20. Local reachability

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if:

i) Δ involutive

ii) Δ contains span$\{g_i, \dots, g_m\}$

iii) Δ is f-invariant and g-invariant

<u>Then</u>: $\exists$ a local coordinate transformation $z = \Phi(x)$ s.t. the control system can be expressed as:

$$\begin{cases} \dot{\xi}_1 = f_1(\xi_1, \xi_2) + \sum_{i=1}^{m} g_i(\xi_1, \xi_2)\, u_i \\ \dot{\xi}_2 = f_2(\xi_2) \\ y_i = h_i(\xi_1, \xi_2) \end{cases}$$

$$\xi_1 = (z_1, \dots, z_d) \qquad \xi_2 = (z_{d+1}, \dots, z_n)$$

Reachability in linear systems

A state is reachable at time t starting from $x(0)=0$ with the control u.

reachable set:

$$\mathcal{R} = \mathrm{Im}\{B, AB, A^2B, \dots, A^{n-1}B\}$$

$\mathcal{R}$ satisfies

(i) $A\mathcal{R} \subset \mathcal{R}$

(ii) $\mathrm{Im}\{B\} \subset \mathcal{R}$

(iii) $\mathcal{R}$ is the smallest subspace satisfying i and ii

$\mathcal{R} = \langle A \mid \mathrm{Im}\{B\} \rangle$ contains B and it's invariant wrt A

Reachable state at time t starting from x_0

$$\mathcal{R}_t(x_0) = \{x \in \mathbb{R}^n : x(t) = e^{At}x_0 + v,\ v \in \mathcal{R}\}$$

Reachability in NL systems

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The smallest distribution which contains Δ and is invariant under $\tau_1, \dots, \tau_q$

Algorithm for the smallest distribution (Lemma)

$$\Delta_0 = \Delta$$

$$\Delta_k = \Delta_{k-1} + \sum_{i=1}^{q} [\tau_i, \Delta_{k-1}] \subset \langle \tau_1, \dots, \tau_q | \Delta \rangle \quad \forall k$$

if there exists k^* s.t. $\Delta_{k^*} = \Delta_{k^*+1}$, then

$$\Delta_{k^*} = \langle \tau_1, \dots, \tau_q | \Delta \rangle$$

In our case we need the smallest Δ invariant under f and g and containing g:

$$\Delta_R = \langle f, g \,|\, \mathrm{span}\{g\} \rangle$$

And using the Lemma, we find k^* s.t.

$$\rho(\Delta_{k^*}) = \rho(\Delta_{k^*+1}) \Longleftrightarrow \Delta_{k^*} = \Delta_{k^*+1}$$

$$\begin{cases} \Delta_0 = \mathrm{span}\{g\} \\ \Delta_k = \Delta_{k-1} + [f, \Delta_{k-1}] + [g, \Delta_{k-1}] \end{cases}$$

$$\left(\text{in linear case} \quad \begin{aligned} \Delta_0 &= B \\ \Delta_1 &= B + AB \\ &\vdots \\ \Delta_k &= \Delta_{k-1} + A\Delta_{k-1} \end{aligned} \right)$$