

Maths refreshing course

HSLU, Semester 1

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September 2, 2024

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Part I

Lesson 1

1 Algebraic definitions

- $\mathbb{N} :=$ Natural numbers
- $\mathbb{Z} :=$ Integer numbers
- $\mathbb{Q} :=$ Rational numbers
- $\mathbb{R} :=$ Real numbers

We have that:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

2 Prime numbers

A prime number is a natural number which can be divided only by itself or 1.

$$n \in \mathbb{N}, n \neq \{0, 1\}$$

3 Positive powers

Let $a \in \mathbb{R}, n \in \mathbb{R}^*$ and $a \in \mathbb{R}$

$$3^1 := 3$$

$$3^2 := 3 \cdot 3$$

$$3^{23} := 3 \cdot 3 \cdot \dots \cdot 3 \text{ (23 times)}$$

3.1 Property 1

Let $a, b \in \mathbb{R}, n, m \in \mathbb{N}$, then

$$a^n \cdot a^m = a^{n+m}$$

3.2 Property 2

Let $a, b \in \mathbb{R}, n \in \mathbb{N}$, then

$$(a \cdot b)^n = a^n \cdot b^n$$

Notation: The power a^n , a is the base and n is the exponent.

3.3 Property 3

Let $a \in \mathbb{R}, m, n \in \mathbb{N}^*$, then

$$(a^n)^m = a^{n \cdot m}, \text{ which is } \neq a^{(n^m)}$$

4 Fractions

Notation 1: $a \cdot b = a \times b = ab$; $\frac{a}{b} = a \div b = a : b$

Notation 2: a is called numerator, b is called denominator.

Notation 3: $\frac{a}{b}$, $a, b \in \mathbb{R}$, $b \neq 0$

4.1 Property 1

Let $a, b, c, d \in \mathbb{R}$, $a, b \neq 0$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

4.2 Property 2

Let $a, b, c, d \in \mathbb{R}$, $a, b \neq 0$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

4.3 Property 3

Let $a, b, c, d \in \mathbb{R}$, $a, b \neq 0$

$$\frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot d \pm c \cdot b}{b \cdot d}$$

5 Negative powers

5.1 Definition

$$\forall a \in \mathbb{R}, a \neq 0; \quad a^{-1} := \frac{1}{a}$$

5.2 Property 4

Let $\forall n \in \mathbb{N}$, $\forall a \in \mathbb{R}$

$$a^{-n} = \left(\frac{1}{a}\right)^n$$

This property implies that $\forall z \in \mathbb{Z}$, $\forall a \in \mathbb{R}$, $a \neq 0$
We can compute a^z

5.3 Property 5

Let $\forall a \in \mathbb{R}$, $a \neq 0$, $\forall n, m \in \mathbb{Z}$, then

$$\frac{a^n}{a^m} = a^{n-m}$$

Consequences:

1. Properties 1, 2 and 3 also hold for integral exponents:

$$\forall a \in \mathbb{R}, \forall n, m \in \mathbb{Z} \Rightarrow a^n \cdot a^m = a^{n+m}$$

$$\forall b \in \mathbb{R}, (a \cdot b)^n = a^n \cdot b^n$$

$$(a^n)^m = a^{n \cdot m}$$

2. $\forall a \in \mathbb{R}^*, a^0 = a^{1-1} = \frac{a^1}{a^1} = 1 \Rightarrow a^0 = 1$

6 Fractions and percentages (and back)

$$\alpha \in \mathbb{R}, n\% \text{ of } \alpha \iff \frac{n}{100} \cdot \alpha$$

Part II

Lesson 2