Robotics 2

Robot Interaction with the Environment

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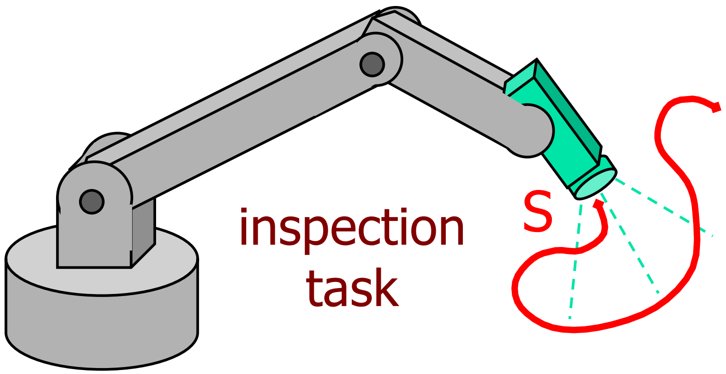
Robot-environment interaction

a robot (end-effector) may interact with the environment

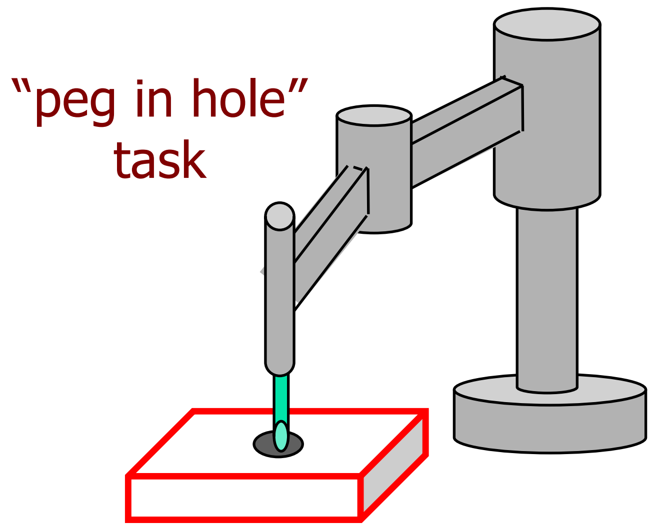
- modifying the state of the environment (e.g., pick-and-place operations)
- exchanging forces (e.g., assembly or surface finishing tasks)

control of free motion



sensors: position (encoders)
at the joints* or
vision at the Cartesian level

control of compliant motion



sensors: as before +
6D force/torque
(at the robot wrist)

*and velocity (by numerical differentiation
or, more rarely, with tachos)

Robot compliance

PASSIVE ⟺ ACTIVE

PASSIVE

robot end-effector equipped with mechatronic devices that "comply" with the generalized forces applied at the TCP = Tool Center Point

RCC = Remote Center of Compliance device

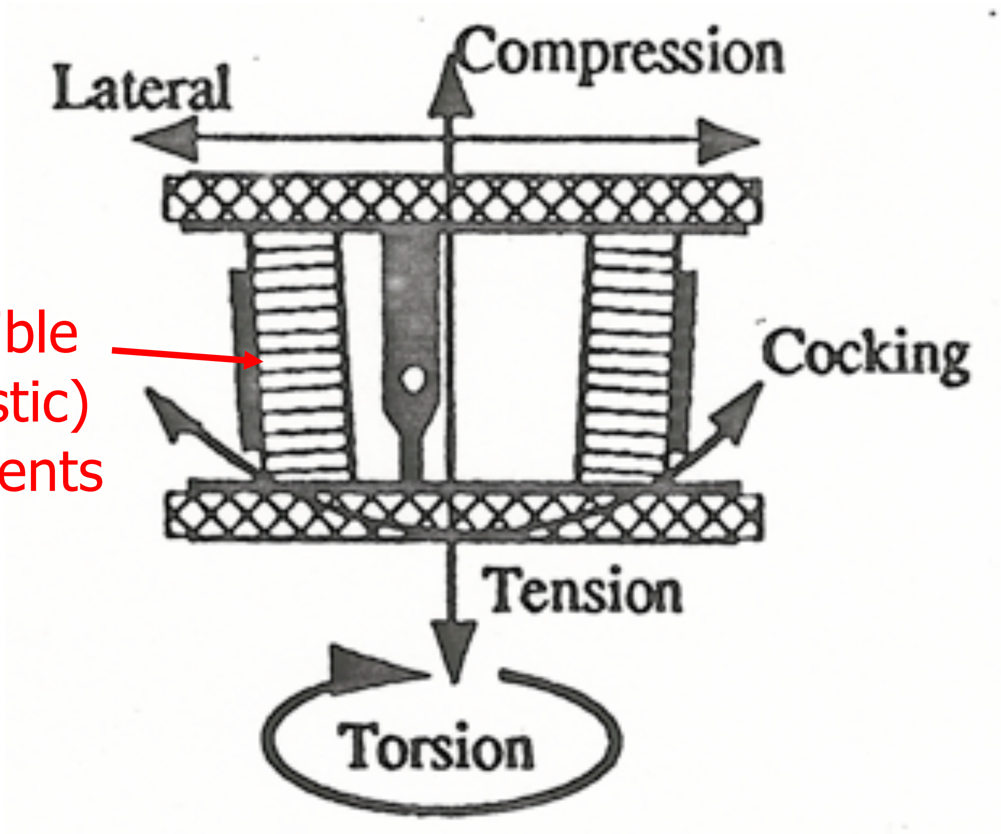
ACTIVE

robot is moved by a control law so as to react in a desired way to generalized forces applied at the TCP (typically measured by a F/T sensor)

- admittance control
  contact forces ⇒ velocity commands
- stiffness/compliance control
  contact displacements ⇒ force commands
- impedance control
  contact displacements ⇔ contact forces

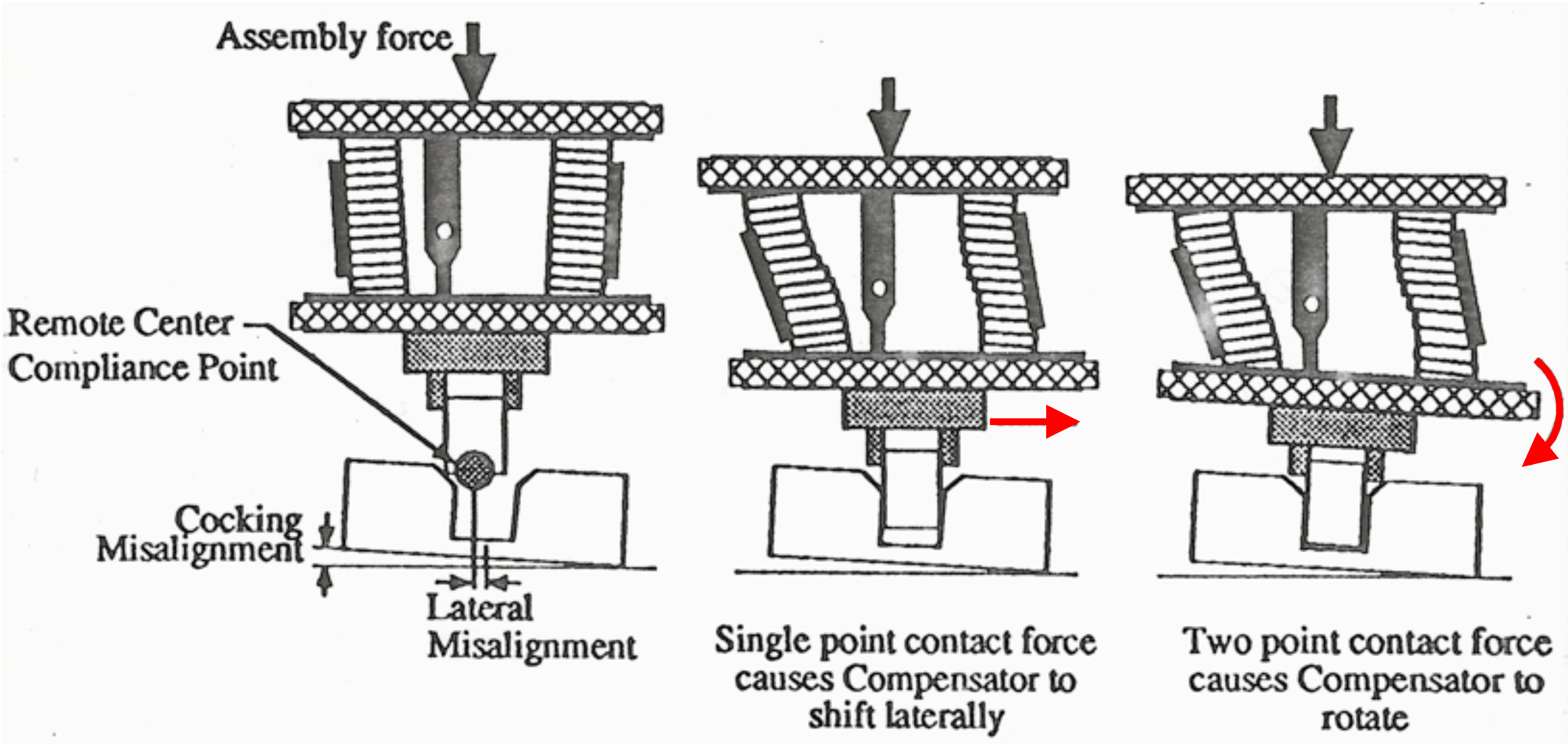
RCC device

flexible (elastic) elements

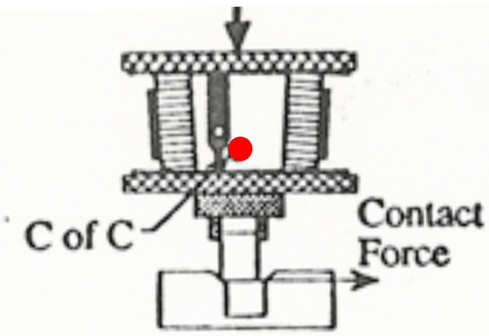


RCC models of different size by ATI

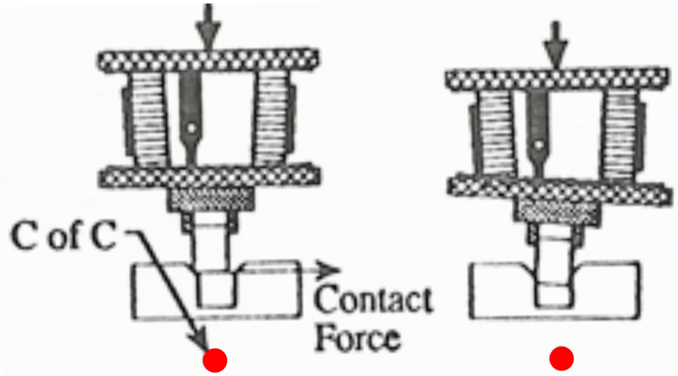
RCC behavior
in case of misalignment errors in assembly tasks



Effects of RCC positioning



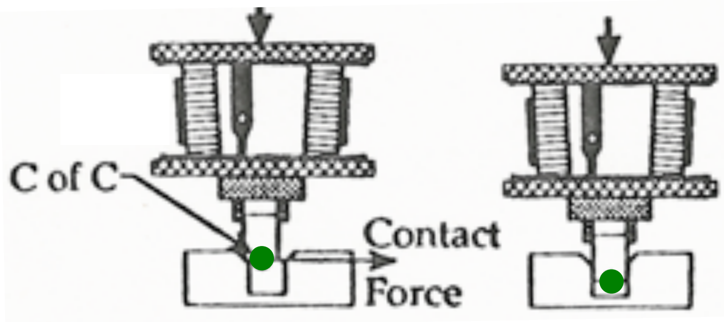
With the C of C far above the point of contact a lateral contact force causes the part to enter at an angle, causing a two point contact.



With the C of C far below the point of contact the part enters at an angle causing two point contact

too high...

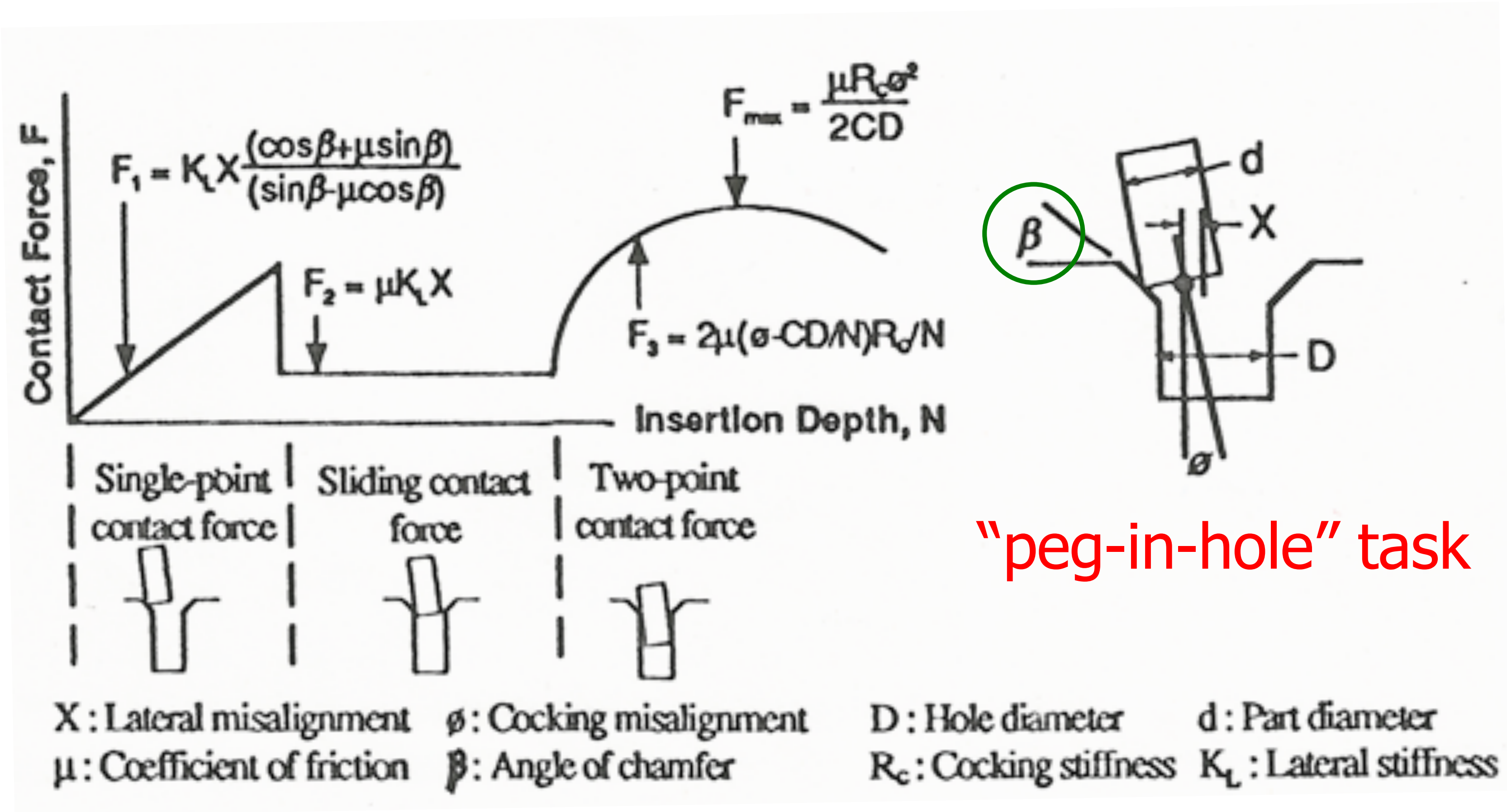
too low...



When the C of C is near the contact point the part enters correctly

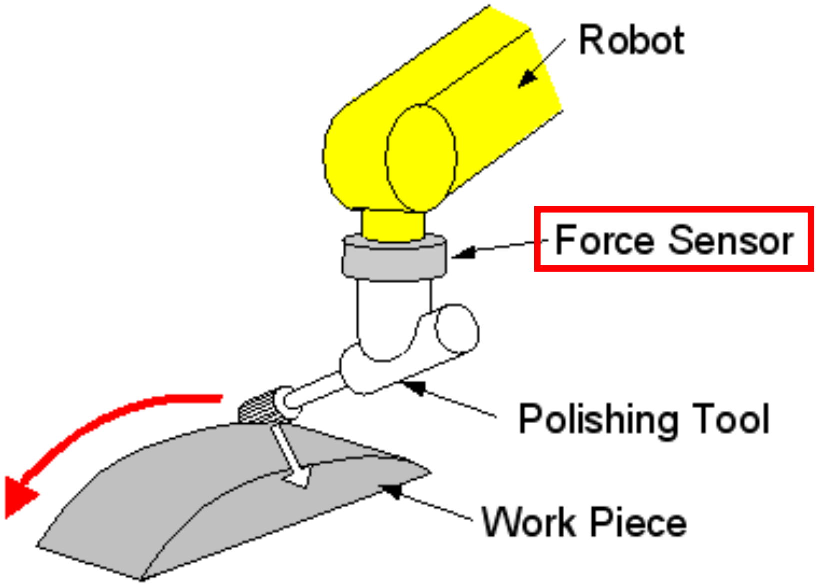
correct!
(TCP = RCC)

Typical evolution of assembly forces



chamfer angle β = to ease the insertion,
related also to the tolerances of the hole

Active compliance
for contour following

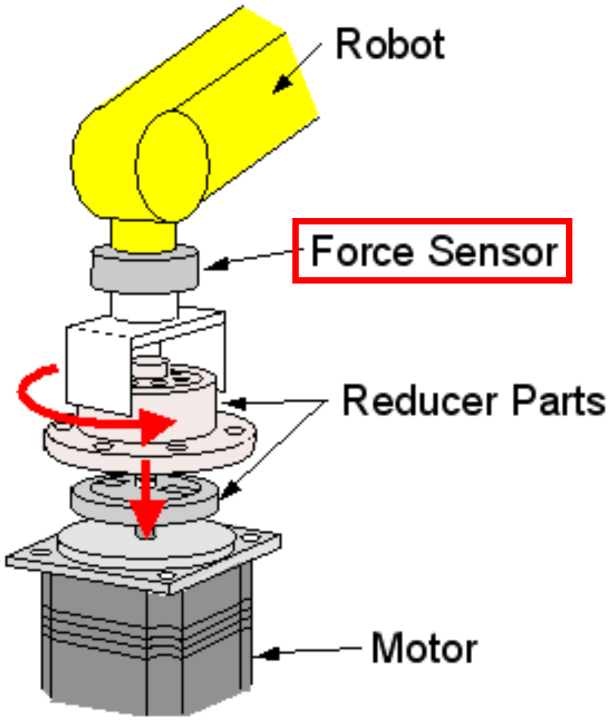


Following with constant pushing force

Washstand

Metal Cabinet

Active compliance
"matching" of mechanical parts



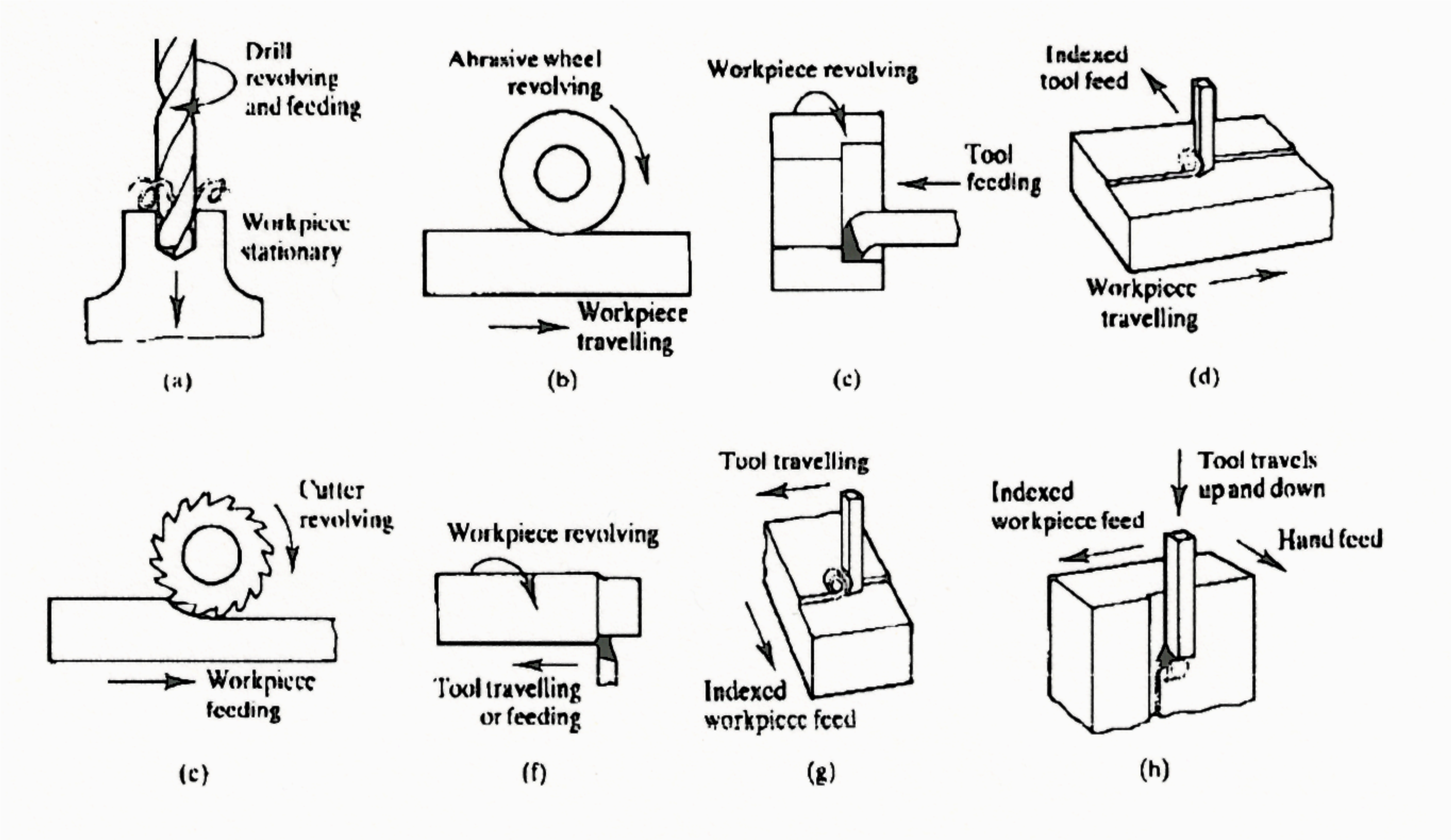
Phase matching by force sensing

Gear Parts

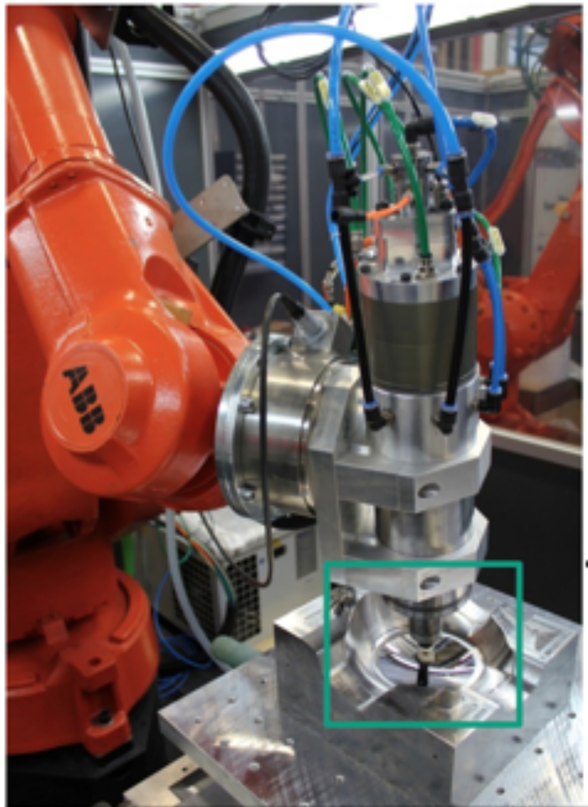
Tasks with environment interaction

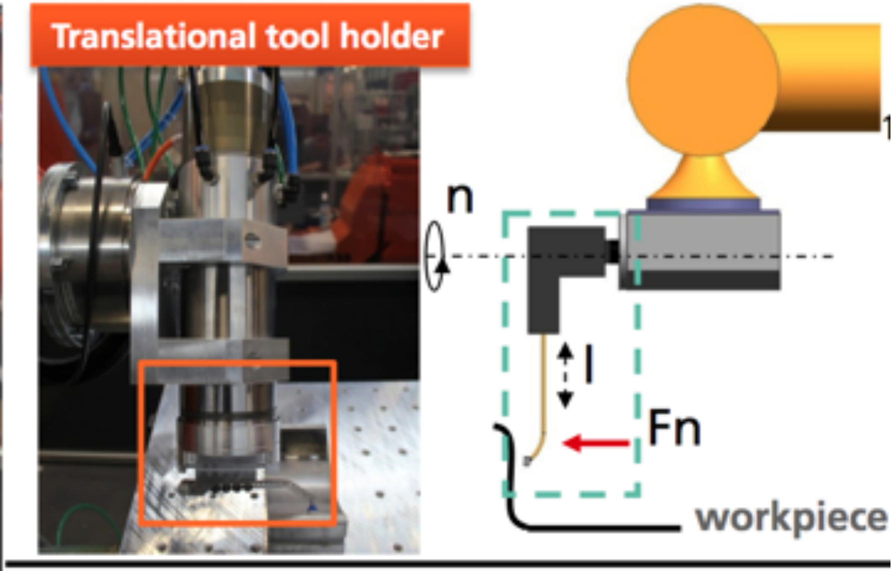
- mechanical machining
  - deburring, surface finishing, polishing, assembly,...
- tele-manipulation
  - force feedback improves performance of human operators in master-slave systems
- contact exploration for shape identification
  - force and velocity/vision sensor fusion allow 2D/3D geometric identification of unknown objects and their contour following
- dexterous robot hands
  - power grasp and fine in-hand manipulation require force/motion cooperation and coordinated control of the multiple fingers
- cooperation of multi-manipulator systems
  - the environment includes one of more other robots with their own dynamic behaviors
- physical human-robot interaction
  - humans as active, dynamic environments that need to be handled under full safety premises …

Examples of mechanical machining

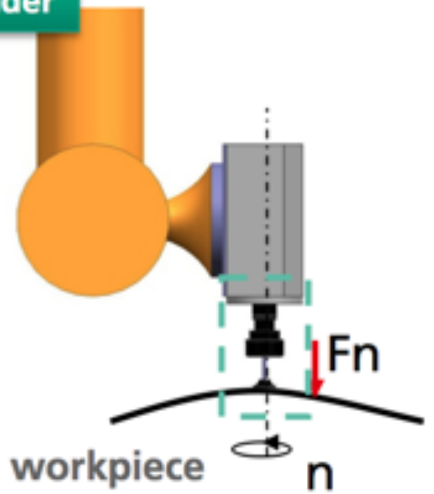


Abrasive finishing of surfaces





Rotational tool holder

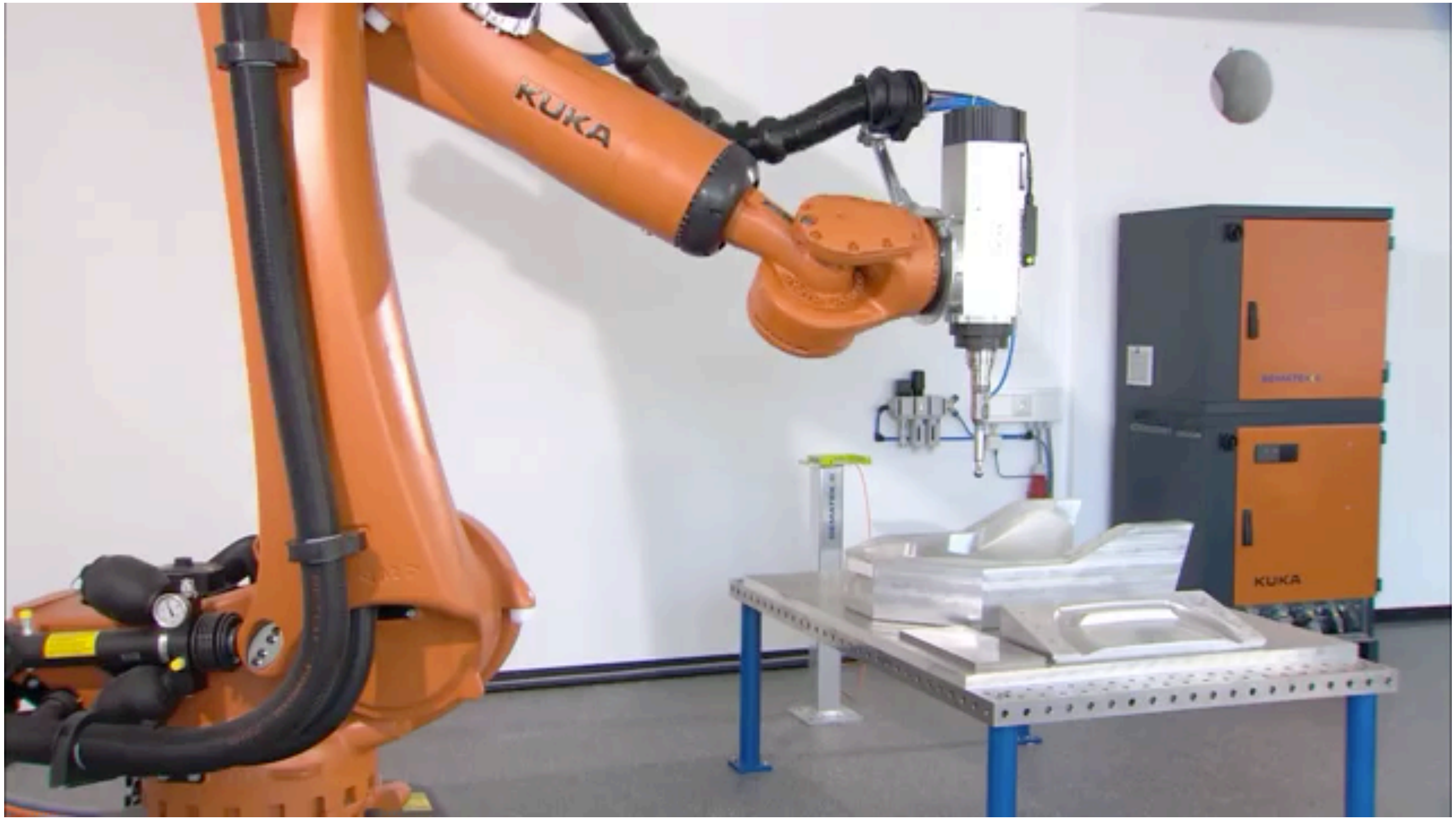


Main properties:

- synchronous motor
- rotation : 100 - 36.000 rpm
- power : 6 kW
- mass : 16 kg
- automated tool exchanger
- pneumatic canals for the force control (x3)

Abrasive finishing of surfaces

video



technological processes: cold forging of surfaces
and hammer peening by pneumatic machine

Non-contact surface finishing

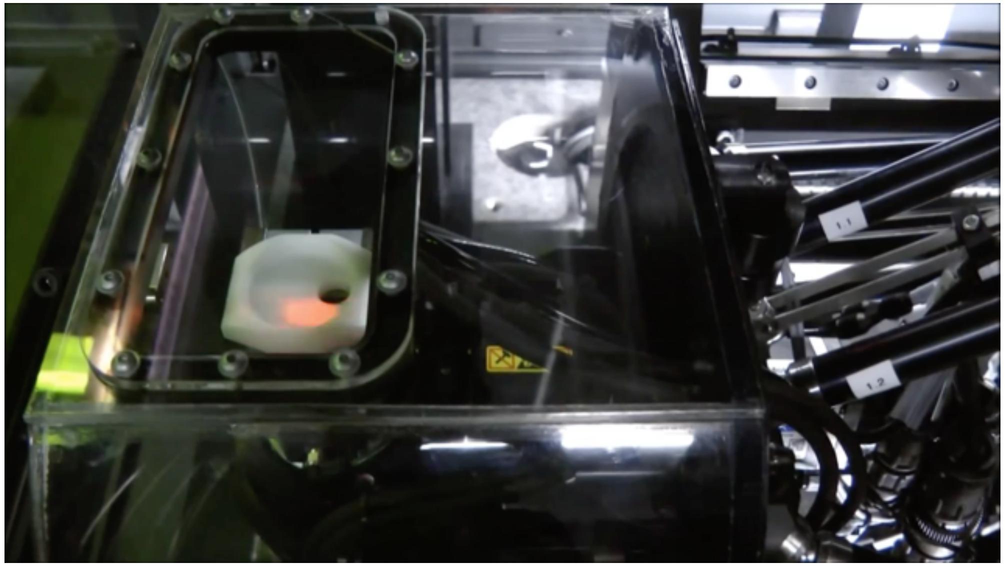
video

Fluid Jet technology

SYMPLEXITY

H2020 EU project for the
Factory of the Future (FoF)

Pulsed Laser technology



video

In all cases ...

- for physical interaction tasks, the desired motion specification and execution should be integrated with complementary data for the desired force

  ➔ hybrid planning and control objectives

- the exchanged forces/torques at the contact(s) with the environment can be explicitly set under control or simply kept limited in an indirect way

Evolution of control approaches
a bit of history from the late 70's-mid '80s ...

- explicit control of forces/torques only [Whitney]
  - used in quasi-static operations (assembly) in order to avoid deadlocks during part insertion
- active admittance and compliance control [Paul, Shimano, Salisbury]
  - contact forces handled through position (stiffness) or velocity (damping) control of the robot end-effector
  - robot reacts as a compressed spring (with damper) in selected/all directions
- impedance control [Hogan]
  - a desired dynamic behavior is imposed to the robot-environment interaction, e.g., a "model" with forces acting on a mass-spring-damper
  - mimics the human arm behavior moving in an unknown environment
- hybrid force-motion control [Mason]
  - decomposes the task space in complementary sets of directions where either force or motion is controlled, based on
    - a purely kinematic robot model [Raibert, Craig]
    - the actual dynamic model of the robot [Khatib]

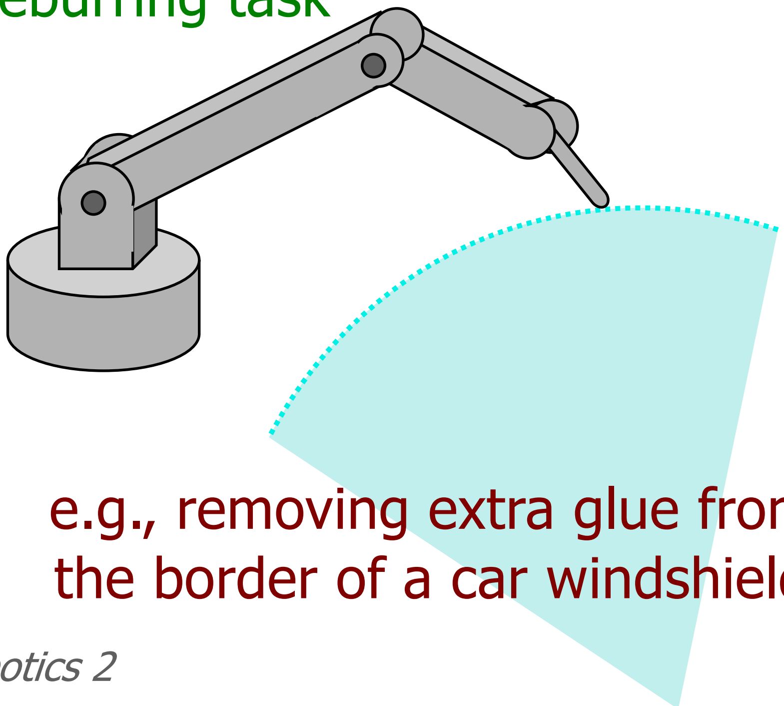
appropriate for fast and accurate motion in dynamic interaction...

Interaction tasks of interest

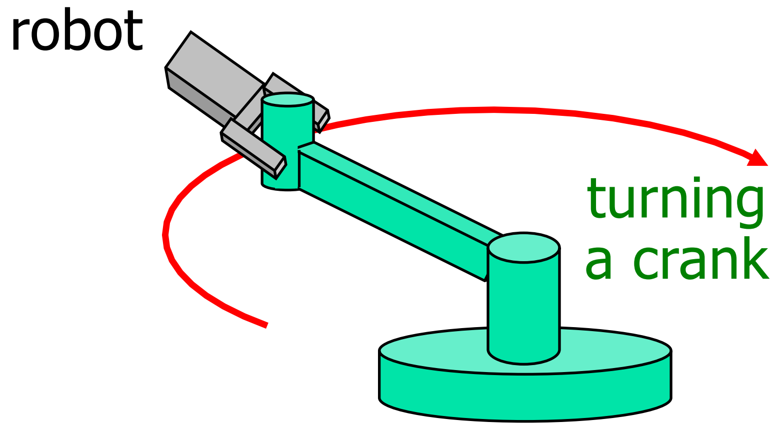
interaction tasks with the environment that require

- accurate following/reproduction by the robot end-effector of desired trajectories (even at high speed) defined on the surface of objects
- control of forces/torques applied at the contact with environments having low (soft) or high (rigid) stiffness

deburring task

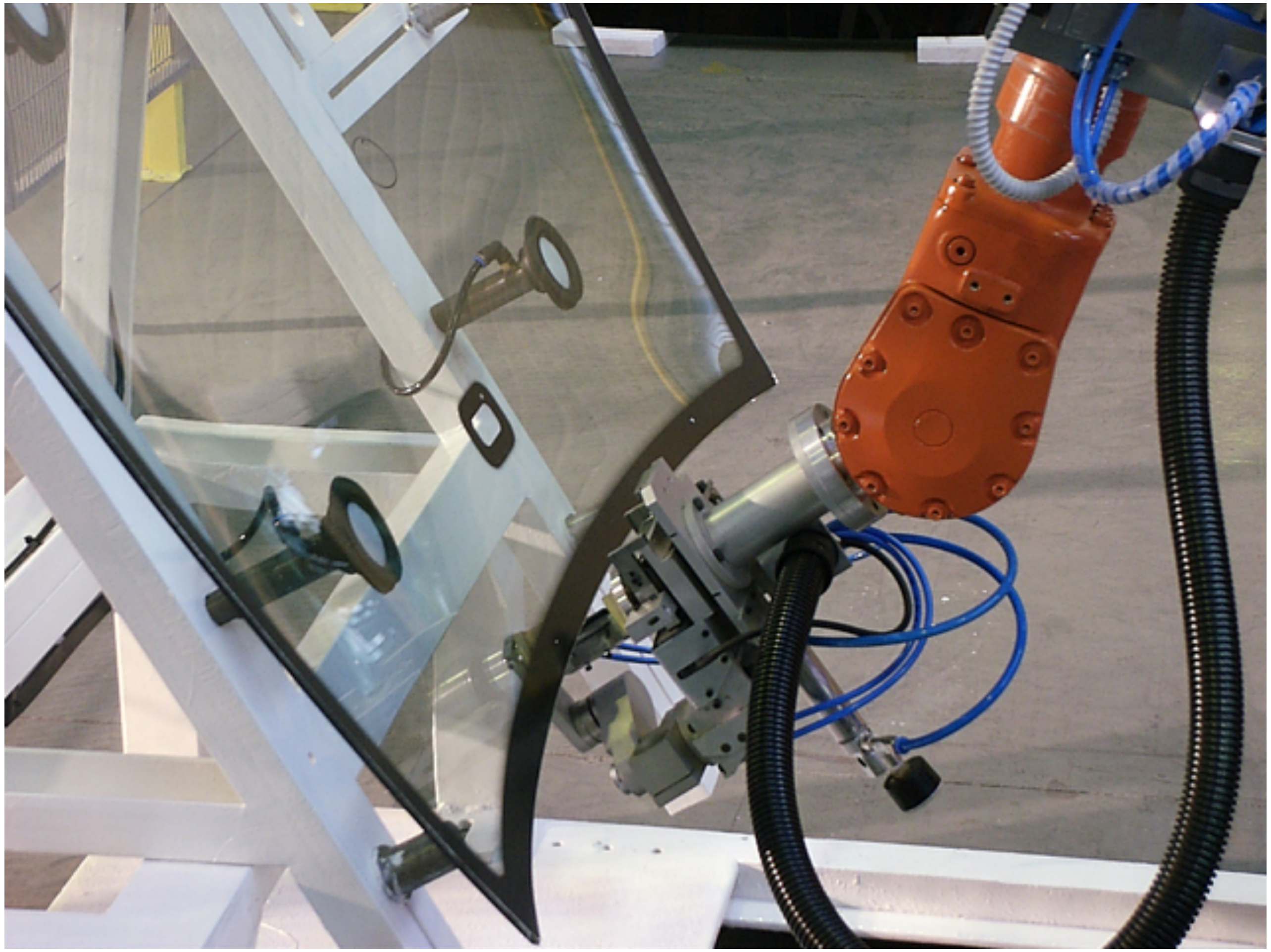


e.g., removing extra glue from the border of a car windshield



e.g., opening a door

Robotized deburring of windshields



c/o ABB Excellence Center in Cecchina (Roma), 2002

Impedance vs. Hybrid control

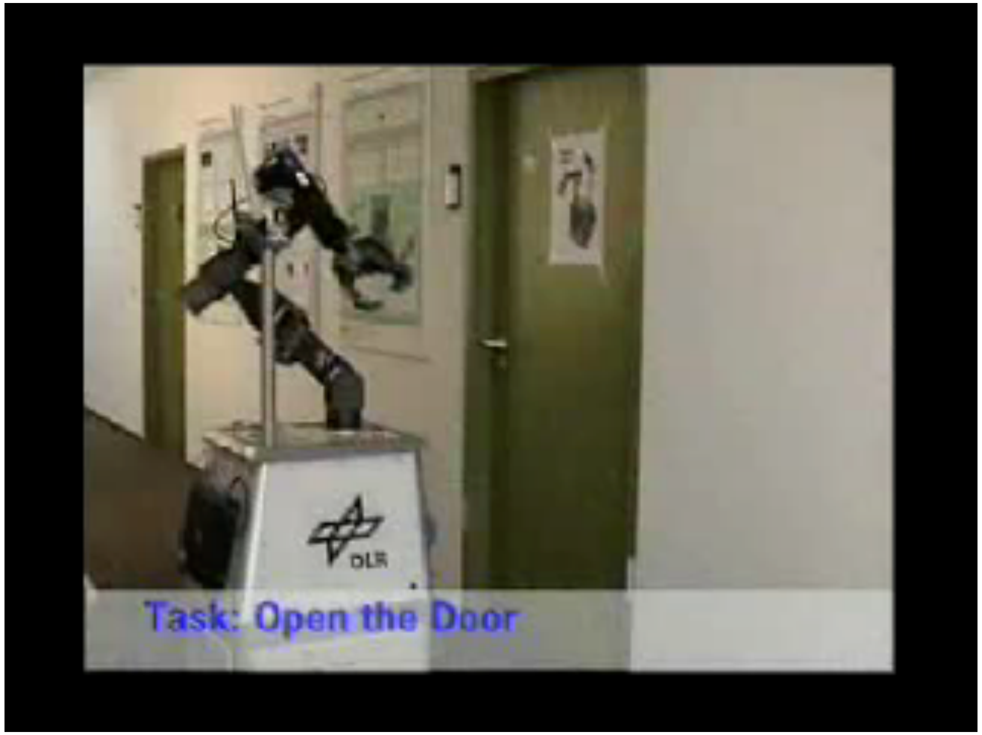
environment model (⬌ domain of control application)

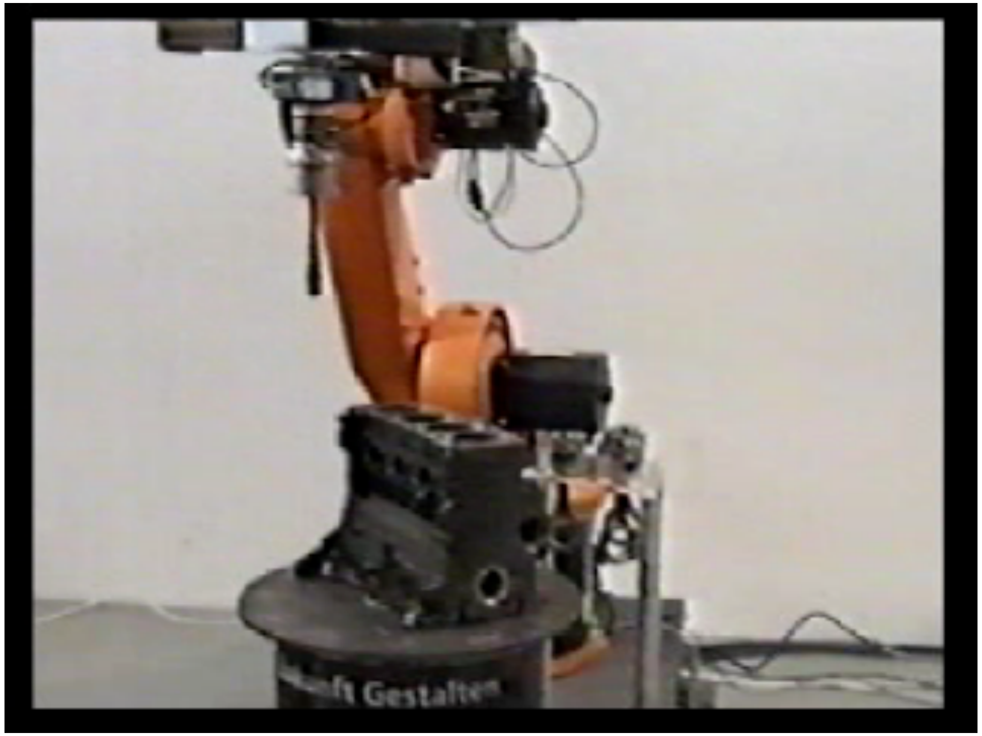
impedance control	hybrid force/motion control
▪ environment = mechanical system undergoing small but finite deformations	▪ a rigid environment reduces the degrees of freedom of the robot when in (bi-/uni-lateral) contact
▪ contact forces arise as the result of a balance of two coupled dynamic systems (robot+environment)	▪ contact forces result from attempts to violate geometric constraints imposed by the environment
➔ desired dynamic characteristics are assigned to the force/motion interaction	➔ task space is decomposed in sets of directions where only motion or only reaction forces are feasible

- the required level of knowledge about the environment geometry is only *apparently* different between the two control approaches
- however, measuring contact forces may not be needed in impedance control, while it always necessary in hybrid force/motion control

Impedance vs. Hybrid control

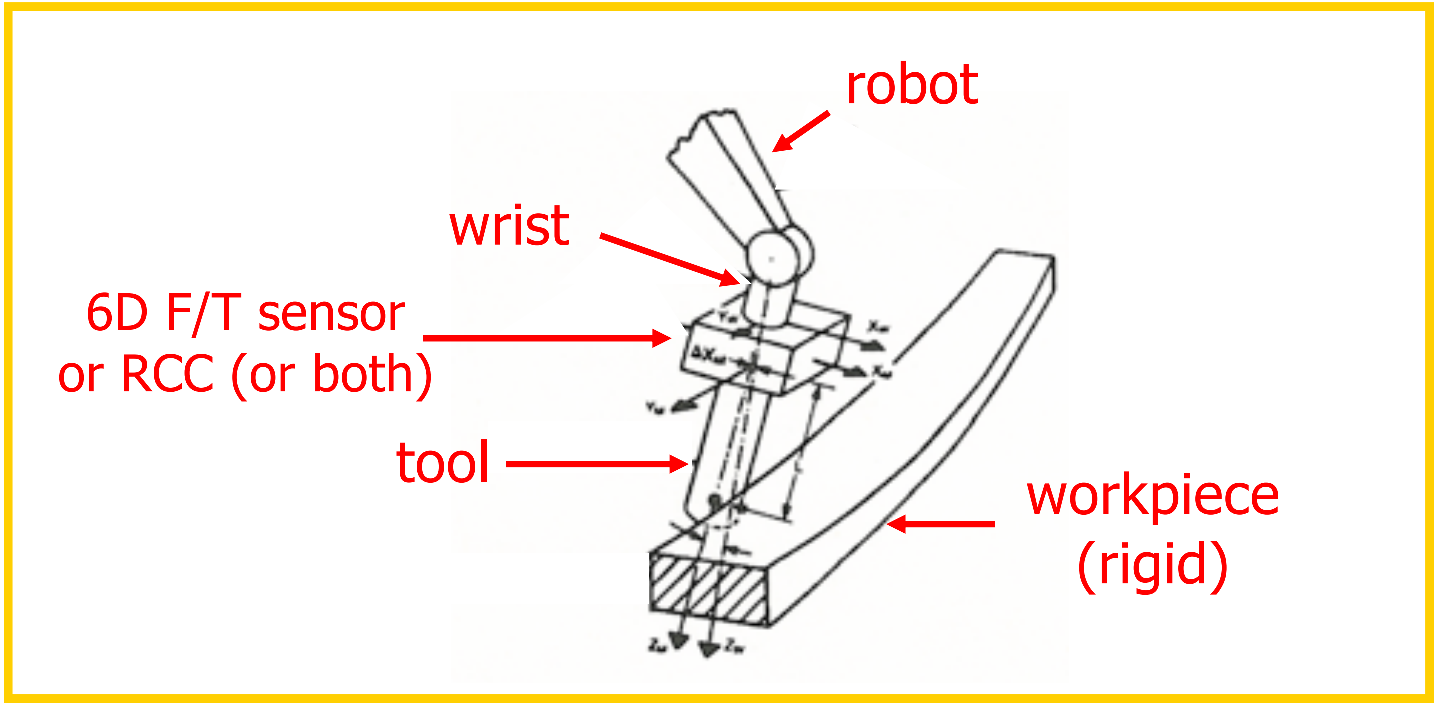
- opening a door with a mobile manipulator under impedance control
- piston insertion in a motor based on hybrid control of force-position (visual)





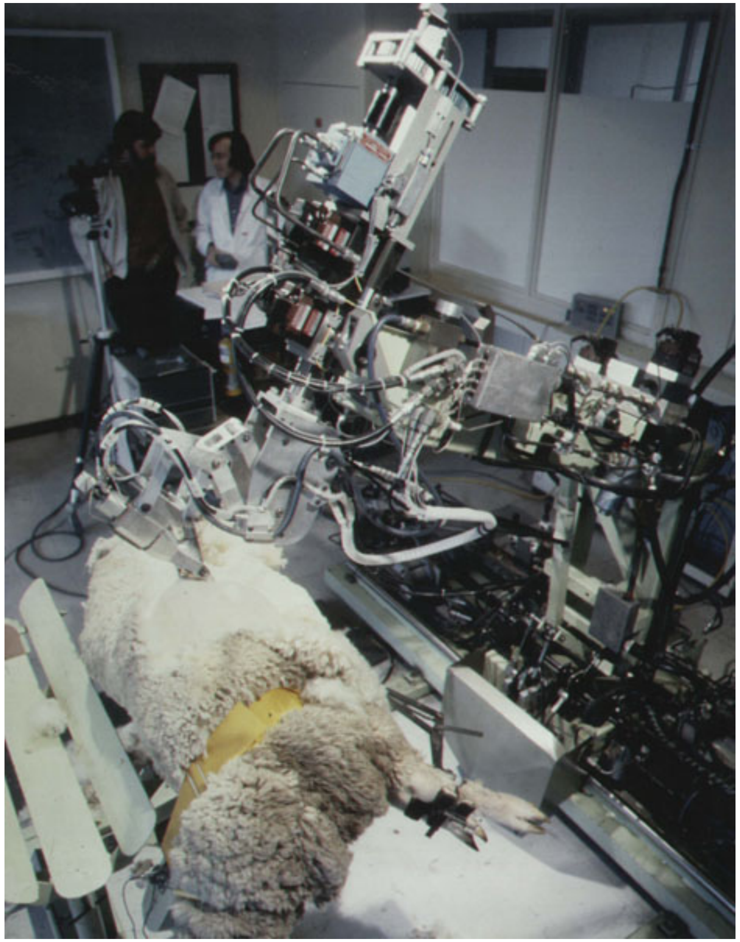
video video

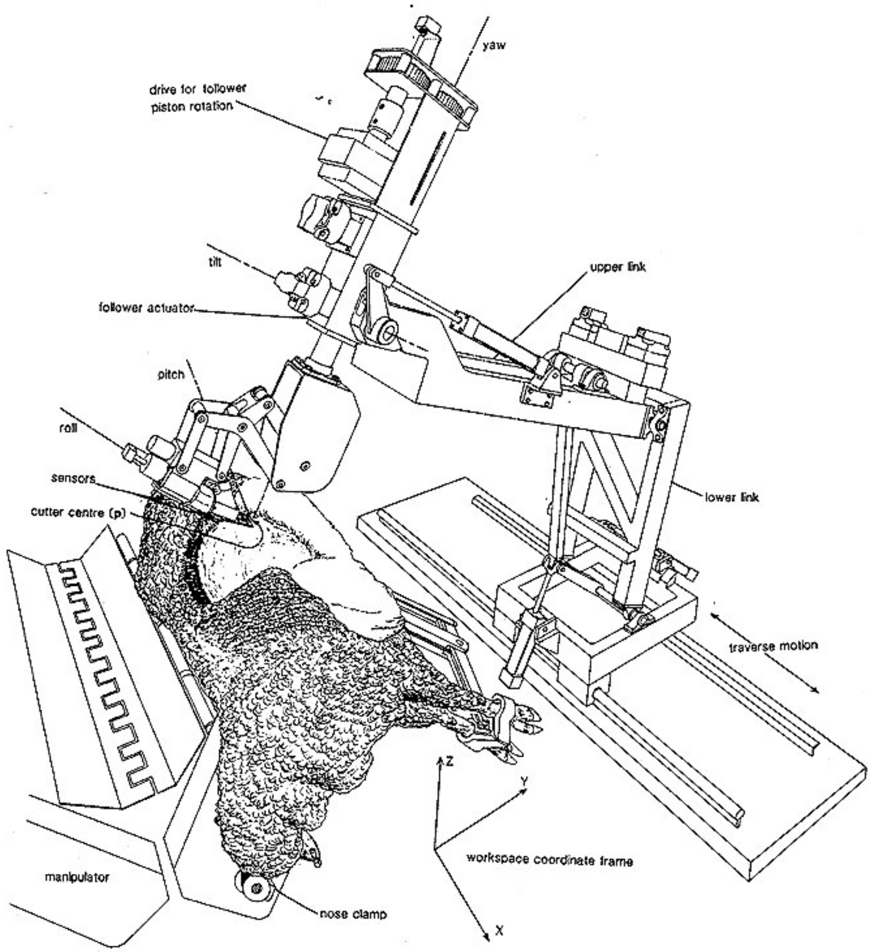
A typical constrained situation …



the robot end-effector follows in a stable and accurate way the geometric profile of a very stiff workpiece, while applying a desired contact force

An unusual compliant situation …

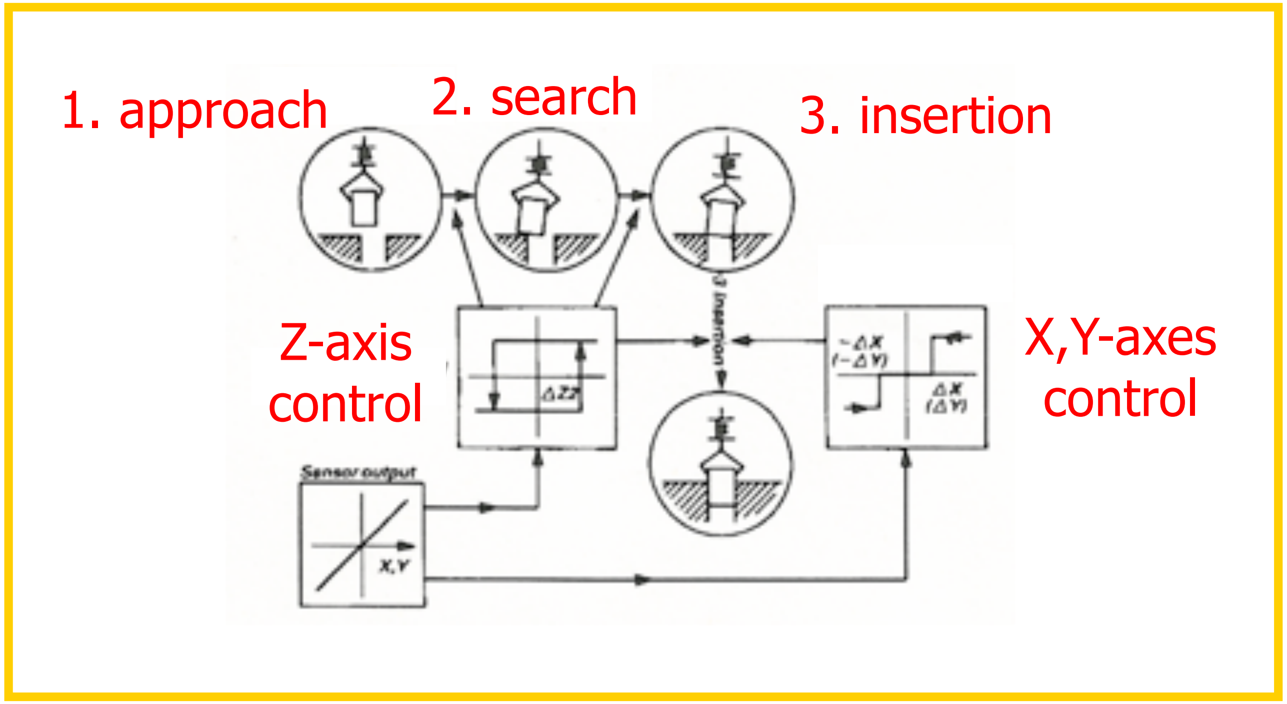




Trevelyan (AUS): Oracle robotic system in a test dated 1981

…is the sheep happy?

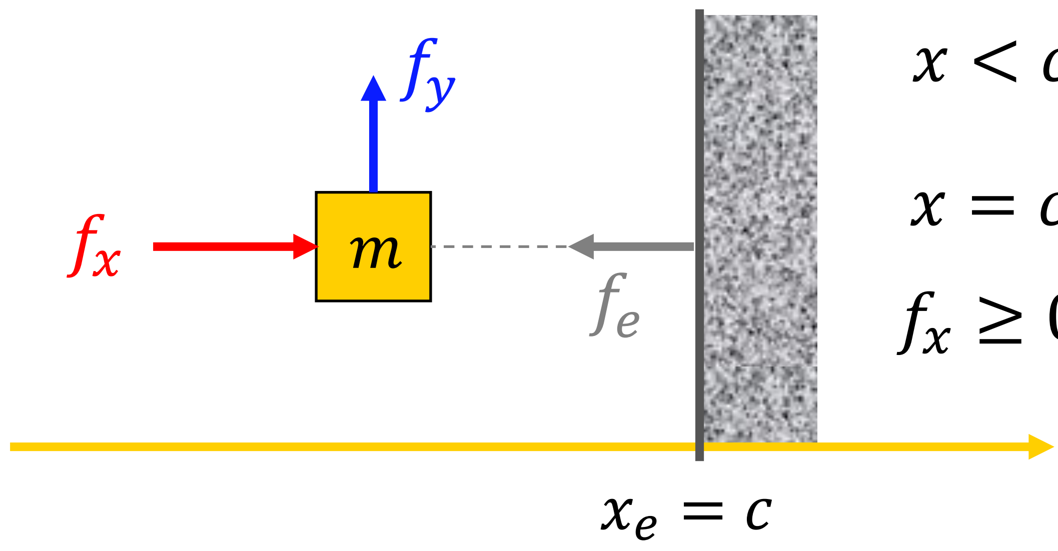
A mixed interaction situation



processing/reasoning on force measurements leads to a sequence of fine motions
⇒ correct completion of insertion task with help of (sufficiently large) passive compliance

Ideally constrained contact situation

a first possible modeling choice for very stiff environments



$$x < c \quad \Rightarrow \quad f_e = 0$$

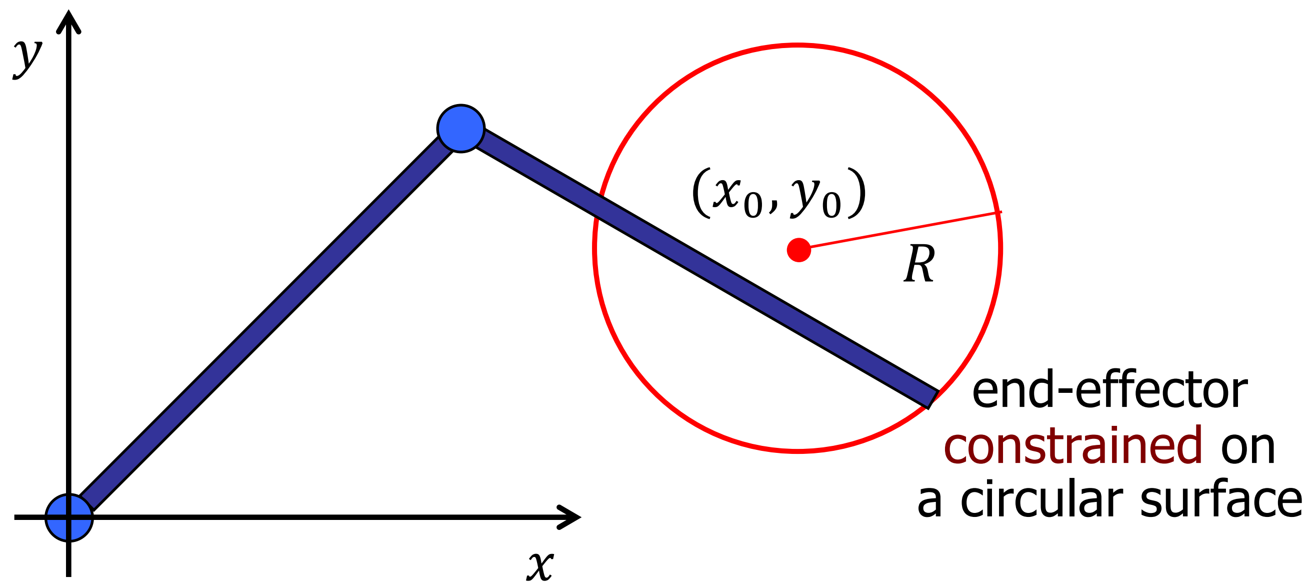
$$\left.\begin{array}{l} x = c \\ f_x \geq 0 \end{array}\right\} \Rightarrow \quad \begin{array}{l} \dot{x} = 0 \\ \ddot{x} = 0 \end{array} \quad \Rightarrow \quad \boxed{f_e = -f_x}$$

$$\begin{cases} m\ddot{x} = f_x + f_e \\ m\ddot{y} = f_y \end{cases}$$

"ideal" = robot (sketched here as a Cartesian mass)
+ environment are both infinitely STIFF
(and without friction at the contact)

In more complex situations

- how can we describe more complex contact situations, where the end-effector of an articulated robot (not yet reduced to a Cartesian mass via feedback linearization control) is constrained to move on an environment surface with nonlinear geometry?
- example: a planar 2R robot with end-effector moving on a circle



Constrained robot dynamics - 1

- consider a robot in free space described by its Lagrange dynamic model and a task output function (e.g., the end-effector pose)

$$\boxed{M(q)\ddot{q} + c(q,\dot{q}) + g(q) = u} \qquad \boxed{r = f(q)} \qquad q \in \mathbb{R}^N$$

- suppose that the task variables are subject to $M < N$ (bilateral) geometric constraints in the general form $k(r) = 0$ and define

$$h(q) = k\big(f(q)\big) = 0$$

- the constrained robot dynamics can be derived using again the Lagrange formalism, by defining an augmented Lagrangian as

$$L_a(q,\dot{q},\lambda) = L(q,\dot{q}) + \lambda^T h(q) = T(q,\dot{q}) - U(q) + \lambda^T h(q)$$

where the Lagrange multipliers λ (a M-dimensional vector) can be interpreted as the generalized forces that arise at the contact when attempting to violate the constraints

Constrained robot dynamics - 2

- applying the Euler-Lagrange equations in the extended space of generalized coordinates q AND multipliers λ yields

$$\frac{d}{dt}\left(\frac{\partial L_a}{\partial \dot{q}}\right)^T - \left(\frac{\partial L_a}{\partial q}\right)^T = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)^T - \left(\frac{\partial L}{\partial q}\right)^T - \left(\frac{\partial}{\partial q}(\lambda^T h(q))\right)^T = u$$

$$\left(\frac{\partial L_a}{\partial \lambda}\right)^T = h(q) = 0$$

$\longleftarrow$ contact forces do NOT produce work

$$\Rightarrow \begin{cases} M(q)\ddot{q} + c(q,\dot{q}) + g(q) = u + A^T(q)\lambda & (\star) \\ \text{subject to} \quad h(q) = 0 \end{cases}$$

where we defined the Jacobian of the constraints as the matrix

$$A(q) = \frac{\partial h(q)}{\partial q}$$

that will be assumed of full row rank $(= M)$

Constrained robot dynamics - 3

- we can eliminate the appearance of the multipliers as follows
  - differentiate the constraints twice w.r.t. time

$$h(q) = 0 \;\Rightarrow\; \dot{h} = \frac{\partial h(q)}{\partial q}\dot{q} = A(q)\dot{q} = 0 \;\Rightarrow\; \ddot{h} = A(q)\ddot{q} + \dot{A}(q)\dot{q} = 0$$

  - substitute the joint accelerations from the dynamic model (★) (dropping dependencies)

$$AM^{-1}(u + A^T\lambda - c - g) + \dot{A}\dot{q} = 0$$

  - solve for the multipliers

invertible $M \times M$ matrix, when A is full rank

$$\lambda = (AM^{-1}A^T)^{-1}\left(AM^{-1}(c + g - u) - \dot{A}\dot{q}\right)$$
$$= \left(A_M^{\#}\right)^T (c + g - u) - (AM^{-1}A^T)^{-1}\dot{A}\dot{q}$$

the inertia-weighted pseudoinverse of the constraint Jacobian A

constraint forces λ are uniquely determined by the robot state $(q, \dot{q})$ and input u !!

to be replaced in the dynamic model...

Constrained robot dynamics - 4

- the final constrained dynamic model can be rewritten as

$$\boxed{M(q)\ddot{q} = \underbrace{\left[I - A^T(q)\left(A_M^{\#}(q)\right)^T\right]}_{\text{dynamically consistent projection matrix}}(u - c(q,\dot{q}) - g(q)) - M(q)A_M^{\#}(q)\dot{A}(q)\dot{q}}$$

where $A_M^{\#}(q) = M^{-1}(q)A^T(q)(A(q)M^{-1}(q)A^T(q))^{-1}$ and with

$$\lambda = \left(A_M^{\#}(q)\right)^T(c(q,\dot{q}) + g(q) - u) - \left(A(q)M^{-1}(q)A^T(q)\right)^{-1}\dot{A}(q)\dot{q}$$

- if the robot state $(q(0),\dot{q}(0))$ at time $t=0$ satisfies the constraints, i.e.,

$$h\big(q(0)\big) = 0, \qquad A\big(q(0)\big)\dot{q}(0) = 0$$

then the robot evolution described by the above dynamics will be consistent with the constraints for all $t \geq 0$ and for any $u(t)$

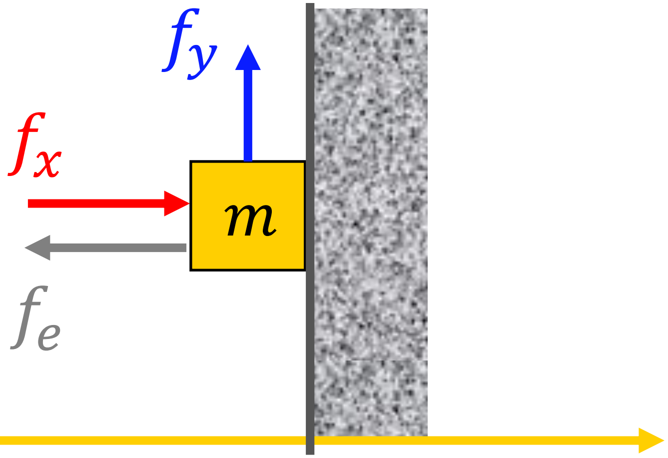
- this is a useful simulation model (constrained direct dynamics)

Example – ideal mass

constrained robot dynamics

$$q = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$u = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$



$x = c$

$M\ddot{q} = u$ robot dynamics in free motion

$$M = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$h(q) = x - c = 0 \quad \Rightarrow \quad A(q) = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad \Rightarrow \quad A_M^{\#}(q) = \cdots = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\left(I - A^T(q)\left(A_M^{\#}(q)\right)^T\right) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$ dynamically consistent projection matrix

$$\lambda = -\left(A_M^{\#}(q)\right)^T u = -\begin{pmatrix} 1 & 0 \end{pmatrix} u = -f_x$$ multiplier (contact force f_e)

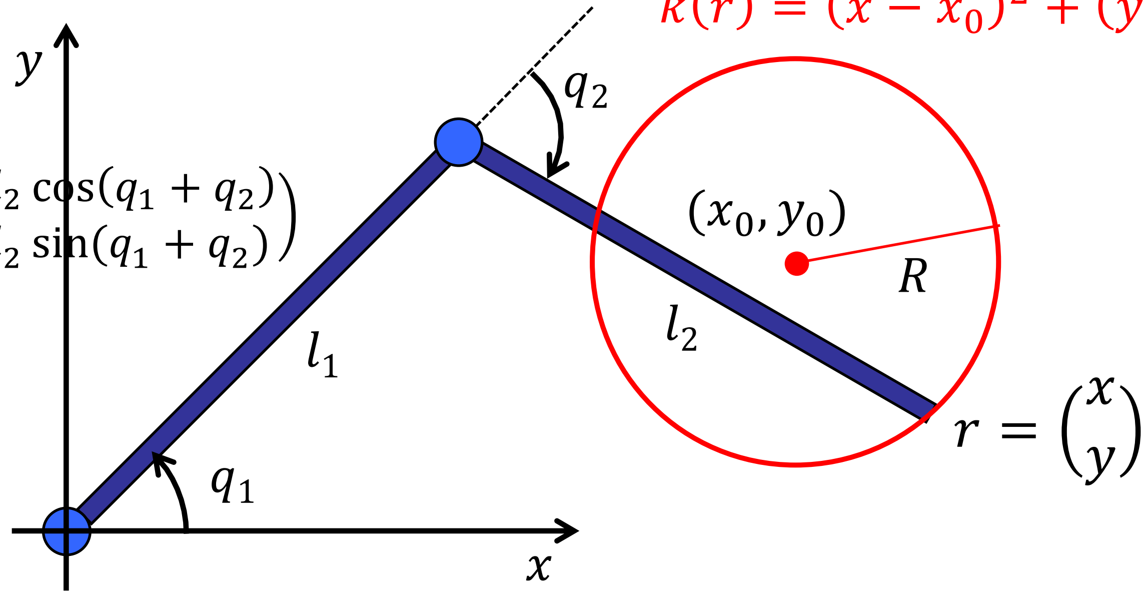
$$M\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = M\ddot{q} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} u = \begin{pmatrix} 0 \\ f_y \end{pmatrix}$$ constrained robot dynamics

Example – planar 2R robot

constrained robot dynamics

$$k(r) = (x - x_0)^2 + (y - y_0)^2 - R^2 = 0$$

$$r = f(q) = \begin{pmatrix} l_1 \cos q_1 + l_2 \cos(q_1 + q_2) \\ l_1 \sin q_1 + l_2 \sin(q_1 + q_2) \end{pmatrix}$$



$$h(q) = k(f(q)) =$$

$$(l_1 \cos q_1 + l_2 \cos(q_1 + q_2) - x_0)^2 + (l_1 \sin q_1 + l_2 \sin(q_1 + q_2) - y_0)^2 - R^2 = 0$$

$$\dot{h} = \frac{\partial k}{\partial r}\frac{\partial r}{\partial q}\dot{q} = [2(x - x_0) \quad 2(y - y_0)]\, J_r(q)\dot{q}$$

$$= [2(l_1 c_1 + l_2 c_{12} - x_0) \quad 2(l_1 s_1 + l_2 s_{12} - y_0)]\, J_r(q)\dot{q} = A(q)\dot{q}$$

Reduced robot dynamics - 1

- by imposing M constraints $h(q) = 0$ on the N generalized coordinates q, it is also possible to reduce the description of the constrained robot dynamics to a $N - M$ dimensional configuration space
- start from constraint matrix $A(q)$ and select a matrix $D(q)$ such that

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix} \text{ is a nonsingular } N{\times}N \text{ matrix} \;\Rightarrow\; \begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = \begin{pmatrix} E(q) & F(q) \end{pmatrix}$$

- define the $(N - M)$-dimensional vector of pseudo-velocities v as the linear combination (at a given q) of the robot generalized velocities

$$v = D(q)\dot{q} \quad \Rightarrow \quad \dot{v} = D(q)\ddot{q} + \dot{D}(q)\dot{q}$$

- inverse relationships (from “pseudo” to “generalized” velocities and accelerations) are given by

$$\dot{q} = F(q)v \qquad \ddot{q} = F(q)\dot{v} - \big(E(q)\dot{A}(q) + F(q)\dot{D}(q)\big)F(q)v$$

$\Leftrightarrow$ properties of block products in inverse matrices have been used for eliminating the appearance of $\dot{F}$ (often F is only known numerically)

Reduced robot dynamics – 2

whiteboard …

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = \begin{pmatrix} E(q) & F(q) \end{pmatrix}$$

a number of properties from this definition…

three useful identities!

two matrix inverse products

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix} \begin{pmatrix} E(q) & F(q) \end{pmatrix} = \begin{pmatrix} A(q)E(q) & A(q)F(q) \\ D(q)E(q) & D(q)F(q) \end{pmatrix} = \begin{pmatrix} I_{M\times M} & 0 \\ 0 & I_{(N-M)\times(N-M)} \end{pmatrix}$$

$$\begin{pmatrix} E(q) & F(q) \end{pmatrix} \begin{pmatrix} A(q) \\ D(q) \end{pmatrix} = E(q)A(q) + F(q)D(q) = I_{N\times N}$$

➡ differentiating w.r.t. time $\qquad \dot{E}A + E\dot{A} + \dot{F}D + F\dot{D} = 0 \quad \triangleleft$

from pseudo-velocity $v = D(q)\dot{q}$
since F is a right inverse of the
full row rank matrix D $(DF = I)$

➡

$$\dot{q} = F(q)v = D^T(q)\left(D(q)D^T(q)\right)^{-1}v$$

(in fact $D\dot{q} = DFv = v$)

➡ differentiating w.r.t. time $\dot{q} = F(q)v$

$$\ddot{q} = F\dot{v} + \dot{F}v = F\dot{v} + (\dot{F}D)\dot{q} \overset{(\triangleleft)}{=} F\dot{v} - \left(\cancel{\dot{E}A} + E\dot{A} + F\dot{D}\right)Fv$$

$$= F(q)\dot{v} - \left(E(q)\dot{A}(q) + F(q)\dot{D}(q)\right)F(q)v$$

Reduced robot dynamics - 3

- consider again the dynamic model (★), dropping dependencies

$$M\ddot{q} + c + g = u + A^T\lambda$$

- since $AE = I$, multiplying on the left by E^T isolates the multipliers

$$E^T(M\ddot{q} + c + g - u) = \lambda$$

- since $AF = 0$, multiplying on the left by F^T eliminates the multipliers

$$F^T M\ddot{q} = F^T(u - c - g)$$

- substituting in the latter the generalized accelerations and velocities with the pseudo-accelerations and pseudo-velocities leads finally to

invertible $(N-M)\times(N-M)$ positive definite matrix

$$\boxed{(F^T M F)\dot{v} = F^T\left(u - c - g + M\left(E\dot{A} + F\dot{D}\right)Fv\right)}$$

which is the reduced $(N-M)$-dimensional dynamic model

- similarly, the expression of the multipliers becomes

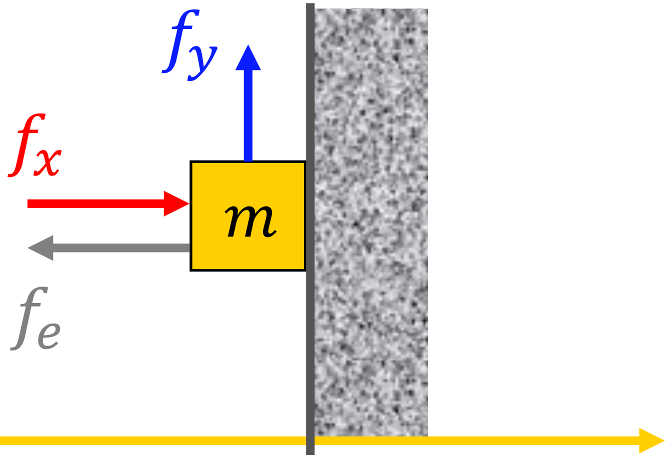
$$\lambda = E^T\left(MF\dot{v} - M\left(E\dot{A} + F\dot{D}\right)Fv + c + g - u\right) \quad (\S)$$

Example – ideal mass

reduced robot dynamics

$$q = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$u = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$



$x = c$

$$M\ddot{q} = u$$ robot dynamics in free motion

$$M = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}$$

$$h(q) = x - c = 0 \;\Rightarrow\; A = (1 \quad 0) \;\Rightarrow\; \begin{pmatrix} A \\ D \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} E & F \end{pmatrix}$$

$$\Rightarrow \quad v = D\dot{q} = \dot{y}$$ pseudo-velocity

$$\lambda = E^T(MF\dot{v} - u)$$

$$= (1 \quad 0)\left(\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\ddot{y} - \begin{pmatrix} f_x \\ f_y \end{pmatrix}\right) = -(1 \quad 0)\begin{pmatrix} f_x \\ f_y \end{pmatrix} = -f_x$$

multiplier (contact force f_e)

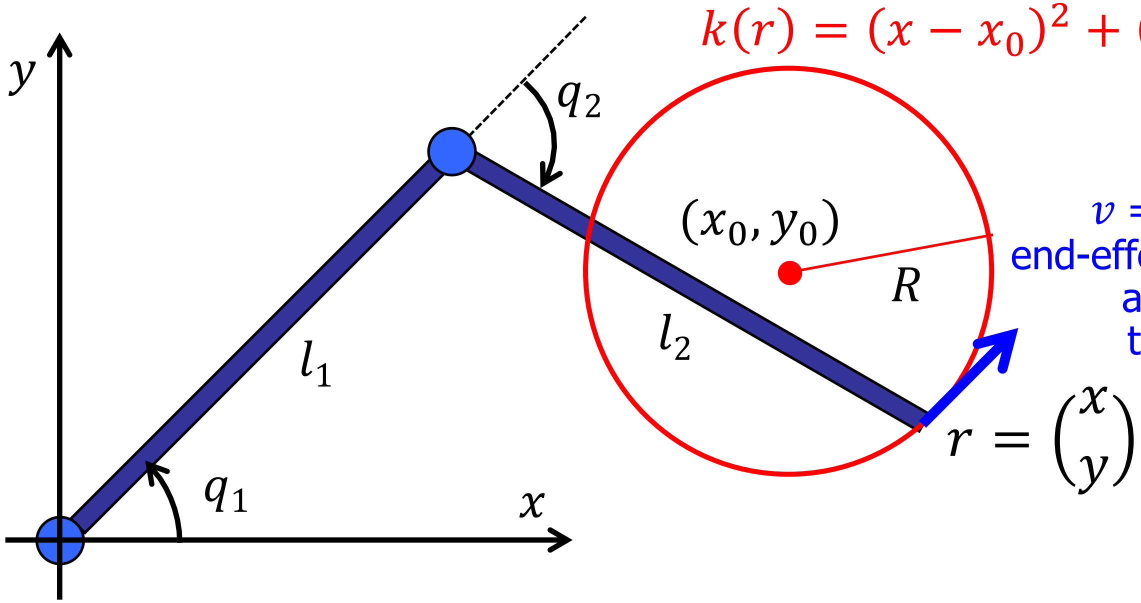
$$(F^T M F)\dot{v} = (0 \quad 1)\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\dot{v} = m\ddot{y} = f_y = F^T u$$

reduced robot dynamics

Example – planar 2R robot

reduced robot dynamics

$$k(r) = (x - x_0)^2 + (y - y_0)^2 - R^2 = 0$$



v = (scalar) value of end-effector velocity **reduced** along the tangent to the constraint

robot Jacobian

$$A(q) = [2(x - x_0) \quad 2(y - y_0)]\, J_r(q)$$
$$= [2(l_1 c_1 + l_2 c_{12} - x_0) \quad 2(l_1 s_1 + l_2 s_{12} - y_0)]\, J_r(q)$$

a feasible selection of matrix $D(q)$

out of robot singularities

$$D(q) = \left[-\frac{1}{2}(y - y_0) \quad \frac{1}{2}(x - x_0)\right] J_r(q) \quad \Rightarrow \quad \det\begin{pmatrix} A(q) \\ D(q) \end{pmatrix} = R^2 \cdot \det J_r(q) \neq 0$$

$$\begin{pmatrix} A(q) \\ D(q) \end{pmatrix}^{-1} = (E(q) \quad F(q)) \quad \Rightarrow \quad \boxed{v} = D(q)\dot{q} \quad \Rightarrow \quad \dot{q} = F(q)v = J_r^{-1}(q) \begin{pmatrix} -2(y - y_0)/R^2 \\ 2(x - x_0)/R^2 \end{pmatrix} v$$

a scalar

Control based on reduced robot dynamics

- the reduced $N-M$ dynamic expressions are more compact but also more complex and less used for simulation purposes than the N-dimensional constrained dynamics
- however, they are useful for control design (reduced inverse dynamics)
- in fact, it is straightforward to verify that the feedback linearizing control law

$$u = \left(c + g - M\left(E\dot{A} + F\dot{D}\right)Fv\right) + MFu_1 - A^T u_2$$

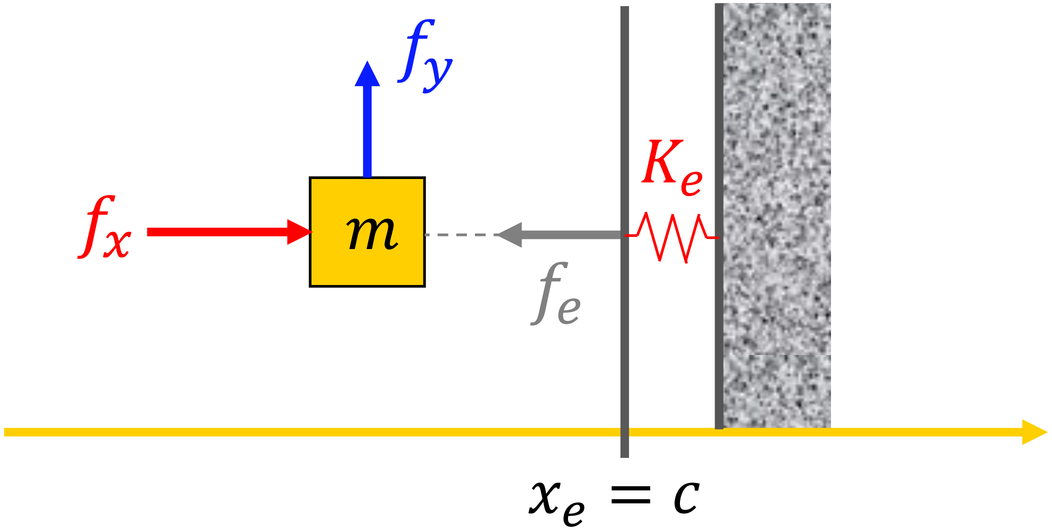
applied to the reduced robot dynamics and to the expression (§) of the multipliers leads to the closed-loop system

$$\dot{v} = u_1 \qquad\qquad \lambda = u_2$$

Note: these are exactly in the form of the ideal mass example of slide #24, with $v = \dot{y}$, $u_1 = f_y/m$, $\lambda = f_e$, $u_2 = -f_x$ (being $N = 2$, $M = 1$, $N - M = 1$)

Compliant contact situation

a second possible modeling choice for softer environments



compliance/impedance control (in all directions) is here a good choice that allows to handle

- uncertain position
- uncertain orientation

of the wall

$$\begin{cases} m\ddot{x} = f_x + f_e \\ m\ddot{y} = f_y \end{cases} \qquad \begin{cases} x < c \;\Rightarrow\; f_e = 0 \\ x \geq c \;\Rightarrow\; f_e = K_e(x - c) \end{cases}$$

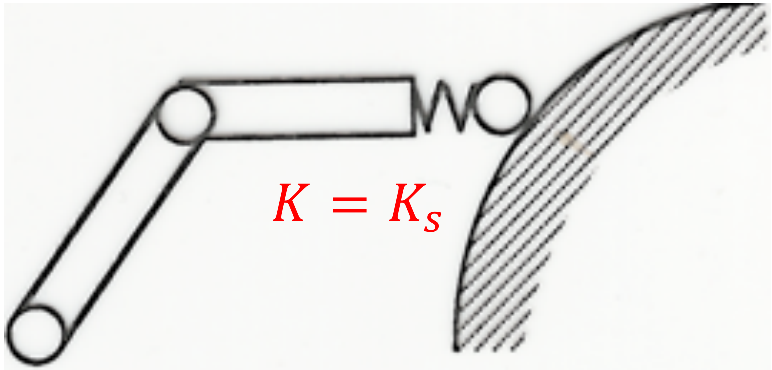
with $K_e > 0$ being the stiffness of the environment

Robot-environment contact types

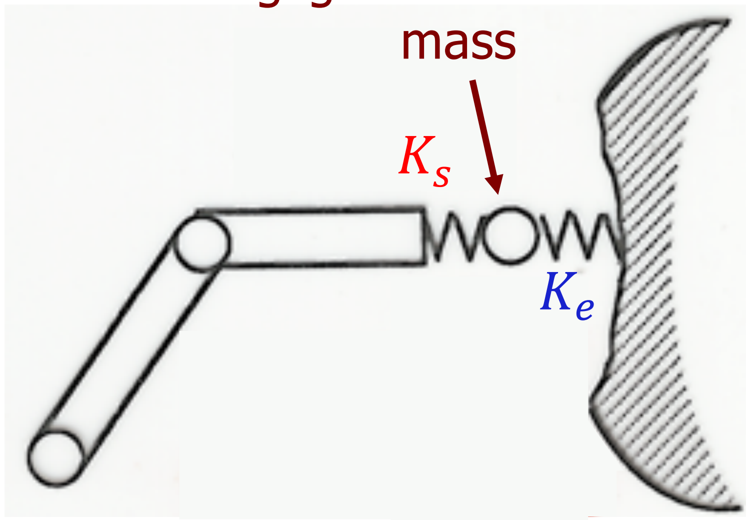
modeled by a single elastic constant

compliant
force sensor

rigid environment

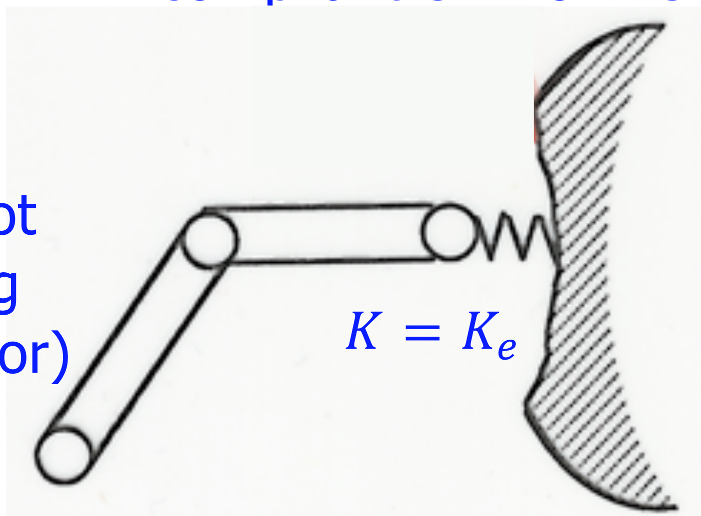


negligible intermediate
mass



compliant environment

rigid robot
(including
force sensor)

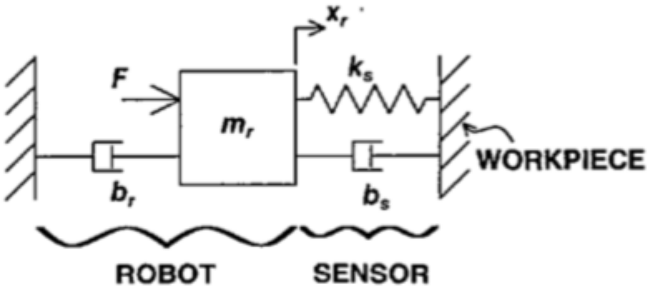


$$\frac{1}{K} = \frac{1}{K_s} + \frac{1}{K_e} \Rightarrow K = \frac{K_s K_e}{K_s + K_e}$$

series of springs =
sum of compliances
(inverse of stiffnesses)

Force control
1-dof robot-environment linear dynamic models

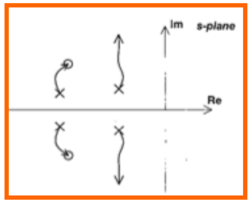
- with a force sensor (having stiffness k_s and damping b_s) measuring the contact force F_c
- stability analysis of a proportional control loop for regulation of the contact force (to a desired constant value F_d) can be made using the root-locus method (for a varying k_f)
- by including/excluding work-piece compliance and/or robot (transmission) compliance

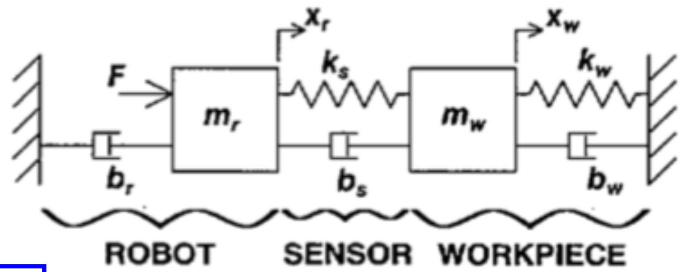


$F_c = k_s x_r$

stable
for all k_f

+ work-piece
compliance

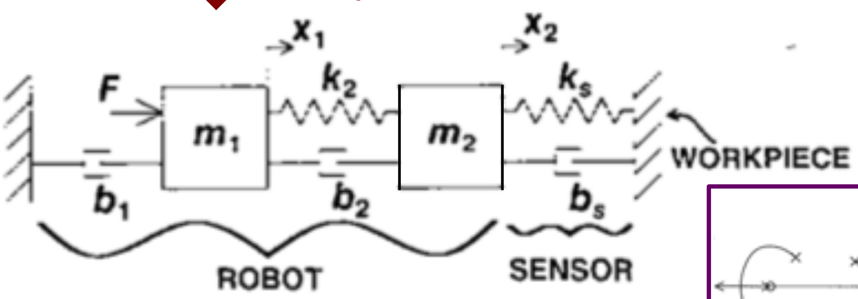




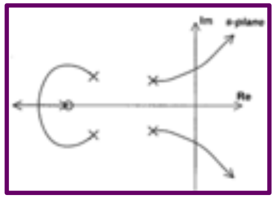
$$F = k_f(F_d - F_c), \quad k_f > 0$$

$F_c = k_s(x_r - x_w)$

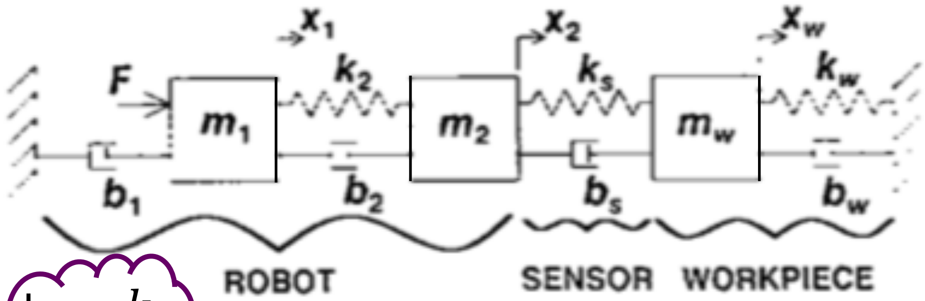
+ robot
compliance



$F_c = k_s x_2$



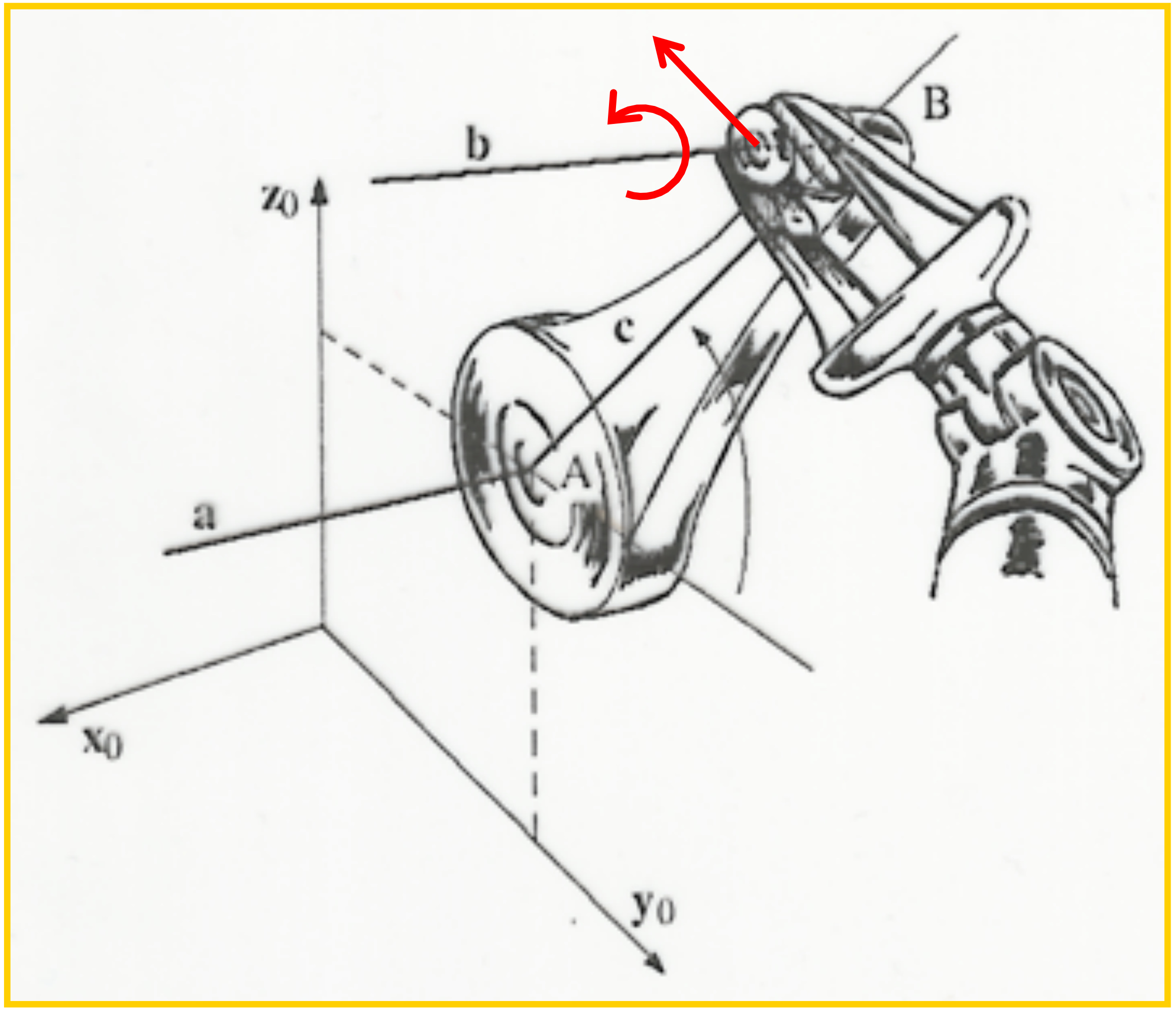
large k_f
$\Downarrow$
unstable



$F_c = k_2(x_2 - x_w)$

(see the paper by S. Eppinger, W.P. Seering: IEEE Control Systems Mag., 1987; available as extra material on the course web)

Tasks requiring hybrid control



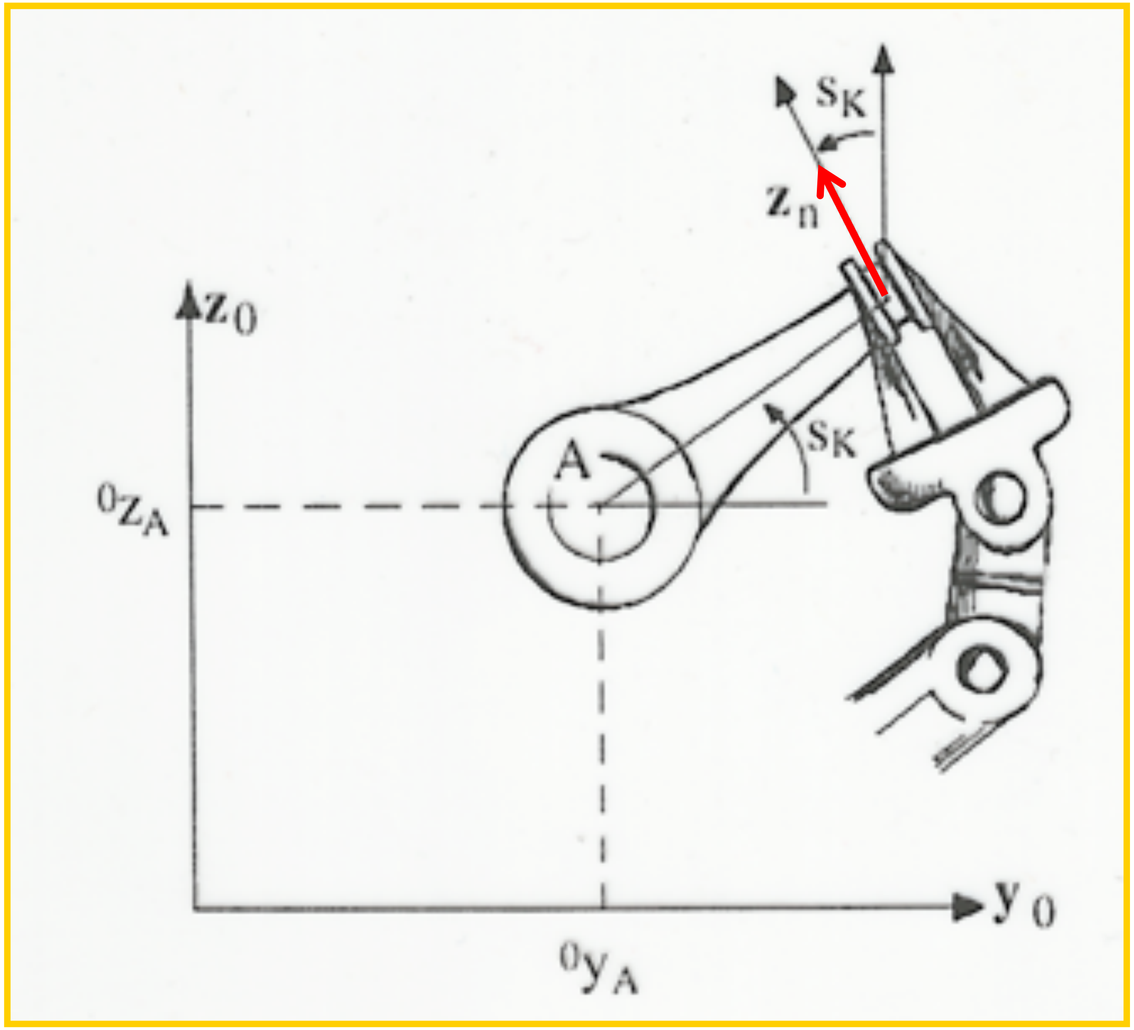
two generalized **directions** of instantaneous free motion at the contact: *tangential velocity & angular velocity* around handle axis

↕

four directions of generalized reaction forces at the contact

the robot should turn a crank having a free-spinning handle

Tasks requiring hybrid control



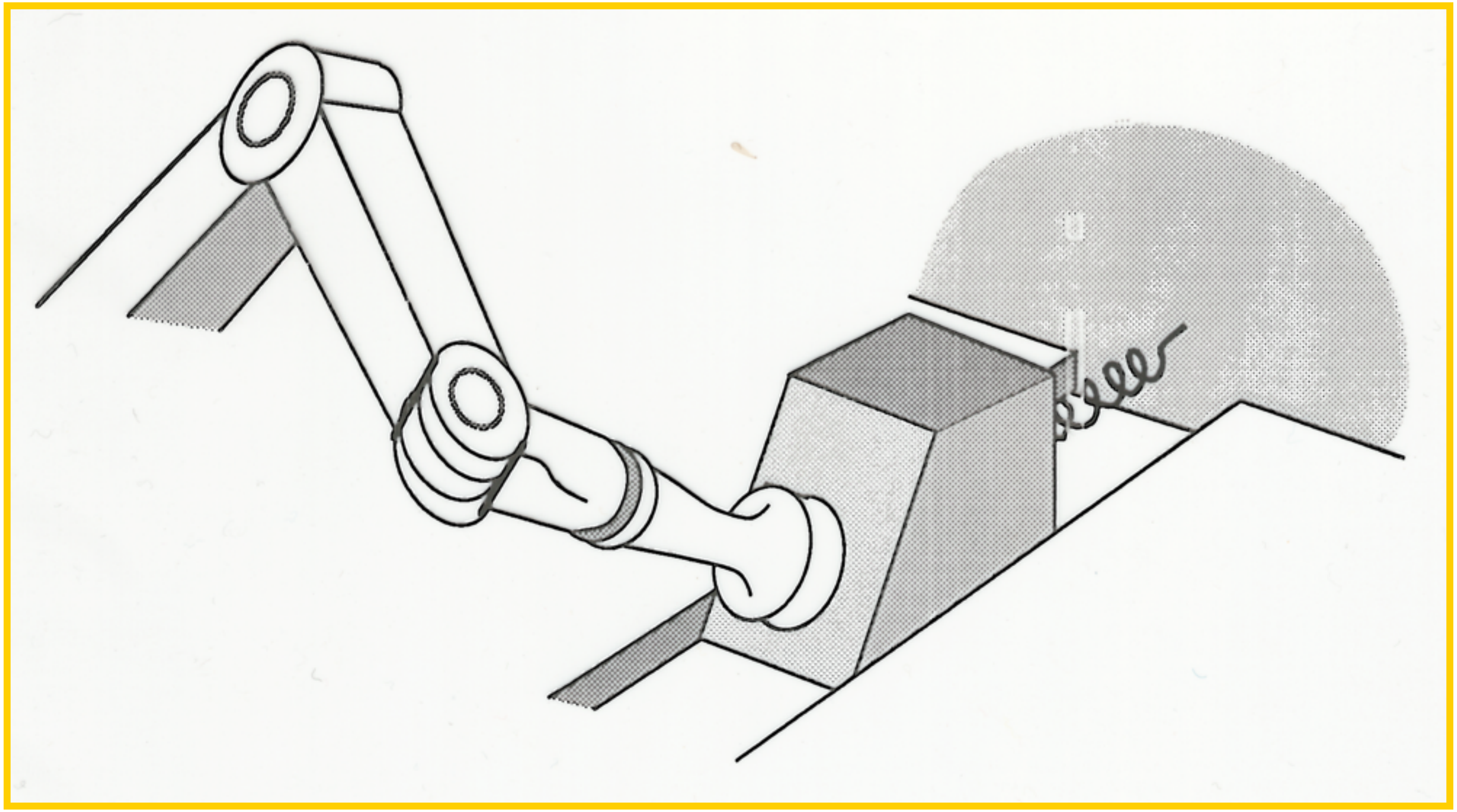
one direction **only**
of instantaneous
free motion
at the contact:
tangential velocity

↕

five directions
of generalized
reaction forces
at the contact

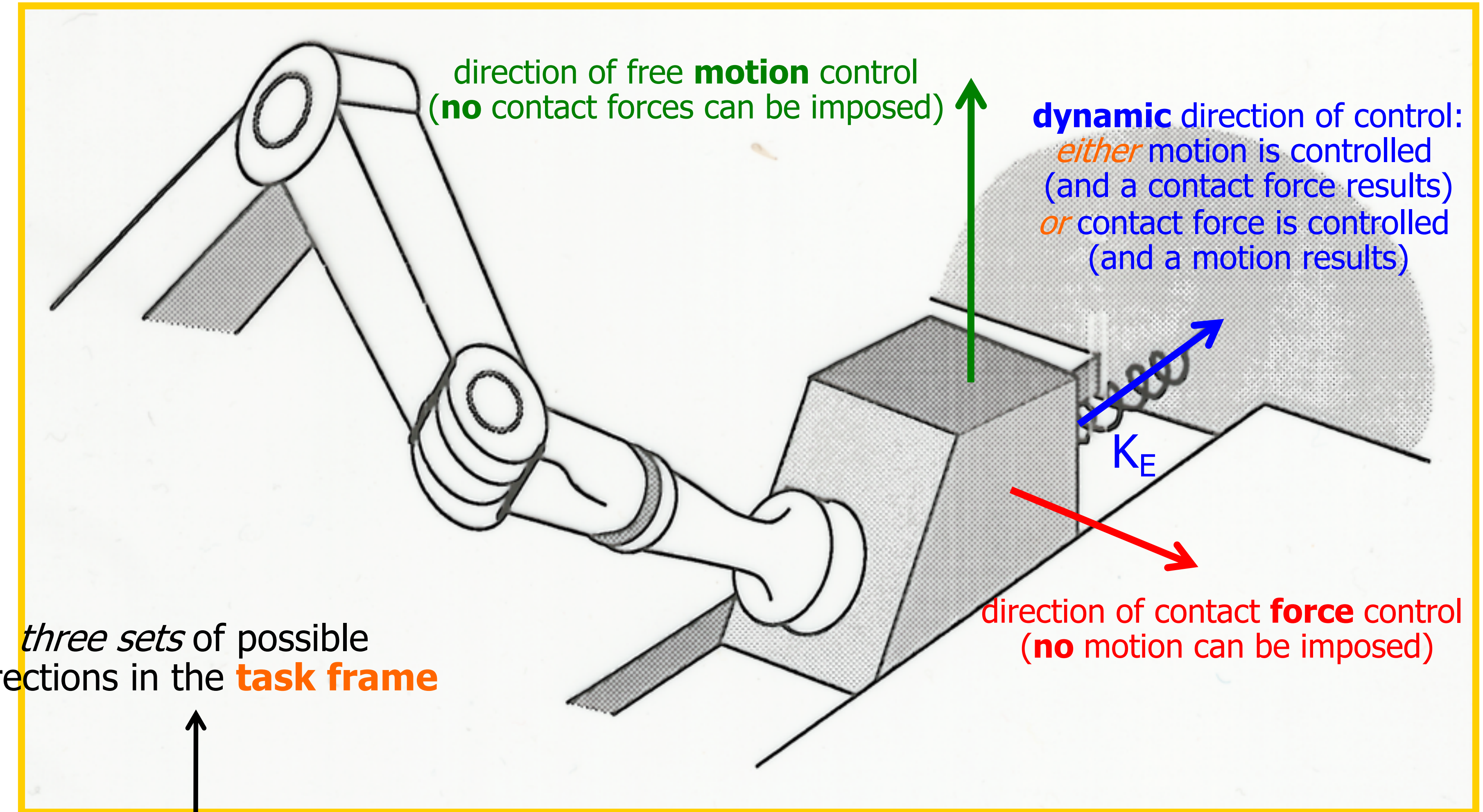
the robot should turn a crank
having a fixed handle

Tasks requiring hybrid control



the robot should push a mass
elastically coupled to a wall and constrained in a guide

Tasks requiring hybrid control



three sets of possible directions in the **task frame**

generalized hybrid modeling and control for dynamic environments

A. De Luca, C. Manes: IEEE Trans. Robotics and Automation, vol. 10, no. 4, 1994