Add 2: introduction to I/O decoupling problem (kernels) (from NL context)

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Looking at a normed system having relative degree $\{r_1, \dots, r_m\}$ on which the noninteractive feedback will be imposed, we will consequently find that its outputs $y_i(t)$, for $1 \leq i \leq m$, are related to the input by expressions of the form

$$y_i(t) = \varphi_i(t)\, \xi^i(0) + \int_0^t k_i(t-s)\, v_i(s)\, ds$$

where $\varphi_i(t) = \left(1 + \frac{t^2}{2} \cdots \frac{t^{r_i - 1}}{(r_i-1)!}\right)$ $\quad k_i(t) = \frac{t^{r_i-1}}{(r_i-1)!}$

and $\xi^i(0)$ represents the value at $t=0$ of certain components of the state vector in the normal form.

The response is always given by the sum of the response under zero input (function of the time and initial condition) and of of a response depending on the input and not on the initial state, which is linear in the input.

The structure of the response is:

$$y(t) = Q(t, x^0) + \sum_{i=1}^{m} \int_0^t w_i(t-\tau_i)\, v_i(\tau_i)\, d\tau_i$$

Comparing this with the general expression of the Volterra series expansion of the input-output response of a NL system, one may conclude that a system having relative degree $\{r_1, \dots, r_m\}$ subject to a noninteractive feedback is characterized by an output response in which the first order kernels $w_i(t, \tau_i)$ depend only on the difference $(t - \tau_i)$ and not on x_0, and all the kernels of higher order than one are vanishing.

Given a kernel $w_i(t, \tau_i)$ it is easily found that a necessary and sufficient condition for this kernel to be independent of x_0 and dependent only on $t - \tau_i$ is that

$$L_{\tilde{g}_i} L_{\tilde{f}}^{k} h_j(x) = \text{independent of } x \quad \forall k \geq 0 \quad \forall\, 1 \leq i, j \leq m$$

In general this condition is not satisfied and this aspect can be fixed with feedback.