Solutions of non-linear equations

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Lab of Fundamentals of Computational Mathematics

Summary

First exercise

Exercise

Complete the script `equation_solvers_wip.py` by substituting ??? with a correct piece of code.

Second exercise

Exercise

Let $f(x) = (x+1)log(x^2+1)$. In a new script:

1. Plot the function and spot the unique negative root z.
2. Apply the bisection method (import it from the other script) with $a = z - \pi/2$ and $b = z + 0.5$.
3. Print in exponential format the relative error between z and the approximation achieved by the bisection scheme.

Third exercise

Exercise

1. Apply the Newton-Raphson method (import it from the other script) with $x0 = a$.
2. Print in exponential format the relative error between z and the approximation achieved by the Newton-Raphson scheme.

Estimating the convergence order

We recall a practical way to estimate the convergence order. Observe that for *sufficiently large* k, we have $e_{k+1} \approx q e_k^p,\ e_k \approx q e_{k-1}^p$, which leads to

$$\frac{e_{k+1}}{e_k} \approx \left(\frac{e_k}{e_{k-1}}\right)^p,$$

and then

$$p \approx \frac{\log(e_{k+1}/e_k)}{\log(e_k/e_{k-1}|)} = \frac{\log(|x_{k+1}-\alpha|/|x_k-\alpha|)}{\log(|x_k-\alpha|/|x_{k-1}-\alpha|)}.$$

Since usually we do not know α, one considers the expression

$$p \approx \frac{\log(|x_{k+1}-x_k|/|x_k-x_{k-1}|)}{\log(|x_k-x_{k-1}|/|x_{k-1}-x_{k-2}|)}.$$

Fourth exercise

Exercise

For both bisection and Newton-Raphson, print the estimated convergence order computed via the presented formula.

Fifth exercise

Exercise

What happens if we apply Newton-Raphson with $x0 = -0.5$? By using again Newton-Raphson, provide an estimate of the critical value $c \in \mathbb{R}$ such that if $x0 \in (z, c)$ the method converges to z.

Sixth exercise

Exercise

Let $f(x) = \sin(\pi x) - 4x - 1$. In a new script, after plotting the function to spot (approximately) the root of f, find a proper function g to employ the fixed point scheme and to estimate the sought root.