INSURANCE DEMAND UNDER MORAL HAZARD

Probability of accident can be affected by behavior of insured

Behavior of insured may be difficult to observe for insurance company

Assume

- $\pi(x)$ = probability of accident
- x = action of insured; level of prevention; expressed in terms of monetary cost of prevention; UNOBSERVABLE (moral hazard) to insurance company; $x \geq 0$; (notice that if x is unobservable, then also $\pi(x)$ is unobservable)
- $\pi'(x) < 0$ the higher the level of prevention the smaller the prob. of accident
- Fair insurance

What happens if insurance company sets premium γ independent of x?

$$
\begin{aligned}
&\max_{\{K,x\}} V(K,x) \\
= \quad & \pi(x)\,U\left(\underbrace{W-D+K(1-\gamma)-x}_{C_2}\right)+(1-\pi(x))\,U\left(\underbrace{W-\gamma K-x}_{C_1}\right) \\
& s.t.\ K \leq D; x \geq 0
\end{aligned}
$$

Kuhn-Tucker conditions (corner conditions)

$$
\frac{\partial V}{\partial x}=\pi'(x)\left[U(C_2)-U(C_1)\right]-\pi(x)\left[U'(C_2)-U'(C_1)\right]-U'(C_1)\left\{\begin{array}{l}\leq 0 \text{ if } x=0\\ =0 \text{ if } x>0\end{array}\right. \tag{1}
$$

$$
\frac{\partial V}{\partial K}=\pi(x)\,U'(C_2)(1-\gamma)-\gamma(1-\pi(x))\,U'(C_1)=0 \tag{2}
$$

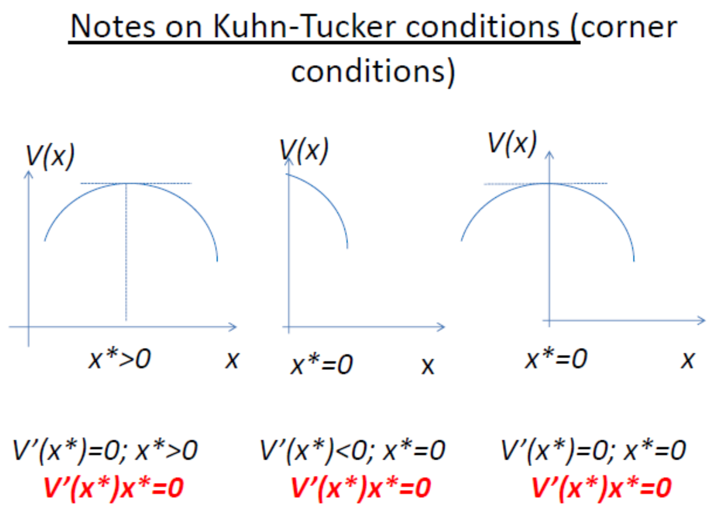

Figure 1: Kuhn-Tucker conditions

Back to model

Fair insurance requires that premium per unit = probability of damage

But now prob. of damage is endogenous; function of x

Must therefore calculate the level of x that will result in equilibrium; x^*

Fair insurance condition is

$$\gamma = \pi(x^*) \tag{3}$$

Remark: Notice that $\gamma = \pi(x^*)$ is a FIXED NUMBER, not a variable. The individual can buy insurance at the premium per unit $= \gamma = \pi(x^*)$; then individual decides the level of prevention, but unit premium will not change. Insurance cannot observe x but computes which x will be chosen as a function of incentives, that is as a function of γ.

Timing

1. Insurance company offers contract: $\gamma = \pi(x^*)$ and $K \leq D$
2. Individual sees the terms of the contract and chooses K, x and pays premium.
3. Nature determines whether accident happens or not. Damage liquidation happens in case of accident.

From equations (2) and (3) it follows that

$$U'(C_2) = U'(C_1)$$

that is full insurance.

Given this, then equation (1) becomes

$$-U'(C_1) \leq 0 \text{ if } x = 0$$
$$-U'(C_1) = 0 \text{ if } x > 0$$

But from utility theory we know that it cannot be

$$-U'(C_1) = 0 \Leftrightarrow U'(C_1) = 0$$

Hence it must be

$$-U'(C_1) < 0 \Leftrightarrow U'(C_1) > 0$$

which implies

$$x^* = 0$$

Meaning: the level of prevention chosen by individual is ZERO.

Thus the resulting probability of acccident is HIGH; $\pi(0)$; recall infact that $\pi'(x) < 0$.

The fair premium is HIGH $\gamma = \pi(0)$.

What would happen if insurance company could observe level of prevention x?

Insurance company could establish a unit premium which changes (no longer a fixed number) as a function of $\pi(x)$

That is $\gamma = \pi(x)$

Timing

1. Insurance company offers contract: $K \leq D$; insurance knows that it will be able to observe x when the individual will choose it and thus to offer a premium $\gamma = \pi(x)$
2. Individual sees the terms of the contract and chooses K, x, and pays premium.
3. Nature determines whether accident happens or not. Damage liquidation happens in case of accident.

Consumer chooses

$$\max_{\{K,x\}} V(K,x) =$$

$$\pi(x)\,U\left(\underbrace{W - D + K - \pi(x)\,K - x}_{C_2}\right) + (1-\pi(x))\,U\left(\underbrace{W - \pi(x)\,K - x}_{C_1}\right)$$

$$s.t.\ K \leq D; x \geq 0$$

First order conditions

$$\frac{\partial V}{\partial x} = \pi'(x)\left[U(C_2) - U(C_1)\right] - \qquad \pi(x)\left(1 + \pi'(x)K\right)U'(C_2) - \tag{4}$$
$$-\left[1 - \pi(x)\right]\left(1 + \pi'(x)K\right)U'(C_1) \quad = \quad 0$$

$$\frac{\partial V}{\partial K} = \pi(x)\left[1 - \pi(x)\right]U'(C_2) - \pi(x)\left[1 - \pi(x)\right]U'(C_1) = 0 \tag{5}$$

Thus equation (5) => $U'(C_2) = U'(C_1)$ => full insurance $K^* = D$.

From equation (4) using $K^* = D$, we have

$$\begin{aligned}\frac{\partial V}{\partial x} = \pi'(x)\left[U(C_2) - U(C_1)\right] - \quad & \pi(x)\left(1 + \pi'(x)K\right)U'(C_2) - \\ - \left[1 - \pi(x)\right]\left(1 + \pi'(x)K\right)U'(C_1) \quad = \quad & 0\end{aligned}$$

=>

$$1 + \pi'(x)D = 0$$

$$\underbrace{1}_{\text{marginal cost of prevention}} \quad = \quad \underbrace{-\overbrace{\pi'(x)}^{<0}D}_{\text{marginal gain for society}}$$

Marginal cost of prevention; $1 = dx/dx$

Marginal gain from prevention; determined multiplying the reduction in the prob. of accident $(\pi'(x))$ times the loss from accident (D)

$$1 + \pi' (x) D = 0$$

implicitly determines x^*, the level of prevention that equates marginal cost of prevention = marginal gain from prevention

Since in choosing x individual FULLY internalizes the consequences of his actions (because he pays $\gamma = \pi(x)$), the choice that he makes is SOCIALLY EFFICIENT.

Individual bears FULL cost of his actions

FIRST BEST

- insurance is full; $K^* = D$
- the level of prevention chosen, x^*, is such that for society marginal cost of prevention = marginal gain from prevention; $1 + \pi'(x^*) D = 0$
- insurance is fair; $\gamma = \pi(x^*)$

Observations

- in general x^*, resulting from $1 + \pi' (x^*) D = 0$ is such $x^* > 0$,
- compare with $x^* = 0$ when x is unobservale; thus $\pi (x^* > 0) < \pi (x^* = 0)$
- information improves efficiency

How can insurance company increase level of prevention above $x^* = 0$ if it cannot observe x?

Insurance company can demand COINSURANCE

Split the problem in two parts:
- mathematical solution
- implementation of the mathematical solution

Mathematical solution

Consider the program $\wp$

$$\max_{\{K,x\}} V(K,x) =$$

$$\pi(x)\,U\left(\underbrace{W - D + K - \gamma K - x}_{C_2}\right) + (1 - \pi(x))\,U\left(\underbrace{W - \gamma K - x}_{C_1}\right)$$

$$s.t.\ K \leq D; x \geq 0$$

$$\pi(x^*) = \gamma \quad \text{fair insurance}$$

$$\pi'(x)\left[U(C_2) - U(C_1)\right] - \pi(x)\left[U'(C_2) - U'(C_1)\right] - U'(C_1) = 0 \qquad (6)$$

Constraint (6)

$$\pi'(x)\left[U(C_2) - U(C_1)\right] - \pi(x)\left[U'(C_2) - U'(C_1)\right] - U'(C_1) = 0$$

it is (1) satisfied with =0; => $x^* > 0$

In words: In program $\wp$ we ask the consumer to

- choose the K and x that he wants
- given premium γ, that will be fair for the level of prevention that results in equilibrium
- provided that he chooses a positive level of prevention; (1) satisfied with =0 =>$x^* > 0$

Focus on constraint (6)

- For (6) to be satisfied it must be $U'(C_2) \neq U'(C_1)$
- Why? Because when (1) was satisfied with < 0, then $U'(C_2) = U'(C_1) =>$ $U(C_2) = U(C_1)$

$$\pi'(x)\left[U(C_2) - U(C_1)\right] - \pi(x)\left[U'(C_2) - U'(C_1)\right] - U'(C_1) < 0$$

Focus on constraint (6) (cont.ed)

- So we established that $U'(C_2) \neq U'(C_1)$
- But will it be $U'(C_2) > U'(C_1)$ or $U'(C_2) < U'(C_1)$?
- It must be $U'(C_2) > U'(C_1)$ because $U(C_2) < U(C_1), => C_2 < C_1$; recall that must $K \leq D$
- Program $\wp$ determines: $K^*, x^*, \pi(x^*) = \gamma$
- $C_2 < C_1 => D > K^*$ individual is not fully insured. Call $K^* = \bar{K}$
- $\bar{K} < D$ means coinsurance

Focus on constraint (6) (cont.ed)

- Amount of coinsurance $D - \bar{K} > 0$
- Second best solution: it results from trade-off between the desire to induce a positive level of effort and desire to insure a risk averse individual. This compromise results in less than full insurance: coinsurance
- Food for thought: constraint (6) means that individual chooses level of prevention satisfiying a self-selection constraint, since it satisfies its FOC with equality
- Constraint (6) is (1) satisfied with =0; => $x^* > 0$; this is true of course only if parameters of problem are such $V'(x^*) = 0$, and $x^* > 0$; if instead parameters of problem are such that $V'(x^*) = 0$, and $x^* = 0$ then coinsurance will not induce positive prevention.

Implementation of the mathematical solution

How can insurance company implent the allocation $K^* = \bar{K}, x^*, \pi(x^*) = \gamma$? (remember insurance company does not observe x)

It can offer the individual an insurance contract specified as follow

- unit premium $\pi(x^*) = \gamma$
- $K \leq \bar{K}$

and let consumer choose x and K

That is

$$
\begin{aligned}
& \max_{\{K,x\}} V\left(K,x\right) \\
= \; & \pi\left(x\right) U\left(W-D+K-\gamma K-x\right)+\left(1-\pi\left(x\right)\right) U\left(W-\gamma K-x\right) \\
& \text{s.t. } \; x \geq 0 \\
& \gamma=\pi\left(x^{*}\right) \\
& K \leq \bar{K}<D
\end{aligned}
$$

Since $\bar{K} < D$ then individual is (partially) exposed to consequences of his actions, and will choose a positive level of prevention. Less than first best, though.

$K \leq \bar{K} < D$ means coinsurance

Coinsurance (partially) exposes individual to consequences of his actions; hence it mitigates moral hazard.

Like in the case of demand of insurance with adverse selection, also under moral hazard contract is defined in terms of price ($\gamma = \pi(x^*)$) and quantity ($K \leq \bar{K}$).

COINSURANCE IN THE REAL WORLD

source:
https://www.scontopolizza.it/blog/primo-rischio-assoluto-differenza-assicurazione-valore-intero/

1. **First absolute loss** (in Italian, Primo Rischio Assoluto). Suppose you choose to insure $K = \bar{K} < D$ and, in different model where damage is variable, incurred loss is $d < \bar{K}$, then you are entitled to receive d. Example: you insure you property against theft for $K = \bar{K}$ even if you potentially stand to lose up to D. Suppose theft happens and you lose $d < K$; you are entitled to receive d. Numerical example: Fire insurance with max 65K euro (**insured** value) on apartment worth 100K. Suppose in case of fire the damage you suffer is 20K; then you are entitled to receive FULL value of the damage 20K. (same applies for damages < 65K). No need to specify insurable value, 100K in this case.

2. **Full value insurance** (in Italian, Valore Intero). Numerical example: max: 65.000 euro; value of the apart. 100.000 euro (**insurable** value, declared). It follows that the you are not insuring (100.000-65.000)/100.000 = 35% of the value. Thus 35% is proportional rule that is going to be applied to the damage to liquidate you. If the damage is 20.000 euro, you are entitled to receive 20.000 – 7.000 (35% x 20.000) = 13.000 euro. Hence you are suffering a loss of 7.000 euro.

3. **First relative loss.** (In Italian, Primo Rischio Relativo) It is between 1 and 2. We need to know 2 values: **insured** value (65.000) and **insurable** value; Cost to to replace the good = 100.000. Fire insurance. Suppose you suffer damage 10.000. If you chose insurable value 100.000 = cost to replace good, then you get 10.000 back. However, if you choose insurable value 70.000, in case of damage 10.000 you receive (100.000-30.000)/100.000 = 70% of 10.000, namely 7.000. Of course you can never receive more than 65.000. Comments: a) The **rationale** is to induce the insured to assess constantly the ratio between the insured the declared value, renewal after renewal; b) the **premium** is intermediate between first absolute risk and first value insurance. The largest is the ratio between the amount insured and the value declared, the more the premium is close to that at first absolute risk.

EXERCISE ON DEMAND OF INSURANCE UNDER MORAL HAZARD

This question is inspired to the model of insurance demand under moral hazard seen in class and uses its assumptions and notation unless otherwise specified. A risk-neutral insurance company offers an insurance policy to a risk-averse individual whose wealth (W=100) is exposed to a damage (D=80) with probability $\pi(x) = \frac{1}{2}$ exp(-x) where x $\geq$ 0 is the monetary cost of the prevention level exerted by the individual, unobservable to the insurance company. Insurance premium per unit γ will be fair on the basis of the probability of damage that results in equilibrium, that is $\gamma = \pi(x^*)$. At the unit premium $\gamma = \pi(x^*)$ the individual can buy insurance coverage K up to K $\leq$ D. The utility function of the individual is ln(consumption) so that its Von Neuman-Morgestern expected utility is

$$V(K,x) = \pi(x)\ln(C2) + (1-\pi(x))\ln(C1)$$

where C2 and C1 denote consumption with and without damage, respectively.

a) Find the level of prevention that the individual chooses, the premium that it will pay, and the insurance coverage that it will buy. (neglect the case in which the individual remains in autarky)

b) Suppose now that prevention level exerted by the individual is observable to the insurance company that can therefore tie the unit premium $\gamma = \pi(x)$ to the prevention exerted by the individual. Find the level of prevention that the individual chooses, the unit premium that it will pay, and the insurance coverage that it will buy.

S O L U T I O N

a) We know that under the above assumptions (see lecture notes) dV/dx < 0 so that x* = 0.

Hence $\pi(x^*) = \frac{1}{2}$ exp(-x*) = $\frac{1}{2}$. Hence $\gamma = \pi(x^*)$=1/2. From the FOC for dV/dK = 0 it

follows that

$\pi(x)$U'(C2)(1- γ)- γ(1- $\pi(x)$) U'(C1) = 0.

Thus using $\gamma = \pi(x^*)$=1/2 and U(C)= ln(C) => U'(C) = 1/C, we have that

(1/4)[1/(20 +K– K/2)] = (1/4)[1/(100 – K/2)] =>

1/(20 + K/2) = 1/(100 – K/2)

100 – K/2 = 20 + K/2 => K* = 80

To recap: x*=0; $\gamma = \pi\,(x^{*})$=1/2; K* = 80.

b) We know (see lecture notes) that under observability of the prevention level we will have

K* = 80 and the FOC for x satisfies 1 + $\pi'\,(x)$D = 0. Hence 1 = - $\pi'\,(x)$80. Since

$\pi\,(x) = \frac{1}{2}$ exp(-x), then $\pi'\,(x)$ = - $\frac{1}{2}$ exp(-x).

Therefore we have 1 = $\frac{1}{2}$ exp(-x)80 =>

1/40 = exp(-x) => ln(1/40) = -x = > - ln(1) + ln(40) = x* = ln(40) = 3.688

Thus from $\pi\,(x) = \frac{1}{2}$ exp(-x)

=> 1/40 = exp(-x) => 1/80 = (1/2)exp(-x) => π(ln(40)) = γ=1/80.