Technical Mechanics
HSLU, Semester 2

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(Adapted from `TECHMECH_Notes.pdf`)

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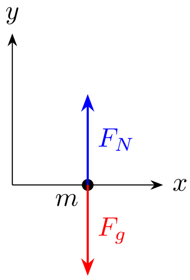
Contents

1 Static system

A body of mass m subject to gravity F_g and a normal reaction F_N on a flat surface. In the static case:

$$\sum F_y = F_N - F_g = 0, \quad \sum F_x = 0$$



2 Dynamic system

For a body of mass m under a resultant force F_{res}, the acceleration is

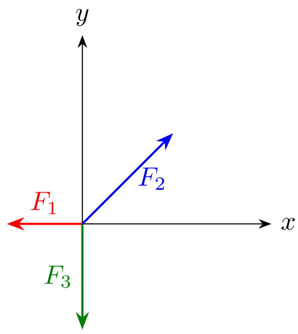
$$a = \frac{F_{\text{res}}}{m}$$

Example with wind F_w acting horizontally and a rope tension or reaction F_R:

$$\sum F_y = 0, \quad \sum F_x = F_w - F_R = 0 \quad \text{(if static in the horizontal direction)}$$

3 Force directions and resultants

Suppose there are three forces F_1, F_2, F_3 in different directions.



We can write equilibrium as

$$\sum F_x = -F_1 + F_2 \cos(\alpha), \quad \sum F_y = -F_3 + F_2 \sin(\alpha)$$

For instance, if $\alpha = 45^\circ$ and $F_2 = 100\,\text{N}$:

$$F_1 = F_2 \cos 45^\circ = 70.7\,\text{N}, \quad F_3 = F_2 \sin 45^\circ = 70.7\,\text{N}$$

4 Ropes

Key property: Ropes can only carry tensile forces, not compressive or bending forces.

4.1 Static vs. dynamic with wind

- In a static system with a rope supporting a mass m, $\sum F_y = 0$, $\sum F_x = 0$
- In a dynamic system with wind F_w, $\sum F_y = 0$, $\sum F_x = F_w$

5 Moments and couples

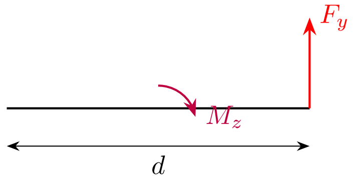
5.1 Moment

Moment (torque) is created by a force acting at a distance from a pivot (or reference point):

$$M_z = F_x\,d_x \quad \text{or} \quad M_z = F_y\,d_y.$$

5.2 Couple

Couple is formed by two equal and opposite forces whose lines of action do not coincide, creating a pure moment.

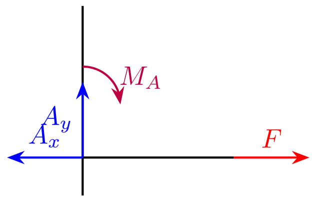


6 Free Body Diagram (FBD)

Procedure:

- Isolate the body from its surroundings.
- Replace each support or contact with the appropriate reaction forces (and possibly moments).
- Apply equilibrium equations:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0.$$



7 Supports

Every blocked degree of freedom (DOF) introduces a reaction (either a force or a moment). In 2D, each point can have up to 3 DOFs:

$$\text{(1) Translation in } x, \quad \text{(2) Translation in } y, \quad \text{(3) Rotation about } z.$$

Types of supports:

- Pin/Hinge: Fixes x and y, allows rotation. (Reactions: A_x, A_y)
- Roller: Often fixes y but allows translation in x and rotation. (Reaction: A_y)
- Fixed/Wall support: Fixes x, y, and rotation. (Reactions: A_x, A_y, M_A)

8 Examples of Beams or Shelves

(1) Simply supported beam with two pinned supports. (2) Cantilever with a fixed end and free end. (3) Beam with supports used for bending tests or balance boards.

Small FBD Exercises:

- (a) Two vertical forces F at different points, sum up in y-direction, etc.
- (b) Two horizontal forces, $\sum F_x = 2F$, $\sum M = 0$, etc.
- (c) Summation of vertical forces $F_1 + F_2 = 2F$, etc.
- (d) Force at 135° from horizontal, decompose into F_x and F_y, check moments.

9 Multi-Body Systems

Sometimes we have multiple bodies connected at joints, each with its own free-body diagram.

Example

Let $F = 2000\,\mathrm{N}$, $a = 7\,\mathrm{m}$, $b = 2\,\mathrm{m}$, $c = 6\,\mathrm{m}$, $d = 3\,\mathrm{m}$.

1. Draw the FBD of the entire system.
2. Write equilibrium equations for the unknown reactions $F(A_x)$, $F(A_y)$, $F(B_x)$, $F(B_y)$, etc.
3. Solve for magnitudes and directions.
4. Calculate internal forces at the joints if needed.

10 Constraints and Statical Determinacy

- **Statically determinate**: Number of independent equilibrium equations = number of unknowns.
- **Statically indeterminate**: Equations < unknowns.
- **Statically overdeterminate**: Equations > unknowns.

Examples:

- A table with 4 legs on rollers (4 legs $\times$ 3 DOF each = 12 unknowns, but only 3 equilibrium equations in 2D) $\Rightarrow$ statically indeterminate.
- A rod supported by a hinge and a rope (3 unknowns total, 3 equations in 2D) $\Rightarrow$ statically determinate.
- A rod fixed on both ends (6 unknowns, but only 3 equations in 2D) $\Rightarrow$ statically indeterminate.
- A shoe on the ground without slipping (3 DOF, 2 unknowns, friction plus normal) $\Rightarrow$ possibly overdeterminate if friction is large, etc.

11 Internal Forces

To find internal forces (normal, shear, bending moment, etc.), we can make a virtual cut and apply equilibrium to one side of the cut:

$$N = \text{internal normal force}, \quad Q = \text{internal shear force}, \quad M = \text{bending moment}.$$

12 Shear/Moment/Tension Diagrams

Procedure:

1. Draw the overall FBD, solve for external support reactions.
2. "Cut" the beam (or member) at various sections x and solve for the internal forces/moments at each cut to plot $N(x)$, $Q(x)$, $M(x)$.

13 Stress and Bending

Stress (σ) is needed to evaluate safety. It differs for each load case:

$$\sigma_{\text{tensile}} = \frac{F_{\text{int}}}{A}, \quad \sigma_{\text{compressive}}(\text{same formula, different sign}), \quad \tau_{\text{shear}} = \frac{F_{\text{shear}}}{A}.$$

Bending combines tensile, compressive, and possibly shear stress across a cross-section.

Strain (ε) is the internal shape change:

$$\varepsilon_{\text{tensile}} = \frac{\Delta l}{l_0}, \quad \varepsilon_{\text{compressive}} = \frac{\Delta l}{l_0}, \quad \gamma_{\text{shear}} = \frac{\Delta s}{\Delta h}.$$

$$\sigma = E\,\varepsilon,$$

where E is Young's modulus (in MPa or GPa).

14 Some Young's Modulus Values

$$E_{\text{steel}} \approx 210{,}000\,\text{MPa} = 210\,\text{GPa}, \quad E_{\text{aluminium}} \approx 68{,}000\,\text{MPa} = 68\,\text{GPa}, \quad E_{\text{polymer}} \approx 2{,}100\,\text{MPa} = 2.1\,\text{GPa}.$$

15 Safety Calculation

A common requirement is:

$$\sigma_{\text{int}} < \sigma_{\text{max,admissible}}, \quad \varepsilon_{\text{int}} < \varepsilon_{\text{max,admissible}},$$

where allowable (admissible) stresses and strains come from material data and/or a chosen safety factor.