26. Observer with linear error dynamics

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Consider a non linear system without input with linear output

$$\begin{cases} \dot{x} = f(x) \\ y = h(x) \end{cases}$$

Under coordinate transformation becomes

$$\begin{cases} \dot{z} = \dfrac{\partial \phi}{\partial x} f(x) \Big|_{x = \phi^{-1}(z)} = \tilde{f}(z) = Az + K(Cz) \\ y = \tilde{h}(x) = h(\phi^{-1}(z)) = Cz \end{cases}$$

where (A, C) observable pair and k vector field of the real variable function $y = Cz$

Then, we can define an observer of the form

$$\dot{\xi} = (A + kC)\xi - ky + k(y)$$

the observation error is $e = \xi - z$, and $\dot{e} = \dot{\xi} - \dot{z}$

$$\begin{aligned} \dot{e} &= (A + kC)\xi - ky + \cancel{k(y)} - Az - \cancel{k(y)} \\ &= (A + kC)\xi - kCz - Az = \\ &= (A + kC)\xi - (A - kC)z = (A + kC)e \end{aligned}$$

$$e(t) = e^{(A+kC)t} e_0 \qquad e_0 = \xi_0 - z_0$$

the error dynamics is linear and spectrally assignable.

It is possible to assign K such that:

$$\sigma(A + KC) \subset \mathbb{C}^-$$

$$\Rightarrow \xi(t) \longrightarrow z(t) \qquad \text{reconstruction convergence}$$

$$\hat{x}(t) = \phi^{-1}(\xi)$$

Observer linearization Problem (OLP)

Given the previous system (without input), and an initial state x^0, find, if possible, a neighborhood U_0 of x_0, a coordinate transformation $z = \Phi(x)$ on U_0 and an output a coordinate transformation $z = \Phi(x)$ on U_0 and an output injection $k(y)$ in such a way that the error dynamics be linear.

This problem is the dual of the feedback linearization problem.

Defined:

$$\left[\frac{\partial \phi}{\partial x} g(x) \right] \Bigg|_{x = \phi^{-1}(z)} = Az + k\, Cz$$

$$h\left(\Phi^{-1}(z)\right) = Cz \qquad \text{for all } z \in \Phi(U_0)$$

for some (A, C) satisfying:

$$\rho \begin{pmatrix} C \\ CA \\ \cdots \\ CA^{n-1} \end{pmatrix} = n$$

Necessary condition for OLP solvability:

OLP solvable iff:

$$\operatorname{rank} \begin{pmatrix} dh \\ \cdots \\ dL_g^{n-1} h \end{pmatrix} \Bigg|_{x_0} = n$$

which is equivalent to the well known condition for linear systems:

$$\operatorname{rank} \begin{pmatrix} C \\ CA \\ \cdots \\ CA^{n-1} \end{pmatrix} = n \qquad \text{i.e. full rank of the observability matrix.}$$

And in linear systems this condition is also sufficient.

Theorem:

the observer linearization problem is solvable if and only if:

(i) $\exists\, \tau(x)$: $\begin{pmatrix} dh \\ \cdots \\ dL_g^{n-1} h(x) \end{pmatrix} \tau(x) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

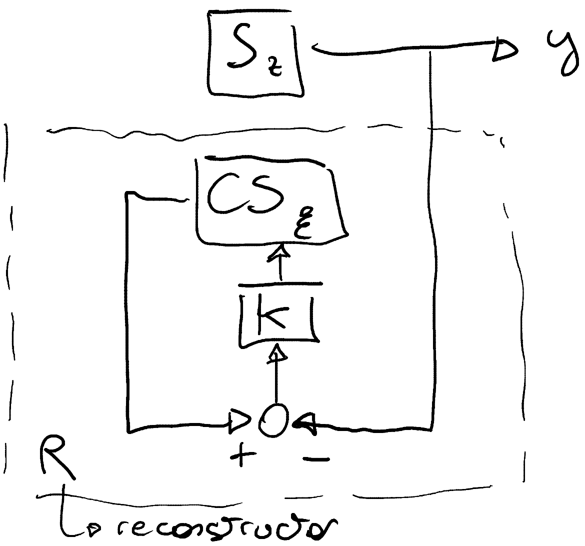
(ii) $\tau(x)$ is such that $[ad_f^i \tau, ad_f^j \tau] = 0 \quad 0 \le i,j \le n-1$

About the structure of the observer

$$\dot z = Az + k(y) \quad y = Cz \qquad (A,C) \text{ observable}$$

$$\dot\xi = A\xi + K(C\xi - y) + k(y)$$
$$= (A+KC)\xi - Ky - k(y)$$

$$\dot e = \dot\xi - \dot z = (A+KC)e$$



the condition (ii) assures the existence of the coordinates transformation $z = \Phi(x)$.

In fact $\begin{pmatrix} dh \\ \cdots \\ dL_f^{n-1}h \end{pmatrix} \tau(x) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$ assures the solvability of:

$$\left(\tau, -ad_f\tau, \dots, (-1)^{n-1} \cdot ad_f^{n-1}\tau\right)\Big|_{x=\phi^{-1}(z)} = \frac{\partial \phi^{-1}}{\partial z}$$

Full procedure

1. check $\rho\begin{pmatrix} dh \\ \cdots \\ dL_f^{n-1}h \end{pmatrix} = n$

$(dL_f^{n-1}h)$

2. compute $\tau(x) \ni \begin{pmatrix} dh \\ \cdots \\ dL_f^{n-1}h \end{pmatrix} \tau(x) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

3. compute $(\tau, \dots, (-1)^{n-1} ad_f^{n-1}\tau) = (t_1(x), \dots, t_n(x)) = \left(\frac{\partial \phi(x)}{\partial x}\right)^{-1}$
and verify that $[t_i, t_j] = 0 \ \forall i,j \ j \neq i$ (CNES)

4. $T(x) = \left(\frac{\partial \phi}{\partial x}\right)^{-1}$

5. solve in $\phi(x)$: $\frac{\partial \phi(x)}{\partial x} = T^{-1}(x)$

6. Compute the system in the $z = \phi(x)$ coordinates

7. $\dot{\xi} = (A + KC)\xi - Ky + \kappa(y)$ for a suitable K that assigns the dynamics of the reconstructor $(\xi(t) \to z(t))$

8. $\hat{x}(t) = \phi^{-1}(\xi(t))$