# 7. Zeros

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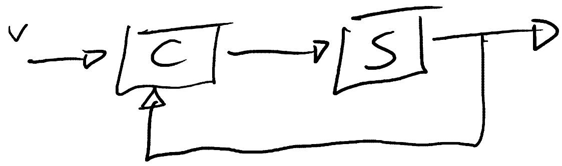


$$S: \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$

$$C: \begin{cases} \dot{z} = (A - KC + BF) z + Mv + Ky \\ u = Fz + Nv \end{cases}$$

if $M = B$, $N = I \Rightarrow C$: observer + feedback

zeros are the values of $s$ such that:

$$\begin{pmatrix} sI - A & -B \\ C & D \end{pmatrix} \begin{pmatrix} x_0 \\ u_0 \end{pmatrix} = 0$$

$$\begin{cases} (sI - A) x_0 - Bu_0 = 0 \\ Cx_0 + Du_0 = 0 \end{cases} \qquad (x_0, u_0) \neq (0, 0)$$

$$\begin{cases} x_0 = (sI - A)^{-1} B u_0 \\ \underbrace{(C (sI - A)^{-1} B + D) u_0}_{\neq 0} = 0 \end{cases}$$

zeros specify the dynamics of the system and characterize the filtering properties.
They are the complex numbers that annihilate the determinant of the system matrix.

State feedback does not modify the zeros

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad u = Fx + v \Rightarrow S_F = \begin{cases} \dot{x} = (A + BF) x + Bv \\ y = Cx \end{cases}$$

$$\underset{(1)}{\begin{pmatrix} sI - A - BF & -B \\ C & 0 \end{pmatrix}} = \underset{(2)}{\begin{pmatrix} sI - A & -B \\ C & 0 \end{pmatrix}} \begin{pmatrix} I & 0 \\ F & I \end{pmatrix}$$

det (1) = det (2) so the feedback doesn't modify the zeros

Reachability remains unchanged

$\{ \dot{x} = Ax + Bu$

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$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \qquad u = Fx + Gv \Rightarrow \dot{x} = (A+BF)x + BGv$$
$$|G| \neq 0$$

$$\mathcal{R} = \left( BG \;\vdots\; (A+BF)BG \;\vdots\; \cdots \right)$$
$$(A+BF)BG \to ABG + BFBG$$

$BFBG \rightarrow$ already belongs to the span of $BG$

For what concerns the zeros of the compensator $C$ $(p=q=1)$

$$C: \begin{cases} \dot{z} = (A - KC + BF)z + Mv + Ky \\ u = Fz + Nv \end{cases}$$

$$\det \begin{pmatrix} sI - A + KC - BF & -M \\ F & N \end{pmatrix} = \det\left( \begin{pmatrix} sI - A + KC - BF & -\frac{M}{N} \\ F & \frac{N}{N} \end{pmatrix} \begin{pmatrix} I & 0 \\ -F & I \end{pmatrix} \right)$$

$$= \det\left(sI - A + KC - BF + \frac{M}{N}F\right)$$

if $A - KC + BF = A_c$ and $\frac{M}{N} = L$

for any fixed pair $(F, K)$ the zeros of $C$ coincide with the ones in the unobservable subsystem of the pair $(A_c, F)$ + others arbitrary.

$\rightarrow = \det(sI - A_c + LF)$

zeros are the poles of the reconstructor

**Remark:** If all zeros are $< 0$ the system is said to be minimum phase.

$$\det \begin{pmatrix} sI - A & -B \\ C & D \end{pmatrix} = \det(sI - A) \cdot \det\left(D + C(sI-A)^{-1}B\right)$$

$$\rightarrow \det\left(D + C(sI-A)^{-1}B\right) = \frac{\det \begin{pmatrix} sI - A & -B \\ C & D \end{pmatrix}}{\det(sI - A)} = \frac{N(s)}{D(s)}$$