Robotics 2

Impedance Control

Prof. Alessandro De Luca

Dipartimento di Ingegneria Informatica
Automatica e Gestionale Antonio Ruberti

Sapienza
Università di Roma

Impedance control

- imposes a desired dynamic behavior to the interaction between robot end-effector and environment
- the desired performance is specified through a generalized dynamic impedance, namely a complete set of mass-spring-damper equations (typically chosen as linear and decoupled, but also nonlinear)
- a model describing how reaction forces are generated in association with environment deformation is not explicitly required
- suited for tasks in which contact forces should be "kept small", while their accurate regulation is not mandatory
- since a control loop based on force error is missing, contact forces are only indirectly assigned by controlling position
- the choice of a specific stiffness in the impedance model along a Cartesian direction results in a trade-off between contact forces and position accuracy in that direction

Dynamic model of a robot in contact

$q \in \mathbb{R}^n$

$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) = u + J^T(q)F$$

generalized Cartesian force

forces

torques

linear velocity

angular velocity

"geometric" Jacobian

"analytic" Jacobian

derivative of Euler angles

$F = \begin{pmatrix} f \\ m \end{pmatrix} \in \mathbb{R}^m$ performing work on $V = \begin{pmatrix} v \\ \omega \end{pmatrix} = J(q)\dot{q} \neq \dot{x} = \begin{pmatrix} \dot{p} \\ \dot{\phi} \end{pmatrix} = J_a(q)\dot{q}$

direct kinematics

$$J_a(q) = \frac{\partial f(q)}{\partial q} = T_a(\phi)\, J(q) \quad \Rightarrow \quad \dot{x} = T_a(\phi)\, V$$

$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) = u + J_a^T(q)F_a$$

with

$$F_a = T_a^{-T}(\phi)\, F$$

generalized forces performing work on $\dot{x}$

Dynamic model in Cartesian coordinates

assuming
$n = m$

$$M_x(q)\ddot{x} + S_x(q,\dot{q})\dot{x} + g_x(q) = J_a^{-T}(q)u + F_a$$

with

$$M_x(q) = J_a^{-T}(q)M(q)J_a^{-1}(q)$$

$$S_x(q,\dot{q}) = J_a^{-T}(q)S(q,\dot{q})J_a^{-1}(q) - M_x(q)\dot{J}_a(q)J_a^{-1}(q)$$

$$g_x(q) = J_a^{-T}(q)g(q)$$

... and the usual structural properties

- $M_x > 0$, if J_a is non-singular
- $\dot{M}_x - 2S_x$ is skew-symmetric, if $\dot{M} - 2S$ satisfies the same property
- the Cartesian dynamic model of the robot can be linearly parameterized in terms of a set of dynamic coefficients

Design of the control law

designed in two steps:

1. feedback linearization in the Cartesian space (with force measure)

$$\boxed{u = J_a^T(q)\left[M_x(q)a + S_x(q,\dot{q})\dot{x} + g_x(q) - F_a\right]}$$

$$\Rightarrow \quad \boxed{\ddot{x} = a} \qquad \text{closed-loop system}$$

2. imposition of a dynamic impedance model

most of the times it is "decoupled" (diagonal matrices)

$$\boxed{M_m(\ddot{x} - \ddot{x}_d) + D_m(\dot{x} - \dot{x}_d) + K_m(x - x_d) = F_a}$$

desired (apparent) inertia (> 0) — desired damping (≥ 0) — desired stiffness (> 0) — external forces from the environment

is realized by choosing

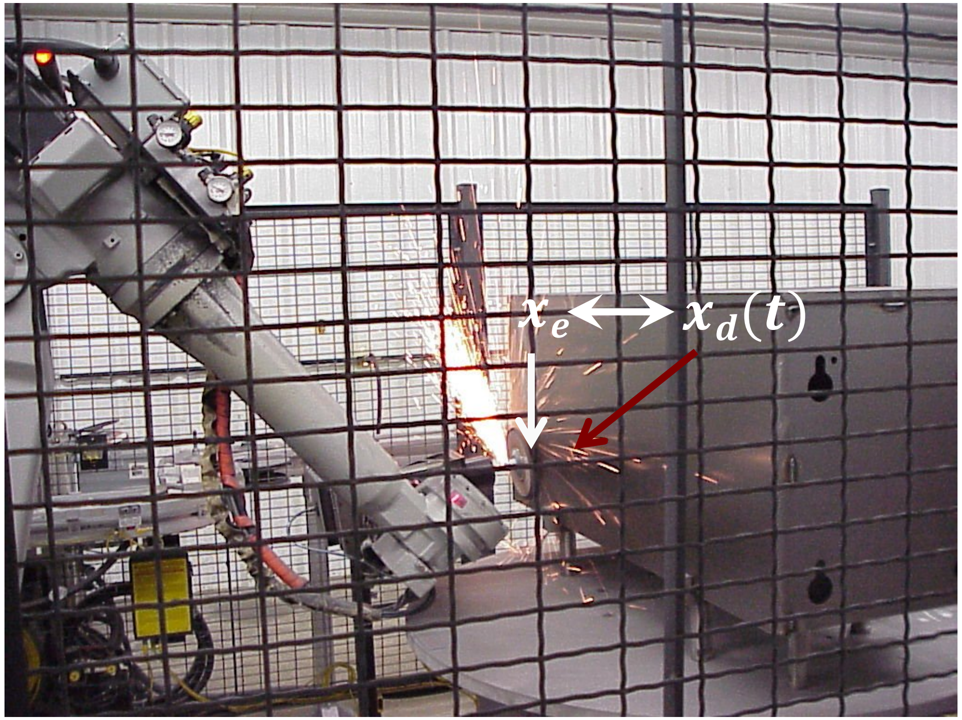
$$\boxed{a = \ddot{x}_d + M_m^{-1}\left[D_m(\dot{x}_d - \dot{x}) + K_m(x_d - x) + F_a\right]}$$

Note: $x_d(t)$ is the desired motion, which typically "slightly penetrates" inside the compliant environment (inducing contact forces)...

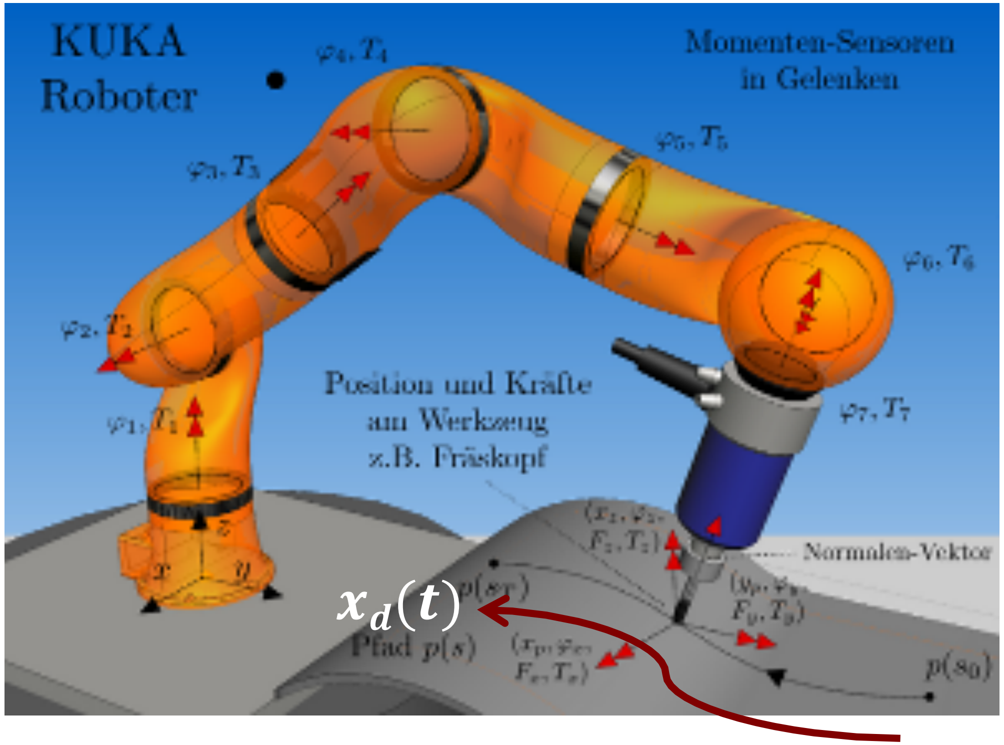
Examples of desired reference x_d in impedance/compliance control

$$\boxed{M_m(\ddot{x} - \ddot{x}_d) + D_m(\dot{x} - \dot{x}_d) + K_m(x - x_d) = F_a}$$

the desired motion $\boldsymbol{x_d(t)}$ is slightly inside the environment (keeping thus contact)



robot in grinding task

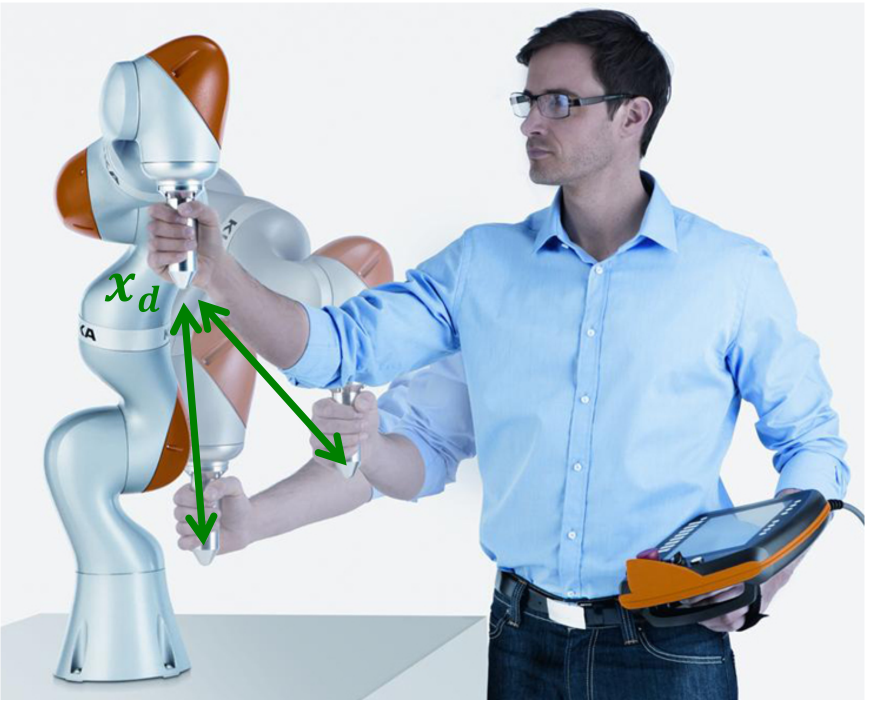


robot writing on a surface

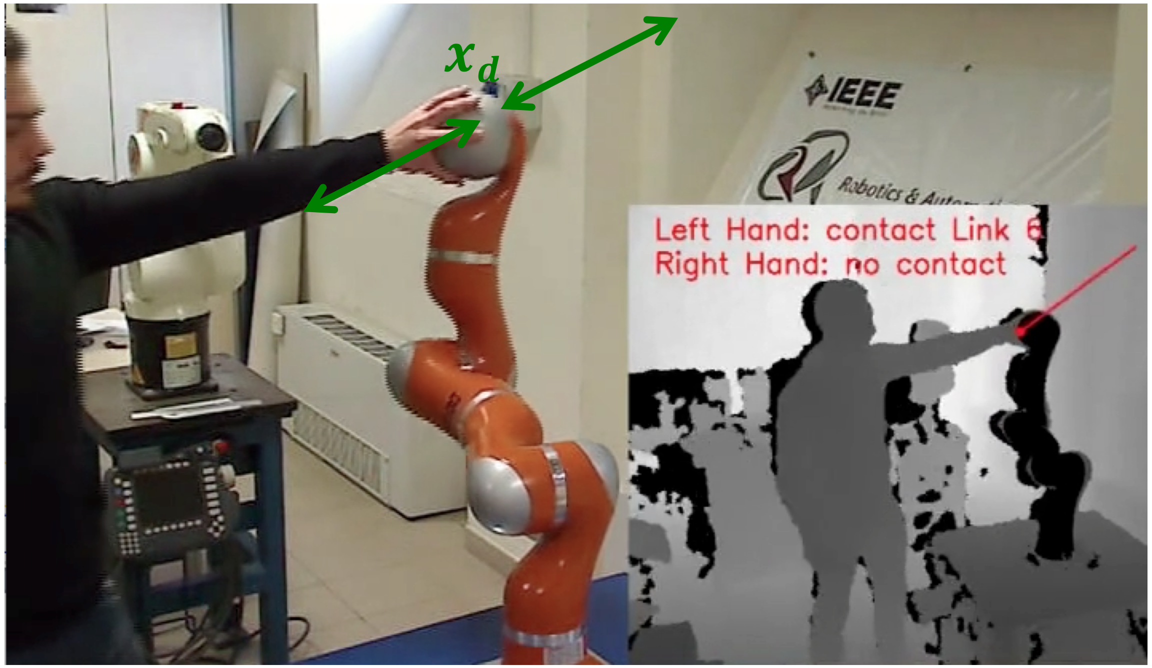
Examples of desired reference x_d
in impedance/compliance control

$$\boxed{M_m(\ddot{x} - \ddot{x}_d) + D_m(\dot{x} - \dot{x}_d) + K_m(x - x_d) = F_a}$$

constant desired pose $\boldsymbol{x_d}$ is the free Cartesian
rest position in a human-robot interaction task



KUKA iiwa robot with human operator



KUKA LWR robot in pHRI (collaboration)

Control law in joint coordinates

substituting and simplifying…

$$\boxed{\begin{aligned} u = M(q)J_a^{-1}(q)\{\ddot{x}_d - \dot{J}_a(q)\dot{q} + M_m^{-1}[D_m(\dot{x}_d - \dot{x}) + K_m(x_d - x)]\} \\ + S(q,\dot{q})\dot{q} + g(q) + J_a^T(q)\underbrace{[M_x(q)M_m^{-1} - I]}_{\text{matrix weighting the measured contact forces}}F_a \end{aligned}}$$

- the following identity holds for the term involving contact forces

$$J_a^T(q)[M_x(q)M_m^{-1} - I]F_a = [M(q)J_a^{-1}(q)M_m^{-1} - J_a^T(q)]F_a$$

which eliminates from the control law also the appearance of the last remaining Cartesian quantity (the Cartesian inertia matrix)

- while the control design is based on dynamic analysis and desired (impedance) behavior described in the Cartesian space, the final control implementation is always at the robot joint level

Choice of the impedance model

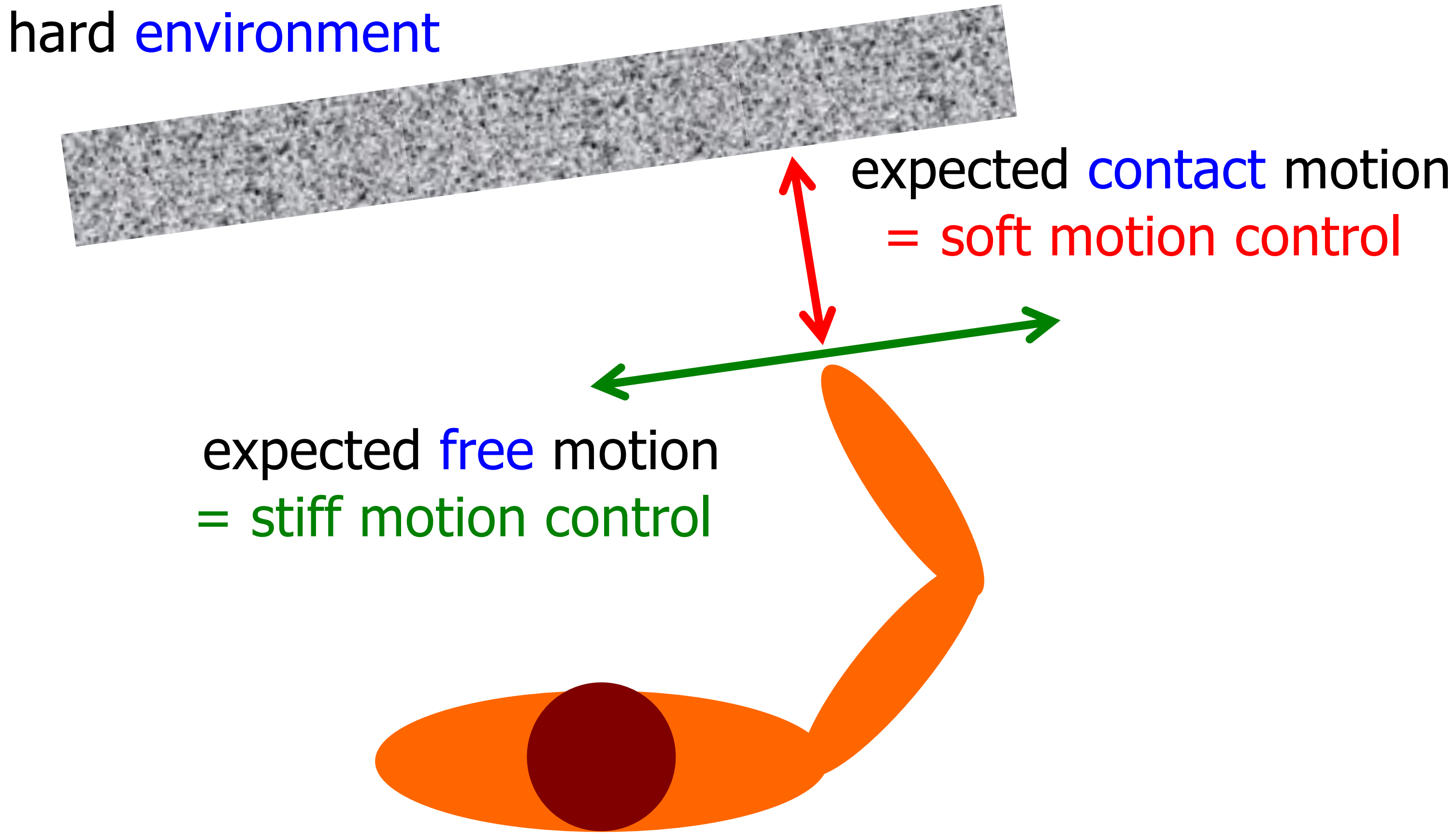
rationale ...

- avoid large impact forces due to uncertain geometric characteristics (position, orientation) of the environment
- adapt/match to the dynamic characteristics of the environment (in particular, of its estimated stiffness) in a complementary way
- mimic the behavior of a human arm
  - ➔ fast and stiff in "free" motion, slow and compliant in "guarded" motion

⬇

- large $M_{m,i}$ and small $K_{m,i}$ in Cartesian directions where contact is foreseen (➔ low contact forces)
- large $K_{m,i}$ and small $M_{m,i}$ in Cartesian directions that are supposed to be free (➔ good tracking of desired motion trajectory)
- damping coefficients $D_{m,i}$ are used then to shape transient behaviors

Human arm behavior



in the selected i-th Cartesian direction:
the stiffer is the environment, the softer is the chosen model stiffness $K_{m,i}$

A notable simplification - 1

choose the apparent inertia equal to the natural Cartesian inertia of the robot

$$M_m = M_x(q) = J_a^{-T}(q)M(q)J_a^{-1}(q)$$

then, the control law becomes

$$u = M(q)J_a^{-1}(q)\{\ddot{x}_d - \dot{J}_a(q)\dot{q}\} + S(q,\dot{q})\dot{q} + g(q) + J_a^T(q)[D_m(\dot{x}_d - \dot{x}) + K_m(x_d - x)]$$

WITHOUT contact force feedback! (a F/T sensor is no longer needed…)

this is a pure motion control applied also during interaction,
but designed so as to keep limited contact forces at the end-effector level
(as before, K_m is chosen as a function of the expected environment stiffness)

A notable simplification - 2

technical issue: if the impedance model (now, nonlinear) is still supposed to represent a real mechanical system, then in correspondence to a desired non-constant inertia ($M_x(q)$) there should be Coriolis and centrifugal terms...

$$\boxed{M_x(q)(\ddot{x}-\ddot{x}_d)+(S_x(q,\dot{q})+D_m)(\dot{x}-\dot{x}_d)+K_m(x-x_d)=F_a}$$

nonlinear impedance model (“only” gravity terms disappear)

redoing computations, the control law becomes

$$\boxed{\begin{aligned} u = {} & M(q)J_a^{-1}(q)\{\ddot{x}_d-\dot{J}_a(q)J_a^{-1}(q)\dot{x}_d\}+S(q,\dot{q})J_a^{-1}(q)\dot{x}_d+g(q) \\ & +J_a^T(q)[D_m(\dot{x}_d-\dot{x})+K_m(x_d-x)] \end{aligned}}$$

which is indeed slightly more complex, but has the following advantages:

- guarantee of asymptotic convergence to zero tracking error (on $x_d(t)$) when $F_a=0$ (no contact situation) ⇒ Lyapunov + skew-symmetry of $\dot{M}_x-2S_x$
- further simplifications when x_d is constant

Cartesian regulation revisited
(without contact, $F_a = 0$)

when x_d is constant ($\dot{x}_d = 0$, $\ddot{x}_d = 0$), from the previous expression we get the control law

$$\boxed{u = g(q) + J_a^T(q)[K_m(x_d - x) - D_m \dot{x}]} \quad (\star)$$

Cartesian PD control with gravity cancellation…

when $F_a = 0$ (absence of contact), we know already that this control law ensures asymptotic stability of x_d, provided $J_a(q)$ has full rank

proof (alternative)

Lyapunov candidate $V_1 = \frac{1}{2}\dot{x}^T M_x(q)\dot{x} + \frac{1}{2}(x_d - x)^T K_m (x_d - x)$

$$\Rightarrow \quad \dot{V}_1 = \dot{x}^T M_x(q)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}_x(q)\dot{x} - \dot{x}^T K_m (x_d - x) = \cdots = -\dot{x}^T D_m \dot{x} \leq 0$$

using skew-symmetry of $\dot{M}_x - 2S_x$ and $g_x = J_a^{-T} g$

Cartesian stiffness control
(with contact, $F_a \neq 0$)

when $F_a \neq 0$, convergence to x_d is not assured
(it may not even be a closed-loop equilibrium…)

- for analysis, assume an elastic contact model for the environment

$$\boxed{F_a = K_e(x_e - x)}$$ with stiffness $K_e \geq 0$ and rest position x_e

- closed-loop system behavior

Lyapunov candidate

$$V_2 = \frac{1}{2}\dot{x}^T M_x(q)\dot{x} + \frac{1}{2}(x_d - x)^T K_m (x_d - x) + \frac{1}{2}(x_e - x)^T K_e (x_e - x)$$

$$= V_1 + \frac{1}{2}(x_e - x)^T K_e (x_e - x)$$

$$\Rightarrow \quad \dot{V}_2 = \dot{x}^T M_x(q)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}_x(q)\dot{x} - \dot{x}^T K_m (x_d - x) - \dot{x}^T K_e (x_e - x)$$

$$= \cdots = -\dot{x}^T D_m \dot{x} + \dot{x}^T (F_a - K_e(x_e - x)) = -\dot{x}^T D_m \dot{x} \leq 0$$

Stability analysis (with $F_a \neq 0$)

when $\dot{x} = \ddot{x} = 0$, at a closed-loop system equilibrium it is

$$K_m(x_d - x) + K_e(x_e - x) = 0$$

which has the unique solution

$$\boxed{x = (K_m + K_e)^{-1}(K_m x_d + K_e x_e) =: x_E}$$

(check that the Lyapunov candidate V_2 has in fact its minimum in x_E!)

LaSalle ➡ x_E asymptotically stable equilibrium

$$\boxed{x_E \approx \begin{cases} x_e \text{ for } K_e \gg K_m \text{ (rigid environment)} \\ x_d \text{ for } K_m \gg K_e \text{ (rigid controller)} \end{cases}}$$

Note: the Cartesian stiffness control law (★) is often called compliance control in the literature

Active equivalent of RCC device

IF

- displacements from the desired position x_d are small, namely

$$(x_d - x) \approx J_a(q_d - q)$$

- $g(q) = 0$ (gravity is compensated/cancelled, e.g., mechanically)
- $D_m = 0$

THEN

$$\boxed{u = J_a^T(q) K_m J_a(q_d - q) = K(q)(q_d - q)}$$

$$\begin{tikzcd}
\Delta x \arrow[r, "K_m"] & F \arrow[d, "J_a^T(q)"] \\
\Delta q \arrow[u, "J_a(q)"] \arrow[r, "K(q)"'] & u
\end{tikzcd}$$

constant Cartesian-level stiffness K_m
corresponds to
variable joint-level stiffness $K(q)$
(and vice versa on compliance)

$$\begin{tikzcd}
\Delta x \arrow[d, "J_a^{-1}(q)"'] & F \arrow[l, "C(x)"'] \\
\Delta q & u \arrow[u, "J_a^{-T}(q)"'] \arrow[l, "C_m"]
\end{tikzcd}$$

is the "active" counterpart of a Remote Center of Compliance (RCC) device

Admittance control

- in some cases, we don't have access to low-level robot torque (or motor current) commands $\Rightarrow$ closed control architecture
- for handling the interaction with the environment, one uses then admittance control: contact forces $\Rightarrow$ velocity commands
- implementation (with compliant matrices C)
  - at the velocity or incremental position level
  - in the joint or Cartesian (or task) space

$$u_c = J^T(q)F_c \longrightarrow \dot{q} = C_q u_c \longrightarrow \boxed{\dot{q} = C_q\, J^T(q)F_c} \qquad C_q \geq 0$$

$$\updownarrow$$

Δq (to be added to the current q)

$$F_c \longrightarrow \dot{x} = C_x F_c \longrightarrow \boxed{\dot{q} = J^{-1}(q)C_x F_c} \qquad C_x \geq 0$$

$$\updownarrow$$

(in case of redundancy) $J^{\#}(q)$