5. Necessary Hypotheses

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Linearity :

given $x_1(t) = \varphi(t, t_0, x_1(t_0), u_1|_{[t_0,t)})$ $x_2(t) = \varphi(t, t_0, x_2(t_0), u_2|_{[t_0,t)})$

computing from t_0 to t with initial state $\alpha x_1(t_0) + \beta x_2(t_0)$

and input $\alpha u|_{[t_0,t)} + \beta u|_{[t_0,t)}$

obtaining $x(t) = \varphi(t, t_0, \alpha x_1(t_0) + \beta x_2(t_0), \alpha u|_{[t_0,t)} + \beta u|_{[t_0,t)})$

If it is equal to $x(t) = \alpha x_1(t) + \beta x_2(t)$ $\forall \alpha, \beta, x(t_0), u|_{[t_0,t)}$

then the linearity holds.

- **Particular case:**

$\alpha = \beta = 1$ $x_1(t_0) = x(t_0)$ $u_1|_{[t_0,t)} = 0$

$x_2(t_0) = 0$ $u_2|_{[t_0,t)} = u|_{[t_0,t)}$

$$x(t) = \varphi(t, t_0, x_0, u) = \underbrace{\varphi(t, t_0, x_0, 0)}_{\text{free evolution}} + \underbrace{\varphi(t, t_0, 0, u)}_{\text{forced evolution}}$$

$$y(t) = \eta(t, x(t), 0) + \eta(t, 0, u(t))$$

Stationariety :

given $x_1(t) = \varphi(t, t_0, x(t_0), u|_{[t_0,t)})$

$x_2(t) = \varphi(t+\Delta t, t_0+\Delta t, x(t_0), u_{\Delta t}|_{[t_0+\Delta t, t+\Delta t)})$

if for any Δt $x_1(t) = x_2(t)$ $\forall t$ then stationariety holds

(for the output $y(t) = (t+\Delta t, x(t), u_{\Delta t}(t))$)

- **Particular case**

$\Delta t = -t_0 \Rightarrow x(t) = (t-t_0, 0, x(t_0), u_{-t_0}|_{[0, t-t_0)})$

$\Delta t = -t \Rightarrow y(t) = \eta(0, x(t), u(t))$

We pay attention on the overall duration of the experiment $t - t_0$ and we lose dependencies of the starting instant t_0 and on the observation instant on the output