27. Lyapunov theorem in LTI

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if $\forall Q > 0$ symmetric there exists a unique $P > 0$ symm such that:

$$A^TQ + QA = -P$$

then $\sigma(A) \subset \mathbb{C}^- \Rightarrow$ the system is GAS

Proof:

- **Sufficiency:** $V(x) = x^TQx \Rightarrow \dot{V}(x) = \dot{x}^TQx + x^TQ\dot{x} =$
$= x^T(A^TQ + QA)x$
$= -x^TPx < 0$

- **Necessity:** assuming $\dot{x} = Ax$ GAS

$$Q = \int_0^\infty e^{A^Tt} P \cdot e^{At} dt$$

$$\Rightarrow A^TQ + QA = \int_0^\infty \frac{d}{dt}\left(e^{A^Tt} P e^{At}\right) dt$$

the integral is symmetric $\rightarrow$ solution is symmetric

$$= \left[e^{A^Tt} P e^{At}\right]_0^\infty = -P$$

- **Uniqueness:** By using the Sylvester criterion, $A^TQ + QA = -P$ has a unique solution Q