23. Asymptotic model matching (AMMP)

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Sometimes $y_R(t)$ cannot be a fixed function but the output of a reference model described by the equations

$$w \longrightarrow \boxed{(A,B,C)} \longrightarrow y_M \qquad \text{Model:} \begin{cases} \dot{\xi} = A\xi + Bw \\ y_M = C\xi \end{cases}$$

Problem:

Compute a feedback over S

$$S: \begin{cases} \dot{x} = f + gu \\ y = h \end{cases}$$

in such a way that $y(t) \to y_M(t) \quad \forall w$

to solve the problem one could in principle use reference reproduction y_M from the output of the model.

$$y_M^{(i)}(t) = CA^i\xi(t) + CA^{i-1}Bw(t) + \dots + CBw^{(i-1)}(t)$$

If we suppose that $CB = \dots = CA^{r-2}B = 0$

i.e., we are assuming that the relative degree of the model is greater than the one of the system ($r_M \geq r$)

then

$$y_M^{(i)}(t) = CA^i\xi(t) \qquad i = 0, \dots, r-1$$

$$y_M^{(r)}(t) = CA^r\xi(t) + \underbrace{CA^{r-1}Bw}_{\neq 0 \text{ if } r_M = r \text{ (otherwise } = 0)}$$

in this way:

$$u = \frac{1}{L_g L_f^{r-1} h}\left(-L_f^r h - CA^r\xi + CA^{r-1}Bw(t) - \sum_{i=1}^{r} \alpha_{i-1}^* \left(L_f^i h - CA^i\xi\right)\right)$$

$\underline{\alpha}$ Hurwitz

Considering the outputs' expressions:

$$y_M(t) = Ce^{At}\xi(0) + \int^{t} Ce^{A(t-\tau)}Bw(\tau)\,d\tau$$

$$y_M(t) = Ce^{At}\xi(0) + \int_0^t Ce^{A(t-\tau)}Bw(\tau)\,d\tau$$

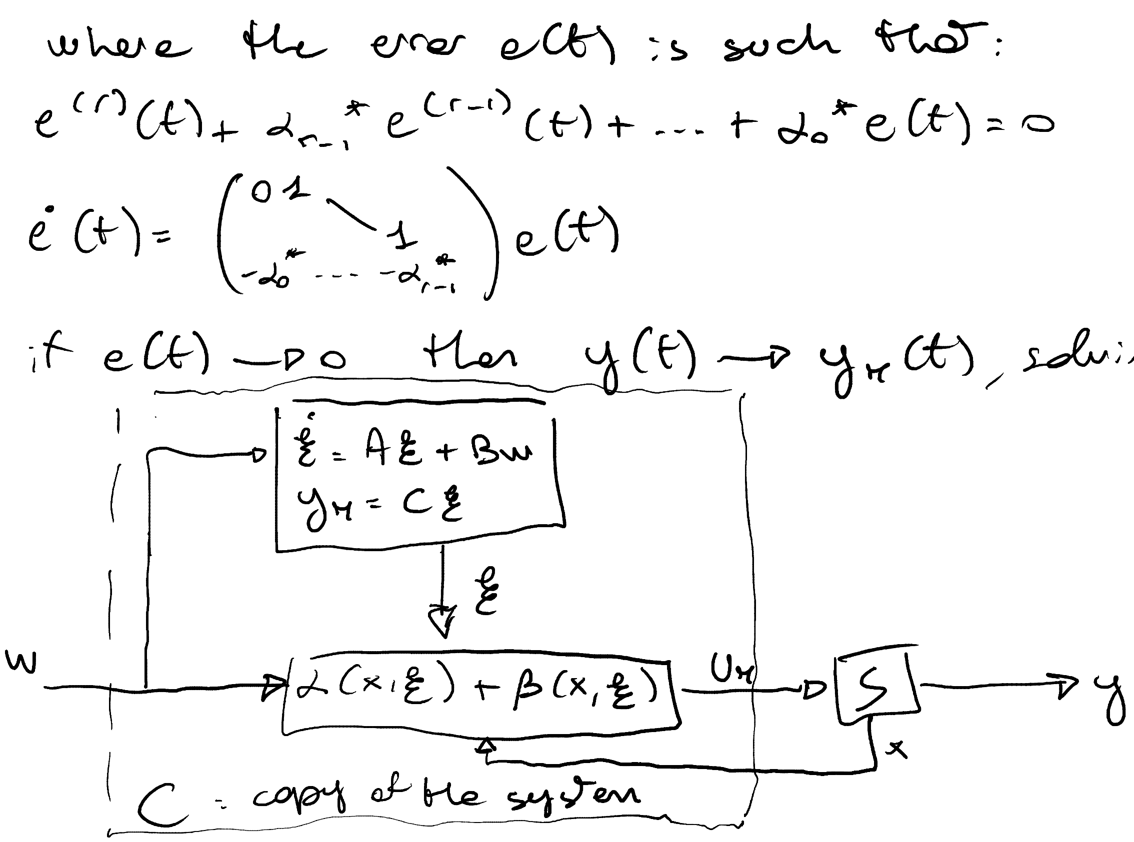
$$y(t) = e(t) + y_M(t)$$
$$= e(t) + Ce^{At}\xi(0) + \int_0^t Ce^{A(t-\tau)}Bw(\tau)\,d\tau$$

where the error $e(t)$ is such that:

$$e^{(r)}(t) + \alpha_{r-1}^* e^{(r-1)}(t) + \dots + \alpha_0^* e(t) = 0$$

$$\dot{e}(t) = \begin{pmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ -\alpha_0^* & \cdots & & -\alpha_{r-1}^* \end{pmatrix} e(t)$$

if $e(t) \to 0$ then $y(t) \to y_M(t)$, solving the problem.



$$\dot{\xi} = A\xi + Bw = \gamma(x,\xi) + \delta(x,\xi)w$$
$$U_M = \alpha(x,\xi) + \beta(x,\xi) \qquad \text{if } r_M > r \quad \beta = 0$$

In conclusion:

$$\begin{cases} \dot{\xi} = A\xi + Bw \\ \dot{e} = \begin{pmatrix} 0 & 1 & & \\ & & \ddots & 1 \\ -\alpha_0^* & \cdots & & -\alpha_{r-1}^* \end{pmatrix} e \\ \dot{\eta} = q(z,\eta) = q(L\xi + \underline{e}, \eta) \end{cases}$$ $\to$ dynamics of the inverse system

where: $L = \begin{pmatrix} C \\ \vdots \\ CA^{r-1} \end{pmatrix}$ $\quad y_M^{(i)} = CA^{(i)}\xi$

$e^{(i)} = y^{(i)} - y_r^{(i)}$ $\qquad$ $z = \begin{pmatrix} y \\ \vdots \\ y^{(r-1)} \end{pmatrix}$

The stability of the inverse dynamics is responsible for the stability of the whole control system.