14. Linear Tangent Model

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$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x,u) \end{cases} \qquad \begin{matrix} f(x_e, u_e) = 0 \\ h(x_e, u_e) \neq 0 \end{matrix}$$

for small variations of x and u

$$\begin{cases} \Delta x = x - x_e \\ \Delta u = u - u_e \\ \Delta y = y - y_e \end{cases}$$

$$\Rightarrow \dot{\Delta x} = f(x_e, u_e) + \frac{\partial f}{\partial x}\bigg|_{(x_e,u_e)} \overset{\Delta x}{(x - x_e)} + \frac{\partial f}{\partial x}\bigg|_{(x_e,u_e)} (u - u_e) + \ldots \rightarrow 0$$

we don't care about the NL dependence

$$\Delta \dot{x} \cong \underbrace{\overbrace{\frac{\partial f}{\partial x}\bigg|_{(x_e,u_e)}}^{A}}_{n \times n} \Delta x + \underbrace{\overbrace{\frac{\partial f}{\partial u}\bigg|_{(x_e,u_e)}}^{B}}_{n \times p} \Delta u$$

Variation around x_e

$$\Delta y \cong \overbrace{\frac{\partial h}{\partial x}\bigg|_{(x_e,u_e)}}^{C} \Delta x + \overbrace{\frac{\partial h}{\partial u}\bigg|_{(x_e,u_e)}}^{D} \Delta u$$

$$\begin{cases} \Delta \dot{x} = A \Delta x + B \Delta u \\ \Delta y = C \Delta x + D \Delta u \end{cases}$$

Used to linearize a NL model.
The solution of the LTM gives the approximation of the Volterra Kernels.

$$A = \frac{\partial f}{\partial x}\bigg|_{x_e} = J_f(x_e) \qquad B = g(x_e)$$

$$C = \frac{\partial h}{\partial x}\bigg|_{x_e} = J_h(x_e) \qquad D = h(x_e)$$

The solution of the approximated model coincides with the NL's one

We consider the point at the origin $x_e = 0$ because any equilibrium can be shifted to the origin with
$\tilde{x} = x - x_e$