17. Lie brackets

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Sort of measure of the commutability of Lie derivatives

f, g vector fields

$$[f, g](x) = \frac{\partial g}{\partial x} f(x) - \frac{\partial f}{\partial x} g(x)$$

$$L_{[f,g]} \lambda = L_f L_g \lambda - L_g L_f \lambda$$

In this case:

$$[\tau_1(x), \tau_2(x)] = \frac{\partial \tau_2}{\partial x} \tau_1(x) - \frac{\partial \tau_1}{\partial x} \tau_2(x)$$

$$L_{[\tau_1, \tau_2]} \lambda = \frac{\partial \lambda}{\partial x} \left(\frac{\partial \tau_2}{\partial x} \tau_1 - \frac{\partial \tau_1}{\partial x} \tau_2 \right)$$

Involutivity property

A distribution is involutive if

$\forall \tau_i, \tau_j \in \Delta \Rightarrow [\tau_i, \tau_j] \in \Delta$

and I can write it as $\alpha \tau_i + \beta \tau_j$

$$[\tau_1, \tau_2] = -[\tau_2, \tau_1]$$