Preambule

Theory box

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Formula box

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Lab/examples box

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1 Fluids as energy carriers

1.1 Fluid state variables and properties

Formulas

1.1.1 State variables

Density

$$\rho \triangleq \frac{m}{V} \left[\frac{kg}{m^3}\right] \tag{1}$$

Specific volume

$$v \triangleq \frac{V}{m} = \frac{1}{\rho} \left[\frac{m^3}{kg}\right] \tag{2}$$

1.1.2 Viscosity

Kinematic viscosity

$$\nu \triangleq \frac{\eta}{\rho} \left[\frac{m^2}{s}\right] \tag{3}$$

Dynamic viscosity

$$\eta \triangleq \nu \cdot \rho \left[Pa \cdot s = \frac{Ns}{m^2} = \frac{kg}{m \cdot s}\right] \tag{4}$$

1.1.3 Real and ideal fluid

Real fluid	**Ideal fluid**
variable density ($\Delta\rho \neq 0$)	incompressible ($\Delta\rho = 0$)
friction ($\eta > 0, \nu > 0$)	frictionless ($\eta = 0, \nu = 0$)

1.1.4 Compressibility

Mach number

$$M \triangleq \frac{u}{c} \tag{5}$$

where:
- M is the Mach number [-]
  $M \lesssim 0.3$: incompressible flow
- u is the flow velocity [m/s]
- c is the speed of sound in the fluid [m/s]

and:
- $c_{\mathrm{w}}^{20°} = 1484$ m/s
- $c_{\mathrm{a}}^{20°} = 343$ m/s

1.2 Laminar and turbulent flow

Reynolds number

$$Re = \frac{v \cdot L}{\nu} = \frac{\rho \cdot v \cdot L}{\eta} [-] \tag{6}$$

where:
- v is the mean flow velocity [m/s]
- L is the characteristic length [m]

Re values
- $Re < 2000$: laminar flow
- $Re \simeq 2300$: critical point
- $2000 < Re < 4000$: transitional regime
- $Re \geq 4000$: turbulent flow

1.3 Pressure and velocity

Pressure

1.3.1 Total pressure

In addition to the static pressure p_{stat}, there is also the dynamic pressure p_{dyn} and the total pressure p_{tot}:

$$p_{\mathrm{tot}} = p_{\mathrm{stat}} + p_{\mathrm{dyn}} \tag{7}$$

1.3.2 Absolute pressure

Absolute pressure p_{abs} refers to the pressure in a vacuum $p_{\mathrm{vaacum}} = 0Pa$ while relative pressure p_{rel} can refer to any chosen reference pressure p_{ref}.

$$p_{\mathrm{abs}} = p_{\mathrm{rel}} - p_{\mathrm{ref}} \tag{8}$$

1.3.3 Velocity

Velocity is a vector quantity:

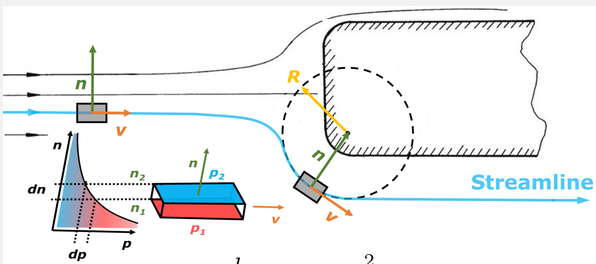
$$\vec{v} = (v_x v_y v_z) \tag{9}$$

The magnitude is given by:

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{10}$$

1.4 Curvature pressure formula

Deflection motion of a fluid element around a blunt body

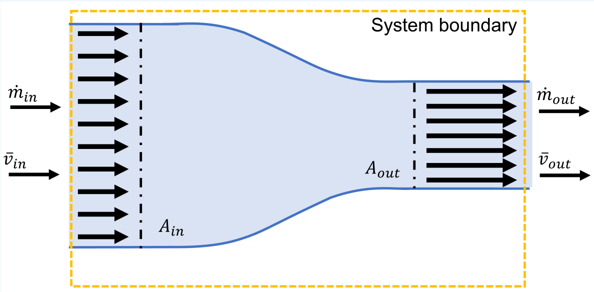


$$\frac{dp}{dn} = -\rho \cdot \frac{v^2}{R} \tag{11}$$

2 Mass conservation

2.1 Continuity equation / Mass conservation

Continuity equation



2.1.1 Steady mass-flow

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \tag{12}$$

2.1.2 Incompressible fluid

$$\dot{m} = \rho \dot{V} \quad \Longrightarrow \quad \dot{V}_{\text{in}} = \dot{V}_{\text{out}} \tag{13}$$

2.1.3 Streamline theory

$$\dot{V} = \bar{v}\, A \quad \Longrightarrow \quad \bar{v}_{\text{in}}\, A_{\text{in}} = \bar{v}_{\text{out}}\, A_{\text{out}} \tag{14}$$

3 Energy conservation

3.1 Fluid mechanical energy conservation

Derivation of the Bernoulli equation

$$\dot{m}_1 \left(\frac{p_1}{\rho} + \frac{v_1^2}{2} + g z_1 \right) = \dot{m}_2 \left(\frac{p_2}{\rho} + \frac{v_2^2}{2} + g z_2 \right) \tag{15}$$

This derivation is based on the assumption that the system has:

- steady flow
- ideal fluid
- adiabatic process
- no work in or out of the system
- 1D streamline flow

3.1.1 Energy flow

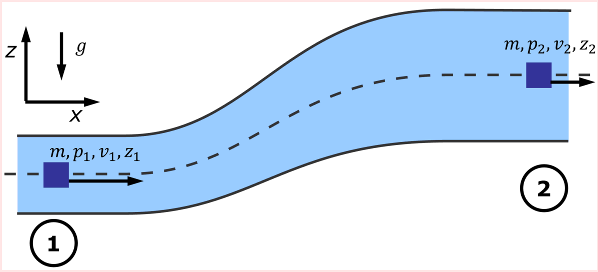
$$\frac{dE}{dt} = \underbrace{\sum P + \sum \dot{Q}}_{\substack{\text{Energy flow} \\ \text{across system boundary}}} + \underbrace{\sum_{in} \left[\dot{m}^{\swarrow} \cdot \left(h^{\swarrow} + \frac{v^{2\swarrow}}{2} + g z^{\swarrow} \right) \right]}_{\substack{\text{Energy transfer} \\ \text{mass in}}} - \underbrace{\sum_{out} \left[\dot{m}^{\nearrow} \cdot \left(h^{\nearrow} + \frac{v^{2\nearrow}}{2} + g z^{\nearrow} \right) \right]}_{\substack{\text{Energy transfer} \\ \text{mass out}}} \tag{16}$$

3.1.2 Outflow formula according to Torricelli

$$g z_1 = \frac{v_2^2}{2} \Longrightarrow v_2 = \sqrt{2 g \Delta z} \tag{17}$$

3.2 Bernoulli equation

Specific energy equation



$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + g z_1 = \frac{p_2}{\rho} + \frac{v_2^2}{2} + g z_2 = \text{const.} \left[\frac{J}{kg} \right] \tag{18}$$

3.2.1 Alternative forms

Pressure equation

$$p_1 + \frac{\rho v_1^2}{2} + \rho g z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g z_2 = \text{const.} \, [Pa] \tag{19}$$

Height equation

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 = \text{const.} \, [m] \tag{20}$$

True energy equation

The Bernoulli equation states that the sum of these energies is constant along a streamline.

3.2.2 Pressure energy

$$E_p = m \cdot \frac{p}{\rho} \, [J] \tag{21}$$

3.2.3 Kinetic energy

$$E_{\text{kin}} = m \cdot \frac{v^2}{2} \, [J] \tag{22}$$

3.2.4 Potential energy

$$E_{\text{pot}} = m \cdot g \cdot z \, [J] \tag{23}$$

3.2.5 Energy conservation

$$E_{p,1} + E_{\text{kin},1} + E_{\text{pot},1} = E_{p,2} + E_{\text{kin},2} + E_{\text{pot},2}$$

$$m \left(\frac{p_1}{\rho} + \frac{v_1^2}{2} + g z_1 \right) = m \left(\frac{p_2}{\rho} + \frac{v_2^2}{2} + g z_2 \right) \tag{24}$$

3.3 Hydrostatics

Fundamental law of hydrostatics

$$p = p_0 + \rho g h = \text{const.} \, [\text{Pa}] \tag{25}$$

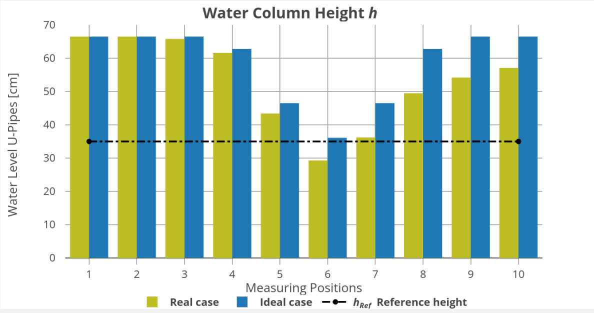
derived from:

$$p = p_0 + \frac{F_g}{A} = p_0 + \frac{mg}{A} = p_0 + \frac{\rho h A g}{A} \tag{26}$$

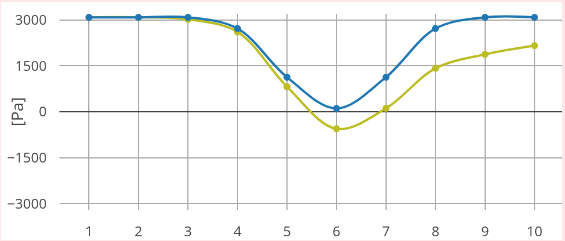
3.4 Venturi effect experiment

Venturi effect

Height – pressure difference at $\dot{V} = 6$ l/s

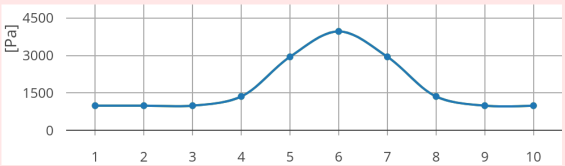


Relative static pressure p_{rel}



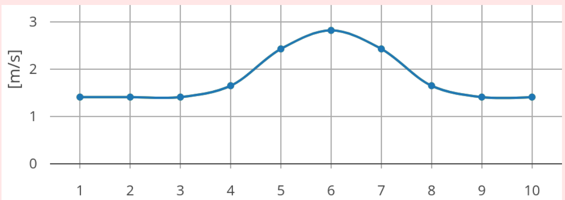
$$p_{\text{rel}} = p_{\text{hydro}} = \rho g \left(h - h_{\text{ref}}\right) \qquad (27)$$

Dynamic pressure p_{dyn}



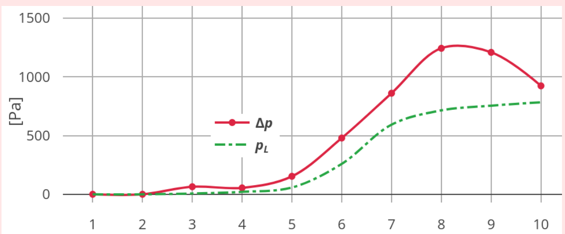
$$p_{\text{dyn}} = \rho \frac{v^2}{2} \qquad (28)$$

Dynamic pressure v



$$v = \frac{\dot{V}}{A} \qquad (29)$$

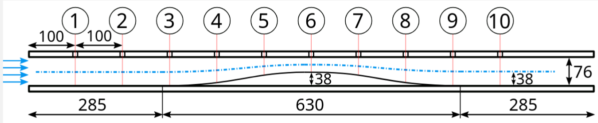
Pressure difference Δp



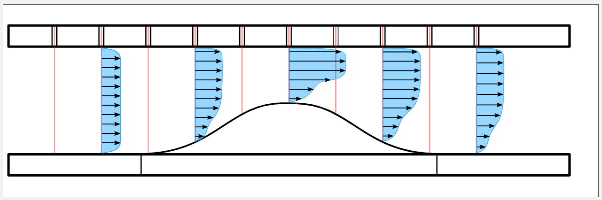
$$\Delta p = p_{\text{NoFric}} - p_{\text{real}} \Longrightarrow p_V \sim v^2 \qquad (30)$$

Venturi effect

Measurement points

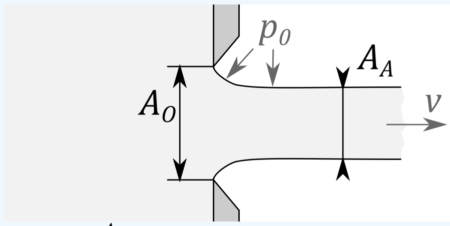


Measurement shear flow



3.5 Contraction coefficient

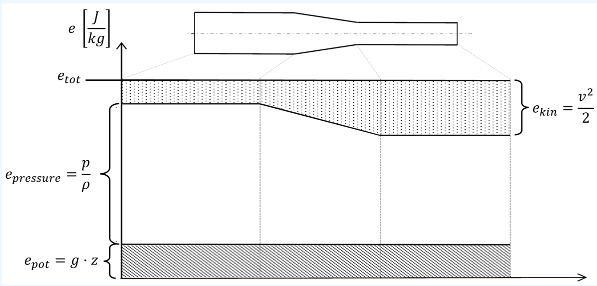
Outflow contraction coefficient α



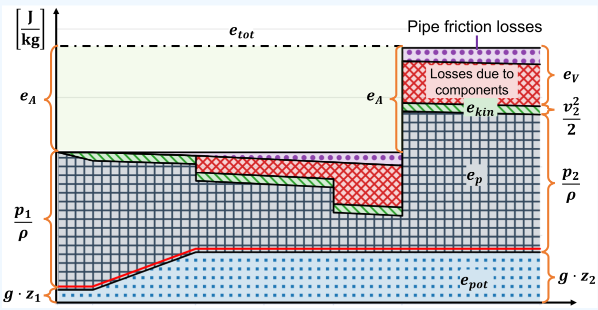
$$\alpha = \frac{A_{\text{actual}}}{A_{\text{opening}}} = \frac{\pi}{2+\pi} \approx 0.611[-] \qquad (31)$$

3.6 Energy line diagram

Ideal fluid energy line diagram



Extended energy line diagram



3.7 Extended Bernoulli equation

Extension of the Bernoulli equation

$$\frac{p_1}{\rho} + \frac{v_1^2}{2} + gz_1 + e_A = \frac{p_2}{\rho} + \frac{v_2^2}{2} + gz_2 + e_V \left[\frac{J}{kg}\right]$$
$$E_{p,1} + K_1 + U_1 + E_A = E_{p,2} + K_2 + U_2 + E_V\,[J] \tag{32}$$

3.7.1 Additional terms

Work term e_A

$$e_A = \frac{p_A}{\rho} = gz_A = \frac{E_A}{m} = \frac{P_A}{\dot{m}} \left[\frac{J}{kg}\right] \tag{33}$$

where:
e_A: work term [J/kg] E_A: energy difference [J]
p_A: pressure diff [Pa] P_A: power difference [W]
z_A: height difference [m]

If energy is added to the fluid along a streamline from point 1 to point 2 (eg. a pump), the total energy at point 2 becomes higher than at point 1.

Sign convention
$\mathbf{e_A > 0}$: work is done on the fluid
→ energy is added to the fluid (eg. pump);

$\mathbf{e_A < 0}$: work is done by the fluid
→ energy is extracted from the fluid (eg. turbine).

Pump and turbine work Y

In the pressure equation, the pressure p_A increase (or decrease with a turbine) can be read directly at the working term, hence:

$$e_A = \frac{E_A}{m} = Y = H \cdot g = \frac{p_A}{\rho} \left[\frac{J}{kg}\right] \tag{34}$$

The hydraulic power P_{hyd} is then given by:

$$P_{\text{hyd}} = \dot{m} \cdot Y = \dot{V} \cdot \rho \cdot Y = \rho \cdot \dot{V} \cdot g \cdot H\,[W] \tag{35}$$

Specific loss term e_V

$$e_V = \frac{p_V}{\rho} = gz_V = \frac{E_V}{m} = \frac{P_V}{\dot{m}} \left[\frac{J}{kg}\right] \tag{36}$$

where:
e_V: loss term [J/kg] E_V: energy loss [J]
p_V: pressure diff [Pa] P_V: power loss [W]
z_V: height loss [m]

The effects of a viscous fluid along a stramline from point 1 to point 2 are taken into account by e_V.

Pressure loss Δp_V

$$\Delta p_V = e_V \cdot \rho = \frac{E_V \cdot \rho}{m} = g \cdot z_V \cdot \rho = \zeta \cdot \rho \cdot \frac{v^2}{2}\,[Pa] \tag{37}$$

3.8 Loss behavior in turbolent flows

Zeta value

$$\zeta = \frac{2 \cdot \Delta p_V}{\rho \cdot v^2} \tag{38}$$

Total pressure loss

If multiple losses occur in a system due to sequentially connected hydraulic components, the ttal loss $\Delta p_{V,\text{tot}}$ is given by the sum of the individual losses:

$$\Delta p_{V,\text{tot}} = \sum_i \Delta p_{V,i} = \sum_i \zeta_i \cdot \rho \cdot \frac{v_i^2}{2}\,[Pa] \tag{39}$$

$$\Delta p_{V,\text{tot}} = \rho \cdot \frac{v^2}{2} \cdot \sum_i \zeta_i = \rho \cdot \frac{v^2}{2} \cdot \zeta_{\text{tot}}\,[Pa] \tag{40}$$