

Energies, fluids & processes – Laboratory

HSLU, Semester 2

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Last update: April 15, 2025

Contents

1	Introduction to energies, fluids, and processes	3
1.1	Energy forms	3
2	Fluids as energy carriers	3
2.1	Fluid definition	3
2.1.1	Properties of a fluid	3
2.2	Real and ideal fluids	4
2.2.1	Real fluid	4
2.2.2	Ideal fluid	4
2.3	Technical application flows	5
2.3.1	Internal flow (flow through)	5
2.3.2	External flow (flow around)	5
2.4	Forces for fluid motion	5
2.4.1	1D flow in x direction	5
2.5	Laminar and turbulent flow	6
2.5.1	Reynolds number	6
2.5.2	Critical Reynolds number	6
2.5.3	Flow pressure in curvatures	7
2.6	Compressible and incompressible flow	7
2.6.1	Compressible flow	7
2.6.2	Incompressible flow	7
3	Mass conservation	9
4	Energy conservation	10
4.1	Bernoulli equations	10
4.1.1	Energy conservation	10
4.1.2	10
4.2	1st law of thermodynamics: General energy conservation equation	11
4.3	Examples of Bernoulli equation	11
4.3.1	Horizontal streamtube	11
4.3.2	Flow out of a tank	11
4.4	Hydrostatic equation	11
5	Energy grade line diagram	12
6	Pipe flows	13
6.1	Horizontal pipe flow	13
6.2	Laminar pipe flow	13
6.2.1	Velocity profile	13
6.2.2	Pressure loss	13
6.3	Influence of surface roughness on pipe flows	14
6.3.1	$\delta > k$	14
6.3.2	$\delta \ll k$	14
6.3.3	$\delta \approx k$	14

6.4 Moody diagram	15
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1 Introduction to energies, fluids, and processes

Energy exists in different forms and can neither be destroyed nor generated, but only transformed.

1.1 Energy forms

- Potential energy: $E = mgh$
- Kinetic energy: $E = \frac{1}{2}mv^2$
- Thermal energy: $E = mc_p\Delta T$
- Light energy: $E = h\nu$
- Chemical energy: $E = mH$
- Electrical energy: $E = k \frac{q_1 q_2}{r}$
- Nuclear energy: $E = \Delta mc^2$
- Pressure energy (acoustic): $E = \frac{mp}{\rho}$

2 Fluids as energy carriers

2.1 Fluid definition

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2.1.1 Properties of a fluid

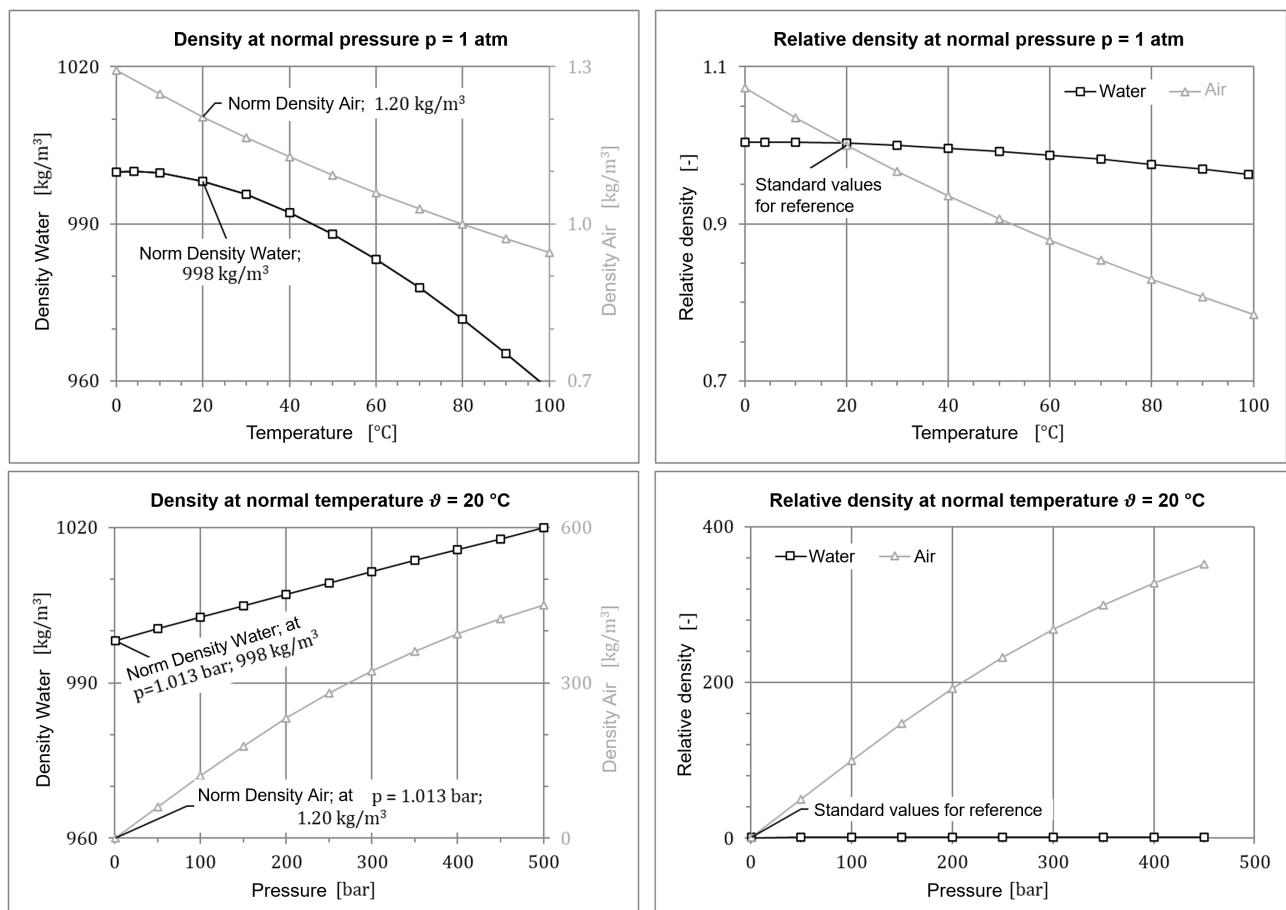
Density ρ

Densitiy is a measure of working potential of a fluid:

$$\rho \triangleq \frac{m}{V} \left[\frac{\text{kg}}{\text{m}^3} \right]$$

where:

- m = mass;
- V = volume.



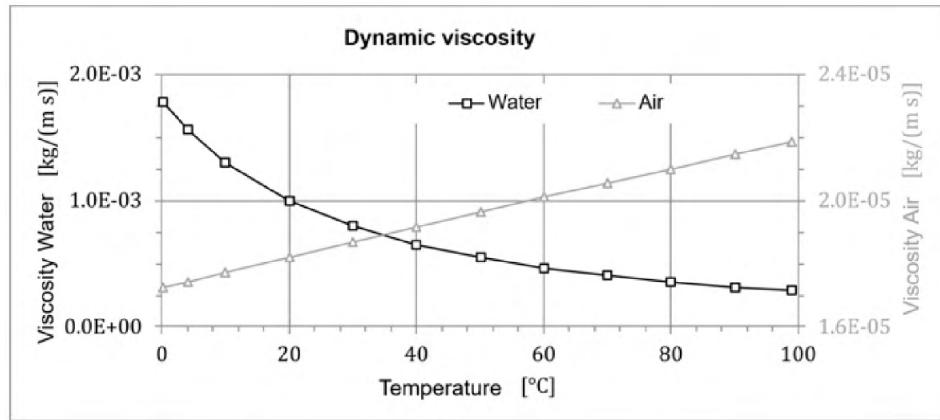
Kinematic viscosity ν

Viscosity is a measure of the specific loss capacity of a fluid:

$$\nu \triangleq \frac{\mu}{\rho} \left[\frac{N \cdot s}{m^2} = Pa \cdot s \right]$$

where:

- μ = dynamic viscosity
- ρ = density



Viscosity of a liquid fluid **decreases** with increasing temperature, while viscosity of a gaseous fluid **increases** with increasing temperature.

Remark: $\nu \propto \frac{1}{T}$

Compressibility

An increase in pressure on a given fluid mass causes compression and thus lead to a reduction in volume.

Mach number is a non-dimensional number that relates the fluid velocity to the sound velocity (in air):

$$M = \frac{u}{c}$$

Note: Since Mach number normally is very small, it can be neglected from calculations.

2.2 Real and ideal fluids

2.2.1 Real fluid

All fluids are real fluids and have real fluid properties. This means that they are compressible and exhibit frictional losses during the flow process. Physically, this means they have a viscosity $\nu > 0$.

2.2.2 Ideal fluid

A fluid can be simplified as an ideal fluid assuming a constant density (incompressible) and a viscosity $\nu = 0$ (frictionless).

2.3 Technical application flows

2.3.1 Internal flow (flow through)

Fluids that flow through a body (pipes, ducts, machines, ...).

Internal losses (such as friction, pressure, and fluid force) are relevant for the calculation of internal flows.

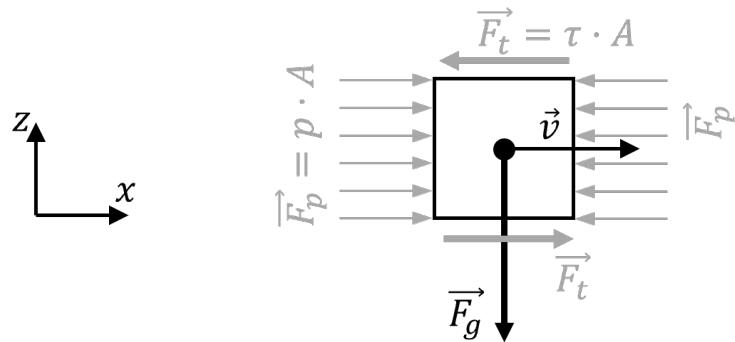
2.3.2 External flow (flow around)

Fluids that flow around bodies (motor vehicles, aircraft, buildings, ...).

External losses (such as velocity, pressure, density, and temperature near and far from bodies) are relevant for the calculation of external flows and aerodynamics.

2.4 Forces for fluid motion

2.4.1 1D flow in x direction



Surface forces act on the interfaces of a fluid body and are introduced by direct contact of the environment. Fluids also cause surface forces on their surroundings.

Forces decomposition

Surface forces:

- $F_t = \tau \cdot A$: shear force (tangential to the surface);
- $F_p = p \cdot A$: fluid pressure force.

Body forces:

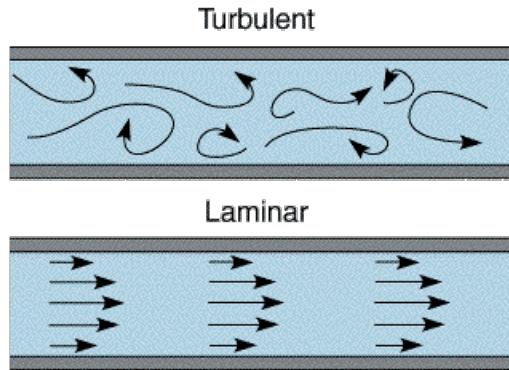
- $F_g = F \cdot g \cdot \cos \theta$: gravitational force (perpendicular to the surface);
- $F_n = -F \cdot g \cdot \cos \theta$: normal force (perpendicular to the surface);
- F_v : inertial force.

Inertial forces will always destabilize the flow field.

Viscous forces will always stabilize the flow field.

2.5 Laminar and turbulent flow

A flow that flows in an orderly manner is called laminar flow. In contrast, flows with vortices are called turbulent flow.



2.5.1 Reynolds number

Reynolds number is a non-dimensional number that makes the distinction between laminar and turbulent flows possible. The Reynolds number is given by the relation between inertial forces and viscous forces:

$$Re = \frac{v \cdot L}{\nu}$$

where:

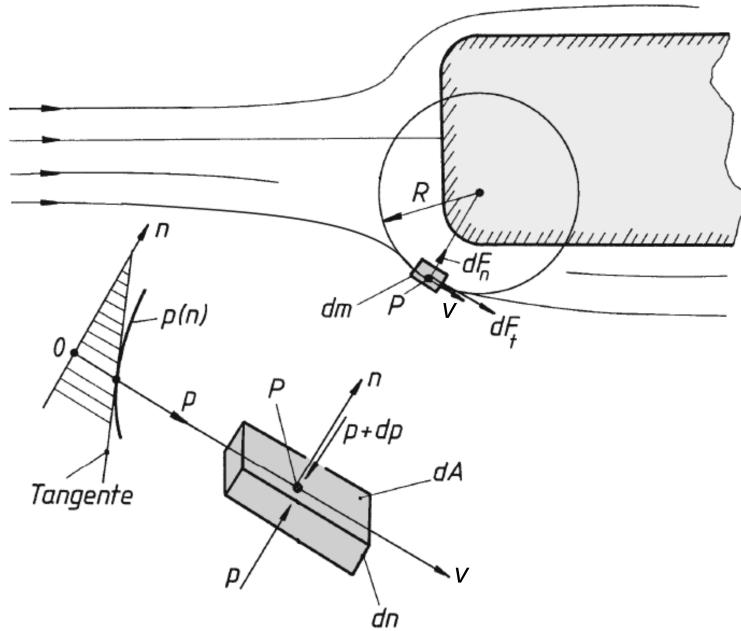
- v : velocity $\left[\frac{\text{m}}{\text{s}}\right]$;
- L : characteristic length [m];
- ν : kinematic viscosity $\left[\frac{\text{m}^2}{\text{s}}\right]$.

2.5.2 Critical Reynolds number

The transition from laminar to turbulent flow and it's determined by the critical Reynolds number:

$Re > 2300 \Rightarrow$ turbulent flow
$Re = 2300 \Rightarrow$ critical point
$Re < 2300 \Rightarrow$ laminar flow

2.5.3 Flow pressure in curvatures



Force balance of the system:

$$dF_n = -dA((p + dp) - p) = dm \cdot a_n$$

where:

- R : radius of the curvature
- $a_n = \frac{v^2}{R}$
- $dm = g \cdot dA \cdot dn$

Pressure in the curvature formulation:

$$\frac{dp}{dn} = -g \cdot \frac{v^2}{R}$$

2.6 Compressible and incompressible flow

2.6.1 Compressible flow

In compressible flows, the density of the fluid changes so much that the density change cannot be neglected.

2.6.2 Incompressible flow

Fluid flows can be considered incompressible at sufficiently low velocities. For ideal gases, the speed of sound can be calculated from the state variables and the fluid properties to:

$$c = \sqrt{\kappa \cdot R_i \cdot T}$$

If the Mach number is below 0.3, the gas flow can be considered incompressible.

$$Ma = \frac{v}{c} = \frac{v}{\sqrt{\kappa \cdot R_i \cdot T}}$$

where:

- v : fluid velocity $\left[\frac{m}{s}\right]$;
- c : speed of sound $\left[\frac{m}{s}\right]$;

- κ : isentropic exponent [−];
- R_i : individual gas constant $\left[\frac{J}{kg \cdot K} \right]$;
- T : temperature [K].

3 Mass conservation

4 Energy conservation

$$\frac{dE}{dt} = \underbrace{\sum P + \sum \dot{Q}}_{\substack{\text{Energy flow} \\ \text{across system boundary}}} + \underbrace{\sum_{in} \left[\dot{m}^{\leftarrow} \cdot \left(h^{\leftarrow} + \frac{v^{2\leftarrow}}{2} + g \cdot z^{\leftarrow} \right) \right]}_{\substack{\text{Energy} \\ \text{transfer mass}}} - \underbrace{\sum_{out} \left[\dot{m}^{\rightarrow} \cdot \left(h^{\rightarrow} + \frac{v^{2\rightarrow}}{2} + g \cdot z^{\rightarrow} \right) \right]}_{\substack{\text{Energy transfer} \\ \text{escaping mass}}}$$

where:

- E : total energy of the system;
- P : power;
- \dot{Q} : heat flow;
- \dot{m} : mass flow entering/leaving the system;
- h : enthalpy of the entering/leaving mass flow;
- v : velocity of the entering/leaving mass flow;
- z : height of the entering/leaving mass flow.

4.1 Bernoulli equations

4.1.1 Energy conservation

$$0 = \sum_{in} \left[\dot{m}^{\leftarrow} \cdot \left(h^{\leftarrow} + \frac{v^{2\leftarrow}}{2} + g \cdot z^{\leftarrow} \right) \right] - \sum_{out} \left[\dot{m}^{\rightarrow} \cdot \left(h^{\rightarrow} + \frac{v^{2\rightarrow}}{2} + g \cdot z^{\rightarrow} \right) \right]$$

4.1.2 ...

4.2 1st law of thermodynamics: General energy conservation equation

In the case of stationary flow:

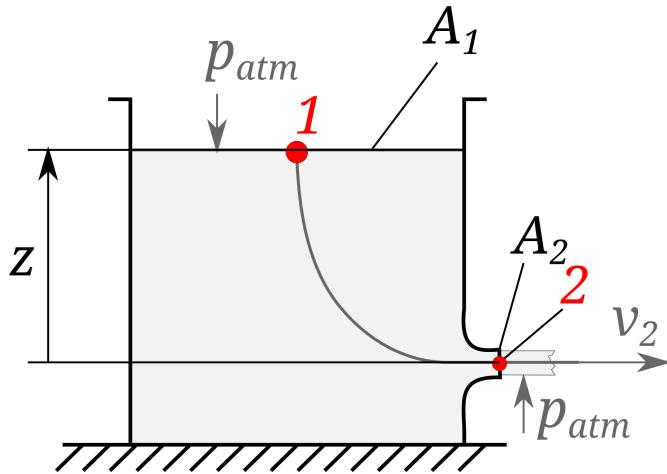
4.3 Examples of Bernoulli equation

4.3.1 Horizontal streamtube

Since the pipe is horizontal, $z_1 = z_2$, and since point (2) is a stagnation point, we have $\frac{v_2^2}{2} = 0$, then the specific form of Bernoulli can be simplified to:

$$p_1 + g \cdot \frac{v_1^2}{2} = p_2$$

4.3.2 Flow out of a tank



Setting the reference point at the top of the tank, we have $z_1 = 0$ and $z_2 = z$, which brings the velocity at the top of the tank to be zero, so $\frac{v_1^2}{2} = 0$. Since $p_1 = p_2 = p_{atm}$, the specific form of Bernoulli can be simplified to:

$$g \cdot z = \frac{v_2^2}{2} \implies v_2 = \sqrt{2gz}$$

The difference between the pressure p_2 at the exit of the tank and the atmospheric pressure p_{atm} changes because of the contraction number α :

$$\alpha = \frac{A_2^*}{A_2} \dots$$

4.4 Hydrostatic equation

The hydrostatic equation is a special case of the Bernoulli equation, where the velocity is zero:

$$p_2 = p_1 + \rho g z_1$$

5 Energy grade line diagram

6 Pipe flows

6.1 Horizontal pipe flow

For the velocity profile of a horizontal pipe flow, the mean velocity and the height of the pipe are constant, hence losses are only due to friction:

$$e_v = \frac{\Delta p}{\rho} = \frac{p_2 - p_1}{\rho} = \zeta \frac{v^2}{2}$$

6.2 Laminar pipe flow

6.2.1 Velocity profile

For the velocity profile of a laminar pipe flow, the mean velocity v_m is exactly half of the maximum velocity v_{max} at the center of the pipe axis ($r = 0$): where:

$$v(r) = \frac{p_1 - p_2}{3\eta \cdot l} \cdot (R^2 - r^2)$$

- R : radius of the pipe;
- r : distance from the center of the pipe;
- η : dynamic viscosity;
- l : length of the pipe.

The pressure loss of a laminar pipe is described by the Hagen-Poiseuille equation, which is a function that can be calculated setting the center of the pipe as the reference point:

$$\begin{aligned} v_{max} &= \frac{p_1 - p_2}{4\eta \cdot l} \cdot (R^2 - 0) \\ v_m &= \frac{v_{max}}{2} = \frac{p_1 - p_2}{8\eta \cdot l} \cdot R^2 \\ v_m &= \frac{p_1 - p_2}{32\eta \cdot l} \cdot d^2 \end{aligned}$$

$$\Delta p = 32v_m \cdot \eta \cdot \frac{l}{d^2}$$

6.2.2 Pressure loss

Flow losses in pipeline systems consist of pressure losses in straight pipes, curved pipes, and in fittings:

$$\Delta p = \lambda \cdot \frac{l}{d} \cdot \rho \cdot \frac{v_m^2}{2}$$

The resistance coefficient λ also incorporates the characteristics of the flow. If the flow is **laminar**, surface roughness effects plays no role, as the strong influence of viscous forces in the fluid smooths out these effects:

$$\begin{aligned} \lambda \cdot \frac{l}{d} \cdot \rho \cdot \frac{v_m^2}{2} &= 32v_m \cdot \eta \cdot \frac{l}{d^2} \\ \lambda &= \frac{64 \cdot \eta}{v_m \cdot d \cdot \rho} = \frac{64\nu}{v_m \cdot d} \end{aligned}$$

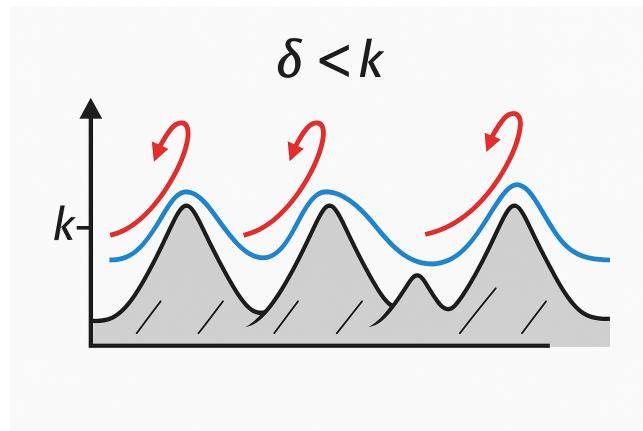
$$\lambda = \frac{64}{Re}$$

6.3 Influence of surface roughness on pipe flows

6.3.1 $\delta > k$

picture 1

6.3.2 $\delta \ll k$

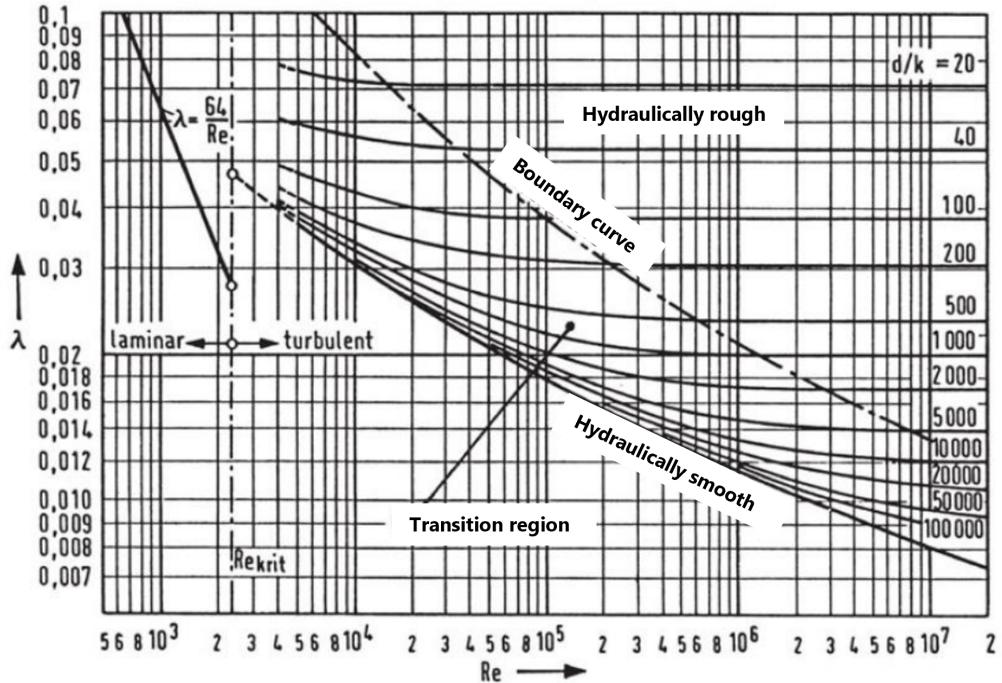


6.3.3 $\delta \approx k$

picture 3

6.4 Moody diagram

Moody diagram is a graph that shows the relationship between the Reynolds number and the friction factor λ for different types of flow (laminar, transitional, and turbulent).



- Laminar part: $\Delta p_v \sim v$ and constant line of $\frac{64}{Re}$;
- Transitional part: $\Delta p_v \sim v^x$ (x is between 1 and 2) and $\delta \approx k$;
- Turbulent part: $\Delta p_v \sim v^2$ and $\delta \gg k$.