

# Maths refreshing course

## HSLU, Semester 1

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September 3, 2024

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## Part I

# Lesson 1

## 1 Algebraic definitions

- $\mathbb{N} :=$  Natural numbers (including 0)
- $\mathbb{Z} :=$  Integer numbers
- $\mathbb{Q} :=$  Rational numbers
- $\mathbb{R} :=$  Real numbers

Notation: The “\*” symbol means that the set does not include 0.

We have that:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

## 2 Prime numbers

A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

$$n \in \mathbb{N}, n \neq \{0, 1\}$$

## 3 Positive powers

Let  $a \in \mathbb{R}, n \in \mathbb{R}^*$  and  $a \in \mathbb{R}$ , then

$$a^1 := a \quad | \quad a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}}$$

### 3.1 Property 1

Let  $a, b \in \mathbb{R}, n, m \in \mathbb{N}$ , then

$$a^n \cdot a^m = a^{n+m}$$

### 3.2 Property 2

Let  $a, b \in \mathbb{R}, n \in \mathbb{N}$ , then

$$(a \cdot b)^n = a^n \cdot b^n$$

Notation: The power  $a^n$ ,  $a$  is the base and  $n$  is the exponent.

### 3.3 Property 3

Let  $a \in \mathbb{R}, m, n \in \mathbb{N}^*$ , then

$$(a^n)^m = a^{n \cdot m}, \text{ which is } \neq a^{(n^m)}$$

## 4 Fractions

Notation 1:  $a \cdot b = a \times b = ab$     |     $\frac{a}{b} = a \div b = a : b$

Notation 2: “ $a$ ” is called numerator, “ $b$ ” is called denominator.

Notation 3:  $\frac{a}{b}$ ,  $a, b \in \mathbb{R}$ ,  $b \neq 0$

### 4.1 Property 1

Let  $a, b \in \mathbb{R}^*$  and  $c, d \in \mathbb{R}$ , then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

### 4.2 Property 2

Let  $a, b \in \mathbb{R}^*$  and  $c, d \in \mathbb{R}$ , then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

### 4.3 Property 3

Let  $a, b \in \mathbb{R}^*$  and  $c, d \in \mathbb{R}$ , then

$$\frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot d \pm c \cdot b}{b \cdot d}$$

## 5 Negative powers

### 5.1 Definition

$$\forall a \in \mathbb{R}^*; \quad a^{-1} := \frac{1}{a}$$

### 5.2 Property 4

Let  $\forall n \in \mathbb{N}$ ,  $\forall a \in \mathbb{R}$ , then

$$a^{-n} = \left(\frac{1}{a}\right)^n$$

This property implies that  $\forall z \in \mathbb{Z}$ ,  $\forall a \in \mathbb{R}$ ,  $z \neq 0$   
We can compute  $a^z$

### 5.3 Property 5

Let  $\forall a \in \mathbb{R}$ ,  $a \neq 0$ ,  $\forall n, m \in \mathbb{Z}$ , then

$$\frac{a^n}{a^m} = a^{n-m}$$

Consequences:

1. Properties 1, 2 and 3 also hold for integer exponents:

- $\forall a \in \mathbb{R}, \forall n, m \in \mathbb{Z} \Rightarrow a^n \cdot a^m = a^{n+m}$
- $\forall b \in \mathbb{R}, (a \cdot b)^n = a^n \cdot b^n$
- $(a^n)^m = a^{n \cdot m}$

2.  $\forall a \in \mathbb{R}^*, a^0 = a^{1-1} = \frac{a^1}{a^1} = 1 \Rightarrow a^0 = 1$

## 6 Fractions and percentages (and back)

$\alpha \in \mathbb{R}, n\% \text{ of } \alpha \iff \frac{n}{100} \cdot \alpha$

## Part II

# Lesson 2

## 7 Symbols

Let  $a, b \in \mathbb{R}$ , then

- $a = b \rightarrow$  equality;
- $a \neq b \rightarrow$  inequality ( $a$  is not equal to  $b$ );
- $a < b \rightarrow$  less than ( $a$  is strictly less than  $b$ );
- $a \leq b \rightarrow$  less than or equal to ( $a$  is less than or equal to  $b$ );
- $a > b \rightarrow$  greater than ( $a$  is strictly greater than  $b$ );
- $a \geq b \rightarrow$  greater than or equal to ( $a$  is greater than or equal to  $b$ ).

Example:  $x \in \mathbb{R}, x \geq 2 \rightarrow 2 \leq x < \infty$

## 8 Brackets

( ) Parenthesis (round brackets) (1)

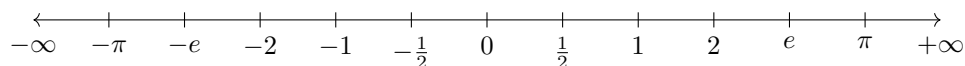
[ ] Square brackets (2)

{ } Braces (3)

## 9 Latin notations

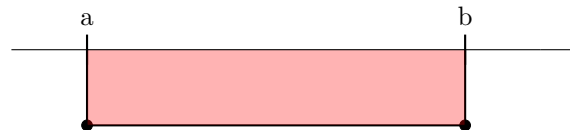
- e.g. = for example;
- i.e. = that is / that implies;
- q.e.d. ( $\square$ ) = quod erat demonstrandum (we finally prove it).

## 10 The real line

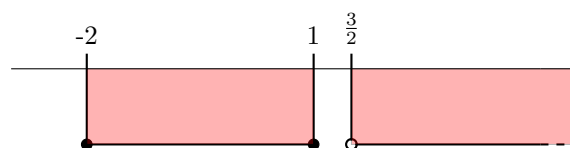


### 10.1 Exercises

1)  $\forall a, b, x \in \mathbb{R}, a \leq x \leq b$



2)  $\forall x \in \mathbb{R}, x \in ]-2, -1] \cup ]\frac{3}{2}, +\infty]$



Notation: The union of two or more intervals where  $x \in \mathbb{R}$  is denoted by the symbol  $\cup$  or  $\vee$ .

## 11 Properties of real numbers

### 11.1 Property 1 - Closure of “+” and “.”

$$\forall x, y \in \mathbb{R}$$

$$x + y \in \mathbb{R}$$

$$x \cdot y \in \mathbb{R}$$

Remark: for  $\forall x \in \mathbb{Z}$ , closure does not hold for division.

### 11.2 Property 2 - Commutativity

$$\forall x, y \in \mathbb{R}$$

$$x + y = y + x$$

$$x \cdot y = y \cdot x$$

Remark: commutativity does not hold for divisions and subtractions.

### 11.3 Property 3 - Associative

$$\forall x, y, z \in \mathbb{R}$$

$$x + (y + z) = (x + y) + z$$

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

Remark: associativity does not hold for divisions and subtractions.

### 11.4 Property 4 - Distributive

$$\forall x, y, z \in \mathbb{R}$$

$$x(y \pm z) = xy \pm xz$$

### 11.5 Property 5 - Identity

$$\forall x \in \mathbb{R}$$

a)  $0 + x = x$

b)  $1 \cdot x = x$

Remark:  $\forall x \in \mathbb{R}$ ,  $x \cdot 0 = 0$  is not an identity property.

### 11.6 Property 6 - Inverses and opposites

$$\forall x \in \mathbb{R}$$

a)  $x + (-x) = 0$  (additive inverse)

b) when  $x \neq 0$ ,  $x \cdot \frac{1}{x} = 1$  (multiplicative inverse or opposite)

Remark 1:  $\forall x \in \mathbb{N}$  does not exist either inverse nor opposite.

Remark 2:  $\forall x \in \mathbb{Z}$  has inverses, but not opposites.

## 12 The order of operations

1. Perform all operations inside grouping symbols beginning with the innermost set:  
( ) inside brackets operations;
2. Perform all exponential operations as you come to them, moving left-to-right:  
 $x^a$ ;
3. Perform all multiplications and divisions as you come to them, moving left-to-right:  
“.” and “÷”;
4. Perform all additions and subtractions as you come to them, moving left-to-right:  
“+” and “-”;
5. When the level of priority is the same (e.g. multiplications and divisions) solve them as you come to them.

## 13 Signed numbers

A number is denoted as positive if it is directly preceded by a + sign or no sign at all.

A number is denoted as negative if it is directly preceded by a - sign.

$\forall x \in \mathbb{R}$

$$-(-x) = x \qquad +(-x) = -x \qquad +(+x) = x \qquad -(+x) = -x$$

## 14 Absolute value

Let  $x \in \mathbb{R}$ , then

$$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

### 14.1 Property

$\forall x \in \mathbb{R}$

$$|x| > 0 \quad \text{if } x \neq 0$$

$$|x| = 0 \quad \text{if } x = 0$$