6. Separation principle

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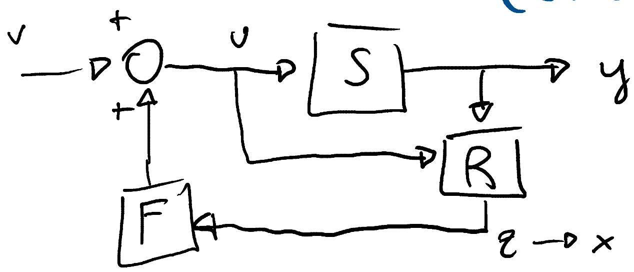
→ Dynamic compensator (observer + feedback)



$$S: \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad R: \dot{z} = (A-KC)z + Bu + Ky$$

picking $u = Fz + v$

$$\begin{cases} \dot{x} = Ax + BFz + Bv \\ \dot{z} = KCx + (A-KC)z + BFz + Bv \\ y = (C \;\; 0)\begin{pmatrix} x \\ z \end{pmatrix} \end{cases}$$

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A & BF \\ KC & A-KC+BF \end{pmatrix}\begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} B \\ B \end{pmatrix} v$$

using the coordinate transformation

$$T = \begin{pmatrix} I & -I \\ 0 & I \end{pmatrix} \qquad T^{-1} = \begin{pmatrix} I & I \\ 0 & I \end{pmatrix}$$

$$\tilde{A} = TAT^{-1} = \left(\begin{array}{c|c} A-KC & 0 \\ \hline KC & A+BF \end{array}\right) \qquad \tilde{B} = TB = \begin{pmatrix} 0 \\ B \end{pmatrix}$$

$$\tilde{C} = CT^{-1} = (C_1 \;\; C_2)$$

$$\begin{cases} \sigma(A-KC) \text{ eigenvalues of the reconstructor} \\ \sigma(A+BF) \text{ eigenvalues of the feedback} \end{cases}$$

the decoupled structure of $\tilde{A}$ allows to approach the 2 problems separately

... eigenvalues separately

$$W(t) = (C_1 \; C_2)\, e^{\begin{pmatrix} A-KC & 0 \\ KC & A+BF \end{pmatrix} t} \begin{pmatrix} 0 \\ B \end{pmatrix} = C e^{(A+BF)t} B$$

coincides with the impulse response of the system without observer