HOW TO MEASURE RISK AVERSION

Notes from

Gollier, The Economics of risk and time, pp. 36-37

What is the percentage of your wealth (suppose your wealth is 2000 EUR) that you are willing to pay to eliminate a risk to gain or lose a given percentage α of your wealth ($\alpha = \{10\%, 30\%\}$) with the same probability?

Suppose we get answers from a group of people. The answers imply no assumption about theory. How to interpret these answers?

To interpret the answers we must make assumptions about the theory, that is about preferences of the individual.

Suppose that the individual has utility function

$$\begin{aligned} U(W) &= \frac{W^{1-R}-1}{1-R} \text{ if } R \neq 1 \\ U(W) &= \ln(W) \text{ if } R = 1 \end{aligned}$$

Observe:

$$\begin{aligned} U'(W) &= \frac{1-R}{1-R} W^{-R} = W^{-R} \\ U''(W) &= -RW^{-R-1} \\ -\frac{WU''(W)}{U'(W)} &= -\frac{-RW^{-R-1} \cdot W}{W^{-R}} = -\frac{-RW^{-R}}{W^{-R}} = R \end{aligned}$$

Thus R is the (constant) relative risk aversion coefficient.

CRRA = Constant Relative Risk Aversion.

$$\underbrace{(0.5)\,\frac{(1-\alpha)^{1-R}-1}{1-R}+(0.5)\,\frac{(1+\alpha)^{1-R}-1}{1-R}}_{\text{Expected utility}} = \underbrace{\frac{(1-\rho)^{1-R}-1}{1-R}}_{\text{Certainty equivalent utility}}$$

The answer to the question we posed before, is ρ namely the percentage of your wealth that your are willing to pay to eliminate this risk.

Observe that $1-\rho$ is the certainty equivalent income, since 1 is the average wealth

$$(0.5)\,(1-\alpha)+(0.5)\,(1+\alpha)=1,$$

and ρ is the risk premium.

To illustrate, take the case of $R = 1$ and $\alpha = 10\%$:

$$\begin{aligned}
&(0.5)\ln(1-0.1) + (0.5)\ln(1+0.1) = \\
&\ln(0.9)^{1/2} + \ln(1.1)^{1/2} = \ln(0.9 \cdot 1.1)^{1/2} = \\
&\ln(0.9949874) \simeq \ln(0.995)
\end{aligned}$$

Thus

$$(0.5)\ln(0.9) + (0.5)\ln(1.1) \simeq \ln(0.995) = \ln(1-0.005)$$

so that the risk premium is

$$0.005 = 0.5\% = \rho$$

which is what this individual is willing to pay to eliminate this risk.

For values of $\alpha = \{10\%, 30\%\}$ and different values of R we obtain the following matrix

R	$\alpha = 10\%$	$\alpha = 30\%$
0.5	0.3%	2.3%
1	0.5%	4.6%
4	2%	16%
10	4.4%	24.4%
40	8.4%	28.7%

To illustrate: the entry 0.5% says that an individual with Relative Risk Aversion =1 facing the risk of gaining/losing 10% of her wealth with the same probability is willing to pay 0.5% of her wealth to eliminate this risk.

Remarks

1. Many studies on large samples of population show that to avoid the risk of gaining/losing 10% with 50:50 chance, people are willing to pay between 0.5% and 2% of their wealth; hence R is between **1 and 4.**
2. A coefficient $R \geq 10$ looks "strange"; in fact it implies that you are willing to spend $\geq$ 4.4% to avoid losing 10%, and willing to spend $\geq$ 24.4% to avoid losing 30%.
3. Risk premiun is sensitive to **size** of initial wealth; in our example initial wealth was 2000 EUR. The larger the initial wealth the more you are willing to pay, studies show.
4. Studies show that people tend to be risk neutral for small risks (Rabin Paradox)

Empirical measure of risk aversion

Source Zweifel and Eisen, Insurance Economics , Springer Verlag, 2012

From Arrow-Pratt approximation, risk premium depends positively on risk aversion. In turn, demand of insurance depends positively on risk premium. Hence, to figure out demand of insurance it is important to know how risk aversion changes in population.

Risk aversion likely to vary within a population. Three imporant factors:

1. **Initial wealth**

   - final wealth depends on shocks + demand of insurance, hence it is endogenous. Initial wealh is exogenous in many cases. Focus on initial wealth
   - generally confirmed by emprical studies HP of Arrow and Pratt that Absolute risk aversion declines with wealth; $\frac{dA}{dW} < 0$
   - From $R = WA(W)$ it follows that

   $$\frac{dR}{dW} = A(W) + \underbrace{\frac{dA}{dW}}_{<0} W$$

   even if $\frac{dA}{dW} < 0$ sign of $\frac{dR}{dW}$ is in principle indetermined. Most empirical studies indicate to $\frac{dR}{dW} < 0$. For example richer people allocate a larger % of their wealth to riskier assets (Blake 1996). Other studies (Morris and Suarez 1983, Siegel and Hogan 1982) distinguish bewteen richer and poorer groups and find that $\frac{dR}{dW} < 0$ among the richer and $\frac{dR}{dW} > 0$ among the poorer.

2. **Age**

    - many studies show that risk aversion is U shaped w.r.t. age: first declines and then increases in the 55-70 age group; intuitive: people find it difficult to make up for a shortfall in income and wealth as they approach or pass retirement.

3. **Gender**

    - Countless studies show that women tend be more risk averse than men: e.g. women tend to have more health insurance coverage, more life insurance coverage.
    - However, these studies show **real data**, not ceteris paribus comparisons. These differences may reflect different starting points: e.g. women may have lower incomes/wealth; in controlled experiments where differences are controlled for, gender-specific differences in risk preferences disappear.

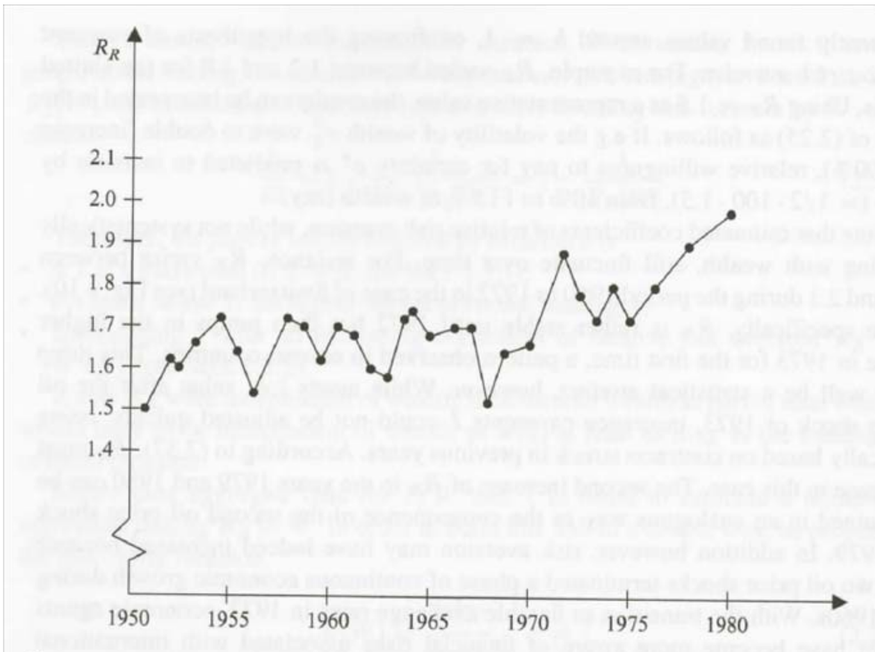

Fig. 2.10 Coefficient of relative risk aversion, Switzerland (1950–1980)
Source: Szpiro (1986b)

Source Zweifel and Eisen, Insurance Economics , Springer Verlag, 2012

PROSPECT THEORY

Notes from Campell, Lo, MacKinlay, The Econometrics of Financial Markets, Princeton University Press, 1997

- Behavioral models of decision making.
- The most famous formulation, alternative to the Von Neuman Morgestern exepected utility is the Prospect theory (Kahneman and Tversky 1979).
- Instead of defining preference over consumption, preference are defined over **gains** and **losses** relative to some benchmark outcome.
- A key feature of this theory is that losses are given greater weight than gains.

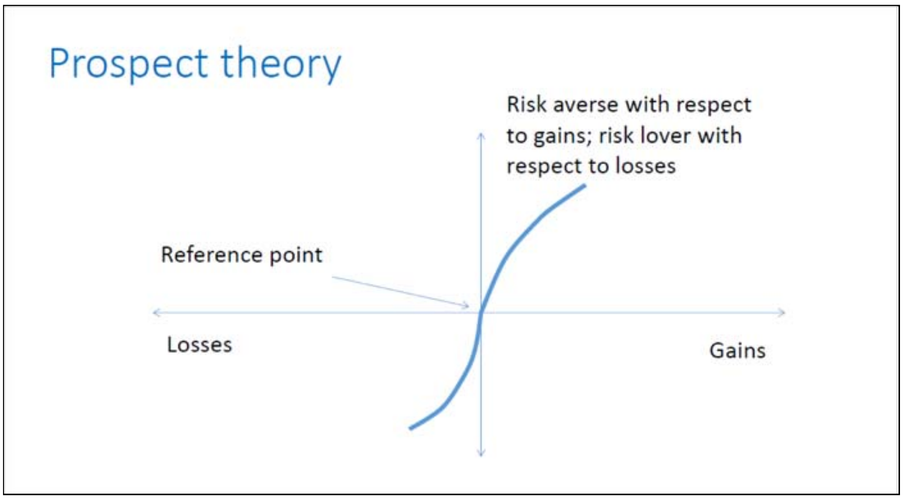

Figure 1: Prospect theory

If x is a random variable that is **positive** for gains and **negative** for losses, then the utility might be represented by

$$
\begin{aligned}
U(W) &= \frac{W^{1-\gamma_1}-1}{1-\gamma_1} \text{ if } x \geq 0 \\
U(W) &= \lambda\frac{W^{1-\gamma_2}-1}{1-\gamma_2} \text{ if } x < 0
\end{aligned}
$$

where γ_1, γ_2 are curvature parameters for gains and losses, which may differ from one another, and $\lambda > 1$ measures the extent of loss aversion, the greater weight given to losses than gains