11. Properties of kernels

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- **Stationarity**

$$\forall \bar{t} : W_n(t, \tau_1, \dots, \tau_n ; x_0) = W_n(t+\bar{t}, \tau_1+\bar{t}, \dots, \tau_n+\bar{t} ; x_0)$$

- **Factorization**

$$W_1(t, \tau_1, t_0 ; x_0) = W_{11}(t-\tau_1)$$

$$W_2(t, \tau_1, \tau_2, t_0 ; x_0) = W_{21}(t-\tau_1)\, W_{22}(\tau_1-\tau_2)$$

$$\vdots$$

$$W_n(t, \tau_1, \dots, \tau_n, t_0 ; x_0) = W_{n1}(t-\tau_1) \dots W_{nn}(\tau_{n-1}-\tau_n)$$

- **Separability**

Stationarity and factorization implies separability of linear representation:

$\exists$ matrices of functions $Q(\cdot)$ and $P(\cdot)$ such that

$$W_1(t-\tau_1) = Q(t)\, P(\tau_1) \qquad t \geq \tau_1$$

$$= C e^{(t-\tau_1)} \cdot B$$

$$\vdots$$

$$W_{ij}(\tau_{j-1}-\tau_j) = Q_{ij}(\tau_{j-1})\, P_{ij}(\tau_j) = C_{ij}\, e^{(\tau_{j-1}-\tau_j) A_{ij}}\, B_{ij}$$