29. State space exact linearization

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Input-to-state (u-x) feedback linearization

Given

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 = f(x) + G(x)u$$

$$\begin{matrix} y_1 = h_1(x) \\ y_2 = h_2(x) \end{matrix} \Rightarrow y = H(x) \qquad \rho(dH) = 2 \qquad x(0) = x_0$$

We are looking for

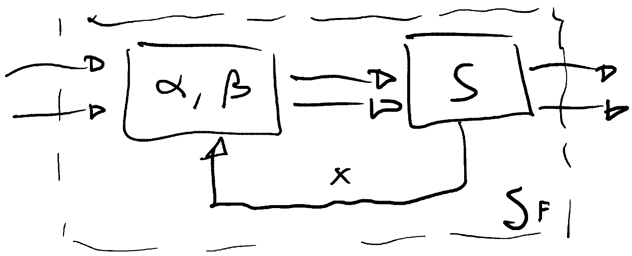
$$\alpha(x) = \begin{pmatrix} \alpha_1(x) \\ \alpha_2(x) \end{pmatrix} \qquad \beta(x) = \begin{pmatrix} \beta_{11}(x) & \beta_{12}(x) \\ \beta_{21}(x) & \beta_{22}(x) \end{pmatrix} \qquad |\beta(x)| \neq 0$$

and $\Phi(x)$ coordinate transformation such that

$$\frac{\partial \Phi(x)}{\partial x}(f + g\alpha)\Big|_{x=\phi^{-1}(z)} = \bar{f}(z) = Az$$

$$\frac{\partial \Phi(x)}{\partial x} G(x)\beta(x)\Big|_{x=\phi^{-1}(z)} = \underset{n\times m}{\bar{G}(z)} = B$$

with $\rho = (B \,\vdots\, AB \,\vdots\, \dots \,\vdots\, A^{n-1}B) = n$



S_F linear and reachable

Prop: Sufficient condition for u-x feedback linearization is that there exist $(\varphi_1, \dots, \varphi_m)$ function $\varphi_i : \mathbb{R}^n \to \mathbb{R}$ such that the system has strong relative degree with respect to them, i.e.:

$$\det \begin{pmatrix} L_{g_1} L_f^{r_{\varphi_1}-1} h_1 & L_{g_2} L_f^{r_{\varphi_1}-1} h_1 \\ L_{g_1} L_f^{r_{\varphi_2}-1} h_2 & L_g L_f^{r_{\varphi_2}-1} h_2 \end{pmatrix} = \det(A_\varphi(x)) \neq 0$$

$$\alpha(x) = -A_\varphi^{-1}(x) \qquad \beta(x) = A_\varphi^{-1}(x)$$

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$$b(x) = \begin{pmatrix} L_f^{r_{\varphi_1}} h_1 \\ L_f^{r_{\varphi_2}} h_2 \end{pmatrix}$$

$$u(x) = A_\varphi^{-1}(x)\left(-b(x) + v\right)$$

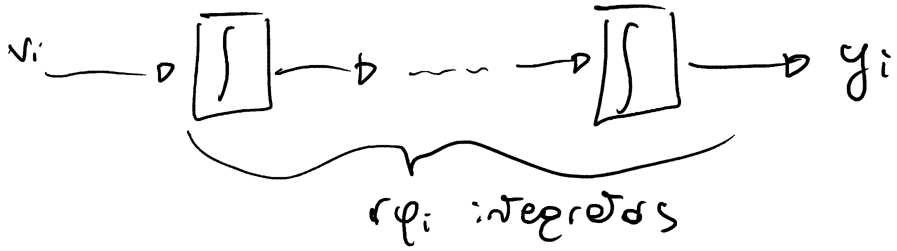
In the z coordinates

$$\begin{array}{c} r_{\varphi_1} \\ r_{\varphi_2} \end{array} \begin{pmatrix} z_1 \\ \hline z_2 \end{pmatrix} = \begin{pmatrix} h_1 \\ \vdots \\ L_f^{r_{\varphi_1}-1} h_1 \\ \hline h_2 \\ \vdots \\ L_f^{r_{\varphi_2}-1} h_2 \end{pmatrix}$$

$$\Downarrow$$

$$\begin{cases} \dot z_1 = A_0^1 z_1 + B_0^1 v_1 \\ \dot z_2 = A_0^2 z_2 + B_0^2 v_2 \end{cases} \qquad A_0^i = \begin{pmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ & & & 1 \\ 0 & & & 0 \end{pmatrix} \qquad B_0^i = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

i.e. for each i-th there is a chain of r_{φ_i} integrators



$$A_0 = \begin{pmatrix} A_0^1 & 0 \\ 0 & A_0^2 \end{pmatrix} \qquad B = \begin{pmatrix} B_0^1 & 0 \\ 0 & B_0^2 \end{pmatrix}$$

$$\rho\left(B \;\; AB \;\; \dots \;\; A^{n-1}B\right) = r_{\varphi_1} + r_{\varphi_2} = n$$

Note that different choices of φ_i can solve the problem.

Geometric condition for solvability

$G_0 = \text{span}\{g_1, g_2\}$

$G_1 = \text{span}\{g_1, g_2, ad_f g_1, ad_f g_2\}$

$G_2 = \text{span}\{g_1, g_2, ad_f g_1, ad_f g_2, ad_f^2 g_1, ad_f^2 g_2\}$

$G_1 = \text{span}\{g_1, g_2, ad_f g_1, ad_f g_2\}$

$G_2 = \text{span}\{g_1, g_2, ad_f g_1, ad_f g_2, ad_f^2 g_1, ad_f^2 g_2\}$

$G_j = \text{span}\{g_1, g_2, \dots, ad_f^k g_i \quad 0 < k < j \; , \; i = 1, 2\}$

Theorem: Assume $\text{rank}(G) = m$ the u-x feedback linearization problem is solvable if and only if:

(i) $i = 0, \dots, n-1$ the G_i's have constant dimensions

(ii) G_{n-1} has dimension n;

(iii) $i = 0, \dots, n-2$ $\quad \bar{G}_i = G_i$ (involutiveness)