Mathematics 1A
HSLU, Semester 1

Matteo Frongillo

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Contents

Part I

Week 1

1 The set theory

1.1 Definition of a set

A set is a collection of objects or elements.

Remark: The collection of all sets is not a set.

1.2 Logical symbols

1.2.1 Definition

Braces and the definition symbol ":=" are used to define a set giving all its elements:

$$\boxed{A := \{a, b, c, d, e\}}$$

1.2.2 Equal

In this case, the equal symbol means that the set A is equal to the set B:

$$\boxed{A = B}$$

1.2.3 Belongs to

The symbols $\in$ and $\ni$ describe an element which is part of the set:

$$\boxed{a \in A \Longleftrightarrow A \ni a}$$

1.2.4 Does not belong to

The symbols $\notin$ mean that an element does not belong to the set:

$$\boxed{f \notin A}$$

1.2.5 Inclusion and contains

The symbols $\subset$ and $\supset$ mean that a set has another set included in its set:

$$\boxed{\mathbb{N} \subset \mathbb{Z} \Longleftrightarrow \mathbb{Z} \supset \mathbb{N}}$$

1.2.6 For all/any

The symbol $\forall$ means that we are considering any type of element:

$$\boxed{\forall x \in \mathbb{R},\ x > 0}$$

In this case, we've defined a new set.

1.2.7 Implication

The symbol $\Rightarrow$ means that by setting a rule, we imply an event or an action:

$$\boxed{\text{if } x = 1 \Longrightarrow x \in \mathbb{N}, \text{ but if } x \in \mathbb{N} \text{ we do not know if } x = 1}$$

With the implication, it is <u>sufficient</u> to claim action "A" in order to claim action "B".

1.2.8 Inference

The symbol $\Leftarrow$ means that by having an event or an action, we have a rule.

$$\boxed{x \in \mathbb{R}^+ \Longleftarrow x > 0}$$

With the inference, it is <u>necessary</u> to have claim the action "A" in order to have claim the action "B"

1.2.9 If and only if

The symbol $\Leftrightarrow$ means that two events happen simultaneously (double implication):

$$\boxed{x \in \mathbb{N},\ x \neq 0 \Longleftrightarrow x \in \mathbb{N}^*}$$

<u>Example</u>:

$x = 2 \Longleftrightarrow x^2 - 4x + 4 = 0$

<u>Proof</u>:

$x = 2 \Rightarrow (x-2) = 0 \Rightarrow (x-2)^2 = 0 \cdot (x-2) \Rightarrow x^2 - 4x + 4 = 0$

This happens **because** $x = 2$

1.3 Numerical sets

- $\mathbb{N} :=$ Natural numbers (including 0);
- $\mathbb{Z} :=$ Integer numbers;
- $\mathbb{Q} :=$ Rational numbers;
- $\mathbb{R} :=$ Real numbers $:= \mathbb{Q} \cup \{\text{irrational numbers}\}$.

<u>Notation</u>: The "$*$" symbol means that the set does not include 0.

1.3.1 Inclusion of sets

$$\boxed{\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}}$$

$B := \{\pi, 1, -1, 0\}$;
$C := \{\pi, 1\}$;
$D := \{\pi\}$.

Then we write some examples: $\pi \in B,\ D \subset B,\ C \subset B,\ B \not\subset C,\ 0 \in B,\ 0 \notin C$.

2 Intervals

Intervals describe what happens between two or more elements.

2.1 Examples

2.1.1 Interval sets

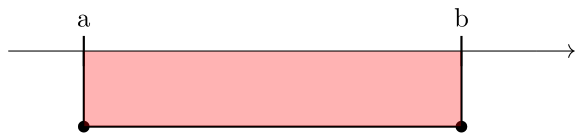
We have 4 cases:

- $(a, b) = \{\forall x \in \mathbb{R} \mid a < x < b\}$;
- $[a, b) = \{\forall x \in \mathbb{R} \mid a \leq x < b\}$;
- $(a, b] = \{\forall x \in \mathbb{R} \mid a < x \leq b\}$;
- $[a, b] = \{\forall x \in \mathbb{R} \mid a \leq x \leq b\}$.

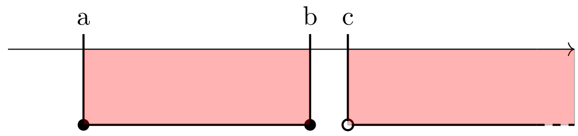
Notation: a and b are often called the "end points" of the interval;

2.1.2 Graphical examples

1) $\forall x \in \mathbb{R},\ x \in [a, b]$



2) $\forall x \in \mathbb{R},\ x \in\ [a, b]\ \cup\ ]c, +\infty[$



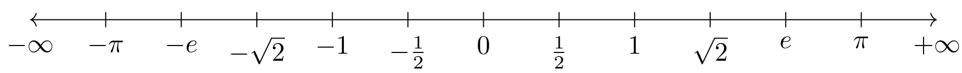
Notation: The union of two or more intervals where $x \in \mathbb{R}$ is denoted by the symbol $\cup$.

3 The extended line

In the real line $\mathbb{R}$ we add $\pm\infty$.

Real line: $(-\infty, +\infty) = \mathbb{R}$

Extended real line: $[-\infty, +\infty] = \overline{\mathbb{R}}$



Remark: $\pm\infty \notin \mathbb{R}$

3.1 Properties

$$\boxed{\forall x \in \mathbb{R} \ \mid\ \infty > x \ \mid\ -\infty < 0}$$

3.2 Operation in the extended line

If $a, b \in \mathbb{R}$, then $a + b,\ a - b,\ a \cdot b,\ \frac{a}{b}$ (with $b \neq 0$) stay the same

3.2.1 Additions

Let $\forall a \in \mathbb{R}$:

- $a + \infty := \infty;$
- $a - \infty := -\infty;$
- $+\infty + \infty := +\infty;$
- $-\infty - \infty := -\infty;$
- $+\infty - \infty :=$ undefined.

3.2.2 Moltiplications

Let $\forall a \in \mathbb{R}$:

- $+\infty \cdot +\infty := +\infty;$
- $-\infty \cdot +\infty := -\infty;$
- $-\infty \cdot (-\infty) := \infty;$
- $a \cdot \infty := \begin{cases} a > 0 & +\infty \\ a < 0 & -\infty \\ a = 0 & \text{undefined} \end{cases}$
- $a \cdot (-\infty) := \begin{cases} a > 0 & -\infty \\ a < 0 & +\infty \\ a = 0 & \text{undefined} \end{cases}$
- $\frac{a}{+\infty} = \frac{a}{-\infty} := 0;$
- $\frac{+\infty}{a} := \begin{cases} a > 0 & +\infty \\ a < 0 & -\infty \\ a = 0 & +\infty \end{cases}$
- $\frac{-\infty}{a} := \begin{cases} a > 0 & -\infty \\ a < 0 & +\infty \\ a = 0 & -\infty \end{cases}$
- $\frac{\infty}{\infty} :=$ undefined.