



Robotics 2

Iterative Learning for Gravity Compensation

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Control goal

- regulation of arbitrary equilibrium configurations in the **presence of gravity**
 - **without** explicit knowledge of robot dynamic coefficients (nor of the structure of the gravity term)
 - **without** the need of “high” position gain
 - **without** complex conditions on the control gains
- based on an **iterative control scheme** that uses
 1. PD control on joint position error + **constant** feedforward term
 2. iterative **update** of the feedforward term at successive steady-state conditions
- derive **sufficient conditions** for the global convergence of the iterative scheme with zero final error



Preliminaries

- robot dynamic model

$$M(q)\ddot{q} + c(q, \dot{q}) + g(q) = u$$

- available bound on the gradient of the gravity term

$$\left\| \frac{\partial g(q)}{\partial q} \right\| \leq \alpha$$

- regulation attempted with a joint-based PD law (without gravity cancellation nor compensation)

$$u = K_P(q_d - q) - K_D\dot{q} \quad K_P > 0, K_D > 0$$

- at steady state, there is a **non-zero error** left

$$q = \bar{q}, \dot{q} = 0 \quad g(\bar{q}) = K_P(q_d - \bar{q}) \quad \bar{e} = q_d - \bar{q} \neq 0$$



Iterative control scheme

- **control law** at the i -th iteration (for $i = 1, 2, \dots$)

$$u = \gamma K_P (q_d - q) - K_D \dot{q} + u_{i-1} \quad \gamma > 0$$

with a constant compensation term u_{i-1} (**feedforward**)

- $K_P > 0, K_D > 0$ are chosen **diagonal** for simplicity
- q_0 is the initial robot configuration
- $u_0 = 0$ is the 'easiest' initialization of the feedforward term
- at the **steady state** of the i -th iteration ($q = q_i, \dot{q} = 0$), one has

$$g(q_i) = \gamma K_P (q_d - q_i) + u_{i-1}$$

- **update law** of the compensation term (for next iteration)

$$u_i = \gamma K_P (q_d - q_i) + u_{i-1} \quad [= g(q_i)]$$

← **for implementation** → [**for analysis**]



Convergence analysis

Theorem

(a) $\lambda_{\min}(K_P) > \alpha$

(b) $\gamma \geq 2$

guarantee that the sequence $\{q_0, q_1, q_2, \dots\}$ converges to q_d (and $\dot{q} = 0$) from **any** initial value q_0 (and \dot{q}_0), i.e., **globally**

- condition (a) is sufficient for the global asymptotic stability of the desired equilibrium state when using

$$u = K_P(q_d - q) - K_D\dot{q} + g(q_d)$$

with a **known** gravity term and diagonal gain matrices

- the additional **sufficient** condition (b) guarantees the convergence of the iterative scheme, yielding

$$\lim_{i \rightarrow \infty} \|u_i\| = g(q_d)$$



Proof

- let $e_i = q_d - q_i$ be the error at the end of the i -th iteration; based on the update law, it is $u_i = g(q_i)$ and thus

$$\begin{aligned}\|u_i - u_{i-1}\| &= \|g(q_i) - g(q_{i-1})\| \leq \alpha \|q_i - q_{i-1}\| \\ &\leq \alpha (\|e_i\| + \|e_{i-1}\|)\end{aligned}$$

- on the other hand, from the update law it is

$$\|u_i - u_{i-1}\| = \gamma \|K_P e_i\|$$

- combining the two above relations under (a), we have

$$\gamma \alpha \|e_i\| < \gamma \lambda_{\min}(K_P) \|e_i\| \leq \gamma \|K_P e_i\| \leq \alpha (\|e_i\| + \|e_{i-1}\|)$$

$$\text{or } \|e_i\| < \frac{1}{\gamma} (\|e_i\| + \|e_{i-1}\|)$$



Proof (cont)

- condition (b) guarantees that the error sequence $\{e_0, e_1, e_2, \dots\}$

$$\|e_i\| < \frac{\frac{1}{\gamma}}{1 - \frac{1}{\gamma}} \|e_{i-1}\| = \frac{1}{\gamma - 1} \|e_{i-1}\|$$

is a **contraction mapping**, so that

$$\lim_{i \rightarrow \infty} \|e_i\| = 0$$

with asymptotic convergence from any initial state



⇒ the **robot progressively approaches** the desired configuration through **successive steady-state conditions**

- K_P and K_D affect each transient phase
- coefficient γ drives the convergence rate of intermediate steady states to the final one



Remarks

- combining (a) and (b), the sufficient condition only requires the **doubling** of the proportional gain w.r.t. the known gravity case

$$\hat{K}_P = \gamma K_P \quad \rightarrow \quad \boxed{\lambda_{\min}(\hat{K}_P) > 2\alpha}$$

- for a diagonal \hat{K}_P , this condition implies a (positive) lower bound on the single diagonal elements of the matrix
- again, it is only a **sufficient** condition
 - the scheme may converge even if this condition is violated ...
- the scheme can be interpreted as using an **integral term**
 - updated only in correspondence of a **discrete sequence of time instants**
 - with guaranteed **global** convergence (and implicit stability)



Numerical results

- 3R robot with uniform links, moving in the vertical plane

$$l_1 = l_2 = l_3 = 0.5 \text{ [m]}$$

$$m_1 = 30, m_2 = 20, m_3 = 10 \text{ [kg]} \quad \rightarrow \quad \alpha \cong 400$$

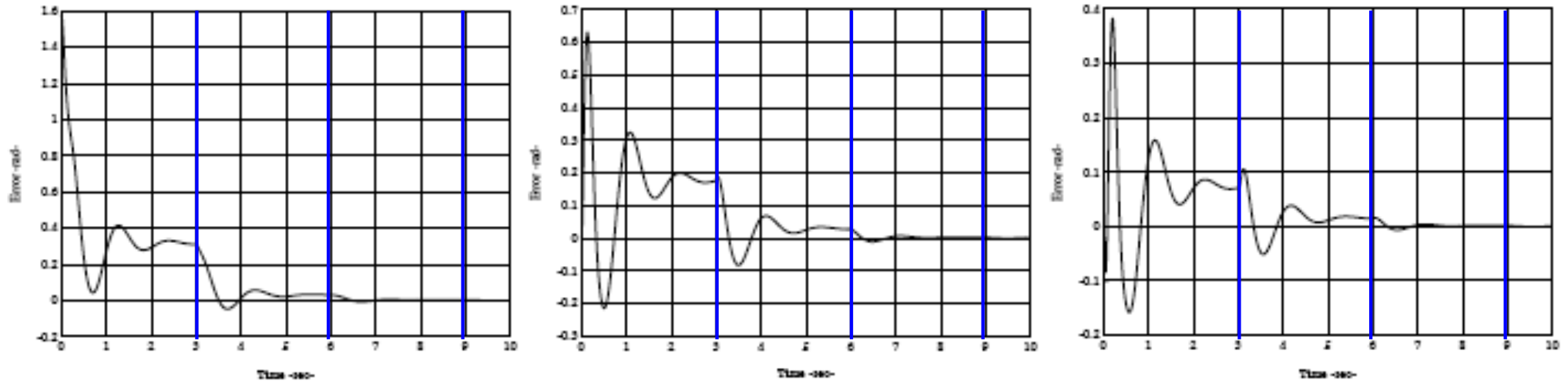
- with saturations of the actuating torques

$$U_{1,\max} = 800, U_{2,\max} = 400, U_{3,\max} = 200 \text{ [Nm]}$$

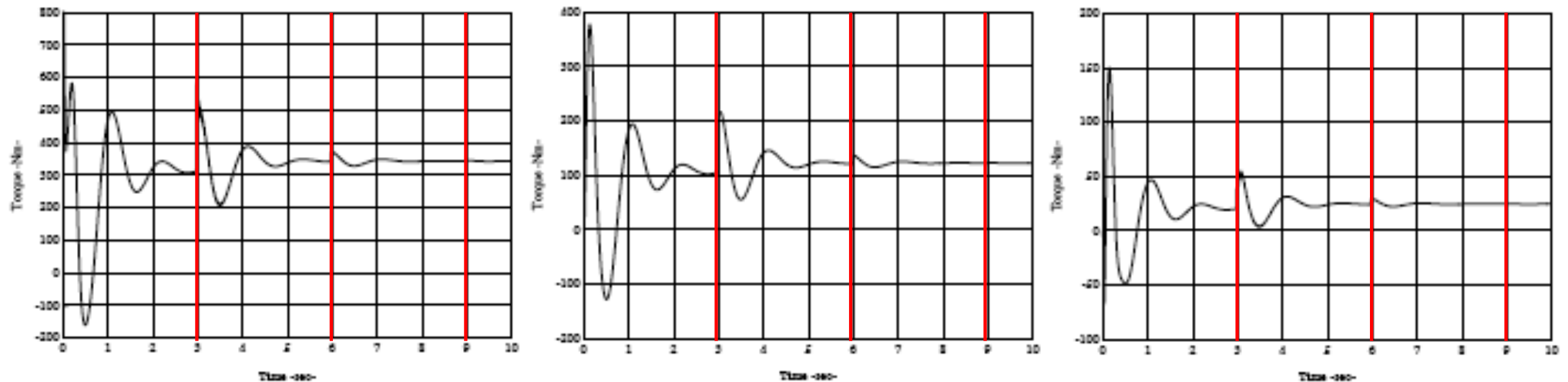
- three cases, from the downward position $q_0 = (0, 0, 0)$

$$\begin{array}{ll} \text{I:} & q_d = (\pi/2, 0, 0) \\ \text{II:} & q_d = (3\pi/4, 0, 0) \\ \text{III:} & q_d = (3\pi/4, 0, 0) \end{array} \left\{ \begin{array}{l} \hat{K}_P = \text{diag}\{1000, 600, 280\} \\ K_D = \text{diag}\{200, 100, 20\} \end{array} \right. \quad \left\{ \begin{array}{l} \hat{K}_P = \text{diag}\{500, 500, 500\} \\ K_D = \text{as before} \end{array} \right.$$

Case I: $q_d = (\pi/2, 0, 0)$



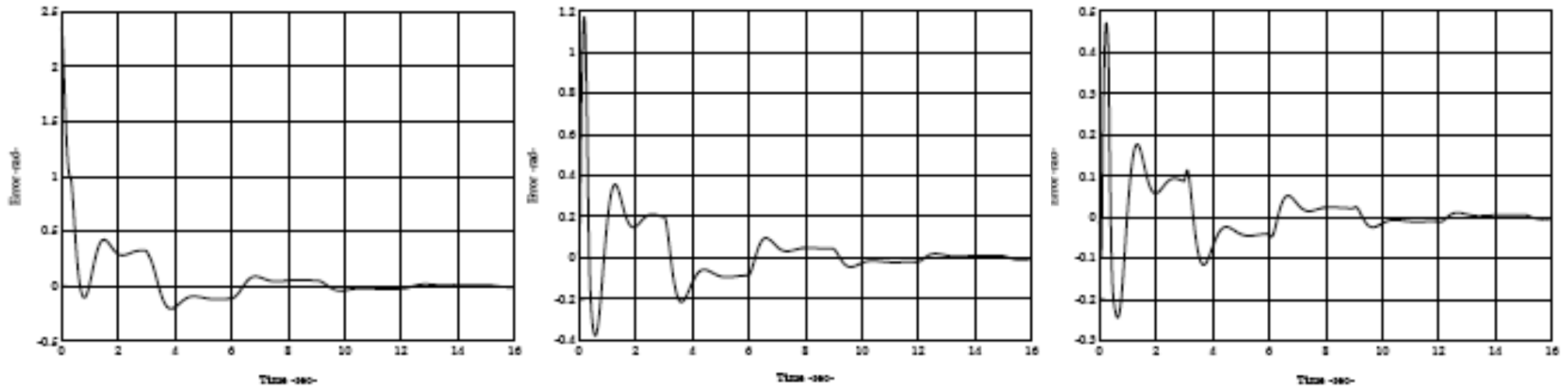
joint position errors (zero after 3 updates)



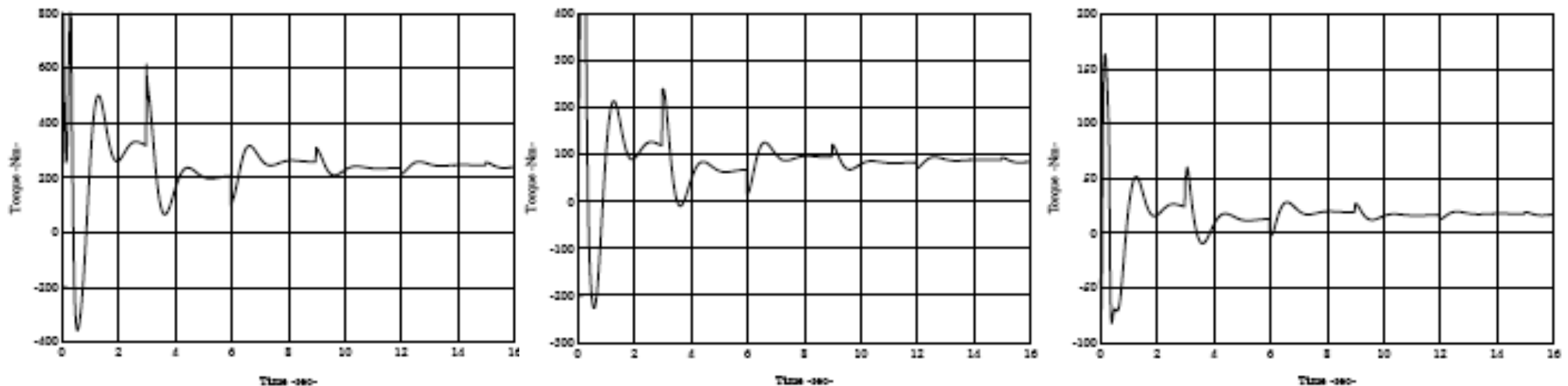
control torques



Case II: $q_d = (3\pi/4, 0, 0)$

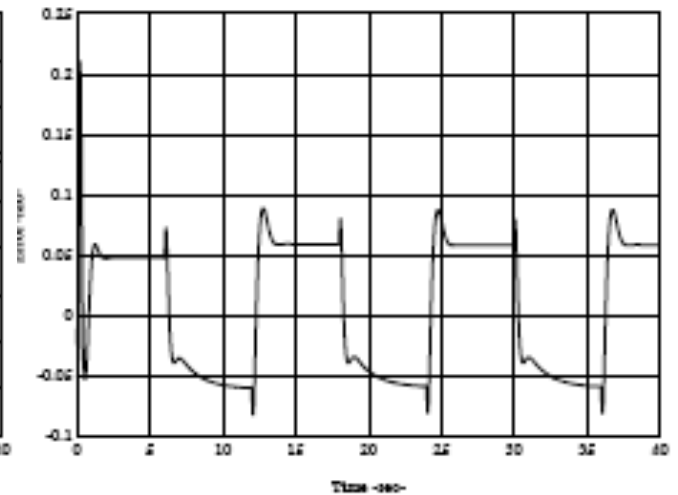
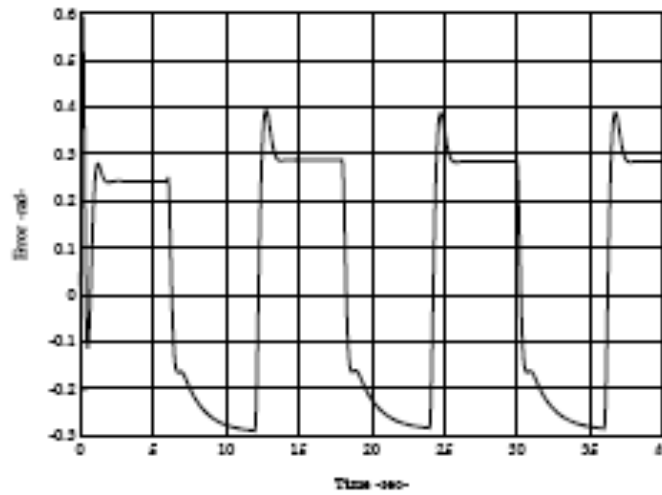
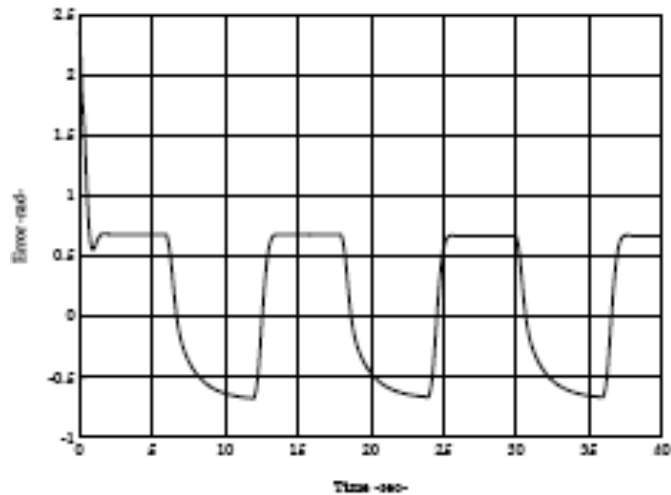


joint position errors (zero after 5 updates)

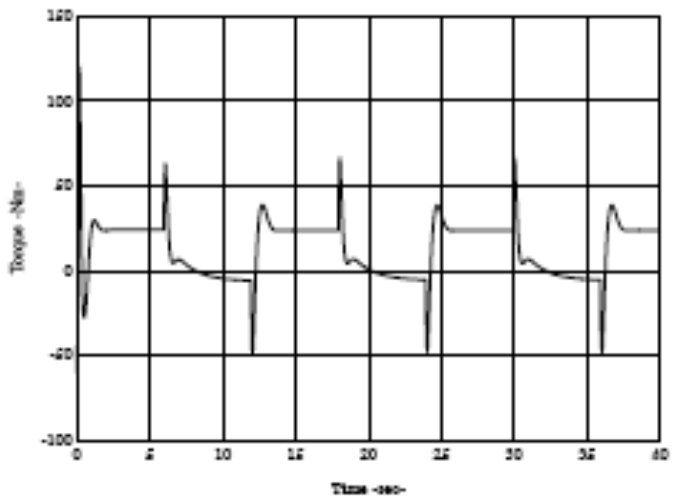
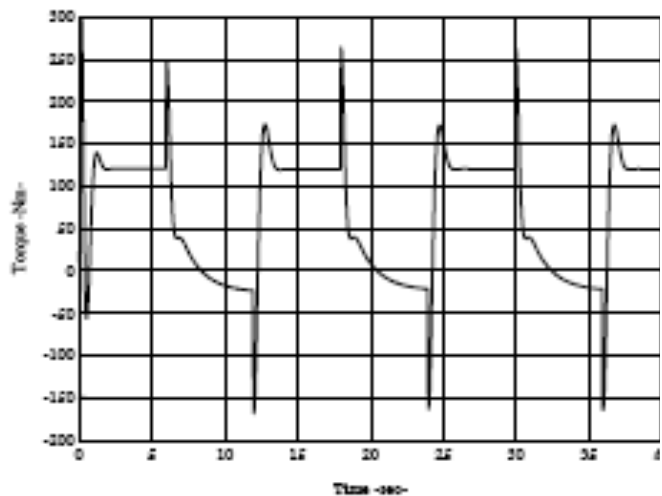
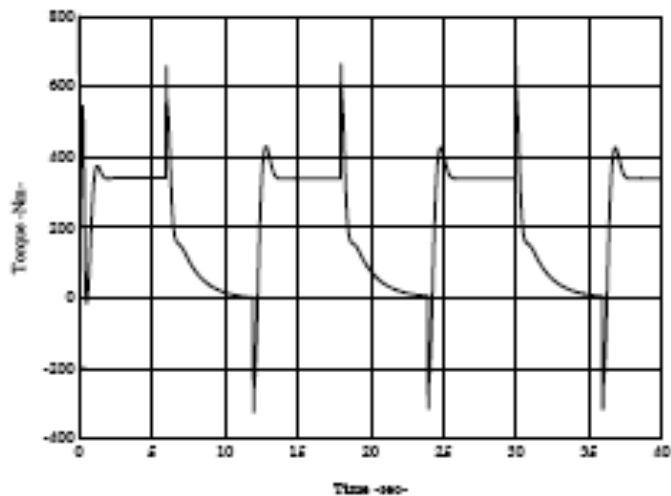


control torques

Case III: $q_d = (3\pi/4, 0, 0)$, reduced gains



joint position errors (limit cycles, no convergence!)



control torques



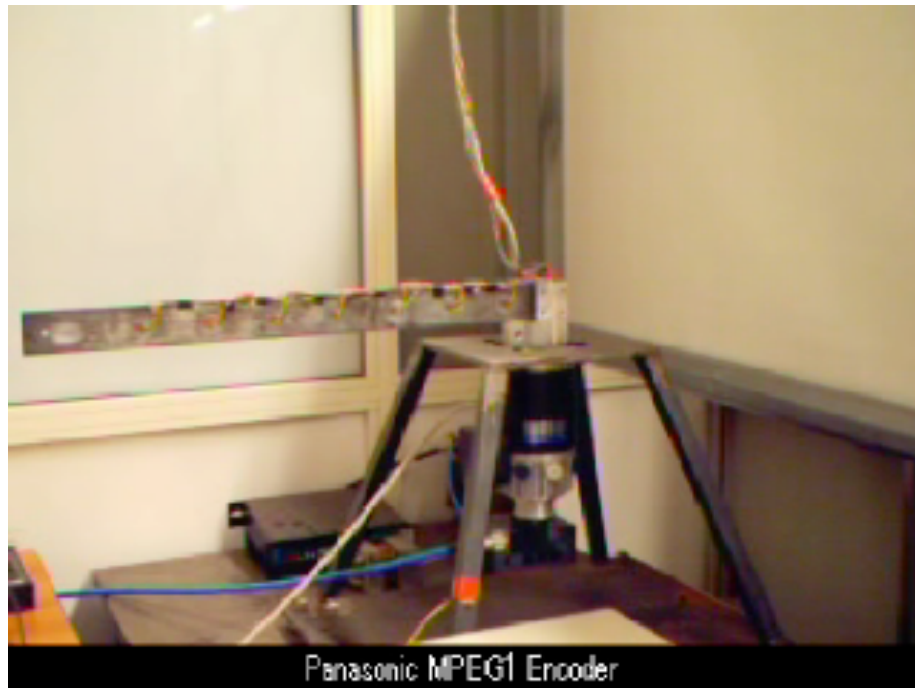
Final comments

- only **few iterations** are needed for obtaining convergence, **learning the correct gravity compensation** at the desired q_d
- **sufficiency** of the condition on the P gain
 - even if violated, convergence can **still be** obtained (first two cases); otherwise, a limit motion cycle takes place between two equilibrium configurations that are **both incorrect** (as in the third case)
 - this shows how 'distant' is sufficiency from **necessity**
- analysis can be refined to get lower bounds on the K_{p_i} (diagonal case) that are smaller, but still sufficient for convergence
 - intuitively, lower values for K_{p_i} should be sufficient for distal joints
- in practice, update of the feedforward term occurs when the robot is **close enough to a steady state** (joint velocities and position variations are below **suitable thresholds**)

Control experiments with flexible robots without gravity

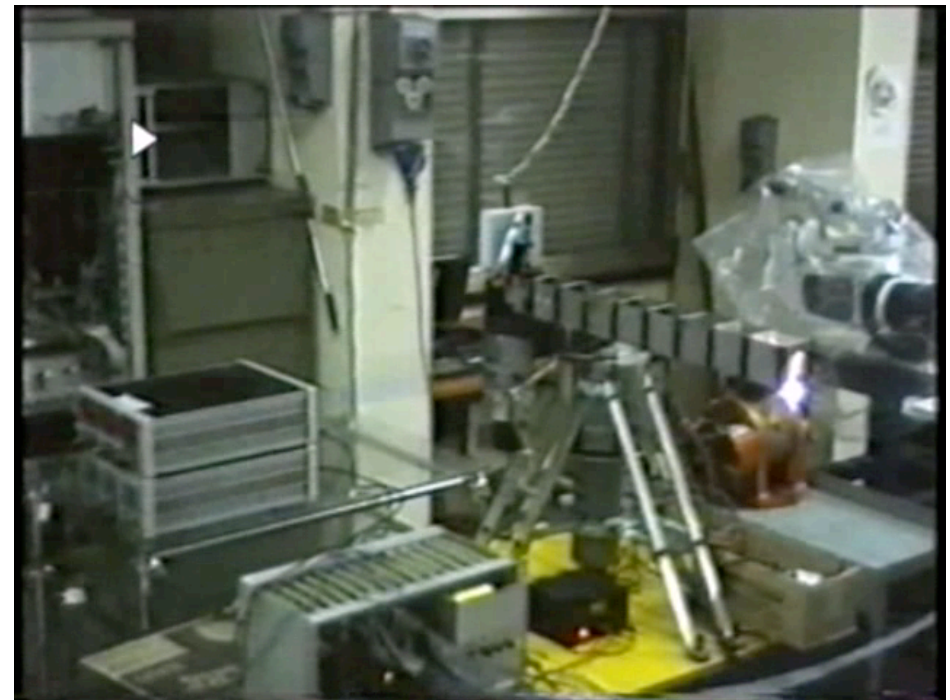


video

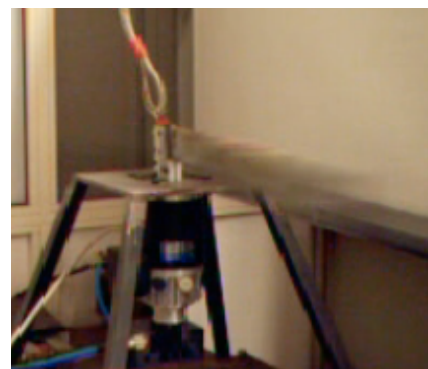
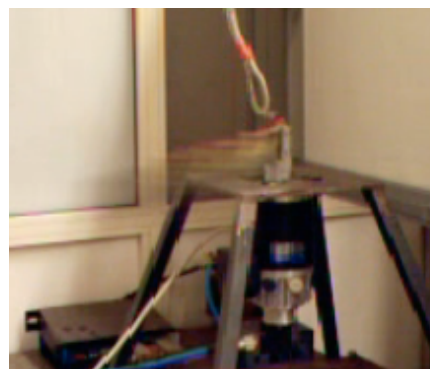


rest-to-rest maneuver in given motion time
for a single flexible link (PD + feedforward)

video

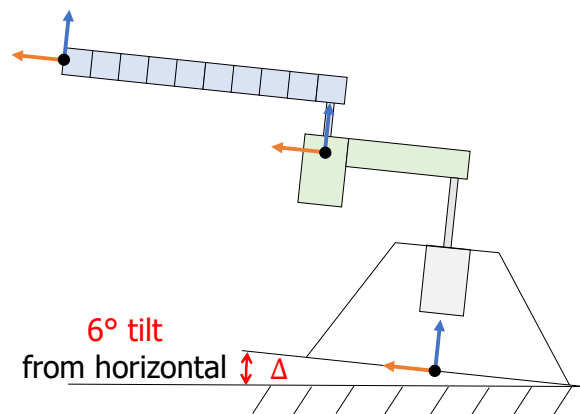


end-effector trajectory tracking for FlexArm
—a planar 2R robot with flexible forearm

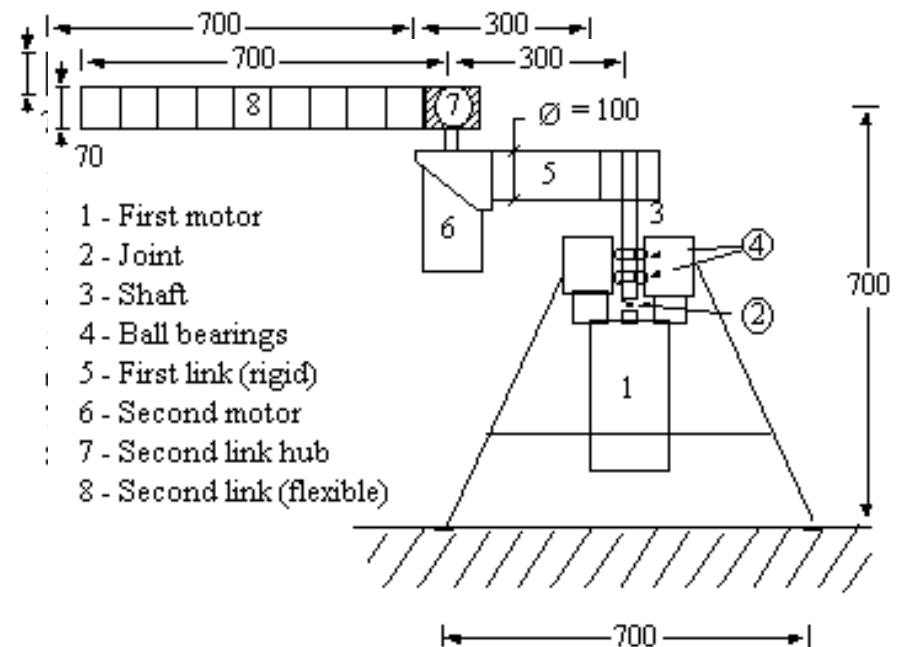
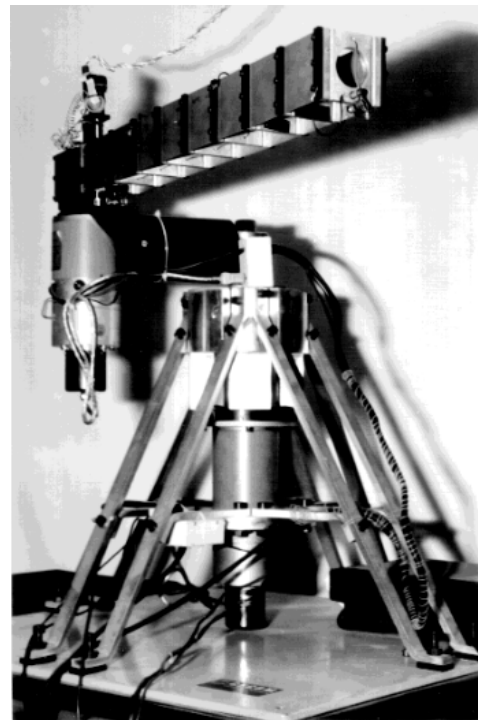


Extension to flexible robots

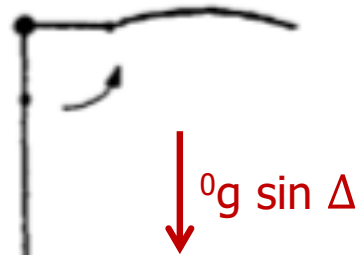
- the same iterative learning control approach has been extended to position regulation in robots with **flexible joints and/or links** under gravity
 - at the motor/joint level
 - at the Cartesian level (end-effector tip position, **beyond** flexibility), using a **double iterative** scheme
- experimentally validated on the **Two-link FlexArm @ DIS** (now DIAG!)



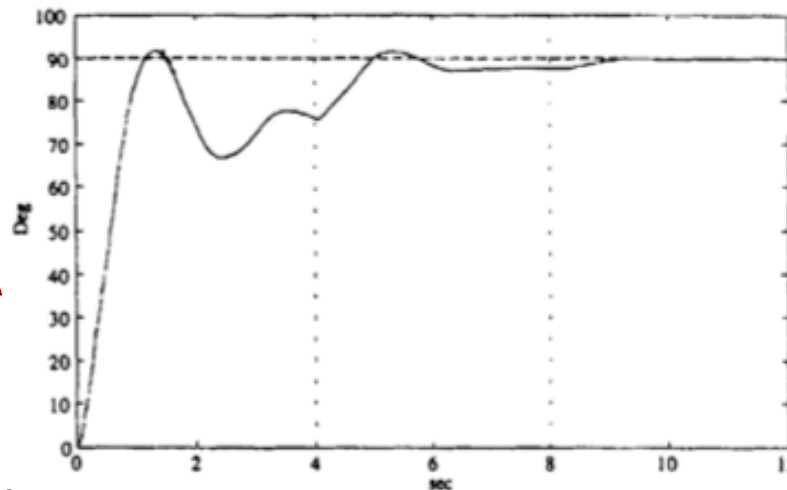
with supporting base
tilted by approx $\Delta = 6^\circ$
(inclusion of gravity)



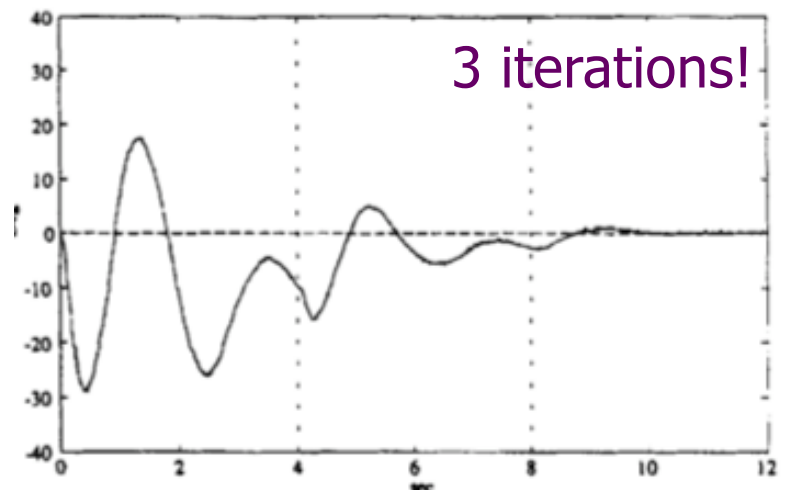
Experimental results for tip regulation



motion task:
 $(0^\circ, 0^\circ) \Rightarrow (90^\circ, 0^\circ)$

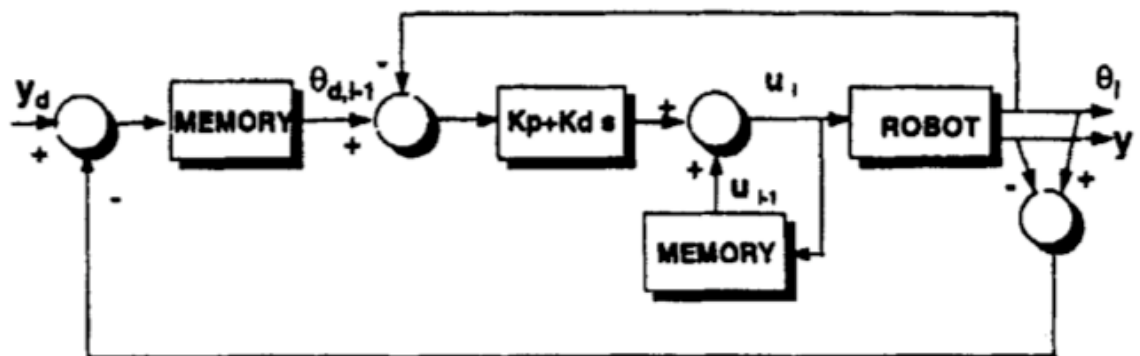


first link position



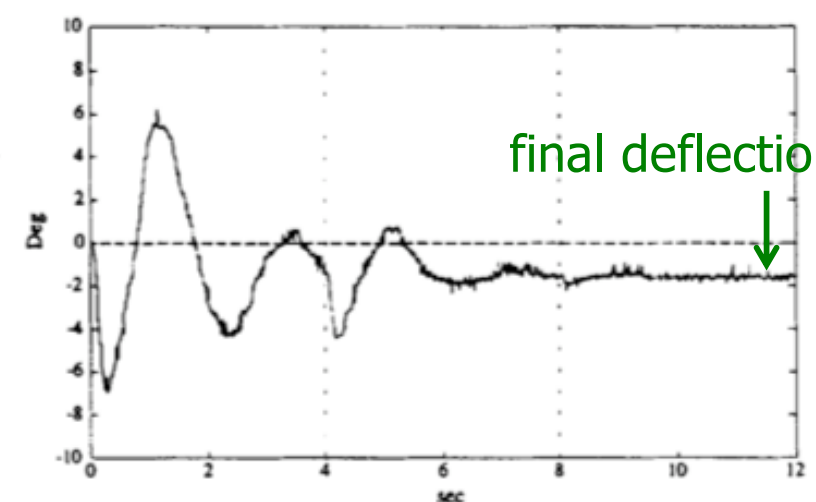
3 iterations!

tip angle w.r.t. first link



double iterative scheme

De Luca, Panzieri: Int J Adapt Cont & Sign Proc, 1996
 (factor $\gamma \rightarrow 1/\beta$ in the original paper)



final deflection

second link deflection