30. Variable gradient method

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Construction of a Lyapunov function:

Considering $\dot{x} = f(x)$

$$\dot{V}(x) = \frac{\partial V}{\partial x} \cdot f(x) = G(x) f(x)$$

where $G(x)$ denotes the GRADIENT of V.

The procedure reduces to compute the row vector $G(x)$ s.t. it is the gradient of a scalar positive definite function and in order to obtain $\dot{V}(x) \leq 0$ at least.

Condition:

$G(x)$ is the gradient of a function iff:

$$\frac{\partial g_i}{\partial x_j} = \frac{\partial g_j}{\partial x_i} \quad \forall i,j = 1 \dots n \quad \text{(i.e. its Jacobian is symmetric)}$$

Therefore one starts by choosing $G(x)$ s.t. $G(x) f(x)$ is ND

Then the function $V(x)$ is computed through the integration formula of an exact differential:

$$V(x) = \int_0^x G(y)\,dy = \int_0^{x_n} \sum_{i=1} g_i(y)\,dy$$

which can be reduced to integration along the axes:

$$V(x) = \int_0^{x_1} g_1(y, 0, \dots, 0)\,dy + \int_0^{x_2} g_2(x_1, y, 0 \dots, 0)\,dy + \dots$$

$$\dots + \int_0^{x_n} g_n(x_1, \dots, x_n, y)\,dy$$

The resulting Lyapunov function will be $V(x) > 0$ and $\dot{V}(x) < 0$, therefore the equilibrium is AS