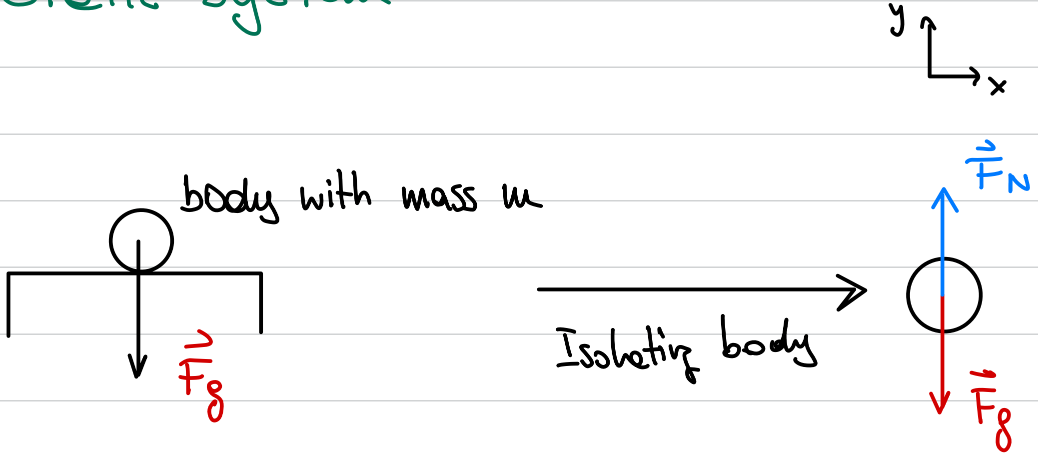
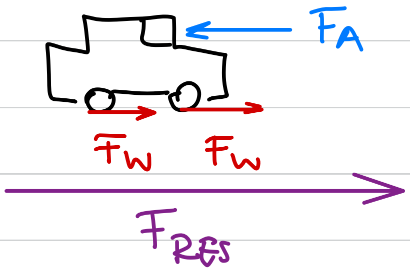
Static system



$$\Sigma F_y = F_N - F_g = 0$$

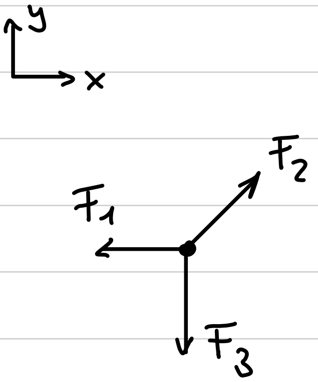
$$\Sigma F_x = 0$$

Dynamic system



$$a = \frac{F_{RES}}{m}$$

Force directions and resultants



$$\Sigma F_x = -F_1 + F_2 \cos(\alpha)$$

$$\Sigma F_y = -F_3 + F_2 \sin(\alpha)$$

Let's assume $\alpha = 45°$ and $F_2 = 100\ N$:

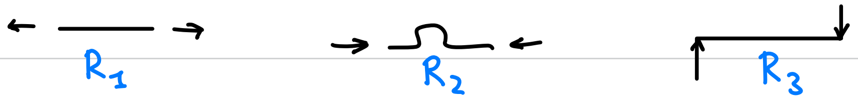
$$F_1 = F_2 \cos 45° = 70{,}7\ N$$

$$F_3 = F_2 \sin 45° = 70{,}7\ N$$

Ropes

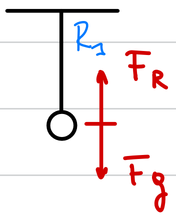
Ropes only can take tensile forces and NOT compressive forces

Pull ✓ Push ✗ Bend ✗



• Isolated ropes

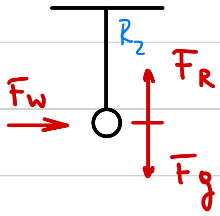
STATIC



$\Sigma F_y = 0$

$\Sigma F_x = 0$

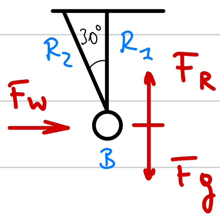
DYNAMIC (with wind)



$\Sigma F_y = 0$

$\Sigma F_x = F_w$ } For make it static, we have to add more ropes

STATIC from a D. state



let's assume: $F_w = 50\ N$

$m_B = 200\ kg$

$\Sigma F_x = F_w - F_{R_1} \cdot \cos(-30°)$

$\Sigma F_y = 0$

$\Sigma F_x = 50N - F_{R_2} \sin 30° = 0$

$\Sigma F_y = F_{R_1} - F_g + F_{R_2} \cos 30° =$

$F_{R_2} = 50N / \sin 30° = 100\ N$

$F_y = F_{R_1} - F_g + F_{R_2} \cos 30°$

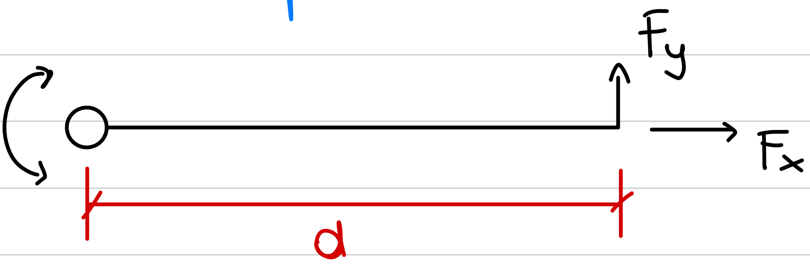
$F_g = mg = 200kg \cdot 9{,}81\ m/s^2 = 1962\ N$

$F_{R_1} = F_g - F_{R_2} \cos(30°) = 1962N - 86{,}6\ N = 1875{,}4\ N$

$F_y = 0$

Moments and couple

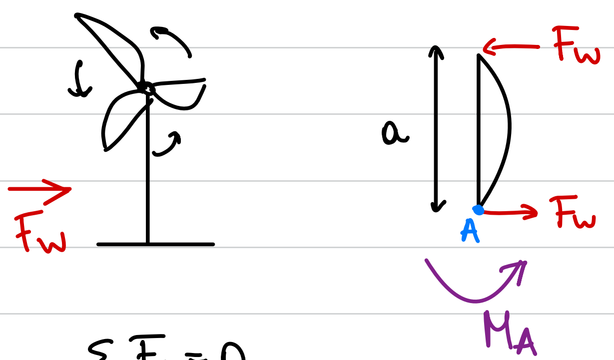
Couple:



A couple is created by a force applied at a distance

$$\boxed{\begin{aligned} M_z &= F_x d_x \\ M_z &= F_y d_y \end{aligned}}$$

$\Downarrow$ Example

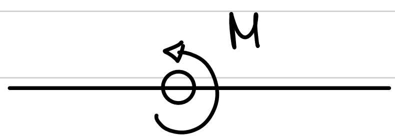


$\Sigma F_A = 0$

$M_A = F_w \cdot d \rightarrow M = \begin{cases} F_x \cdot d_y \\ F_y \cdot d_x \end{cases}$

$M [Nm]$

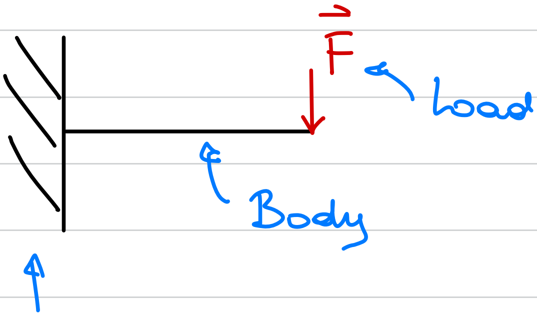
Moments:



A moment is created by an engine and acts at one single point

Free body diagram (FBD)

For each mechanical problem, drawing a FBD is needed!



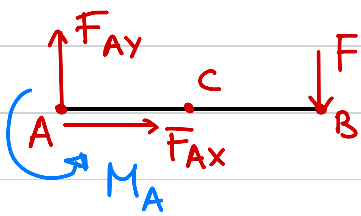
(1)

Boundary ← We need to replace the boundary condition by forces and moments

Boundary conditions can be created by:

- touching bodies
- hinges and fixations
- environmental forces (pressure, gravity)

In (1), we isolate the body:



$$\Sigma F_x = 0 = A_x$$

$$\Sigma F_y = 0 = -F + A_y$$

$$\Sigma M_A = -F \cdot dx + M_A$$

$$\Sigma M_B = M_A - A_y \cdot dx = 0$$

$$\Sigma M_C = M_A - F \cdot \tfrac{1}{2} dx - A_y \cdot \tfrac{1}{2} dx$$

$$M_A - \left(\frac{A_x \cdot dx - B_x \cdot dx}{2} \right)$$

Supports

Every blocked degree of freedom (DOF) needs to be replaced by a force or a moment

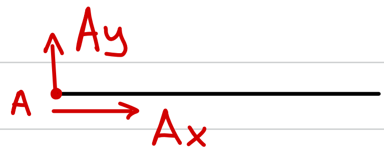
- Rotation blocked → M
- Translation blocked → F

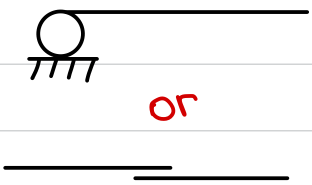
In 2Ds systems there are 3 DOF for each point:

$$\boxed{\begin{cases} 1)\ \text{Translation in } x \\ 2)\ \text{Translation in } y \\ 3)\ \text{Rotation around } z \end{cases}}$$

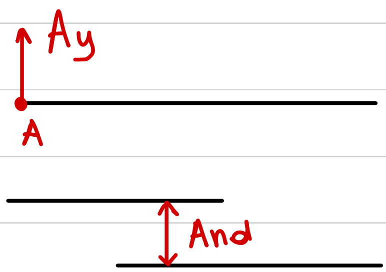
Types of supports

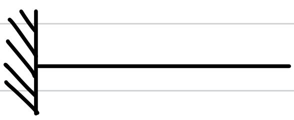
Hinges fix x, y and allow rotation



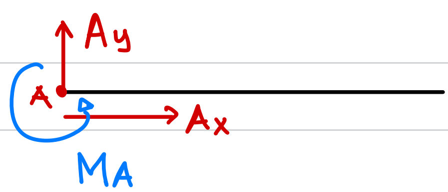


Rollers or two horizontal surfaces fix y and allow rotation and traslation in x

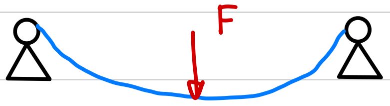




Wall fixtures / Fixed supports fix x, y and rotations

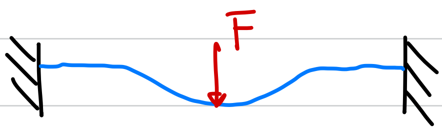


1)



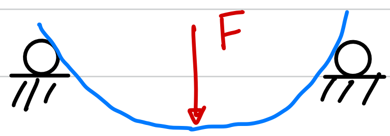
→ Used for shelves

2)



→ Used for shelves

3)

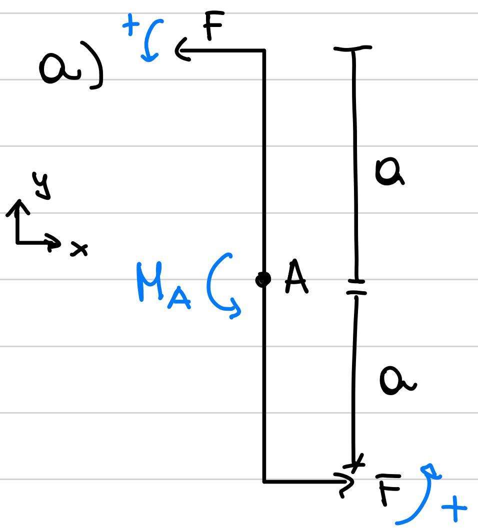


→ used for bending tests, balance boards

---

exercises:

a)

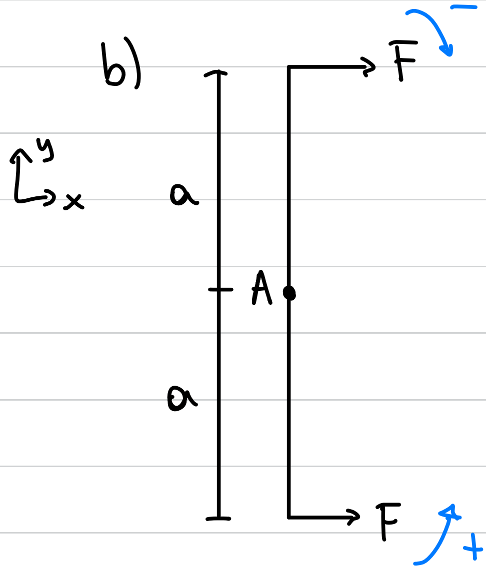


$$\Sigma F_x = F - F = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M_A = F \cdot a + F \cdot a = 2Fa$$

b)

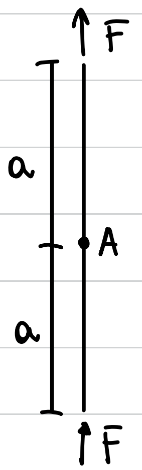


$$\Sigma F_x = 2F$$

$$\Sigma F_y = 0$$

$$\Sigma M_A = +Fa - Fa = 0$$

c)

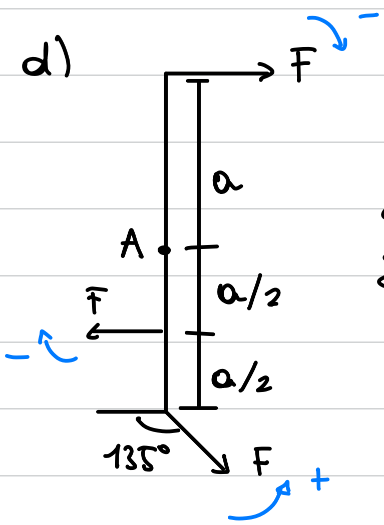


$$\Sigma F_y = F + F = 2F$$

$$\Sigma F_x = 0$$

$$\Sigma M_A = 0$$

d)



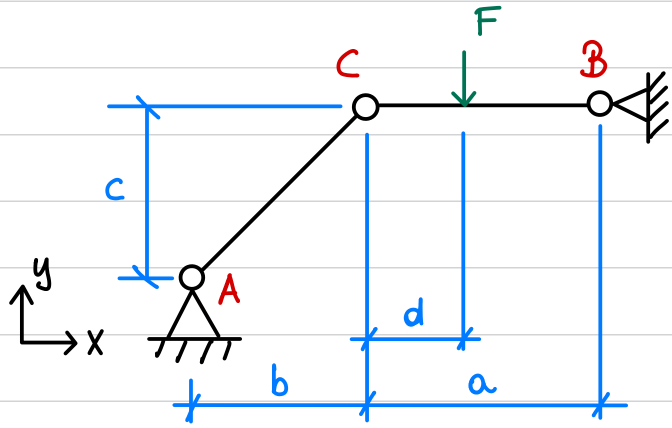
$$\Sigma F_x = \cancel{F} - \cancel{F} + F\cos 45$$

$$\Sigma F_y = -F\sin 45$$

$$\Sigma M_A = -Fa - F \cdot a/2 + F \cdot \cos 45 \cdot a$$

Multi - body systems

Two bodies can have several FBD's:



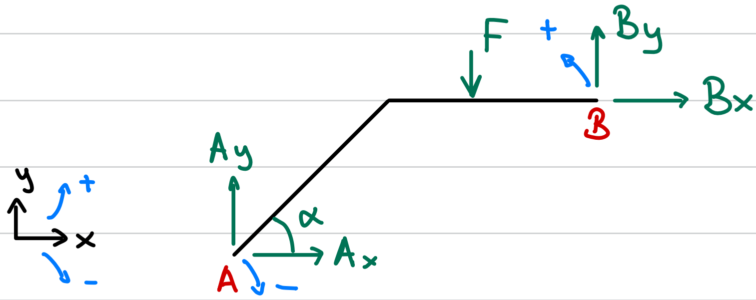
Example

Let:

$F = 2000\ N$, $a = 7\ m$,

$b = 2\ m$, $c = 6\ m$, $d = 3\ m$.

Step 1: Set up the FBD of the entire system:



Step 2: Equilibrium equations for $F\begin{pmatrix} A_x, B_x \\ A_y, B_y \end{pmatrix}$ seen from point B:

$$\Sigma F_x = F(A_x) + F(B_x) = 0$$

$$\Sigma F_y = F(A_y) + F(B_y) - F$$

where:

$F(A_x) = F_A \cdot \cos\alpha$

$F(A_y) = F_A \cdot \sin\alpha$

$$\Sigma M_B = F(A_x) \cdot c - F(A_y)(a+b) + F(a-d) = 0$$

Step 3: Magnitude / Direction:

$$\tan\alpha = \frac{c}{b}$$

Step 4: Final calculations:

$\Sigma M_B = 0$

$0 = F_A \cos\alpha \cdot 6 - F_A \sin\alpha\,(7+2) + 2000\,(7-3)$

$F_A = \underline{1204{,}7\ N}$

$F(A_x) = F_A \cos\alpha = \underline{381\ N}$

$F(A_y) = F_A \sin\alpha = \underline{1142{,}8\ N}$

$\Sigma F_x = 0$

$F(A_x) + F(B_x) = 0$

$F(B_x) = \underline{-381\ N}$

$\Sigma F_y = 0$

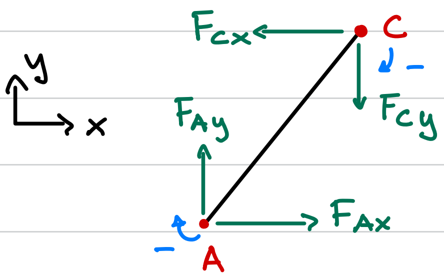
$F(A_y) + F(B_y) - F = 0$

$F(B_y) = F - F(A_y)$

$F(B_y) = 2000\ N - 1142{,}8\ N = \underline{857{,}1\ N}$

Step 5: Forces in the joint

Body 1:



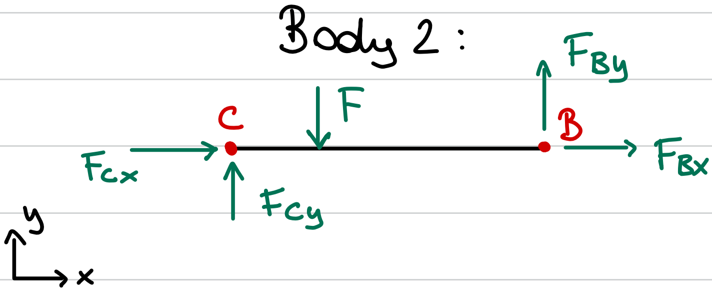
$\Sigma F_x = F_{Ax} - F_{Cx} = 0$

$F_{Cx} = F_{Ax} = \underline{381\ N}$

$\Sigma F_y = F_{Ay} - F_{Cy} = 0$

$F_{Cy} = F_{Ay} = \underline{1141{,}9\ N}$

Body 2:



$\Sigma F_x = F_{Bx} + F_{Cx} = 0$

$F_{Cx} = -F_{Bx} = \underline{381\ N}$

$\Sigma F_y = F_{By} + F_{Cy} - F$

$F_{Cy} = F - F_{By} = \underline{1141{,}9\ N}$

Constrains and Statical Determinancy

Statically determinate:

No. of eq. = No. of unknowns

Support forces = DOF

Statically indeterminate:

No. of eq. < No. of unknowns

support forces < DOF

Statically overdeterminated

No. of eq. > No. of unknowns

support forces > DOF

Examples:

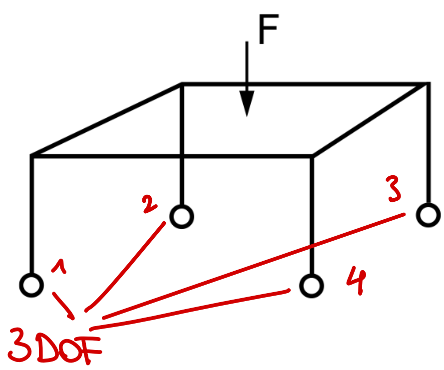
a) Table with 4 legs. All 4 legs on rollers on a flat floor.

3 DOF / leg

4 Legs

⇓

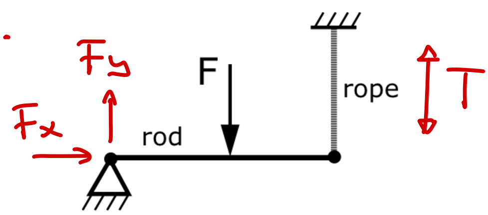
Statically indet.



b) A rod supported by a hinge and a rope

$F_x, F_y, T = 3$ unk.

2D → 3 DOF



3DOF = 3 unk

⇓

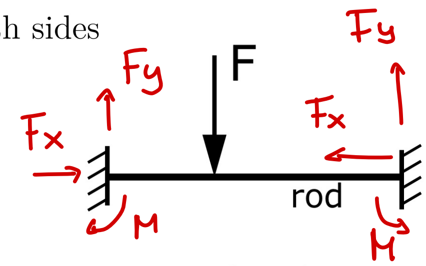
Statically det.

c) A rod fixed on both sides

6 unk, 3DOF

⇓

statically ind.

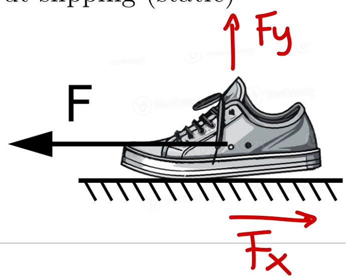


d) A shoe on the ground without slipping (static)

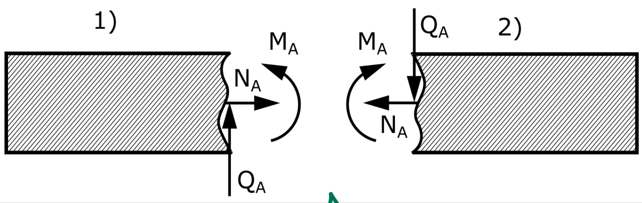
3DOF, 2 unk.

⇓

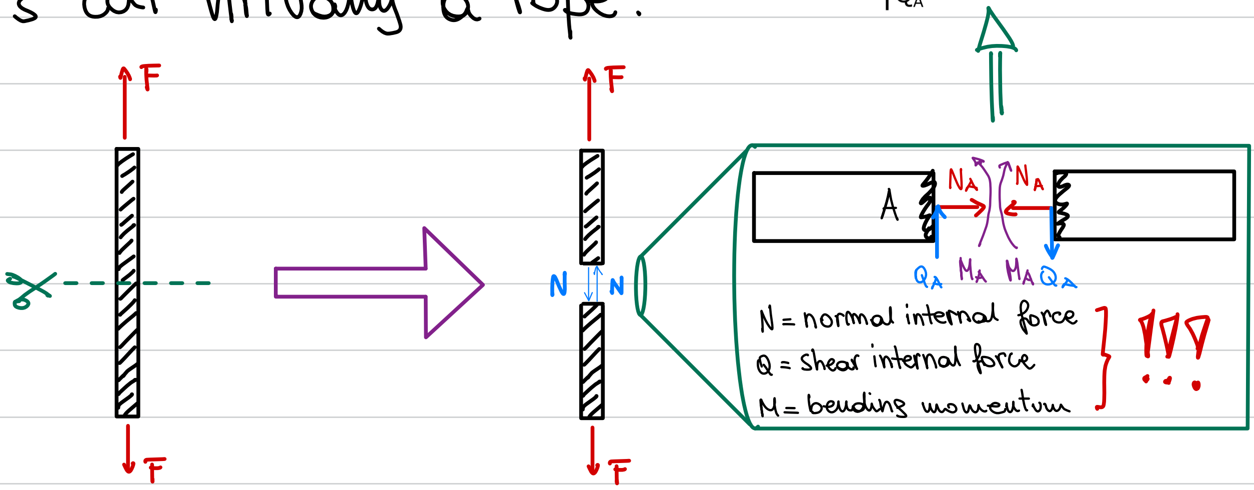
statically overdet



Internal forces



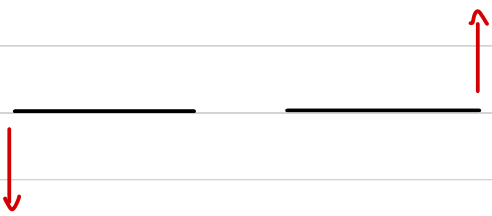
Let's cut virtually a rope:



Reactions with less unknowns than equations:

Missing N:

Missing Q:



Missing M:

What do we use for what?

1. System FBD and equilibrium: Determining external forces and support reactions
2. Body isolation in a multi-body system: Determining interface and reaction forces support reactions
3. Internal forces: Determining stress and evaluate the safety

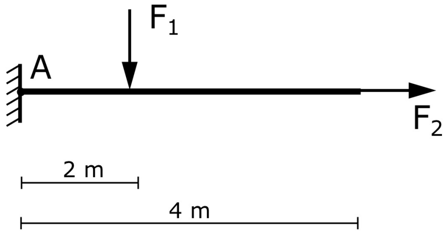
Shear / Moment / Tension diagram

Procedure:

Step 1: FBD diagram, calculate support reactions

Step 2: Calculate internal forces and moment

Example

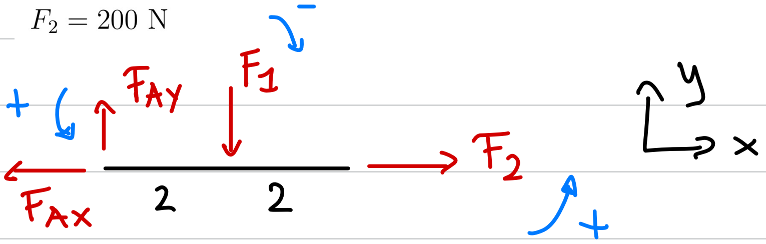


Given:

$F_1 = 100$ N

$F_2 = 200$ N

FBD :

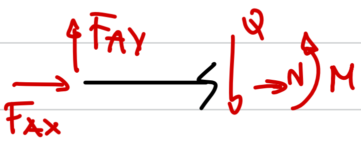


$$\Sigma F_x = -F_{AX} + F_2 = 0 \Rightarrow F_{AX} = F_2 = 200\,N$$

$$\Sigma F_y = F_{Ay} - F_1 = 0 \Rightarrow F_{Ay} = F_1 = 100\,N$$

$$\Sigma M_A = -F_1 \cdot 2m + F_2 \cdot 4m = 200\,N$$

Cut 1: $0m < x < 2m$

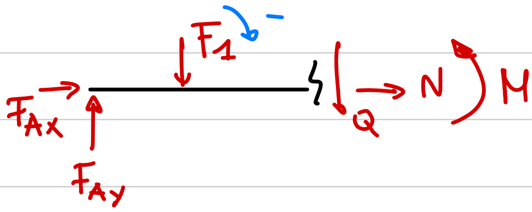


$$N_1 = F_{AX} = 200N$$

$$Q_1 = -F_{AY} = -100\,N$$

$$\Sigma M = M_x + Q_1 x$$

Cut 2: $2m < x < 4m$



$$N_2 = F_{AX} = 200N$$

$$Q_2 = -F_{Ay} + F_1 = 0N$$

$$\Sigma M = M_x - M_{F_1} + Q_2 x$$

Cut 3:

$$N_3 =$$

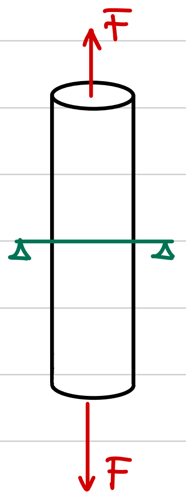
Stress and bending

Stress:

- It is needed to evaluate the safety
- It is calculated differently for each load case
  - Tensile (pure tensile load - stress)
  - Compressive (pure compressive load - stress)

  } calculated in the same way
  - Bending (tensile + compressive + shear stress)
  - Shear (pure shear stress)
  - Torsion (pure shear stress)

Tensile and compressive stress

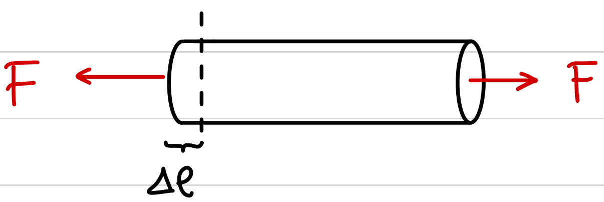


$$\boxed{\sigma_{Tensile} = \frac{F_{int}}{A} \quad \left[MPa = \frac{N}{mm^2}\right]}$$

Stress: internal loads incl. geometry

Strain

strain: internal shape changes



$$\boxed{\begin{aligned} \varepsilon_{Tensile} &= \frac{\Delta l}{l_0} \quad [-] \\ \varepsilon_{Compressive} &= \frac{\Delta l}{l_0} \quad [-] \\ \gamma_{Shear} &= \frac{\Delta S}{\Delta h} \quad [-] \end{aligned}}$$

$$\boxed{\sigma = E \cdot \varepsilon}$$

E = young's modulus [MPa] or [GPA]

Some young's modulus

$E_{steel} = 210'000 \text{ MPa} = 210 \text{ GPa}$

$E_{Aluminium} = 68'000 \text{ MPa} = 68 \text{ GPa}$

$E_{PA\ (Polymer)} = 2'100 \text{ MPa} = 2{,}1 \text{ GPa}$

Safety calculation

$$\left.\begin{aligned} \sigma_{int} &< \sigma_{max,\ admissible} \\ \varepsilon_{int} &< \varepsilon_{max,\ admissible} \end{aligned}\right\} \text{material data} + \text{safety factor}$$