29. Krusowsky theorem

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Let $J_f(x)$ denote the Jacobian of $f(x)$.

- If $\left(J_f^T(x) + J_f(x)\right) < 0$ in $S(x_e, r)$ then x_e is AS
- If in addition the previous property holds in $S(x_e, \infty)$ and $V(x) = f^T(x) f(x)$ is radially unbounded then $x_e = 0$ is GAS

Proof: $V(x) = f^T(x) f(x)$ is clearly positive definite, moreover

$$\begin{aligned}\dot{V}(x) &= \dot{f}^T(x) f(x) + f^T(x) \dot{f}(x) \\ &= \dot{x}^T J_f^T(x) \dot{x} + \dot{x}^T J_f(x) \dot{x} \\ &= \dot{x}^T \left(J_f^T(x) + J_f(x) \right) \dot{x} < 0\end{aligned}$$