Structured Sparsity in Numerical Optimisation

Université Côte D'Azur, Inria, France

Matteo Frigo

Contents

1. Mathematics

Proximal Operator, FISTA, Regularisation

Model Design

Sparsity, Hierarchy, Lasso

3. Brain Imaging

Tractography, dMRI, Connectomics

Mathematics

Mathematics 1/4

Numerical Optimisation

- $ightharpoonup \mathcal{H}$ is a set
- Find

$$x^* = \underset{x \in \mathcal{H}}{\operatorname{argmin}} \Phi(x)$$

Mathematics 2/4

Numerical Optimisation

- $\vdash \mathcal{H}$ is a set
- $\Phi: \mathcal{H} \to \mathbb{R}$
- Find

$$x^* = \underset{x \in \mathcal{H}}{\operatorname{argmin}} \Phi(x)$$

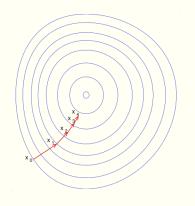
Our setting:

- $\mathcal{H} = \mathbb{R}^d$
- $\Phi(x) := f(x) + g(x)$
 - f(x) is convex and has *L*-Lipschitz continuous gradient
 - g(x) is convex and lower semi-continuous
- Find

$$x \in \mathbb{R}^d = \operatorname*{argmin}_{x \in \mathbb{R}^d} f(x) + g(x)$$

Mathematics 2

Trivial case



Smooth case: g(x) = 0

$$x^* = \operatorname*{argmin}_{x \in \mathbb{R}^d} f(x).$$

Scheme:

$$x^+ = x - \gamma \nabla f(x)$$

where $\gamma = \frac{1}{L}$

Mathematics 3/4

Constrained optimisation

Let
$$C \subset \mathbb{R}^d$$
,
$$x^* = \operatorname*{argmin}_{x \in C} f(x)$$

Mathematics 4/4

Constrained optimisation

Let $C \subset \mathbb{R}^d$,

$$x^* = \operatorname*{argmin}_{x \in \mathcal{C}} f(x)$$

Unconstrained formulation:

$$x^* = \operatorname*{argmin}_{x \in \mathbb{R}^d} f(x) + \iota_{\mathcal{C}}(x)$$

where

$$\iota_{\mathcal{C}}(x) = \begin{cases} 0 & \text{if } x \in \mathcal{C} \\ +\infty & \text{if } x \notin \mathcal{C} \end{cases}$$

Mathematics 4/4

Constrained optimisation

Let $C \subset \mathbb{R}^d$,

$$x^* = \operatorname*{argmin}_{x \in \mathcal{C}} f(x)$$

Unconstrained formulation:

$$x^* = \operatorname*{argmin}_{x \in \mathbb{R}^d} f(x) + \iota_{\mathcal{C}}(x)$$

where

$$\iota_{\mathcal{C}}(x) = \begin{cases} 0 & \text{if } x \in \mathcal{C} \\ +\infty & \text{if } x \notin \mathcal{C} \end{cases}$$

No smoothness, no continuity

Mathematics 4/4