

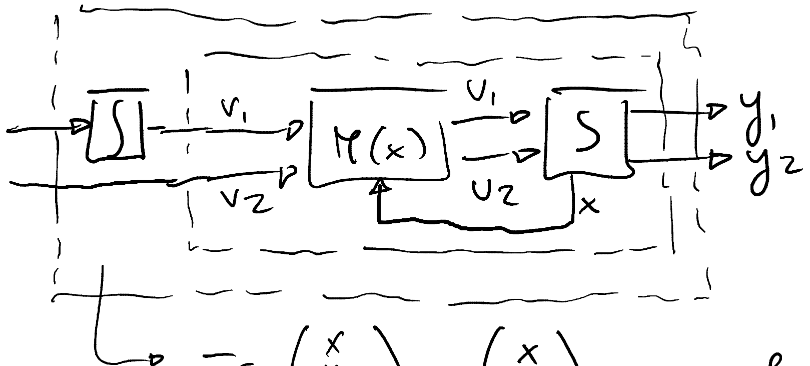
This algorithm provides an iterative procedure for possible achieving of strong vector relative degree under feedback and dynamic extension.

At each step of the procedure a state feedback, in order to obtain the desired structure for $A(x)$, and integrators over the first input channels are added to the system, as specified below:

1. Given S, compute $A(x)$ such that $\rho(A(x)) = k < m$
2. Compute $M(x) \ni A(x) \cdot M(x) = \begin{pmatrix} I_{k \times k} & 0 \\ \# & 0 \end{pmatrix}$

   and $u = M(x) \cdot v$

3. add integrators over $v_1, \dots, v_k$



$$\bar{x} = \begin{pmatrix} x \\ v_1 \\ \vdots \\ v_k \end{pmatrix} = \begin{pmatrix} x \\ \xi_1 \end{pmatrix} \qquad \xi_1 = \begin{pmatrix} v_1 \\ \vdots \\ v_k \end{pmatrix}$$

and compute the corresponding decoupling matrix $\bar{A}(x)$

Such a new decoupling matrix is characterized by a vector relative degree which is

$$\bar{r} = (r_1 + 1 \ , \ r_2 + 1) \qquad |\bar{r}|_c = |r|_c + m$$

Recalling that, at the generic step of the procedure:

$\bar{m} = n + k$ with $k < m$

$\bar{n}$: dimension of the state such that

$$n - |r|_c > \bar{n} - |\bar{r}|_c > \dots > 0$$

4. If $|\bar{A}(x)| \neq 0$ then the system is right invertible and the procedure is concluded.
   Otherwise $\rho(A(x)) = k$

5. Verify whether the condition $n - |r|_c > m - k$ is verified.
   If yes then retry the procedure going back to step 2.
   If no then the algorithm fails.

The failure of the condition implies the failure of

$$\sum_{i=1}^{m} \bar{r}_i < \bar{n}$$

i.e. the possibility of achieving the strong vector relative degree.