Maths refreshing course
HSLU, Semester 1

Matteo Frongillo

September 2, 2024

Contents

Part I

Lesson 1

1 Algebraic definitions

- $\mathbb{N}$:= Natural numbers (including 0)
- $\mathbb{Z}$:= Integer numbers
- $\mathbb{Q}$:= Rational numbers
- $\mathbb{R}$:= Real numbers

Notation: The "*" symbol means that the set does not include 0.

We have that:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

2 Prime numbers

A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

$$\boxed{n \in \mathbb{N},\ n \neq \{0, 1\}}$$

3 Positive powers

Let $a \in \mathbb{R}, n \in \mathbb{R}^*$ and $a \subset \mathbb{R}$, then

$$\boxed{a^1 := a \quad | \quad a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}}$$

3.1 Property 1

Let $a, b \in \mathbb{R},\ n, m \in \mathbb{N}$, then

$$\boxed{a^n \cdot a^m = a^{n+m}}$$

3.2 Property 2

Let $a, b \in \mathbb{R},\ n \in \mathbb{N}$, then

$$\boxed{(a \cdot b)^n = a^n \cdot b^n}$$

Notation: The power a^n, a is the base and n is the exponent.

3.3 Property 3

Let $a \in \mathbb{R},\ m, n \in \mathbb{N}^*$, then

$$\boxed{(a^n)^m = a^{n \cdot m}, \text{ which is} \neq a^{(n^m)}}$$

4 Fractions

Notation 1: $a \cdot b = a \times b = ab \quad | \quad \frac{a}{b} = a \div b = a : b$

Notation 2: "a" is called numerator, "b" is called denominator.

Notation 3: $\frac{a}{b},\ a, b \in \mathbb{R},\ b \neq 0$

4.1 Property 1

Let $a, b \in \mathbb{R}^*$ and $c, d \in \mathbb{R}$, then

$$\boxed{\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}}$$

4.2 Property 2

Let $a, b \in \mathbb{R}^*$ and $c, d \in \mathbb{R}$, then

$$\boxed{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}}$$

4.3 Property 3

Let $a, b \in \mathbb{R}^*$ and $c, d \in \mathbb{R}$, then

$$\boxed{\frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot d \pm c \cdot b}{b \cdot d}}$$

5 Negative powers

5.1 Definition

$$\boxed{\forall a \in \mathbb{R}^*; \quad a^{-1} := \frac{1}{a}}$$

5.2 Property 4

Let $\forall n \in \mathbb{N},\ \forall a \in \mathbb{R}$, then

$$\boxed{a^{-n} = \left(\frac{1}{a}\right)^n}$$

This property implies that $\forall z \in \mathbb{Z},\ \forall a \in \mathbb{R},\ z \neq 0$
We can compute a^z

5.3 Property 5

Let $\forall a \in \mathbb{R},\ a \neq 0,\ \forall n, m \in \mathbb{Z}$, then

$$\boxed{\frac{a^n}{a^m} = a^{n-m}}$$

Consequences:

1. Properties 1, 2 and 3 also hold for integer exponents:
   - $\forall a \in \mathbb{R},\ \forall n, m \in \mathbb{Z} \Rightarrow a^n \cdot a^m = a^{n+m}$
   - $\forall b \in \mathbb{R},\ (a \cdot b)^n = a^n \cdot b^n$
   - $(a^n)^m = a^{n \cdot m}$
2. $\forall a \in \mathbb{R}^*,\ a^0 = a^{1-1} = \frac{a^1}{a^1} = 1 \Rightarrow a^0 = 1$

6 Fractions and percentages (and back)

$\alpha \in \mathbb{R},\ n\%$ of $\alpha \iff \frac{n}{100} \cdot \alpha$

Part II

Lesson 2