19_06.pdf

sabato 18 luglio 2020 13:08

19_06.pdf

Nonlinear Systems & Control
Part II
06/06/19

Student: ____________________

Student ID: ____________________

---

1. Verify whether the system

$$\dot{x} = \begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & -2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & -3 \end{pmatrix} x + \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{pmatrix}$$

is asymptotically stabilizable by output feedback.

√ 2. Given the system

$$\begin{aligned} \dot{x}_1 &= -x_1^5 + x_2 \\ \dot{x}_2 &= -x_1 - x_2 + x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= x_2^2(x_1 - x_3) + u \\ y &= x_3 \end{aligned}$$

(a) compute, if any, a feedback making the origin of the system locally asymptotically stable based on input-output feedback linearization;

(b) compute the zero-dynamics.

(c) compute, if any, a feedback making the origin of the system globally asymptotically stable based on backstepping.

(d) What do the two closed-loop systems in points (a) and (c) share?

3. Maximal input-state feedback linearizability.

4. The problem of the nonlinear observer with linear error dynamics.

5. The discretization problem: from linear to nonlinear.

(2) compute if any, a feedback, making the origin LAS, based on u-y FL

$$\begin{cases} \dot{x}_1 = -x_1^5 + x_2 \\ \dot{x}_2 = -x_1 - x_2 + x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = x_2^2(x_1 - x_3) + u \\ y = x_3 \end{cases} \qquad f = \begin{pmatrix} -x_1^5 + x_2 \\ -x_1 - x_2 + x_3 \\ x_4 \\ x_2^2(x_1 - x_3) \end{pmatrix}$$

$$g = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \ldots \end{pmatrix} \qquad h = x_3$$

$$|\ y = x_3 \qquad g = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad h = x_3$$

$$\frac{\partial h}{\partial x} = (0\ 0\ 1\ 0)$$

$$y = x_3$$
$$\dot{y} = \dot{x}_3 = x_4$$
$$\ddot{y} = \dot{x}_4 = x_2^2(x_1 - x_3) + u \longrightarrow r = 2$$

Change of coordinates

$$z_1 = h = x_3$$
$$z_2 = L_f h = (0\ 0\ 1\ 0)\cdot f = x_4$$

$$\begin{matrix} r \\ n-r \end{matrix}\begin{pmatrix} z \\ \hline \eta \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ \hline \eta_1 \\ \eta_2 \end{pmatrix}$$

$$\eta_1, \eta_2 \quad \text{s.t.} \quad \nabla\varphi \cdot g = 0$$

$$\left(\frac{\partial\varphi}{\partial x_1}\ \frac{\partial\varphi}{\partial x_2}\ \frac{\partial\varphi}{\partial x_3}\ \frac{\partial\varphi}{\partial x_4}\right)\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$\frac{\partial\varphi}{\partial x_4} = 0 \Rightarrow \eta_1 = x_1, \quad \eta_2 = x_2$$

Normal form:

$$\begin{cases} \dot{z}_1 = \dot{x}_3 = x_4 = z_2 \\ \dot{z}_2 = \dot{x}_4 = x_2^2(x_1 - x_3) + u = \eta_2^2(\eta_1 - z_1) + u \\ \dot{\eta}_1 = \dot{x}_1 = -x_1^5 + x_2 = -\eta_1^5 + \eta_2 \\ \dot{\eta}_2 = \dot{x}_2 = -x_1 - x_2 + x_3 = -\eta_1 - \eta_2 + z_1 \end{cases}$$

$$u = \frac{-L_f^2 h}{L_g L_f h} + \frac{v}{L_g L_f h} = -\eta_2^2(\eta_1 - z_1) + v$$

$$\Rightarrow \dot{z}_2 = v$$

Zero dynamics

$Z = \{x \in \mathbb{R}^n \text{ s.t. } y(t) \equiv 0\} = \{x \in \mathbb{R}^n \text{ s.t. } x_3 = 0\}$

$z = 0, v = 0$

$$\begin{cases} \dot{\eta}_1 = -\eta_1^5 + \eta_2 \\ \dot{\eta}_2 = -\eta_1 - \eta_2 \end{cases}$$

$$Q = \frac{\partial q(0,\eta)}{\partial \eta}\bigg|_{\eta=0} = \begin{pmatrix} -5\eta_1^4 & 1 \\ -1 & -1 \end{pmatrix}\bigg|_0 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\begin{vmatrix} -\lambda & 1 \\ -1 & -1-\lambda \end{vmatrix} = -\lambda(-1-\lambda) + 1 = 0$$

$$\lambda + \lambda^2 + 1 = 0$$

$$\lambda_{12} = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}i}{2}$$

$Re[\lambda_i] < 0$

So the dynamics is LAS and the closed loop is stable.

Compute if any a fb making origin GAS by backstepping

$$\begin{cases} \dot{x}_1 = -x_1^5 + x_2 \\ \dot{x}_2 = -x_1 - x_2 + x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = x_2^2(x_1 - x_3) + v \\ y = x_3 \end{cases}$$

① $\dot{x}_1 = -x_1^5 + x_2$

choosing $x_2 = \gamma(x_1)$ such that $\dot{V}_1(x_1) < 0$

$V_1(x_1) = \frac{1}{2} x_1^2 \Rightarrow \dot{V}_1(x_1) = \dot{x}_1 x_1 = -x_1^6 + x_2 x_1$

by choosing $\gamma(x_1) = 0$

$\dot{V}_1(x_1) = -x_1^6 < 0 \Rightarrow$ GAS $\forall x_1 \neq 0$

② $e_1 = x_2 - \gamma(x_1) = x_2$

no charge of coordinates in this step

$\dot{x}_1 = -x_1^5 + x_2$

$\dot{x}_2 = x_3$

choosing $x_3 = \gamma(x_1, x_2)$ s.t. $\dot{V}_2(x_1, x_2) < 0$

$V_2(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2)$

$\dot{V}_2(x_1, x_2) = \dot{x}_1 x_1 + \dot{x}_2 x_2 = -x_1^6 + x_2 x_1 + x_2 x_3$

by choosing $\gamma(x_1, x_2) = -x_1 - x_2$

$\dot{V}_2 = \underbrace{-x_1^6}_{<0} + \cancel{x_2 x_1} - \cancel{x_2 x_1} \underbrace{- x_2^2}_{<0} < 0 \Rightarrow$ GAS $\forall x_1, x_2 \neq 0$

③ $e_2 = x_3 - \gamma(x_1, x_2) = x_3 + x_1 + x_2 \Rightarrow x_3 = e_2 - x_1 - x_2$

charge of coordinates

$\dot{x}_1 = -x_1^5 + x_2$

$\dot{x}_2 = e_2 - x_1 - x_2$

$\dot{x}_3 = x_4 = \dot{e}_2$

choosing $x_4 = \gamma(x_1, x_2, e_2)$ s.t. $\dot{V}_3 < 0$

$V_3(x_1, x_2, e_2) = \frac{1}{2}(x_1^2 + x_2^2 + e_2^2)$

$$\dot{V}_3 = \dot{x}_1 x_1 + \dot{x}_2 x_2 + \dot{e}_2 e_2 =$$
$$= -x_1^6 + \cancel{x_2 x_1} + e_2 x_2 - \cancel{x_1 x_2} - x_2^2 + x_4 e_2$$

by choosing $\gamma(x_1, x_2, e_2) = -e_2 - x_2$

$$\dot{V}_3 = -x_1^6 - x_2^2 - e_2^2 < 0 \Rightarrow \text{GAS} \ \forall x_1, x_2, e_2 \neq 0$$

(4) $e_3 = x_4 - \gamma(x_1, x_2, e_2) = x_4 + e_2 + x_2$

change of coordinates $\quad x_4 = e_3 - e_2 - x_2$

$$\dot{x}_1 = -x_1^5 + x_2$$
$$\dot{x}_2 = e_2 - x_1 - x_2$$
$$\dot{e}_2 = e_3 - e_2 - x_2$$
$$\dot{x}_4 = x_2^2(x_1 - x_3) + u = \dot{e}_3$$

by choosing $u = -x_2^2(x_1 - x_3) + v$

$$\dot{e}_3 = v$$

$$V(x_1, x_2, e_2, e_3) = \frac{1}{2}(x_1^2 + x_2^2 + e_2^2 + e_3^2)$$

$$\dot{V} = \dot{x}_1 x_1 + \dot{x}_2 x_2 + \dot{e}_2 e_2 + \dot{e}_3 e_3 =$$
$$= -x_1^6 + \cancel{x_1 x_2} + \cancel{e_2 x_2} - \cancel{x_1 x_2} - x_2^2 + e_3 e_2 - e_2^2 - \cancel{e_2 x_2} + e_3 v$$

by choosing $v = -e_2 - e_3$

one obtains

$$\dot{V} = -x_1^6 - x_2^2 - e_2^2 - e_3^2 < 0 \Rightarrow \text{GAS}$$