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Nonlinear Systems & Control
Part II
25/07/17

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1. Consider a system with transfer function $W(s) = \frac{1}{s^3+s^2+s+1}$ and affected by a constant disturbance $w \in \mathbb{R}$. Characterize, if any, all the action of disturbances that can be decoupled under feedback and compute the decoupling feedback

2. Consider the system

$$\begin{aligned}
\dot{x}_1 &= x_2 + p(x)w \\
\dot{x}_2 &= -\frac{2x_2x_3x_4}{x_3^2+1} + \frac{u_1}{x_3^2+1} \\
\dot{x}_3 &= x_3 + x_4^2 \\
\dot{x}_4 &= x_3x_2^2 + u_2 + u_1 \\
y_1 &= x_2 \\
y_2 &= x_4
\end{aligned}$$

where $w(t) \in \mathbb{R}$ is a bounded disturbance and $p(x)$ is a smooth vectorfield.

A. Assuming that no disturbance is affecting the system (i.e., $p(x) \equiv 0$)

1. verify whether the vector relative degree is defined;
2. compute the feedback $u(x)$ solving the input/output linearization problem;
3. Compute the zero-dynamics;
4. Discuss the stability of the closed-loop system.

B. When $p(x)$ is generally non zero

1. does the feedback $u(x)$ computed in **A.2** solve the disturbance-decoupling problem?
2. what about its applicability?

3. Compute a feedback yielding global asymptotic stability of the equilibrium of the system

$$\begin{aligned}
\dot{x}_1 &= -e^{2x_2}x_1 \\
\dot{x}_2 &= u
\end{aligned}$$

4. Consider a system $x = f(x) + g(x)u$ possessing an equilibrium at the origin and assume that the pair (A, B) with $A = \frac{\partial f}{\partial x}(0)$ and $B = g(0)$ is not controllable. Is it possible to locally asymptotically stabilize the system through a linear state feedback?

5. Consider the system

$$\begin{aligned} \dot{x}_1 &= -x_1^3 + x_2^2 \\ \dot{x}_2 &= x_2^2 u - x_1 x_2 \\ y &= x_2^3 \end{aligned}$$

A. Is it passive? Is it lossless? *Hint*: use a quadratic storage function.

B. Is the output feedback $u = -y$ achieving global asymptotic stability of the equilibrium?

1. Characterize, if any, all the action of disturbance that can be decoupled under Fb and compute the decoupling fb.

$$W(s) = \frac{1}{s^3 + s^2 + s + 1} \longrightarrow m = 0 \quad n = 3$$

$$W(s) = \frac{b_0 + b_1 s + \dots + b_m s^m}{a_0 + a_1 s + \dots + s^n}$$

companion forms

$$A = \begin{pmatrix} 0 & 1 & & \\ & & \ddots & 1 \\ -a_0 & \dots & & -a_{n-1} \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

$$C = (b_0 \dots b_m \; 0 - 0)$$

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad C = (1 \; 0 \; 0)$$

relative degree

$$r = 1 \qquad CB = (1 \; 0 \; 0) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$r=2 \quad CAB = (1\ 0\ 0)\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (1\ 0\ 0)\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 0$

$r=3 \quad CA^2B = CA \cdot AB = (0\ 1\ 0)\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 1 \neq 0$

$r = 3 = n$

Compute $V^* = \ker \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \ker \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

so $V^* = \text{span}\left\{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\right\}$

the DDP is not solvable

$\Rightarrow \forall D$ s.t $\text{Im} D \in \text{span}\left\{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\right\}$ the DDP is solvable

but the only D is $D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, so DDP is not solvable

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(2)

$$\begin{cases} \dot{x}_1 = x_2 + p(x)w \\ \dot{x}_2 = -\dfrac{2x_2x_3x_4}{x_3^2+1} + \dfrac{u_1}{x_3^2+1} \\ \dot{x}_3 = x_3 + x_4^2 \\ \dot{x}_4 = x_3 x_2^2 + u_2 + u_1 \\ y_1 = x_2 \\ y_2 = x_4 \end{cases}$$

$g_1 = \begin{pmatrix} 0 \\ \frac{1}{x_3^2+1} \\ 0 \\ 1 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$dh_1 = (0\ 1\ 0\ 0)$

$dh_2 = (0\ 0\ 0\ 1)$

2.1 Assuming no disturbance ($p(x)=0$)

- Compute the vector relative degree

$r_1 = 1 \quad L_{g_1} h_1 = (0\ 1\ 0\ 0)\begin{pmatrix} 0 \\ \frac{1}{x_3^2+1} \\ 0 \\ 1 \end{pmatrix} = \frac{1}{x_3^2+1} \neq 0$

$$L_{g_2} h_1 = (0\ 1\ 0\ 0)\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = 0$$

$$\dot{y}_1 = \dot{x}_2 = \frac{-2x_2x_3x_4}{x_3^2+1} + \frac{u_1}{x_3^2+1}$$

- $r_2 = 1$ $\quad L_{g_1} h_2 = (0\ 0\ 0\ 1)\begin{pmatrix}0\\ \frac{1}{x_3^2+1}\\0\\1\end{pmatrix} = 1 \neq 0$

$$L_{g_2} h_2 = (0\ 0\ 0\ 1)\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = 1 \neq 0$$

$r_2 = 1$

- Compute the feedback which solves the I/O linearization

$$A(x) = \begin{pmatrix} L_{g_1}h_1 & L_{g_2}h_1 \\ L_{g_1}h_2 & L_{g_2}h_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{x_3^2+1} & 0 \\ 1 & 1 \end{pmatrix}$$

$$|A(x)| = \frac{1}{x_3^2+1} \neq 0 \rightarrow A(x) \text{ is invertible}$$

Therefore I can apply the control

$u = A^{-1}(x)(v - M(x))$, setting $M(x) = \begin{pmatrix} L_f h_1 \\ L_f h_2 \end{pmatrix}$

$$L_f h_1 = (0\ 1\ 0\ 0)\begin{pmatrix} * \\ -\frac{2x_2x_3x_4}{x_3^2+1} \\ * \\ * \end{pmatrix} = -\frac{2x_2x_3x_4}{x_3^2+1}$$

$$L_f h_2 = (0\ 0\ 0\ 1)\begin{pmatrix} * \\ * \\ * \\ x_3x_2^2 \end{pmatrix} = x_3x_2^2$$

So $M(x) = \begin{pmatrix} -\frac{2x_2x_3x_4}{x_3^2+1} \\ x_3x_2^2 \end{pmatrix}$

So $\mu(x) = \begin{pmatrix} -\dfrac{2x_2x_3x_4}{x_3^2+1} \\ x_3x_2^2 \end{pmatrix}$

Coord. transf. :

$$\left.\begin{pmatrix} z \\ \hline \xi \\ \hline \eta_1 \\ \hline \eta_2 \end{pmatrix}\right\} n-r_1-r_2 = \begin{pmatrix} h_1 \\ \vdots \\ L_f^{r_1-1} h \\ \hline h_2 \\ \vdots \\ L_f^{r_2-1} h \\ \hline \varphi_3 \\ \hline \varphi_4 \end{pmatrix} = \begin{pmatrix} h_1 \\ \hline h_2 \\ \hline \varphi_3 \\ \hline \varphi_4 \end{pmatrix} = \begin{pmatrix} x_2 \\ \hline x_4 \\ \hline \varphi_3 \\ \hline \varphi_4 \end{pmatrix}$$

find $\eta_{1,2}$ s.t. $\nabla\varphi_3 \cdot g_1 = 0$ & $\nabla\varphi_4 \cdot g_2 = 0$

$$\begin{pmatrix} \frac{\partial\varphi_3}{\partial x_1} & \frac{\partial\varphi_3}{\partial x_2} & \frac{\partial\varphi_3}{\partial x_3} & \frac{\partial\varphi_4}{\partial x_4} \end{pmatrix} \begin{pmatrix} 0 \\ \frac{1}{x_3^2+1} \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\frac{\partial\varphi_3}{\partial x_2} \cdot \frac{1}{x_3^2+1} + \frac{\partial\varphi_4}{\partial x_4} = 0 \qquad \eta_1 = x_1$$

$$\begin{pmatrix} \frac{\partial\varphi_4}{\partial x_1} & \frac{\partial\varphi_4}{\partial x_2} & \frac{\partial\varphi_4}{\partial x_3} & \frac{\partial\varphi_4}{\partial x_4} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\frac{\partial\varphi_4}{\partial x_4} = 0 \qquad \eta_2 = x_3$$

So $x_1 = \eta_1$, $x_2 = z$, $x_3 = \eta_2$, $x_4 = \xi$

$$\begin{cases} \dot z = \dot x_2 = -\dfrac{2x_2x_3x_4}{x_3^2+1} + \dfrac{u_1}{x_3^2+1} = -\dfrac{2z\eta_2\xi}{\eta_2^2+1} + \dfrac{u_1}{\eta_2^2+1} \\ \dot\xi = \dot x_4 = x_3x_2^2 + u_2 + u_1 = \eta_2 z^2 + u_2 + u_1 \\ \dot\eta_1 = \dot x_1 = x_2 = z \\ \dot\eta_2 = \dot x_3 = x_3 + x_4^2 = \eta_2 + \xi^2 \end{cases}$$

by choosing

$$u = \begin{pmatrix} [v_1 + 2z\eta_2\xi](\eta_2^2+1) \\ v_2 - \eta_2 z_2^2 - (v_1 + 2z\eta_2\xi)(\eta_2^2+1) \end{pmatrix}$$

one obtains:

$$\begin{cases} \dot{z} = v_1 \\ \dot{\xi} = v_2 \\ \dot{\eta}_1 = z \\ \dot{\eta}_2 = \eta_2 + \xi^2 \end{cases}$$

· Zero dynamics

$$Z = \{ x \in \mathbb{R}^n \text{ s.t. } x_2 = 0 \ \& \ x_4 = 0 \} \Rightarrow z = 0, \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} \dot{\eta}_1 = 0 \\ \dot{\eta}_2 = \eta_2 \end{cases} \quad \text{is the zero dynamics}$$

$$Q = \left.\frac{\partial q(0,\eta)}{\partial \eta}\right|_0 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} = \lambda(1-\lambda) = 0$$

$\lambda_1 = 0 \quad \lambda_2 = 1$ I can't conclude anything

2.2 Assume now $p(x) \neq 0$

· Does the feedback found before solve the DDP?

if $p(x)$ s.t. $\Delta^* = \ker\left(d \begin{pmatrix} h_1 \\ L_g^{r_1-1} h_1 \\ h_2 \\ L_g^{r_2-1} h_2 \end{pmatrix} \right)$, then $u(x)$ solves DDP

$$\Delta^* = \ker \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \ker \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightsquigarrow \begin{matrix} x_2 = 0 \\ x_4 = 0 \end{matrix} \rightsquigarrow \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\Delta^* = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$

· Is it applicable?

It's applicable since $p(x) = \begin{pmatrix} p(x) \\ 0 \\ 0 \\ 0 \end{pmatrix} \subset \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\} \subset \Delta^*$

It's applicable since $p(x) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \subset \text{span}\left\{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\right\} \subset \Delta^*$

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(3) Compute a feedback yelding GAS

$$\begin{cases} \dot{x}_1 = -e^{2x_2} x_1 \\ \dot{x}_2 = u \end{cases}$$

(1) $x_2 = \gamma(x_1)$ s.t. $\dot{V}(x_1) < 0$

$V(x_1) = \frac{1}{2} x_1^2 \Rightarrow \dot{V}(x_1) = \dot{x}_1 x_1 = -x_1^2 e^{2x_2}$

Setting $\gamma(x_1) = 0 \Rightarrow \dot{V}(x_1) = -x_1^2 < 0$ GAS $\forall x_1 \neq 0$

(2) $e = x_2 - \gamma(x_1) \Rightarrow e = x_2$

therefore no change of coordinates is necessary

$$\begin{cases} \dot{x}_1 = -e^{2x_2} x_1 \\ \dot{x}_2 = u \end{cases}$$

$V_2(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2)$

$\dot{V}_2 = \dot{x}_1 x_1 + \dot{x}_2 x_2 = -e^{2x_2} x_1^2 + x_2 u$

setting $u = -x_2$

$$\dot{V}_2 = \underbrace{-\underbrace{e^{2x_2}}_{\text{positive quantity}} x_1^2}_{<0} \underbrace{- x_2^2}_{<0} < 0 \Rightarrow \text{GAS}$$

(5.B) Given the input verify if it achieves GAS

$$\begin{cases} \dot{x}_1 = -x_1^3 + x_2^2 \\ \dot{x}_2 = x_2^2 u - x_1 x_2 \\ y = x_2^3 \end{cases} \qquad u = -y$$

With the input the system becomes:

With the input the system becomes:

$$\begin{cases} \dot{x}_1 = -x_1^3 + x_2^2 \\ \dot{x}_2 = -x_2^5 - x_1 x_2 \\ y = x_2^3 \end{cases}$$

$$V = \frac{1}{2}\left(x_1^2 + x_2^2\right)$$

$$\dot{V} = \dot{x}_1 x_1 + \dot{x}_2 x_2 = \overset{<0}{-x_1^4} + \cancel{x_2^2 x_1} \overset{<0}{- x_2^6} - \cancel{x_1 x_2^2}$$

$u = -y$ achieves GAS