DEMAND FOR INSURANCE

Outline

1. Contingent goods and Expected utility
2. Risk aversion
3. Nature & Origin of insurance activity
4. Model of demand for insurance
5. Property and casualty (P&C) insurance in the real world
6. Demand of insurance and behavioral economics
7. Generalization to *n* states of nature
8. Undifersifiable risks
9. Exercises
10. Appendix: Affine transformation

1. Contingent goods and Expected utility

Uncertainty – Contingent goods

Introduction to decisions under uncertainty

Probability distribution:

- Events and the probability that they happen

EXAMPLE : - EVENTS (states of nature) Probability

Rain tomorrow	π_1
No rain tomorrow	$1-\pi_1$

Space of events

- In the simple example above we assume that we know the events space, that is we can list all events that can happen and assign them a probability
- In reality this is not true
- Think of very important events that nobody had anticipated, let alone given a probability of happening: 9/11/2001, Covid-19.

Von-Neuman & Morgestern Expected utility

c_i = Consumption in state of nature i , i =1,…,n

$U(c_i)$ = Utility of consumer from c_i

i= 1,2

$$V(c_1, c_2, \pi_1, \pi_2) = \pi_1 U(c_1) + \pi_2 U(c_2)$$

$$\pi_1 + \pi_2 = 1$$

Expected utility

Unique up to affine transformation (see chapter on affine transf.): can multiply by positive constant and add constant; not unique to monotonic transformation like utility under certainty (e.g. cannot take lnU)

State-dependent utility; advanced topic 1/2

- In VNM expected utility the U is the same in all the states of nature.
- Decision maker cares only about distribution of monetary payoffs.
- Does not care about the causes of payoff.
- This may be a good assumption if the event is price of a security in portfolio

State-dependent … 2/2

- If the cause of the payoff is one's state of health this assumption is not good
- Hence consider also that decision maker cares not only about monetary returns (x_s) but also about states of nature that caused them
- Hence U depends on state s and becomes $U_s(x_s)$
- Expected utility representation becomes $\Sigma_s p_s U_s(x_s)$
- See MasColell et al. pp. 199 and ff. for assumptions about its existence.

Subjective probabilty theory; advanced topic (SPT)

Savage 1954, ½

- Up to now risk is regarded as an objective fact
- In many applications people have subjective estimates of risk
- SPT generalizes VNM

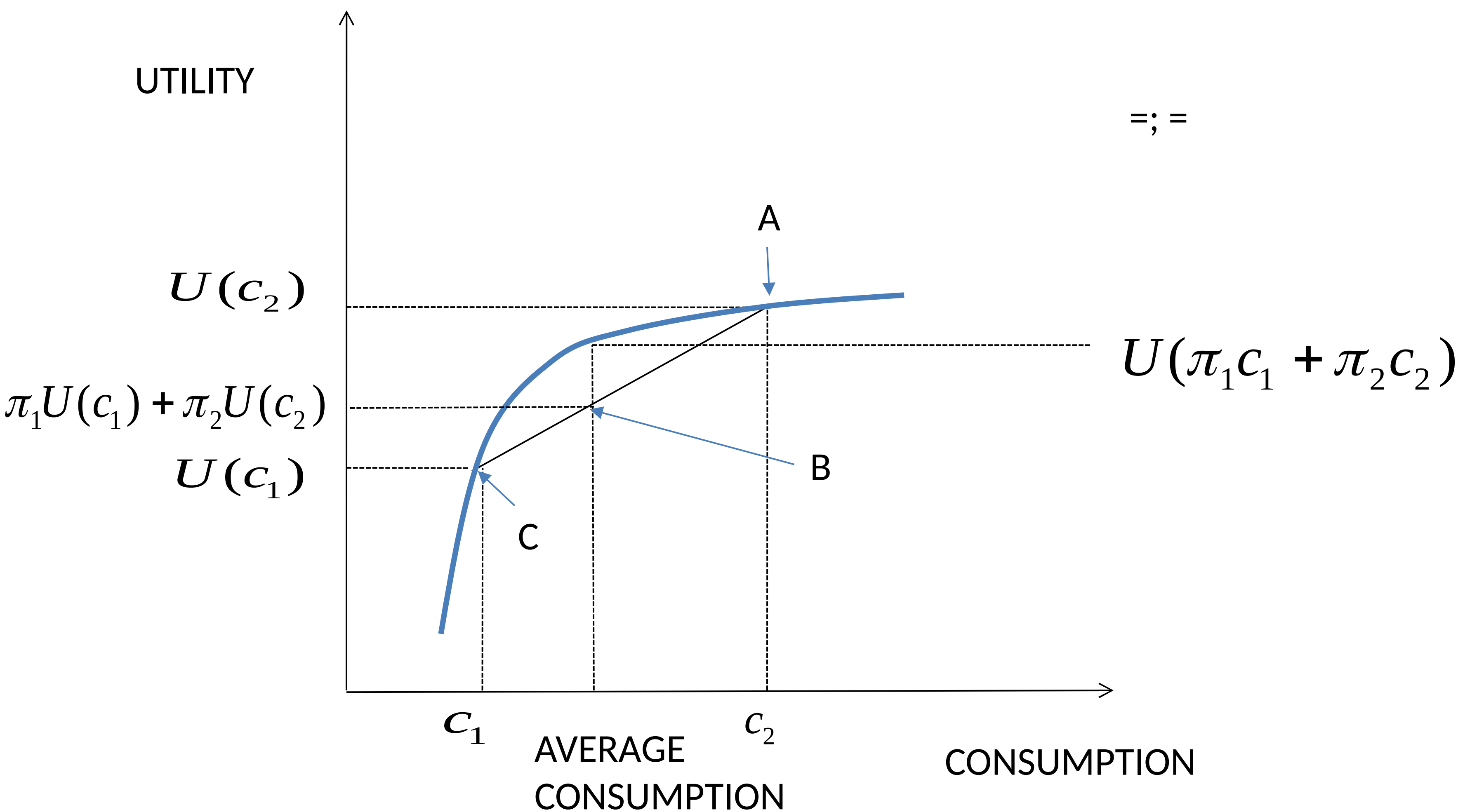
<u>SPT</u> 2/2

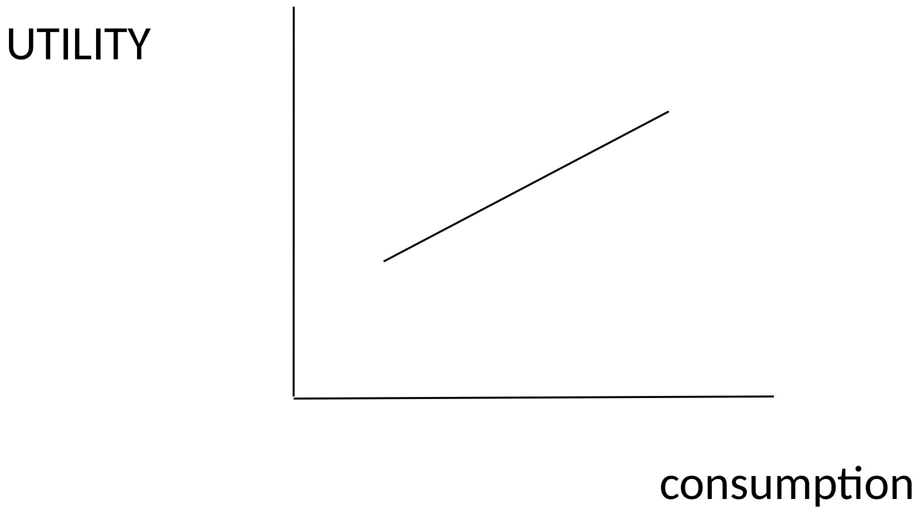
- SPT = even if states are not associated with objective probabilities, consistency-like restrictions on preferences still imply that decision maker behaves AS IF utilities are assigned to outcomes + prob. are assigned to states + decisions are made taking expected utilities
- See MasColell et al. pp. 205 and ff. for assumptions about its existence.

2. Risk aversion

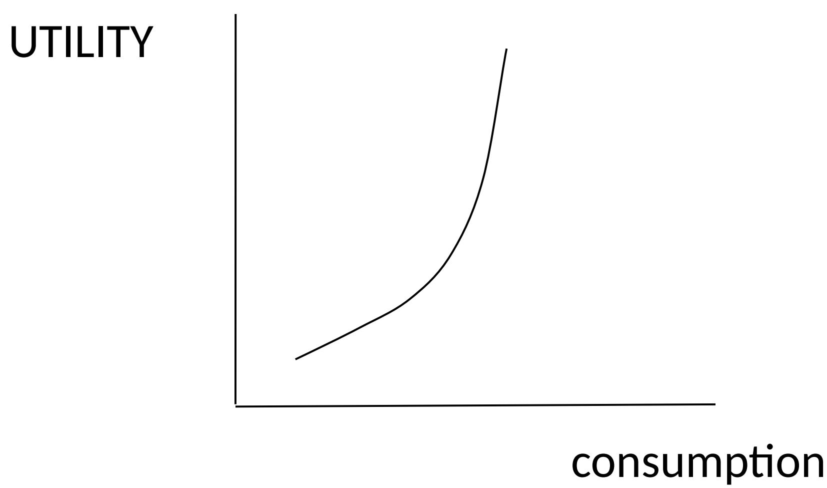
Risk aversion: U''<0

$$U(\pi_1 c_1 + \pi_2 c_2) > \pi_1 U(c_1) + \pi_2 U(c_2) = V(c_1, c_2, \pi_1, \pi_2)$$





Risk neutral; U’’=0



Risk lover; U’’>0

3. Nature and Origin of insurance

What is an insurance company?

- Organizations that take on many largely <u>independent</u> risks
- By doing so they are able to forecast with a «good» degree of accuracy probability that an event (e.g. accident) will occur over a <u>large number</u> of insured individuals and underwite these <u>risks</u>: take premia and liquidate damage
- Law of large numbers allows to <u>diversify</u> away these risks either directly or through re-insurance; means insurance company behaves as risk neutral

(Weak) Law of large numbers

X r.v. with $E(x) = \mu$

$$\left\{ x_1, \ldots\ldots, x_n \right\} \rightarrow \frac{1}{n} \sum_{i=1}^{n} x_i = \bar{x}_n$$

$\bar{x}_n$ = sample mean of a random sample with n elements.

$$\forall \varepsilon > 0, \forall 0 < \delta < 1 \qquad \exists\ n : \forall m \geq n$$

$$\Pr\left[\left| \bar{x}_m - \mu \right| < \varepsilon \right] \geq 1 - \delta$$

It means that I can make arbitrarily large the probability that the difference $\left| \bar{x}_m - \mu \right|$ is very small.

Risk ≠ uncertainty 1/2

- Distinction between risk and uncertainty goes back to Frank Knight in 1921
- Risk: situations where we do not know outcome of a given situation, but can accurately measure odds.
- This is what insurance companies do by exploiting law of large numbers (likelihood of airplane accident 1 in 20 million takeoffs, life expectancy tables, etc.)

Risk ≠ uncertainty 2/2

- Risk: Things we know that we don't know = Known unknowns ≠ uncertainty: things we don't know that we don't know = Unknown unknowns (black swam)
- Uncertainty: situations where we cannot know all information we need in order to have good idea of distribution (conditions of airline industry 30 yrs from now)
- Despite this distinction we will use risk and uncertainty interchangebly in this course

<u>Robust vs resilient</u> <u>(curiosity)</u>

source Brunnermeir, The Resilient Society, 2021

The Oak is Robust: capable to resist to shock of some strength; but it breaks down for bigger shocks and does not come back once it breaks down

The Reed is Resilient: bends with shocks (with wind) but does not break down and bounces back

Robust vs resilient

source Brunnermeir, The Resilient Society, 2021

- Robust
  - blocks most shocks (knowns/unknowns)
  - breaks after barrier is overcome, never bounces back
- Resilient
  - Suffers impact
  - Bounces back
  - Reacts to shock
  - Volatility Paradox: learning how to be resilient through small shocks (immune system in sterile vs not-sterile environments)

Risk avoidance vs resilience (curiosity)

source Brunnermeir, The Resilient Society, 2021

Risk avoidance

- Aims to min variance
- Static

Resilience

- Managing risk
- Dynamic
- Mean reversion

Dimensions of resilience

- Training, Human capital
- Redundancies (global value chains, etc.)
- Buffers (bank capital, insurance reserve)

How insurance companies were born? (1/2)

1. In ancient Greece and Rome insurance was provided by occupational guilds
2. More recently, workers' cooperatives (around religious organizations, trade unions) where members pay premia giving right to obtain indemnity in case of accident

How insurance … (2/2)

3. German kings forced their subjects to buy house fire insurance; fire is negative externality; if one wood house catches fire, nearby houses are at risk; hence Gov't intervention

4. Maritime activities

Insure ship + cargo against ship wreck

– Assicurazioni Generali (Trieste, 1831)

– LLOYD'S (London, 1771)

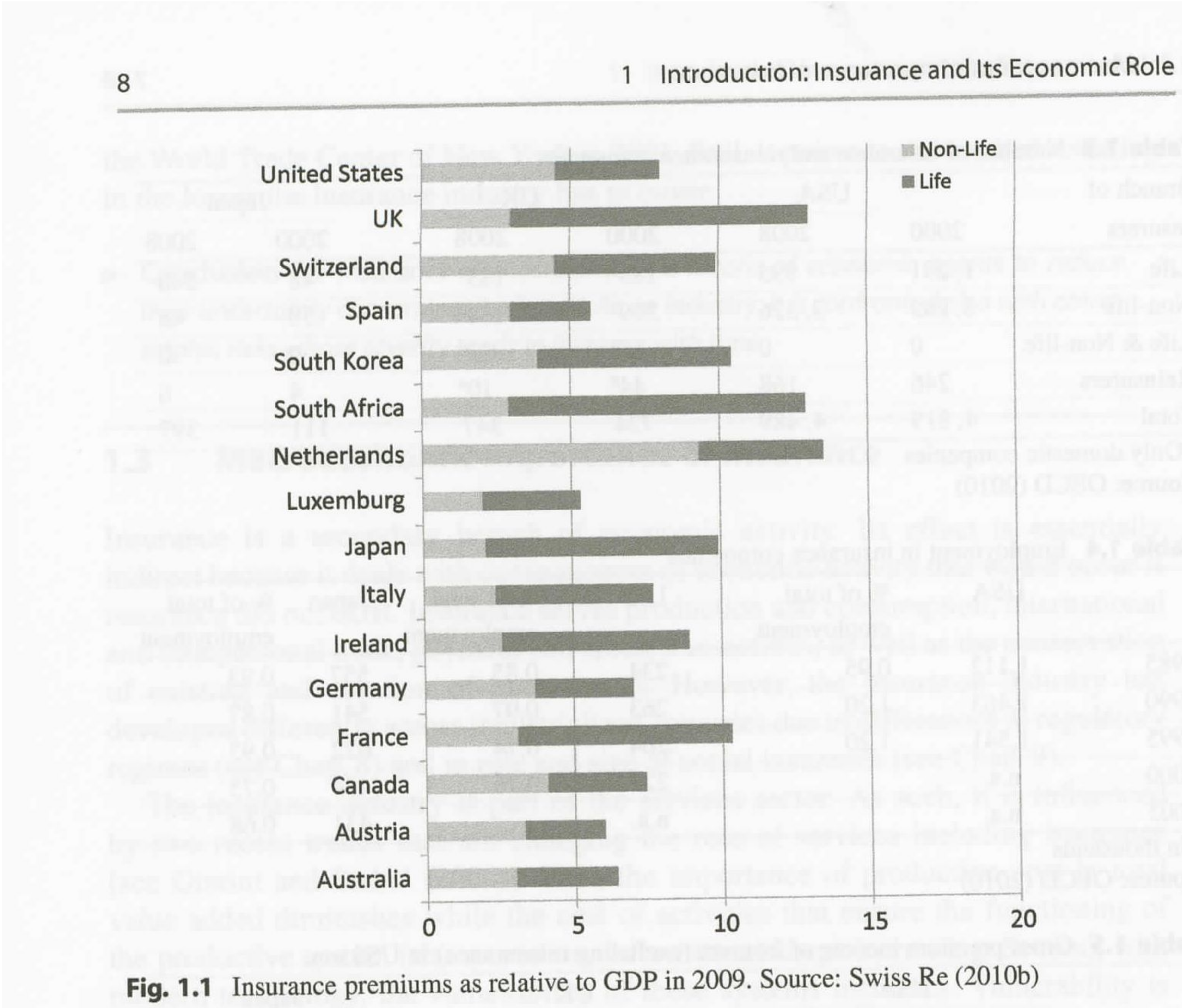
Lloyds of London

source Besanko and Breutingan, Microeconomics

- Group of individuals (Society of Lloyds) who did business at Lloyds coffee house agreed to commit their personal wealth to underwrite any losses incurred by group members and their customers
- Group that paid insurance premiums to society included shipowners, merchants, and building owners
- Basic principle of insurance: A group of people who have not sustained losses provides money to compensate other people who have sustained losses

How important is insurance? (1/3)

Source Zweifel and Eisen, Insurance Economics, Springer Verlag, 2012



Premium is a revenue concept.

Life/non-life insurance.

Non-life = property & casualty

How important … (2/3)

Source Zweifel and Eisen, Insurance Economics , Springer Verlag, 2012

Table 1.6 GDP share of insurance and banks, in percent

	UK		Germany		France	
	Insurance	Banks	Insurance	Banks	Insurance	Banks
2000	1.6	2.7	1.4	2.3	1.1	2.8
2007	1.8	3.4	1.5	2.1	1.3	2.4

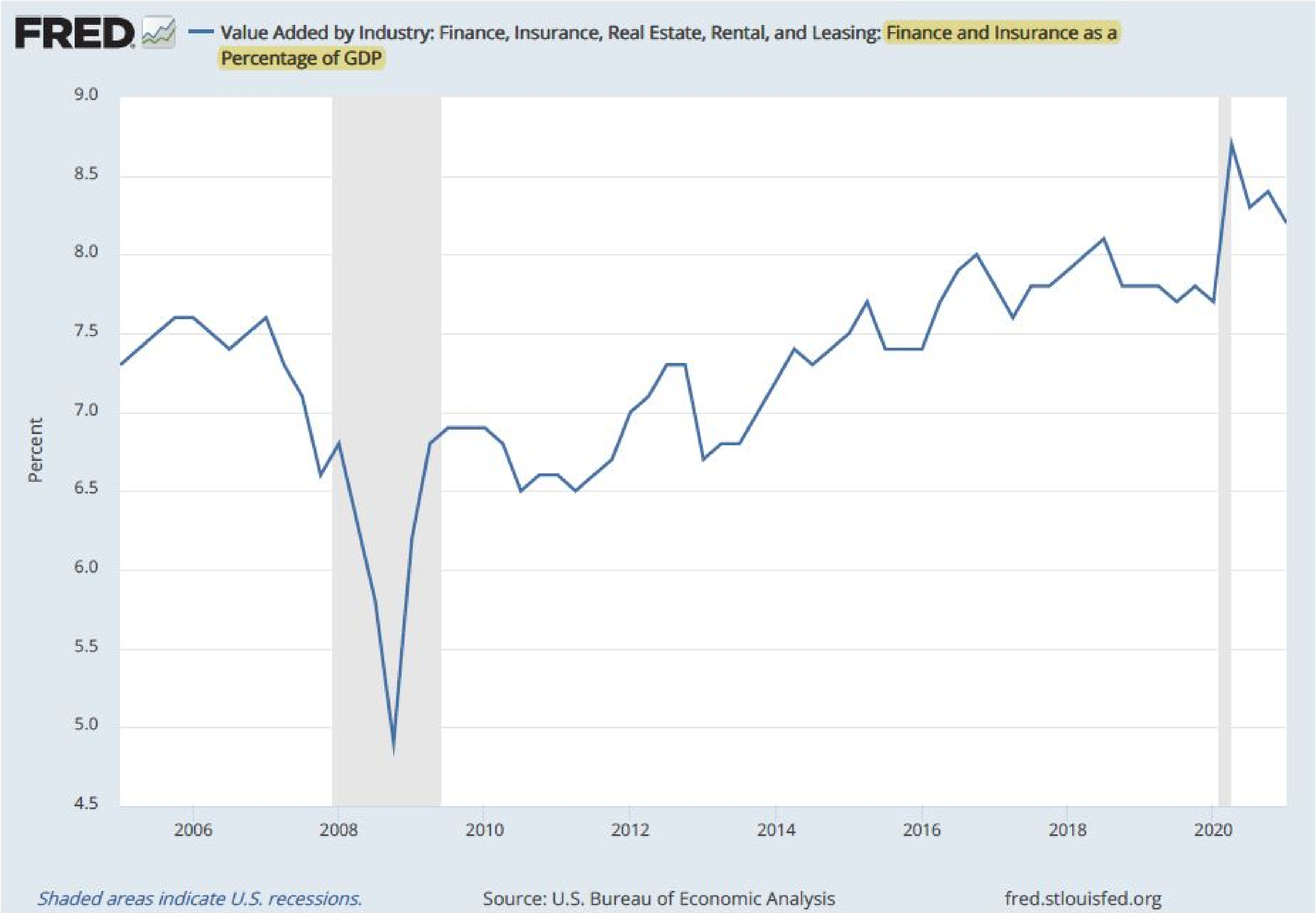
Source: Eurostat (2010)

GDP is a value added concept

In U.S. Finance and Insurance share of GDP was 8.2% Q1 2021,
Source https://fred.stlouisfed.org/series/VAPGDPFI.

How important … (3/3)

Source https://fred.stlouisfed.org/series/VAPGDPFI



Life vs non-life insurance

Non-life (property & casualty)

- Insurance payment is restoration after damage
  - Property damage: flood, fire, theft, etc.
  - Casualty: civil liability like in car insurance
- Short time span; typically 1 yr
- Damage can be repeated

Life

- Insurance payment is set (or can be set)
- Medium to long term period (some for residual life of insured)
- Event cannot be repeated

Life insurance

- Subject pays in advance 1 (or >1) premia to company that commits to pay 1 (or >1) sums if certain events linked to survival or death of individuals/s
- In the policy we have:
  - Insurance company
  - Subject that signs policy and pays premium/a
  - Insured: person/s to whom insured events refer
  - Beneficiary: subject to whom insured payments go

<u>a. Classical life insurance contracts</u>

- Are <u>death</u> protection instruments, provide cash on death or maturity
- <u>Term insurance</u>: pays lump sum if death of policyholder occurs <u>by end of specified term</u>.
- <u>Whole life insurance</u>: pays lump sum upon death of policyholder whenever it occurs
- <u>Endowment insurance</u>: pays lump sum either upon death of policyholder or at end of specified term, which ever comes first. If policyholder dies, sum is paid just as under term insurance; if policyholder survives, sum insured is treated as maturing investment

b. Life annuities

- Offer regular <u>series of payments</u> to recipient (annuitant)
- Life annuity = payments continue until death of the annuitant
- Term life annuity = payments for a fixed period
- Rationale: often purchased to provide retirement income

Types of insurance benefits

- Fixed: benefits/premia predetermined (change in a prespecifed way); risk borne by company
- Flexible: risk borne by insured
  - Unit linked: benefits linked to performance of fund
  - Index linked: benefits linked to performance of index

4. Model of demand for insurance. Non-Life

Demand for insurance: property

- W = wealth/consumption with no accident (event)
- D = amount of damage in case of accident; D<W
- $0 < \pi$ = probability of accident;
  - π assumed given here
  - With moral hazard π is endogenous, function of action of insured
  - With adverse selection there are many π; $\pi^L < \pi^H$

Demand… (cont.ed)

- $1 - \pi$ = probability that no accident happens
- k = quantity of insurance bought (indemnity paid in case of accident; liquidation); $k \geq 0$; choice variable
- $0 < \gamma$ = insurance premium per unit of insurance bought

Demand… (cont.ed)

- In reality it may be difficult to know exactly value of π
- leads to premium risk
- That is risk that expenses for accidents > premia
- More serious if risk is new (environmental risk) vs old (car accidents)

Demand… (cont.ed)

- In reality Damage must be assessed, which is a costly activity
- Insurtech aims, among other things, to lower assessment cost + speed liquidation.
- Example: external car damages → pictures sent to insurance directly by insured → car parts have a code → easy to figure out replacement cost + liquidation of damage w/o human intervention → lower cost

Demand… (cont.ed)

- So far Damage = Event
- In reality Damage ≠ Event, conceptually
- Assessing damage is costly/lengthy
- Parametric insurance is based on notion that assessing (some) event, is simpler/cheaper than assessing damage
- Example: insurance against bad weather that ruins your vacation; blockchain certifies event + electronic transmission of info + liquidation in few days w/o human intervention

Back to model. Find optimal amount of insurance that consumer with U'' < 0 chooses

$$\max_k V = \pi U(W - D + k - \gamma k) + (1 - \pi) U(W - \gamma k)$$

$$\frac{\partial V}{\partial k} = 0 \qquad (1 - \gamma)\pi U'(W - D + k - \gamma k) - \gamma(1 - \pi) U'(W - \gamma k) = 0$$

Marginal utility

$$\frac{U'(W - D + k^* - \gamma k^*)}{U'(W - \gamma k^*)} = \frac{1 - \pi}{\pi} \frac{\gamma}{1 - \gamma}; (1)$$

Profits of insurance company

$$\pi(\gamma k - k) + (1 - \pi)\gamma k$$

Assume perfect competition (mutual) among insurance companies => Expected profit = 0

$$\pi(\gamma k - k) + (1 - \pi)\gamma k = 0$$

If k ≠ 0 then $$(1 - \pi)\gamma = (1 - \gamma)\pi$$

$$\frac{(1 - \pi)}{\pi} = \frac{(1 - \gamma)}{\gamma} \Rightarrow \gamma = \pi$$

Premium is actuarially fair .

The cost of the policy = the probability of the damage.

Remark: *k* does not enter in the determination of γ; premium is independent of the quantity insurend.

Quantity of insurance demanded if $\gamma = \pi$? From eq. (1) we have

$$\frac{U'(W - D + k^* - \gamma k^*)}{U'(W - \gamma k^*)} = \frac{\pi}{1-\pi}\frac{1-\pi}{\pi} = 1$$

$$U'(W - D + k^* - \gamma k^*) = U'(W - \gamma k^*)$$

$$\Rightarrow \quad W - D + k^* - \gamma k^* = W - \gamma k^*$$

$$k^* = D$$

Comments

- If insurance is fair, the individual will buy a quantity of insurance k^* equal to the damage D
- Thus he will be fully insured
- The individual can transfer wealth/consumption from state 1 (no accident) to state 2 (accident) at the rate $1-\gamma$ to γ

Toward a graphical represantation

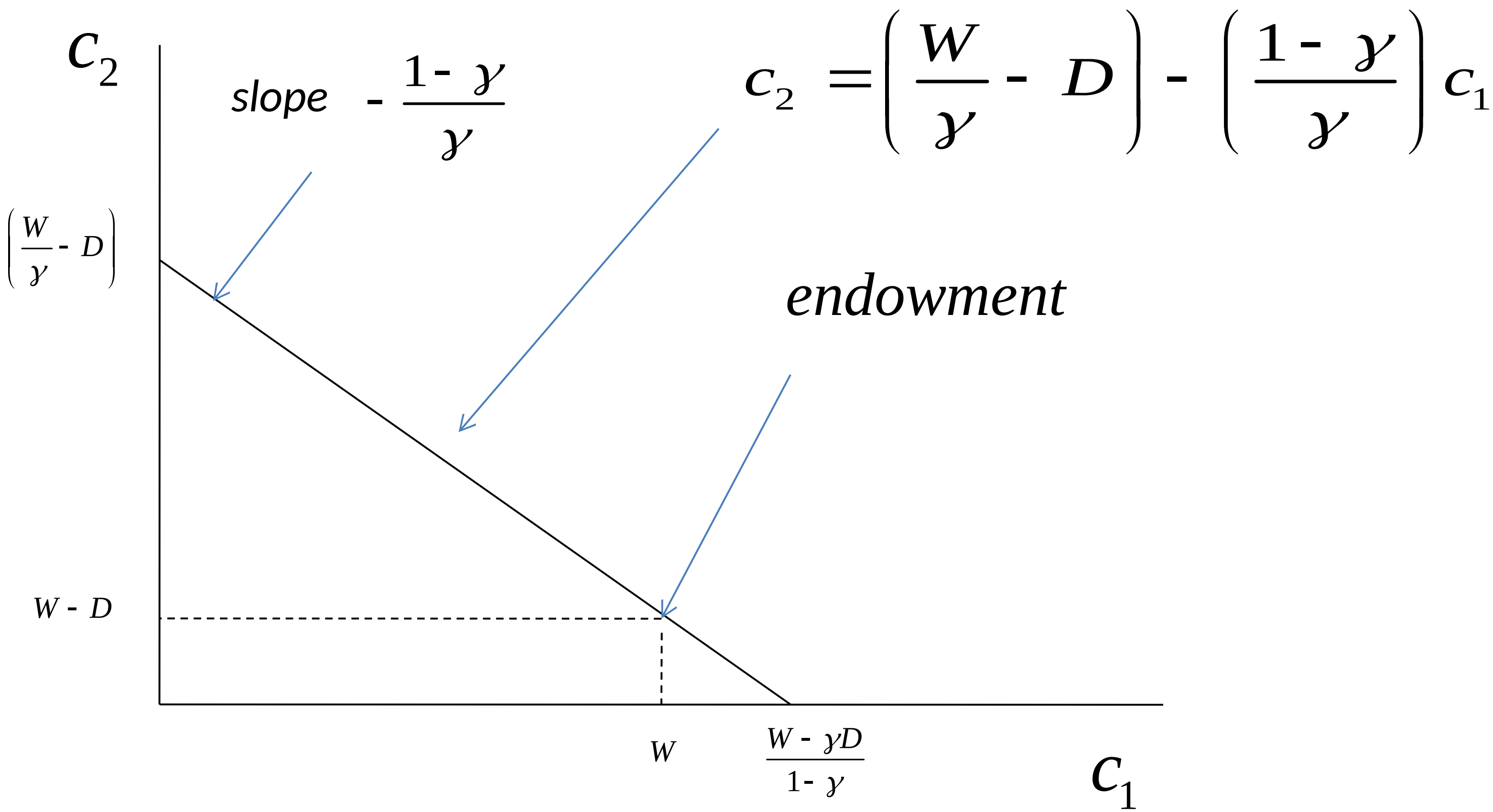
$$c_1 = W - \gamma k \quad \Rightarrow \quad \gamma k = W - c_1$$

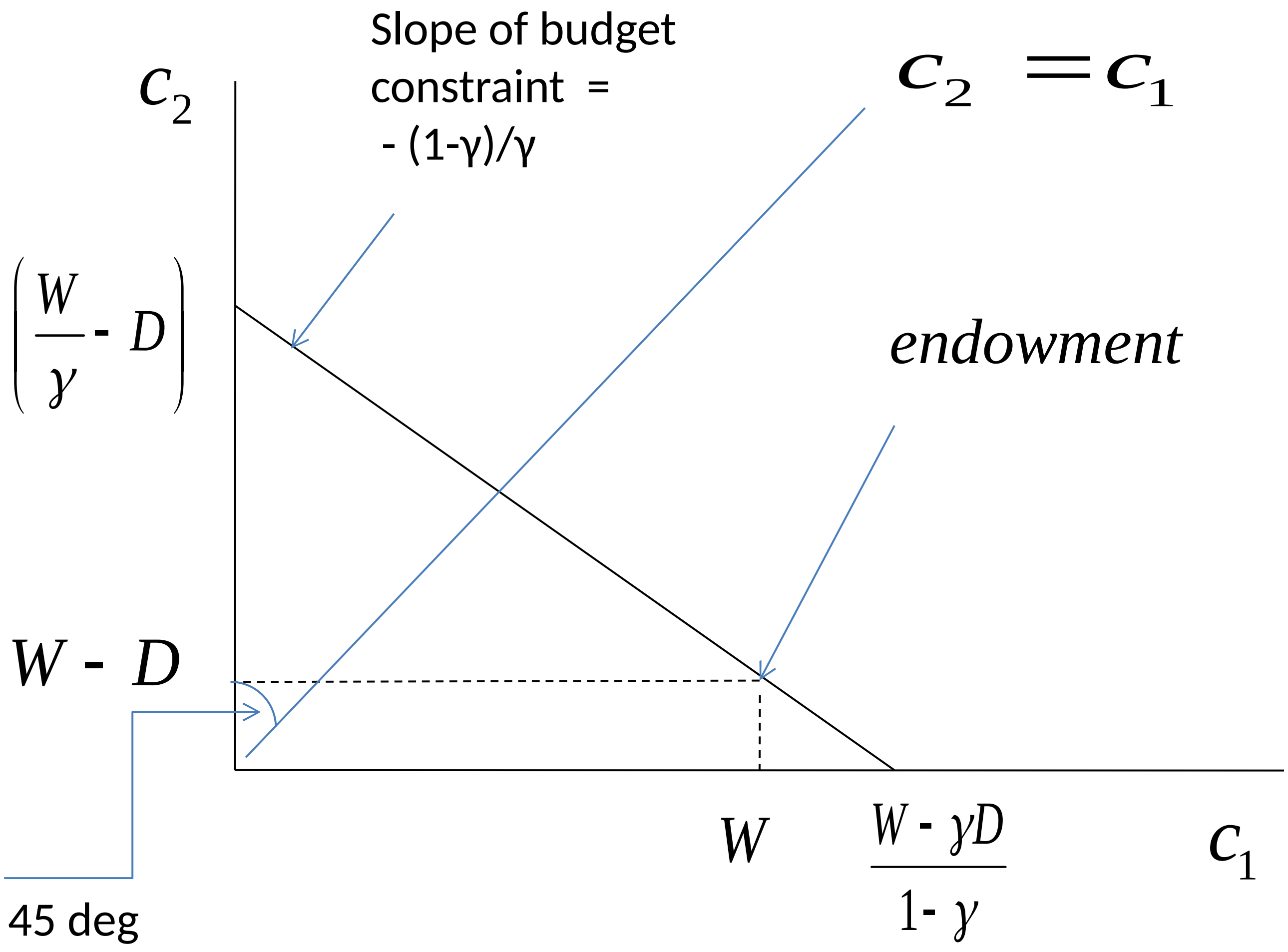
$$k = \frac{W}{\gamma} - \frac{c_1}{\gamma}$$

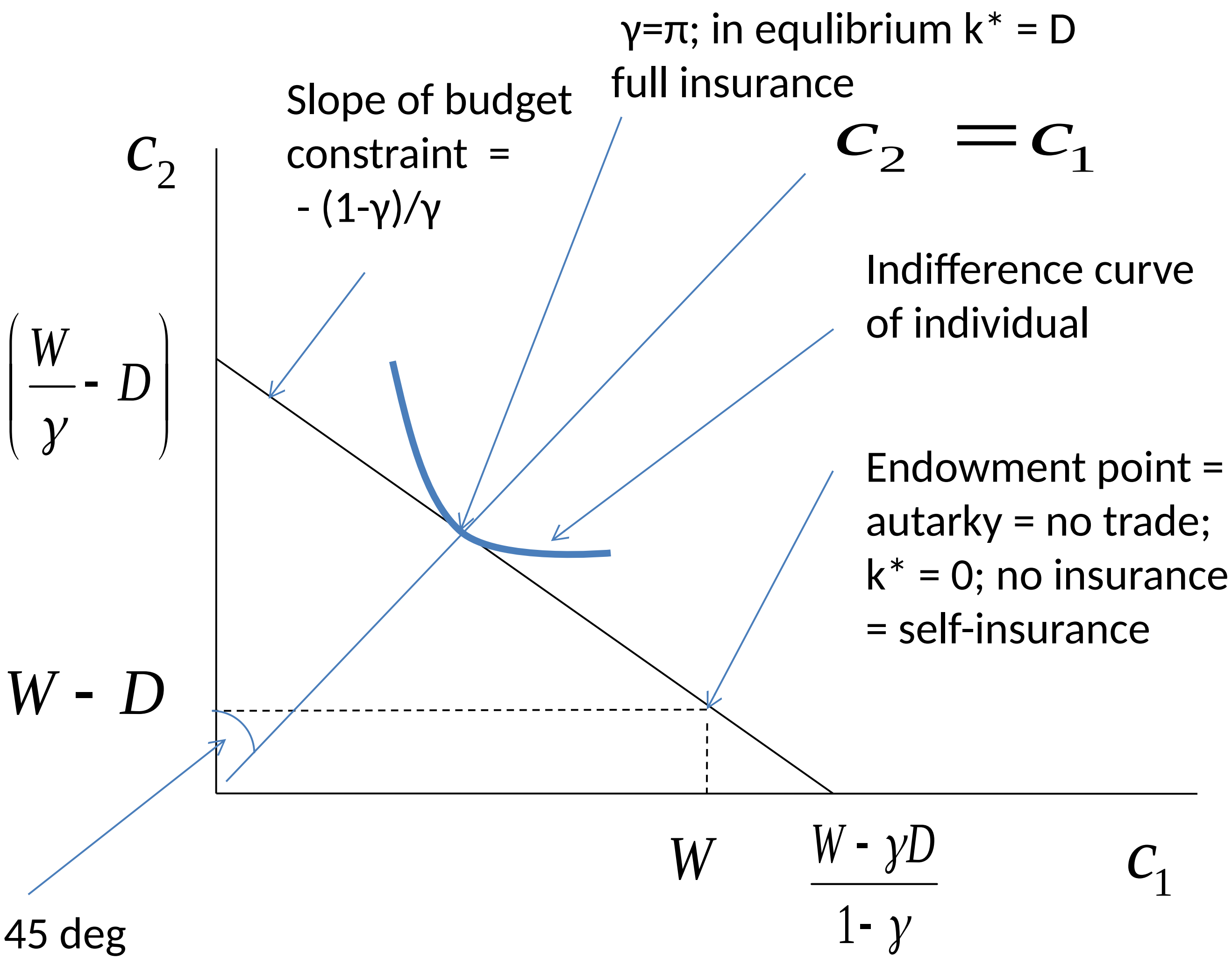
$$c_2 = W - D + k - \gamma k$$

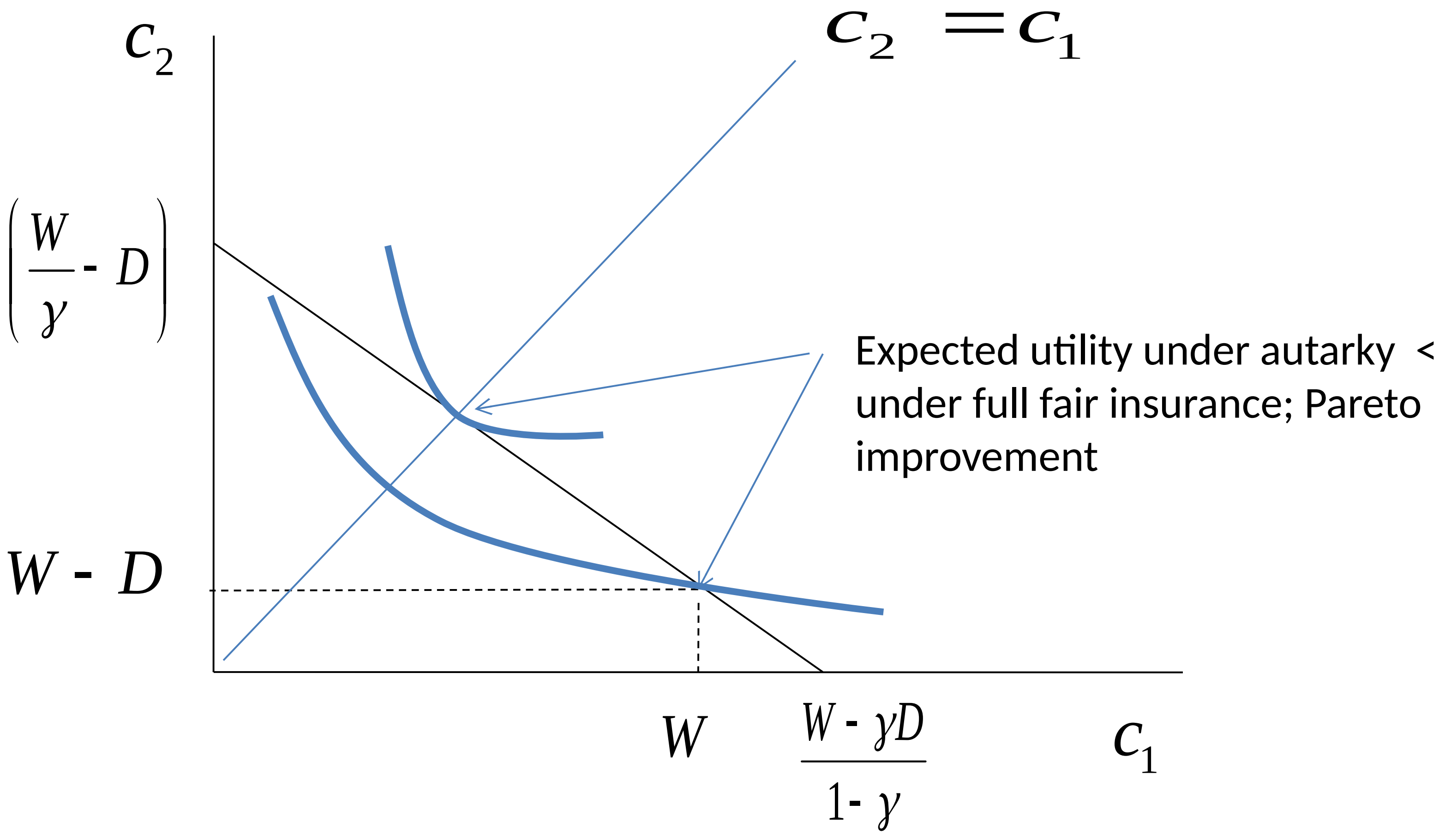
$$= W - D + k(1 - \gamma) = W - D + (1 - \gamma)\left(\frac{W}{\gamma} - \frac{c_1}{\gamma}\right)$$

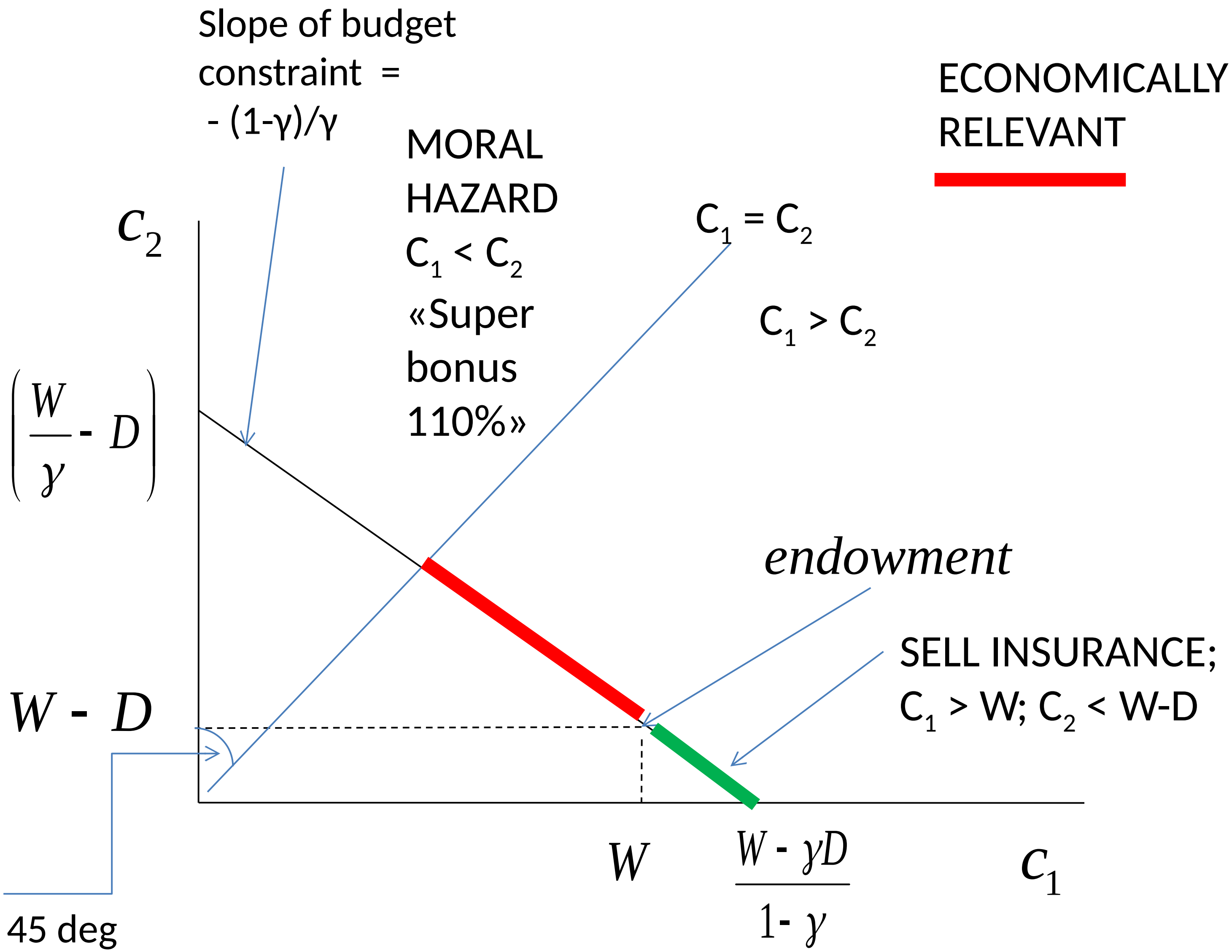
$$c_2 = \left(\frac{W}{\gamma} - D\right) - \left(\frac{1 - \gamma}{\gamma}\right) c_1$$


c_2
slope $-\frac{1-\gamma}{\gamma}$
$c_2 = \left(\frac{W}{\gamma} - D\right) - \left(\frac{1-\gamma}{\gamma}\right) c_1$
$\left(\frac{W}{\gamma} - D\right)$
endowment
$W - D$
W
$\frac{W - \gamma D}{1-\gamma}$
c_1

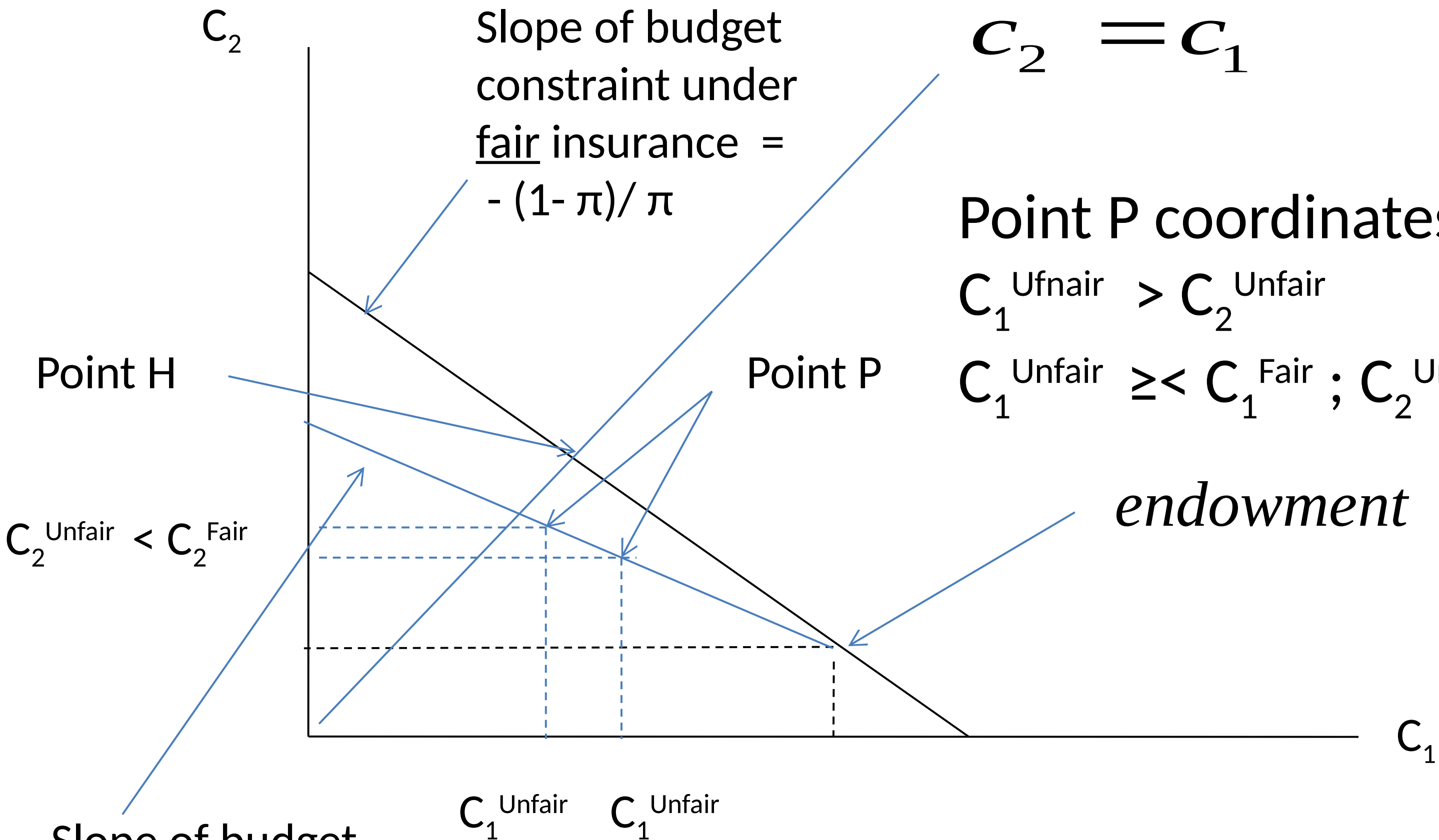

Slope of budget constraint = - (1-γ)/γ
$c_2 = c_1$
c_2
$\left(\frac{W}{\gamma} - D\right)$
endowment
$W - D$
W
$\frac{W - \gamma D}{1 - \gamma}$
c_1
45 deg


γ=π; in equlibrium k* = D
full insurance
Slope of budget
constraint =
- (1-γ)/γ
$c_2 = c_1$
c_2
Indifference curve
of individual
$\left(\frac{W}{\gamma} - D\right)$
Endowment point =
autarky = no trade;
k* = 0; no insurance
= self-insurance
$W - D$
W
$\frac{W - \gamma D}{1 - \gamma}$
c_1
45 deg


$c_2 = c_1$
c_2
$\left(\frac{W}{\gamma} - D\right)$
$W - D$
W
$\frac{W - \gamma D}{1 - \gamma}$
c_1
Expected utility under autarky < under full fair insurance; Pareto improvement


Slope of budget
constraint =
- (1-γ)/γ
MORAL
HAZARD
$C_1 < C_2$
«Super
bonus
110%»
ECONOMICALLY
RELEVANT
$C_1 = C_2$
$C_1 > C_2$
c_2
$\left(\frac{W}{\gamma} - D\right)$
endowment
SELL INSURANCE;
$C_1 > W$; $C_2 < W-D$
$W - D$
W
$\frac{W - \gamma D}{1-\gamma}$
c_1
45 deg

Under unfair insurance ($\pi < \gamma$) the expected profit of the insurance company is > 0



Point P coordinates:

$C_1^{Ufnair} > C_2^{Unfair}$

$C_1^{Unfair} \gtreqless C_1^{Fair}$; $C_2^{Unfair} < C_2^{Fair}$

Slope of budget constraint under unfair insurance = - (1- γ)/ γ

Point H coordinates:

$C_1^{Fair} = C_2^{Fair}$

Under <u>unfair</u> insurance ($\pi < \gamma$) …

$C_2^{Fair} = W - D + K(1\text{-}\pi)$; $C_1^{Fair} = W - \pi K$

$C_2^{Unfair} = W - D + K(1\text{-}\gamma)$; $C_1^{Unfair} = W - \gamma K$

Expected profit of insurance under <u>fair</u> =

$$\pi(\pi K\text{-}K) + (1\text{-}\ \pi)\pi K = 0$$

Expected profit of insurance under <u>unfair</u> =

$$\pi(\gamma K\text{-}K) + (1\text{-}\ \pi)\gamma K = \gamma K - \pi K > 0$$

since $\pi < \gamma$

Under <u>unfair</u> insurance ($\pi < \gamma$) …

Why insurance may be unfair?

- Market power
- Administrative costs
- Safety loading; see the ruin of the insurer
- In anticipation of moral hazard; MH implies < effort than first best; when effort ↘, premium ↗

Different types of unfair insurance

- Proportional (per unit) loading $\gamma = (1+\lambda)\pi$; where λ is the loading factor; this is the case in the picture above
- Fixed loading total pemium = $\pi K + c$ where $c>0$ is the loading (K=quantity insured)

5. Property&Casualty Insurance in the real world

5.1 Real world policies are not «actuarially fair»

5.2 Inverted productive cycle

5.3 Where insurance companies invest premia?

5.4 Balance sheet of insurance company

5.5 Reinsurance

5.6 Regulation of insurance

5.1 Real policies are <u>not</u> «actuarially fair»

- In practice P&C insurance policies are never actuarially fair because on top of expected losses insurance company incurs, for example, administrative expenses ($c > 0$)
- With «fixed loading» due to admin. expenses, zero expected profit condition of a real-world insurance company is $\gamma K - \pi K - c = 0$
- Insurance companies use concept of <u>**combined ratio**</u> = (incurred losses + admin. expenses)/(earned premia)

Real policies … (cont.ed)

- If we assume that incurred losses = expected losses (πK), and we observe that earned premium = γK, then zero expected profit condition of a real world insurance company is:

Combined ratio = $(\pi K+c)/\gamma K = 1$

- When combined ratio < 1 company gains, when it is > 1 it loses
- **Combined ratio is one of most important indicators of profitability in insurance industry.**
- (revenues from portfolio of assets is negligible)

Real policies … (cont.ed)

- Combined ratio is also expressed as:
  combined ratio = loss ratio + expense ratio
- Loss ratio = incurred losses/earned premia; expense ratio = admin. expenses/earned premia

5.2 Inverted productive cycle

- Key feature of insurance industry is inverted productive cycle:
  - first ins. companies sell policies & collect upfront premiums (so-called «float»)
  - then bear costs of damages if they happen
- Opposite of all other industries, where first you bear costs (investment, staff, raw material, etc.), then you sell, and finally you get paid.

Implications of inverted cycle

1. Costs of an insurance company are stochastic (much more than in any other business)
2. Hence pricing is based on uncertain costs
3. Receiving lots of cash upfront («float») implies that financial management is crucial.

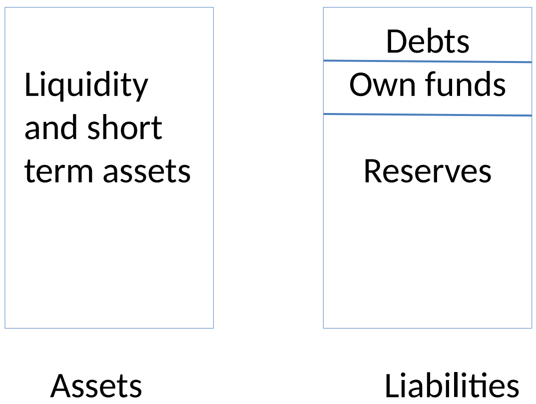
5.3 Where P&C ins. companies invest premia?

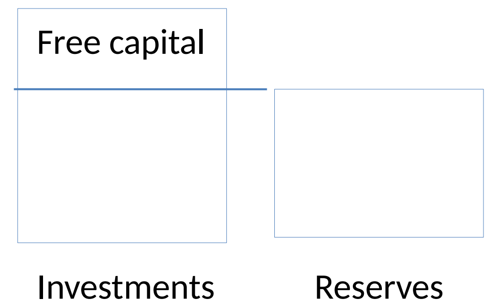
Example: Investment portfolio of Intesa Sanpaolo Assicura; Total ≈ 1 bn EUR Source: Relazioni e Bilancio 2018			
Government Bonds	82.6%	Largely fixed rate; Almost all BBB; Largely Italian	≈ 30% < 1 year ≈ 52.6% between 1 and 5 years
Corporate Bonds	0.7%		
Stocks	0.1%		
Mutual Funds	16.6%		
	100%		

Combined ratio ≤ 1 means that insurance company makes profit just with core business, w/o the contribution of investments portfolio.

5.4 Balance sheet of P&C

Index of risk coverage:
Investments/Reserves >
1 = sound





- Reserves
  - to guarantee policies issued to insured
  - <u>Premia reserves</u>: for losses not yet incurred
  - <u>Damage reserves</u>: for losses already incurred but of uncertain amount
- Free capital: resources that can be invested w/o constraints to protect insured

5.5 Reinsurance (RI)

Source: Zweifel and Eisen, Insurance Economics, Springer, 2012

- Insurer has choice or retaining risk or <u>transferring</u> it at least in part to RI
- Risks that insurer bears and that it could in part transfer
    a. Loss risk = risk that losses are on average > than expected
    b. Probability risk = probability of loss > than calculated
    c. Distribution risk = density function used to calculate premium is wrong

Functions (main) of RI

1. **Risk transfer** = RI relieves insurance company of risks a, b, c.
2. RI ↗ underwriting capacity (ability to sell policies to individuals/firms). Underwiting a risk exposes insurance company to possibility that future payments > reserves + equity. To keep prob. of insolvency to desirable (or regulatory) level there are three alternatives
   a. Renounce the business
   b. Take only a share of business = coinsurance; costly because of contract preparation, monitoring, execution
   c. Purchase RI coverage designed to cap loss payments to amount that does not jeopardize solvency

Functions …

3. For <u>given amount of equity</u>, <u>RI allows to underwrite more risk</u>. To see that RI has same effect of <u>equity</u> consider that after RI premia ↘ from 100 to 90.

Suppose solvency margin (equity/premia) of 10%, then minimum equity ↘ from 10 to 9. That is RI allows to ↗ leverage (similar to securitization)

Functions …

4. Reserve smoothing: underwriting risks with many standard deviation > expected value could entail so large safety loading that insurance may lose business.

Instead of ↗ reserves, insurance company could rely on RI to cover extremely high losses, allowing to avoid ↗ in reserves + premia.

Functions …

5. Allows to enter new lines of business,

using commercial and technical knowledge of reinsurer,

with the aim of acquiring its know how

Types of RI

1. <u>Proportional</u>: a share α of losses is paid by RI.
   – Very simple
   – Allows to increase ability of insurer to underwrite risks
   – Lowers absolute variability of potential liability, but does not lowers substantially overall risk of the insurance portfolio
   – RI likes it because it allows participate in all the activities of the insurer

Types…

2. Aggregate-excess contract:

primary insurer pays total loss up to a year limit, while excess is paid by RI. So called «stop-loss».

Liability for primary insurer = min(year limit, loss)

RI dislikes it because transfers relate to largest risks, hence expected gains of RI are < for proportional RI

Note: Providing full marginal coverage beyond a deductible, is equivalent to Pareto-optimal contract between risk averse individual and risk neutral company in presence of administrative expenses.

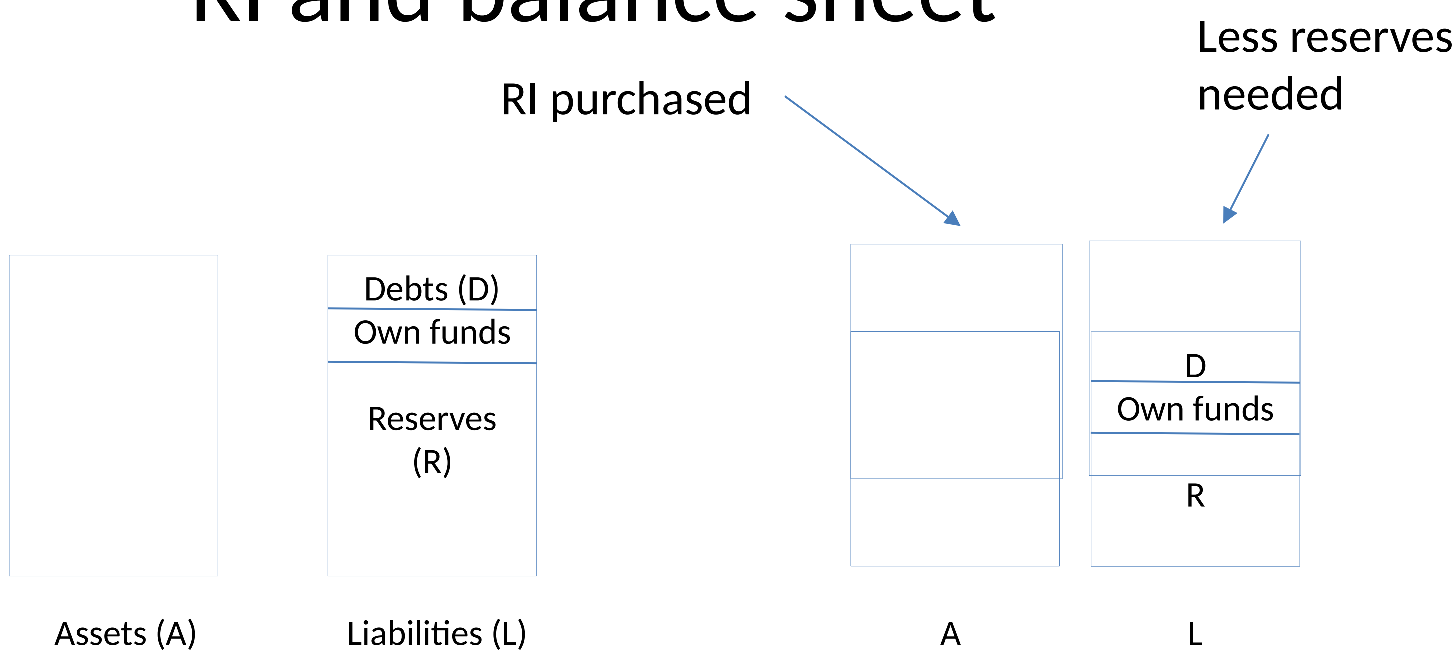
Types …

3. Per-risk excess contract: same as 2 but per contract

4. Per-occurrence excess contract: same as 2 but per event

Also RI can transfer risk to another RI (reinsurance of second level) and so on.

RI and balance sheet



- RI ↘ A and R by same amount. Hence shareholders positions (A- R - D) does not change.
- Under fair premium Expected value [A -R - D] does not change.
- Variance [A - R - D] ↘. Matters if shareholders + debt holders are not perfectly diversified.

Alternative to RI: Catastrophe bond

- CAT bonds are the most common type of Insurance linked securities
- issued by vehicles linked to insurance (or RI) companies
- In normal times pay an interest + capital
- Stop paying interest and debt is cancelled if prespecified events happen (earthquake, natural disaster, tornados, etc.) during the short life of bond and if losses > predetermined amount

Alternative to RI …

- Money raised by issuing CAT bond is placed in an account (collateral account) separate from insurance company
- Money goes to buy low risk securities, out which the interest is paid
- If event happens, the money goes to the insurance company, and helps pay for liquidation of damages caused by event

Alternative to RI …

- Hedge funds and other inst. investors benefit because CAT bonds are an asset class with payoff uncorrelated with economic events
- Insurance companies benefit because CAT bonds allow them to transfer risk to investors and capital mkts hence increasing dramatically RI ability

Alternative to RI …

- Also before CAT bonds, insurance ad RI companies transferred risk to capital mkts, by issuing shares in insurance company itself (or in RI company)
- This was indirect transfer of risk, not the transfer of specific risk, which is now possible with CAT bonds
- Also RI companies can issue CAT bonds

RI ≠ coinsurance

- RI: risk transfered after it is assumed; in general insured ignores the transfer
- Coinsurance
  - Other insurance companies share part of same risk at time policy contract is signed
  - Or, insured individual shares part of risk; limits moral hazard

5.6 Regulation of insurance

Source: Zweifel and Eisen, Insurance Economics, Springer, 2012

- Solvency II of European Union. Designed before crisis 2007-09, adopted in 2009.
- Similar logic of Basel II for banks
- Three pillars

1. Quantitative requirements regarding solvency capital
2. Supervisory review
3. Disclosure requirements (market)

5.6 Regulation…

- Focus on pillar 1 quantitative capital requirement
- Depends on risk profile of company (more than in Solvency I)
- Solvency capital = equity + insurance reserves

5.6 Three layers of solvency capital

Layer A: best estimate of liabilities (BEL) augmented by risk margin

Mkt based: reflecting rate of return on capital a potential buyer of liabilities (policies) would require

-

5.6 Three layers (cont.ed)

Layer B: additional solvency (add-on) capital to reach minimum required capital (MRC)

At discretion of national authorities

When solvency capital falls short of MRC, authority has right to intervene

5.6 Three layers (cont.ed)

Layer C: risk-sensitive additional requirements reflecting operational + underwriting + counterparty default risks

Correlations among these risks taken into account in «standardized» approach

Alternatively, insurers can use internal model reflecting specific risk profile, to be approved by authorities (copied from Basel II)

5.6 Insurance regulation in US

- 1992, National Association of Insurance Commissioners instituted <u>risk-based capital</u> (RBC) principle
- <u>RBC principle</u>: equity + insurance reserves must be set in a way to mantain a target probability (usually 99.5%) with which it can cover claims of policy holders + other losses
- Detailed regulation of risks can be avoided

5.6 Insurance regulation …

4 basics types of risks

1. Asset risk: risk that the subsidiaries lose value
2. Asset risk: risk that bonds, equity, loans fluctuate
3. Underwiting risk: claims can be higher than expected, or pricing may have been inaccurate
4. Business risk: in life insurance a variation of interest rate can cause losses because cash flows related to assets and liabilities have different maturities.

5.6 Systemic risk is one reason banking and insurance are regulated (1/2)

Banks

- GDP share ≈ 3% (VA)
- Connections: payment services, crucial, no substitues
- Liabilities/Equity ≥ 9/1
- Maturity transformation: high duration of assets, low for liabilities

Insurance Companies

- GDP share ≈ 1.5%
- Connections: credit insurance, limited
- Liabilities/Equity ≈ 5/1
- Duration matching

5.6 Systemic risk is … (2/2)

Banks

- Complexity: use of structured products (securitization)
- Governance: dispersed depositors
- Regulation: strong deregulation before Great Financial Crisis

Insurance Companies

- Limited use of structured products
- Disperse policy holders
- Little deregulation

5.7 Deductibles and reimbursments

- A deductible (in Italian «franchigia») of amount «f» means that damages < «f» are not reimbursed
- Damages ≥ «f» are reimbursed
  - for whole amount of damage
  - only for damage > «f»
- Often «f» is presented as a % of the value of the object insured
- Main rationale of deductibles: to prevent small claims whose liquidation entails high administrative costs, hence lower premium
- More on lecture of demand of insurance with moral hazard

5.7 Deductibles …
Damage = D; reimbursment = K; K ≤ D

1. K=D full insurance
2. K = min (D,M) where M is max that insurance will pay; in property insurance = <u>first absolute loss</u>; in casualty insurance (civil liability) is called policies with <u>max guarantee</u>

5.7 Deductibles …

3. $K = \max(0, D - f)$ with absolute deductible «f»
4. $K = D$ if $D \geq f$, $K = 0$ otherwise; relative deductible
5. $K = 0$ if $D \leq f$, $K = D - f$ if $f < D \leq M$; $K = M - f$ if $D > M$ or $K = \min[\max(0, D - f), M - f]$, first absolute risk with absolute deductible
6. $K = (1-\alpha)D$, $0 \leq \alpha < 1$ coinsurance

6. Demand of insurance and behavioral economics

Insurance is <u>not</u> «actuarially fair» in real world

Rewrite eq. (1) as

$$U'(C_2)/U'(C_1) = [(1-\pi)/\pi][\gamma/(1-\gamma)]$$

If **$\gamma > \pi$** then

$[(1-\pi)/\pi][\gamma/(1-\gamma)] > 1 \Rightarrow U'(C_2)/U'(C_1) > 1 \Rightarrow \mathbf{C_2 < C_1}$ since $U''<0$

- That is optimal insurance coverage is not full when insurance is not fair; individuals bear some risk; e.g. deductibles
- This result proved under general conditions by Mossin, Journal Political Economy, 1968.

In practice: we observe high demand for full-coverage policies or very low deductibles

Examples (Shapira and Venezia Journal of Economic Psychology, 2008)

- Almost all cars liability insurance policies provide full coverage or a zero deductible. Typical collision damage waiver (CDW) for rental car costs on average $25 per day, equal to $7200 per year.
- In contrast, comprehensive automobile insurance for one's own car does not cost more than $1000 per year in most locations in US. Why are people willing to pay such high rates for CDW when renting a car?

In practice…

- Deductible on automobile insurance is often as low as $100 and almost always < $500, which means that consumers are insured against losses of $500 or less.
- Example: When Pennsylvania's Insurance Commissioner during 1970s, tried to ↑ minimum auto insurance deductible from $50 to $100, he was forced to withdraw this idea by massive consumer outcry.

In practice…

- Merchants who sell electronic products costing < $200 also offer insurance against loss, for a non-trivial additional cost. Consumer purchases of such insurance do not seem to be rational even when those policies include service.
- Companies offering such warranty stand to make high profit
- According to Harvard Business School case (see Burns, 2004), Circuit City sold electronics at cost and made its profits on extended warranties

In practice…

- The situation is even more striking in medical insurance. US Bureau of Labor Statistics reports that in 1994–1997, 34% of full time employees in no-HMO (health maintenance organizations) medical care organizations had no deductibles in their medical plans
- HMOs typically have no deductibles

How to reconcile theory and practice

- When insurance is not fair:
  - Theory predicts no full coverage (deductibles)
  - In practice we observe wide presence of full coverage and low deductibles
- Braun and Muermann (2004) explain it by aversion to <u>regret</u>
- <u>Framing</u> affects how people evaluate insurance alternatives. Schoemaker (1976) show that when faced with decisions described as insurance against hypothetical <u>losses</u>, subjects chose full coverage over deductibles. When same choices were framed as lotteries (<u>gains</u>), choice was reversed.

How to reconcile …

- Wakker, Thaler, and Tversky (1997) argue that people buy too much insurance since they are averse to "probabilistic insurance"
- Kahneman and Tversky (1979) argued unattractiveness of ''probabilistic insurance'' is related to desire of people to insure against worries rather than against actual damages. Based on difficulty to conceive potential situations that may arise if one does not have full coverage.

How to reconcile …

- Full-coverage policies provide anchor for thinking about insurance problems because such policies are easy to envision and need to calculate expected damages is reduced.
- When offered menu of policies with different deductibles, people may find it convenient to think about policies with small deductibles; these are close in price to a full-coverage policy.
- With high deductibles, people may exhibit the bias of the anchoring heuristic (Tversky & Kahneman, 1974).

How to reconcile …

- In this case the anchoring heuristic works like that:
- In estimating reasonable price for a policy with a deductible, they often
- anchor on deductible amount itself,
- subtract it from the price of the full-coverage policy
- and in setting the price of the policy with the deductible they do not adjust the price enough upwards to take into account the fact that actual damage amounts are probabilistic.

How to reconcile …

- Thus, as deductible ↗ in value, people anchor on it and their estimate of a reasonable price of such a policy departs to a larger degree than is warranted from the price of the full-coverage policy.
- That is: <u>people consider the absolute value of the deductible not it expected value equivalent</u>.

Anchoring and adjustment heuristic

The anchoring and adjustment heuristic was first theorized by Tversky and Kahneman (1974)

In one of their first studies, participants were asked to compute, within 5 seconds, the product of the numbers one through eight, either as

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$$

or reversed as

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1.$$

Anchoring and …

Because participants did not have enough time to calculate full answer,
they had to make an estimate after their first few multiplications (anchor)

When the sequence started with small numbers the median estimate was **512**;
when the sequence started with larger numbers, the median estimate was **2,250**. (The correct answer is 40,320.)

Anchoring and …

In another study by Tversky and Kahneman, participants observed a roulette wheel that was predetermined to stop on either 10 or 65.

Participants were then asked to guess the percentage of the United Nations that were African nations.

Participants whose wheel stopped on 10 guessed lower values (25% on average) than participants whose wheel stopped at 65 (45% on average).

The pattern has held in other experiments for a wide variety of different subjects of estimation.

7. Generalization

Generalization to *n>2* states of nature

Equation (1) can be written as

$$\pi_2 U'(C_2)/[\pi_1 U'(C_1)] = p_2/p_1 \qquad (2)$$

where $p_2 = \gamma$, $p_1 = 1 - \gamma$, $\pi_2 = \pi$; $\pi_1 = 1 - \pi$.

Rewrite (2) as (expected marginal utility per EUR)

$$\pi_2 U'(C_2)/p_2 = \pi_1 U'(C_1)/p_1$$

Generalization …

Suppose to have *n>2* states of nature.

The fundamental theorem of optimal risk sharing states that (in case of interior solution)

$$\pi_1 U'(C_1)/p_1 = \pi_2 U'(C_2)/p_2 = \ldots = \pi_n U'(C_n)/p_n$$

Generalization …

Meaning

The individual will choose to consume the contingent goods in such a way that the expected marginal utility per euro is equal in all the states of nature (in case of interior solution)

8. Un-difersifiable risks

Undiversifiable risks

- Insurance works well when risks are independent so that they can be diversified away via law of large numbers
- Two notable examples of lack of independence are
  - COVID 19 Virus
  - House bubble collapse

COVID 19 Virus

source: Wall Street Journal March 30, 2020

- Firms typically buy <u>business-interruption insurance policies</u> (BIP)
- To cover lost income following hurricanes, fire or events that cause physical damage to their property
- Insurance industry inserted <u>exclusions</u> into most standard policies following SARS scare of early 2000s to deny claims "due to virus or bacteria".

Covid-19…

- As a result of Covid many firms that had bought BIP suffered because of virus exclusion
- Virus exclusion has economic logic, though
- “Insurance works well and remains affordable when a relatively small number of claims are spread across a broader group, and therefore it is not typically well suited for a global pandemic where virtually every policyholder suffers significant losses at the same time for an extended period” (National Association of Insurance Commissioners, Wall Street Journal March 30, 2020) .

Housing collapse in US during Great Recession (1/10)

source Besanko and Breutingan, Microeconomics

- A notable example of when <u>independence does not</u> hold involved housing mortgage industry in 2008–2009
- To understand this example, first describe *mortgage securitization* and *credit default swaps*

Housing … (2/10)

- When bank lends money to buy home, homeowner is promising to make monthly payments for life of mortgage (usually 30 years)
- However, there is risk that homeowner will stop making those payments, e.g. because of job loss
- *Incidentally, bank obliges you buy Credit Protection Insurance (CPI); no CPI → no mortgage*
- *This is a case where bank can <u>legally</u> condition loan to you buying other financial services; lots of regulations to prevent abuses; opposite of "baciate" where bank lends money if you buy shares of bank itself*

Housing … (3/10)

- In typical economic times, risk that homeowner defaults on mortgage is <u>independent</u> of risk of default on mortgages issued to other homeowners
- If one mortgage defaults, homeowners with other mortgages typically keep making their payments to bank
- Law of Large Numbers applies

Housing … (4/10)

- Bank charges small margin to all mortgage holders, as a form of insurance for when one mortgage goes into default
- Actually, mortgage industry spreads risks of mortgage even more broadly through *mortgage securitization*
- In US banks sell their mortgages to companies such as Fannie Mae (Federal National Mortgage Association), federally sponsored corporations

Housing … (5/10)

- Fannie Mae issues *mortgage-backed securities*, similar to bonds, the value of which depends on monthly payments on thousands of mortgages
- Investors and mutual funds buy these securities as part of their portfolios
- Thus, risks from thousands of individual mortgages (theoretically independent of each other) are combined, and the joint risk is then spread over many investors via capital mkts.

Housing … (6/10)

- In early 2000s, investors could also purchase collections of mortgage-backed securities known as *collateralized debt obligations* (CDOs)
- CDOs are groups of mortgage-backed securities, segmented (tranched) according to riskiness of underlying mortgages.

Housing … (7/10)

- Even with spreading of risks, some investors in mortgage-backed securities or CDOs sought to purchase insurance on their investments
- This insurance is called *credit-default swap* (CDS)
- CDS protects owner of bond or CDO against risk of default
- In effect, <u>CDS is an insurance policy</u> on bond or CDO
- An important issuer of CDS was insurance firm AIG.

Housing … (8/10)

- Suppliers of CDS like AIG counted on risk <u>independence</u>
- In late 2000s, such <u>independence broke down</u>
- Between 1997-2005, U.S. housing mkt experienced dramatic ↗ ↗ in prices
- By early 2000s, mkt was in speculative bubble; many individuals invested large % of personal wealth in their homes
- Banks also greatly ↗ extent to which they were willing to issue “subprime” mortgages (i.e. mortgages to subjects that were <u>too</u> risky)

Housing … (9/10)

- In 2006, bubble deflated, housing prices began ↘, to point that many homeowners owed <u>more</u> on their mortgage than current mkt value of their home
- Observe that house prices never ↘ at the same time in all States of US before; hence ↘ in one State could be compensated by ↗ in other States (<u>independence at work</u>)
- Furthermore, interest rates on adjustable rate subprime mortgages "reset" (↗) from low "teaser rates" (designed to attract borrowers)
- This triggered large scale mortgage defaults in 2006 <u>(defaults stopped to be independent</u>)

Housing ... (10/10)

- As rate of defaults ↗ ↗ in 2006-7, not only did holders of mortgage-backed securities and CDOs experience losses, so too did insurers of those securities such as AIG
- AIG failed — bailed out by U.S. Gov't — because it had inadequate reserves to pay claims of those to whom it had sold CDS
- These developments took many by surprise, including ratings agencies (Moody's and Standard&Poor's) that had given **AAA (??)** to CDOs consisting of bonds containing subprime mortgages.

Conclusion

- In both cases (housing collapse and in Covid-19) those un-diversifiable risks were borne by the state that can borrow from future generations
- There is a difference
  - Housing collapse is largely due to MH, speculation, poor regulation
  - Covid-19 is exogenous shock, of which nobody bears responsibility

9. Exercises

Intermediate Exam, November 21, 2012
Question 1

Consider the model of demand for insurance under full information seen in class and use the same notation unless otherwise specified: U= utility function, $U'>0$, $U''<0$, W = initial wealth, D = damage, γ = premium per unit, K = quantity insured, π = probability of the damage. Suppose $\gamma=\pi$, so that we know that $K^*=D$, that is full insurance is optimal.

Prove analytically, <u>not graphically</u>, that buying full insurance increases the expected utility of the individual w.r.t. autarky, where, recall, autarky means $K^*=0$.

Intermediate Exam, November 21, 2012
Question 1 . S O L U T I O N

Autarky gives **EU(random wealth)** = $\pi U(W\text{-}D)+(1\text{-}\ \pi)U(W)$.

Denote EW the average of the random wealth.

Observe that $W\text{-}\pi D = EW = \pi(W\text{-}D) + (1\text{-}\ \pi)W$. Observe that EW is not stochastic.

Thus full insurance gives

$\boldsymbol{\pi U(W\text{-}D\text{+}D\text{-}\ \pi D)+(1\text{-}\ \pi)U(W\text{-}\pi D))} = \pi U(W\text{-}\pi D)+(1\text{-}\ \pi)U(W\text{-}\pi D))$ = **EU(EW)**

Since $U''<0$ it follows that **EU(random wealth)** < U(EW)=**EU(EW))** since EW is not stochastic.

Question 1, June 2019

Consider the standard problem of demand of insurance seen in class.

An expected utility maximizer with utility function U(wealth) = ln(wealth) has initial wealth $W=100$, faces the risk of suffering a damage $D=80$, with probability $\pi=½$ and with complementary probability she does not suffer any damage.

The individual can purchase insurance $0 \leq K \leq D$ at the premium per unit $\gamma=5/6$. Denote by W_i, $i=1,2$ her wealth without and with damage, respectively.

Using Khun-Tucker theorem find mathematically the optimal amount of insurance K^* that she buys.

Question 1, June 2019: SOLUTION

Insurance is unfair hence $K^*<D$. The derivative w.r.t. K of the expected utility is

$dV/dK = (1-\gamma)\pi U'(W_2) - \gamma(1-\pi)U'(W_1)$ where W_i $i=1,2$ denotes wealth without and with damage, respectively, and V denotes the expected utility function.

By Khun-Tucker we want $dV/dK \leq 0$. It will be $dV/dK = 0$ and $K^*>0$; $dV/dK < 0$ and $K^*=0$, or $dV/dK = 0$ and $K^*=0$. This implies

Question 1, June 2019: SOLUTION (cont.ed)

$dV/dK = (1-\gamma)\pi/W_2 - \gamma(1-\pi)/W_1 \leq 0 \Leftrightarrow (1-\gamma)/W_2 - \gamma/W_1 \leq 0 \Leftrightarrow (1/6)/W_2 - (5/6)/W_1 \leq 0 \Leftrightarrow$

$1/W_2 - 5/W_1 \leq 0 \Leftrightarrow W_1 \leq 5W_2 \Leftrightarrow 100 - K5/6 \leq 5(20 + K/6) \Leftrightarrow - K10/6 \leq 0$.

Thus for any $K > 0$ we have that $- K10/6 < 0$. Hence the first derivative of the objective function declines in K for any positive K, so the optimal insurance is **$K^*=0$**.

The intuition is that if the premium is too high the risk averse individual prefers not to buy insurance.

Exam January 23, 2019 entire program
Question 2

Consider the model of demand of insurance under full information and adopt its assumptions and notation. An individual with utility function ln(final wealth) and initial wealth $w=100$ suffers the risk of having a damage $D=10$ with probability $\pi = 50\%$. She can insure this risk by paying a per unit premium γ so that the total premium is γk, where k is quantity insured.

a) Find the maximum γ that this person would be happy to pay to buy full insurance if her best alternative is no insurance at all (autarky).

b) Determine whether the answer that you gave in point a) above entails fair insurance.

SOLUTIONS

a) In autarky she obtains

$50\%\ln(90) + 50\%\ln(100) = \frac{1}{2}(4.499) + \frac{1}{2}(4.605) = 2.249 + 2.302 =$ **4.551.**

Recall that the expected utility is

$V(k, \gamma, \pi, D, w) = \pi\ln(w-D+k-\gamma k) + (1-\pi)\ln(w-\gamma k)$

which becomes

$50\%\ln(90+10-\gamma 10) + 50\%\ln(100-\gamma 10) = 50\%\ln(100-\gamma 10) + 50\%\ln(100-\gamma 10) = \ln(100-\gamma 10)$.

Hence $4.551 = \ln(100-\gamma 10) \Rightarrow e^{4.551} = 100 - \gamma 10 \Rightarrow 94.727 = 100 - \gamma 10 \Rightarrow$

$5.27 = \gamma 10 \Rightarrow$ $\boldsymbol{\gamma = 52.72\%}$

b) At $\gamma = 52.72\%$ insurance is not fair since the prob. of the accident is just 50%

Exam February 16, 2018 ENTIRE PROGRAM Question 2

Consider the following problem of insurance under full information. The main difference w.r.t. what we saw in class is that the insurance company is a risk neutral **monopolist**. The individual is risk averse with Von Neuman Morgenstern utility function of final wealth $U(.)$ with $U'>0$, $U''<0$, initial wealth W, loss in case of accident $D<W$, probability of the damage $1>\pi>0$ and probability that the damage does not happen $1-\pi$. Insurance is voluntary, so the monopolist can charge the per unit premium γ that it wants but the individual will buy insurance $K\leq D$ only if her expected utility from insuring is not smaller than the best alternative which is not insuring at all (autarky).

Questions

a) write the expected profit of the monopolist insurance
b) write the participation constraint of the individual
c) determine the amount of insurance K that the monopolist will offer
d) given your answer to point c above, determine analytically whether insurance is fair or not (hint: we don't have to find the γ that the monopolist will charge, but only whether it will be fair or not and why)
e) is the solution that we found above Pareto efficient? Please explain.
f) please explain your results by drawing the relevant pictures.

SOLUTION

a. $\pi(\gamma K-K)+(1-\pi)\gamma K = (\gamma-\pi)K$

b. $\pi U(W-\gamma K-D+K)+(1-\pi)U(W-\gamma K) \geq \pi U(W-D)+(1-\pi)U(W)$

c. max w.r.t. γ and K the function $(\gamma-\pi)K$ under the constraint that

$$\pi U(W-\gamma K-D+K)+(1-\pi)U(W-\gamma K) \geq \pi U(W-D)+(1-\pi)U(W)$$

Write the Lagrangean

$$Z=(\gamma-\pi)K+\lambda[\pi U(W-\gamma K-D+K)+(1-\pi)U(W-\gamma K)-\pi U(W-D)-(1-\pi)U(W)]$$

Denote $W-\gamma K-D+K = C_2$ and $W-\gamma K=C_1$

SOLUTION

Recall $K \leq D$, $\gamma \geq \pi$. Taking the derivative of the Lagrangean we obtain:

(1) $\delta Z/\delta K = \gamma - \pi + \lambda[\pi(1-\gamma)U'(C_2) - (1-\pi)\gamma U'(C_1)] = 0$

(2) $\delta Z/\delta \gamma = K + \lambda[-\pi K U'(C_2) - K(1-\pi)U'(C_1)] = 0$ if K different from 0 =>

$1 = \lambda[\pi U'(C_2) + (1-\pi)U'(C_1)] \Rightarrow \lambda > 0$

SOLUTION

From (2) it follows that (1) becomes

γ- π + $[\pi(1-\gamma)U'(C_2)-(1-\pi)\gamma U'(C_1)]/[\pi U'(C_2)+(1-\pi)U'(C_1)] = 0$ (3)

Replacing $U'(C_2) = U'(C_1)$ in (3) we see that (3)=0.

Hence the solution is

$U'(C_2) = U'(C_1) \Rightarrow$ **K*=D** full insurance.

SOLUTION

d. Insurance is not fair, because even if full, it gives the same satisfaction of autarky. In fact, suppose insurance is fair, i.e. $\gamma = \pi$, then using $K^* = D$ the expected utility of the individual becomes

$$\pi U(W - \pi D) + (1 - \pi) U(W - \pi D) = U(W - \pi D).$$

Observe that $W - \pi D = \pi(W - D) + (1 - \pi)W$ = expected wealth, which is not stochastic. Therefore

$$U(W - \pi D) = U(\text{expected wealth}).$$

Since the function U is concave then U(expected wealth) > Expected U(random wealth) = $\pi U(W-D) + (1 - \pi)U(W)]$.

SOLUTION

But we know that $\lambda>0$ hence that

$$[\pi U(W-\gamma K-D+K)+(1-\pi)U(W-\gamma K)-\pi U(W-D)-(1-\pi)U(W)] = 0 ,$$

that is

$$\pi U(W-\gamma D)+(1-\pi)U(W-\gamma D) = \pi U(W-D)+(1-\pi)U(W) <$$
$$\pi U(W-\pi D)+(1-\pi)U(W-\pi D) = U(W-\pi D)$$

which is what the individual would have obtained under fair insurance.

SOLUTION

e. The solution above is Pareto efficient because it is not possible to improve the satisfaction of any player without lowering that of another. Unfair insurance does not distort the quantity of insurance bought by the individual (or offered by the insurance) which is like in perfect competition

f. Graphically, in the space of contingent goods (consumption with damage – vertical axis, and consumption without damage – horizontal axis) draw an indifference curve of the insured from the endowment point of the individual, up to the 45^0 line

Question 2. Intermediate Exam, November 21, 2012

- Insurance policies may differ from one another in a number of regards. Some have a deductible feature whereby the first X euro of loss is not reimbursed. Others insurance policies have a coinsurance (co-payment) feature whereby the insured must pay a fraction $0<\alpha<1$ of any loss.
- Imagine that a risk-averse consumer with wealth W faces the risk of a minor accident with probability p_1, of a major accident with probability p_2, and has no accident with probability $1-p_1-p_2$. It cannot have a major and minor accident at the same time. The losses from the minor and from the major accidents are A and B, respectively, with $B > A > X$.

Question 2. Intermediate … (cont.ed)

- The consumer must choose between a deductible and a coinsurance policy.
- X and α are selected in such a way that the expected value of the loss for the insurance company is equal for both policies and it is equal to the <u>overall</u> <u>premium</u> R for each policy:

  $R = p_1(A-X) + p_2(B-X) = p_1(1-\alpha)A + p_2(1-\alpha)B,$

  which implies that $X(p_1+p_2) = p_1\alpha A + p_2\alpha B.$
- Determine whether the consumer chooses the deductible policy or the coinsurance policy.

SOLUTION

The consumer will always choose the deductible policy because it yields a higher expected utility; in fact

$$p_1U(W\text{-}X\text{-}R)+ p_2U(W\text{-}X\text{-}R)+(1\text{-}p_1\text{-} p_2)U(W\text{-}R)>$$

$$p_1U(W\text{-}\alpha A\text{-}R)+ p_2U(W\text{-}\alpha B\text{-}R) + (1\text{-}p_1\text{-} p_2)U(W\text{-}R)$$

Subtract $(1\text{-}p_1\text{-}p_2)U(W\text{-}R)$ from each side, divide by p_1+p_2, and finally let $Q_1= p_1/(p_1+p_2)$ and
$Q_2= p_2/(p_1+p_2)$ where $Q_1+Q_2 =1$ to get:

SOLUTION

$U(W-X-R) > Q_1U(W-\alpha A-R) + Q_2U(W-\alpha B-R)$

which must hold for any risk-averse consumer since by assumption

$X(p_1+p_2) = p_1\alpha A + p_2\alpha B \Rightarrow$

$X = Q_1\alpha A + Q_2\alpha B.$

C O M M E N T to S O L U T I O N

This is a special case of general result proved by Arrow (1974) and Raviv (1979): when there are administrative expenses, and individual can choose amount of coverage (liquidation, K) of a variable damage (D), <u>the Pareto optimal insurance contract has deductible and no co-payment beyond that deductible.</u>

<u>Intuition</u>: deductible is fixed cost that covers admin. expenses; beyond that the liquidation increases <u>in step</u> with coverage less deductible; thus individual faces the correct choice at the margin as to the amount of coverage K to choose. Co-payment $\alpha>0$ instead, distorts the marginal choice of coverage because even if insurance is fair ($\pi=\gamma$) the premium per unit becomes unfair at $\gamma + \alpha$ to cover admin. Expenses.

10. Appendix: affine transformation

(Positive) affine transformation

$V(\pi, C_1, C_2) = \pi U(C_1) + (1-\pi)U(C_2)$

Positive Affine transf. : a>0, b any real number

Denote X= aU(C) +b

$Z = \pi(aU(C_1) + b) + (1-\pi)(aU(C_2) + b)$

Absolute risk aversion of

U: $-U''(C)/U(C)'$

X: $-aU''(C)/[aU'(C)] = -U''(C)/U(C)'$ **unchanged**

Same for relative risk aversion coeff.

Counterexample that a monotonic transformation alters risk aversion

Suppose $U(C) = \ln C \Rightarrow U' > 0, U'' < 0$

risk averse; RRA =1.

Take $V(C) = \exp[\ln (C)] = C \Rightarrow V' > 0, V'' = 0$ RRA = 0; risk neutral

Example of affine transformation with constant relative risk aversion

$U(C) = [C^{1-R} - 1]/[1-R]$, $R<1$

$U' = C^{-R}$, $U'' = -RC^{-R-1}$; $- CU''/ U' = C\, RC^{-R-1}/ C^{-R} = R$

Positive Affine transf:

multiply $U(C)$ by $1-R = a$, add $[1-R]/[1-R] = b$,

$V(C) = C^{1-R}[1-R]/[1-R] - [1-R]/[1-R] + [1-R]/[1-R]$

$= C^{1-R}$

(if $R>1$, multiply by $-(1-R)$, positive affine transform.)

Why can't we measure risk aversion by 2nd derivate of utility?

- After all the sign of U'' determines whether risk averse/neutral/lover.
- Suppose U'' measures of risk aversion.
- If we take an affine trasformation of a generic U(.), e.g. V(.) = kU(.)+b, (with $0<k<\infty$, b any real number) consumer behavior will not change.
- Instead the candidate measure of risk aversion (that is U'') will change.

Why <u>can't</u> …

- Here is how
- U(.)=> U'(.)=> U''(.)
- V(.)=>V'(.)=kU'(.)=>V''(.)=kU''(.) hence individual would look like more/less risk averse; contradiction
- If instead we normalize U'' by U', then

  -U''/U'= -kU''/[kU'] = -V''/V'