Robotics 2

Adaptive Trajectory Control

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Motivation and approach

- need of adaptation in robot motion control laws
  - large uncertainty on the robot dynamic parameters
  - poor knowledge of the inertial payload
- characteristics of direct adaptive control
  - direct aim is to bring to zero the state trajectory error, i.e., position and velocity errors
  - no need to estimate on line the true values of the dynamic coefficients of the robot (as opposed to indirect adaptive control)
- main tool and methodology
  - linear parametrization of robot dynamics
  - nonlinear control law of the dynamic type (the controller has its own 'states')

Summary of robot parameters

- parameters assumed to be known
  - kinematic description based, e.g., on Denavit-Hartenberg parameters ($\{\alpha_i, d_i, a_i, i = 1, \dots, N\}$ in case of all revolute joints), including link lengths (kinematic calibration)
- uncertain parameters that can be identified off line
  - masses m_i, positions r_{ci} of CoMs, and inertia matrices I_i of each link, appearing in combinations (dynamic coefficients) $\Rightarrow p \ll 10 \times N$
- parameters that are (slowly) varying during operation
  - viscous F_{Vi}, dry F_{Si}, and stiction F_{Di} friction at each joint $\Rightarrow 1 \div 3 \times N$
- unknown and abruptly changing parameters
  - mass, CoM, inertia matrix of the payload w.r.t. the tool center point

$\Downarrow$

when a payload is firmly attached to the robot E-E, only the 10 parameters of the last link are modified, influencing however most part of the robot dynamics

Goal of adaptive control

- given a twice-differentiable desired joint trajectory $q_d(t)$
  - with known desired velocity $\dot{q}_d(t)$ and acceleration $\ddot{q}_d(t)$
  - possibly obtained by kinematic inversion + joint interpolation
- execute this trajectory under large dynamic uncertainties
  - with a trajectory tracking error vanishing asymptotically

$$e = q_d - q \longrightarrow 0 \qquad \dot{e} = \dot{q}_d - \dot{q} \longrightarrow 0$$

  - guaranteeing global stability, no matter how far are the initial estimates of the unknown/uncertain parameters from their true values and how large is the initial trajectory error

- identification is not of particular concern: in general, the estimates of dynamic coefficients will not to converge to the true ones!
- if this convergence is a specific extra requirement, then one should use (more complex) indirect adaptive schemes

Linear parameterization

$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) + F_V\dot{q} = u$$

- there exists always a (p-dimensional) vector a of dynamic coefficients, so that the robot model takes the linear form

$$Y(q,\dot{q},\ddot{q})\, a = u$$

- vector a contains only unknown or uncertain coefficients
- each component of a is in general a combination of the robot physical parameters (not necessarily all of them)
- the model regression matrix Y depends linearly on $\ddot{q}$, quadratically on $\dot{q}$ (for the terms related to kinetic energy), and nonlinearly (trigonometrically) on q

Trajectory controllers
based on model estimates

- inverse dynamics feedforward (FFW) + PD (linear) control

$$u = \underbrace{\hat{M}(q_d)\ddot{q}_d + \hat{S}(q_d, \dot{q}_d)\dot{q}_d + \hat{g}(q_d) + \hat{F}_V\dot{q}_d}_{\hat{u}_d} + K_P e + K_D \dot{e}$$

- (nonlinear) control based on feedback linearization (FBL)

$$u = \hat{M}(q)(\ddot{q}_d + K_P e + K_D \dot{e}) + \hat{S}(q, \dot{q})\dot{q} + \hat{g}(q) + \hat{F}_V\dot{q}$$

$$\boxed{\hat{M}, \hat{S}, \hat{g}, \hat{F}_V \quad \Longleftrightarrow \quad \text{estimate } \hat{a}}$$

- approximate estimates of dynamic coefficients may lead to instability with FBL due to temporary 'non-positive' PD gains (e.g., $\hat{M}(q)K_P < 0$!)
- not easy to turn these laws in adaptive schemes: inertia inversion/use of acceleration (FBL); bounds on PD gains (FFW)

A control law easily made 'adaptive'

- nonlinear trajectory tracking control (without cancellations) having global asymptotic stabilization properties

$$u = \hat{M}(q)\ddot{q}_d + \hat{S}(q,\dot{q})\dot{q}_d + \hat{g}(q) + \hat{F}_V\dot{q}_d + K_P e + K_D\dot{e}$$

- a natural adaptive version would require ...

$$\dot{\hat{a}} = \text{designing a suitable update law (in continuous time)}$$

  - without extra assumptions, it can be shown only that joint velocities become eventually "clamped" to those of the desired trajectory (zero velocity error), but a permanent residual position error is left

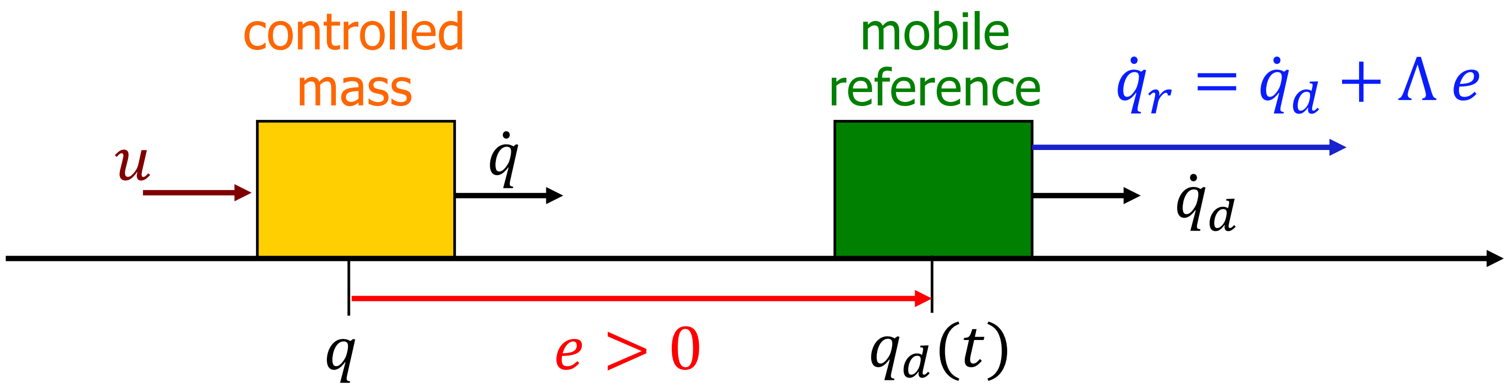
- idea: on-line modification with a reference velocity

$$\dot{q}_d \quad \longrightarrow \quad \boxed{\dot{q}_r = \dot{q}_d + \Lambda(q_d - q)} \qquad \Lambda > 0$$

typically $\Lambda = K_D^{-1}K_P$ (all matrices will be chosen diagonal)

Intuitive interpretation of $\dot{q}_r$

- elementary case
  - a mass 'lagging behind its mobile reference ($e > 0$) on a linear rail



➡ 'enhanced' velocity error $s = \dot{q}_r - \dot{q} > \dot{q}_d - \dot{q} = \dot{e}$

$$u = K_D s = K_D(\dot{q}_r - \dot{q}) = K_D(\dot{q}_d + \Lambda\, e - \dot{q}) = K_D\dot{e} + \underbrace{K_D\Lambda}_{K_P}\, e$$

  - a mass 'leading in front' of its mobile reference ($e < 0$)

➡ in a symmetric way, a 'reduced' velocity error will appear ($s < \dot{e}$)

Adaptive control law design

- substituting $\dot{q}_r = \dot{q}_d + \Lambda e$, $\ddot{q}_r = \ddot{q}_d + \Lambda\dot{e}$ in the previous nonlinear controller for trajectory tracking

$$\boxed{\begin{aligned} u &= \widehat{M}(q)\ddot{q}_r + \hat{S}(q,\dot{q})\dot{q}_r + \hat{g}(q) + \widehat{F}_V\dot{q}_r + K_P e + K_D\dot{e} \\ &= Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\hat{a} \qquad + \qquad K_P e + K_D\dot{e} \end{aligned}}$$

dynamic parameterization of the control law using current estimates (note here the 4 arguments in $Y(\cdot)$!)

PD stabilization (diagonal matrices, >0)

- update law for the estimates of the dynamic coefficients ($\hat{a}$ becomes the p-dimensional state of the dynamic controller)

$$\boxed{\dot{\hat{a}} = \Gamma Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)\underbrace{(\dot{q}_r - \dot{q})}_{s}}$$

'modified' velocity error

$\Gamma > 0$ (diagonal)

estimation gains (variation rate of estimates)

Asymptotic stability of trajectory error

Theorem

The introduced adaptive controller makes the tracking error along the desired trajectory globally asymptotically stable

$$e = q_d - q \longrightarrow 0,\ \dot{e} = \dot{q}_d - \dot{q} \longrightarrow 0$$

Proof

- a Lyapunov candidate for the closed-loop system (robot + dynamic controller) is given by

$$V = \frac{1}{2} s^T M(q) s + \frac{1}{2} e^T R e + \frac{1}{2} \tilde{a}^T \Gamma^{-1} \tilde{a} \ \geq 0$$

$$s = \dot{q}_r - \dot{q}\ (= \dot{e} + \Lambda e) \qquad R > 0 \qquad \tilde{a} = a - \hat{a}$$

modified velocity error | constant matrix (to be specified later) | error in parametric estimation

$$V = 0 \quad \Longleftrightarrow \quad \hat{a} = a,\ \ q = q_d,\ \ s = 0 \ \ (\Rightarrow\ \dot{q} = \dot{q}_d)$$

Proof (cont)

- the time derivative of V is

$$\dot{V} = \frac{1}{2} s^T \dot{M}(q) s + s^T M(q)\dot{s} + e^T R \dot{e} - \tilde{a}^T \Gamma^{-1} \dot{\hat{a}}$$

since $\dot{\tilde{a}} = -\dot{\hat{a}}$ $(\dot{a} = 0)$

- the closed-loop dynamics is given by

$$M(q)\ddot{q} + S(q,\dot{q})\dot{q} + g(q) + F_V \dot{q} =$$
$$= \hat{M}(q)\ddot{q}_r + \hat{S}(q,\dot{q})\dot{q}_r + \hat{g}(q) + \hat{F}_V \dot{q}_r + K_P e + K_D \dot{e}$$

subtracting the two sides from $M(q)\ddot{q}_r + S(q,\dot{q})\dot{q}_r + g(q) + F_V \dot{q}_r$ leads to

$$M(q)\dot{s} + (S(q,\dot{q}) + F_V)s =$$
$$= \tilde{M}(q)\ddot{q}_r + \tilde{S}(q,\dot{q})\dot{q}_r + \tilde{g}(q) + \tilde{F}_V \dot{q}_r - K_P e - K_D \dot{e}$$

with $\tilde{M} = M - \hat{M},\ \tilde{S} = S - \hat{S},\ \tilde{g} = g - \hat{g},\ \tilde{F}_V = F_V - \hat{F}_V$

Proof (cont)

- from the property of linearity in the dynamic coefficients, it follows

$$M(q)\dot{s} + (S(q,\dot{q}) + F_V)s = Y(q,\dot{q},\dot{q}_r,\ddot{q}_r)\tilde{a} - K_P e - K_D \dot{e}$$

- substituting in $\dot{V}$, together with $\dot{\hat{a}} = \Gamma Y^T s$, and using the skew-symmetry of matrix $\dot{M} - 2S$ we obtain

$$\dot{V} = \frac{1}{2} s^T [\dot{M}(q) \cancel{- 2S(q,\dot{q})}] s - s^T F_V s + \cancel{s^T Y \tilde{a}}$$
$$- s^T (K_P e + K_D \dot{e}) + e^T R \dot{e} - \cancel{\tilde{a}^T Y^T s}$$
$$= -s^T F_V s - s^T (K_P e + K_D \dot{e}) + e^T R \dot{e}$$

- replacing $s = \dot{e} + \Lambda\, e$ and being $F_V = F_V^T$ (diagonal)

$$\dot{V} = -e^T (\Lambda^T F_V \Lambda + \Lambda^T K_P) e$$
$$- e^T (2\Lambda^T F_V + \Lambda^T K_D + K_P - R) \dot{e} - \dot{e}^T (F_V + K_D) \dot{e}$$

a complete quadratic form in $e, \dot{e}$!

Proof (end)

- defining now (all matrices are diagonal!)

$$\Lambda = K_D^{-1} K_P > 0 \qquad R = 2K_P\,(I + K_D^{-1} F_V) > 0$$

leads to

$$\begin{aligned}\dot{V} &= -e^T \Lambda^T (F_V + K_D) \Lambda e - \dot{e}^T (F_V + K_D) \dot{e} \\ &= -e^T K_P K_D^{-1} (F_V + K_D) K_D^{-1} K_P e - \dot{e}^T (F_V + K_D) \dot{e} \le 0\end{aligned}$$

and thus

$$\boxed{\dot{V} = 0 \quad \Leftrightarrow \quad e = \dot{e} = 0}$$

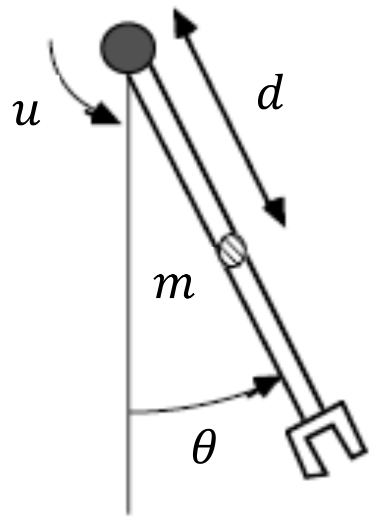
the thesis follows from Barbalat lemma + LaSalle theorem

the set of states of convergence has zero trajectory error and a constant value for $\hat{a}$, not necessarily the true one ($\tilde{a} \neq 0$)

Remarks

- if the desired trajectory $q_d(t)$ is persistently exciting, then also the estimates of the dynamic coefficients converge to their true values
- condition of persistent excitation
  - for linear systems: # of frequency components in the desired trajectory should be at least twice as large as # of unknown coefficients
  - for nonlinear systems: the condition can be checked only a posteriori (a certain motion integral should be permanently lower bounded)
- in case of known absence of (viscous) friction ($F_V \equiv 0$), the same proof applies (a bit easier in the final part)
- the adaptive controller does not require the inverse of the inertia matrix (true or estimated), nor the actual robot acceleration (only the desired acceleration), nor further lower bounds on $K_P > 0, K_D > 0$
- adaptation can be also used only for a subset of dynamic coefficients, the remaining being known ($Ya = Y_{adapt}\hat{a}_{adapt} + Y_{known}a_{known}$)
- the non-adaptive version (using accurate estimates) is a static tracking controller based on the passivity property of robot dynamics

Case study: Single-link under gravity



model $I\ddot{\theta} + mgd\sin\theta + f_V\dot{\theta} = u$ (with friction)

linear parameterization

$$Y(\theta,\dot{\theta},\ddot{\theta})a = \begin{bmatrix}\ddot{\theta} & \sin\theta & \dot{\theta}\end{bmatrix}\begin{bmatrix} I \\ mgd \\ f_V \end{bmatrix} = u$$

adaptive controller

$$e = \theta_d - \theta$$

$$\dot{\theta}_r = \dot{\theta}_d + \frac{k_P}{k_D}e \qquad \Lambda > 0$$

$$\gamma_i > 0, i = 1,2,3$$

$$u = \hat{I}\,\ddot{\theta} + \widehat{mgd}\sin\theta + \hat{f}_V\dot{\theta} + k_P e + k_D\dot{e}$$

$$\dot{\hat{a}} = \begin{pmatrix} \dot{\hat{I}} \\ \dot{\widehat{mgd}} \\ \dot{\hat{f}}_V \end{pmatrix} = \begin{pmatrix} \gamma_1\ddot{\theta}_r \\ \gamma_2\sin\theta \\ \gamma_3\dot{\theta}_r \end{pmatrix}(\dot{\theta}_r - \dot{\theta})$$

Simulation data

- real dynamic coefficients

$$I = 7.5, \qquad mgd = 6, \qquad f_V = 1$$

- initial estimates

$$\hat{I} = 5, \qquad \widehat{mgd} = 5, \qquad \hat{f}_V = 2$$

- control parameters

$$k_P = 25, \qquad k_D = 10, \qquad \gamma_i = 5, \qquad i = 1,2,3$$

- test trajectories (starting with $\theta(0) = 0, \dot{\theta}(0) = 0$)
  - first
  - second

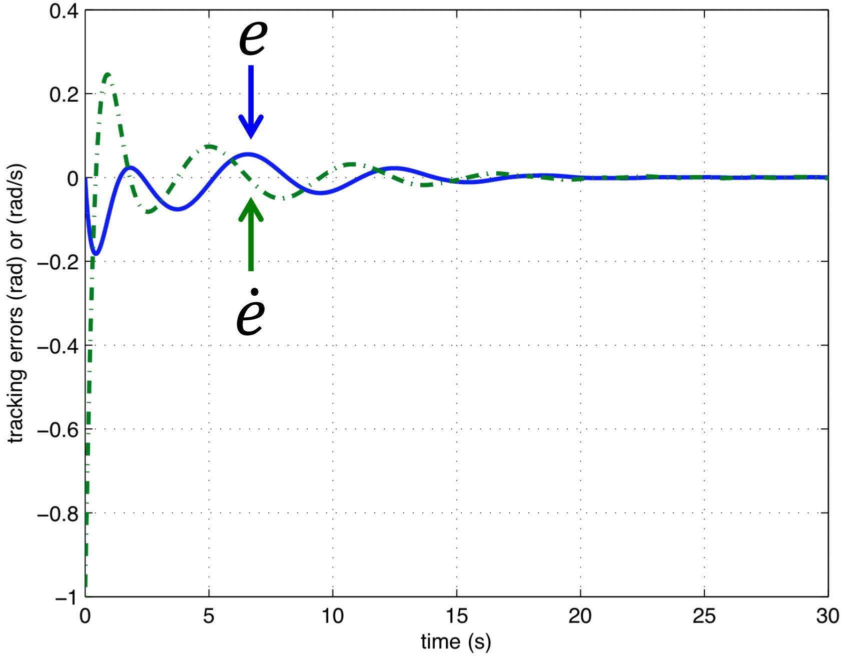
$$\theta_d(t) = -\sin t$$

Note: same test trajectories used also for robust control

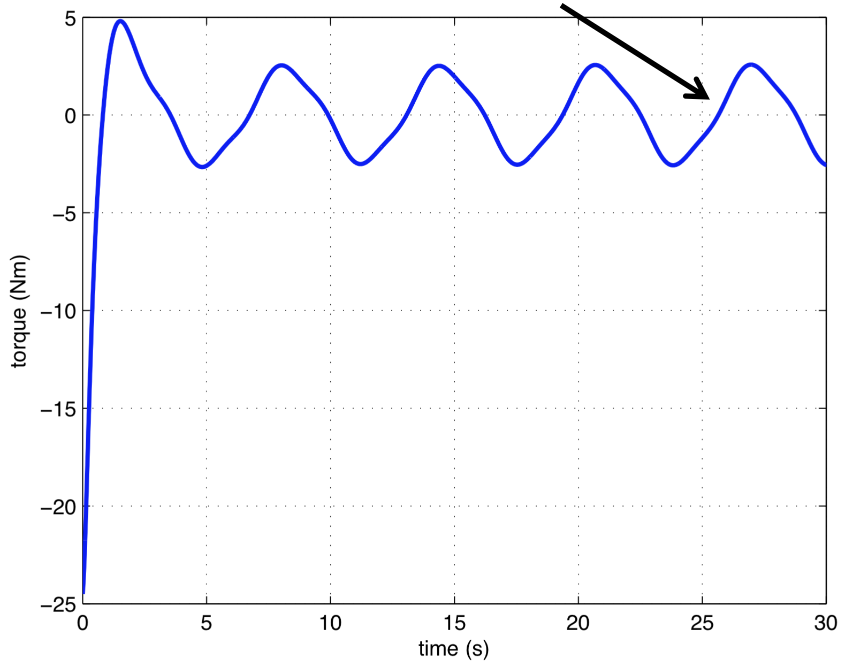
$\ddot{\theta}_d(t) =$ (periodic) bang-bang acceleration profile with $A = 1$ rad/s², $\omega = 1$ rad/s

Results
first trajectory

note the nonlinear system dynamics
(no sinusoidal regime at steady state!)



position and velocity errors

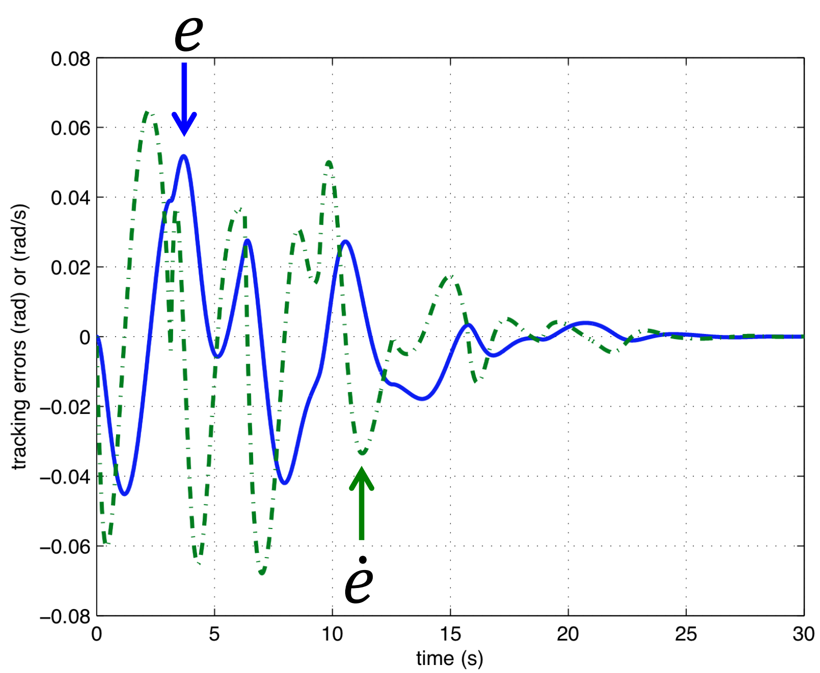


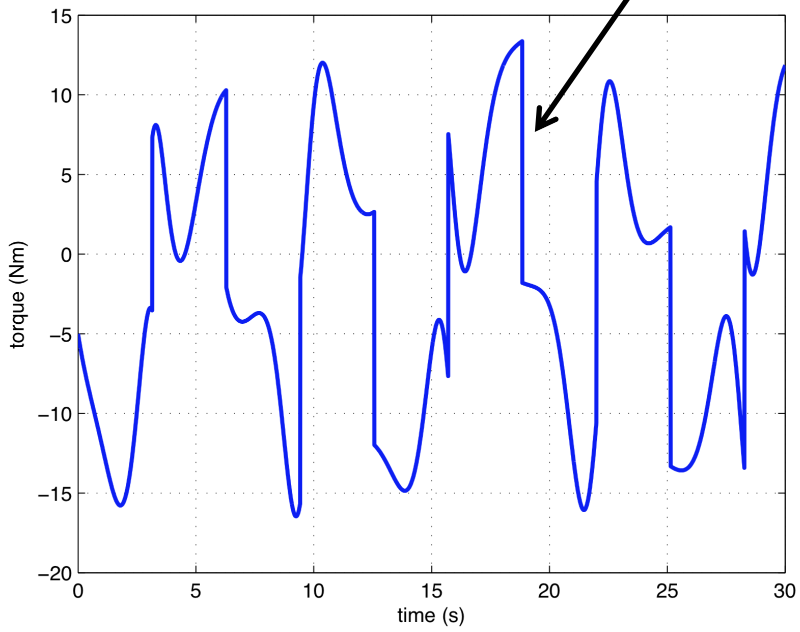
control torque

$$\theta_d(t) = -\sin t$$

Results
second trajectory

note the torque discontinuities
(due to those of the desired acceleration)



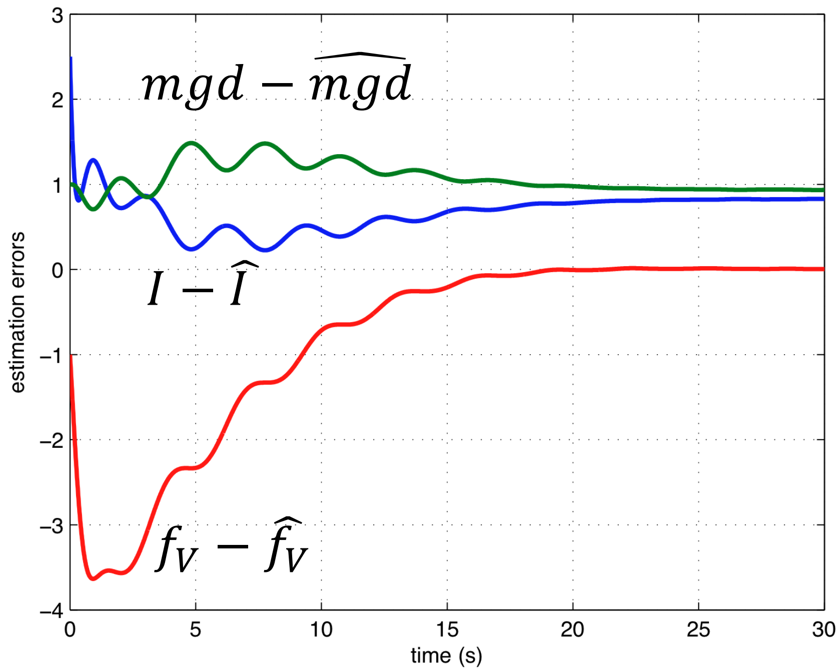


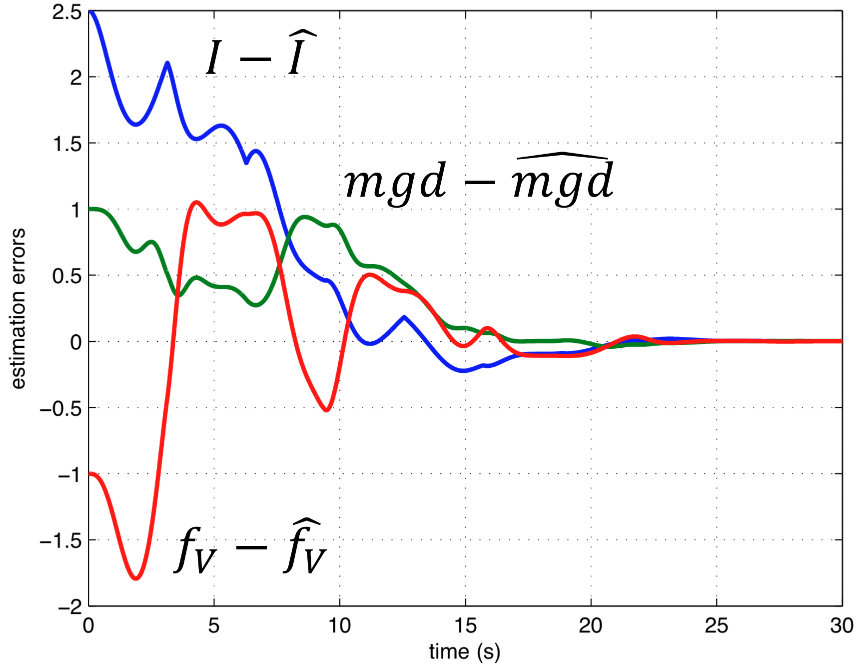
position and velocity errors

control torque

$$\ddot{\theta}_d(t) = \text{(periodic) bang-bang acceleration profile}$$

Estimates of dynamic coefficients





errors $\tilde{a} = a - \hat{a}$

first trajectory

only the estimate of the viscous friction coefficient converges to the true value

second trajectory

all three estimates of dynamic coefficients converge to their true values

A special case: Adaptive regulation

- adaptation in case q_d is constant
- no special simplifications for the presented adaptive control law (designed for the general tracking case…)

$$u = \widehat{M}(q)\ddot{q}_r + \hat{S}(q,\dot{q})\dot{q}_r + \hat{g}(q) + \widehat{F}_v\dot{q}_r + K_P e + K_D\dot{e}$$

$$\dot{\hat{a}} = \Gamma Y^T(q,\dot{q},\dot{q}_r,\ddot{q}_r)(\dot{q}_r - \dot{q})$$

since $\dot{q}_r = \Lambda(q_d - q)$ and $\ddot{q}_r = -\Lambda\dot{q}$ do not vanish!

- a different case would be the availability of an adaptive version of the trajectory tracking controller

$$u = \widehat{M}(q)\ddot{q}_d + \hat{S}(q,\dot{q})\dot{q}_d + \hat{g}(q) + \widehat{F}_v\dot{q}_d + K_P e + K_D\dot{e}$$

since, when q_d collapses to a constant, only the adaptation of the gravity term would be left over (which is what one would naturally expect…)

An efficient adaptive regulator

- use a linear parameterization of the gravity term only

$$g(q) = G(q)a_g$$

with a p_g-dimensional vector a_g

- an adaptive regulator yielding global asymptotic stability of the equilibrium state $(q_d, 0)$ is provided by

$$u = G(q)\hat{a}_g + K_P(q_d - q) - K_D\dot{q}$$

$$\dot{\hat{a}}_g = \gamma G^T(q)\left(\frac{2e}{1 + 2\|e\|^2} - \beta\dot{q}\right), \qquad \gamma > 0$$

where $e = q_d - q$, $K_P > 0$, $K_D > 0$ (symmetric), and $\beta > 0$ is chosen sufficiently large

(see paper by P. Tomei, IEEE TRA, 1991; available as extra material on the course web)

An adaptive regulator

Sketch of asymptotic stability analysis

- use the function

$$V = \frac{\beta}{2}\left(\dot{q}^T M(q)\dot{q} + e^T K_P e\right) - \frac{2\dot{q}^T M(q) e}{1 + 2\|e\|^2} + \frac{1}{2}\left(\hat{a}_g - a_g\right)^T\left(\hat{a}_g - a_g\right)$$

- a sufficient condition for V to be a Lyapunov candidate is that

$$\beta > \frac{2M_M}{\sqrt{M_m K_{P,m}}}$$

- a sufficient condition which guarantees also that

$$\dot{V} = \cdots \leq -a\|e\|^2 - b\|\dot{q}\|^2 \leq 0, \qquad a > 0, b > 0$$

is

$$\|S(q,\dot{q})\| \leq \alpha_S \dot{q}$$

$$\beta > \max\left\{\frac{2M_M}{\sqrt{M_m K_{P,m}}}, \frac{1}{K_{D,m}}\left(\frac{K_{D,m}^2}{2K_{P,m}} + 4M_M + \frac{\alpha_S}{\sqrt{2}}\right)\right\}$$

Note: for any symmetric, positive definite matrix A

$$A_M = \lambda_{\max}(A) = \sqrt{\lambda_{\max}(A^T A)} = \|A\|$$
$$A_m = \lambda_{\min}(A)$$

and thus, e.g., $\frac{1}{2}\,\dot{q}^T M(q)\dot{q} \geq \frac{1}{2} M_m \|\dot{q}\|^2$