4. Representation with the state

giovedì 4 giugno 2020 22:37

x_0 needs to be limited at the different instant of time for letting the state to contain all the informations "on the past"

$$S = \{T, U \times Y, X, \Sigma_x\}$$

$(x_0, u_0) \to y_0$ at t_0

and $\forall t_1 \geq t_0$ $\quad (u_0, y_0)|_{T(t_1)} \in \Sigma(t_1)$

Then $(u_0, y_0)|_{T(t_1)}$ corresponds to a value of the parameters at t_1 in $\mathcal{X}_{t_1}$

If $\mathcal{X}_{t_0}, \mathcal{X}_{t_1} \subset X$, is assumed that the state x_1 such that

$$(x_1, u_0|_{T(t_1)}) \to y_0|_{T(t_1)}$$

is linked to x_0 by a function

$x_1 = x(t_1) = \varphi(t_1, t_0, x_0, u_0)$ with φ strictly causal, i.e. only $u_{[t_0, t_1)}$ is needed

X space of parameters

u input value $\quad \mathcal{U} \subset U^T$

$$(T \times T)^* = \{(t_1, t_0) : t \geq t_0,\ t, t_0 \in T\}$$

φ : transition function

$$(T \times T)^* \times X \times U \to X \text{ satisfies } P_1, P_2, P_3$$

$x(t) = \varphi(t, t_0, x_0, u)$:

P_1 Consistency : in order to have continuity for $\forall t \geq t_0$ we observe the system in $t = t_0$

$$\Rightarrow x(t) = \varphi(t_0, t_0, x(t_0), \underbrace{0}_{\text{null input}}) = x(t_0)$$

if $x(t) = x(t_0)$ we have consistency, in fact the system in t_0 remains in t_0

P_2 Causality : $\forall t \in T, \forall u \in \mathcal{U} \quad u_{[t_0, t)} = u'_{[t_0, t)}$

$$\Rightarrow \varphi(t, t_0, x_0, u) = \varphi(t, t_0, x_0, u')$$

P_3 **Separation:** $\forall (t, t_0), \forall x_0, \forall u$

$$t > t_1 > t_0 \longrightarrow \varphi(t, t_0, x_0, u_{[t_0, t)}) =$$
$$= \varphi(t, t_1, \varphi(t_1, t_0, x_0, u_{[t_0, t_1)}), u_{[t_1, t)})$$

Output:

$\forall t_0 \quad y_0(t) = \Pi_{t_0}(x_0, u_0)(t) \quad t \geq t_0$

and depends on u_0 over $[t_0, t]$ due to causality

η: transformation function

$\eta : T \times X \times U \longrightarrow Y$

$y(t) = \eta(t, x(t), u(t))$