25. Direct Lyapunov theorem

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Let $x_e = 0$ be an equilibrium for $\dot{x} = f(x)$ and $x_e \in D \subset \mathbb{R}^n$.

Let $V: D \to \mathbb{R}$ C^1 function defined in D, if:

(i) $V(x_e) = V(0) = 0$

(ii) $V(x) > 0 \quad \forall x \neq 0 \in D$

(iii) $\dot{V}(x) = \left[\frac{\partial V}{\partial x_1}, \dots, \frac{\partial V}{\partial x_n}\right] \begin{bmatrix} f_1(x) \\ \vdots \\ f_n(x) \end{bmatrix} = \frac{\partial V}{\partial x} f(x) = L_f V \leq 0 \quad \forall x \in D$

then x_e is stable.

iii bis) if $\underset{\substack{\uparrow \\ \text{strict}}}{\dot{V}(x) < 0}$ $\forall x \neq 0 \in D$ $\quad x_e = 0$ is AS

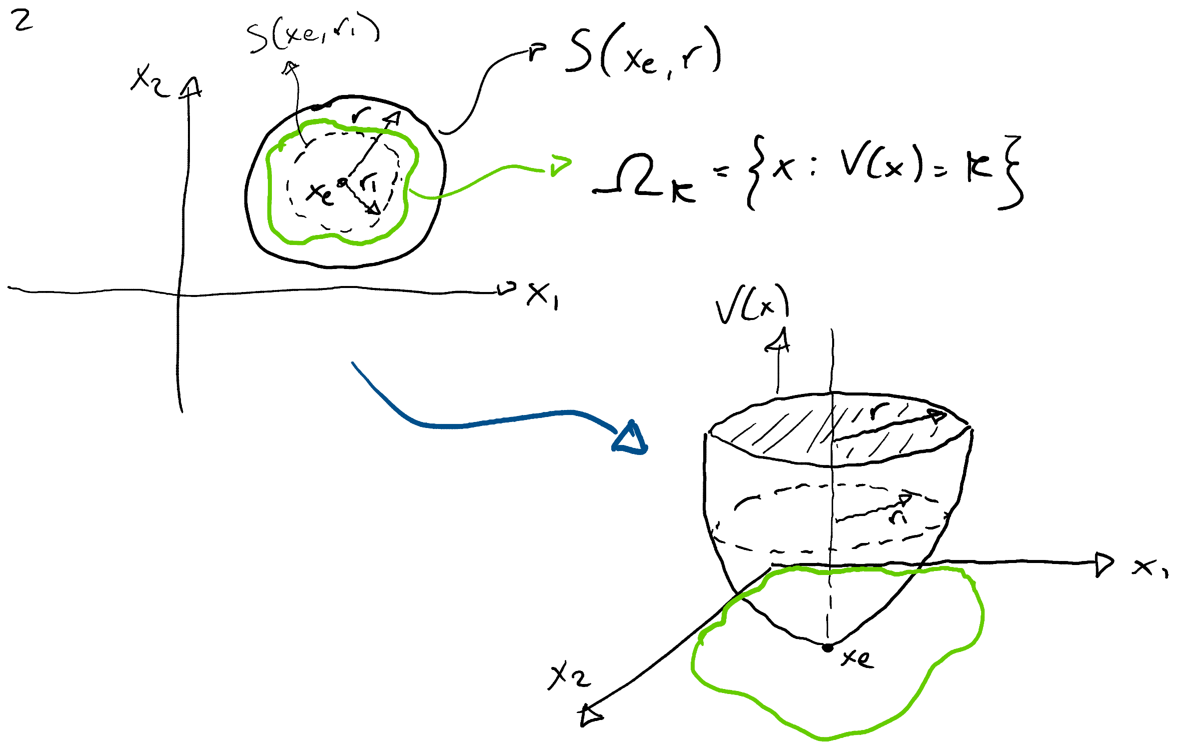
and $V(x)$ is said Lyapunov function (LF)

CNES: x_e is stable if $\exists$ a LF

x_e is AS if $\exists$ an ALF

Proof:

$n = 2$



Because of $L_f V \leq 0$ always, I know that the evolution is confined in Ω_k.

Therefore I can find an $r > 0$ s.t. the ball $\Omega_K \subset S(x_e, r)$
an $r_1 > 0$ s.t. the ball $S(x_e, r_1) \subset \Omega_K$.
By using ε in place of r and δ_ε in place of r_1 the stability definition holds:
the evolution remains confined in Ω_K $\forall t$.
Moreover, if $L_f V < 0$ (strictly decreasing) the evolution goes through x_e and the AS definition holds, and $S(x_e, r_1) = S(x_e, \delta_\varepsilon)$ gives an estimate of the region of attraction

• Lyapunov function

The basic positive-definite function is the quadratic function:

can be $I_{n \times n}$

$$V(x) = (x - x_e)^T Q \, (x - x_e)$$

with Q symmetric satisfying the Sylvester conditions

$$Q = \begin{array}{c} \begin{array}{ccc} M_1 & M_2 & M_3 \end{array} \\ \begin{pmatrix} q_{11} & q_{12} & q_{13} & \cdots \\ q_{21} & q_{22} & q_{23} & \cdots \\ q_{31} & q_{32} & q_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \end{array}$$

$|M_i| > 0$
$i = 1, \ldots, n$

M_i: principal diagonal submatrices

$$x^T M x = x^T \left(\frac{M + M^T}{2} \right) x$$