Electrical Engineering
HSLU, Semester 2

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Contents

Part I

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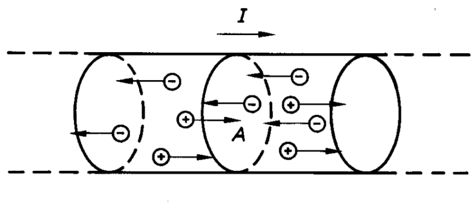
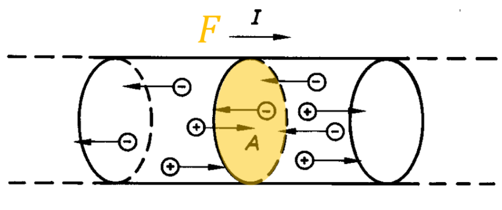
1 ...

1.1 Current strength or current “I”



$$I\ [A] = \frac{\text{el. charge}}{t}$$

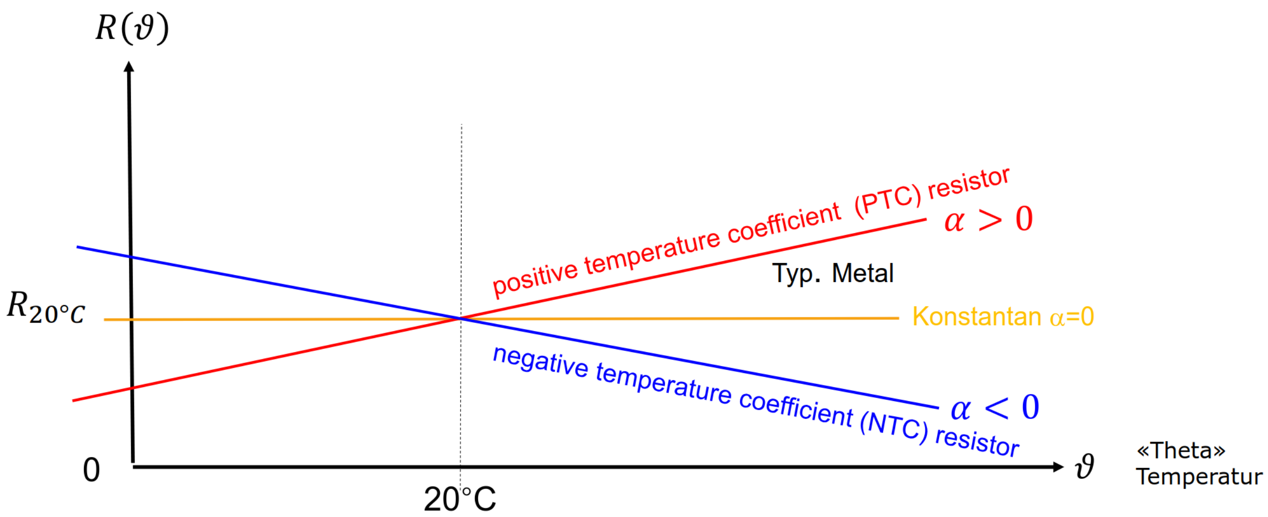
1.2 Current density “J”



The current density indicates how large the current per cross-sectional area (F) is:

$$J\ [\frac{A}{mm^2}] = \frac{I}{F}$$

1.3 Temperature dependence of the resistance



Depending on the material, the resistance can increase, remain the same or decrease with temperature. In ET+L we calculate using the linear approach.

$$\boxed{R(\vartheta) = R_{20}(1 + \alpha(\vartheta - 20°\text{C})) = R_{20}(1 + \alpha\Delta T)}$$

1.4 Object properties

The resistance indicates the voltage required for a current. In addition to the material, the cross-sectional area and also the length are decisive factors.

$$R = \frac{U}{I}$$

1.5 Reciprocal quantities

1.5.1 Specific resistance

To describe material properties, the resistance per length and cross-sectional area is specified (precondition: homogeneous conductor, direct current):

$$\rho\,[\frac{\Omega \cdot mm^2}{m}] = R \cdot \frac{A}{l}$$

1.5.2 Conductance

1.5.3 Specific conductivity

2 Gravitational fields

2.1 Between bodies

$$\boxed{F_1 = F_2 = G\frac{m_1 m_2}{d^2}}$$

2.2 Between particles

2.2.1 Coulomb's law

It calculates the amount of force between two electrically charged particles at rest:

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}$$

where:

- F: Force [N];
- q: Charge [As];
- ε_0: absolute permittivity $= 8.8542 \cdot 10^{-12}$ [As/Vm].

2.3 Electric field and force on a charge Q

2.3.1 Homogeneous electric fields

$$\boxed{E = \frac{U}{d}}$$

where:

- E: electric field strength [V/m];
- U: voltage [V];
- d: distance of the electrodes [m].

2.3.2 Force on a point charge

$$\boxed{F = Q \cdot E}$$

where:

- E: electric field strength [V/m];
- Q: charge [As];
- F: force [N].

3 Capacitance and Capacitor

3.1 Capacitor

A capacitor is a device in which the capacitance is used.

3.2 Capacitance

Capacitance C is the **capability** to store electric charge. It is measured by the charge divided by the applied voltage:

$$\boxed{C = \frac{Q}{U}}$$

where:

- Q: charge [As];
- U: voltage [V];
- C: capacitance [As/V = F (Farad)].

3.2.1 Capacitance of a plate capacitor

$$\boxed{C = \varepsilon \cdot \frac{A}{d}}$$

where:

- A: plate area (one side) [m^2];
- d: distance between plates [m];
- C: capacitance [F].

Permittivity

$$\boxed{\varepsilon = \varepsilon_r \cdot \varepsilon_0}$$

- ε_r: relative permittivity of the dielectric, relative to the air;
- ε_0: absolute permittivity [As/Vm].

3.2.2 Energy in a capacitor

If a capacitor is discharged with a constant current, the voltage decreases linearly:

$$\boxed{\int_0^{t_{\text{empty}}} U(t) \cdot I \, dt = I \cdot U_0 = \frac{I \cdot U_0 \cdot t_{\text{empty}}}{2}}$$

Or, simplified:

$$\boxed{W = \frac{1}{2} C \cdot U_0^2}$$

where:

- W: energy [J or Ws];
- U_0: initial voltage [V];
- C: capacitance [F].

3.3 Capacitors in parallel connection

Capacitances connected in parallel add up:

$$\boxed{C_{\text{tot}} = \frac{\sum_n Q_n}{U} = \sum_n C_n}$$

or

$$\boxed{C = \frac{\varepsilon \cdot \left(\sum_n A_n\right)}{d} = \sum_n C_n}$$

3.4 Capacitors in series connection

In a series connection, the reciprocal of the total capacitance is the sum of the reciprocals of the individual capacitances:

$$\boxed{\frac{1}{C_{\text{tot}}} = \sum_n \frac{1}{C_n}}$$

where:

- C_{tot}: total capacitance [F];
- C_n: capacitance of the n-th capacitor [F].

4 Transient Analysis in RC Circuits

4.1 Charging of a Capacitor

When a capacitor is charged through a resistor, the voltage across it increases exponentially:

$$\boxed{U_C(t) = U_0 \cdot \left(1 - e^{-t/(R \cdot C)}\right)}$$

with the time constant defined as:

$$\boxed{\tau = R \cdot C}$$

where:

- $U_C(t)$: voltage across the capacitor at time t [V];
- U_0: applied voltage [V];
- R: resistance [Ω];
- C: capacitance [F];
- τ: time constant [s].

4.2 Discharging of a Capacitor

When a charged capacitor discharges through a resistor, the voltage decays exponentially:

$$\boxed{U_C(t) = U_0 \cdot e^{-t/(R \cdot C)}}$$

and the discharging current is:

$$\boxed{I(t) = \frac{U_0}{R} \cdot e^{-t/(R \cdot C)}}$$

4.3 Transitional phase

$$\boxed{f(t) = A + \Delta \cdot \left(1 - e^{t/\tau}\right) = A + (B - A) \cdot (1 - e^{1/\tau})}$$

5 Additional Topics

5.1 Energy Stored in a Capacitor

The energy stored in a capacitor is given by:

$$\boxed{W = \frac{1}{2} C \cdot U_0^2}$$

where:

- W: energy [J];
- C: capacitance [F];
- U_0: voltage [V].

5.2 Charge–Voltage Relationship

For an ideal capacitor, the relationship between charge and voltage is:

$$\boxed{Q = C \cdot U}$$

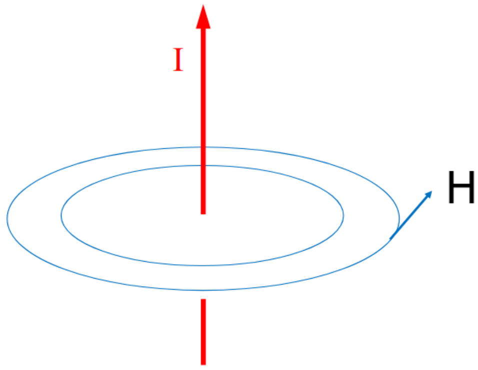
Moreover, the current is the time derivative of the charge:

$$\boxed{I = \frac{dQ}{dt} = C \cdot \frac{dU}{dt}}$$

Note that the voltage across an ideal capacitor cannot change instantaneously.

6 Electromagnetic fields

6.1 Hans Christian Ørsted Observation



1. The magnetic field lines encircle the current-carrying conductor;
2. The magnetic field lines lie in a plane perpendicular to the current-carrying wire;
3. If the direction of the current is reversed, the direction of the magnetic field lines is also reversed;
4. The strength of the field is directly proportional to the magnitude of the current;
5. The strength of the field at any point is inversely proportional to the distance of the point from the wire.

6.2 Definitions and formuals

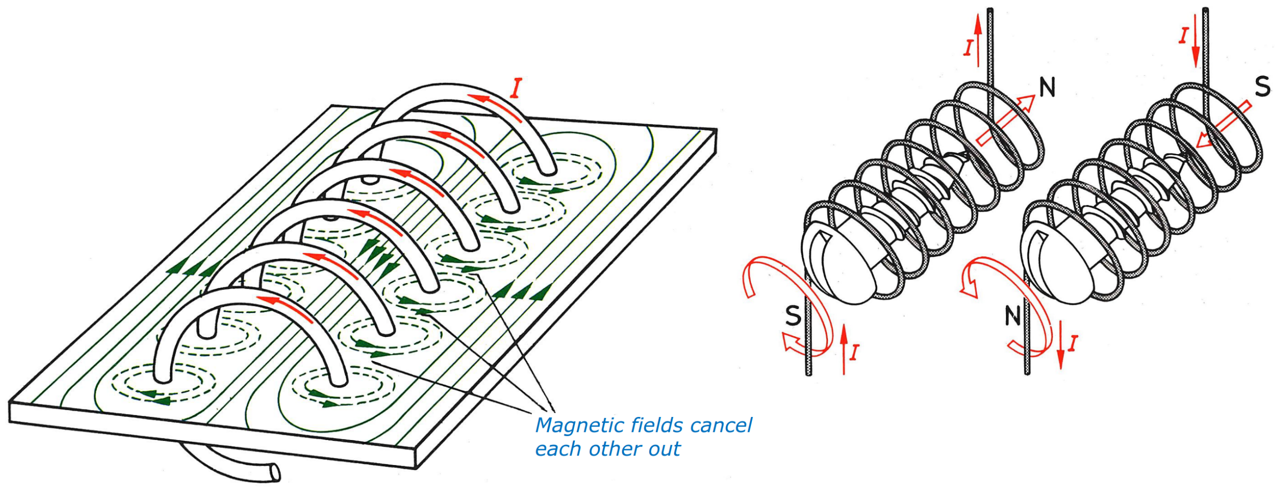
6.2.1 Magnetomotive force

$$\boxed{\theta = N \cdot I}$$

6.2.2 Ampère's circuital law

$$\boxed{\theta = \oint \overrightarrow{H(s)} \cdot \mathrm{d}\vec{s}}$$

6.2.3 Magnetic field in a coil



6.2.4 Magnetic flux density

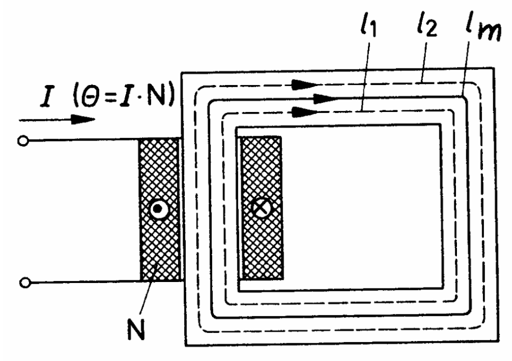
$$\boxed{B = \frac{\Phi}{A} = \mu \cdot H = \mu_0 \mu_r \cdot H}$$

where:

- B: magnetic flux density [T = Vs/m^2];
- Φ: magnetic flux [Wb];
- A: area [m^2];
- μ: magnetic permeability [H/m = Vs/Am];
- H: magnetic field strength [A/m];
- μ_0: magnetic constant [$4\pi \cdot 10^{-7}$ Vs/Am];
- μ_r: relative permeability.

Note: Φ is the sum of all B-field lines through the cross section A

6.2.5 Magnetic field strength in coil with iron core



$$\boxed{H = \frac{N \cdot I}{l_m} = \frac{\Theta}{l_m}}$$

where:

- H: magnetic field strength [A/m];
- N: number of turns;
- I: current [A];
- l_m: median field line length [m];
- Θ: magnetomotive force [A].

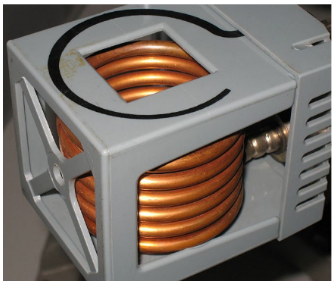
6.2.6 Magnetic relative permeability μ

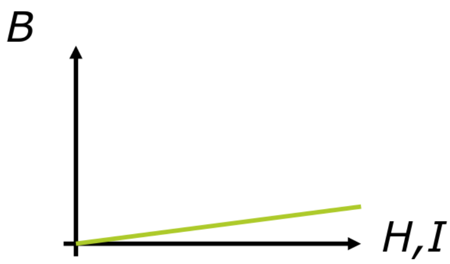
Permeability is a measure for the ability to conduct magnetic field lines:

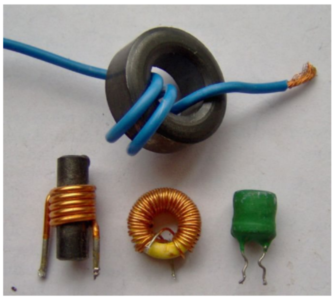
Material	$\mu_\mathbf{r}$
Air	1
Pure iron	up to 250'000
Electrical steel	500 ... 7000
Steel	40 ... 7000
Water	0.99991

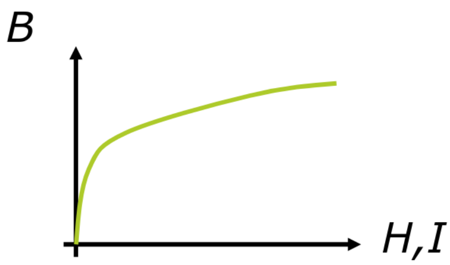
6.2.7 Coils with and without iron core

The magnetization curve of a coil without a core is linear, but there is significantly less flux density B than with an iron core.



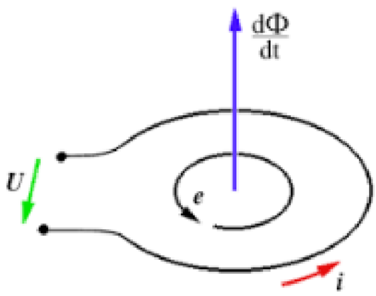


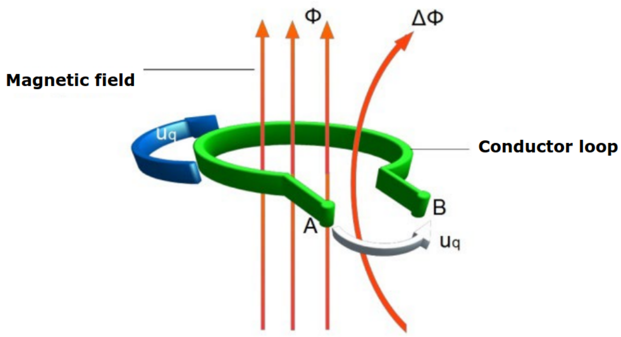




6.2.8 Law of induction and inductance

Changing magnetic flux generates a voltage





Phenomenon: a changing magnetic flux Φ induces a voltage in a conductor loop around it:

$$\boxed{U = -N \cdot \frac{\mathrm{d}\Phi}{\mathrm{d}t}}$$

6.2.9 Inductance and induction

Inductance L is the capability to generate a magnetic field. It is measured by the voltage divided by the rate of change of current over time. It is a measure of the magnetic “capacity” of an arrangement of conductors (e.g. coil) and can be compared to the capacity C of a capacitor. It indicates how much magnetic flux per ampere is generated.

$$\boxed{L = \frac{N \cdot \Phi}{I} = \frac{U}{\frac{\Delta I}{\Delta t}}}$$

where:

- L: inductance [H = Vs/A];
- N: number of turns;
- Φ: magnetic flux [Wb];
- I: current [A];
- U: voltage [V].

6.2.10 Inductivity of a very long coil

The inductance of a very long coil can be calculated approximately with:

$$\boxed{L = \frac{\mu \cdot N^2 \cdot A}{l}}$$

where:

- L: inductance [H = Vs/A];
- μ: magnetic permeability [Vs/Am];
- N: number of turns;
- A: cross-section of the coil [m^2];
- l: length [m].

6.2.11 Energy stored in an inductor

Since a variable magnetic field induces a voltage in which a current can also flow, the magnetic field must contain energy:

$$\boxed{W = \frac{1}{2} L \cdot I^2}$$

where:

- W: work, energy [J = Ws];
- L: inductance [H = Vs/A];
- I: current [A].

6.2.12 Current-voltage relationship of an inductor

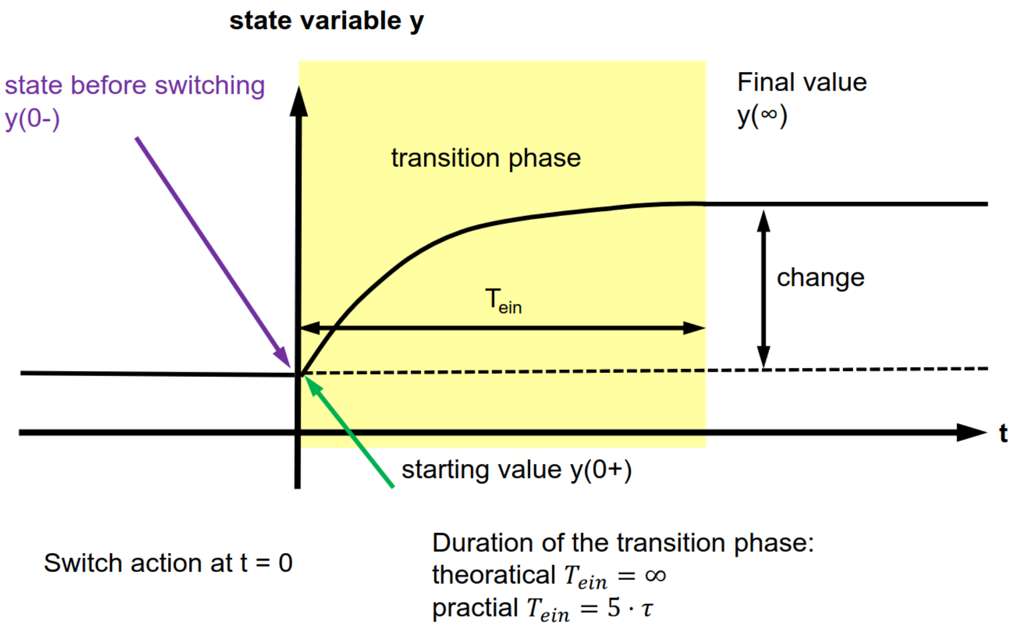
The current-voltage relationship of an inductor is:

$$\boxed{U = L \cdot \frac{\mathrm{d}I}{\mathrm{d}t}}$$

Special case:

$$\boxed{0 = L \cdot \frac{\mathrm{d}I}{\mathrm{d}t} \rightarrow u_c = 0}$$

6.2.13 Transient analysis



1. The state variable $y(t)$ is the variable that cannot change instantaneously. For the inductor, this is $i_L(t)$. The state just before the switch action:
$$y(0^-) = i_L(0^-).$$
2. The starting value is the state immediately before the switch action:
$$y(0^+) = i_L(0) = i_L(0^-).$$
That is, the state variable i_L keeps the value from $t = 0^-$.
3. The final value is the value long after the switch action:
$$y(\infty) = i_L(\infty),$$
which is practically reached after 5τ.
4. The transient is described by the function of time:
$$y(t) = \text{final value} + \big(\text{starting value} - \text{final value}\big) \exp\left(-\frac{t}{\tau}\right).$$
Hence,
$$i_L(t) = i_L(\infty) + \Big(i_L(0+) - i_L(\infty)\Big) \exp\left(-\frac{t}{\tau}\right).$$

Time constant τ for an inductor

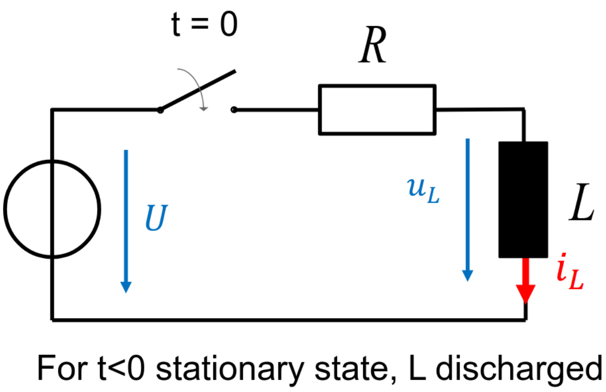
$$\boxed{\tau = \frac{L}{R}}$$

where:

- τ: time constant [s];
- L: inductance [H];
- R: resistance [Ω].

6.3 Examples

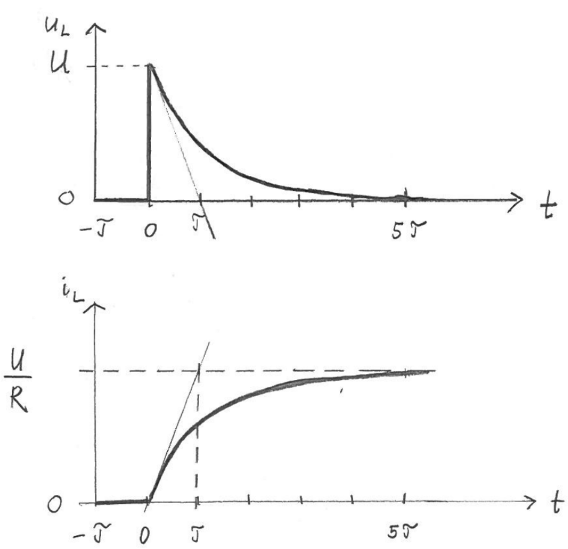
6.3.1 Charging an inductor in a RL-network



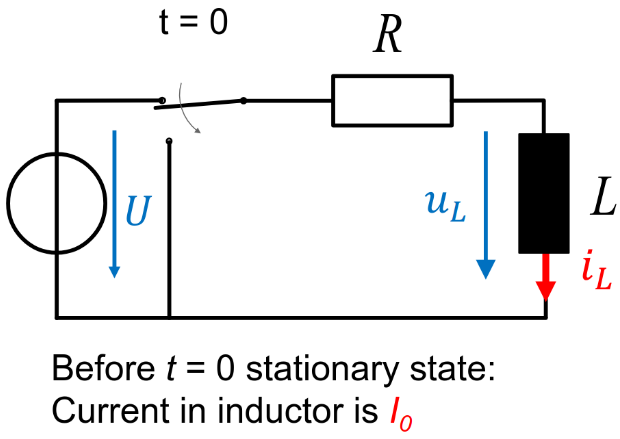
Calculations

$$\boxed{\begin{aligned} i_L &= \frac{U}{R} \cdot \left(1 - \exp\left(-\frac{t}{\tau}\right)\right) \\ u_L &= U \cdot \exp\left(-\frac{t}{\tau}\right) \end{aligned}}$$

Graphical representation



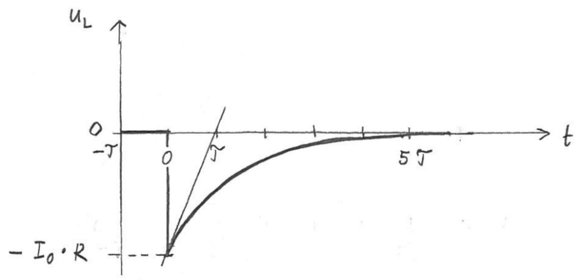
6.3.2 Discharging an inductor in a RL-network

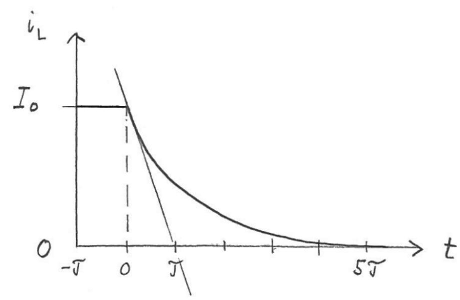


Calculations

$$\boxed{\begin{aligned} i_L &= I_0 \cdot \exp\left(-\frac{t}{\tau}\right) \\ u_L &= -I_0 \cdot R \cdot \exp\left(-\frac{t}{\tau}\right) \end{aligned}}$$

Graphical representation





7 Alternating current (AC)

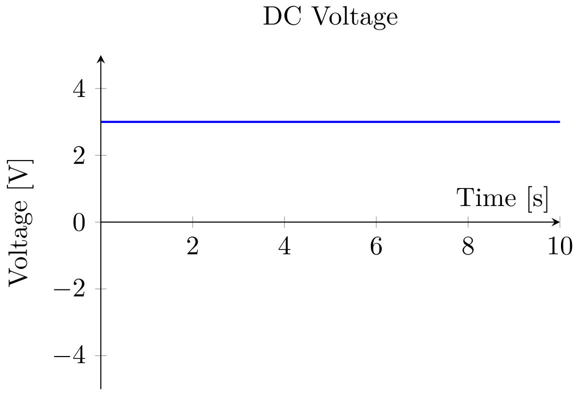
7.1 Generation of alternating current / voltage

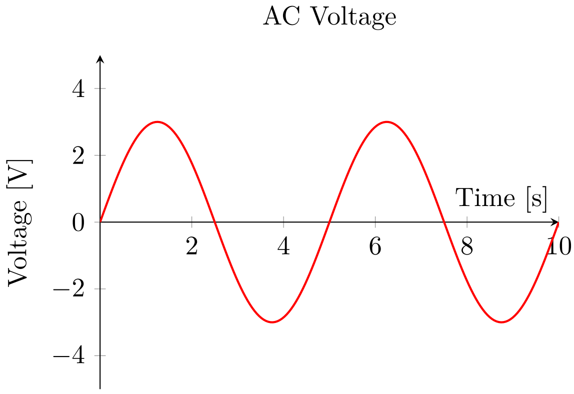
$$\boxed{U = -N \cdot \frac{\Delta\Phi}{\Delta t}}$$

where:

- U: voltage [V];
- N: number of turns;
- Φ: magnetic flux [Wb].

7.2 Comparison of AC and DC





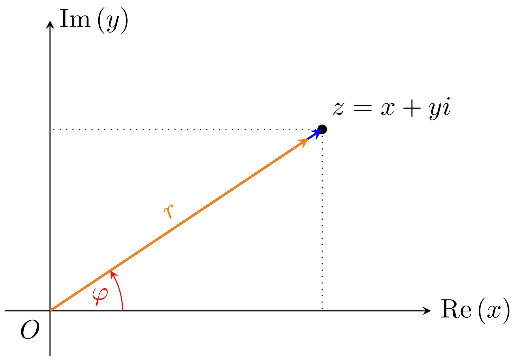
7.2.1 Advantages of AC

- Simple voltage transformation;
- Efficient trransmission;
- Easier generation;
- Compatibility with electric motors.

7.2.2 Disadvantages of AC

- Complexity in storage;
- Higher risk of shock;
- Complex circuits;
- Higher rectification costs.

7.3 Phasors



$$\boxed{z = x + yi = r\angle\varphi}$$

7.4 Oscillation as a function of the angle

Sinusoidal voltage has an instantaneous value $u(t)$ or u for every time t.

After a period of time T, the curve repeats itself.

$$\boxed{u(t) = \widehat{U}\sin(\omega \cdot t)}$$

7.5 Zero phase angle φ

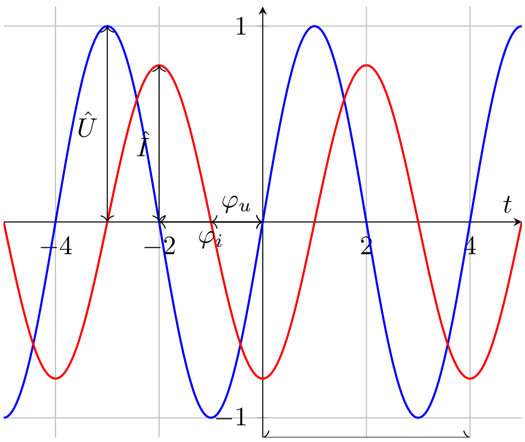
$$\boxed{u(t) = \widehat{U}\sin(\omega \cdot t + \varphi_u)}$$

7.5.1 Phase shift $\Delta\varphi$ between two signals

$$\boxed{\begin{aligned} u(t) &= \widehat{U}\sin(\omega \cdot t + \varphi_u) \\ i(t) &= \widehat{I}\sin(\omega \cdot t + \varphi_i) \end{aligned}}$$

The phase shift between two signals is the difference between the their zero phase signals:

$$\boxed{\Delta\varphi = \varphi_u - \varphi_i}$$



7.6 Power in a sinusoidal signal and effective value

7.6.1 Instantaneous power

The instantaneous power $p(t)$ is the actual power at a specific time t and is the product of the voltage $u(t)$ and the current $i(t)$ at that moment:

$$\boxed{p(t) = u(t) \cdot i(t) = \frac{u(t)^2}{R} = i(t)^2 \cdot R}$$

The active power P corresponds to the mean value of the instantaneous powerr $p(t)$ averaged over a period T:

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt}$$

7.6.2 Effective value

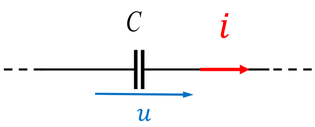
The effective value U_{eff} of a sinusoidal signal is the voltage that would generate the same power in a resistor as the sinusoidal signal:

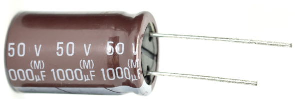
$$\boxed{U_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T u(t)^2\,dt} = \frac{\widehat{U}}{\sqrt{2}}}$$

The same can be applied to the effective value I_{eff}:

$$\boxed{I_{\text{eff}} = \frac{\widehat{I}}{\sqrt{2}}}$$

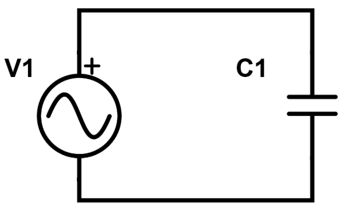
7.7 Relationship between current and voltage on a capacitor

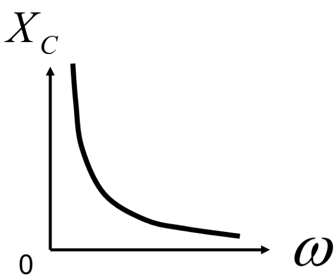




$$\boxed{i = C \cdot \frac{\Delta u}{\Delta t}}$$

7.8 Capacitive reactance X_c



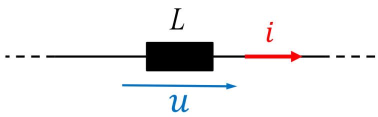


$$\boxed{X_c = \frac{1}{\omega \cdot C}}$$

where:

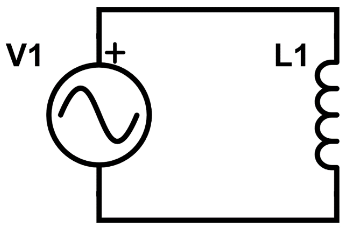
- X_c: capacitive reactance [Ohm];
- ω: angular frequency [rad/s];
- C: capacitance [F = As/V].

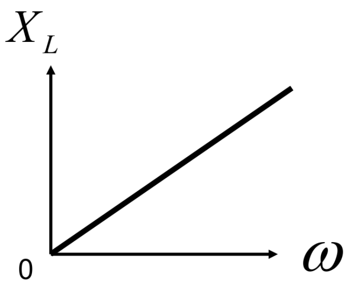
7.9 Relationship between current and voltage on an ideal inductor



$$\boxed{u = L \cdot \frac{\Delta i}{\Delta t}}$$

7.10 Inductive reactance X_L





$$\boxed{X_L = \omega \cdot L}$$

where:

- X_L: inductive reactance [Ohm];
- L: inductance [H];
- ω: angular frequency [rad/s];