21. Local observability

martedì 9 giugno 2020 19:17

Linear case

Observability puts in light the possibility of reconstruct the state.

- Undistinguishability

  x_a and x_b are undistinguishable at time t_0 if:

  $$y(t, \varphi(t, t_0, x_a, u), u(t)) = y(t, \varphi(t, t_0, x_b, u), u(t)) \quad \forall u, \forall t$$

- Inexistence of undistinguishable states:

  I can always reconstruct the state,

  If this is true I can say that:

  $$Ce^{A(t-t_0)} x_a = Ce^{A(t-t_0)} x_b$$

  $$Ce^{A(t-t_0)} (x_a - x_b) = 0 \Longrightarrow x_a = x_b$$

  x_a and x_b are unobservable if and only if their difference is undistinguishable with the null vector

$$\mathcal{U} = \{ x \in \mathbb{R}^n : ce^{At} x = 0 \} = \ker \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$ set of unobservable states

$\mathcal{U}$ satisfies

(i) $A\mathcal{U} \subset \mathcal{U}$

(ii) $\mathcal{U} \subset \ker[C]$

(iii) $\mathcal{U}$ is the biggest subspace satisfying (i) and (ii)

There exists $Tx = z$ such that $T^{-1} = \left(\begin{array}{c|c} \text{base of } \mathcal{U} & V \end{array} \right)$

Non Linear case

$$\Delta_S = \ker[\nabla \lambda] = \left(\frac{\partial \lambda}{\partial x_1}, \dots, \frac{\partial \lambda}{\partial x_n} \right)$$ "covector field"

$$\Omega^{\perp}(x) = \Delta_S$$

$\Omega = \text{span}\{\omega_1, \omega_2, \dots, \omega_d\} \rightarrow$ codistribution

$\Omega_0 = \Omega$

$\Omega_k = \Omega_{k-1} + \sum_{i=1}^{q} L_{\tau_i} \Omega_{k-1} \subset \langle \tau_1, \tau_2, \dots, \tau_q | \Omega \rangle \ \forall k$

if there exists k^* such that $\Omega_{k^*} = \Omega_{k^*+1}$, then

$$\Omega_{k^*} = \langle \tau_1, \dots, \tau_q | \Omega \rangle$$

is the smallest codistribution which contains Ω, invariant under $\tau_1, \dots \tau_q$.

By duality $\Omega_{k^*}^{\perp}$ is the largest distribution invariant under $\tau_1, \dots, \tau_q$ contained in $\Omega^{\perp}$

(iiii) Largest distribution invariant under f and g and contained in $\ker[dh]$

$$\Omega^{\perp}_{u} = \langle f, g | \text{span}\{dh\} \rangle^{\perp}$$

$$\begin{cases} \Omega_0 = \text{span}\{dh\} \\ \Omega_k = \Omega_{k-1} + L_f \Omega_{k-1} + L_g \Omega_{k-1} \end{cases}$$