9. Time invariant Finite dimensional Continuous time state representations

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Assume X, U, Y vector spaces with dimension n, p, q.
Consider a strictly causal, finite dimension, stationary system.
The generic IMPLICIT representation is

$$\Delta x(t) = f(x(t), u(t))$$
$$y(t) = h(x(t))$$

the generating function $f(x,u)$ and the output function $h(x)$ can take different structures

Linear

$$\begin{cases} f(x,u) = Ax + Bu \\ h(x) = Cx \end{cases}$$

f linear on $X \times U$
h linear on X

A, B, C are matrices of dimension $(n \times n), (n \times p), (p \times n)$

Bilinear

$$\begin{cases} f(x,u) = Ax + Bu + \sum_{i=1}^{p} N_i x u_i = f_1(x,u) + f_2(x,u) \\ h(x) = Cx \end{cases}$$

f_1 linear on $X \times U$
f_2 bilinear on $X \times U$
h linear on X

$A, N_1, \dots, N_p$ $(n \times n)$ matrices

State Affine

$$\begin{cases} f(x,u) = A_0(u) + A_1(u)x \\ h(x) = Cx \end{cases}$$

f affine w.r.t. x $\forall u$ fixed
h linear on X

$A_0(\cdot), A_1(\cdot)$ analytic, C $(q \times n)$ matrix

Input Affine

$$\begin{cases} f(x,u) = f(x) + g(x)u \\ h(x) = h(x) \end{cases}$$

g linear wrt u for any fixed x
f, g, h are C^{∞} or analytic