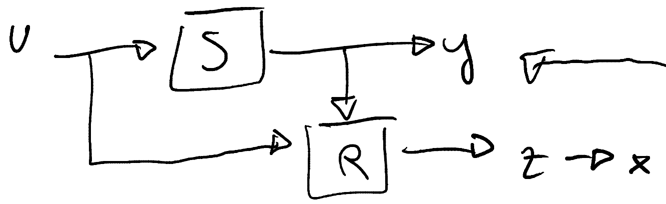
→ estimate of the internal state

Given $S: \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$ $x(0) \in \mathbb{R}^n$

and $R: \begin{cases} \dot{\omega} = F\omega + Gu + Ky \\ z = H\omega + Ly \end{cases}$ such that $z(t) \to x(t)$

the problem is the tuning of the velocity of the reconstruction and under which conditions z converges to x.



Introducing $Q \in \mathbb{R}^{q \times q}$ such that:

$y(t) = Cx(t)$

$\tilde{y}(t) = QCx = (I \;\; C_2)x$ $\quad y \to \boxed{Q} \to \tilde{y}$ $\quad |Q| \neq 0$

the output can be transformed in order to obtain the structure $\tilde{y} = (I \;\; C_2)x$

$\tilde{y} = x_1 + C_2 x_2$

$\tilde{y} - x_1 = C_2 x_2$

An estimate $\hat{x}_1 = \tilde{y}(t) - C\hat{x}_2$, since $\hat{x}_2 \to x_2$, therefore $\hat{x}_1 \to x_1$

Full state observer

$H = I \quad L = 0$

$\begin{cases} \dot{z} = Fz + Gu + Ky \\ \dot{x} = Ax + Bu \end{cases}$ $\quad z(t) - x(t) = \overset{\text{error}}{\mathcal{E}}(t)$

In order to compute F, G, K to make $\mathcal{E}(t) \to 0$ the dynamics of the error must be analyzed:

$\dot{\mathcal{E}}(t) = \dot{z}(t) - \dot{x}(t) \to \dot{\mathcal{E}}(t) = Fz - Ax + KCx + Gu - Bu$

The dynamics of the error must be studied:

$\dot{\xi}(t) = \dot{z}(t) - \dot{x}(t) \rightarrow \dot{\xi}(t) = Fz - Ax + KCx + Gu - Bu$

choosing $G = B$, $\dot{\xi}$ does not depend on u

$\dot{\xi} = Fz - Ax + KCx = Fz - (A-KC)x$

choosing now $F = A - KC$

$\dot{\xi} = (A-KC)\xi \rightarrow \xi(t) = e^{(A-KC)t}\xi_0$ which goes to 0 iff $\sigma(A-KC) \subset \mathbb{C}^-$

The problem becomes a problem of eigenvalues assignment

$\dot{z} = (A-KC)z + Bu + Ky$

→ speed of convergence depends on the eigenvalues of $(A-KC)$

→ the observer problem is solvable iff $\sigma(A-KC)$ can be placed wherever I want.

→ the problem can be solved iff (A, C) fully observable

then

$$F = -\gamma p^*(A^T) = -K^T \rightarrow K = p^*(A)\gamma^T$$

↳ last column of the inverse of the observability matrix

Observability condition:

$\rho(C^T \,|\, A^TC^T \,|\, \dots \,|\, A^{T^{n-1}}C^T) = n$

holds because:

$$\sigma(A-KC) = \sigma(A-KC)^T = \sigma(A^T + C^T(-K)^T)$$

Partial state observer

Used when (A, C) not fully observable

$\exists T: TAT^{-1} = \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} \quad CT^{-1} = (0 \;\; C_2)$

$$A_0 - KC_0 = \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} - \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}(0 \;\; C_2) =$$

$$\begin{pmatrix} 0 & A_{22} \end{pmatrix} \begin{pmatrix} K_2 \end{pmatrix} \begin{pmatrix} 0 & C_2 \end{pmatrix}$$

unobservable

$$= \begin{pmatrix} A_{11} & A_{12} - K C_2 \\ 0 & A_{22} - K_2 C_2 \end{pmatrix}$$

↳ observable

The problem is solvable iff $\sigma(A_{11}) \subset \mathbb{C}^-$