MECHANICS OF MOBILE ROBOTS

miércoles, 2 de octubre de 2019 8:04 a. m.

- Outline:
    - Wheels
    - Kinematics structures
    - Mobility and constraints
    - Nonholonomic Constraints
        - Rolling coin example

- Mobile robots (ground):
    - Wheeled: (WMR wheelede mobile robots)
        - One main body (base or chassis) that is in contact with the ground via wheels
        - They can have trailers in addition to the main body, otherwise WMR are single-body robots.
    - Legged:
        - Several bodies:
            - Trunk
            - Limbs
            - Head
        - Contact with the ground happens via feet
    - Ground robots may also include other forms of locomotion
        - Snake robots.

WHEELS

Kind	Description
Fixed	• One axis of rotation • The wheel cannot change orientation with respect to the chassis
Orientable (steerable)	• Two axis of rotation that meet at the center of the wheel • Both axis are actuated and the orientation of the wheel wrt the chassis is controlled
Caster	• Two axis of rotation that do not meet at the center of the wheel • Orientation of the wheel wrt is variable but not controlled.

Differential: It is a device that distributes the speed among the two wheels of an axl using the same torque on both. If you don't add a differential to a car the car will slip

BALANCE OF MOBILE ROBOTS

- **Statical balance**: the projection of the robot CoM on the ground must fall **inside** the **polygon of support**.
    - Wheeled robots need at least 3 wheels for balances. Casters are usually used as a 3rd wheel that are only used for support.

KINEMATIC STRUCTURES

Structure	Components
Differential-drive mobile robot	• Wheels:

	• 2 fixed wheels connected with a differential drive • 1 caster wheel • Motors: • 2 motors, one on each of the fixed wheels
Synchro-drive mobile robot	• Wheels: • 3 orientable wheels • Motors: • One motor that rotates all wheels simultaneously • One motor that rolls all wheels simultaneously
Tricycle	• Wheels: • 2 fixed wheels connected via a mechanical axel • 1 orientable wheel that steers the robot • Motors: • One motor to power the rear axel • One motor tho steer the front wheel
Car-like	• Wheels: • 2 fixed wheels connected via a mechanical axel • 2 orientable wheels connected via a mechanichal axel • Motors: • One to power the motion (rear or front) • One to steer the front wheels
Omnidirectional	• Wheels: • 3 (actuated) caster wheels • Motors: • One on every wheel

MOBILITY AND CONSTRAINTS

- **Global mobility** is guaranteed: they can go anywhere
- **Local mobility** is restricted: there are some instantaneous motions are not possible.

Constraints in mechanical systems:
- Kinds of constraints:
  - **Equality**: bilateral constraints
    - **Geometric** constraint: is an equality which includes only the configuration q
      - $h_i(q) = 0$
      - $i = 0, \dots, k$
      - k is the number of constraints
      - C is n-dimensional
      - The set of admissible configurations is $(n-k)$-dimensional
      - They represents a **global mobility** limitation. This means that some coordinates can be eliminated and only $(n-k)$ parameters are required to describe the motion.
        - This is done using the **implicit function theorem** for eliminating k variables by solving $h(q) = 0$. This woks only locally.
        - For a global solution, a better choice of generalized coordinates for the constrained system is necessary.
    - **Kinematic** constraint: is an equality that includes the configuration q and its time derivative $\dot{q}$
      - $a_i(q, \dot{q}) = 0$

- $i = 0, \dots, k$
- k is the number of constraints
- C is n-dimensional
- These constraints are usually found in the form $a_i^T(q)\dot{q} = 0$. Pfeffien constraints
  - Nonlinear in q
  - Linear in $\dot{q}$
- In matrix form:
  - $\begin{pmatrix} a_1^T(q) \\ a_2^T(q) \\ \dots \\ a_k^T(q) \end{pmatrix} \dot{q} = 0$
  - $A^T(q)\dot{q} = 0$
    - Admissible generalized velocities must belong to $N(A^T(q))$
  - k-rows and n-columns
- This constraints represent a **local mobility** limitation. However we don't know anything about the global mobility limitations.

- Relationship between geometric and kinematic constraints:
  - Every geometric constraint implies a kinematic constraint.
    - $h(q) = 0 \rightarrow \frac{dh(q)}{dt} = 0 \rightarrow \frac{\partial h}{\partial q}\dot{q} = 0 \rightarrow a^T(q)\dot{q} = 0$
    - $\frac{\partial h}{\partial t}$ is the transpose of the gradient.
    - Geometrically this means that the admissible velocities must be orthogonal to the gradient vector.
  - In other words, if the mobility is globally limited then it is also locally limited.
  - Does a kinematic constraint imply a geometric constraint?
    - A kinematic constraint **may or may not** imply a geometric constraint
    - Example:
      - $C = R^2$
      - $\begin{pmatrix} q_2 \\ q_1 \end{pmatrix}^T \dot{q} = 0 \rightarrow q_2\dot{q_1} + q_1\dot{q_2} = 0$
      - Consider $q_1 q_2 = c$, where c is a constant. After differentiation wrt time we get:
        - $q_2\dot{q_1} + q_1\dot{q_2} = 0$
      - Geometrically this represent a hyperbola and c, which becomes the integration constant, is determined by the initial conditions of the robot.
        - $q_1(t_0)q_2(t_0) = c$
      - In this case the kinematic constraint implies a geometric constraint
    - Example:
      - $C = R^2$
      - $\begin{pmatrix} q_1 \\ q_2 \end{pmatrix}^T \dot{q} = 0 \rightarrow q_1\dot{q_1} + q_2\dot{q_2} = 0$
      - Consider $q_1^2 + q_2^2 = c$. After differentiation we get:
        - $q_1\dot{q_1} + q_2\dot{q_2} = 0$
      - Geometrically the function represents a circle and, again, c is determined by the initial conditions of the robot.
      - In this case the kinematic constraint implies a geometric constraint.
    - Example:
      - $C = R^3$
      - $\begin{pmatrix} \sin(q_3) \\ -\cos(q_3) \\ 0 \end{pmatrix}^T \dot{q} = 0$

    - This is **not integrable**, so there are not global mobility restrictions associated with this constraint.
  - In general, **all linear kinematic constraints** are integrable and represent a global mobility constraints.
    - $a^T(q)\dot{q} = 0$ may be integrable if:
      - $\exists h(q): \dfrac{\partial h}{\partial q} = a(q)$
    - Generalizing:
      - $\dfrac{\partial h}{\partial q} = \gamma(q) a^T(q)$
      - The condition is that the integrating factor $\gamma(q)$ should not be zero.
  - **HOLONOMIC**: kinematic constraints that can be integrated to geometric constraints.
    - Restrict both local and global mobility.
  - **NON-HOLONOMIC**: kinematic constraints that cannot be integrated to geometric constraints. Non-holonomic robots are subject to at least one non-holonomic constraint.
    - Restrict only local mobility.
  - Rolling coin (disk) example - Non-holonomic constraint:There is a coin rolling upright on a flat surface. To define the configuration we need the position of the contact point on the surface and the orientation.
    - $q = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$
    - $C = SE(2)$
    - Constraints:
      - Pure rolling:
        - $\dot{x}\sin(\theta) - \dot{y}\cos(\theta) = \begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{bmatrix} \dot{q} = 0$
        - It is a Pfeffien constraint
        - Geometrically it means that $\begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \\ 0 \end{bmatrix}$ is orthogonal to the velocity vector $\dot{q}$.
    - Is it holonomic? If we can prove that the system can go anywhere then the constraint is NH (i.e. the system is controllable)
    - Admissible velocities: $N - k = 2$ Dimensional space.
      - $\dot{q} \in N(a^T(q))$
      - $N(a^T(q)) = \left[\begin{pmatrix} cos\theta \\ \sin(\theta) \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right]$
      - The first vector represents rolling and the second represents rotation.
      - The null space will then span any linear combination of these two vectors, which means that it can reach any point in the plane. So the constraint is **Non-holonomic**.

- **Inequality**: unilateral constraints

Autonomous and Mobile Robotics

Prof. Giuseppe Oriolo

Wheeled Mobile Robots 1

Mechanics of Mobile Robots

companion slides for the blackboard lecture

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Ground Locomotion

contact with the ground via
- wheels
- legs

wheels → wheeled mobile robot (WMR) → 1 rigid body (base, chassis) + wheels

legs → legged (mobile) robot (LR) → several rigid bodies (arms, torso...) → some of them: feet

Balance ("not falling")

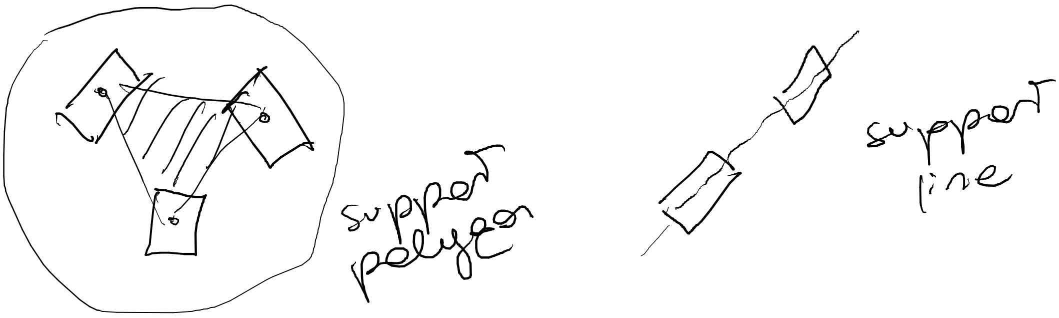
→ static: the CoM of the robot falls (once projected) within the support polygon

→ dynamical

→ when walking

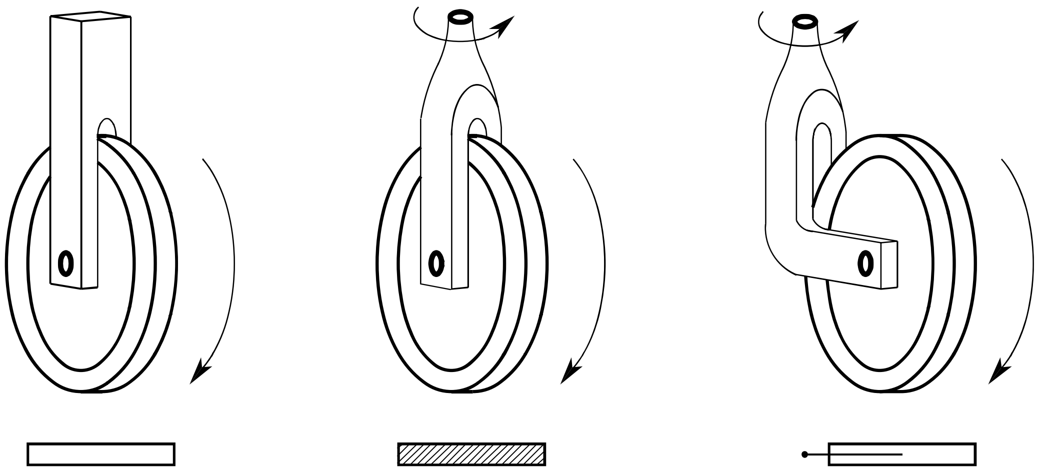
Balance in WMR

each wheel → 1 point contact with ground
to get an ideal support polygon, need 3 wheels



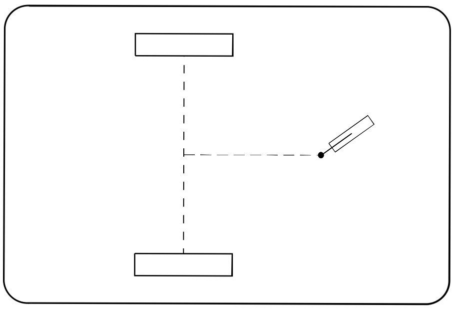
wheels

three basic types



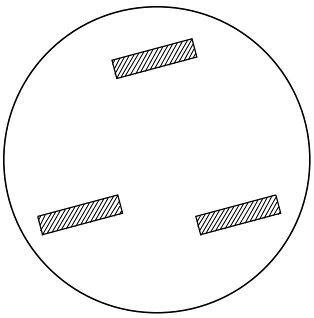
adds static stability (static balance)

kinematic structures



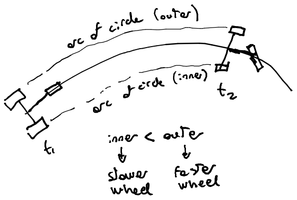
differential-drive mobile robot

kinematic structures

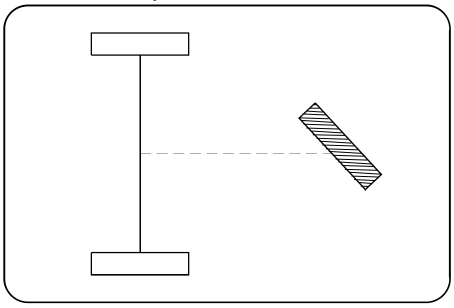


synchro-drive mobile robot

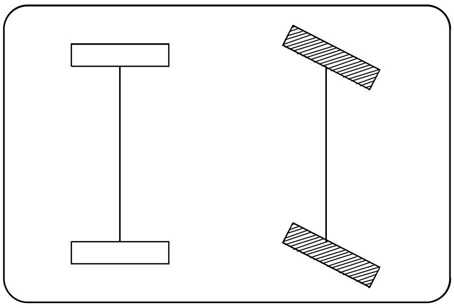
kinematic structures



need a differential allows different wheel velocity with the same driving torque

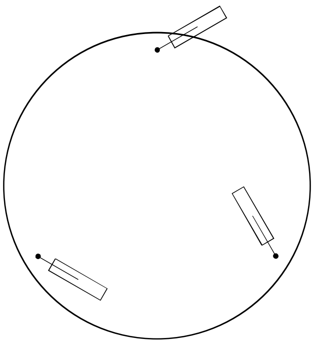


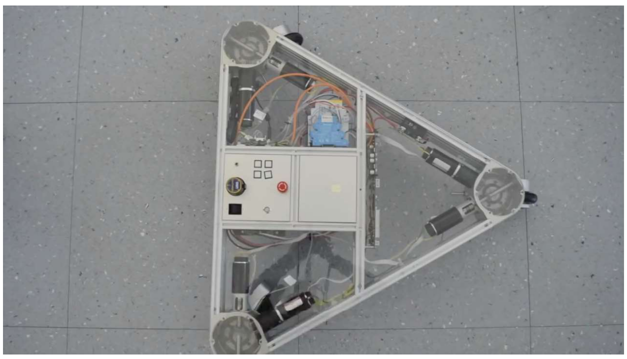
tricycle



car-like

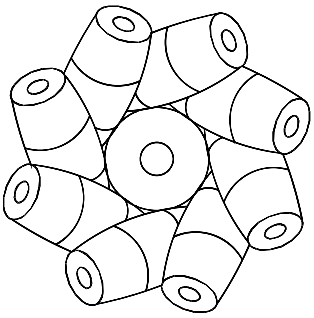
kinematic structures





omnidirectional mobile robot with
3 (actuated) caster wheels

kinematic structures



Mecanum (Swedish) wheels can be also used to build omnidirectional mobile robots

Constraints & Mobility

robot configuration $q \in \mathcal{C}$ → n-dimensional → manifold that locally looks like Euclidean space

└→ n components (generalized coordinates)

constraints:

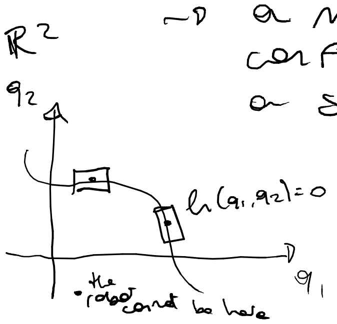
- geometrical $h(q) = 0$ (GC)
- kinematic $a(q, \dot{q}) = 0$ (KC)

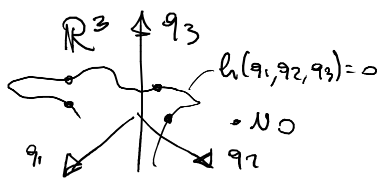
GC → it limits configurations

$h_i(q) = 0 \quad i = 1, \dots, k \; (< n)$

→ only involves the configuration q

→ a mobility limitation the robot can only assume configurations in $\mathcal{C}$ that satisfy (GC) i.e. a subset of $\mathcal{C}$ of dimension $n-k$





→ A global mobility limitation

Can we redefine q so that we only use $n-k$ coords? Yes

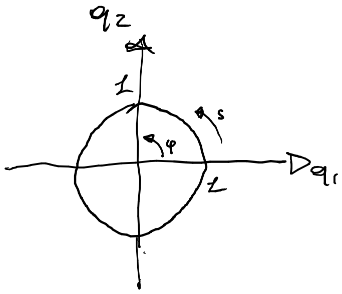
Implicit function theorem

$h(q) = 0$

it is locally possible to make it explicit as

$q_1 = \varphi(q_2, \dots, q_n)$

provided that some singularity conditions



- example $q \in \mathbb{R}^2$ $\quad q_1^2 + q_2^2 = 1$ (GC) unit circle

$q_2 = \pm\sqrt{1 - q_1^2}$ express q_2 as $\varphi(q_1)$

locally I can solve saying "only + in the upper half circle"

- Globally: better change variables

In this case → arc length s (ascissa curvilinea) or phase length φ

- as we did for the manipulators

KC $\quad a_i(q, \dot{q}) = 0 \quad i = 1, \dots, k$

therefore also velocity is involved

Typically linear in $\dot{q}$ → $a_i^T(q)\,\dot{q} = 0 \Rightarrow \overbrace{(a_1(q) \dots a_n(q))}^{\text{Non linear components}}\, \underset{\text{linear}}{\dot{q}} = 0$

arrange them like this

$$\begin{pmatrix} a_1^T(q) \\ a_2^T(q) \\ \vdots \\ a_k^T(q) \end{pmatrix} \dot{q} = 0$$

$$k\{\underbrace{A^T}_{n}(q)\,\dot{q} = 0 \qquad \text{Pfaffian form of KC (linear in } \dot{q})$$

- **Mobility limitation**

at each q, the admissible $\dot{q}$ must belong to $\mathcal{N}(A^T(q))$ (null space)

a linear space of dimension $n - k$
↳ hyper plane

→ local mobility limitation : it is limiting the possible motion, not the configuration itself

Relationship GC / KC

a GC always implies a KC

$\boxed{h(q)=0}$ GC continuous satisfaction $\frac{\partial h(q)}{\partial t}=0$

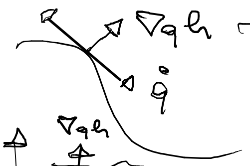
$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial q}\dot{q} = 0 \Rightarrow \underbrace{\left(\frac{\partial h}{\partial q_1} \quad \frac{\partial h}{\partial q_2}\right)}_{\text{transpose of the gradient}} = \nabla_q^T h$$

$$\boxed{(\nabla_q^T h)\,\dot{q} = 0} \quad \text{Pfaffian KC}$$

$(a^T(q))$

geometrical interpretation: if we have GC $h(q)=0$ then necessarily $\dot{q}$ must be orthogonal to $\underbrace{\nabla_q h}_{\text{the normal to the surface}}$

$\mathbb{R}^2$



$\nabla_q h \to$ i.e. it must be tangent to the surface

$\mathbb{R}^3$

$\to$ tangent plane to the surface at q

- a KC does not always implies a GC

$q_1\dot{q}_1 + q_2\dot{q}_2 = 0$ in $\mathbb{R}^2$

$$\underbrace{(q_1 \quad q_2)}_{A^T(q)} \begin{pmatrix}\dot{q}_1 \\ \dot{q}_2\end{pmatrix} = 0$$

$A^T(q)\cdot\dot{q}$ Pfaffian constraint

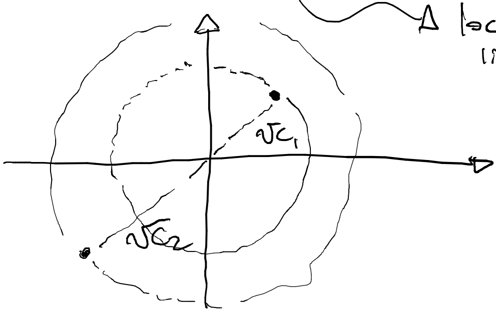
Can this be written as $h(q)=0$ ($\to$ GC) for some h, i.e., is it integrable? Yes it is clearly integrable

$$q_1^2 + q_2^2 = c \xrightarrow{\text{derivative}} \cancel{2}q_1\dot{q}_1 + \cancel{2}q_2\dot{q}_2 = 0 \quad \text{the original KC}$$

$\boxed{\dot{q}_1 q_1 + q_2\dot{q}_2 = 0}$ can be integrated as $\boxed{q_1^2+q_2^2=c}$

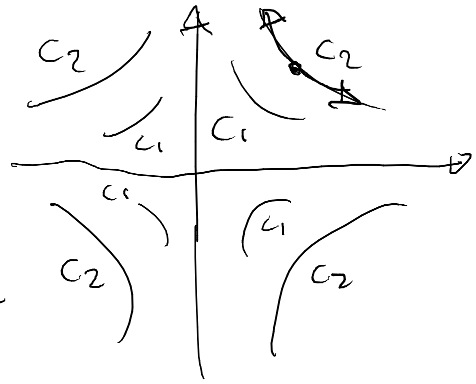
$\to$ local mobility limitation

$\to$ global mobility limitation: the robot is constrained to move along that level circle



$q_1 q_2 = c$

$\dot{q}_1 q_2 + \dot{q}_1 q_2 = 0$



robot moves on this hyperbole

the level curves are FOLIATION in $\mathbb{R}^2$

in $\mathbb{R}^3$ $\quad (\sin q_3 \ \ -\cos q_3 \ \ 0) \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = 0$

$\sin q_3 \dot{q}_1 - \cos q_3 \dot{q}_2 = 0 \rightarrow$ Is this integrable? No

recapitulate

$a^T(q)\dot{q} = 0 \quad$ a KC (Pfaffian)

- if there exists a $h(q)$ such that $\frac{\partial h^T}{\partial q} = \gamma(q)\, a^T(q)$ then KC can be integrated as $h(q) = c$

  integrating factor $\neq 0$

  KC is called integrable of HOLONOMIC (olonomo)

  $\rightarrow$ The KC implies a GLOBAL mobility limitation

- if there exists no such function, then KC cannot be integrated:
  KC is said NON-INTEGRABLE or NON-HOLONOMIC (anolonomo)

  $\rightarrow$ The KC remains a purely LOCAL mobility limitation

- How to tell if

  $a_i^T(q)\dot{q} = 0$ is $< \begin{matrix} H \\ NH \end{matrix}$

Frobenius Theorem (integrability of 1 forms)

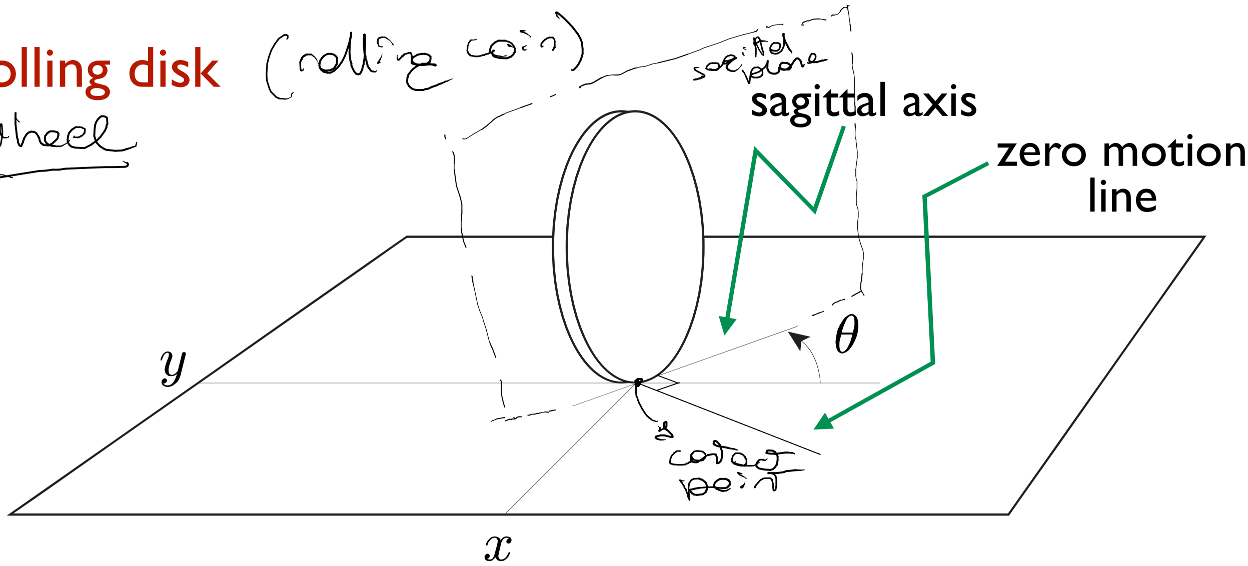
- a different viewpoint

Let's look at mobility (global) and ask ourselves if robot can go anywhere

example of nonholonomic constraint

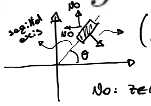
rolling disk (rolling coin)

wheel



generalized coordinates $\quad \boldsymbol{q} = [\,x \quad y \quad \theta\,]^T$

the vector $\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}$ is orthogonal to $\begin{pmatrix}\sin\theta\\ \cos\theta\end{pmatrix}$



$\begin{pmatrix}\sin\theta\\ \cos\theta\end{pmatrix}$ ⇒ Cartesian velocity is always along the sagittal axis

No: zero motion line

pure rolling $\quad \dot{x}\sin\theta - \dot{y}\cos\theta = [\,\sin\theta \quad -\cos\theta \quad 0\,]\,\dot{\boldsymbol{q}} = 0$

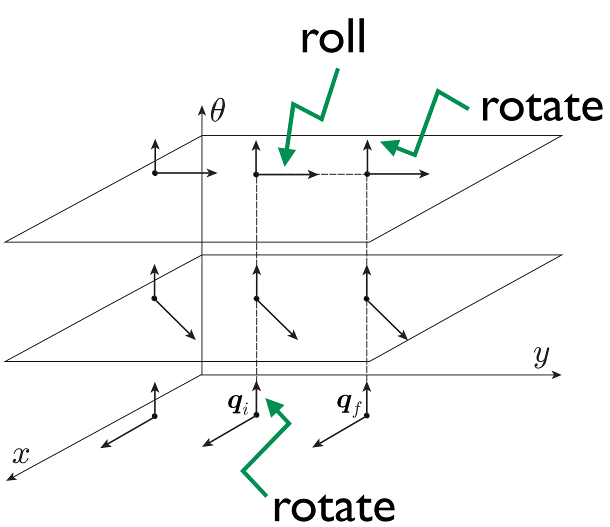
is this integrable ?

↳ constraint of zero motion line (rolling without slipping)

Can the wheel go anywhere in C_i without violating the constraint? YES

the disk can go from any initial to any final configuration

e.g.
1. rotate so as to align with the final position
2. roll up to the final position
3. rotate up to the final orientation



hence, the rolling constraint is nonholonomic we proved that the rolling without slipping constraint is Non-holonomic by exhibiting a manouver that steers the car between any two configurations

("CONSTRUCTIVE CONTROLLABILITY").

↓

used only
because the case is simple

In general I use an algebraic test.