Structured Sparsity in Numerical Optimisation

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by

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Contents

Mathematics

Numerical Optimisation

- $\mathcal{H}$ is a set
- $\Phi : \mathcal{H} \to \mathbb{R}$
- Find

$$x^* = \operatorname*{argmin}_{x \in \mathcal{H}} \Phi(x)$$

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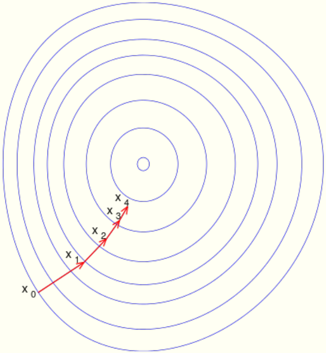
Our setting:

- $\mathcal{H} = \mathbb{R}^d$
- $\Phi(x) := f(x) + g(x)$
  - $f(x)$ is convex and has L-Lipschitz continuous gradient
  - $g(x)$ is convex and lower semi-continuous
- Find

$$x \in \mathbb{R}^d = \operatorname*{argmin}_{x \in \mathbb{R}^d} f(x) + g(x)$$

Trivial case



Smooth case: $g(x) = 0$

$$x^* = \underset{x \in \mathbb{R}^d}{\operatorname{argmin}} f(x).$$

Scheme:

$$x^+ = x - \gamma \nabla f(x)$$

where $\gamma = \frac{1}{L}$

Constrained optimisation

Let $C \subset \mathbb{R}^d$,

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Unconstrained formulation:

$$x^* = \operatorname*{argmin}_{x \in \mathbb{R}^d} f(x) + \iota_C(x)$$

where

$$\iota_C(x) = \begin{cases} 0 & \text{if } x \in C \\ +\infty & \text{if } x \notin C \end{cases}$$

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No smoothness, no continuity