7. Linear Time-Invariant state-space representation

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(x, φ, η) is said to be <u>time-invariant</u> if

$\forall (t, t_0), \forall x_0, \forall u$:

$$\Delta_\delta \varphi(t, t_0, x_0, u) = \varphi(t+\delta, t_0+\delta, x_0, \Delta_\delta u_{[t_0+\delta, t+\delta)})$$

$$\Delta_\delta \eta(t, x, u) = \eta(t+\delta, x, u)$$

$$\Downarrow$$

$$\begin{cases} x(t) = \varphi(t-t_0, 0, x_0, u) \\ y(t) = \eta(0, x, u) \end{cases}$$

- **Discrete time** $\longrightarrow$ from explicit to implicit

$$\begin{cases} x(t) = \Phi(t-t_0)x_0 + \sum_{\tau=t_0}^{t-1} H(t-\tau)u(\tau) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$$= \Psi(t-t_0)x_0 + \sum_{\tau=t_0}^{t} W(t-\tau)u(\tau)$$

$$W(t-\tau) = \begin{cases} CH(t-\tau) & t > \tau \\ D & t = \tau \end{cases}$$

equivalently $t = t+1$, $t_0 = t$

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) & x(t_0) = x_0 \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$A = \Phi(1) \quad B = H(1) \quad C = \Psi(0) \quad D = W(0)$

$\Phi(t-t_0) = A^{t-t_0} \quad H(t-\tau) = A^{t-\tau-1}B$

$\Psi(t-t_0) = CA^{t-t_0} \quad W(t-\tau) = CA^{t-\tau-1}B$

- **Continuous time** $\longrightarrow$ from explicit to implicit

$$x(t) = \Phi(t-t_0)x_0 + \int_{t_0}^{t} H(t-\tau)u(\tau)\,d\tau$$

$$x(t) = \Phi(t-t_0)x_0 + \int_{t_0} H(t-\tau)u(\tau)d\tau$$

$$y(t) = Cx(t) + Du(t)$$
$$= \Psi(t-t_0)x_0 + \int_{t_0}^{t} W(t-\tau)u(\tau)\,d\tau$$

$$\Psi(t-t_0) = C\Phi(t-t_0)$$
$$W(t-\tau) = CH(t-\tau) + D\delta(t-\tau)$$

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

Where $A = \frac{\partial}{\partial t}\Phi(t)\Big|_{t=0}$ $\quad B = H(0) \quad C = \Psi(0)$

$$D = \int_{t-\varepsilon}^{t+\varepsilon}\left(W(t-\tau) - CH(t-\tau)\right)d\tau$$

Since $\Phi(t) = e^{At} = \sum \frac{t^k}{k!}A^k \quad H(t) = e^{At}B$

$\Psi(t) = Ce^{At} \quad W(t) = Ce^{At}B + D\delta(t)$