22. Disturbance Decoupling

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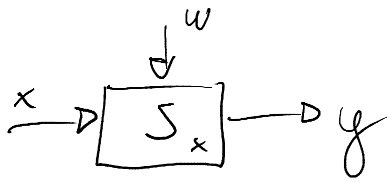
$$\begin{cases} \dot{x} = f(x) + g(x)u + p(x)w \\ y = h(x) \end{cases}$$



Δ distribution such that $\rho(\Delta) = k$ and $\Delta = \bar{\Delta}$ (involutive closure)

the main idea is to use a feedback to reject the action of the disturbance w.

• Linear Case

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

$(*)$ $\exists\, \mathcal{V}: A\mathcal{V} \subset \mathcal{V}$ & $\mathrm{Im}[B] \subset \mathcal{V} \subset \ker[C]$

the $(*)$ condition tells us that $\mathcal{R} \subset \mathcal{U}$. It is possible to find a partition of the system such that

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} 0 & C_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_2 x_2$$

In such a way that the output y does not depend on the input u.

Consider now:

$$\begin{cases} \dot{x} = Ax + Bu + Pw \leftarrow \text{disturbance} \\ y = Cx \end{cases}$$

$(**)$ $\exists\, \mathcal{V}: A\mathcal{V} \in \mathcal{V}$ & $\mathrm{Im}[P] \subset \mathcal{V} \subset \ker[C]$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u + \begin{pmatrix} P_1 \\ 0 \end{pmatrix} w$$

$$y = \begin{pmatrix} 0 & C_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = C_2 x_2$$

$\Rightarrow$ $(**)$ ensures that y does not depend on w

Consider the system under the state feedback

$u = Fx + Gv \qquad |G| \neq 0$

$$\begin{cases} \dot{x} = (A + BF)x + BGv \\ y = Cx \end{cases}$$

i) the reachable set does not change (guaranteed by $|G| \neq 0$)

$$\mathcal{R} = \mathrm{Im}[BG \mid (A+BF)BG \mid \cdots \mid (A+BF)^{n-1}BG] \equiv$$
$$\equiv \mathrm{Im}[B \mid AB \mid \cdots \mid A^{n-1}B]$$

ii) the zeros of the system do not change

What is changing is the set of unobservable states:

$$\ker \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \not\equiv \ker \begin{bmatrix} C \\ C(A+BF) \\ \cdots \\ C(A+BF)^{n-1} \end{bmatrix}$$

Introducing the disturbance

$$\begin{cases} \dot{x} = (A+BF)x + BGv + Pw \\ y = Cx \end{cases}$$

DDP is solvable iff is true the condition (**):

$\exists F : (A+BF)\mathcal{V} \subset \mathcal{V}$ & $\mathrm{Im}[P] \subset \mathcal{V} \subset \ker[C]$

Lemma: $\mathcal{V}$ is (A,B)-invariant iff $A\mathcal{V} \subset \mathcal{V} \subset \mathrm{Im}[B]$

def: $\mathcal{V}$ is (A,B)-invariant iff $\exists f : (A+BF)\mathcal{V} \subset \mathcal{V}$

Non Linear case

def: Δ is (f,g)-invariant iff:

$$[f,\Delta] \subset \Delta + \mathrm{span}\{g\} \;\&\; [g,\Delta] \subset \Delta + \mathrm{span}\{g\}$$

def: Δ is (f,g)-controlled invariant iff:

given $\alpha(x), \beta(x) : \mathbb{R}^n \mapsto \mathbb{R}$ $[f+g\alpha, \Delta] \subset \Delta$ & $[f+g\beta, \Delta] \subset \Delta$

DDP is solvable iff:

i) Δ is (f,g)-invariant

ii) $P \subset \Delta \subset \ker[dh]$

or if we call Δ^* the largest controlled invariant distribution contained in $\ker[dh]$, the problem is solvable if and only if

$$P \in \Delta^*$$

the non linear system under feedback becomes:

$$\begin{cases} \dot{x} = (f(x) + g(x)\alpha(x)) + g(x)\beta(x)v + P(x)w \\ y = h(x) \end{cases}$$

$$
\begin{cases}
\dot{x} = \left(f(x) + g(x)\alpha(x) \right) + g(x)\beta(x)v + P(x)w \\
y = h(x)
\end{cases}
$$