Add 1: maximizing unobservability

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Suppose $p=q=1$ for simplicity.

Since under $u = Fx + v$ the zeros of P_F are not modified one has

$$P(s) = \frac{N(s)}{D(s)} \xrightarrow{F} P_F(s) = \frac{N(s)}{D_F(s)}$$

m = degree of $N(s)$

n = degree of $D(s)$ ($D_F(s)$)

Assuming $P(s)$ reachable and observable ($P(s)$, $D(s)$ prime) a loss of observability over $P(s)$ corresponds to the presence of a common factor between $N(s)$ and $D_F(s)$, therefore it can be obtained only by cancelling zeros.

This can be done by assigning to $D_F(s)$ zeros (eigenvalues of the system) coincident with zeros of $N(s)$.

Cancelling $N(s)$ (all the zeros) one obtains maximal loss of observability.

If $P(s)$ has no zeros ($N(s) = k$) it is not possible to get unobservability under feedback and the problem cannot be solved.

If zeros of $P(s)$ have not all negative real part, the feedback F^* cannot be applied because it assigns eigenvalues which are not AS.

Thus, maximal unobservability is the one that can be generated by cancelling stable zeros.

Since (supposing always $p=q=1$ but can be extended to MIMO square with strong vector relative degree)

DDP is solvable iff $\mathrm{Im}(D) \subset V^*$

with V^* maximal (A,B) invariant subspace contained in $\mathrm{Ker}(C)$.

$V^*(s)$ can be computed in this way:

$$P(s) = c(sI-A)^{-1}b = \frac{N^+(s)N^-(s)}{D(s)} = \frac{N^-(s)}{D(s)} \cdot N^+(s)$$

$N^+(s)$ non minimum phase zeros

$D(s)$ $\qquad$ $\overline{D(s)}$

$N^+(s)$ non minimum phase zeros

$N^-(s)$ minimum phase zeros

$$P^-(s) = \frac{N^-(s)}{D(s)}$$

Computing from a realization of $P^-(s)$

$$A^- = \begin{pmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ -a_0 & \cdots & \cdots & -a_{n-1} \end{pmatrix} \qquad B^- = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \qquad C^- = \begin{pmatrix} b_0^- \cdots b_m^- & 0 \cdots 0 \end{pmatrix}$$

V^*_- is the max (A^-, B^-) invariant in ker C^-, and it coincides with V^* associated to $P(s)$