26. Indirect Lyapunov theorem

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Given a non linear sys $\dot{x} = f(x)$, $f(x_e) = 0$, $A = J_f(x_e)$

\# $A = \left.\frac{\partial f}{\partial x}\right|_{x = x_e}$ LTM matrix

(i) if $\sigma(A) \subset \mathbb{C}^-$, x_e is LAS

(ii) if $\sigma(A) \cap \mathbb{C}^+ \neq 0$, x_e is unstable

(iii) if $\sigma(A) = 0 \Rightarrow$ undefined

Lemma if $\det(\overset{A}{J_f(x_e)}) \neq 0$ (non-sing) then x_e is an isolated equilibrium and $f(x_e) = 0$

Proof: Suppose $\exists\, x_e' : f(x_e') = f(x_e) = 0$

LTM $\Rightarrow \cancel{f(x_e')} = \cancel{f(x_e)} + J_f(x_e)(x_e' - x_e) + h(x_e' - x_e) = 0$

$$J_f(x_e)(x_e' - x_e) + h(x_e' - x_e) = 0$$

with x_e' varying $\Rightarrow$ both addendums must be 0 which would contradict that $|J_f(x_e)| \neq 0$

(*) if there are some eigenvalues with 0 real part, linearization fails to determine stability of the origin

(**) AS of the origin for the linear approximation (which is always global) only implies LAS of x_e for the nonlinear system