# 32. Global Normal Forms

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All the previous procedures have a local character but there are conditions and design methodologies by which is possible to extend their concepts and reach globally defined solutions.

Consider a SISO system

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \qquad f(0)=0 \quad h(0)=0 \qquad u = \alpha(x)$$

$f(x)$, $g(x)$ smooth vector fields, $h(x)$ smooth function.

**Prop:** this system has Uniform Relative degree if it has relative degree $r$ at each $x_0 \in \mathbb{R}^n$. So $r$ is defined everywhere and

$z^* = \{x \in \mathbb{R}^n : h(x) = \dots = L_f^{r-1} h(x) = 0\}$

is a smooth embedded submanifold of dimension $(n-r)$

and each component of $z^*$ is a maximal submanifold of the nonsingular and involutive distribution

$$\Delta^* = \ker d \begin{pmatrix} h(x) \\ \dots \\ L_f^{r-1} h(x) \end{pmatrix}$$

**Prop:** Suppose the system has uniform relative degree $r$.

Set

$$\alpha(x) = \frac{-L_f^r h(x)}{L_g L_f^{r-1} h(x)} \qquad \beta(x) = \frac{1}{L_g L_f^{r-1} h(x)}$$

And consider the globally defined vector fields

$\tilde{f}(x) = f(x) + g(x)\alpha(x) \qquad \tilde{g}(x) = g(x)\beta(x)$

Suppose the vector fields

$\tau_i = (-1)^{i-1} ad_{\tilde{f}}^{i-1} \tilde{g}(x), \quad 1 \le i \le r$

are complete that is solution always defined.

$\tau_i = (-1) \; ad_{\tilde f}\, \tilde g(x), \quad 1 \le i \le r$

are complete, that is, solution always defined.

Thus, $Z^*$ is connected.

Moreover there exists a smooth mapping

$\Phi : Z^* \times \mathbb{R}^r \rightarrow \mathbb{R}^n$ globally defined (with global inverse)

$$\Phi^{-1}(x) = \begin{array}{c} \begin{pmatrix} z \\ \hline \xi_1 \\ \vdots \\ \xi_r \end{pmatrix} \end{array} \begin{array}{l} n-r \\ \\ r \\ \\ \end{array} = \begin{pmatrix} \psi_1(x) \\ \hline h(x) \\ \vdots \\ L_f^{r-1} h(x) \end{pmatrix}$$

such that the system, with the control law

$$u = \alpha(x) + \beta(x) v$$

takes the form

$$\begin{aligned} \dot z &= f_0(z, \xi_1, \ldots, \xi_r) \\ \dot \xi_1 &= \xi_2 \\ &\vdots \\ \dot \xi_{r-1} &= \xi_r \\ \dot \xi_r &= v \\ y &= \xi_1 \end{aligned}$$

If, in addition $[\tau_i, \tau_j] = 0 \quad \forall \, 1 \le i, j \le r$

this new system assumes the special form

$$\begin{aligned} \dot z &= f_0(z, \xi_1) \\ \dot \xi_1 &= \xi_2 \\ &\vdots \\ \dot \xi_{r-1} &= \xi_r \\ \dot \xi_r &= v \\ y &= \xi_1 \end{aligned}$$

Moreover, if $r = n$, the system is linear, controllable and observable