

# Electrical Engineering

## HSLU, Semester 2

Matteo Frongillo

Last update: April 25, 2025

## Contents

<b>I Lectures</b>	<b>3</b>
<b>1 Current and voltage</b>	<b>3</b>
1.1 Current strength or current $I$ . . . . .	3
1.2 Current density $J$ . . . . .	3
1.3 Temperature dependence of the resistance . . . . .	3
1.4 Object properties . . . . .	4
1.5 Reciprocal quantities . . . . .	4
1.5.1 Specific resistance . . . . .	4
1.5.2 Conductance . . . . .	4
1.5.3 Specific conductivity . . . . .	4
<b>2 Gravitational fields</b>	<b>5</b>
2.1 Between bodies . . . . .	5
2.2 Between particles . . . . .	5
2.2.1 Coulomb's law . . . . .	5
2.3 Electric field and force on a charge $Q$ . . . . .	5
2.3.1 Homogeneous electric fields . . . . .	5
2.3.2 Force on a point charge . . . . .	5
<b>3 Capacitance and Capacitor</b>	<b>6</b>
3.1 Capacitor . . . . .	6
3.2 Capacitance . . . . .	6
3.2.1 Capacitance of a plate capacitor . . . . .	6
3.2.2 Energy in a capacitor . . . . .	6
3.3 Capacitors in parallel connection . . . . .	7
3.4 Capacitors in series connection . . . . .	7
<b>4 Transient Analysis in RC Circuits</b>	<b>7</b>
4.1 Charging of a Capacitor . . . . .	7
4.2 Discharging of a Capacitor . . . . .	7
4.3 Transitional phase . . . . .	8
<b>5 Additional Topics</b>	<b>8</b>
5.1 Energy Stored in a Capacitor . . . . .	8
5.2 Charge–Voltage Relationship . . . . .	8
<b>6 Electromagnetic fields</b>	<b>9</b>
6.1 Hans Christian Ørsted Observation . . . . .	9
6.2 Definitions and formuals . . . . .	9
6.2.1 Magnetomotive force . . . . .	9
6.2.2 Ampère's circuital law . . . . .	9
6.2.3 Magnetic field in a coil . . . . .	9
6.2.4 Magnetic flux density . . . . .	10

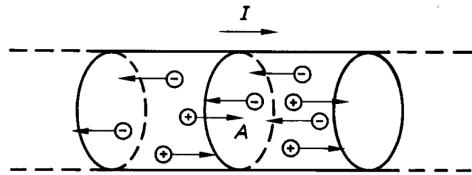
6.2.5	Magnetic field strength in coil with iron core . . . . .	10
6.2.6	Magnetic relative permeability $\mu$ . . . . .	10
6.2.7	Coils with and without iron core . . . . .	11
6.2.8	Law of induction and inductance . . . . .	11
6.2.9	Inductance and induction . . . . .	12
6.2.10	Inductivity of a very long coil . . . . .	12
6.2.11	Energy stored in an inductor . . . . .	12
6.2.12	Current-voltage relationship of an inductor . . . . .	13
6.2.13	Transient analysis . . . . .	13
6.3	Examples . . . . .	14
6.3.1	Charging an inductor in a RL-network . . . . .	14
6.3.2	Discharging an inductor in a RL-network . . . . .	15
<b>7</b>	<b>Alternating current (AC)</b>	<b>16</b>
7.1	Generation of alternating current / voltage . . . . .	16
7.2	Comparison of AC and DC . . . . .	16
7.2.1	Advantages of AC . . . . .	16
7.2.2	Disadvantages of AC . . . . .	16
7.3	Phasors . . . . .	16
7.4	Oscillation as a function of the angle . . . . .	17
7.5	Zero phase angle $\varphi$ . . . . .	17
7.5.1	Phase shift $\Delta\varphi$ between two signals . . . . .	17
7.6	Power in a sinusoidal signal and effective value . . . . .	17
7.6.1	Instantaneous power . . . . .	17
7.6.2	Effective value . . . . .	18
7.7	Relationship between current and voltage on a capacitor . . . . .	18
7.8	Capacitive reactance $X_c$ . . . . .	18
7.9	Relationship between current and voltage on an ideal inductor . . . . .	19
7.10	Inductive reactance $X_L$ . . . . .	19
7.11	Vectors properties . . . . .	20
7.11.1	Multiply . . . . .	20
7.11.2	Divide . . . . .	20
7.12	Impedance $Z$ . . . . .	20
7.12.1	Types of impedance . . . . .	20
7.12.2	Graphical representation . . . . .	21
7.13	Admittance $Y$ . . . . .	21
7.14	Current and voltage relations . . . . .	21
7.14.1	Resistor $R$ . . . . .	21
7.14.2	Capacitor $C$ . . . . .	21
7.14.3	Inductor $L$ . . . . .	22
7.15	Impedance and admittance phasor with R, C and L . . . . .	23
7.15.1	Series connection . . . . .	23
7.15.2	Parallel connection . . . . .	24
7.16	AC newtwork analysis . . . . .	25
7.16.1	Kirchhoff's current law (KCL) . . . . .	25
7.16.2	Kirchhoff's voltage law (KVL) . . . . .	25
7.16.3	Voltage and current phasor relationship for circuit elements . . . . .	25

# Part I

## Lectures

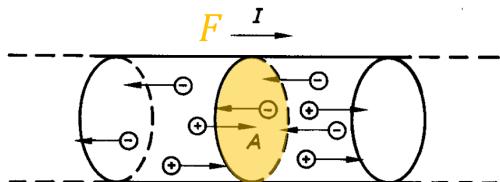
### 1 Current and voltage

#### 1.1 Current strength or current $I$



$$I [A] = \frac{\text{el. charge}}{t}$$

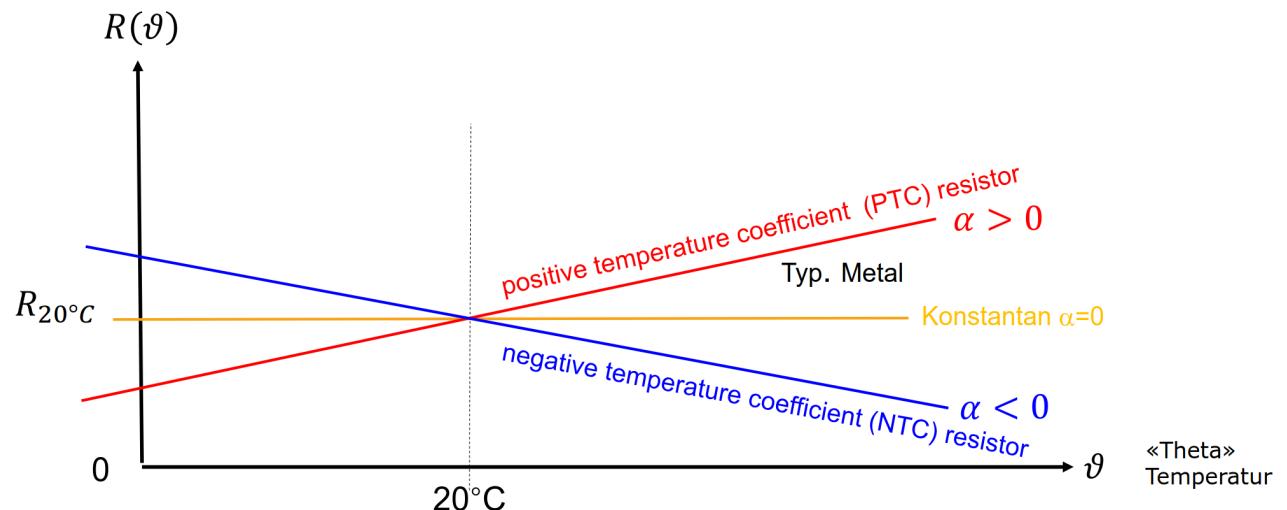
#### 1.2 Current density $J$



The current density indicates how large the current per cross-sectional area ( $F$ ) is:

$$J \left[ \frac{A}{mm^2} \right] = \frac{I}{F}$$

#### 1.3 Temperature dependence of the resistance



Depending on the material, the resistance can increase, remain the same or decrease with temperature. In ET+L we calculate using the linear approach.

$$R(\vartheta) = R_{20^\circ\text{C}}(1 + \alpha(\vartheta - 20^\circ\text{C})) = R_{20}(1 + \alpha\Delta T)$$

## 1.4 Object properties

The resistance indicates the voltage required for a current. In addition to the material, the cross-sectional area and also the length are decisive factors.

$$R = \frac{U}{I}$$

## 1.5 Reciprocal quantities

### 1.5.1 Specific resistance

To describe material properties, the resistance per length and cross-sectional area is specified (precondition: homogeneous conductor, direct current):

$$\rho \left[ \frac{\Omega \cdot mm^2}{m} \right] = R \cdot \frac{A}{l}$$

### 1.5.2 Conductance

### 1.5.3 Specific conductivity

## 2 Gravitational fields

### 2.1 Between bodies

$$F_1 = F_2 = G \frac{m_1 m_2}{d^2}$$

### 2.2 Between particles

#### 2.2.1 Coulomb's law

It calculates the amount of force between two electrically charged particles at rest:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

where:

- $F$ : Force [N];
- $q$ : Charge [As];
- $\epsilon_0$ : absolute permittivity =  $8.8542 \cdot 10^{-12}$  [As/Vm].

### 2.3 Electric field and force on a charge $Q$

#### 2.3.1 Homogeneous electric fields

$$E = \frac{U}{d}$$

where:

- $E$ : electric field strength [V/m];
- $U$ : voltage [V];
- $d$ : distance of the electrodes [m].

#### 2.3.2 Force on a point charge

$$F = Q \cdot E$$

where:

- $E$ : electric field strength [V/m];
- $Q$ : charge [As];
- $F$ : force [N].

### 3 Capacitance and Capacitor

#### 3.1 Capacitor

A capacitor is a device in which the capacitance is used.

#### 3.2 Capacitance

Capacitance  $C$  is the **capability** to store electric charge. It is measured by the charge divided by the applied voltage:

$$C = \frac{Q}{U}$$

where:

- $Q$ : charge [As];
- $U$ : voltage [V];
- $C$ : capacitance [As/V = F (Farad)].

##### 3.2.1 Capacitance of a plate capacitor

$$C = \varepsilon \cdot \frac{A}{d}$$

where:

- $A$ : plate area (one side) [ $\text{m}^2$ ];
- $d$ : distance between plates [m];
- $C$ : capacitance [F].

#### Permittivity

$$\varepsilon = \varepsilon_r \cdot \varepsilon_0$$

- $\varepsilon_r$ : relative permittivity of the dielectric, relative to the air;
- $\varepsilon_0$ : absolute permittivity [As/Vm].

##### 3.2.2 Energy in a capacitor

If a capacitor is discharged with a constant current, the voltage decreases linearly:

$$\int_0^{t_{\text{empty}}} U(t) \cdot I dt = I \cdot U_0 = \frac{I \cdot U_0 \cdot t_{\text{empty}}}{2}$$

Or, simplified:

$$W = \frac{1}{2} C \cdot U_0^2$$

where:

- $W$ : energy [J or Ws];
- $U_0$ : initial voltage [V];
- $C$ : capacitance [F].

### 3.3 Capacitors in parallel connection

Capacitances connected in parallel add up:

$$C_{\text{tot}} = \frac{\sum_n Q_n}{U} = \sum_n C_n$$

or

$$C = \frac{\varepsilon \cdot (\sum_n A_n)}{d} = \sum_n C_n$$

### 3.4 Capacitors in series connection

In a series connection, the reciprocal of the total capacitance is the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_{\text{tot}}} = \sum_n \frac{1}{C_n}$$

where:

- $C_{\text{tot}}$ : total capacitance [F];
- $C_n$ : capacitance of the  $n$ -th capacitor [F].

## 4 Transient Analysis in RC Circuits

### 4.1 Charging of a Capacitor

When a capacitor is charged through a resistor, the voltage across it increases exponentially:

$$U_C(t) = U_0 \cdot \left(1 - e^{-t/(R \cdot C)}\right)$$

with the time constant defined as:

$$\tau = R \cdot C$$

where:

- $U_C(t)$ : voltage across the capacitor at time  $t$  [V];
- $U_0$ : applied voltage [V];
- $R$ : resistance [ $\Omega$ ];
- $C$ : capacitance [F];
- $\tau$ : time constant [s].

### 4.2 Discharging of a Capacitor

When a charged capacitor discharges through a resistor, the voltage decays exponentially:

$$U_C(t) = U_0 \cdot e^{-t/(R \cdot C)}$$

and the discharging current is:

$$I(t) = \frac{U_0}{R} \cdot e^{-t/(R \cdot C)}$$

### 4.3 Transitional phase

$$f(t) = A + \Delta \cdot \left(1 - e^{t/\tau}\right) = A + (B - A) \cdot (1 - e^{1/\tau})$$

## 5 Additional Topics

### 5.1 Energy Stored in a Capacitor

The energy stored in a capacitor is given by:

$$W = \frac{1}{2}C \cdot U_0^2$$

where:

- $W$ : energy [J];
- $C$ : capacitance [F];
- $U_0$ : voltage [V].

### 5.2 Charge–Voltage Relationship

For an ideal capacitor, the relationship between charge and voltage is:

$$Q = C \cdot U$$

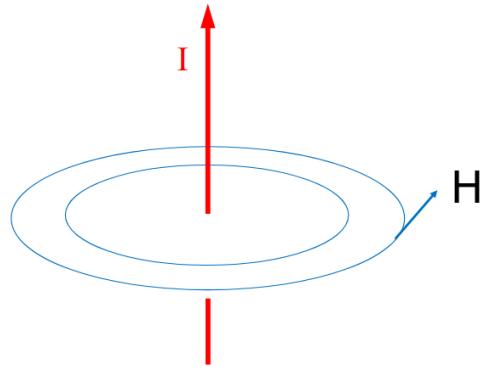
Moreover, the current is the time derivative of the charge:

$$I = \frac{dQ}{dt} = C \cdot \frac{dU}{dt}$$

Note that the voltage across an ideal capacitor cannot change instantaneously.

## 6 Electromagnetic fields

### 6.1 Hans Christian Ørsted Observation



1. The magnetic field lines encircle the current-carrying conductor;
2. The magnetic field lines lie in a plane perpendicular to the current-carrying wire;
3. If the direction of the current is reversed, the direction of the magnetic field lines is also reversed;
4. The strength of the field is directly proportional to the magnitude of the current;
5. The strength of the field at any point is inversely proportional to the distance of the point from the wire.

### 6.2 Definitions and formuals

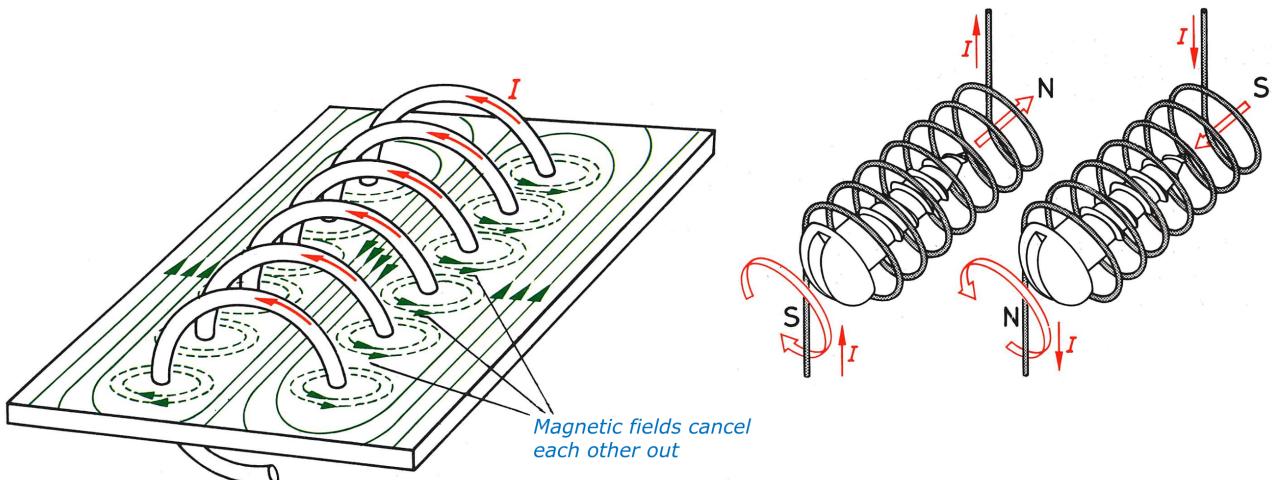
#### 6.2.1 Magnetomotive force

$$\theta = N \cdot I$$

#### 6.2.2 Ampère's circuital law

$$\theta = \oint \overrightarrow{H(s)} \cdot d\vec{s}$$

#### 6.2.3 Magnetic field in a coil



#### 6.2.4 Magnetic flux density

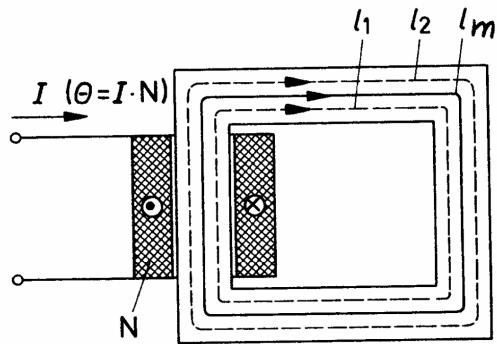
$$B = \frac{\Phi}{A} = \mu \cdot H = \mu_0 \mu_r \cdot H$$

where:

- $B$ : magnetic flux density [ $T = Vs/m^2$ ];
- $\Phi$ : magnetic flux [Wb];
- $A$ : area [ $m^2$ ];
- $\mu$ : magnetic permeability [ $H/m = Vs/Am$ ];
- $H$ : magnetic field strength [ $A/m$ ];
- $\mu_0$ : magnetic constant [ $4\pi \cdot 10^{-7} Vs/Am$ ];
- $\mu_r$ : relative permeability.

Note:  $\Phi$  is the sum of all  $B$ -field lines through the cross section A

#### 6.2.5 Magnetic field strength in coil with iron core



$$H = \frac{N \cdot I}{l_m} = \frac{\Theta}{l_m}$$

where:

- $H$ : magnetic field strength [ $A/m$ ];
- $N$ : number of turns;
- $I$ : current [A];
- $l_m$ : median field line length [m];
- $\Theta$ : magnetomotive force [A].

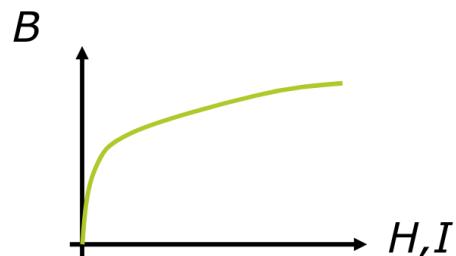
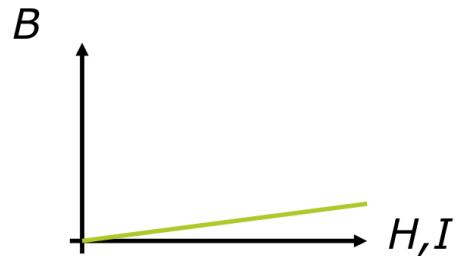
#### 6.2.6 Magnetic relative permeability $\mu$

Permeability is a measure for the ability to conduct magnetic field lines:

Material	$\mu_r$
Air	1
Pure iron	up to 250'000
Electrical steel	500 ... 7000
Steel	40 ... 7000
Water	0.99991

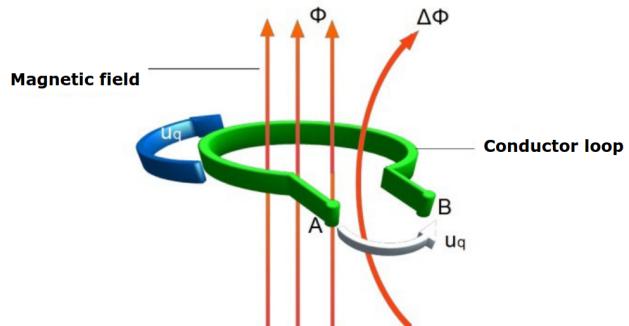
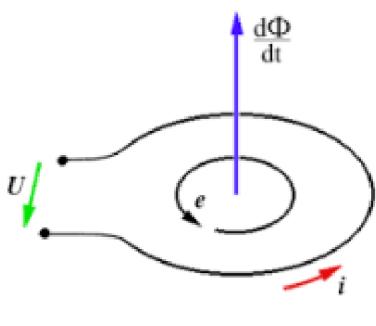
### 6.2.7 Coils with and without iron core

The magnetization curve of a coil without a core is linear, but there is significantly less flux density  $B$  than with an iron core.



### 6.2.8 Law of induction and inductance

Changing magnetic flux generates a voltage



Phenomenon: a changing magnetic flux  $\Phi$  induces a voltage in a conductor loop around it:

$$U = -N \cdot \frac{d\Phi}{dt}$$

### 6.2.9 Inductance and induction

Inductance L is the capability to generate a magnetic field. It is measured by the voltage divided by the rate of change of current over time. It is a measure of the magnetic “capacity” of an arrangement of conductors (e.g. coil) and can be compared to the capacity C of a capacitor. It indicates how much magnetic flux per ampere is generated.

$$L = \frac{N \cdot \Phi}{I} = \frac{U}{\frac{\Delta I}{\Delta t}}$$

where:

- $L$ : inductance [ $\text{H} = \text{Vs/A}$ ];
- $N$ : number of turns;
- $\Phi$ : magnetic flux [ $\text{Wb}$ ];
- $I$ : current [ $\text{A}$ ];
- $U$ : voltage [ $\text{V}$ ].

### 6.2.10 Inductivity of a very long coil

The inductance of a very long coil can be calculated approximately with:

$$L = \frac{\mu \cdot N^2 \cdot A}{l}$$

where:

- $L$ : inductance [ $\text{H} = \text{Vs/A}$ ];
- $\mu$ : magnetic permeability [ $\text{Vs/Am}$ ];
- $N$ : number of turns;
- $A$ : cross-section of the coil [ $\text{m}^2$ ];
- $l$ : length [ $\text{m}$ ].

### 6.2.11 Energy stored in an inductor

Since a variable magnetic field induces a voltage in which a current can also flow, the magnetic field must contain energy:

$$W = \frac{1}{2}L \cdot I^2$$

where:

- $W$ : work, energy [ $\text{J} = \text{Ws}$ ];
- $L$ : inductance [ $\text{H} = \text{Vs/A}$ ];
- $I$ : current [ $\text{A}$ ].

### 6.2.12 Current-voltage relationship of an inductor

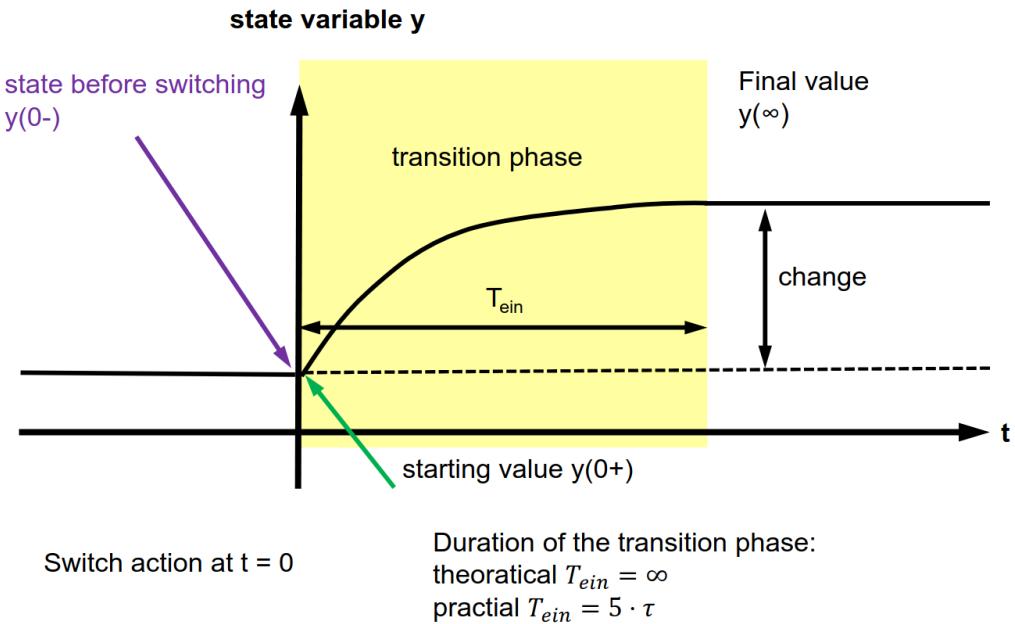
The current-voltage relationship of an inductor is:

$$U = L \cdot \frac{dI}{dt}$$

Special case:

$$0 = L \cdot \frac{dI}{dt} \rightarrow u_c = 0$$

### 6.2.13 Transient analysis



1. The state variable  $y(t)$  is the variable that cannot change instantaneously. For the inductor, this is  $i_L(t)$ .  
The state just before the switch action:

$$y(0^-) = i_L(0^-).$$

2. The starting value is the state immediately before the switch action:

$$y(0^+) = i_L(0) = i_L(0^-).$$

That is, the state variable  $i_L$  keeps the value from  $t = 0^-$ .

3. The final value is the value long after the switch action:

$$y(\infty) = i_L(\infty),$$

which is practically reached after  $5\tau$ .

4. The transient is described by the function of time:

$$y(t) = \text{final value} + (\text{starting value} - \text{final value}) \exp\left(-\frac{t}{\tau}\right).$$

Hence,

$$i_L(t) = i_L(\infty) + (i_L(0^+) - i_L(\infty)) \exp\left(-\frac{t}{\tau}\right).$$

## Time constant $\tau$ for an inductor

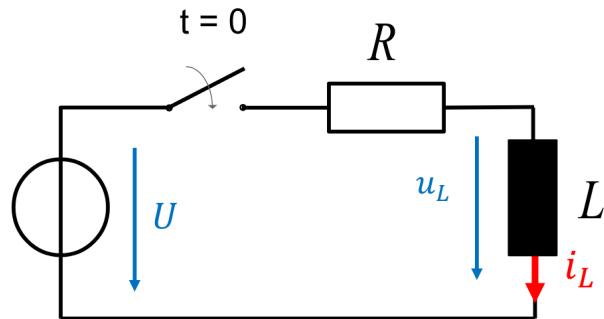
$$\boxed{\tau = \frac{L}{R}}$$

where:

- $\tau$ : time constant [s];
- $L$ : inductance [H];
- $R$ : resistance [ $\Omega$ ].

## 6.3 Examples

### 6.3.1 Charging an inductor in a RL-network



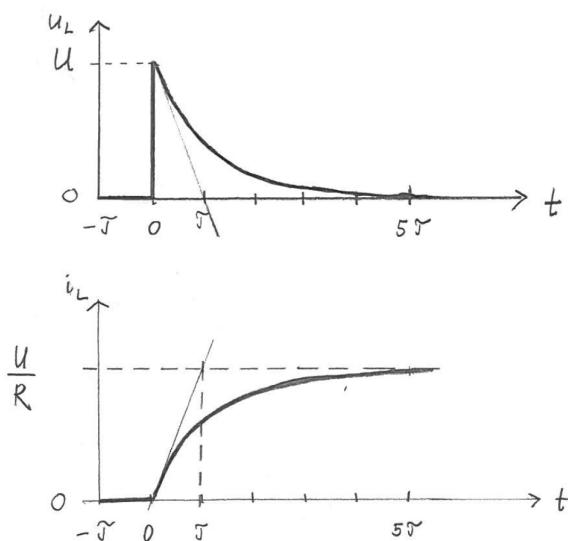
For  $t < 0$  stationary state, L discharged

### Calculations

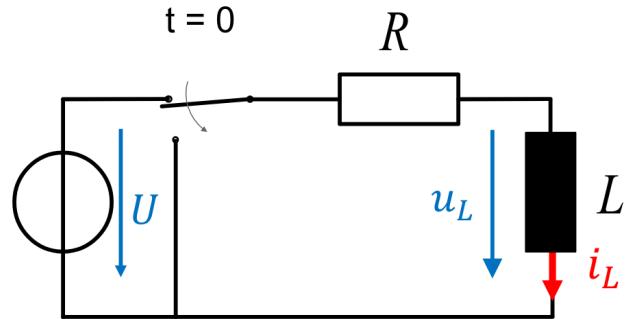
$$\boxed{i_L = \frac{U}{R} \cdot \left( 1 - \exp\left(-\frac{t}{\tau}\right) \right)}$$

$$\boxed{u_L = U \cdot \exp\left(-\frac{t}{\tau}\right)}$$

### Graphical representation



### 6.3.2 Discharging an inductor in a RL-network



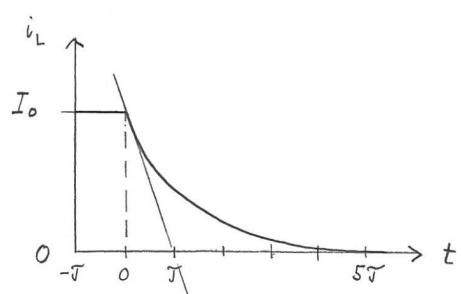
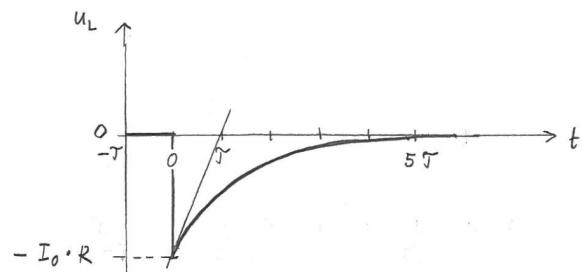
Before  $t = 0$  stationary state:  
Current in inductor is  $I_0$

Calculations

$$i_L = I_0 \cdot \exp\left(-\frac{t}{\tau}\right)$$

$$u_L = -I_0 \cdot R \cdot \exp\left(-\frac{t}{\tau}\right)$$

Graphical representation



## 7 Alternating current (AC)

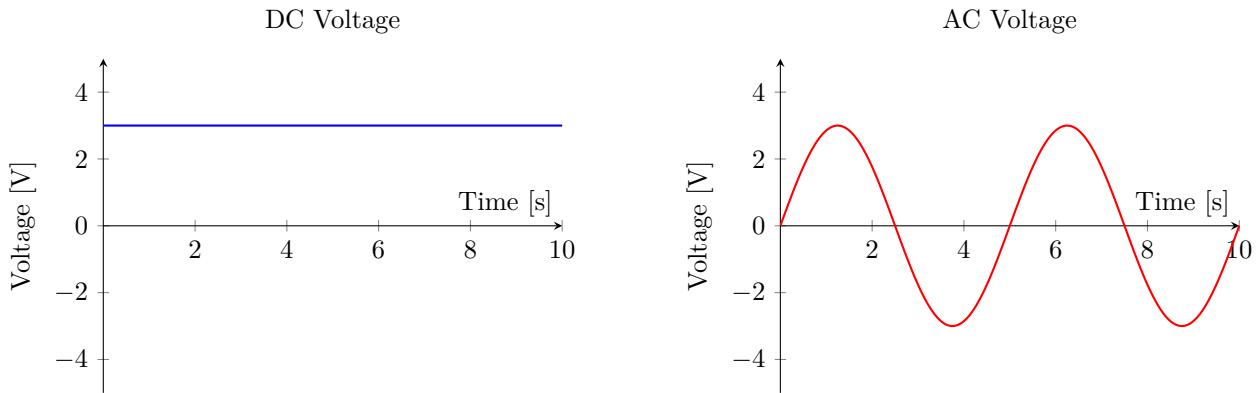
### 7.1 Generation of alternating current / voltage

$$U = -N \cdot \frac{\Delta\Phi}{\Delta t}$$

where:

- $U$ : voltage [V];
- $N$ : number of turns;
- $\Phi$ : magnetic flux [Wb].

### 7.2 Comparison of AC and DC



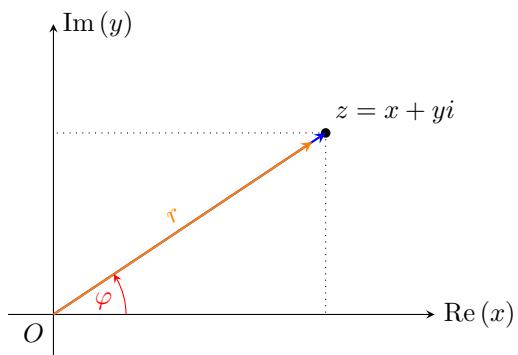
#### 7.2.1 Advantages of AC

- Simple voltage transformation;
- Efficient transmission;
- Easier generation;
- Compatibility with electric motors.

#### 7.2.2 Disadvantages of AC

- Complexity in storage;
- Higher risk of shock;
- Complex circuits;
- Higher rectification costs.

### 7.3 Phasors



$$z = x + yi = r\angle\varphi$$

## 7.4 Oscillation as a function of the angle

Sinusoidal voltage has an instantaneous value  $u(t)$  or  $u$  for every time  $t$ .

After a period of time  $T$ , the curve repeats itself.

$$u(t) = \hat{U} \sin(\omega \cdot t)$$

## 7.5 Zero phase angle $\varphi$

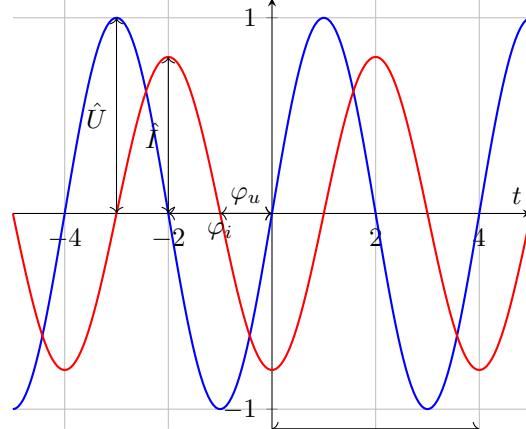
$$u(t) = \hat{U} \sin(\omega \cdot t + \varphi_u)$$

### 7.5.1 Phase shift $\Delta\varphi$ between two signals

$$\begin{aligned} u(t) &= \hat{U} \sin(\omega \cdot t + \varphi_u) \\ i(t) &= \hat{I} \sin(\omega \cdot t + \varphi_i) \end{aligned}$$

The phase shift between two signals is the difference between their zero phase signals:

$$\Delta\varphi = \varphi_u - \varphi_i$$



## 7.6 Power in a sinusoidal signal and effective value

### 7.6.1 Instantaneous power

The instantaneous power  $p(t)$  is the actual power at a specific time  $t$  and is the product of the voltage  $u(t)$  and the current  $i(t)$  at that moment:

$$p(t) = u(t) \cdot i(t) = \frac{u(t)^2}{R} = i(t)^2 \cdot R$$

The active power  $P$  corresponds to the mean value of the instantaneous power  $p(t)$  averaged over a period  $T$ :

$$P = \frac{1}{T} \int_0^T p(t) dt$$

### 7.6.2 Effective value

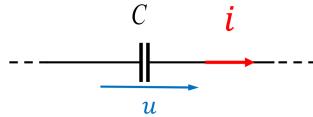
The effective value  $U_{\text{eff}}$  of a sinusoidal signal is the voltage that would generate the same power in a resistor as the sinusoidal signal:

$$U_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T u(t)^2 dt} = \frac{\hat{U}}{\sqrt{2}}$$

The same can be applied to the effective value  $I_{\text{eff}}$ :

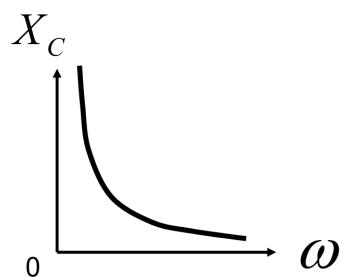
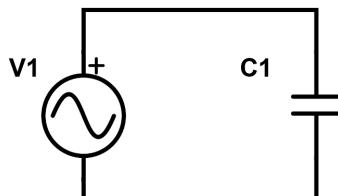
$$I_{\text{eff}} = \frac{\hat{I}}{\sqrt{2}}$$

## 7.7 Relationship between current and voltage on a capacitor



$$i = C \cdot \frac{\Delta u}{\Delta t}$$

## 7.8 Capacitive reactance $X_c$

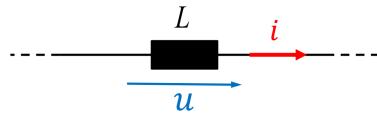


$$X_c = \frac{1}{\omega \cdot C}$$

where:

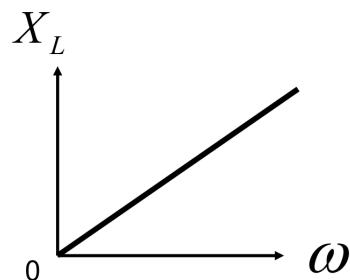
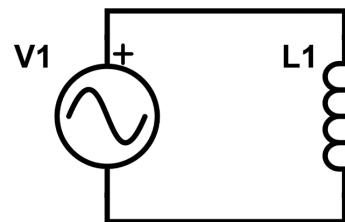
- $X_c$ : capacitive reactance [Ohm];
- $\omega$ : angular frequency [rad/s];
- $C$ : capacitance [ $F = As/V$ ].

## 7.9 Relationship between current and voltage on an ideal inductor



$$u = L \cdot \frac{\Delta i}{\Delta t}$$

## 7.10 Inductive reactance $X_L$



$$X_L = \omega \cdot L$$

where:

- $X_L$ : inductive reactance [Ohm];
- $L$ : inductance [H];
- $\omega$ : angular frequency [rad/s];

## 7.11 Vectors properties

### 7.11.1 Multiply

The magnitude (es. the radius  $r$  in polar representation) is multiplied and the angle is added:

$$\boxed{\begin{aligned} r_c &= r_a \cdot r_b \\ \varphi_c &= \varphi_a + \varphi_b \end{aligned}}$$

### 7.11.2 Divide

The magnitude is devided and the angle is subtracted:

$$\boxed{\begin{aligned} r_c &= \frac{r_a}{r_b} \\ \varphi_c &= \varphi_a - \varphi_b \end{aligned}}$$

## 7.12 Impedance $Z$

The impedance is the ratio of voltage and current phasor. It's a complex number.

$$\boxed{\begin{aligned} Z &= \frac{u(t)}{i(t)} \\ |Z| &= \frac{|u(t)|}{|i(t)|} \\ \angle Z &= \angle u(t) - \angle i(t) = \Delta\varphi \end{aligned}}$$

Therefore, the impedance corresponds to the AC resistance with phase shift:

$$\boxed{\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U \angle \varphi_u}{I \angle \varphi_i} = Z \angle (\varphi_u - \varphi_i) = Z \angle \varphi_Z}$$

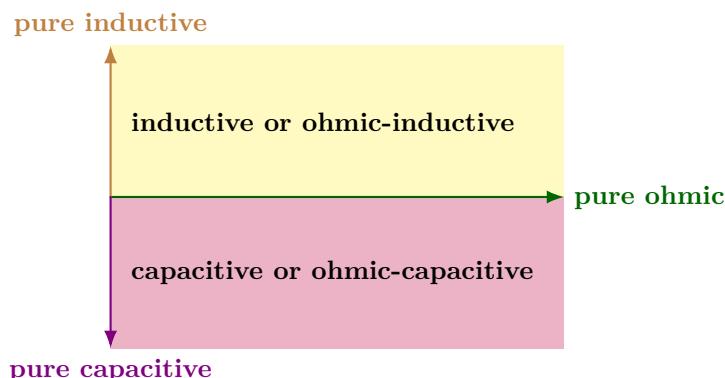
where:

- $Z$ : impedance [Ohm];
- $\underline{U}$ : voltage phasor [V];
- $\underline{I}$ : current phasor [A];
- $\varphi_u$ : phase angle of the voltage [rad];
- $\varphi_i$ : phase angle of the current [rad];
- $\varphi_Z$ : phase angle of the impedance [rad].

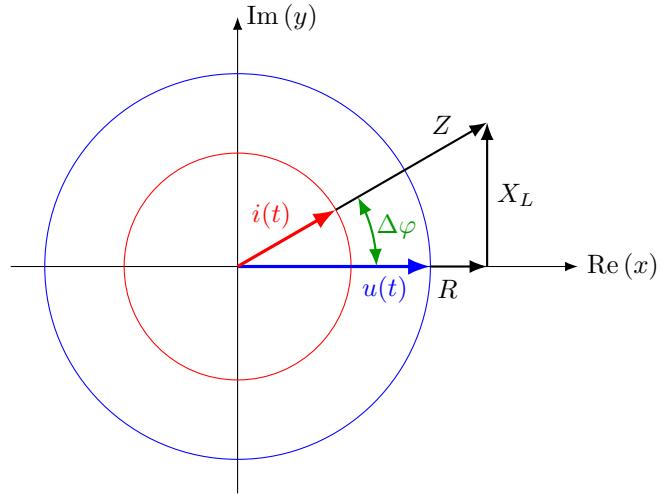
### 7.12.1 Types of impedance

The angle of the impedance  $\varphi_Z$  indicates the type of impedance:

- $\varphi_Z > 0^\circ \rightarrow$  voltage is ahead of current;
- $\varphi_Z = 0^\circ \rightarrow$  voltage and current are in phase;
- $\varphi_Z < 0^\circ \rightarrow$  current is ahead of voltage.



### 7.12.2 Graphical representation



### 7.13 Admittance $Y$

The reciprocal of the impedance  $Z$  is the admittance  $Y$  and thus the ratio of the current and voltage phasor. The admittance therefore corresponds to the AC conductance with phase shift:

$$\boxed{\begin{aligned} \underline{Y} &= \frac{\underline{I}}{\underline{U}} = \frac{I\angle\varphi_i}{U\angle\varphi_u} = Z\angle(\varphi_i - \varphi_u) = Z\angle\varphi_Y \\ |\underline{Y}| &= \frac{1}{|Z|} \implies \varphi_Y = -\varphi_Z \end{aligned}}$$

### 7.14 Current and voltage relations

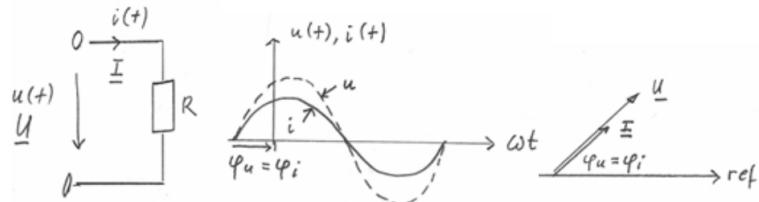
#### 7.14.1 Resistor $R$

**Current-voltage relationship for instantaneous values**

$$\boxed{u_R(t) = R \cdot i_R(t)}$$

**Impedance = Ohmic resistance**

$$\boxed{R = \frac{U_R}{I_R} \angle 0^\circ}$$



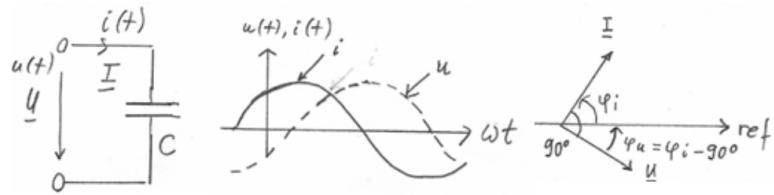
#### 7.14.2 Capacitor $C$

**Current-voltage relationship for instantaneous values**

$$\boxed{i_C(t) = C \cdot \frac{du_C(t)}{dt}}$$

**Impedance = Capacitive reactance**

$$X_C = \frac{U_C}{I_C} = \frac{1}{\omega \cdot C} \angle -90^\circ$$



Current leads the voltage by 90 degrees

**Phase shift between current and voltage**

$$\varphi = \arctan \left( \frac{-X_C}{R} \right)$$

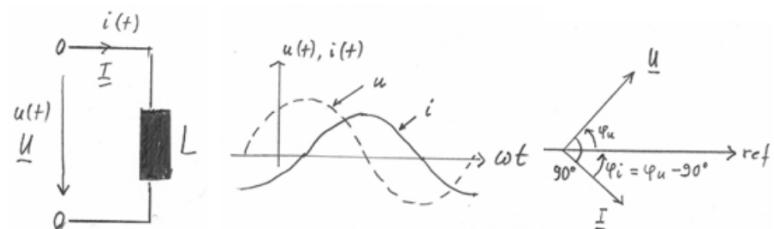
### 7.14.3 Inductor $L$

**Current-voltage relationship for instantaneous values**

$$u(t) = L \cdot \frac{di(t)}{dt}$$

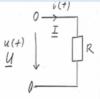
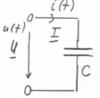
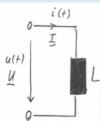
**Impedance = Inductive reactance**

$$X_L = \frac{U_L}{I_L} = \omega \cdot L \angle +90^\circ$$



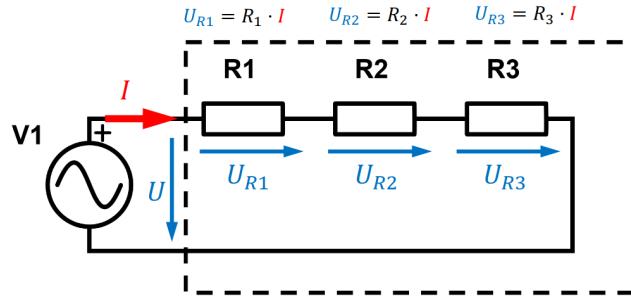
Current lags the voltage by 90 degrees

## 7.15 Impedance and admittance phasor with R, C and L

		<b>Phase shift</b> $\varphi_z = \varphi = \varphi_u - \varphi_i$	<b>Impedance <math>Z</math></b> $\underline{Z} = \frac{U}{I} \angle \varphi_z$	<b>Admittance <math>Y</math></b> $\underline{Y} = \frac{1}{\underline{Z}} = \frac{I}{U} \angle \varphi_Y$
	$\varphi_u = \varphi_i$	$0^\circ$	$\underline{Z}_R = R \angle 0^\circ$	$\underline{Y}_R = \frac{1}{R} \angle 0^\circ$
	$\varphi_u = \varphi_i - 90^\circ$	$-90^\circ$	$\underline{Z}_C = \frac{1}{\omega \cdot C} \angle -90^\circ$	$\underline{Y}_C = \omega \cdot C \angle +90^\circ$
	$\varphi_i = \varphi_u - 90^\circ$	$90^\circ$	$\underline{Z}_L = \omega \cdot L \angle +90^\circ$	$\underline{Y}_L = \frac{1}{\omega \cdot L} \angle -90^\circ$

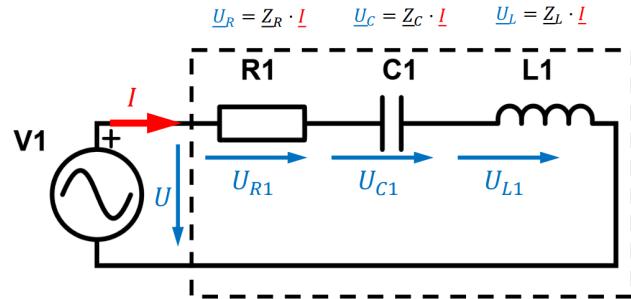
### 7.15.1 Series connection

#### Resistances



$$R_{\text{equi}} = \frac{U}{I} = \frac{U_{R1} + U_{R2} + U_{R3}}{I} = R_1 + R_2 + R_3$$

#### Impedances



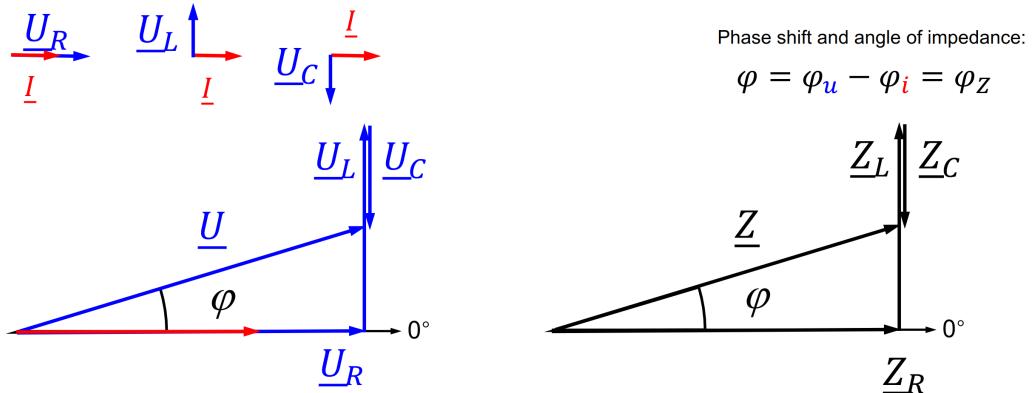
$$Z_{\text{equi}} = \frac{U}{I} = \frac{U_{R1} \angle 0^\circ + U_{C1} \angle -90^\circ + U_{L1} \angle +90^\circ}{I \angle 0^\circ} = \underline{Z}_1 + \underline{Z}_2 + \underline{Z}_3$$

Adding voltages in series connection means adding impedances:

$$\underline{U}_R = \underline{Z}_R \cdot \underline{I} = R \cdot \angle \varphi_i$$

$$\underline{U}_L = \underline{Z}_L \cdot \underline{I} = X_L \angle 90^\circ \cdot \angle \varphi_i$$

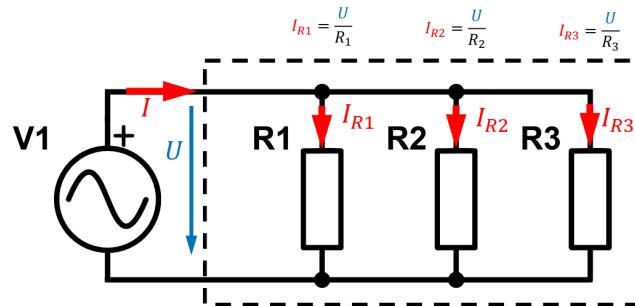
$$\underline{U}_C = \underline{Z}_C \cdot \underline{I} = X_C \angle -90^\circ \cdot \angle \varphi_i$$



$$Z_{\text{eq}} = Z_1 + Z_2 + Z_3$$

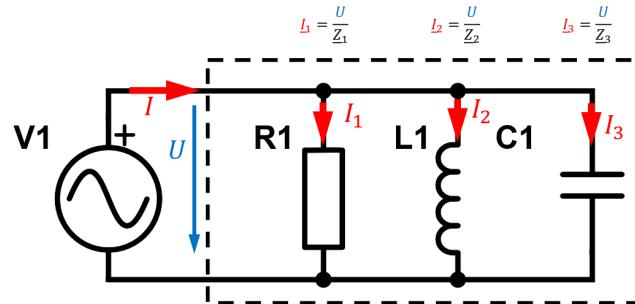
### 7.15.2 Parallel connection

Resistances



$$R_{\text{equi}} = \frac{U}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{G_1 + G_2 + G_3}$$

Impedances



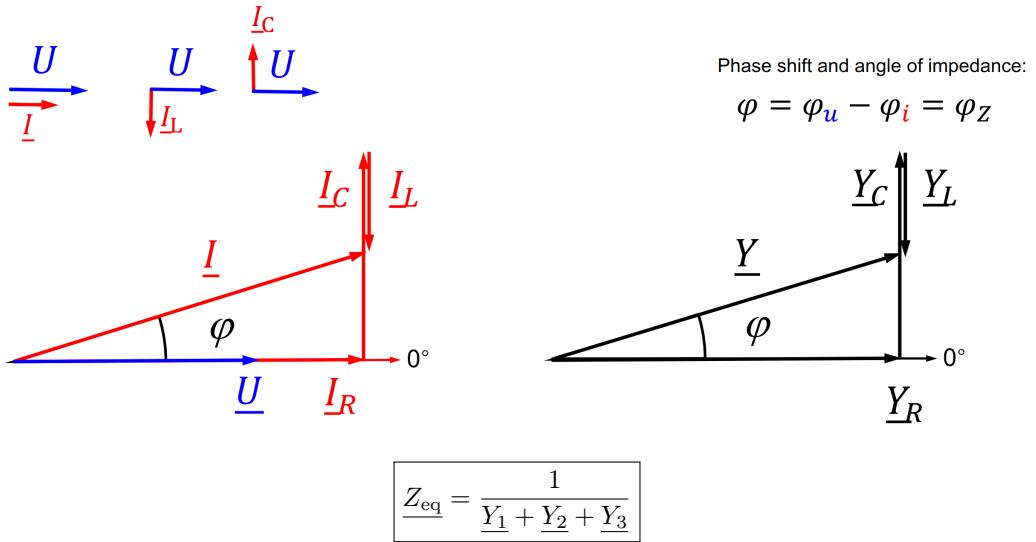
$$Z_{\text{equi}} = \frac{U}{I} = \frac{U}{I \angle 0^\circ} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}} = \frac{1}{Y_1 + Y_2 + Y_3}$$

Adding currents in parallel connection means adding admittances:

$$\underline{I}_R = \frac{\underline{U}_R}{\underline{Z}_R} = \frac{\underline{U}}{R} \cdot \angle \varphi_u$$

$$\underline{I}_L = \frac{\underline{U}_L}{\underline{Z}_L} = \frac{\underline{U}}{X_L} \angle \varphi_u - 90^\circ$$

$$\underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_C} = \frac{\underline{U}}{X_C} \angle \varphi_u + 90^\circ$$



## 7.16 AC newtwork analysis

AC network analysis is similar to DC network analysis but calculated with phasors.

### 7.16.1 Kirchhoff's current law (KCL)

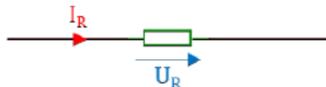
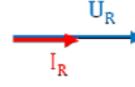
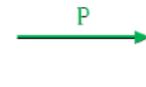
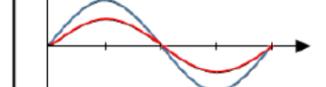
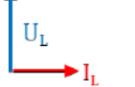
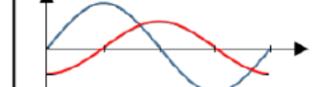
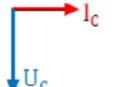
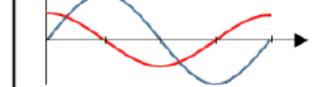
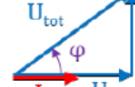
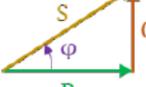
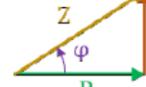
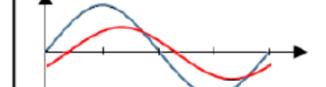
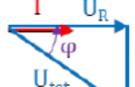
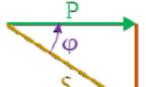
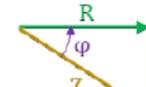
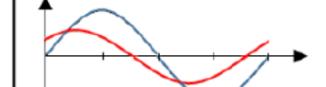
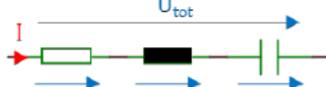
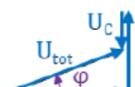
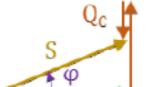
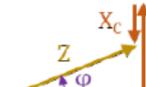
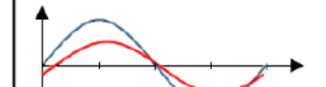
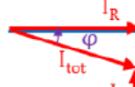
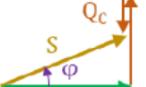
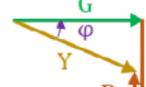
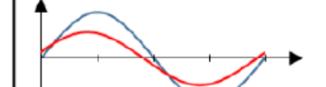
$$\underline{I}_1 + \underline{I}_2 + \dots + \underline{I}_n = \sum_{k=1}^n \underline{I}_k = 0$$

### 7.16.2 Kirchhoff's voltage law (KVL)

$$\underline{U}_1 + \underline{U}_2 + \dots + \underline{U}_n = \sum_{k=1}^n \underline{U}_k = 0$$

### 7.16.3 Voltage and current phasor relationship for circuit elements

$$\underline{U} = \underline{Z}_{\text{Element-type}} \cdot \underline{I}$$

Circuit	Current / Voltage	Power	Impedanz (Admittanz)	Signal sequence
				
				
				
				
				
				
				

Parallel connection: Calculating via the admittance  $\rightarrow G = \frac{1}{Z}$