17. Full exact feedback linearization

giovedì 9 luglio 2020 23:18

U - X - Y EFL

Find a feedback (α, β) and a coordinate transformation Φ trasforming the full system including the output into a linear one.

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \xrightarrow[\Phi]{(\alpha,\beta)} \begin{cases} \dot{z} = Az + Bu \\ y = Cz \end{cases}$$

Theorem Consider a system with relative degree r at $x = x_0$. Suppose $f(x_0) = 0$ and $h(x_0) = 0$.
$\exists\ (\alpha, \beta)$ and $z = \Phi(x)$ solving the full feedback linearization problem if and only if:

(i) $\operatorname{rank}\left(g(x_0), ad_f g(x_0), \dots, ad_f^{n-1} g(x_0)\right) = n$

(ii) $\tilde{f}(x) = f(x) + g(x)\alpha(x)$ and $\tilde{g}(x) = g(x)\beta(x)$

with $\alpha(x) = -\dfrac{L_f^r h(x)}{L_g L_f^{r-1} h(x)}$ $\quad \beta(x) = \dfrac{1}{L_g L_f^{r-1} h(x)}$

are such that

$$[ad_{\tilde f}^i \tilde{g}, ad_{\tilde f}^j \tilde{g}] = 0 \quad \forall i,j \ni 0 \le i,j \le n$$
and all x near x_0

Remark:

- (ii) is the condition that allows a u-x feedback to linearize a u-y feedback linear system.