SW 1: Introduction

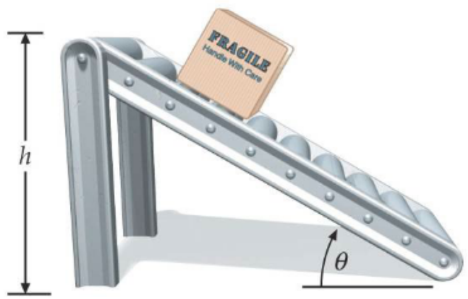
Model's three properties

Mapping: models act as a representation of natural or artificial originals and can be models in turn;

Reduction: models function as abstraction. They do not capture every attribute of the original; instead, they isolate and retain only those attributes relevant to the specific objective, intentionally omitting detail to manage complexity and focus on the problem at hand;

Pragmatic: models function as utilitarian substitutes. They do not replace the original universally but serve as a representative for a specific user (subject), within a defined time frame, and for a particular purpose or operation.

Example



Generaliz.: point mass sliding down an inclined plane;

Mapping: box as mass, conveyor slope as an angle θ, vertical drop as height h, gravity;

Reduction: no structure flexibility, no air movement, no friction, no rollers $\rightarrow$ flat plane;

Pragmatic: it allows a, v_f, t of the box to be calculated, it enables the prediction of how to build a belt mockup.

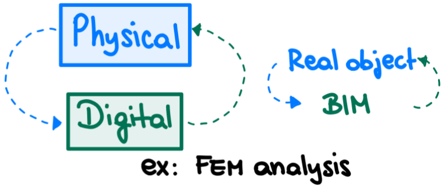
Digital representation

- - - - - - - ► Manual Data Flow (Offline)

——————► Automatic Data Flow (Real-time)

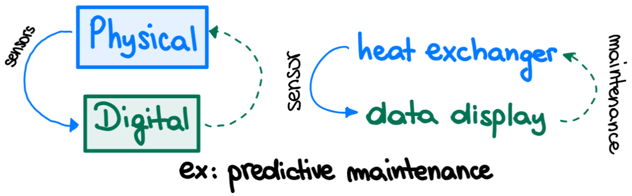
Digital model (simulation)

No direct connection between digital and physical object:



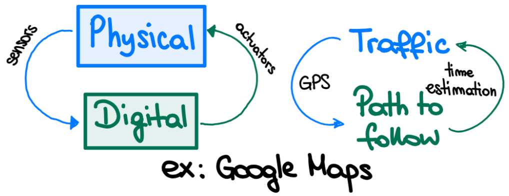
Digital shadow

Unidirectional, automated data flow from physical object to digital model:



Digital twin

Automated data exchange between physical object and model:



Role of time

Stationary behavior

Steady-state operation: $\dot{m}_\alpha = \dot{m}_\omega$

Dynamic behavior

Non stationary/transient/unsteady: $\frac{dm}{dt} = \dot{m}_\alpha - \dot{m}_\omega$

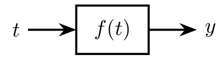
Governing dynamics

Empirical (black box)

Data based, without direct physics link.
(ex: machine learning, fitting of functions)

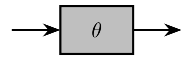
Physics-based (white box)

Based on physical laws.
(ex: conservation of mass)



Grey-box (hybrid)

Combining physics and data parameters.



Role of space

Point model (0D)

Assumes the whole system is perfectly mixed.
(ex: ideal mixer with isotropic distribution)
Software: Excel, MATLAB

Linked point

Connects several simple models together to create a basic network or layout.
(ex: space shown via linking of 0D-models)
Software: Simulink, Modelica

Spatial model (1-3D)

Considers real position of state variables or entities; spatial relationships affect the dynamics.
(ex: real mixer with anisotropic, heterogeneous distribution)
Software: COMSOL, ANSYS, AutoCAD, REVIT

Example with a heat pump

- Purpose: digital shadow $\rightarrow$ automated data;
- Governing dynamics: physics-based $\rightarrow$ based on thermodyn. laws;
- Time: time dependent, dynamic behavior $\rightarrow$ heating load, power of the hp, on/off cycles;
- Space: linked point $\rightarrow$ el. inputs, thermal energy exchange, 4 components to monitor.

Solvability of models

as example, $A = \int_0^2 x^2 \, dx$

Analytical

Closed formula as solution. Only for simple problems.

$$A = \frac{x^3}{3}\Big|_0^2 = \frac{8}{3}$$

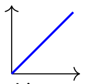
Numerical

Numerical approximation. For complex problems.

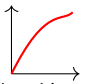
$$A \approx \sum_{i=1}^{n} f(x_i)dx \approx 2.6667$$

Further modelling properties

Linear vs Non-linear

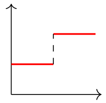


Linear

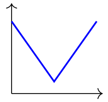


Non-Linear

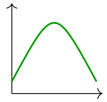
Continuity vs Differentiability



Non-Cont

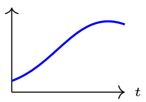


Cont/Non-Diff

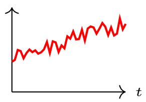


Differentiable

Deterministic vs Stochastic



Deterministic



Stochastic

Modelling approaches

Top-down

Largest components broken down into smaller.
ex: marble block sculpture, railway network.
(+) Efficient model, (−) Misses details

Bottom-up

Individual components combined into larger.
ex: LEGO model, human body.
(+) Detailed model, (−) Complex

SW2: How to model a system

1. Problem formulation
2. Mathematical representation
3. Mathematical analysis
4. Interpretation and evaluation of results

Problem formulation

Task 1 - Defining goals

What do we want to archieve?
How well/closely does our model need to represent reality?
What could be the goals for this specific system?

Task 2 - Characterize the system

What are the relevant parameters and variables of the system?
What are the system boundaries?
What are the inputs and outputs of the system?

Task 3 - Simplify and idealize the system

Still reproduce the significant behaviors of the system, while reducing complexity.
Reduce model to the main parameters and variables (ex. for hp: COP? Max. power? Avg power? Yearly values? Temperature levels?).

Mathematical formulation

Task 1 - Identify fundamental theories and laws

If no laws are available, use ad-hoc or empirical data to derive relationships:
Thermodynamic laws, material properties, ad-hoc
$P_{out} = COP(T_{amb}) \cdot P_{in}$

Task 2 - Derivation of relationships

Derivation of relationships between the