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Nonlinear Systems & Control
Part II
24/03/17

Student: ____________________

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1. Given the linear time invariant system

$$\dot{x} = Ax + Bu + Dd$$
$$y = Cx$$

with

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & -6 & -4 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} -1 & 1 & 0 & 0 \end{pmatrix}$$

   a. characterize all the disturbances that can be decoupled via feedback;
   b. discuss about the solvability of the disturbance-decoupling problem with stability.

2. Prove that a sufficient condition achieve feedback stabilization of a nonlinear system is stabilizability in first approximation.

3. Given the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + \epsilon \sin x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{\epsilon \cos x_3 [x_1 - \epsilon \sin x_3 (1 + x_4^4)] + u}{1 - \epsilon^2 \cos^2 x_3} \\ y &= \begin{pmatrix} 0 & -\frac{2}{3}(\epsilon^2 - 1) & 1 - \epsilon^2 & 0 \end{pmatrix} x \end{aligned}$$

   a. Compute the feedback that solves the input-output linearization problem and stabilizes the input-output dynamics ;
   b. Compute the zero dynamics;
   c. Discuss the stability of the closed-loop system.

4. High gain feedback.

5. Stabilize, if possibile, the system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 - x_2 + x_2 x_3 \\ \dot{x}_3 &= -x_2 + x_1^2 + u. \end{aligned}$$

1 Characterize all the disturbances that can be decoupled via feedback

$$\dot{x} = Ax + Bu + Dd$$
$$y = Cx$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & -6 & -4 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$C = \begin{pmatrix} -1 & 1 & 0 & 0 \end{pmatrix}$$

Compute relative degree : 1^{st} integer s.t. $CA^{r-1}B \neq 0$

$r=1 \quad CB = \begin{pmatrix} -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = 0$

$r=2 \quad CAB = \begin{pmatrix} -1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -4 \end{pmatrix} = 0$

$r=3 \quad CA^2B = CA \cdot AB = \begin{pmatrix} 0 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ -4 \end{pmatrix} = 1 \neq 0$

Compute V^*: $V^* = \ker \begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \ker \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$

$$\begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0$$

$$\begin{cases} -x_1 + x_2 = 0 \\ -x_2 + x_3 = 0 \\ -x_3 + x_4 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = x_2 \\ x_2 = x_3 \\ x_3 = x_4 \end{cases} \rightarrow \begin{pmatrix} x_1 \\ x_1 \\ x_1 \\ x_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$V^* = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

Since for any D such that $\text{Im}\, D \subset \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ the DDP admits a solution, the structure of D must be $D(s) = \begin{pmatrix} s \\ s \\ s \\ s \end{pmatrix}$ with $s \in \mathbb{R}$

The feedback under which the system can be decoupled is:

$$u = F^* x = -\frac{CA^3}{CA^2B} x + v \qquad F^* = \begin{pmatrix} 1 & 4 & 6 & 5 \end{pmatrix}$$

Discuss about solvability of DDP with stability

$$W(s) = C(sI-A)^{-1}B = \dots = \frac{-1+s}{1+4s+6s^2+4s^3+s^4}$$

zero: $s=1 \quad \sigma(z) \in \mathbb{C}^+$

to mantain stability we can't cancel this zero.

$r = 4 = n$

$$\rho\begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{pmatrix} = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ -1 & -4 & -6 & -5 \end{pmatrix} = 4 \quad \text{fully observable}$$

So no disturb assures stability

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③ Compute the Fb which solves the I/O FL prob. and stabilizes the I/O dynamics

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 + \varepsilon \sin(x_3) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{\varepsilon \cos(x_3)\left[x_1 - \varepsilon \sin(x_3)(1+x_4^4)\right] + u}{1-\varepsilon^2\cos^2(x_3)} \\ y = \begin{pmatrix} 0 & -\frac{2}{3}(\varepsilon^2-1) & 1-\varepsilon^2 & 0 \end{pmatrix} x \end{cases} \qquad g = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{1-\varepsilon^2\cos^2(x_3)} \end{pmatrix}$$

1) Compute the I/O feedback linearization

$$\dot{y} = -\frac{2}{3}(\varepsilon^2-1)\dot{x}_2 + (1-\varepsilon^2)\dot{x}_3 = \frac{2}{3}(\varepsilon^2-1)x_1 - \frac{2}{3}(\varepsilon^2-1)\varepsilon\sin(x_3) + (1-\varepsilon^2)x_4$$

$$\ddot{y} = \frac{2}{3}(\varepsilon^2-1)x_2 - \frac{2}{3}(\varepsilon^2-1)(\varepsilon\cos(x_3))x_4 + (1-\varepsilon^2)\left[\frac{\varepsilon\cos(x_3)\ \cdots\ + u}{1-\varepsilon^2\cos^2(x_3)}\right]$$

So $r=2$ for $\varepsilon \neq \pm 1$, otherwise there would be no output

$$u = \left(v - \varepsilon\cos(x_3)\left[x_1 - \varepsilon\sin(x_3)(1+x_4^4)\right]\right)\left(1-\varepsilon^2\cos^2(x_3)\right)$$

with $v = -c_0 z_1 - c_1 z_2 = -c_0 h - c_1 L_f h =$

$$= -c_0\left[-\frac{2}{3}(\varepsilon^2-1)x_2 + (1-\varepsilon^2)x_3\right] - c_1\left[\frac{2}{3}(\varepsilon^2-1)x_1 - \frac{2}{3}(\varepsilon^2-1)\varepsilon\sin(x_3) + (1-\varepsilon^2)x_4\right]$$

c_0, c_1 coeff. of the characteristic polynomial with $\mathrm{Re}(\,) \in \mathbb{C}^-$

Zero dynamics

coord. transf.

$$z_1 = h = -\frac{2}{3}(\varepsilon^2-1)x_2 + (1-\varepsilon^2)x_3$$

$$z_2 = L_f h = \frac{2}{3}(\varepsilon^2-1)x_1 - \frac{2}{3}(\varepsilon^2-1)\varepsilon \sin(x_3) + (1-\varepsilon^2)x_4$$

$\eta_1 = x_1$

$$\eta_2 : \nabla\varphi \cdot g = \frac{\partial \varphi}{\partial x_4} \cdot \left(\frac{1}{1-\varepsilon^2\cos^2(x_3)}\right) = 0$$

$\eta_2 = x_3$

$$x_2 = \frac{z_1 - (1-\varepsilon^2)\eta_2}{-\frac{2}{3}(\varepsilon^2-1)} = \frac{z_1-(1-\varepsilon^2)\eta_2}{\frac{2}{3}(1-\varepsilon^2)} = \frac{3}{2}\frac{z_1}{(1-\varepsilon^2)} - \frac{3}{2}\eta_2$$

$$x_1 = \eta_1, \quad x_3 = \eta_2, \quad x_4 = \frac{\frac{2}{3}(\varepsilon^2-1)\varepsilon\sin(\eta_2) - \frac{2}{3}(\varepsilon^2-1)\eta_1}{(1-\varepsilon^2)}$$

$$= -\frac{\frac{2}{3}\cancel{(\varepsilon^2-1)}(\varepsilon\sin(\eta_2)-\eta_1)}{\cancel{(\varepsilon^2-1)}}$$

$$= -\frac{2}{3}(\varepsilon\sin(\eta_2)-\eta_1)$$

So the zero dynamics is

$$\begin{cases} \dot\eta_1 = \dot x_1 = x_2 = -\frac{3}{2}\eta_2 \\ \dot\eta_2 = \dot x_3 = x_4 = \frac{2}{3}(\eta_1 - \varepsilon\sin(\eta_2)) \end{cases}$$

$$Q = \frac{\partial q(0,\eta)}{\partial \eta}\bigg|_0 = \left.\begin{pmatrix} 0 & -\frac{3}{2} \\ \frac{2}{3} & -\frac{2}{3}\varepsilon\cos(\eta_2) \end{pmatrix}\right|_0 = \begin{pmatrix} 0 & -\frac{3}{2} \\ \frac{2}{3} & -\frac{2}{3}\varepsilon \end{pmatrix}$$

$$\left|\begin{pmatrix} -\lambda & -3/2 \\ \frac{2}{3} & -\frac{2}{3}\varepsilon-\lambda \end{pmatrix}\right| = -\lambda\left(-\frac{2}{3}\varepsilon-\lambda\right) + 1 = +\frac{2}{3}\varepsilon\lambda + \lambda^2 + 1 = 0$$

$$\lambda_{12} = \frac{-\frac{2}{3}\varepsilon \pm \sqrt{\frac{4}{9}\varepsilon^2 - 4}}{2} =$$

for $\varepsilon > 0$

$Re(\lambda_i) < 0$

So the dynamics is LAS and also the closed loop sys is stable.

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(5) Stabilize the [illegible] (if possible)

5 Stabilize the system (if possible)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - x_2 + x_2 x_3 \\ \dot{x}_3 = -x_2 + x_1^2 + u \end{cases}$$

Applying $u = x_2 - x_1^2 + v$ I get $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - x_2 + x_2 x_3 \\ \dot{x}_3 = v \end{cases}$

The first subsystem can be written as

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} x_2 x_3$$

Considering x_3 as a virtual input

I trivially choose $x_3 = \gamma(x_1, x_2) = 0$, so I have

$V(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2) \Rightarrow \dot{V} = x_1 \dot{x}_1 + x_2 \dot{x}_2 = x_1 \cancel{x_2} - x_1 \cancel{x_2} - x_2^2 + x_2^2 x_3$

with $\gamma(x_1, x_2) = x_3 = 0$, I have $\dot{V}(x_1, x_2) = -x_2^2 \leq 0$

Applying La Salle

$\{(x_1, x_2) : \dot{V} = 0\} \equiv \{(x_1, x_2) : x_2 = 0\}$

Hence (x_1, x_2) is GAS.

Then I go on with backstepping:

$x_3 = \gamma(x_1, x_2) = 0$ so $x_3 = e$ (no change of coord.)

$V(x_1, x_2, x_3) = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2)$

$\dot{V}(x_1, x_2, x_3) = x_1 \dot{x}_1 + x_2 \dot{x}_2 + x_3 \dot{x}_3 = \cancel{x_1 x_2} - \cancel{x_1 x_2} + x_2^2 x_3 + x_3 v$

Setting $v = -x_3 - x_2^2$, I get $\dot{V}(x_1, x_2, x_3) = -x_2^2 - x_3^2 \leq 0$

Applying Lasalle again:

$\{(x_1, x_2, x_3) : \dot{V} = 0\} \equiv \{(x_1, x_2, x_3) = x_2 = x_3 = 0\}$

So $(x_1, x_2, x_3) = (0, 0, 0)$ is GAS under $v = -x_3 - x_2^2$