Electrical Engineering
HSLU, Semester 2

Matteo Frongillo

Last update: May 11, 2025

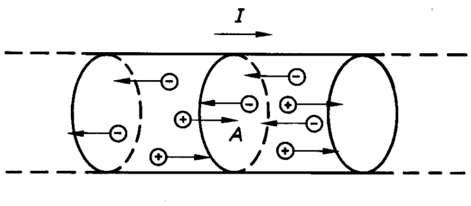
Contents

Part I

Lectures

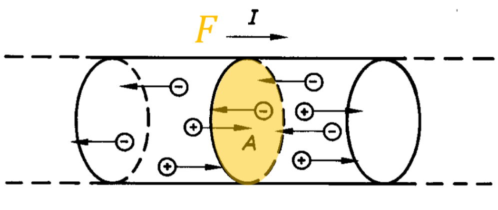
1 Current and voltage

1.1 Current strength or current I



$$I\ [A] = \frac{\text{el. charge}}{t}$$

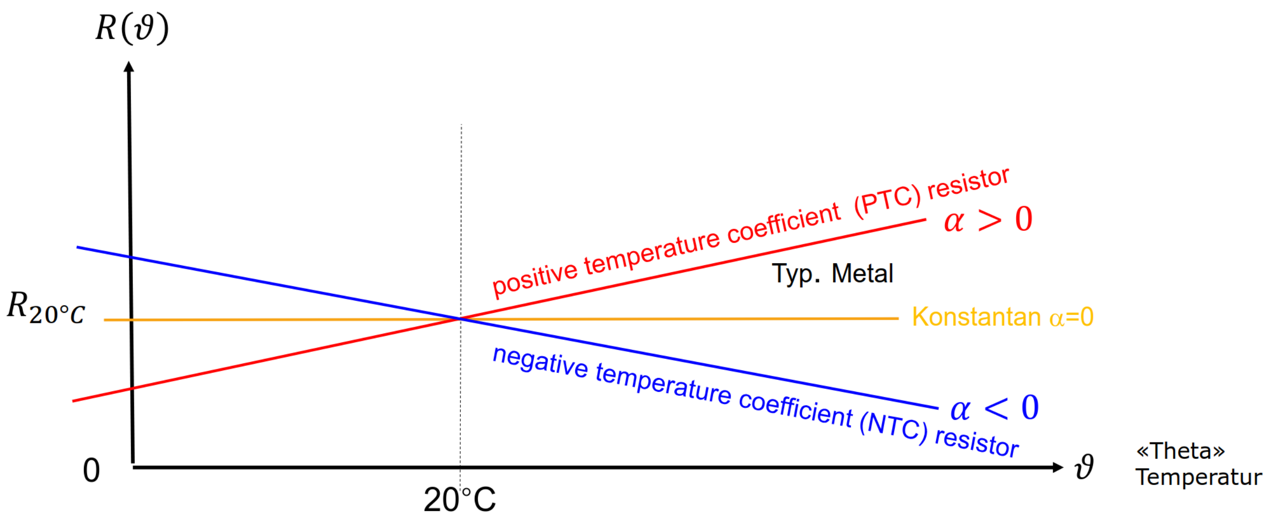
1.2 Current density J



The current density indicates how large the current per cross-sectional area (F) is:

$$J\ [\frac{A}{mm^2}] = \frac{I}{F}$$

1.3 Temperature dependence of the resistance



Depending on the material, the resistance can increase, remain the same or decrease with temperature. In ET+L we calculate using the linear approach.

$$\boxed{R(\vartheta) = R_{20}(1 + \alpha(\vartheta - 20°\text{C})) = R_{20}(1 + \alpha\Delta T)}$$

1.4 Object properties

The resistance indicates the voltage required for a current. In addition to the material, the cross-sectional area and also the length are decisive factors.

$$R = \frac{U}{I}$$

1.5 Reciprocal quantities

1.5.1 Specific resistance

To describe material properties, the resistance per length and cross-sectional area is specified (precondition: homogeneous conductor, direct current):

$$\rho\ [\frac{\Omega \cdot mm^2}{m}] = R \cdot \frac{A}{l}$$

1.5.2 Conductance

1.5.3 Specific conductivity

2 Fields

2.1 Gravitational field between bodies

$$\boxed{F_1 = F_2 = G\frac{m_1 m_2}{d^2}}$$

2.2 Electric field between particles

2.2.1 Coulomb's law

It calculates the amount of force between two electrically charged particles at rest:

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}$$

where:

- F: Force [N];
- q: Charge [As];
- ε_0: absolute permittivity $= 8.8542 \cdot 10^{-12}$ [As/Vm].

2.3 Electric field and force on a charge Q

2.3.1 Homogeneous electric fields

$$\boxed{E = \frac{U}{d}}$$

where:

- E: electric field strength [V/m];
- U: voltage [V];
- d: distance of the electrodes [m].

2.3.2 Force on a point charge

$$\boxed{F = Q \cdot E}$$

where:

- E: electric field strength [V/m];
- Q: charge [As];
- F: force [N].

3 Capacitance and Capacitor

3.1 Capacitor

A capacitor is a device in which the capacitance is used.

3.2 Capacitance

Capacitance C is the **capability** to store electric charge. It is measured by the charge divided by the applied voltage:

$$\boxed{C = \frac{Q}{U}}$$

where:

- Q: charge [As];
- U: voltage [V];
- C: capacitance [As/V = F (Farad)].

3.2.1 Capacitance of a plate capacitor

$$\boxed{C = \varepsilon \cdot \frac{A}{d}}$$

where:

- A: plate area (one side) [m^2];
- d: distance between plates [m];
- C: capacitance [F].

Permittivity

$$\boxed{\varepsilon = \varepsilon_r \cdot \varepsilon_0}$$

- ε_r: relative permittivity of the dielectric, relative to the air;
- ε_0: absolute permittivity [As/Vm].

3.2.2 Energy in a capacitor

If a capacitor is discharged with a constant current, the voltage decreases linearly:

$$\boxed{\int_0^{t_{\text{empty}}} U(t) \cdot I \, dt = \frac{I \cdot U_0 \cdot t_{\text{empty}}}{2}}$$

Or, simplified:

$$\boxed{W = \frac{1}{2} C \cdot U_0^2}$$

where:

- W: energy [J or Ws];
- U_0: initial voltage [V];
- C: capacitance [F].

3.3 Capacitors in parallel connection

Capacitances connected in parallel add up:

$$\boxed{C_{\text{tot}} = \frac{\sum_n Q_n}{U} = \sum_n C_n}$$

or

$$\boxed{C = \frac{\varepsilon \cdot \left(\sum_n A_n\right)}{d} = \sum_n C_n}$$

3.4 Capacitors in series connection

In a series connection, the reciprocal of the total capacitance is the sum of the reciprocals of the individual capacitances:

$$\boxed{\frac{1}{C_{\text{tot}}} = \sum_n \frac{1}{C_n}}$$

where:

- C_{tot}: total capacitance [F];
- C_n: capacitance of the n-th capacitor [F].

4 Transient Analysis in RC Circuits

4.1 Charging of a Capacitor

When a capacitor is charged through a resistor, the voltage across it increases exponentially:

$$\boxed{U_C(t) = U_0 \cdot \left(1 - e^{-t/(R \cdot C)}\right)}$$

with the time constant defined as:

$$\boxed{\tau = R \cdot C}$$

where:

- $U_C(t)$: voltage across the capacitor at time t [V];
- U_0: applied voltage [V];
- R: resistance [Ω];
- C: capacitance [F];
- τ: time constant [s].

4.2 Discharging of a Capacitor

When a charged capacitor discharges through a resistor, the voltage decays exponentially:

$$\boxed{U_C(t) = U_0 \cdot e^{-t/(R \cdot C)}}$$

and the discharging current is:

$$\boxed{I(t) = \frac{U_0}{R} \cdot e^{-t/(R \cdot C)}}$$

4.3 Transitional phase

$$\boxed{f(t) = A + \Delta \cdot \left(1 - e^{-t/\tau}\right) = A + (B - A) \cdot (1 - e^{1/\tau})}$$

5 Additional Topics

5.1 Energy Stored in a Capacitor

The energy stored in a capacitor is given by:

$$\boxed{W = \frac{1}{2} C \cdot U_0^2}$$

where:

- W: energy [J];
- C: capacitance [F];
- U_0: voltage [V].

5.2 Charge–Voltage Relationship

For an ideal capacitor, the relationship between charge and voltage is:

$$\boxed{Q = C \cdot U}$$

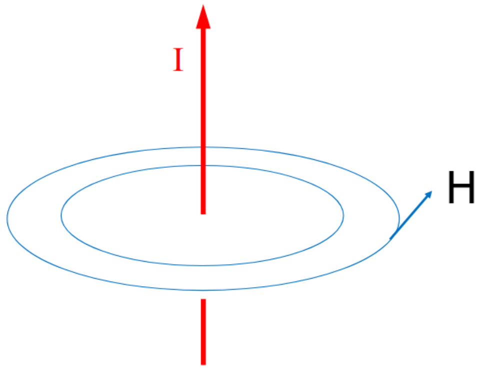
Moreover, the current is the time derivative of the charge:

$$\boxed{I = \frac{dQ}{dt} = C \cdot \frac{dU}{dt}}$$

Note that the voltage across an ideal capacitor cannot change instantaneously.

6 Electromagnetic fields

6.1 Hans Christian Ørsted Observation



1. The magnetic field lines encircle the current-carrying conductor;
2. The magnetic field lines lie in a plane perpendicular to the current-carrying wire;
3. If the direction of the current is reversed, the direction of the magnetic field lines is also reversed;
4. The strength of the field is directly proportional to the magnitude of the current;
5. The strength of the field at any point is inversely proportional to the distance of the point from the wire.

6.2 Definitions and formulas

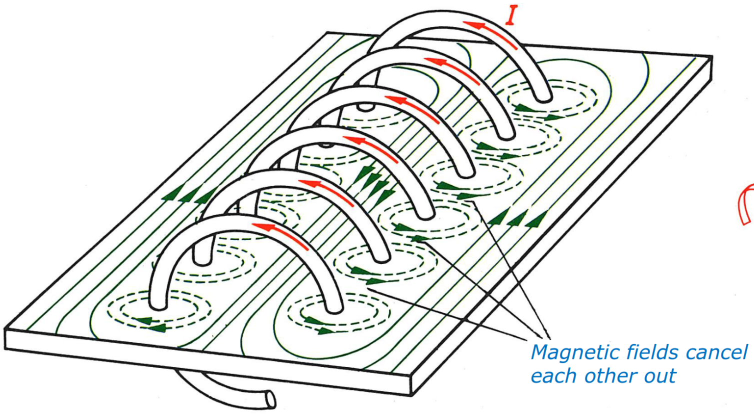
6.2.1 Magnetomotive force

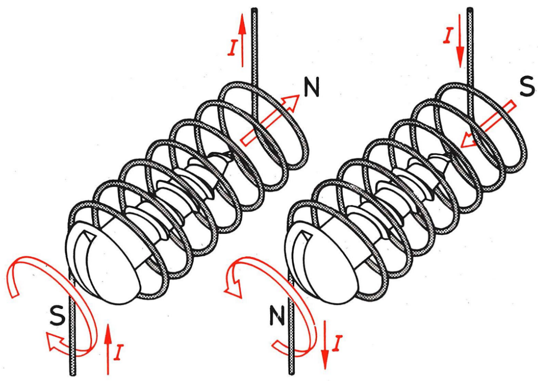
$$\boxed{\theta = N \cdot I}$$

6.2.2 Ampère's circuital law

$$\boxed{\theta = \oint \overrightarrow{H(s)} \cdot \mathrm{d}\vec{s}}$$

6.2.3 Magnetic field in a coil





6.2.4 Magnetic flux density

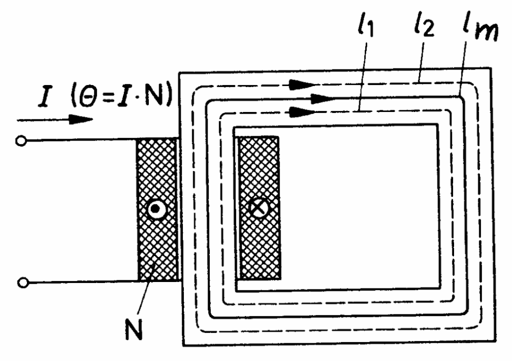
$$\boxed{B = \frac{\Phi}{A} = \mu \cdot H = \mu_0 \mu_r \cdot H}$$

where:

- B: magnetic flux density [T = Vs/m^2];
- Φ: magnetic flux [Wb];
- A: area [m^2];
- μ: magnetic permeability [H/m = Vs/Am];
- H: magnetic field strength [A/m];
- μ_0: magnetic constant [$4\pi \cdot 10^{-7}$ Vs/Am];
- μ_r: relative permeability.

Note: Φ is the sum of all B-field lines through the cross section A

6.2.5 Magnetic field strength in coil with iron core



$$\boxed{H = \frac{N \cdot I}{l_m} = \frac{\Theta}{l_m}}$$

where:

- H: magnetic field strength [A/m];
- N: number of turns;
- I: current [A];
- l_m: median field line length [m];
- Θ: magnetomotive force [A].

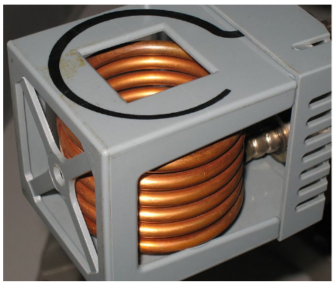
6.2.6 Magnetic relative permeability μ

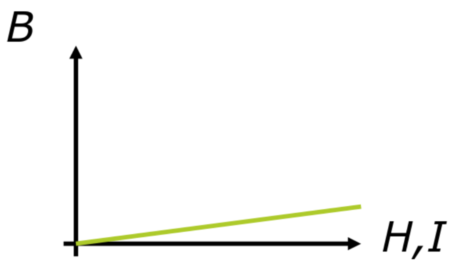
Permeability is a measure for the ability to conduct magnetic field lines:

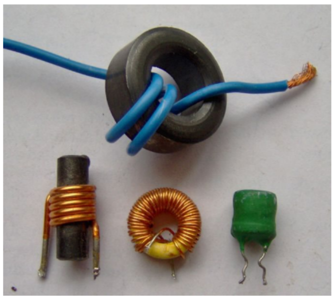
Material	$\mu_\mathbf{r}$
Air	1
Pure iron	up to 250'000
Electrical steel	500 ... 7000
Steel	40 ... 7000
Water	0.99991

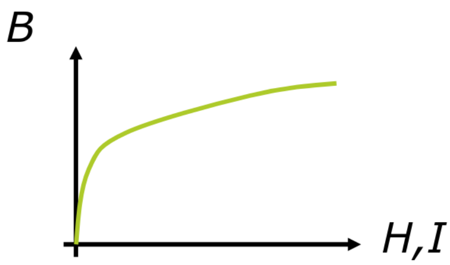
6.2.7 Coils with and without iron core

The magnetization curve of a coil without a core is linear, but there is significantly less flux density B than with an iron core.



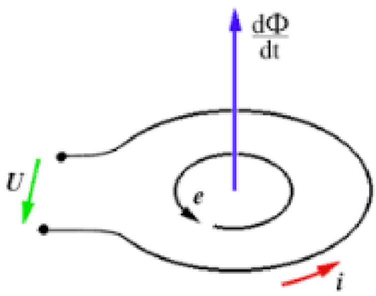


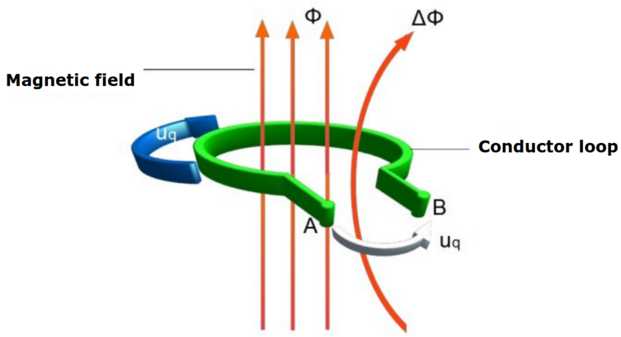




6.2.8 Law of induction and inductance

Changing magnetic flux generates a voltage





Phenomenon: a changing magnetic flux Φ induces a voltage in a conductor loop around it:

$$\boxed{U = -N \cdot \frac{\mathrm{d}\Phi}{\mathrm{d}t}}$$

6.2.9 Inductance and induction

Inductance L is the capability to generate a magnetic field. It is measured by the voltage divided by the rate of change of current over time. It is a measure of the magnetic "capacity" of an arrangement of conductors (e.g. coil) and can be compared to the capacity C of a capacitor. It indicates how much magnetic flux per ampere is generated.

$$\boxed{L = \frac{N \cdot \Phi}{I} = \frac{U}{\frac{\Delta I}{\Delta t}}}$$

where:

- L: inductance [H = Vs/A];
- N: number of turns;
- Φ: magnetic flux [Wb];
- I: current [A];
- U: voltage [V].

6.2.10 Inductivity of a very long coil

The inductance of a very long coil can be calculated approximately with:

$$\boxed{L = \frac{\mu \cdot N^2 \cdot A}{l}}$$

where:

- L: inductance [H = Vs/A];
- μ: magnetic permeability [Vs/Am];
- N: number of turns;
- A: cross-section of the coil [m^2];
- l: length [m].

6.2.11 Energy stored in an inductor

Since a variable magnetic field induces a voltage in which a current can also flow, the magnetic field must contain energy:

$$\boxed{W = \frac{1}{2} L \cdot I^2}$$

where:

- W: work, energy [J = Ws];
- L: inductance [H = Vs/A];
- I: current [A].

6.2.12 Current-voltage relationship of an inductor

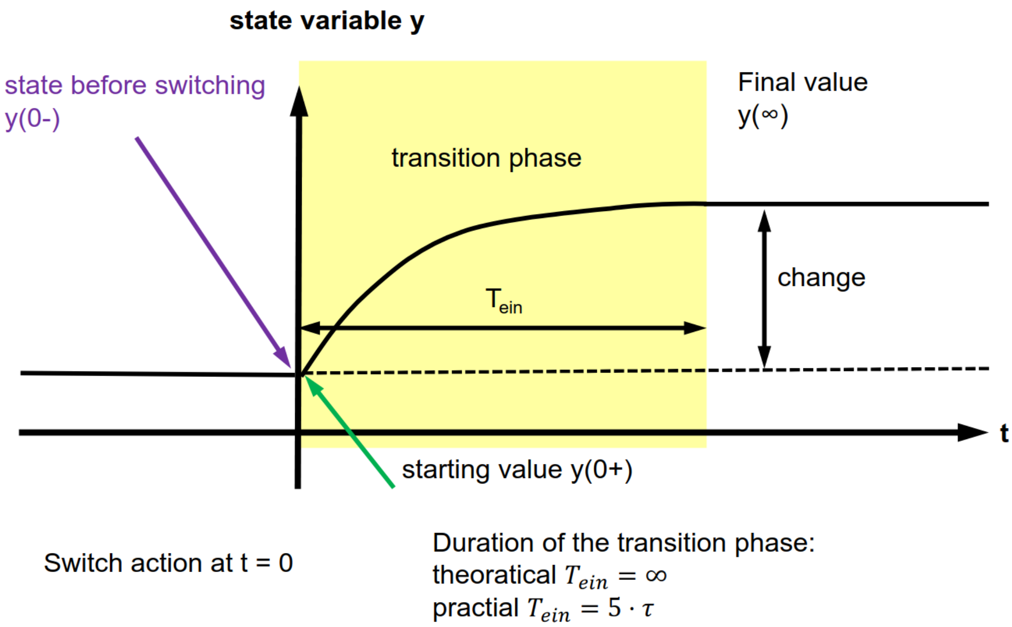
The current-voltage relationship of an inductor is:

$$\boxed{U = L \cdot \frac{\mathrm{d}I}{\mathrm{d}t}}$$

Special case:

$$\boxed{0 = L \cdot \frac{\mathrm{d}I}{\mathrm{d}t} \rightarrow u_c = 0}$$

6.2.13 Transient analysis



1. The state variable $y(t)$ is the variable that cannot change instantaneously. For the inductor, this is $i_L(t)$. The state just before the switch action:
$$y(0^-) = i_L(0^-).$$

2. The starting value is the state immediately before the switch action:
$$y(0^+) = i_L(0) = i_L(0^-).$$
That is, the state variable i_L keeps the value from $t = 0^-$.

3. The final value is the value long after the switch action:
$$y(\infty) = i_L(\infty),$$
which is practically reached after 5τ.

4. The transient is described by the function of time:
$$y(t) = \text{final value} + \big(\text{starting value} - \text{final value}\big) \exp\left(-\frac{t}{\tau}\right).$$
Hence,
$$i_L(t) = i_L(\infty) + \Big(i_L(0+) - i_L(\infty)\Big) \exp\left(-\frac{t}{\tau}\right).$$

Time constant τ for an inductor

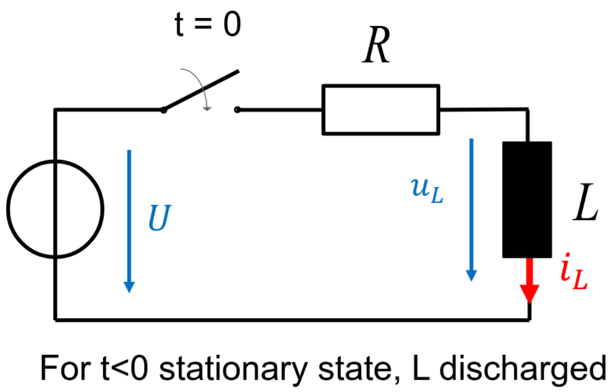
$$\boxed{\tau = \frac{L}{R}}$$

where:

- τ: time constant [s];
- L: inductance [H];
- R: resistance [Ω].

6.3 Examples

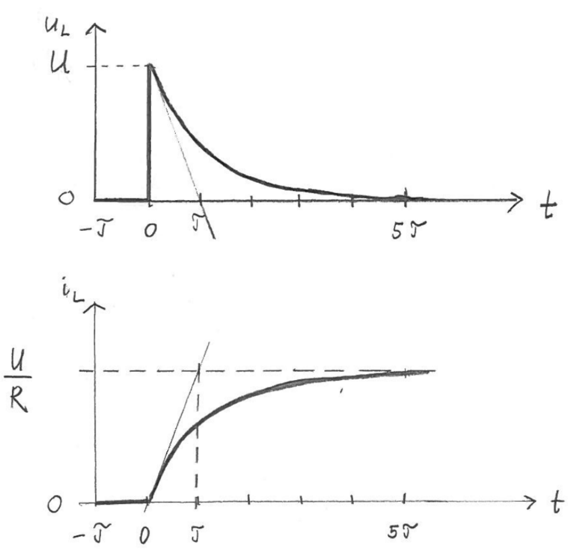
6.3.1 Charging an inductor in a RL-network



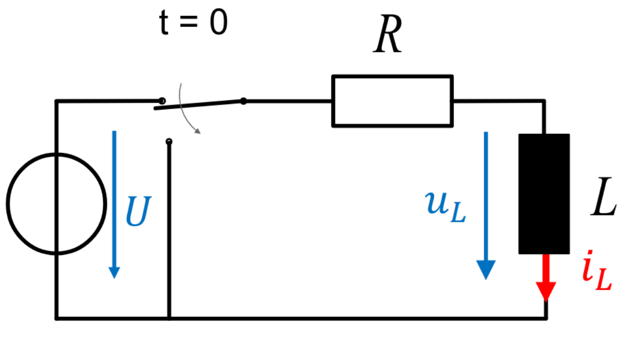
Calculations

$$\boxed{\begin{aligned} i_L &= \frac{U}{R} \cdot \left(1 - \exp\left(-\frac{t}{\tau}\right)\right) \\ u_L &= U \cdot \exp\left(-\frac{t}{\tau}\right) \end{aligned}}$$

Graphical representation



6.3.2 Discharging an inductor in a RL-network

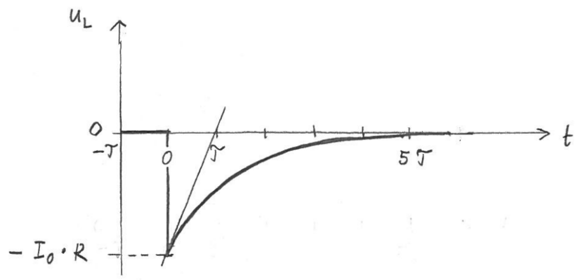


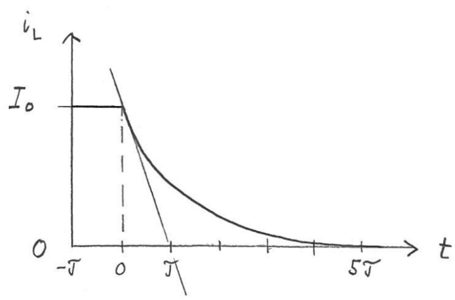
Before t = 0 stationary state:
Current in inductor is I_0

Calculations

$$\boxed{\begin{aligned} i_L &= I_0 \cdot \exp\left(-\frac{t}{\tau}\right) \\ u_L &= -I_0 \cdot R \cdot \exp\left(-\frac{t}{\tau}\right) \end{aligned}}$$

Graphical representation





7 Alternating current (AC)

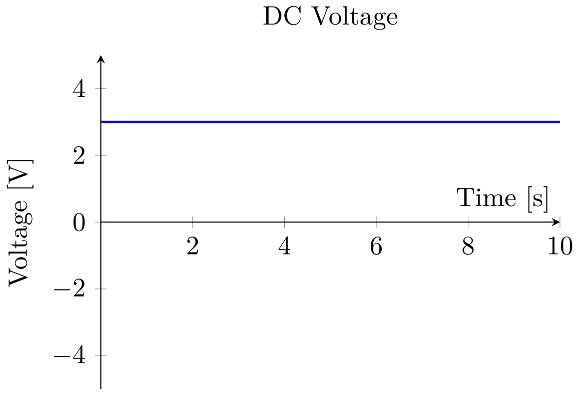
7.1 Generation of alternating current / voltage

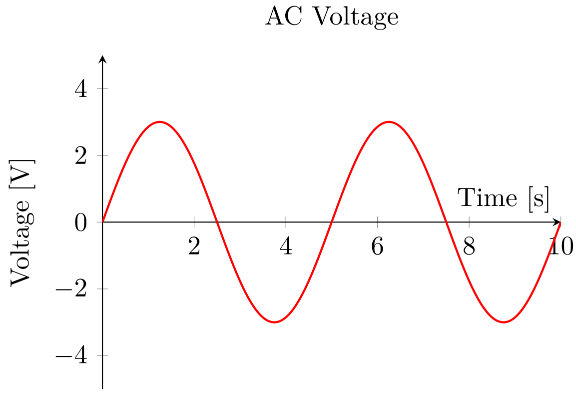
$$\boxed{U = -N \cdot \frac{\Delta\Phi}{\Delta t}}$$

where:

- U: voltage [V];
- N: number of turns;
- Φ: magnetic flux [Wb].

7.2 Comparison of AC and DC





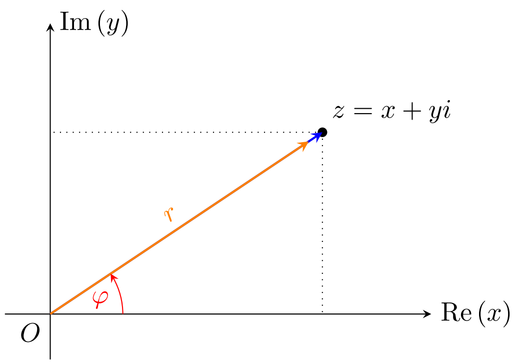
7.2.1 Advantages of AC

- Simple voltage transformation;
- Efficient trransmission;
- Easier generation;
- Compatibility with electric motors.

7.2.2 Disadvantages of AC

- Complexity in storage;
- Higher risk of shock;
- Complex circuits;
- Higher rectification costs.

7.3 Phasors



$$\boxed{z = x + yi = r \angle \varphi}$$

7.4 Oscillation as a function of the angle

Sinusoidal voltage has an instantaneous value $u(t)$ or u for every time t.

After a period of time T, the curve repeats itself.

$$\boxed{u(t) = \widehat{U}\sin(\omega \cdot t)}$$

7.5 Zero phase angle φ

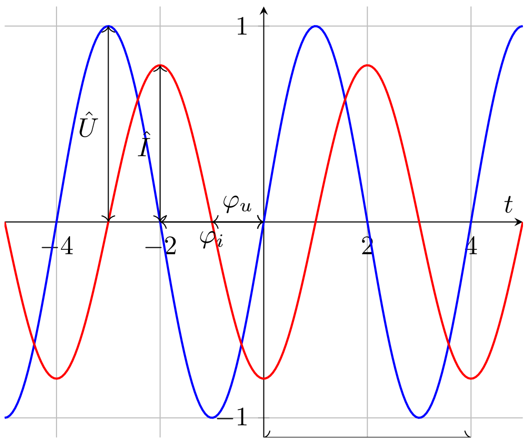
$$\boxed{u(t) = \widehat{U}\sin(\omega \cdot t + \varphi_u)}$$

7.5.1 Phase shift $\Delta\varphi$ between two signals

$$\boxed{\begin{aligned} u(t) &= \widehat{U}\sin(\omega \cdot t + \varphi_u) \\ i(t) &= \widehat{I}\sin(\omega \cdot t + \varphi_i) \end{aligned}}$$

The phase shift between two signals is the difference between the their zero phase signals:

$$\boxed{\Delta\varphi = \varphi_u - \varphi_i}$$



7.6 Power in a sinusoidal signal and effective value

7.6.1 Instantaneous power

The instantaneous power $p(t)$ is the actual power at a specific time t and is the product of the voltage $u(t)$ and the current $i(t)$ at that moment:

$$\boxed{p(t) = u(t) \cdot i(t) = \frac{u(t)^2}{R} = i(t)^2 \cdot R}$$

The active power P corresponds to the mean value of the instantaneous powerr $p(t)$ averaged over a period T:

$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt}$$

7.6.2 Effective value

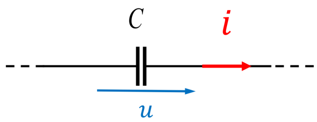
The effective value U_{eff} of a sinusoidal signal is the voltage that would generate the same power in a resistor as the sinusoidal signal:

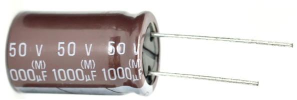
$$\boxed{U_{\text{eff}} = \sqrt{\frac{1}{T}\int_{0}^{T} u(t)^2\,dt} = \frac{\widehat{U}}{\sqrt{2}}}$$

The same can be applied to the effective value I_{eff}:

$$\boxed{I_{\text{eff}} = \frac{\widehat{I}}{\sqrt{2}}}$$

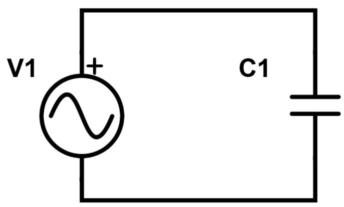
7.7 Relationship between current and voltage on a capacitor

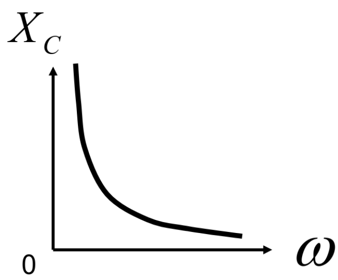




$$\boxed{i = C \cdot \frac{\Delta u}{\Delta t}}$$

7.8 Capacitive reactance X_c



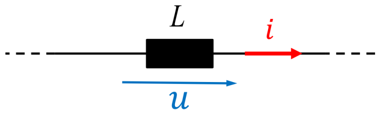


$$\boxed{X_c = \frac{1}{\omega \cdot C}}$$

where:

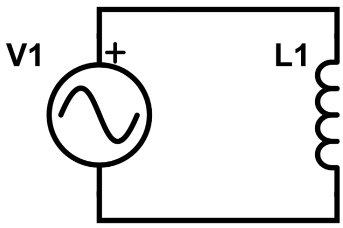
- X_c: capacitive reactance [Ohm];
- ω: angular frequency [rad/s];
- C: capacitance [F = As/V].

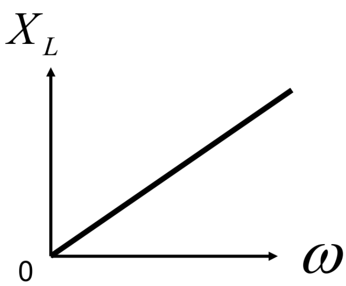
7.9 Relationship between current and voltage on an ideal inductor



$$\boxed{u = L \cdot \frac{\Delta i}{\Delta t}}$$

7.10 Inductive reactance X_L





$$\boxed{X_L = \omega \cdot L}$$

where:

- X_L: inductive reactance [Ohm];
- L: inductance [H];
- ω: angular frequency [rad/s];

7.11 Vectors properties

7.11.1 Multiply

The magnitude (es. the radius r in polar representation) is multiplied and the angle is added:

$$\boxed{\begin{array}{l} r_c = r_a \cdot r_b \\ \varphi_c = \varphi_a + \varphi_b \end{array}}$$

7.11.2 Divide

The magnitude is devided and the angle is subtracted:

$$\boxed{\begin{array}{l} r_c = \dfrac{r_a}{r_b} \\ \varphi_c = \varphi_a - \varphi_b \end{array}}$$

7.12 Impedance Z

The impedance is the ratio of voltage and current phasor. It's a complex number.

$$\boxed{\begin{array}{l} Z = \dfrac{u(t)}{i(t)} \\ |Z| = \dfrac{|u(t)|}{|i(t)|} \\ \angle Z = \angle u(t) - \angle i(t) = \Delta\varphi \end{array}}$$

Therefore, the impedance corresponds to the AC resistance with phase shift:

$$\boxed{\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U\angle\varphi_u}{I\angle\varphi_i} = Z\angle(\varphi_u - \varphi_i) = Z\angle\varphi_Z}$$

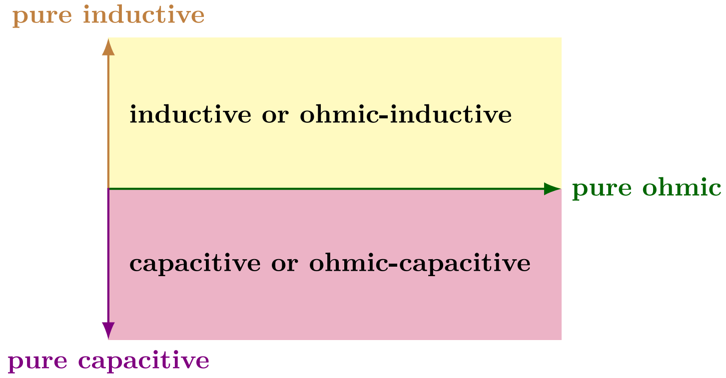
where:

- Z: impedance [Ohm];
- $\underline{U}$: voltage phasor [V];
- $\underline{I}$: current phasor [A];
- φ_u: phase angle of the voltage [rad];
- φ_i: phase angle of the current [rad];
- φ_Z: phase angle of the impedance [rad].

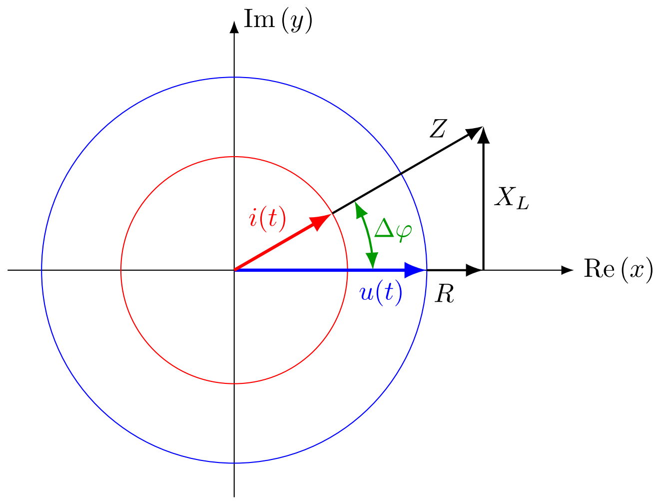
7.12.1 Types of impendance

The angle of the impedance φ_Z indicates the type of impedance:

- $\varphi_Z > 0° \rightarrow$ voltage is ahead of current;
- $\varphi_Z = 0° \rightarrow$ voltage and current are in phase;
- $\varphi_Z < 0° \rightarrow$ current is ahead of voltage.



7.12.2 Graphical representation



7.13 Admittance Y

The reciprocal of the impedance Z is the admittance Y and thus the ratio of the current and voltage phasor. The admittance therefore corresponds to the AC conductance with phase shift:

$$\boxed{\begin{aligned}&\underline{Y} = \frac{\underline{I}}{\underline{U}} = \frac{I\angle\varphi_i}{U\angle\varphi_u} = Y\angle(\varphi_i - \varphi_u) = Y\angle\varphi_Y \\ &|\underline{Y}| = \frac{1}{|\underline{Z}|} \Longrightarrow \varphi_Y = -\varphi_Z\end{aligned}}$$

7.14 Current and voltage relations

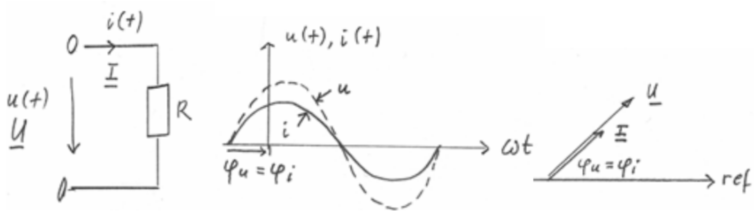
7.14.1 Resistor R

Current-voltage relationship for instantaneous values

$$\boxed{u_R(t) = R \cdot i_R(t)}$$

Impedance = Ohmic resistance

$$\boxed{R = \frac{U_R}{I_R}\angle 0°}$$



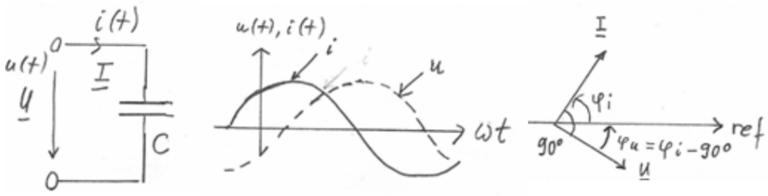
7.14.2 Capacitor C

Current-voltage relationship for instantaneous values

$$\boxed{i_C(t) = C \cdot \frac{du_C(t)}{dt}}$$

Impendance = Capacitive reactance

$$\boxed{X_C = \frac{U_C}{I_C} = \frac{1}{\omega \cdot C} \angle -90°}$$



Current leads the voltage by 90 degrees

Phase shift between current and voltage

$$\boxed{\varphi = \arctan\left(\frac{-X_C}{R}\right)}$$

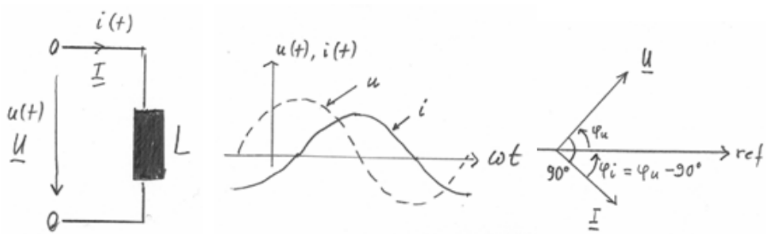
7.14.3 Inductor L

Current-voltage relationship for instantaneous values

$$\boxed{u(t) = L \cdot \frac{di(t)}{dt}}$$

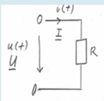
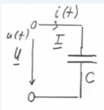
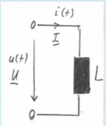
Impendance = Inductive reactance

$$\boxed{X_L = \frac{U_L}{I_L} = \omega \cdot L \angle +90°}$$



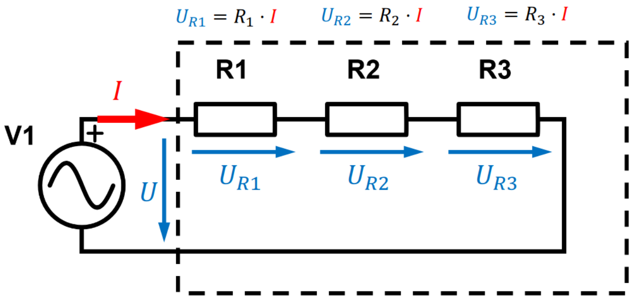
Current lags the voltage by 90 degrees

7.15 Impendance and admittance phasor with R, C and L

		Phase shift $\varphi_z = \varphi = \varphi_U - \varphi_I$	**Impedance $\underline{Z}$** $\underline{Z} = \frac{\underline{U}}{\underline{I}} \angle\varphi_Z$	**Admittance $\underline{Y}$** $\underline{Y} = \frac{1}{\underline{Z}} = \frac{\underline{I}}{\underline{U}} \angle\varphi_Y$
	$\varphi_u = \varphi_i$	$0°$	$\underline{Z}_R = R\angle 0°$	$\underline{Y}_R = \frac{1}{R}\angle 0°$
	$\varphi_u = \varphi_i - 90°$	$-90°$	$\underline{Z}_C = \frac{1}{\omega \cdot C}\angle -90°$	$\underline{Y}_C = \omega \cdot C\angle +90°$
	$\varphi_i = \varphi_u - 90°$	$90°$	$\underline{Z}_L = \omega \cdot L\angle +90°$	$\underline{Y}_L = \frac{1}{\omega \cdot L}\angle -90°$

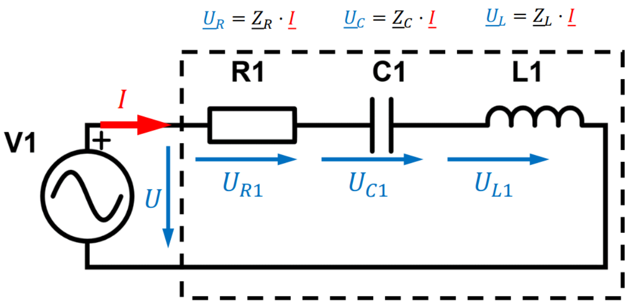
7.15.1 Series connection

Resistances



$$\boxed{R_{\text{equi}} = \frac{U}{I} = \frac{U_{R1} + U_{R2} + U_{R3}}{I} = R_1 + R_2 + R_3}$$

Impedances



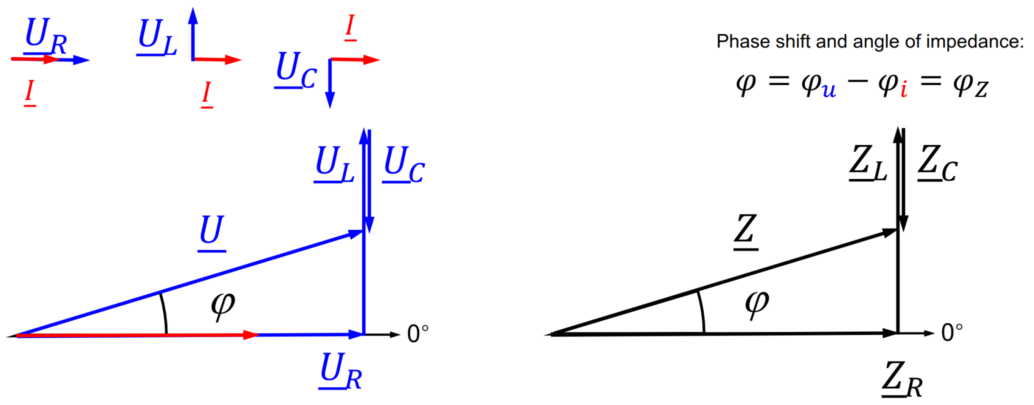
$$\boxed{Z_{\text{equi}} = \frac{\underline{U}}{\underline{I}} = \frac{U_{R1}\angle 0° + U_{C1}\angle -90° + U_{L1}\angle +90°}{I\angle 0°} = \underline{Z_1} + \underline{Z_2} + \underline{Z_3}}$$

Adding voltages in series connection means adding impedances:

$\underline{U_R} = \underline{Z_R} \cdot \underline{I} = R \cdot \angle\varphi_i$

$\underline{U_L} = \underline{Z_L} \cdot \underline{I} = X_L\angle 90° \cdot \angle\varphi_i$

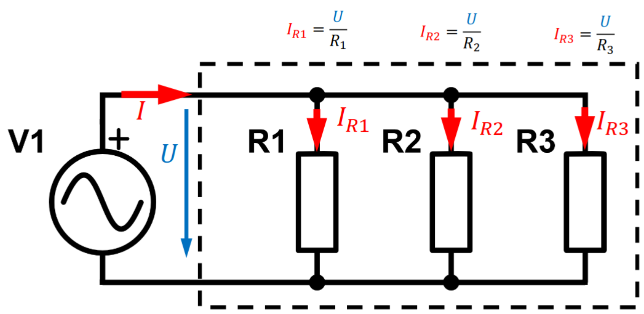
$\underline{U_C} = \underline{Z_C} \cdot \underline{I} = X_C\angle - 90° \cdot \angle\varphi_i$



$$\boxed{Z_{\text{eq}} = Z_1 + Z_2 + Z_3}$$

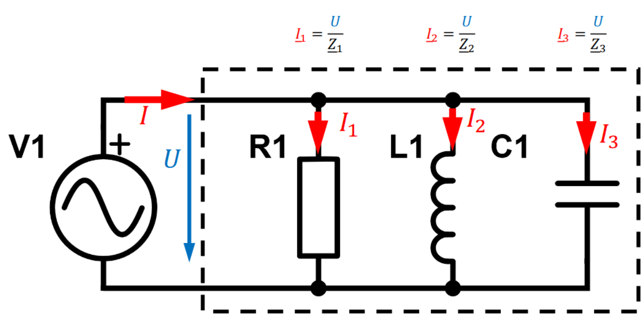
7.15.2 Parallel connection

Resistances



$$\boxed{R_{\text{equi}} = \frac{U}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{G_1 + G_2 + G_3}}$$

Impedances



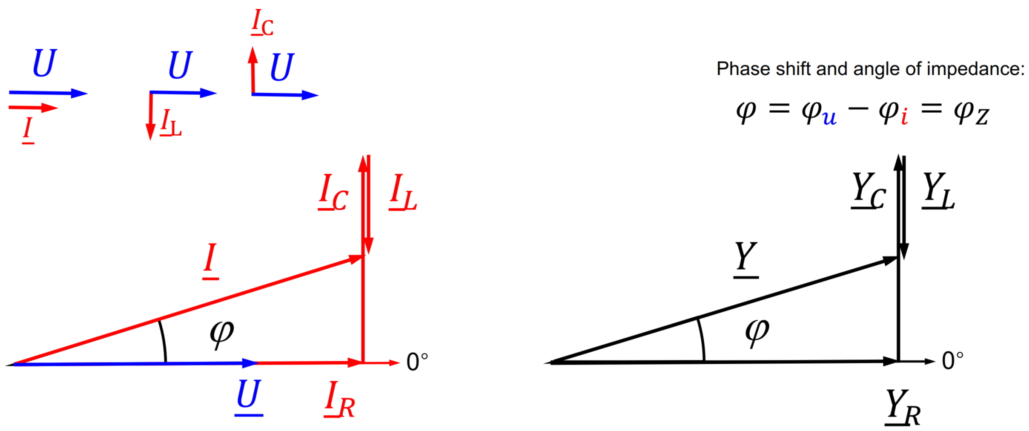
$$\boxed{Z_{\text{equi}} = \frac{\underline{U}}{\underline{I}} = \frac{U}{I\angle 0°} = \frac{1}{\frac{1}{\underline{Z_1}} + \frac{1}{\underline{Z_2}} + \frac{1}{\underline{Z_3}}} = \frac{1}{\underline{Y_1} + \underline{Y_2} + \underline{Y_3}}}$$

Adding currents in parallel connection means adding admittances:

$$\underline{I}_R = \frac{\underline{U}_R}{\underline{Z}_R} = \frac{U}{R} \cdot \angle \varphi_u$$

$$\underline{I}_L = \frac{\underline{U}_L}{\underline{Z}_L} = \frac{U}{X_L} \angle \varphi_u - 90°$$

$$\underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_C} = \frac{U}{X_C} \angle \varphi_u + 90°$$



$$\boxed{\underline{Z}_{\text{eq}} = \frac{1}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3}}$$

7.16 AC newtwork analysis

AC network analysis is similar to DC network analysis but calculated with phasors.

7.16.1 Kirchhoff's current law (KCL)

$$\boxed{\underline{I}_1 + \underline{I}_2 + \ldots + \underline{I}_n = \sum_{k=1}^{n} \underline{I}_k = 0}$$

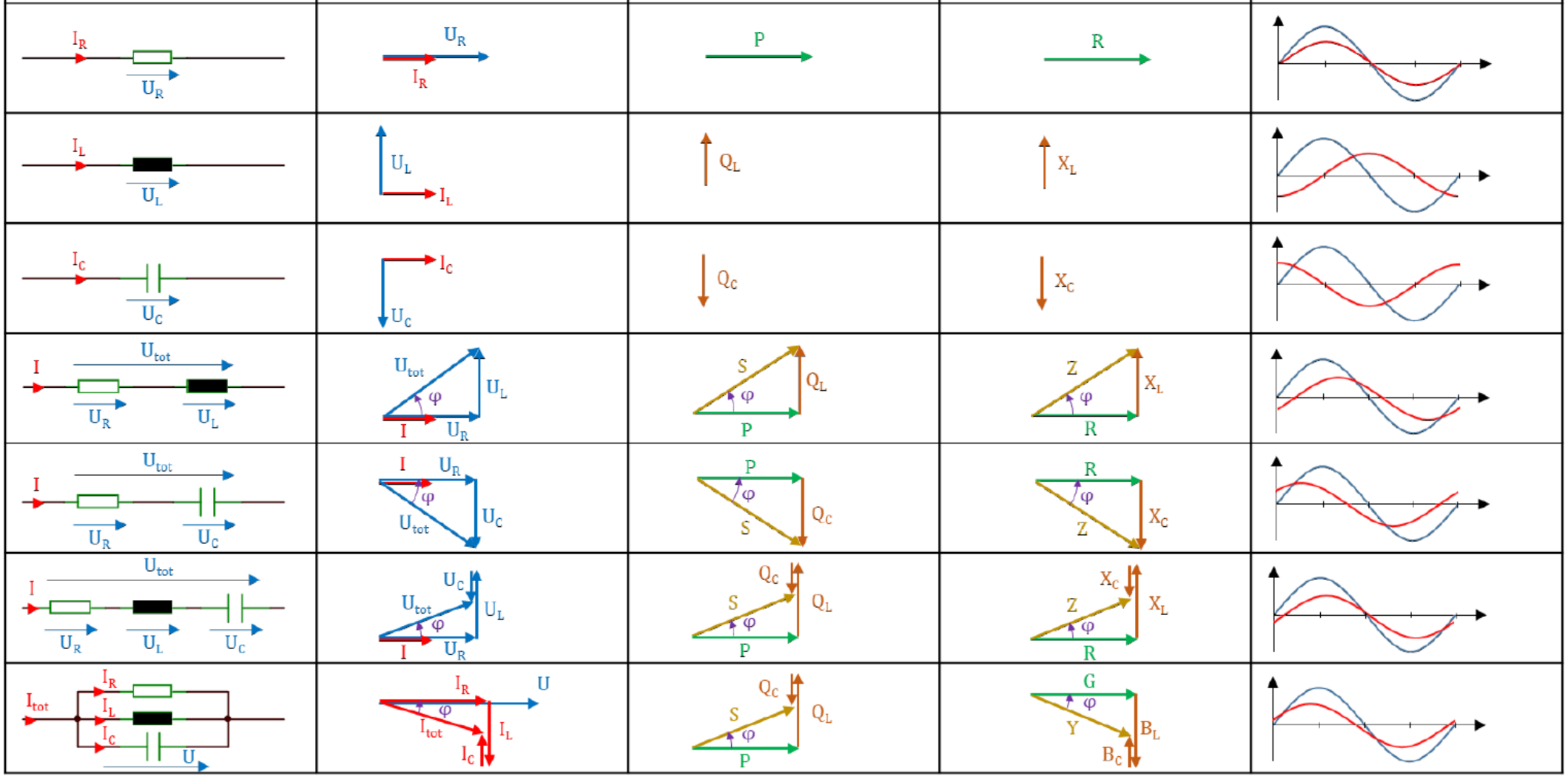
7.16.2 Kirchhoff's voltage law (KVL)

$$\boxed{\underline{U}_1 + \underline{U}_2 + \ldots + \underline{U}_n = \sum_{k=1}^{n} \underline{U}_k = 0}$$

7.16.3 Voltage and current phasor relationship for circuit elements

$$\boxed{\underline{U} = \underline{Z}_{\text{Element-type}} \cdot \underline{I}}$$

Circuit	Current / Voltage	Power	Impedanz (Admittanz)	Signal sequence



Parallel connection: Calculating via the admittance $\rightarrow G = \frac{1}{Z}$

7.17 Power in electrical circuits

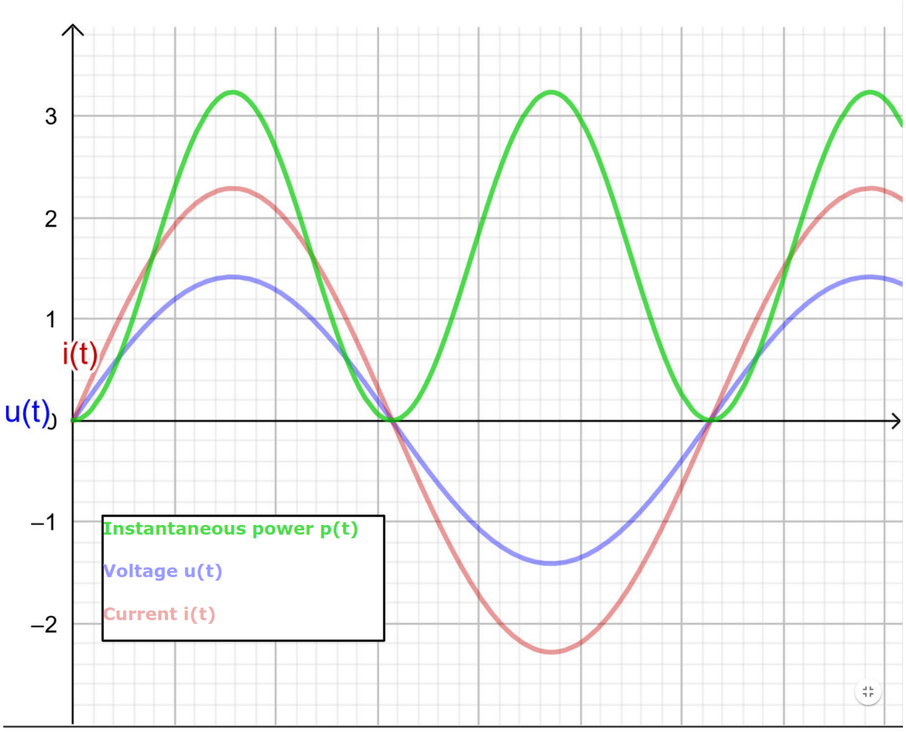
7.17.1 Devices

Passive (load)
For electrical loads, the product of current I and voltage U is **positive**. Electrical power is absorbed and converted into another form of energy (e.g. heat)

Active (power sources)
For electrical sources, the product of current I and voltage U is **negative**. Energy is converted from another form of energy into electrical power and supplied to the network.

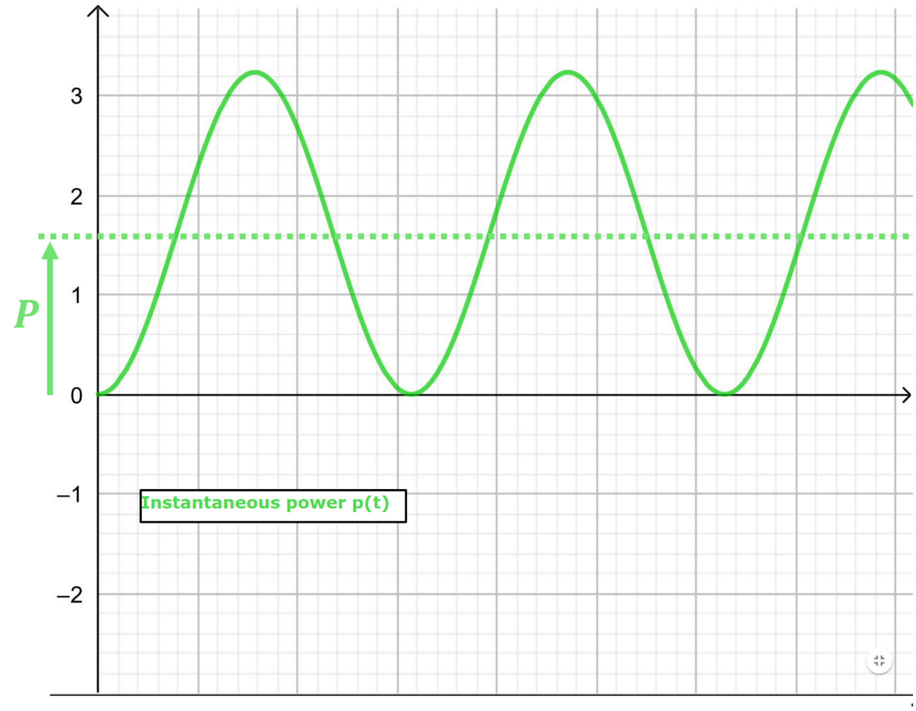
Power in eletrical circuits
The conservation of energy also applies in the electrical circuit.

7.17.2 Instantaneous power



$$\boxed{p(t) = u(t) \cdot i(t) = \frac{u(t)^2}{R} = i(t)^2 \cdot R}$$

7.17.3 Effective power



$$\boxed{P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{U_{\text{eff}}^2}{R} = I_{\text{eff}}^2 \cdot R}$$

7.17.4 Real power on R

When whe have $\varphi = 0°$

$$\boxed{P = U \cdot I}$$

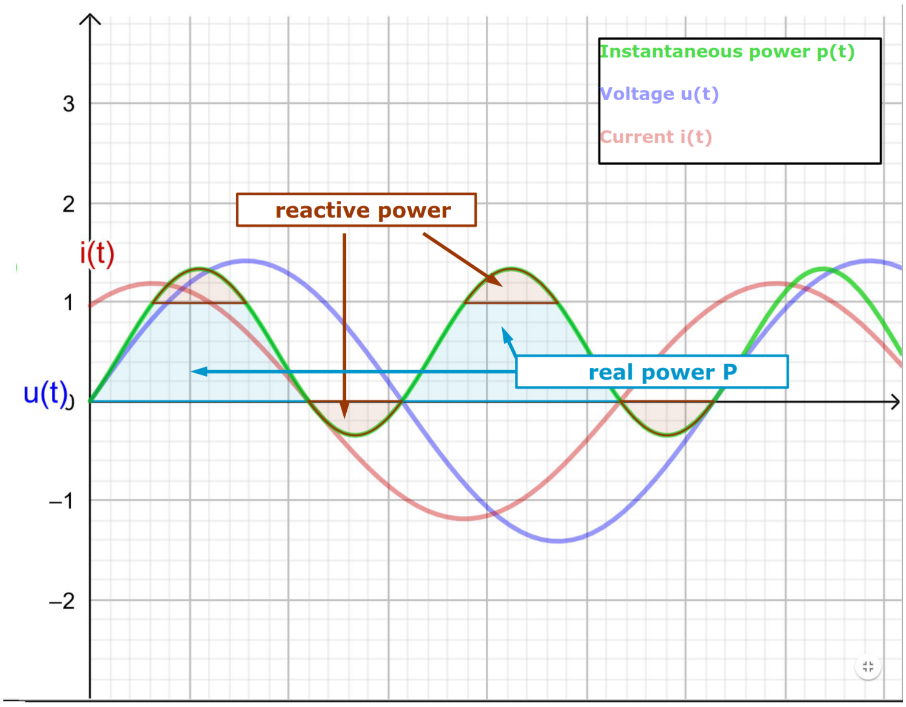
When we have $\varphi = \pm 90°$
With $\varphi = +90°$, the circuit works as a perfect inductor.

With $\varphi = -90°$, the circuit works as a perfect capacitor.

Since the sum of the areas underneath the power curve is zero, the average power is zero:

$$\boxed{P = 0}$$

7.17.5 Instantaneous power $p(t)$ with phase shift (Q)



A phase shift φ between voltage and current leads to positive and negative instantaneous power. The amount of power that is absorbed, stored and released during a period is called **reactive power** Q.

The unit of reactive power is called volt-ampere reactive (var).

The active power P is now lower by the proportion of the oscillating reactive power.

7.17.6 Real power with $0° < \varphi < 90°$

$$\boxed{\begin{aligned} p(t) &= U \cdot I \cdot \cos(\varphi) - U \cdot I \cdot \cos(2\omega t + \varphi) \\ p(t) &= \text{average power } P - \text{apparent power } S \end{aligned}}$$

where:

S: apparent power [VA] $= U \cdot I$

7.17.7 Power factor, performance factor, and power triangle

Power factor

The ratio of the real power P to the apparent power S corresponds to the $\cos\varphi$ and is called the **power factor**.

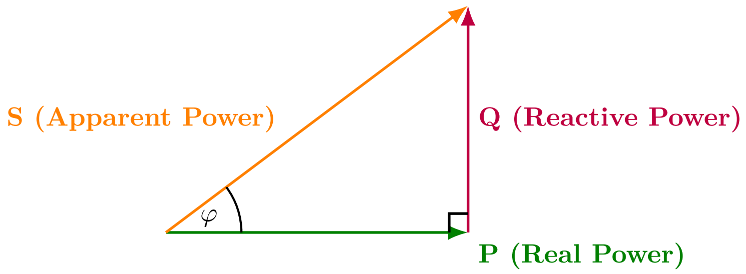
$$\boxed{\cos\varphi = \frac{P}{S}}$$

Performance factor

The performance factor λ is the absolute value of the ratio of the ratio of the real power P to the apparent power S:

$$\boxed{\lambda = \left|\frac{P}{S}\right| = |\cos\varphi|}$$

Power triangle
The real power P, the reactive power Q, and the apparent power S form a rectangular triangle with the angle φ, called **power triangle**



7.17.8 Apparent power S [VA]

$$\boxed{\begin{aligned} &S = U \cdot I \\ &S = \sqrt{P^2 + Q^2} \end{aligned}}$$

7.17.9 Average power P [W]

$$\boxed{\begin{aligned} &P = U \cdot I \cdot \cos\varphi \\ &P = S \cdot \cos\varphi \end{aligned}}$$

7.17.10 Reactive power Q [var]

$$\boxed{\begin{aligned} &Q = U \cdot I \cdot \sin\varphi \\ &Q = S \cdot \sin\varphi \end{aligned}}$$

7.18 Work W and energy E

The power integrated over time results in the work performed W. The ability to perform work is referred to as energy E.

7.18.1 Real energy

$$\boxed{W_W = P \cdot t}$$

7.18.2 Reactive energy

$$\boxed{W_B = Q \cdot t}$$