# Mathematics 1A
# HSLU, Semester 1

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# Part I

# Week 1

## 1 The set theory

### 1.1 Definition of a set

A set is a collection of objects or elements.

Remark: The collection of all sets is not a set.

### 1.2 Logical symbols

#### 1.2.1 Definition

Braces and the definition symbol ":=" are used to define a set giving all its elements:

$$\boxed{A := \{a, b, c, d, e\}}$$

#### 1.2.2 Equal

In this case, the equal symbol means that the set $A$ is equal to the set $B$:

$$\boxed{A = B}$$

#### 1.2.3 Belongs to

The symbols $\in$ and $\ni$ describe an element which is part of the set:

$$\boxed{a \in A \Longleftrightarrow A \ni a}$$

#### 1.2.4 Does not belong to

The symbols $\notin$ mean that an element does not belong to the set:

$$\boxed{f \notin A}$$

#### 1.2.5 Inclusion and contains

The symbols $\subset$ and $\supset$ mean that a set has another set included in its set:

$$\boxed{\mathbb{N} \subset \mathbb{Z} \Longleftrightarrow \mathbb{Z} \supset \mathbb{N}}$$

#### 1.2.6 For all/any

The symbol $\forall$ means that we are considering any type of element:

$$\boxed{\forall x \in \mathbb{R},\ x > 0}$$

In this case, we've defined a new set.

### 1.2.7 Implication

The symbol $\Rightarrow$ means that by setting a rule, we imply an event or an action:

$$\boxed{\text{if } x = 1 \Longrightarrow x \in \mathbb{N}, \text{ but if } x \in \mathbb{N} \text{ we do not know if } x = 1}$$

With the implication, it is sufficient to claim action "A" in order to claim action "B".

### 1.2.8 Inference

The symbol $\Leftarrow$ means that by having an event or an action, we have a rule.

$$\boxed{x \in \mathbb{R}^+ \Longleftarrow x > 0}$$

With the inference, it is necessary to have claim the action “A” in order to have claim the action “B”

### 1.2.9 If and only if

The symbol $\Leftrightarrow$ means that two events happen simultaneously (double implication):

$$\boxed{x \in \mathbb{N},\ x \neq 0 \Longleftrightarrow x \in \mathbb{N}^*}$$

Example:

$x = 2 \Longleftrightarrow x^2 - 4x + 4 = 0$

Proof:

$x = 2 \Rightarrow (x-2) = 0 \Rightarrow (x-2)^2 = 0 \cdot (x-2) \Rightarrow x^2 - 4x + 4 = 0$

This happens **because** $x = 2$

## 1.3 Numerical sets

- $\mathbb{N}$ := Natural numbers (including 0);
- $\mathbb{Z}$ := Integer numbers;
- $\mathbb{Q}$ := Rational numbers;
- $\mathbb{R}$ := Real numbers := $\mathbb{Q} \cup \{\text{irrational numbers}\}$.

Notation: The “$^*$” symbol means that the set does not include 0.

### 1.3.1 Inclusion of sets

$$\boxed{\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}}$$

$B := \{\pi, 1, -1, 0\}$;
$C := \{\pi, 1\}$;
$D := \{\pi\}$.

Then we write some examples: $\pi \in B,\ D \subset B,\ C \subset B,\ B \not\subset C,\ 0 \in B,\ 0 \notin C$.

# 2 Intervals in the real line

Intervals describe what happens between two or more elements.

## 2.1 Examples

### 2.1.1 Interval sets

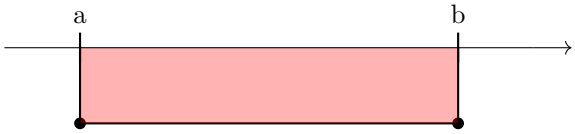
We have 4 cases:

- $(a, b) = \{\forall x \in \mathbb{R} \mid a < x < b\}$;
- $[a, b) = \{\forall x \in \mathbb{R} \mid a \leq x < b\}$;
- $(a, b] = \{\forall x \in \mathbb{R} \mid a < x \leq b\}$;
- $[a, b] = \{\forall x \in \mathbb{R} \mid a \leq x \leq b\}$.

Notation: $a$ and $b$ are often called the "end points" of the interval;

### 2.1.2 Graphical examples

$\forall x \in \mathbb{R},\ x \in [a, b]$

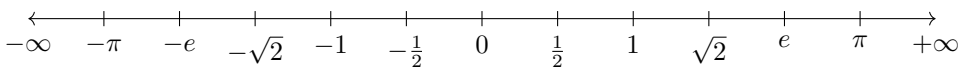



# 3 The extended line

In the real line $\mathbb{R}$ we add $\pm\infty$.

**Real line:** $(-\infty, +\infty) = \mathbb{R}$

**Extended real line:** $[-\infty, +\infty] = \overline{\mathbb{R}}$




Remark: $\pm\infty \notin \mathbb{R}$

## 3.1 Properties

$$\boxed{\forall x \in \mathbb{R} \ \mid \ \infty > x \ \mid \ -\infty < 0}$$

## 3.2 Operation in the extended line

If $a, b \in \mathbb{R}$, then $a + b,\ a - b,\ a \cdot b,\ \frac{a}{b}$ (with $b \neq 0$) stay the same

### 3.2.1 Additions

Let $\forall a \in \mathbb{R}$:

- $a + \infty := \infty$;
- $a - \infty := -\infty$;
- $+\infty + \infty := +\infty$;
- $-\infty - \infty := -\infty$;
- $+\infty - \infty :=$ undefined.

### 3.2.2 Moltiplications

Let $\forall a \in \mathbb{R}$:

- $+\infty \cdot +\infty := +\infty$;
- $-\infty \cdot +\infty := -\infty$;
- $-\infty \cdot (-\infty) := \infty$;
- $a \cdot \infty := \begin{cases} a > 0 & +\infty \\ a < 0 & -\infty \\ a = 0 & \text{undefined} \end{cases}$
- $a \cdot (-\infty) := \begin{cases} a > 0 & -\infty \\ a < 0 & +\infty \\ a = 0 & \text{undefined} \end{cases}$
- $\frac{a}{+\infty} = \frac{a}{-\infty} := 0$;
- $\frac{\pm\infty}{a} := \begin{cases} a > 0 & +\infty \\ a < 0 & -\infty \\ a = 0 & +\infty \end{cases}$
- $\frac{-\infty}{a} := \begin{cases} a > 0 & -\infty \\ a < 0 & +\infty \\ a = 0 & -\infty \end{cases}$
- $\frac{\infty}{\infty} :=$ undefined.

# 4 Intervals including $\pm\infty$

Intervals describe what happens between two or more elements, including $\pm\infty$.

## 4.1 Examples

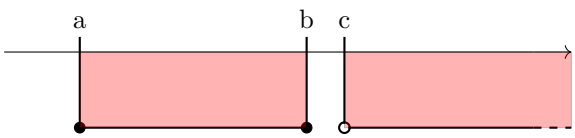
### 4.1.1 Interval sets

Let $a \in \mathbb{R}$, then:

- $(-\infty, a) = \{\forall x \in \mathbb{R} \mid x < a\}$;
- $(a, +\infty) = \{\forall x \in \mathbb{R} \mid x > a\}$;
- $(-\infty, a] = \{\forall x \in \mathbb{R} \mid x \leq a\}$;
- $[a, +\infty] = \{\forall x \in \mathbb{R} \mid x \geq a\}$;
- $(-\infty, +\infty) = \mathbb{R}$;
- $[-\infty, +\infty] = \overline{\mathbb{R}}$.

### 4.1.2 Graphical examples

$\forall x \in \mathbb{R},\ x \in\ [a, b]\ \cup\ ]c, +\infty[$




Notation: The union of two or more intervals where $x \in \mathbb{R}$ is denoted by the symbol $\cup$.

# 5 Union and Intersection

## 5.1 Logical symbols AND, OR and NOT

- $\wedge$ = AND;
- $\vee$ = OR;
- $\neg$ = NOT.

## 5.2 Universe symbol

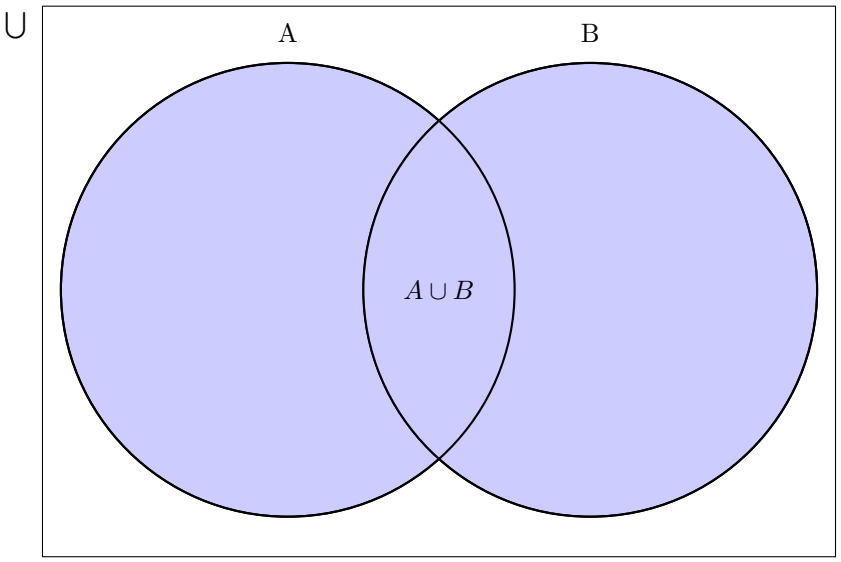
The symbol $\bigcup :=$ Universe describes a big set which contains all sets involved in our discussions (not always).

Let $A, B$ be two sets $[A \subset \bigcup,\ B \subset \bigcup]$. Then:

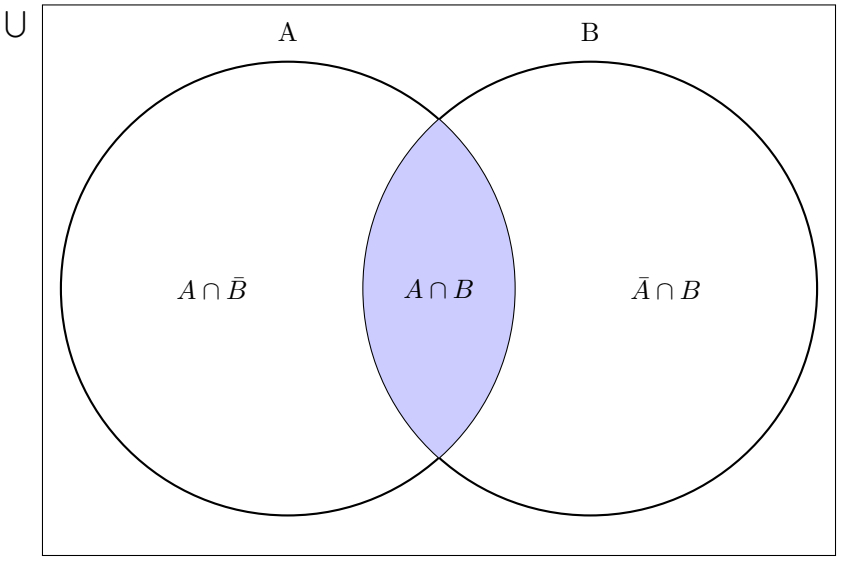
- $A \cup B$ = union of $A$ and $B \Longrightarrow \{\forall x \in \bigcup \mid x \in A \vee x \in B\}$;
- $A \cap B$ = intersection between $A$ and $B \Longrightarrow \{\forall x \in \bigcup \mid x \in A \wedge x \in B\}$.

## 5.3 Venn diagram

### 5.3.1 Union $A \cup B$

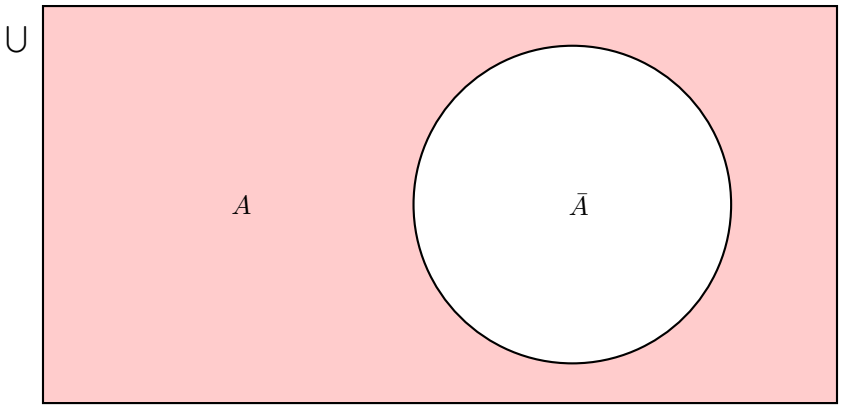



### 5.3.2 Intersection $A \cap B$



### 5.3.3 The complementary element $\overline{A}$

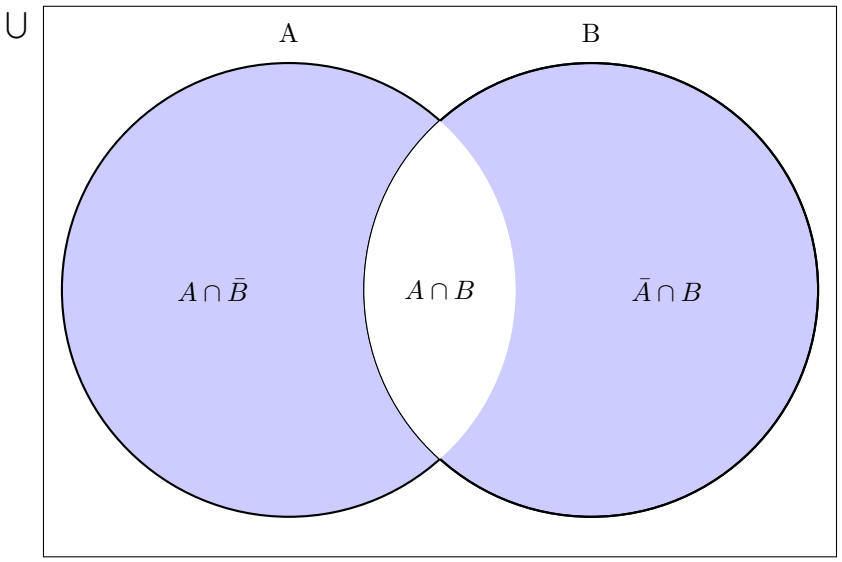
$1 - A = \overline{A}$




### 5.3.4 Difference between sets

$A \setminus B = \{\forall x \in \bigcup \mid x \in A,\ x \notin B\}$

COLORARE TUTTO A PARTE $A \cup B$!!



#### 5.3.5 Empty set $\emptyset$

$\emptyset :=$ the set containing zero elements: $A \cup B = \emptyset$

Definition: $A \triangle B = (A \setminus B) \cup (B \setminus A)$

# 6 The absolute value function

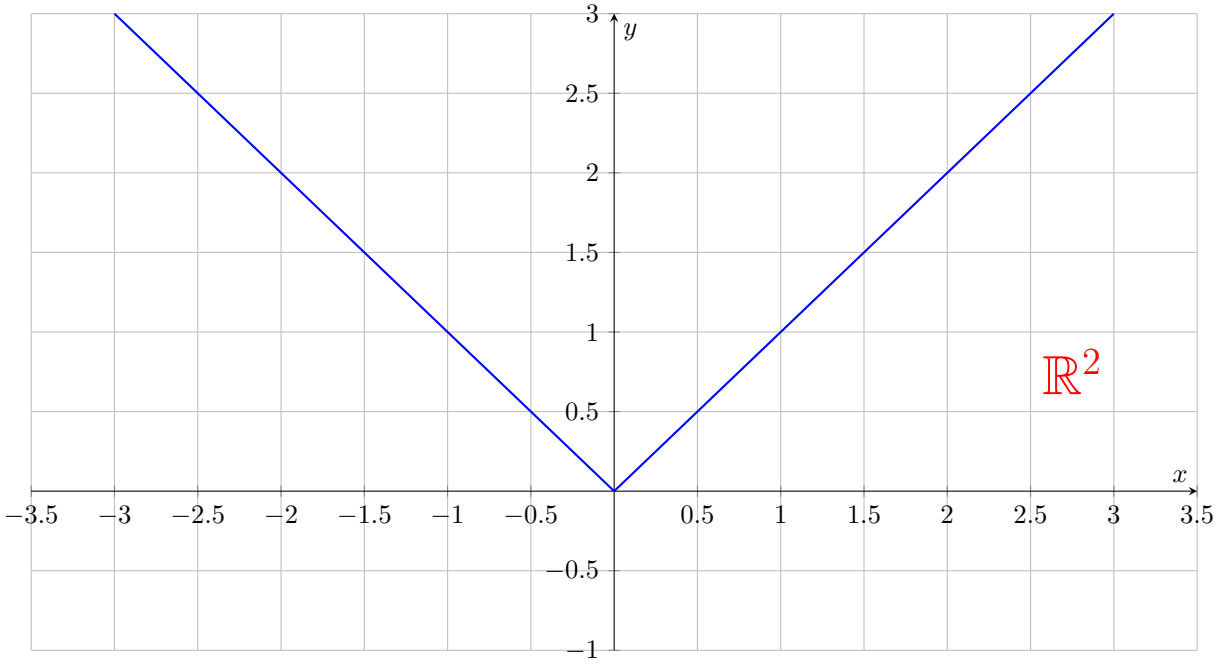
The absolute value is an operator which turns positive the sign of the result of the function.

Let $x \in \mathbb{R}$, then:

$$\boxed{|x| = \begin{cases} x & \text{if} \quad x \geq 0 \\ x & \text{if} \quad -x \leq 0 \end{cases}}$$

## 6.1 Graph of absolute value functions

Let's plot the function $y = |x|$:




## 6.2 Properties

Let $a, b \in \mathbb{R}$, then:

- $|a \cdot b| = |a| \cdot |b|$;
- $|\frac{a}{b}| = \frac{|a|}{|b|} \quad$ for $b \neq 0$;
- $|a \pm b| \neq |a| \pm |b|$.

## 6.3 Triangolar inequalities

Let $a, b \in \mathbb{R}$, then:

$$\boxed{\begin{array}{l} |a|+|b| \geq |a+b| \\ |a|-|b| \leq |a-b| \end{array}}$$