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Nonlinear Systems & Control

Part II

4/07/18

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1. A magnetic suspension system consists of a ball of magnetic material suspended by means of an electromagnet whose current is controlled by feedback from the ball position (which is measured). The equation of motion of the ball is

$$m\ddot{y} = -k\dot{y} + mg + F(y, i)$$

where m is the mass of the ball, $y \geq 0$ is the vertical (downward) position of the ball measured from a reference point (y = 0 when the ball is next to the coil), k is a viscous friction coefficient, g is the gravity acceleration, $F(y, i)$ is the force generated by the electromagnet and i is its electric current. The inductance of the electromagnet depends on the position of the ball and can be modeled as

$$L(y) = L_1 + \frac{L_0}{1 + \frac{y}{a}}$$

where L_0 and L_1 are positive constants. Defining by $E(y, i) = \frac{1}{2}L(y)i^2$ as the energy stored in the electromagnet, the force $F(y, i)$ is given by

$$F(y, i) = \frac{\partial E(y, i)}{\partial y}.$$

When the electric circuit of the coil is driven by a voltage source with voltage v, the Kirchhoff's voltage law gives the relationship

$$v = \dot{\phi}(y, i) + Ri$$

with R being the resistance of the circuit and $\phi(y, i) = L(y)i$ the magnetic flux linkage.

(a) Find a state-space representation of the dynamics when setting the state as $x = \mathrm{col}(x_1, x_2, x_3) = \mathrm{col}(y, \dot{y}, i)$, the control input $u := v$ and output y;

(b) Compute the relative degree of the system and verify it is well defined with respect to physical properties of the dynamics;

(c) Compute, if any, a static feedback $u = \alpha(x) + \beta(x)w$ ensuring input-output feedback linearization;

(d) Compute the zero-dynamics;

(e) Setting w so to stabilize the input-output dynamics, discuss on the stability of the closed-loop system;

(f) Compute, if any, a feedback ensuring the position of the ball to be balanced to a constant $r^* > 0$ via I/O feedback linearization.

√ 2. Given the following system

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\cos x_1 u \\
\dot{x}_3 &= -x_3^3 + x_2^2 + u \\
y &= x_1
\end{aligned}
$$

compute, if any, a high gain feedback making the origin locally asymptotically stable.

3. Illustrate and discuss on the several forms of feedback linearization.

4. Given a nonlinear system

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)u \\
y &= h(x)
\end{aligned}
$$

with an equilibrium at the origin, provide the definition of *zero-dynamics* and discuss on the relationship with the zeros of the corresponding linear tangent model at the origin.

② High gain feedback

$$
\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = -\cos x_1 u \\
\dot{x}_3 = -x_3^3 + x_2^2 + u \\
y = x_1
\end{cases}
\qquad f = \begin{pmatrix} x_2 \\ 0 \\ -x_3^3 + x_2^2 \end{pmatrix} \qquad g = \begin{pmatrix} 0 \\ -\cos x_1 \\ 1 \end{pmatrix} \qquad \frac{\partial h}{\partial x} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}
$$

$$\dot{y} = \dot{x}_1 = x_2 \qquad r \neq 1$$

$$\ddot{y} = \ddot{x}_1 = \dot{x}_2 = -\cos x_1 u \qquad r = 2$$

in fact

$$L_g h = \frac{\partial h}{\partial x} \cdot g = 0 \qquad L_f h = \frac{\partial h}{\partial x} \cdot f = (1\ 0\ 0) \begin{pmatrix} x_2 \\ 0 \\ -x_3^3 + x_2^2 \end{pmatrix} = x_2$$

$$L_g L_f h = (0\ 1\ 0) \begin{pmatrix} 0 \\ -\cos x_1 \\ 1 \end{pmatrix} = -\cos x_1 \longrightarrow r = 2$$

since $r > 1$ a dummy output $w = k(x) \ni r_w = 1$ is needed.

$$w = k(x) = L_f^{r-1} h + \quad + \alpha_1 L_f h + \alpha_0 h$$

Feedback:

$$w = k(x) = L_f^{r-1} h + \dots + \alpha_1 L_f h + \alpha_0 h$$
$$= L_f h + \alpha_0 h = x_2 + \alpha_0 x_1$$

$$\rightarrow \begin{cases} \dot{x} = f(x) + g(x)\,u \\ w = k(x) \end{cases} \qquad u = -k\,w$$

- Normal form:

$$\phi_1(x) = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} h \\ L_f h \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\phi_2(x) \ni L_g \phi_2 = 0 \quad \rightarrow \quad \frac{\partial \phi_2}{\partial x} \cdot \begin{pmatrix} 0 \\ -\cos x_1 \\ 1 \end{pmatrix} = 0$$

$$\begin{pmatrix} \frac{\partial \phi_2}{\partial x_1} & \frac{\partial \phi_2}{\partial x_2} & \frac{\partial \phi_2}{\partial x_3} \end{pmatrix} \cdot g = 0$$

$$-\cos x_1 \frac{\partial \phi_2}{\partial x_2} + \frac{\partial \phi_2}{\partial x_3} = 0$$

$$\frac{\partial \phi_2}{\partial x_3} = \cos x_1 \frac{\partial \phi_2}{\partial x_2} \qquad \phi_2 = x_3 \cos x_1 + x_2$$

$$\phi_2(x) = x_3 \cos x_1 + x_2 = \eta_1$$

$$J_\phi(x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_3 \sin x_1 & 1 & \cos x_1 \end{pmatrix} \qquad \begin{aligned} x_1 &= z_1 \\ x_2 &= z_2 \\ x_3 &= \frac{1}{\cos z_1}(\eta_1 - z_2) \end{aligned}$$

$$\tilde{f}(z,\eta) = \left[J_\phi(x) \cdot f(x) \right]\Big|_{x = \phi^{-1}(z,\eta)} = \begin{pmatrix} z_2 \\ 0 \\ -z_2 \frac{\eta_1 - z_2}{\cos z_1} \sin z_1 - \left(\frac{\eta_1 - z_2}{\cos z_1}\right)^3 \cos z_1 + z_2^2 \cos z_1 \end{pmatrix}$$

$$\tilde{g}(z,\eta) = \begin{pmatrix} 0 \\ -\cos z_1 \\ 0 \end{pmatrix}$$

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\cos z_1\, u \triangleq -\cos z_1 \left(-k\,(x_2 + \alpha_0 x_1)\right) \\ \dot{\eta}_1 = \dots \\ y = z_1 \end{cases}$$

zero dynamics $\quad \dot{\eta}(0,\eta) = -\eta^3$

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zero dynamics $\quad \dot{\eta}(0,\eta) = -\eta^3$

$Q = \frac{\partial q(0,\eta)}{\partial \eta}\Big|_0 = 0 \qquad \sigma(Q) = 0 \longrightarrow$ cannot be stabilized by high gain feedback