24. Stability analysis

mercoledì 10 giugno 2020 16:33

• Linear case

$\dot{x} = Ax$

states of equilibria are the ones which satisfy $Ax_e = 0$
They are all the states which are in the kernel of A and they are a subspace of the state-space.
$x_e = 0$ is always an equilibrium but it's not necessarily unique.

Isolated point: $\exists$ a neighborhood where there are no equilibria

In order to have attractiveness the equilibrium must be isolated.
Attractiveness implies global properties in linear systems.
The stability of a generic equilibrium point coincides with the stability of the origin (if the origin is AS it is also GAS and can be ES only if it is AS)

→ Conditions for stability of a linear system:
$x_e = 0$ is stable iff. $\|\Phi(t)\| < k \quad \forall t$

in fact: $\|x_0\| < \delta$

$\|x(t)\| = \|\Phi(t)\| \, \|x(t_0)\| \leq k \|x_0\| \leq k \delta_\varepsilon = \varepsilon$

being $\Phi(t) = e^{At}$

its norm is bounded if

$$Re(\lambda_i) = \begin{cases} \leq 0 & m_g = 1 \\ < 0 & m_g > 1 \end{cases}$$

→ condition for AS: $Re(\lambda_i) < 0$

• Stability of a motion (evolution)

$$M = \{\underbrace{(t, x_0(t))}_{\text{motion}} ; \; t \geq 0, \; \varphi(t, x_0, u_{0[0,t)}) = x_0(t)\}$$

M is stable wrt perturbation on x_0 if

$$\forall \varepsilon, \exists \delta_\varepsilon : \|x_0 - x_e\| < \delta_\varepsilon \Rightarrow \|x_p(t) - x_0(t)\| < \varepsilon \quad \forall t \geq 0$$

↳ perturbed motion