Electrical Engineering
HSLU, Semester 2

Matteo Frongillo

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Part I

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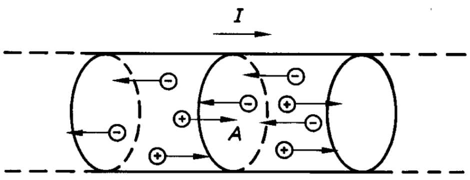
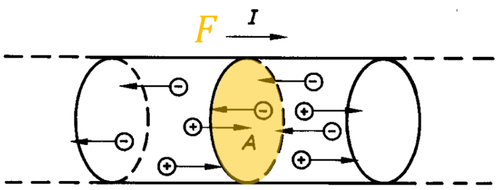
1.1 Current strength or current “I”



$$I\ [A] = \frac{\text{el. charge}}{t}$$

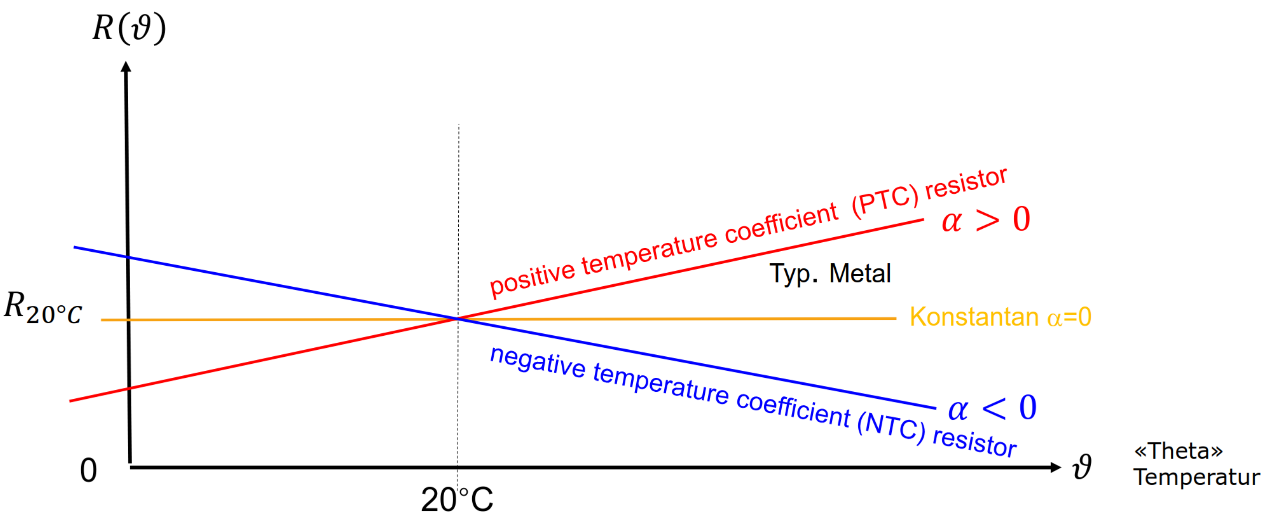
1.2 Current density “J”



The current density indicates how large the current per cross-sectional area (F) is:

$$J\ [\frac{A}{mm^2}] = \frac{I}{F}$$

1.3 Temperature dependence of the resistance



Depending on the material, the resistance can increase, remain the same or decrease with temperature. In ET+L we calculate using the linear approach.

$$\boxed{R(\vartheta) = R_{20}(1 + \alpha(\vartheta - 20°\text{C})) = R_{20}(1 + \alpha\Delta T)}$$

1.4 Object properties

The resistance indicates the voltage required for a current. In addition to the material, the cross-sectional area and also the length are decisive factors.

$$R = \frac{U}{I}$$

1.5 Reciprocal quantities

1.5.1 Specific resistance

To describe material properties, the resistance per length and cross-sectional area is specified (precondition: homogeneous conductor, direct current):

$$\rho\, [\frac{\Omega \cdot mm^2}{m}] = R \cdot \frac{A}{l}$$

1.5.2 Conductance

1.5.3 Specific conductivity

2 Gravitational fields

2.1 Between bodies

$$\boxed{F_1 = F_2 = G\frac{m_1 m_2}{d^2}}$$

2.2 Between particles

2.2.1 Coulomb's law

It calculates the amount of force between two electrically charged particles at rest:

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}}$$

where:

- F: Force [N];
- q: Charge [As];
- ε_0: absolute permittivity $= 8.8542 \cdot 10^{-12}$ [As/Vm].

2.3 Electric field and force on a charge Q

2.3.1 Homogeneous electric fields

$$\boxed{E = \frac{U}{d}}$$

where:

- E: electric field strength [V/m];
- U: voltage [V];
- d: distance of the electrodes [m].

2.3.2 Force on a point charge

$$\boxed{F = Q \cdot E}$$

where:

- E: electric field strength [V/m];
- Q: charge [As];
- F: force [N].

3 Capacitance and Capacitor

3.1 Capacitor

A capacitor is a device in which the capacitance is used.

3.2 Capacitance

Capacitance C is the **capability** to store electric charge. It is measured by the charge divided by the applied voltage:

$$\boxed{C = \frac{Q}{U}}$$

where:

- Q: charge [As];
- U: voltage [V];
- C: capacitance [As/V = F (Farad)].

3.2.1 Capacitance of a plate capacitor

$$\boxed{C = \varepsilon \cdot \frac{A}{d}}$$

where:

- A: plate area (one side) [m^2];
- d: distance between plates [m];
- C: capacitance [F].

Permittivity

$$\boxed{\varepsilon = \varepsilon_r \cdot \varepsilon_0}$$

- ε_r: relative permittivity of the dielectric, relative to the air;
- ε_0: absolute permittivity [As/Vm].

3.2.2 Energy in a capacitor

If a capacitor is discharged with a constant current, the voltage decreases linearly:

$$\boxed{\int_0^{t_{\text{empty}}} U(t) \cdot I \, dt = I \cdot U_0 = \frac{I \cdot U_0 \cdot t_{\text{empty}}}{2}}$$

Or, simplified:

$$\boxed{W = \frac{1}{2} C \cdot U_0^2}$$

where:

- W: energy [J or Ws];
- U_0: initial voltage [V];
- C: capacitance [F].

3.3 Capacitors in parallel connection

Capacitances connected in parallel add up:

$$\boxed{C_{\text{tot}} = \frac{\sum_n Q_n}{U} = \sum_n C_n}$$

or

$$\boxed{C = \frac{\varepsilon \cdot (\sum_n A_n)}{d} = \sum_n C_n}$$

3.4 Capacitors in series connection

In a series connection, the reciprocal of the total capacitance is the sum of the reciprocals of the individual capacitances:

$$\boxed{\frac{1}{C_{\text{tot}}} = \sum_n \frac{1}{C_n}}$$

where:

- C_{tot}: total capacitance [F];
- C_n: capacitance of the n-th capacitor [F].

4 Transient Analysis in RC Circuits

4.1 Charging of a Capacitor

When a capacitor is charged through a resistor, the voltage across it increases exponentially:

$$\boxed{U_C(t) = U_0 \cdot \left(1 - e^{-t/(R \cdot C)}\right)}$$

with the time constant defined as:

$$\boxed{\tau = R \cdot C}$$

where:

- $U_C(t)$: voltage across the capacitor at time t [V];
- U_0: applied voltage [V];
- R: resistance [Ω];
- C: capacitance [F];
- τ: time constant [s].

4.2 Discharging of a Capacitor

When a charged capacitor discharges through a resistor, the voltage decays exponentially:

$$\boxed{U_C(t) = U_0 \cdot e^{-t/(R \cdot C)}}$$

and the discharging current is:

$$\boxed{I(t) = \frac{U_0}{R} \cdot e^{-\frac{t}{(R \cdot C)}}}$$

4.3 Transitional phase

$$\boxed{f(t) = A + \Delta \cdot \left(1 - e^{t/\tau}\right) = A + (B - A) \cdot (1 - e^{1/\tau})}$$

5 Additional Topics

5.1 Energy Stored in a Capacitor

The energy stored in a capacitor is given by:

$$\boxed{W = \frac{1}{2} C \cdot U_0^2}$$

where:

- W: energy [J];
- C: capacitance [F];
- U_0: voltage [V].

5.2 Charge–Voltage Relationship

For an ideal capacitor, the relationship between charge and voltage is:

$$\boxed{Q = C \cdot U}$$

Moreover, the current is the time derivative of the charge:

$$\boxed{I = \frac{dQ}{dt} = C \cdot \frac{dU}{dt}}$$

Note that the voltage across an ideal capacitor cannot change instantaneously.