

Floating point representation and stability

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Lab of Fundamentals of Computational Mathematics

Summary

1 Floating point representation

2 Stability

First exercise

Exercise

The **machine precision** ϵ is the smallest machine number such that

$$1 + \epsilon > 1$$

Write a Python script that computes such value.

Hint: use a while cycle, start from 1 and divide by 2 in each iteration until...

Second exercise

Exercise

The following recursive sequence that can be traced back to Archimede

$$\begin{cases} z_2 = 2 \\ z_{n+1} = 2^{n-1/2} \sqrt{1 - \sqrt{1 - 2^{2-2n} z_n^2}} \quad n \geq 2 \end{cases}$$

converges to π as $n \rightarrow \infty$. Letting $n = 50$, produce a script where:

- 1 Save all the iterations z_2, \dots, z_n in a vector (or list) called `iteration_vec`.
- 2 Plot the relative error $|\text{iteration_vec} - \pi|/\pi$.
- 3 There's something wrong here... modify the recursive sequence in order to obtain a stable computation of π .

