15. Exact linearization problem

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Static state feedback control:

The value of the control at time t depends only on the values, at the same instant of time, of the state x and of the extend reference input. $(u-v)$

Dynamical state feedback control:

The control depends also on a set of additional state variables, i.e. it's the output of an appropriate dynamical system, having its own internal state, driven by x and by the external reference input. $(u-x)$

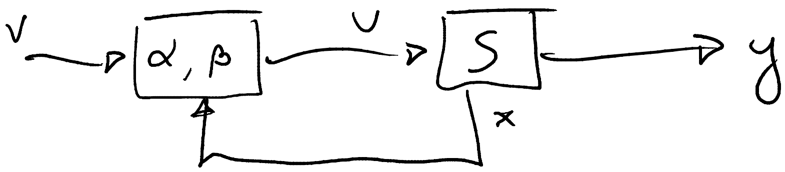
Exact feedback linearization $(u-v)$ Input-Affine regular feedback

The most convenient structure for a static state feedback control is the one in which is

$$u = \alpha(x) + \beta(x)v \qquad |\beta(x)| \neq 0 \quad \text{(regular)}$$

this yelds to a closed-loop characterization of this form

$$\begin{cases} \dot{x} = f(x) + g(x)\alpha(x) + g(x)\beta(x)v \\ y = h(x) \end{cases}$$



The main idea is to use the state feedback in order to transform a nonlinear system into a linear and controllable one.

Suppose $r = n$, then $\exists\, \Phi(x) = \begin{pmatrix} h(x) \\ \cdots \\ L_f^{r-1}(x) \end{pmatrix}$ which puts the system in normal form

$$\dot{z}_1 = z_2$$
$$\cdots$$
$$\dot{z}_{n-1} = z_n$$
$$\dot{z}_n = b(z) + a(z)u$$

Choosing a state feedback control law of the form

$$u = \frac{1}{a(z)}\left(-b(z)+v\right)$$

follows:

$$\dot{z}_n = b(z) + a(z)\,u = \cancel{b(z)} + \cancel{a(z)}\left[\frac{1}{\cancel{a(z)}} - b(z) + v\right] = v$$

And it's linear and controllable (Brunowsky Canonical Form).
It is possible to realize such feedback only if $r = n$.
Looking at $a(z), b(z)$ one can rewrite the feedback as

$$u = \frac{1}{L_f L_g^{n-1} h(x)}\left(-L_g^n h(x) + v\right)$$

which yelds the same linear and controllable system

$$y^{(n)} = L_g^n h(x) + L_f L_g^{n-1} h(x)\,u = \dots = L_g^n h(x) - L_g^n h(x) + v = v$$

It is possible to impose new feedback controls, to assign eigenvalues: $v = Fz$

$$F = [a_0^* \; a_1^* \; \dots \; a_{n-1}^*]$$

therefore the feedback can be written as

$$v = a_0 h(x) + a_1 L_g h(x) + \dots + a_{n-1} L_g^{n-1} h(x)$$

Substituting:

$$u = \frac{1}{L_f L_g^{n-1} h(x)}\left(-L_g^n h(x) + \sum_{i=0}^{n-1} a_i^* L_g^i h(x)\right)$$

Remark:

- if x_0 is an equilibrium point for the non linear system, then $z_0 = \Phi(x_0) = 0$
- if $r = n$ there is no unobservable subsystem

$$u = \frac{-L_g^n h(x) + \sum_0^{n-1} a_i L_g^i h(x)}{L_f L_g^{n-1} h(x)}$$