24. Disturbance Decoupling problem DDP

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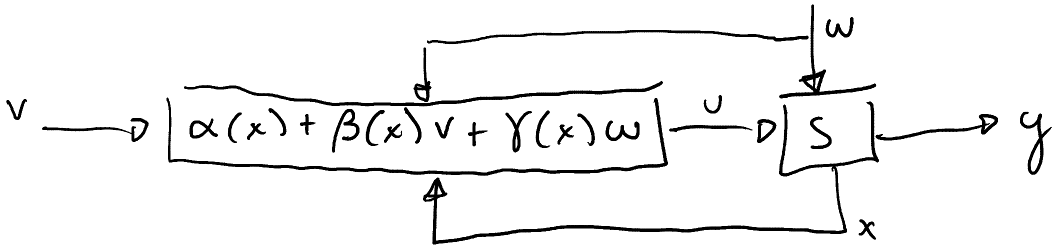
$$S: \begin{cases} \dot{x} = f(x) + g(x)u + p(x)w \\ y = h(x) \end{cases}$$

$$u = \alpha(x) + \beta(x)v \left(+ \gamma(x)w \right)$$ this last term is present only if w is measurable



Theorem: DDP is solvable if and only if

$$L_p L_f^i h(x) = 0 \qquad i = 0, \dots, \begin{cases} r-1 & \text{without perturbation measurement} \\ r-2 & \text{with perturbation measurement} \end{cases}$$

In particular r is related to the $u-y$ link while r_w is the relative degree related to the $w-y$ link whose first integer is such that $L_p L_f^{r_w-1} h \neq 0$.

The two conditions for the solvability can be expressed in terms of both relative degrees, in fact:

- $r-1$ without perturbation measurement $\iff r_w > r$
- $r-2$ with perturbation measurement $\iff r_w = r$

Sometimes it is useful to write the condition of solvability in different way.

Recall that $L_p L_f^i h(x) = \langle dL_f^i h(x), p(x) \rangle$ and define $\Omega = \text{span}\{dh, dL_f h, \dots, dL_f^{r-1} h\}$ codistribution, then it is clear that the condition of solvability is equivalent to:

$$p(x) \in \Omega^\perp(x) \quad \text{for all } x \text{ near } x_0$$

This geometric condition is clearly the same of the linear case, i.e., $D \in \ker \begin{pmatrix} C \\ \vdots \\ CA^{r-1} \end{pmatrix}$.

linear case, i.e., $\Delta \subset \ker\begin{pmatrix} \cdots \\ CA^{r-1}\end{pmatrix}$.

In fact $\Omega^{\perp} = \ker\begin{pmatrix} dh \\ \cdots \\ dL_f^{r-1}h\end{pmatrix}$.

Therefore $p(x) \subset \Omega^{\perp}$ is a necessary and sufficient geometrical condition for DDP.

Proof:

$$\begin{cases} \dot{x} = f + gu + pw \\ y = h \end{cases}$$

assuming $r_w \geq r$

$$y = h$$

$$\dot{y} = L_f h + u \underbrace{L_g h}_{\to 0} + w \underbrace{L_p h}_{\to 0}$$

$$\vdots$$

$$y^{(r-1)} = L_f^{r-1} h + u \underbrace{L_g L_f^{r-2} h}_{\to 0} + w \underbrace{L_p L_f^{r-2} h}_{\to 0}$$

$$y^{(r)} = L_f^{r} h + u L_g L_f^{r-1} h + \left(w L_p L_f^{r-1} h\right)$$

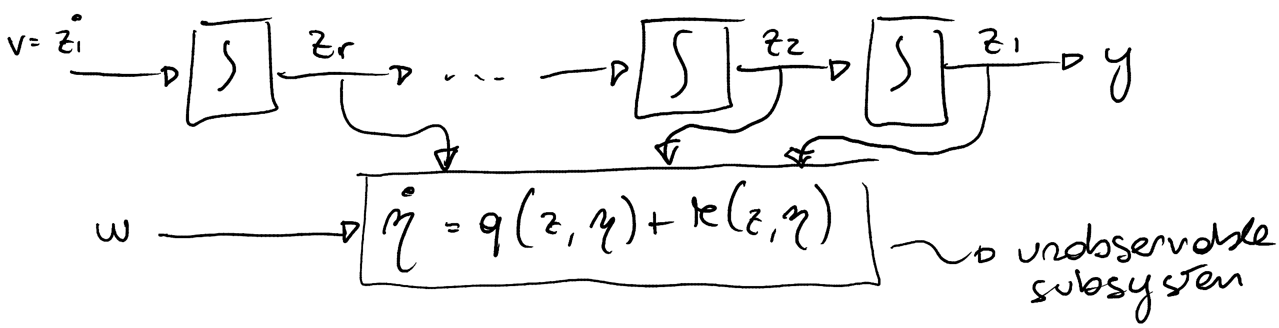
$\to = 0$ if $r_w > r$

$\to \neq 0$ if $r_w = r$

In the (z, η) coordinates the whole system can be seen as:

$$\dot{z} = \begin{pmatrix} z_2 \\ \vdots \\ b + au \end{pmatrix} \qquad u = \frac{-b(z,\eta)}{a(z,\eta)} + \frac{v}{a(z,\eta)}$$

$$\dot{\eta} = q(z,\eta) + k(z,\eta)\, w$$



Measure of the disturbance

If w is available it can be used in the design of the control law.

$$u = \alpha(x) + \beta(x) v + \gamma(x) w$$

Then
$$\begin{cases} \dot{x} = f(x) + g(x)\alpha(x) + g(x)\beta(x) v + \big(g(x)\gamma(x) + p(x)\big) w \\ y = h(x) \end{cases}$$

clearly what is needed is to find a function $\gamma(x)$ such that:

$\big(g(x)\gamma(x) + p(x)\big) \in \Omega^{\perp}(x)$ for all x near x_0 which is equivalent to

$$L_{g\gamma+p} L_f^i h(x) = L_g L_f^i h(x)\gamma(x) + L_p L_f^i h(x) = 0$$

for all $0 \le i \le r-1$ and all x near x_0.

Recalling the definition of relative degree:

$L_p L_f^i h(x) = 0$ for all $0 \le i \le r-2$

$L_p L_f^{r-1} h(x) = - L_g L_f^{r-1} h(x)\gamma(x)$ for all x near x_0

$$\gamma(x) = - \frac{L_p L_f^{r-1} h(x)}{L_g L_f^{r-1} h(x)}$$

$$u = \underbrace{- \frac{L_f^r h}{L_g L_f^{r-1} h}}_{\alpha} + \underbrace{\frac{v}{L_g L_f^{r-1} h}}_{\beta v} \underbrace{\left(- \frac{L_p L_f^{r-1} h}{L_g L_f^{r-1} h} w \right)}_{\gamma w \;\to\; = 0 \text{ if } r_w > r}$$

In conclusion, necessary and sufficient condition for DDP solvability with measure of w is that:

$$\exists \gamma : (g\gamma + p) \in \Omega^{\perp} = \ker \begin{pmatrix} dh \\ \cdots \\ dL_f^{r-1} h \end{pmatrix}$$

$p = -\gamma g + p_1 \qquad p_1 \in \Omega^{\perp}$

$$\rho = -\gamma e + \rho_1 \qquad \rho_1 \in \Omega^\perp$$

$$\Updownarrow$$

$$\rho \in \Omega^\perp + \operatorname{span}\{e\}$$

DDP with stability

Let $\underline{\alpha}$ be Hurwitz, $v = -\alpha \begin{pmatrix} h \\ \cdots \\ L_g^{r-1} h \end{pmatrix} + \bar{v}$

$$u = \frac{1}{a}\left(-b - \sum_{i=1}^{r} \alpha_i^* L_g^i h(x) + \bar{v}\right)$$

$$\begin{cases} \dot{z} = \begin{pmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ -\alpha_0^* & \cdots & & -\alpha_{r-1}^* \end{pmatrix} z + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \bar{v} \\ \dot{\eta} = q(z,\eta) + k(z,\eta)\, w \end{cases}$$

if the zero dynamics is AS (at least LAS)

$\forall \varepsilon\ \exists \delta, k : \|x_0\| < \delta\ \&\ |w(t)| < k\ \&\ |\bar{v}(t)| < k \quad \forall t \geq 0$

$\Rightarrow \|x(t)\| < \varepsilon \quad \forall t \geq 0$