PROBLEM - SET 0

Problem 1. Consider the random experiment of rolling two balanced dice with six faces and sum the two numbers that appears.

(a) Describe the probability space for this random experiment.

(b) Compute the probability to obtain an even number.

Problem 2. Instead of rolling two dice, assume now that we extract at random two balls without replacement from a box that contains six balls numbered from 1 to 6.

(a) Describe the probability space for this random experiment.

(b) Compute the probability to obtain two balls with consecutive numbers.

Problem 3. Let $\Omega = \mathbb{R}$ and define the following subset of 2^{Ω}

$$\mathscr{A} = \{A \subset \mathbb{R} : A \text{ is countable}\} \cup \{A \subset \mathbb{R} : A^c \text{ is countable}\}$$

(a) Prove that $\mathscr{A}$ is a σ–field (it is called the countable/co-countable σ–field)

(b) Prove that $A = (-\infty, 0]$ does not belong to $\mathscr{A}$.

Problem 4. Let $\Omega = \mathbb{N}$ and define

$$\mathscr{A} = \{A \subset \mathbb{N} : A \text{ or } A^c \text{ is finite}\}$$

Show that $\mathscr{A}$ is a field, but not a σ–field.

Problem 5. (a) Prove that the intersections of σ–fields is a σ–field.

(b) Given $\Omega = \{1,2,3,4,5,6\}$, define the minimal σ–field containing the sets $\{1\}$ and $\{2,4\}$.
Recall that given a collection of subsets $\mathscr{C}$ of Ω, the σ–field generated by $\mathscr{C}$, denoted $\sigma(\mathscr{C})$, is the σ–field satisfying:

(i) $\sigma(\mathscr{C}) \supset \mathscr{C}$

(ii) If $\mathscr{B}$ is a σ–field containing $\mathscr{C}$, then $\mathscr{B} \supset \sigma(\mathscr{C})$.