



Robotics 2

Kinematic calibration

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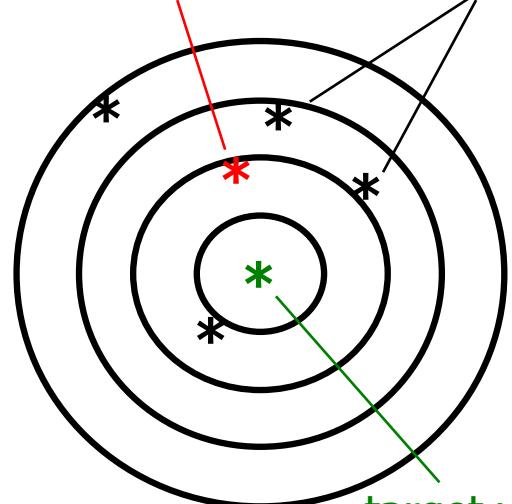




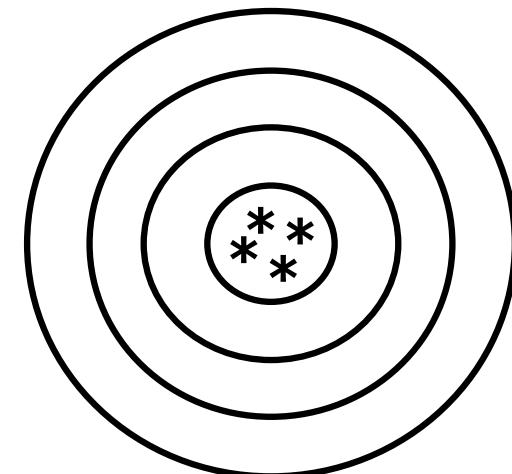
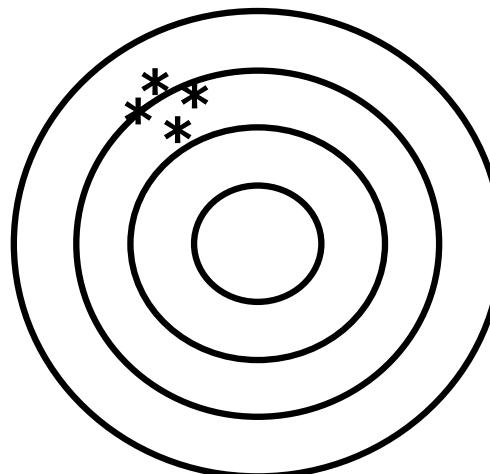
Accuracy and Repeatability

robot as a measuring device

average value measurements



from Robotics 1



low accuracy
low repeatability

low accuracy
high repeatability

high accuracy
high repeatability

better components!

calibration!



Direct kinematics

- nominal set of Denavit-Hartenberg (D-H) parameters

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} \quad \boldsymbol{a} = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \quad \boldsymbol{d} = \begin{pmatrix} d_1 \\ \vdots \\ d_n \end{pmatrix}$$

for simplicity, suppose
an all-revolute joints
manipulator

- nominal direct kinematics

$$\boldsymbol{r}_{nom} = f(\boldsymbol{\alpha}, \boldsymbol{a}, \boldsymbol{d}, \boldsymbol{\theta})$$

θ are typically measured by encoders \rightarrow

$$\boldsymbol{\theta} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix}$$

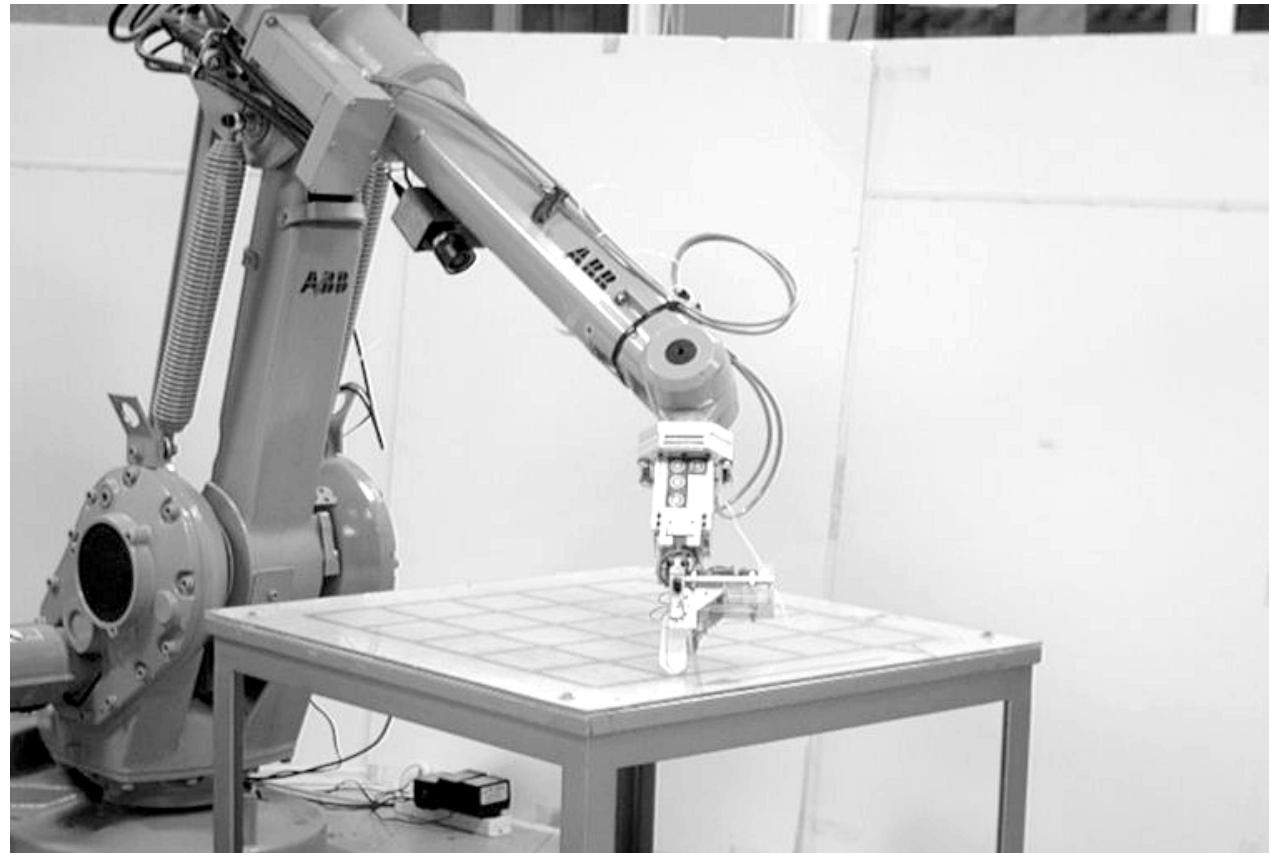


Need for calibration

- tolerances in mechanical construction and in assembly of links/joints imply small errors in actual end-effector pose ($\text{real} \neq \text{nominal}$ parameters)
- encoder mounting on motor axes may not be consistent with the “zero reference” of the robot direct kinematics (joint angle measures are constantly biased)
- errors distributed “along” the arm are amplified, due to the open chain kinematic structure of most robots
- calibration goal: recover as much as possible E-E pose errors by correcting the nominal set of D-H parameters, based on independent external (accurate!) measurements
- experiments to be done once for each robot, before starting operation... (and maybe repeated from time to time)



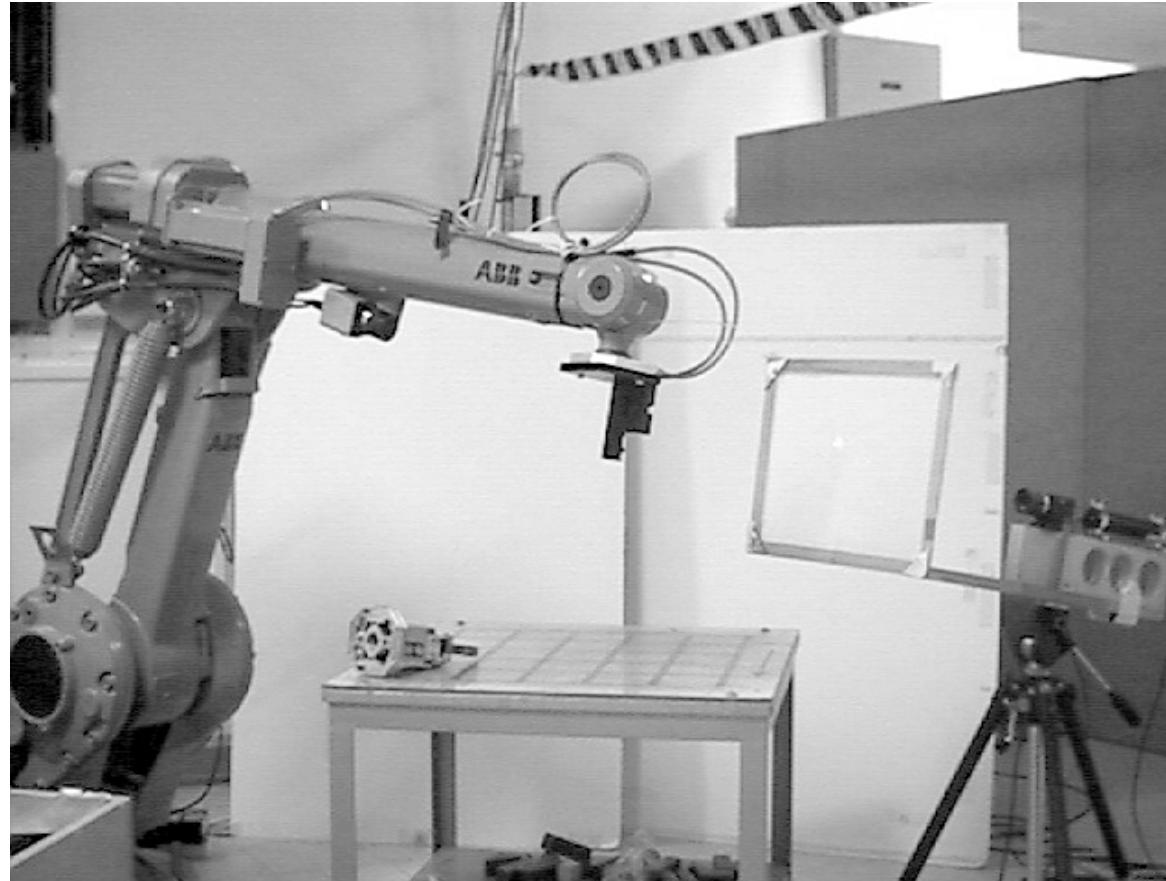
Cartesian measurement systems - 1



calibration table



Cartesian measurement systems - 2



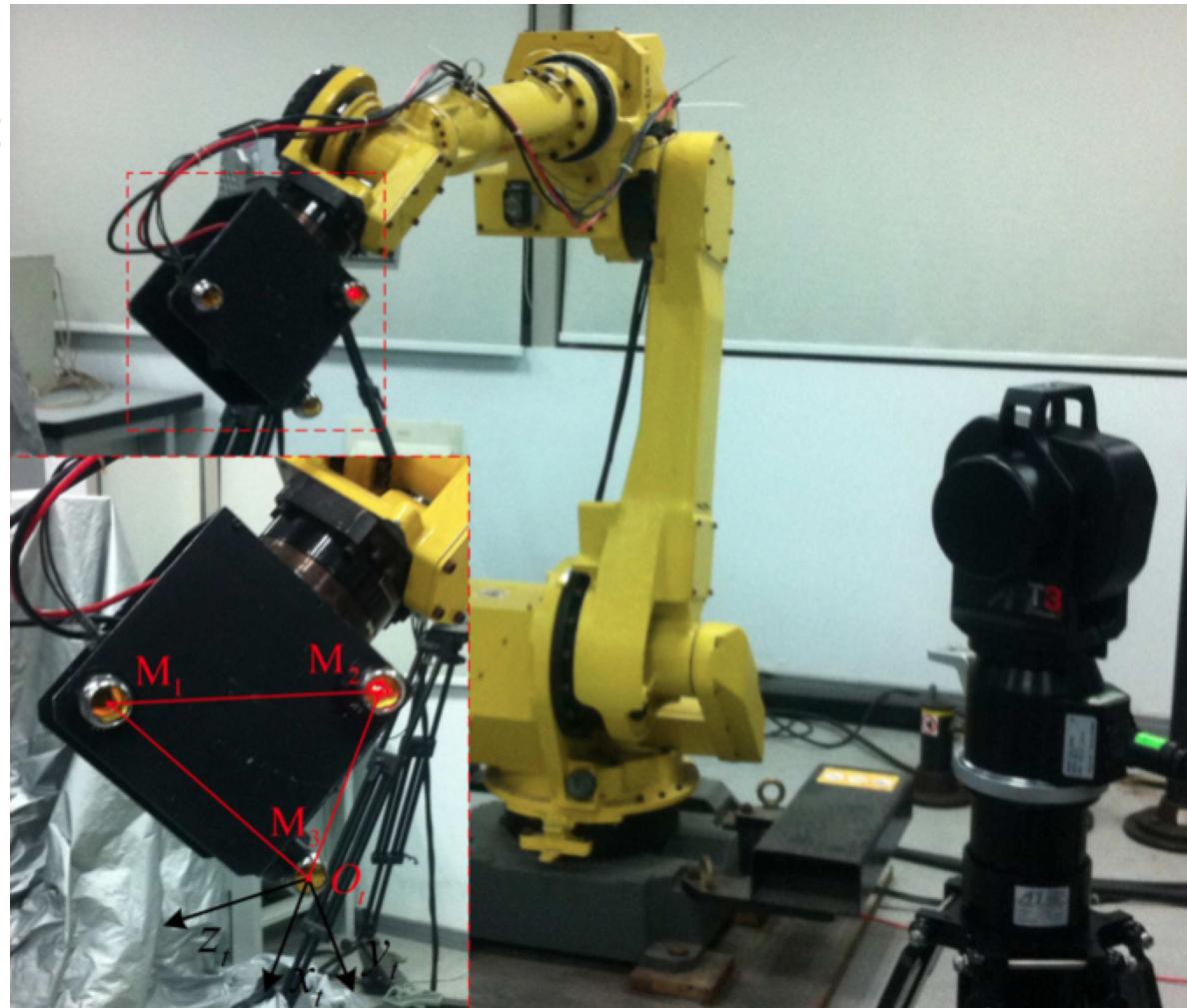
laser/camera system + triangulation



Cartesian measurement systems - 3

FANUC 6R robot
M-710iC/50

3 SMRs
(Spherically-
Mounted
Reflectors)



API
laser tracker III
www.apisensor.com

laser tracker + targets on end-effector



Acquiring data for calibration

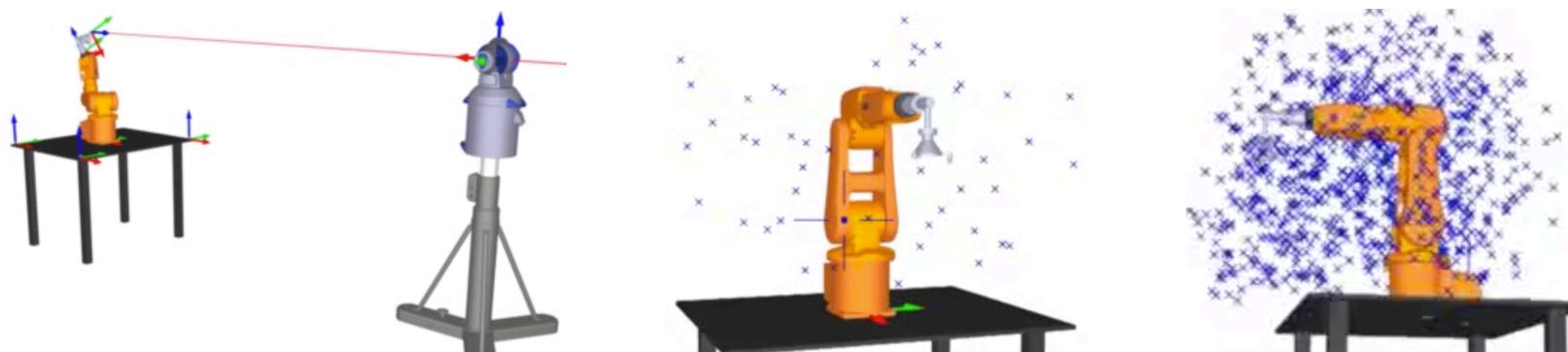
FARO ION
laser tracker



video
@CoRo Lab
ETS Montréal

ABB
IRB 1600
robot

4 SMRs



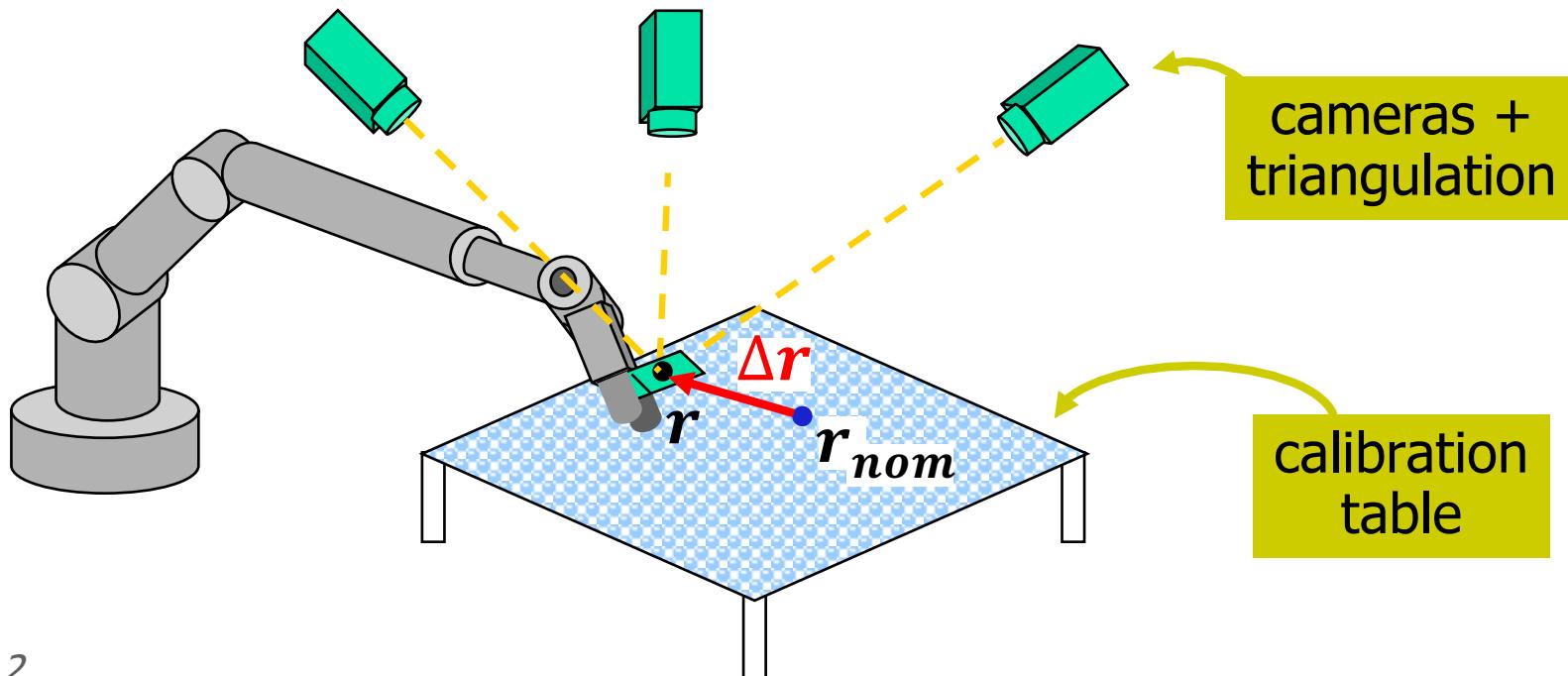


Linearization of direct kinematics

partial Jacobians evaluated in nominal conditions

$$\Delta r = r - r_{nom} = \frac{\partial f}{\partial \alpha} \cdot \Delta \alpha + \frac{\partial f}{\partial a} \cdot \Delta a + \frac{\partial f}{\partial d} \cdot \Delta d + \frac{\partial f}{\partial \theta} \cdot \Delta \theta$$

“small” errors obtained by external measurement system first-order variations





Calibration equation

$$\Delta\varphi = \begin{pmatrix} \Delta\alpha \\ \Delta a \\ \Delta d \\ \Delta\theta \end{pmatrix} \quad \Phi = \begin{pmatrix} \frac{\partial f}{\partial \alpha} & \frac{\partial f}{\partial a} & \frac{\partial f}{\partial d} & \frac{\partial f}{\partial \theta} \end{pmatrix}$$

4n × 1

6 × 4n

$\rightarrow \Delta r = \Phi \cdot \Delta\varphi$

6ℓ × 1

\downarrow

$$\Delta\bar{r} = \begin{pmatrix} \Delta r_1 \\ \Delta r_2 \\ \vdots \\ \Delta r_\ell \end{pmatrix}$$

6ℓ × 4n

\downarrow

$$\bar{\Phi} = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_\ell \end{pmatrix}$$

full column rank
(for sufficiently large ℓ)

ℓ experiments ($\ell \gg n$)

$\rightarrow \Delta\bar{r} = \bar{\Phi} \cdot \Delta\varphi$

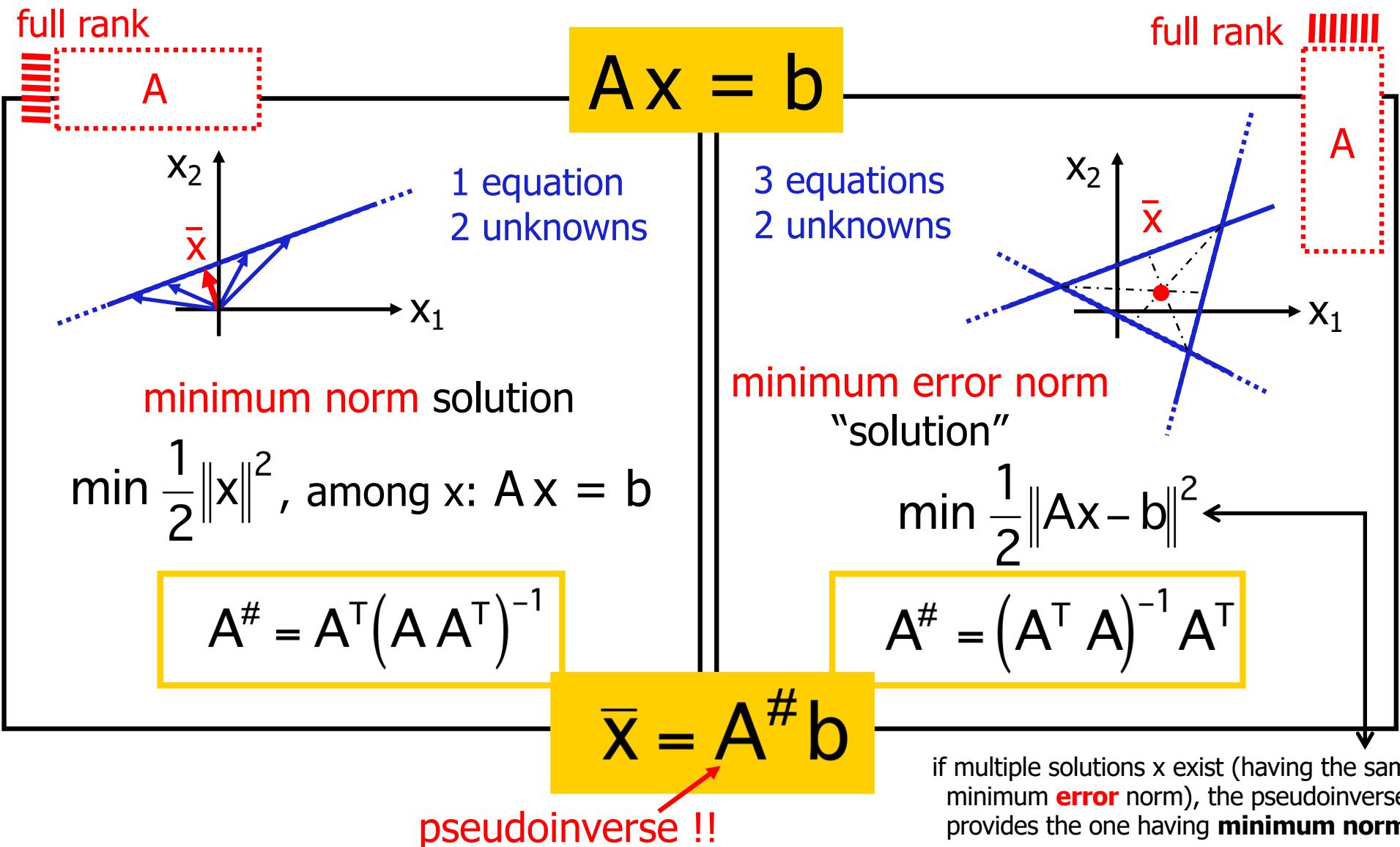
measures

regressor matrix evaluated at nominal parameters

unknowns

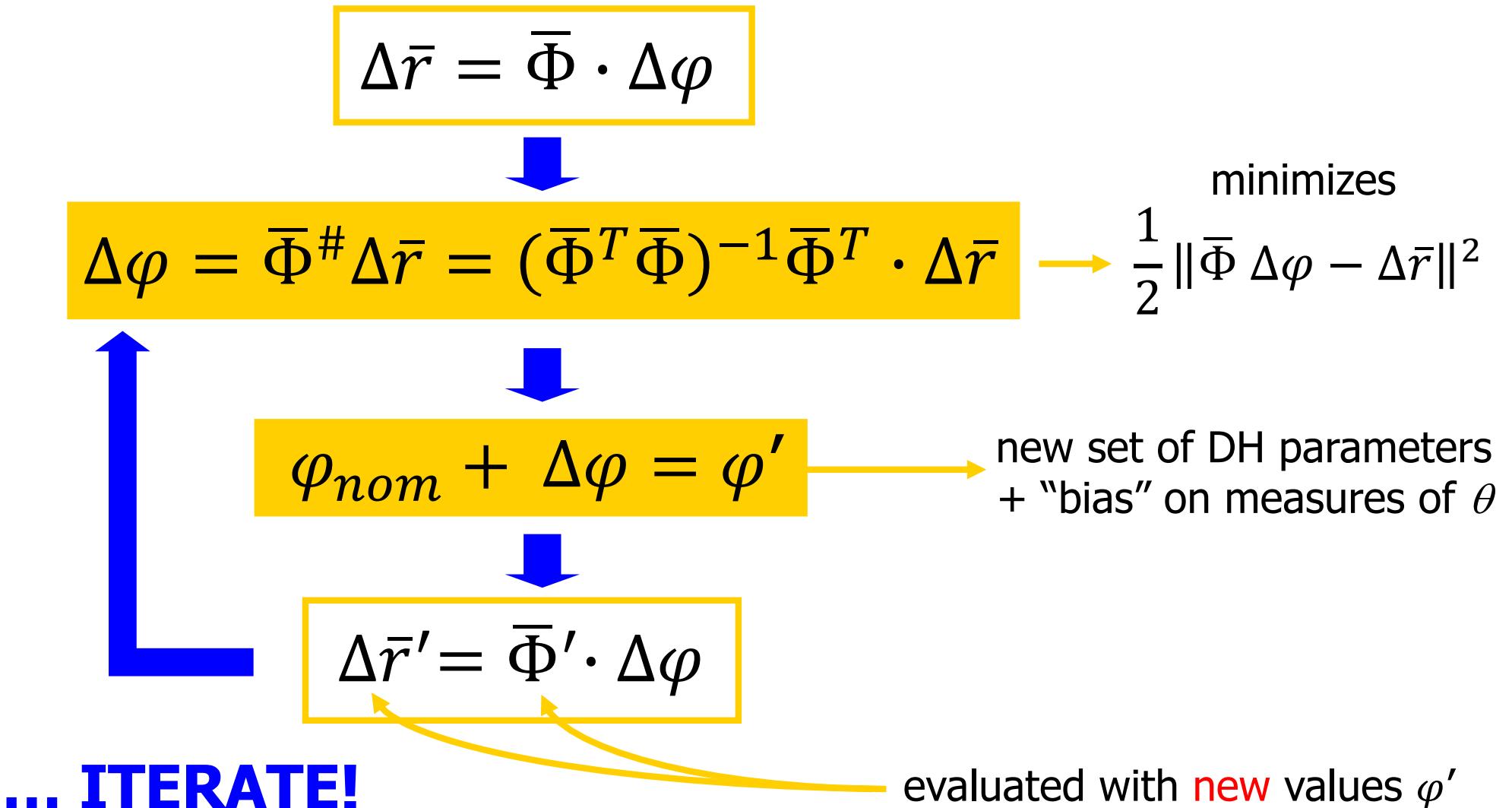


Under- and over-determined systems of linear equations





Calibration algorithm





Improvement by kinematic calibration

- ABB IRB 120 6R industrial robot
- 1000 random configurations (collision-free by simulation)
- 50 arbitrary configurations used for measurement in calibration
- 950 configurations used for validation
- Cartesian position errors

	before calibration	after calibration
Average	1.746 mm	0.193 mm
Median	1.567 mm	0.180 mm
Standard Deviation	1.043 mm	0.085 mm
Min	0.050 mm	0.010 mm
Max	4.423 mm	0.516 mm

- Improvement by **a factor $8 \div 10$**



Final comments

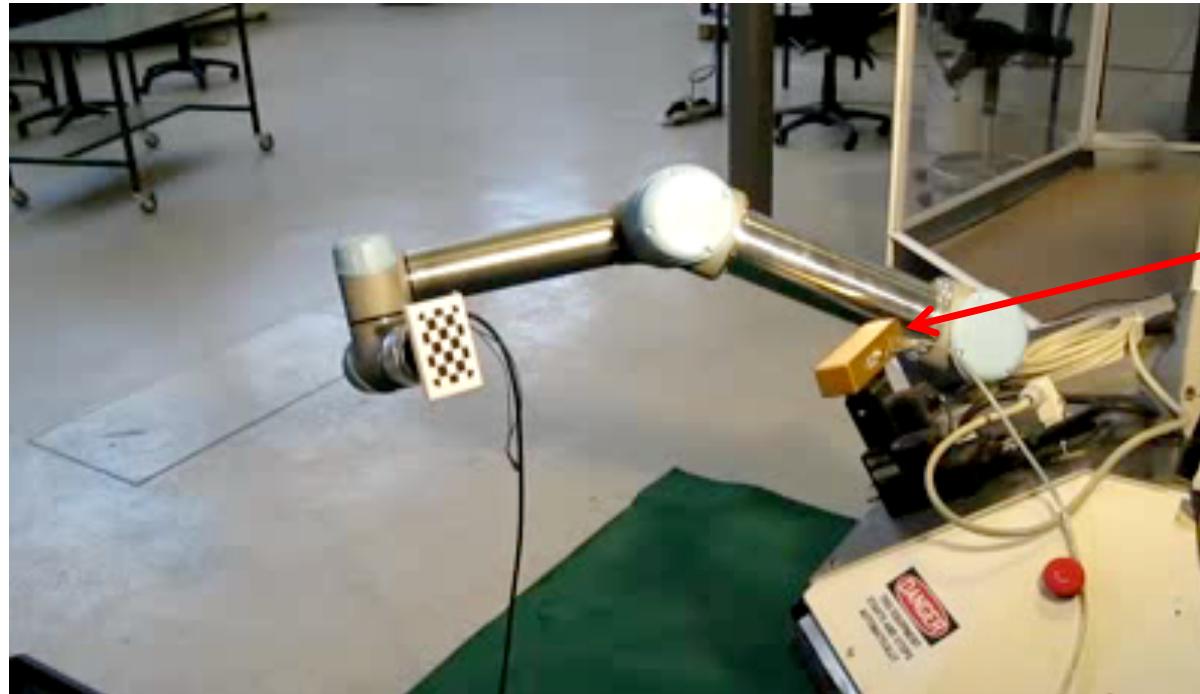
- an **iterative least squares** method
 - original problem is **nonlinear** in the unknowns, then linearized using first-order Taylor expansion
- it is useful to calibrate **first** and **separately** those quantities that are less accurate (typically, the encoder bias)
 - keeping the remaining ones at their nominal values
- **alternative** kinematic descriptions can be used
 - more complex than D-H parameters, but leading to a **better numerical conditioning** of the regressor matrix in calibration algorithm
 - one such description uses the POE (Product Of Exponential) formula
- more in general, **6 base parameters** should also be included
 - to locate 0-th robot frame w.r.t. world coordinate frame (of external sensor)
- accurate calibration/**estimation of real parameters** is a general problem in robotics (and beyond...)
 - for **sensors** (e.g., camera calibration)
 - for **models** (identification of dynamic parameters of a manipulator)



Calibration experiment

in a research environment

video



Videre Design
stereovision
camera

- automatic data acquisition for **simultaneous** calibration of
 - robot-camera transformation
 - DH parameters of the manipulator



Calibration experiment in an industrial setting

FANUC
3D Laser
calibration
(with iR Vision)



video

