Linear Algebra
HSLU, Semester 4

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Last update: February 19, 2026

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1 Vectors

1.1 Linear combination

A sum of scalings of vectors is called a linear combination of the vectors.

Let $\vec{u}$, $\vec{v}$ be vectors, and a, b be scalars, $a, b \in \mathbb{R}$, then:

$$\boxed{a \cdot \vec{u} + b \cdot \vec{v}}$$

Generalizing this to a set of vectors $\vec{u}_1, \ldots, \vec{u}_n$, and scalars $a_1, \ldots, a_n$, we have:

$$\boxed{\sum_{i=1}^{n} a_i \cdot \vec{u}_i}$$

1.2 Cross product (vector product)

The cross product of two vectors $\vec{u}$ and $\vec{v}$ is a vector $\vec{w}$ that is perpendicular to both $\vec{u}$ and $\vec{v}$, and has a magnitude equal to the area of the parallelogram formed by $\vec{u}$ and $\vec{v}$.

Let $\vec{u} = (x, y, z)$ and $\vec{v} = (t, s, q)$

$$\boxed{\vec{u} \times \vec{v} = (yq - sz,\ -(xq - tz),\ xs - yt)}$$

1.3 Unit vectors

Components of the unit vectors of the vector basis are as follow:

$$\boxed{\vec{i} = \vec{e}_x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \vec{j} = \vec{e}_y = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \vec{k} = \vec{e}_z = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

with their norms being $\|\vec{i}\| = \|\vec{j}\| = \|\vec{k}\| = 1$