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Nonlinear Systems & Control
Part II
9/01/17

Student: ____________

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1. Provide necessary and sufficient conditions for achieving input-output decoupling with stability for linear time-invariant systems.
2. Discuss and provide conditions for the problem of maximal feedback linearizability.
3. Given the system

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2^2 \\ \dot{x}_2 &= x_2 + \sin x_1 + u \\ \dot{x}_3 &= -x_3 - \sin x_1 + u \\ y &= x_2 + x_3 \end{aligned}$$

   a. Compute the feedback that solves the input-output linearization problem;
   b. Compute the zero dynamics;
   c. Discuss the stability of the closed-loop system.
4. The dynamic decoupling algorithm (the case $p = q = 2$).
5. √ Stabilize, if possibile, the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 + x_1^2 \\ \dot{x}_3 &= -x_2 + x_1^2 + (1 + x_1^2)u \\ y &= -x_1 + x_3 + x_1^2 \end{aligned}$$

   via high gain feedback.
6. The Sontag-Arstein Theorem.

(5) High gain feedback

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + x_1^2 \\ \dot{x}_3 = -x_2 + x_1^2 + (1+x_1^2)\,u \\ y = -x_1 + x_3 + x_1^2 \end{cases} \qquad g = \begin{pmatrix} 0 \\ 0 \\ 1+x_1^2 \end{pmatrix} \qquad \frac{\partial h(x)}{\partial x} = \begin{pmatrix} -1+2x_1 & 0 & 1 \end{pmatrix}$$

equilibrium at $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

Relative degree

$$L_g h(x) = \frac{\partial h(x)}{\partial x} \cdot g = \begin{pmatrix} -1+2x_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1+x_1^2 \end{pmatrix} = 1 + x_1^2 \neq 0 \qquad r = 1$$

$$\dot{y} = -\dot{x}_1 + \dot{x}_3 + \dot{x}_1^2 = -x_2 - x_2 + x_1^2 + (1+x_1^2)u + x_2^2$$

↳ u appears at the firs derivative of the output

$$\to \exists\, \Phi(x) = \begin{pmatrix} \phi_1(x) \\ \hline \phi_2(x) \end{pmatrix} \begin{matrix} r \\ n-r \end{matrix} \ni \text{ the system is in normal form}$$

$$\phi_1(x) = h(x) = z_1 = -x_1 + x_3 + x_1^2$$

$$\phi_2(x) \ni L_g \phi_2(x) \equiv 0 \quad \wedge \quad |J_\phi(x)| \neq 0$$

$$\phi_2(x) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \longrightarrow \begin{array}{l} x_1 = \eta_1 \\ x_2 = \eta_2 \\ x_3 = z_1 - \eta_1^2 + \eta_1 \end{array}$$

$$J_\phi(x) = \begin{pmatrix} 2x_1 - 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\bar{f}(z) = \left(J_\phi(x) \cdot f(x)\right)\Big|_{x=\phi^{-1}(z,\eta)} = \begin{pmatrix} (2x_1-1)x_2 - x_1 + x_3 + x_1^2 \\ x_2 \\ x_3 + x_1^2 \end{pmatrix}\Bigg|_{x=\phi^{-1}(z,\eta)}$$

$$= \begin{pmatrix} 2\eta_1\eta_2 - \eta_2 - \cancel{\eta_1} + z_1 + \cancel{\eta_1} - \cancel{\eta_1^2} + \cancel{\eta_1^2} \\ \eta_2 \\ z_1 + \eta_1 - \cancel{\eta_1^2} + \cancel{\eta_1^2} \end{pmatrix} = \begin{pmatrix} z_1 + 2\eta_1\eta_2 - \eta_2 \\ \eta_2 \\ z_1 + \eta_1 \end{pmatrix}$$

Normal form:

$$\begin{cases} \dot{z}_1 = z_1 + 2\eta_1\eta_2 - \eta_2 + \varepsilon_1 u \\ \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = z_1 + \eta_1 \\ y = z_1 \end{cases}$$

$$Q = \left(\frac{\partial q(0,\eta)}{\partial \eta}\right)\Bigg|_{(0,0)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$P(\lambda) = +\lambda^2 - 1 = 0 \longrightarrow \lambda_{1,2} = \pm 1$$ unstable zero dynamics

unstable zero dynamics plays the same role of NMP zeros for linear systems

→ the system cannot be stabilized by high gain feedback.

③ Compute the FB that solves the U-Y FL problem

$$\begin{cases} \dot{x}_1 = x_1 + x_2^2 \\ \dot{x}_2 = x_2 + \sin(x_1) + u \\ \dot{x}_3 = \ldots \end{cases} \qquad f = \begin{pmatrix} x_1 + x_2^2 \\ x_2 + \sin(x_1) \\ -x_3 - \sin(x_1) \end{pmatrix} \qquad g = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} \dot{x}_2 = x_2 + \sin(x_1) + u \\ \dot{x}_3 = -x_3 - \sin(x_1) + u \\ y = x_2 + x_3 \end{cases} \qquad f = \begin{pmatrix} x_2 + \sin(x_1) \\ -x_3 - \sin(x_1) \end{pmatrix} \qquad g = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$dh = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}$$

Compute the rel. degree

$$r = 1 \qquad L_g h = \frac{dh}{dx} \cdot g = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 2 \neq 0$$

Calculate the output derivative

$$\dot{y} = \dot{x}_2 + \dot{x}_3 = x_2 + \cancel{\sin(x_1)} + u - x_3 - \cancel{\sin(x_1)} + u$$

$$= x_2 - x_3 + 2u = 0$$

$$\Rightarrow \quad u = \frac{x_3 - x_2}{2} = \frac{L_f h}{L_g h}$$

Coord. transformation $\begin{pmatrix} z \\ \eta \end{pmatrix} \begin{matrix} r = 1 \\ n - r = 2 \end{matrix}$

$z = h = x_2 + x_3$

$\eta_1 =$ avoiding nonlinearities $= x_1$

$$\eta_2 \quad \text{s.t.} \quad \nabla \varphi_2 \, g = \begin{pmatrix} \frac{\partial \varphi_2}{\partial x_1} & \frac{\partial \varphi_2}{\partial x_2} & \frac{\partial \varphi_2}{\partial x_3} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0$$

$$\Rightarrow \frac{\partial \varphi_2}{\partial x_2} + \frac{\partial \varphi_2}{\partial x_3} = 0 \quad \Rightarrow \frac{\partial \varphi_2}{\partial x_2} = -\frac{\partial \varphi_2}{\partial x_3}$$

$$\varphi_2 = x_2 - x_3 \quad \Rightarrow \eta_2 = x_2 - x_3$$

$$+\begin{cases} z = x_2 + x_3 \\ \eta_2 = x_2 - x_3 \end{cases}$$

$$z + \eta_2 = 2x_2$$

$$x_2 = \frac{z + \eta_2}{2}$$

So, the normal form is:

$$\begin{cases} \dot{z} = \dot{x}_2 + \dot{x}_3 = x_2 - x_3 + 2u = \eta_2 + 2u \\ \dot{\eta}_1 = \dot{x}_1 = x_1 + x_2^2 = \eta_1 + \left(\frac{z + \eta_2}{2}\right)^2 \\ \dot{\eta}_2 = \dot{x}_2 - \dot{x}_3 = x_2 + \cancel{\sin(x_1)} \cancel{+u} + x_3 + \cancel{\sin(x_1)} \cancel{-u} = x_2 + x_3 + 2\sin(x_1) \\ \qquad = z + 2\sin(\eta_1) \end{cases}$$

Applying the Fb $u = \frac{-L_f h}{L_g h} + \frac{v}{L_g h} = \frac{-\eta_2 + v}{2}$ ~~*~~ mistake

I obtain

$$\begin{cases} \dot{z} = \cancel{\eta_2} - \cancel{2}\left(\frac{\cancel{\eta_2} + v}{\cancel{2}}\right) = v \\ \dot{\eta}_1 = \eta_1 + \left(\frac{z + \eta_2}{2}\right)^2 \end{cases}$$

$$\begin{cases} \dot{\eta}_1 = \eta_1 + \left(\frac{z+\eta_2}{2}\right)^2 \\ \dot{\eta}_2 = z + 2\sin\eta_1 \end{cases}$$

Compute the zero dynamics

$$Z = \{x \in \mathbb{R}^n : y(t_0) \equiv 0 \quad U^*(x) \text{ s.t. } y(t) \equiv 0\} =$$

$$= \{x \in \mathbb{R}^n \text{ s.t. } \quad x_2 + x_3 = 0\} \qquad \text{if } z=0 \Rightarrow y=0$$

at $z = 0$, $v = 0$

$\Rightarrow \dot{\eta} = q(0, \eta)$

$$\begin{cases} \dot{\eta}_1 = \eta_1 + \frac{\eta_2^2}{4} \\ \dot{\eta}_2 = 2\sin(\eta_1) \end{cases}$$

Discuss the stability of the closed loop system

LTM for $q(0,\eta) \rightarrow Q = \left.\frac{\partial q(0,\eta)}{\partial \eta}\right|_{\eta=0} = \left.\begin{pmatrix} 1 & \frac{1}{2}\eta_2 \\ 2\cos(\eta_1) & 0 \end{pmatrix}\right|_{\eta=0} = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$

$$\begin{vmatrix} 1-\lambda & 0 \\ 2 & -\lambda \end{vmatrix} = -\lambda(1-\lambda) = 0$$

$$\lambda_1 = 1, \quad \lambda_2 = 0$$

$\hookrightarrow$ positive : zero dynamics is unstable