31. La Salle invariance principle

giovedì 11 giugno 2020 16:24

Invariant set:

$M \subset X$ is an invariant set for $\dot{x} = f(x)$ if $\forall$ trajectory $x(t)$ starting from $x_0 \in M$ remains in M.

The La Salle's invariance theorem states that in a compact region where $\dot{V}(x) \le 0$ the evolutions converge to an invariant set.

Theorem:

Let $V(x)$ be a C^1 function, and let Ω be a compact set where $\dot{V}(x) \le 0$.
Denoting E the set of points in Ω where $\dot{V}(x) = 0$ and M the largest invariant set in E, then every solution starting in Ω approaches M as $t \to \infty$.

Corollary

1. If $V(x) > 0$, $\Omega_c = \{x : V(x) \le c\}$ is bounded

2. $x_e = 0$ equilibrium point.
V Lyapunov function: $V > 0$ and C^1 in $S(x_e, r)$
Let $\Omega = \{x : \dot{V}(x) \le 0\}$ and suppose that no solution can stay identically in $E = \{x \in \Omega : \dot{V}(x) = 0\}$ other than the trivial solution $x(t) \equiv 0$.
Then the origin is AS
If $r = \infty$ and radially unbounded of V, then the origin is GAS