22. Asymptotic output tracking (AOTP)

venerdì 10 luglio 2020 18:50

Consider a system in normal form

$$\dot z_1 = z_2$$
$$\dot z_2 = z_3$$
$$\vdots$$
$$\dot z_{r-1} = z_r$$
$$\dot z_r = b(z,\eta) + a(z,\eta)u$$
$$\dot\eta = q(z,\eta)$$
$$y = z_1$$

The aim is to track $y_R(t) \leftarrow y(t)$

Choose

$$u(t) = \frac{1}{a(z,\eta)}\left(y_R^{(r)}(t) - b(z,\eta) - \sum_{i=1}^{r} \alpha_{i-1}^*\left(z_i - y_R^{(i-1)}(t)\right)\right)$$

where $\alpha_0^* \dots \alpha_{r-1}^*$ are real numbers and $\underline{\alpha} = (\alpha_0 \dots \alpha_{r-1})$ is Hurwitz.

$e(t) = y(t) - y_R(t)$ $\qquad z_i = y^{(i-1)}(t)$

with the new considerations, feedback in the (z,η) coordinates can be written as:

$$u(t) = \frac{1}{a(z,\eta)}\left(y_R^{(r)}(t) - b(z,\eta) - \sum_{i=1}^{r} \alpha_{i-1}^* e^{(i-1)}\right)$$

$$\dot z_r = y_R^{(r)} - \sum_{i=1}^{r} \alpha_{i-1}^* e^{(i-1)}$$
$$= y_R^{(r)} - \alpha_{r-1} e^{(r-1)} - \dots - \alpha_1 e^{(1)} - \alpha_0 e$$

$$0 = -y^{(r)} + y_R^{(r)} - \alpha_{r-1} e^{(r-1)} - \dots - \alpha_0 e =$$
$$= e^{(r)} + \alpha_{r-1} e^{(r-1)} + \dots + \alpha_1 e^{(1)} + \alpha_0 e$$

The error function satisfies a linear differential equation of order r whose coefficients can be arbitrarily chosen.

$e(t) \to 0$ i.e. $y(t) \to y_R(t)$

$$\dot{e} = Ae \qquad A = \begin{pmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ -\alpha_0^* & \cdots & & -\alpha_{r-1}^* \end{pmatrix}$$

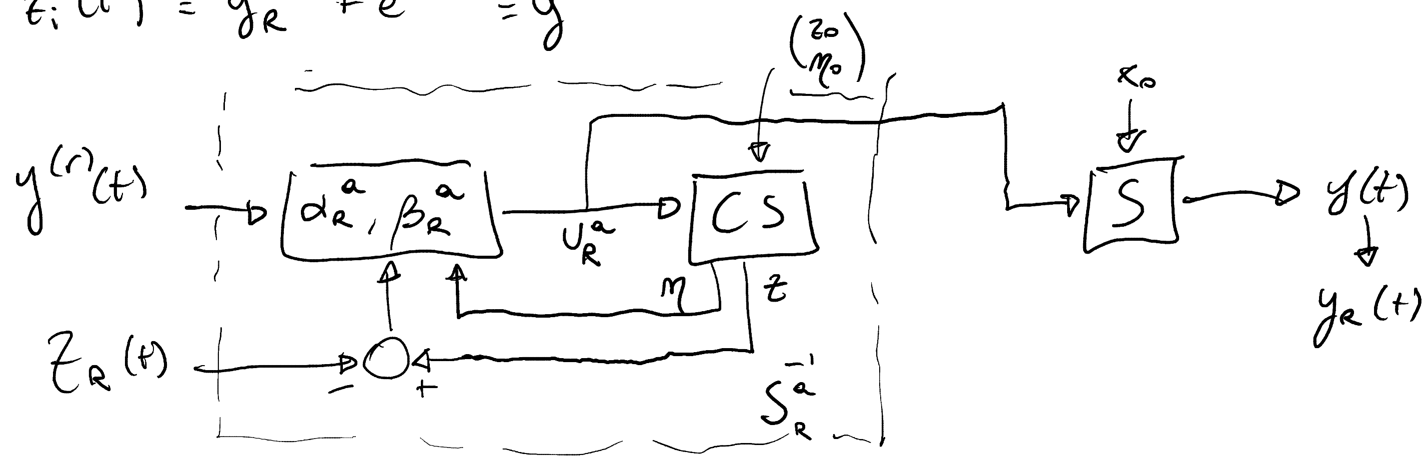
In the x coordinates the feedback is:

$$u(t) = \frac{1}{L_g L_f^{r-1} h(x)} \left(y_R^{(r)}(t) - L_f^r h(x) - \sum_{i=1}^{r} \alpha_{i-1}^* \left(L_f^{i-1} h(x) - y_R^{i-1}(t) \right) \right)$$

$$\underline{Z_R}(t) = \begin{pmatrix} y_R(t) \\ \vdots \\ y_R^{(r-1)}(t) \end{pmatrix} \qquad \underline{e}(t) = \begin{pmatrix} e(t) \\ \vdots \\ e^{(r-1)}(t) \end{pmatrix}$$

since for instance $e^{(r-1)}(t) = y^{(r-1)}(t) - y_R^{(r-1)}(t)$

$$z_i(t) = y_R^{i-1} + e^{i-1} = y^{i-1}$$



$$\begin{cases} \dot{\eta} = q(z, \eta) \\ u_R^a = \frac{1}{a(z,\eta)} \left(-b(z,\eta) + y_R^{(r)}(t) - \sum \alpha_{i-1}^* (z_i - y_R^{i-1}) \right) \end{cases}$$

$$\dot{z} = \begin{pmatrix} 0 & 1 & & \\ & & \ddots & 1 \\ -\alpha_0^* & \cdots & & -\alpha_{r-1}^* \end{pmatrix} z + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} y_R^{(r)}$$

The inverse dynamics in AORP is

$$\dot{\eta} = q(z, \eta)$$
$$\hookrightarrow \underline{Z_R} + \underline{e}$$

For what concerns the boundedness of z and y

For what concerns the boundedness of z and y

Prop: Let $y_R(t), \dots, y_R^{(r-1)}(t)$ be defined and bounded $\forall t$, $\eta_R(t)$, the solution of $\dot{\eta} = q(z_R, \eta)$, $\eta_R(0) = 0$, be defined $\forall t$, bounded and AS. Let $\underline{\alpha}$ be Hurwitz.
Then for $a > 0$, small enough, if
$|z_i(t) - y_R^{(i-1)}(t)| < a \quad i = 1, \dots, r \quad \| \eta(t_0) - \eta_R(t_0) \| < a$
the $z_i(t)$, $\eta_i(t)$ solutions of the system $\forall t \geq t_0$ are bounded.
More precisely:
$\forall \varepsilon > 0, \exists \delta : |z_i(t_0) - y_R^{i-1}(t_0)| < \delta$
$\Rightarrow |z_i(t) - y_R^{(i-1)}(t)| < \varepsilon \quad \forall t \geq t_0$
$\| \eta(t_0) - \eta_R(t_0) \| < \delta \Rightarrow \| \eta(t) - \eta_R(t) \| < \varepsilon \quad \forall t \geq t_0$