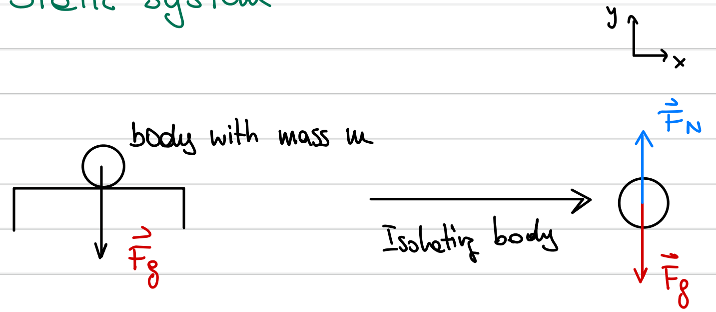
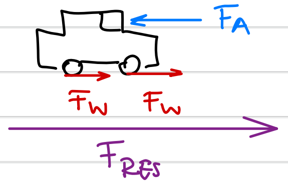
Static system



$\Sigma F_y = F_N - F_g = 0$

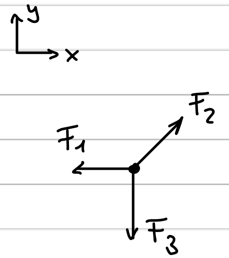
$\Sigma F_x = 0$

Dynamic system



$a = \frac{F_{RES}}{m}$

Force directions and resultants



$\Sigma F_x = -F_1 + F_2 \cos(\alpha)$

$\Sigma F_y = -F_3 + F_2 \sin(\alpha)$

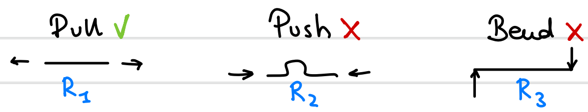
Let's assume $\alpha = 45°$ and $F_2 = 100\ N$:

$F_1 = F_2 \cos 45° = 70{,}7\ N$

$F_3 = F_2 \sin 45° = 70{,}7\ N$

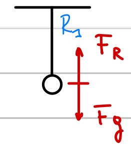
Ropes

Ropes only can take tensile forces and NOT compressive forces



- Isolated ropes

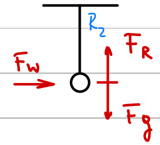
STATIC



$\Sigma F_y = 0$

$\Sigma F_x = 0$

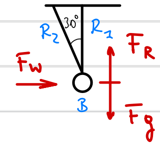
DYNAMIC (with wind)



$\Sigma F_y = 0$

$\Sigma F_x = F_W$ } For make it static, we have to add more ropes

STATIC from a D. State



Let's assume: $F_W = 50\ N$

$m_B = 200\ kg$

$\Sigma F_x = F_W - F_{R_1} \cdot \cos(-30°)$

$\Sigma F_y = 0$

$\Sigma F_x = 50\ N - F_{R_2} \sin 30° = 0$

$\Sigma F_y = F_{R_1} - F_g + F_{R_2} \cos 30° =$

$F_{R_2} = 50\ N / \sin 30° = 100\ N$

$F_y = F_{R_1} - F_g + F_{R_2} \cos 30°$

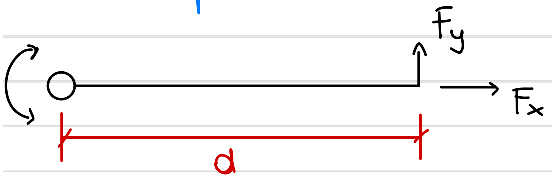
$F_g = mg = 200\ kg \cdot 9{,}81\ m/s^2 = 1962\ N$

$F_{R_1} = F_g - F_{R_2} \cos(30°) = 1962\ N - 86{,}6\ N = 1875{,}4\ N$

$F_y = 0$

Moments and couple

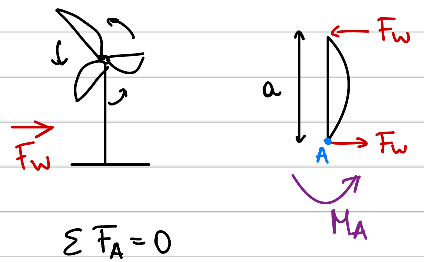
Couple:



A couple is created by a force applied at a distance

$$\boxed{\begin{aligned} M_z &= F_x d_x \\ M_z &= F_y d_y \end{aligned}}$$

$\Downarrow$ Example

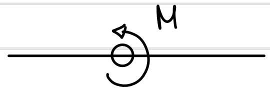


$\Sigma F_A = 0$

$M_A = F_w \cdot d \rightarrow M = \begin{cases} F_x \cdot d_y \\ F_y \cdot d_x \end{cases}$

$M\,[Nm]$

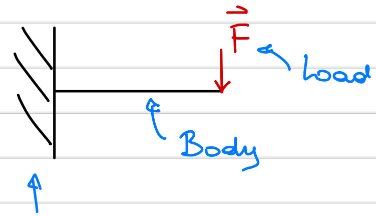
Moments:



A moment is created by an engine and acts at one single point

Free body diagram (FBD)

For each mechanical problem, drawing a FBD is needed!



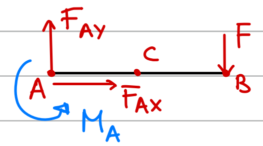
(1)

Boundary ← We need to replace the boundary condition by forces and moments

Boundary conditions can be created by:

- touching bodies
- hinges and fixations
- environmental forces (pressure, gravity)

In (1), we isolate the body:



$\Sigma F_x = 0 = A_x$

$\Sigma F_y = 0 = -F + A_y$

$\Sigma M_A = -F \cdot dx + M_A$

$\Sigma M_B = M_A - A_y \cdot dx = 0$

$\Sigma M_C = M_A - F \cdot \frac{1}{2} dx - A_y \cdot \frac{1}{2} dx$

$M_A - \left(\frac{A_x \cdot dx - B_x \cdot dx}{2} \right)$

Supports

Every blocked degree of freedom (DOF) needs to be replaced by a force or a moment

- Rotation blocked → M

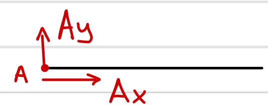
- Translation blocked → F

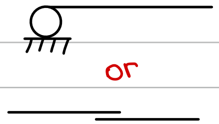
In 2Ds systems there are 3 DOF for each point:

$$\boxed{\begin{cases} 1)\ \text{Translation in } x \\ 2)\ \text{Translation in } y \\ 3)\ \text{Rotation around } z \end{cases}}$$

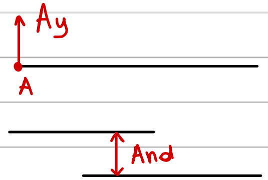
Types of supports

Hinges fix x, y and allow rotation

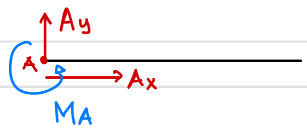




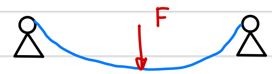
Rollers or two horizontal surfaces fix y and allow rotation and traslation in x



Wall fixtures / Fixed supports fix x, y and rotations

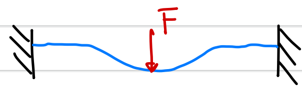


1)



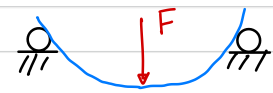
→ Used for shelves

2)



→ Used for shelves

3)

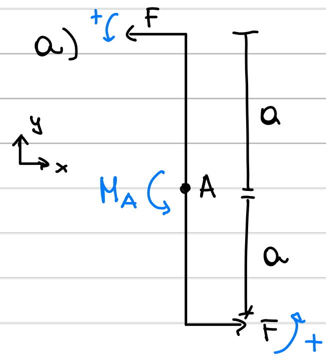


→ used for bending tests, balance boards

---

exercises:

a)

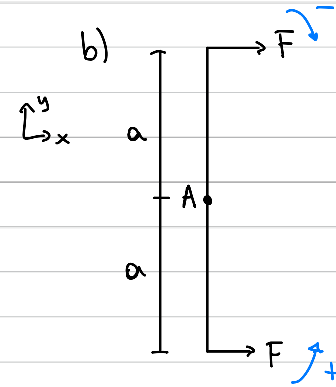


$\Sigma F_x = F - F = 0$

$\Sigma F_y = 0$

$\Sigma M_A = F \cdot a + F \cdot a = 2Fa$

b)

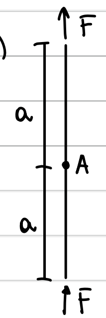


$\Sigma F_x = 2F$

$\Sigma F_y = 0$

$\Sigma M_A = +Fa - Fa = 0$

c)

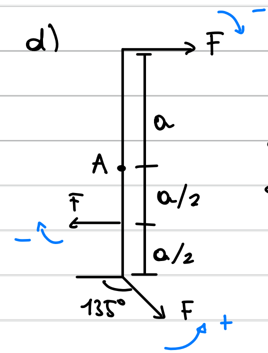


$\Sigma F_y = F + F = 2F$

$\Sigma F_x = 0$

$\Sigma M_A = 0$

d)



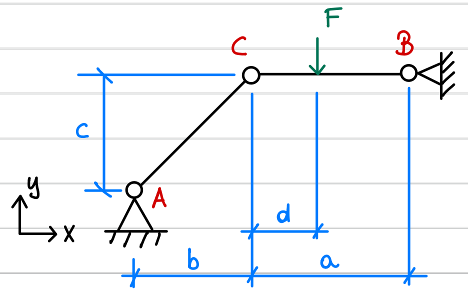
$\Sigma F_x = \cancel{F} - \cancel{F} + F\cos 45$

$\Sigma F_y = -F\sin 45$

$\Sigma M_A = -Fa - F \cdot \frac{a}{2} + F \cdot \cos 45 \cdot a$

Multi - body systems

Two bodies can have several FBD's:



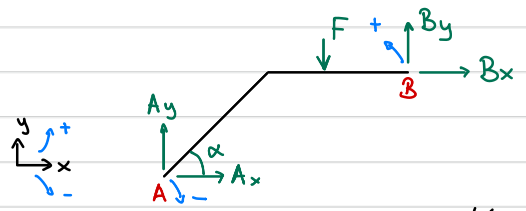
Example

Let:

F = 2000 N, a = 7 m,

b = 2 m, c = 6 m, d = 3 m.

Step 1: Set up the FBD of the entire system:



Step 2: Equilibrium equations for $F\begin{pmatrix} A_x, B_x \\ A_y, B_y \end{pmatrix}$ seen from point B:

$$\Sigma F_x = F(A_x) + F(B_x) = 0$$

$$\Sigma F_y = F(A_y) + F(B_y) - F$$

where:

$F(A_x) = F_A \cdot \cos\alpha$

$F(A_y) = F_A \cdot \sin\alpha$

$$\Sigma M_B = F(A_x)\cdot c - F(A_y)(a+b) + F(a-d) = 0$$

Step 3: Magnitude / Direction:

$$\tan\alpha = \frac{c}{b}$$

Step 4: Final calculations:

$\Sigma M_B = 0$

$0 = F_A \cos\alpha \cdot 6 - F_A \sin\alpha\,(7+2) + 2000\,(7-3)$

$F_A = \underline{1204{,}7\ N}$

$F(A_x) = F_A \cos\alpha = \underline{381\ N}$

$F(A_y) = F_A \sin\alpha = \underline{1142{,}9\ N}$

$\Sigma F_x = 0$

$F(A_x) + F(B_x) = 0$

$F(B_x) = \underline{-381\ N}$

$\Sigma F_y = 0$

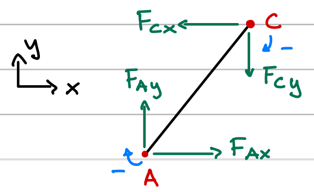
$F(A_y) + F(B_y) - F = 0$

$F(B_y) = F - F(A_y)$

$F(B_y) = 2000\ N - 1142{,}9\ N = \underline{857{,}1\ N}$

Step 5: Forces in the joint

Body 1:



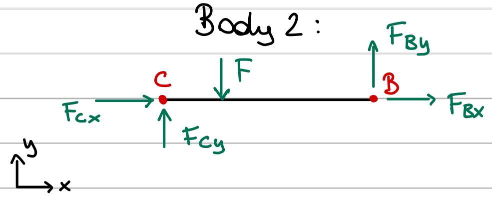
$\Sigma F_x = F_{Ax} - F_{Cx} = 0$

$F_{Cx} = F_{Ax} = \underline{381\ N}$

$\Sigma F_y = F_{Ay} - F_{Cy} = 0$

$F_{Cy} = F_{Ay} = \underline{1141{,}9\ N}$

Body 2:



$\Sigma F_x = F_{Bx} + F_{Cx} = 0$

$F_{Cx} = -F_{Bx} = \underline{381\ N}$

$\Sigma F_y = F_{By} + F_{Cy} - F$

$F_{Cy} = F - F_{By} = \underline{1141{,}9\ N}$

Constrains and Statical Determinancy

Statically determinate:
No. of eq. = No. of unknowns
Support forces = DOF

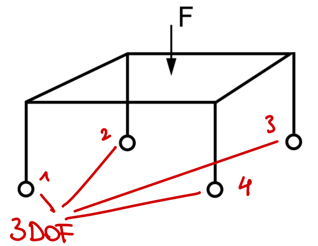
Statically indeterminate:
No. of eq. < No. of unknowns
support forces < DOF

Statically overdeterminated
No. of eq. > No. of unknowns
support forces > DOF

Examples:

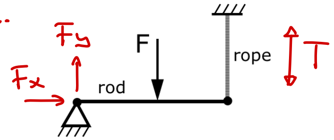
a) Table with 4 legs. All 4 legs on rollers on a flat floor.

3 DOF / leg
4 legs
⇓
Statically indet.



b) A rod supported by a hinge and a rope

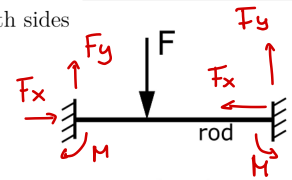
F_x, F_y, T = 3 unk.
2D → 3 DoF



3 DOF = 3 unk
⇓
Statically det.

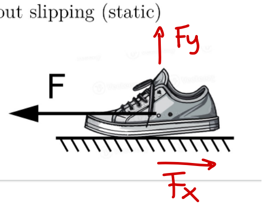
c) A rod fixed on both sides

6 unk, 3 DOF
⇓
Statically ind.

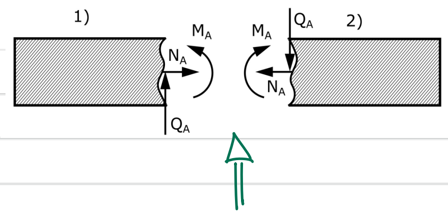


d) A shoe on the ground without slipping (static)

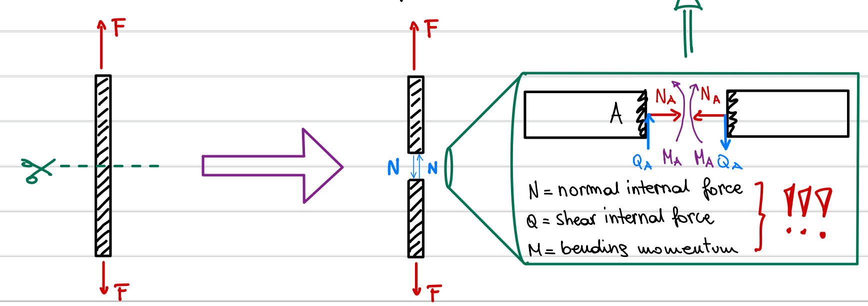
3 DOF, 2 unk.
⇓
statically overdet.



Internal forces



Let's cut virtually a rope:

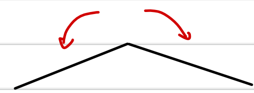


Reactions with less unknowns than equations:

Missing N:

Missing Q:

Missing M:



What do we use for what?

① System FBD and equilibrium: Determining external forces and support reactions

② Body isolation in a multi-body system: Determining interface and reaction forces support reactions

③ Internal forces: Determining stress and evaluate the safety

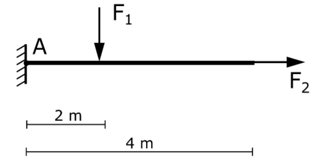
Shear / Moment / Tension diagram

Procedure:

Step 1: FBD diagram, calculate support reactions

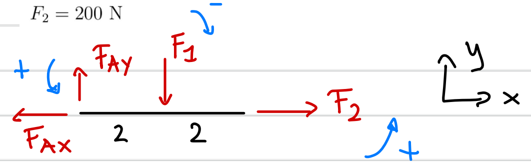
Step 2: Calculate internal forces and moment

Example



Given:
$F_1 = 100$ N
$F_2 = 200$ N

FBD:

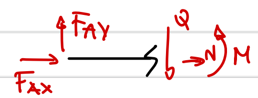


$$\Sigma F_x = -F_{Ax} + F_2 = 0 \Rightarrow F_{Ax} = F_2 = 200\,N$$

$$\Sigma F_y = F_{Ay} - F_1 = 0 \Rightarrow F_{Ay} = F_1 = 100\,N$$

$$\Sigma M_A = -F_1 \cdot 2m + F_2 \cdot 4m = 200\,N$$

Cut 1: $0m < x < 2m$

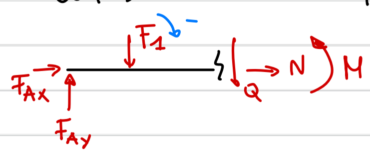


$$N_1 = F_{Ax} = 200N$$

$$Q_1 = -F_{AY} = -100\,N$$

$$\Sigma M = M_A + Q_1 x$$

Cut 2: $2m < x < 4m$

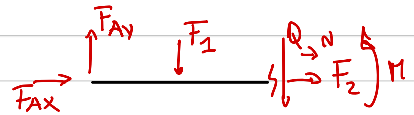


$$N_2 = F_{Ax} = 200N$$

$$Q_2 = -F_{Ay} + F_1 = 0N$$

$$\Sigma M = M_A - M_{F_1} + Q_2 x$$

Cut 3:



$N_3 =$

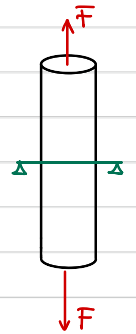
Stress and bending

Stress:

- It is needed to evaluate the safety
- It is calculated differently for each load case
  - Tensile (pure tensile load - stress)
  - Compressive (pure compressive load - stress)
  - Bending (tensile + compressive + shear stress)
  - Shear (pure shear stress)
  - Torsion (pure shear stress)

calculated in the same way

Tensile and compressive stress

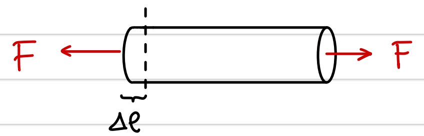


$$\boxed{\sigma_{Tensile} = \frac{F_{int}}{A} \quad \left[MPa = \frac{N}{mm^2}\right]}$$

Stress: internal loads incl. geometry

Strain

strain: internal shape changes



$$\boxed{\begin{aligned} \varepsilon_{Tensile} &= \frac{\Delta \ell}{\ell_0} \quad [-] \\ \varepsilon_{Compressive} &= \frac{\Delta \ell}{\ell_0} \quad [-] \\ \gamma_{Shear} &= \frac{\Delta S}{\Delta h} \quad [-] \end{aligned}}$$

$$\boxed{\sigma = E \cdot \varepsilon}$$

E = young's modulus [MPa] or [GPA]

Some young's modulus

$E_{steel} = 210'000 \text{ MPa} = 210 \text{ GPa}$

$E_{Aluminium} = 68'000 \text{ MPa} = 68 \text{ GPa}$

$E_{PA\ (Polymer)} = 2'100 \text{ MPa} = 2{,}1 \text{ GPa}$

Safety calculation

$\sigma_{int} < \sigma_{max,\ admissible}$

$\varepsilon_{int} < \varepsilon_{max,\ admissible}$

material data + safety factor

Formula Sheet SMT - Diagrams

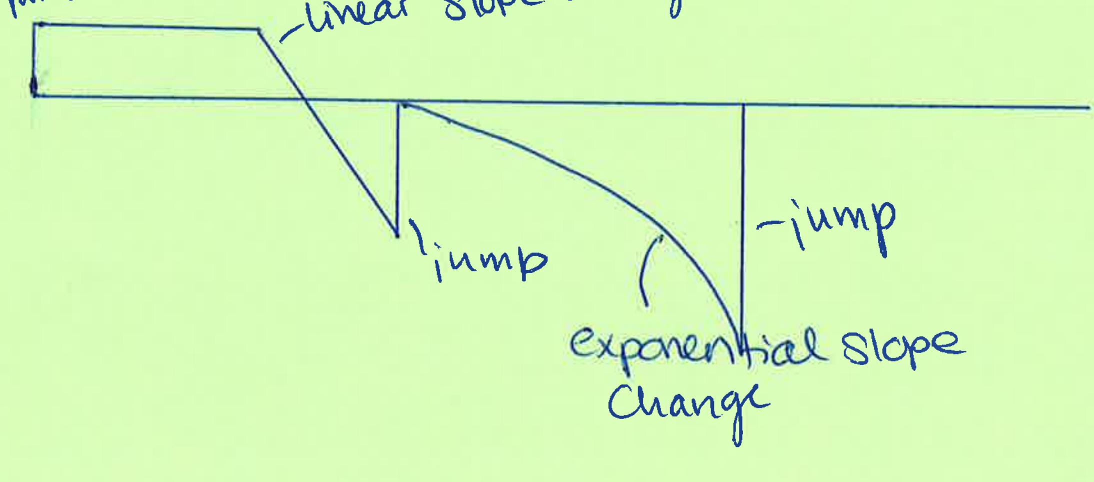
① all diagrams need to be zero at $x = 0$ and at x = part length.

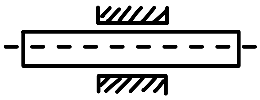
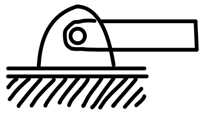
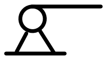
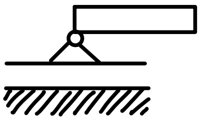
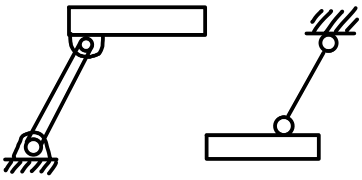
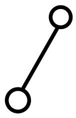
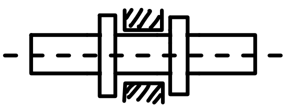
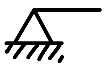
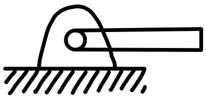
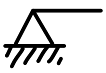
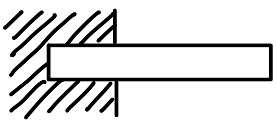
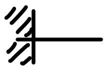
② If a normal force is applied at x...
- → Q makes a jump at x
- → N no change at x
- → M changes its slope in x (linear)

If a parallel force is applied (in line with the body) at x...
- → Q no change at x
- → N makes a jump at x
- → M no change at x

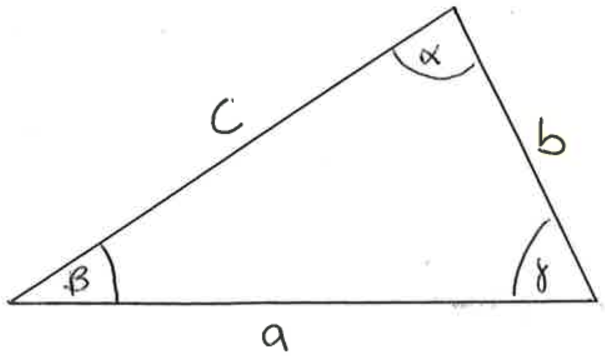
If a moment is applied at x...
- → Q no change at x
- → N no change at x
- → M jumps at x

If a distributed normal force is applied starting at x...
- → Q changes its linear slope at x
- → N no change
- → M changes its exponential slope at x



Type	Symbol	Reaction values		Value factor	
		in plane	in space		
Movable bearings:					
Radial bearing 		F_{AY}	F_{AY}, F_{AZ}	1	2
Slide bearing 		F_{AY}	F_{AZ}	1	1
Roller bearing 		F_{AY}	$(F_{AY}), F_{AZ}$	1	1 (2)
Vibrating rod, cable 		F_A	F_A	1	1
Fixed bearings:					
Thrust and axial bearings 		F_{AX}, F_{AY}	F_{AX}, F_{AY}, F_{AZ}	2	3
Fixed joint 		F_{AX}, F_{AY}	F_{AX}, F_{AY}, F_{AZ}	2	3
Fixed clamping 		F_{AX}, F_{AY}, M_E	M_{EX}, M_{EY}, M_{EZ} F_{AX}, F_{AY}, F_{AZ}	3	6

Formula sheet triangles



Any triangle

Sinus theoreme

$$\sin\alpha : \sin\beta : \sin\gamma = a : b : c$$

$$a = \frac{b}{\sin\beta}\sin\alpha = \frac{c}{\sin\gamma}\sin\alpha$$

$$b = \frac{a}{\sin\alpha}\sin\beta = \frac{c}{\sin\gamma}\sin\beta$$

$$c = \frac{a}{\sin\alpha}\sin\gamma = \frac{b}{\sin\beta}\sin\gamma$$

Tangens theoreme

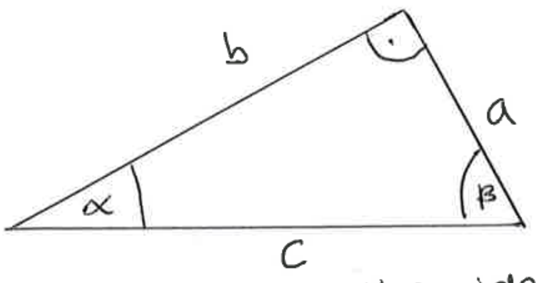
$$\frac{a+b}{a-b} = \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)}$$

$$\frac{a+c}{a-c} = \frac{\tan\left(\frac{\alpha+\gamma}{2}\right)}{\tan\left(\frac{\alpha-\gamma}{2}\right)}$$

$$\frac{b+c}{b-c} = \frac{\tan\left(\frac{\beta+\gamma}{2}\right)}{\tan\left(\frac{\beta-\gamma}{2}\right)}$$

Cosin theoreme

$$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$$
$$b^2 = c^2 + a^2 - 2ac \cdot \cos\beta$$
$$c^2 = a^2 + b^2 - 2ab \cdot \cos\gamma$$



Triangle with right angle (90°)

$$\sin\alpha = \frac{a}{c} = \frac{\text{opposite side}}{\text{hypotenuse}} \longrightarrow \alpha = \sin^{-1}\left(\frac{a}{c}\right)$$

$$\cos\alpha = \frac{b}{c} = \frac{\text{adjacent side}}{\text{hypotenuse}} \longrightarrow \alpha = \cos^{-1}\left(\frac{b}{c}\right)$$

$$\tan\alpha = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}} \rightarrow \alpha = \tan^{-1}\left(\frac{a}{b}\right)$$