15. Invariance

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• Linear systems $\dot{x} = Ax$ $x_0 \in \mathbb{R}^n$

Consider a subspace $V \in \mathbb{R}^n$ and a linear mapping $A: \mathbb{R}^n \to \mathbb{R}^n$

V is invariant under A if $Av \in V$

(where, of course, v is a vector of V)

$x \in V$ belongs to the state-space

$Ax \in V$ does not belong to the state-space. It belongs to the space of the 1-st derivative: Tangent space

Trivial subspaces are $\{0\}$, V.

<u>Invariant subspaces $\subseteq$ eigenspaces</u>

• Non linear systems

$\dot{x} = f(x)$ $x(0) = x_0 \neq 0 \in \mathbb{R}^n$

$f(\cdot)$ vector field: law which associates a function f to each x

$x \to f(x) \in \underbrace{T_x \mathbb{R}^n}_{\text{tangent space of } \mathbb{R}^n \text{ (attached to the point } x)}$

$\longrightarrow$ **Manifold**

in general:

$x \in M$: M differential manifold $\to$ the most general surface structure on which I can define a differential equation

An n-dimensional Manifold is a surface in the linear space $\mathbb{R}^{n+1}$

example: Manifold of dimension 2 : $M \subset \mathbb{R}^3$

It's possible to define a differential equation over a Manifold by defining a <u>vector field</u> that represents the tangent space of M

$f(x) \in T_x M$: $T_x M$ tangent space of M

Given a manifold and a differential equation we can define the evolution on that manifold.

$f(x)$ is said to be integrable at x_0 if the integral curves doesn't intercept themselves (unique solution)

The integral of a vector field $f(x)$ corresponds to the foliation of $f(x)$

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