Group10 Second Assignment (part 2)

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0.1 Train/Test split (time-ordered)

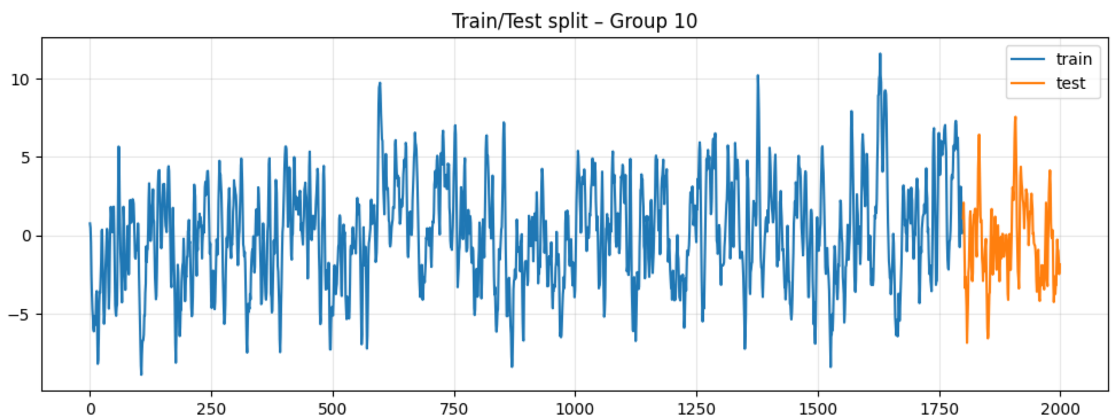
We evaluate forecasting performance out-of-sample using a time-ordered split. The last `H` observations are kept as test set, while the remaining observations are used for training. This mirrors the real forecasting setting where future data are not available at estimation time.

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt

df = pd.read_csv("arma12_seasonal_controlled_15series.csv")
y = df["series_10"].astype(float).to_numpy()

H = 200  # test horizon (typical choice: 100-300 depending on length)
y_train, y_test = y[:-H], y[-H:]

plt.figure(figsize=(12, 4))
plt.plot(y_train, label="train")
plt.plot(np.arange(len(y_train), len(y)), y_test, label="test")
plt.title("Train/Test split - Group 10")
plt.grid(alpha=0.3)
plt.legend()
plt.show()
```



0.2 Model fitting

Following the Box–Jenkins identification performed in the previous assignment, we fit the selected ARMA(p,q) model on the training set.

[2]:
```python
from statsmodels.tsa.arima.model import ARIMA

# ARMA(1,2) = ARIMA(1,0,2)
fit = ARIMA(y_train, order=(1, 0, 2), trend="n").fit()
print(fit.summary())
```

```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 1800
Model:                 ARIMA(1, 0, 2)   Log Likelihood               -2536.299
Date:                Tue, 16 Dec 2025   AIC                           5080.598
Time:                        10:28:11   BIC                           5102.580
Sample:                             0   HQIC                          5088.713
                               - 1800
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1          0.7972      0.017     47.577      0.000       0.764       0.830
ma.L1          0.7062      0.022     31.449      0.000       0.662       0.750
ma.L2          0.5113      0.022     23.595      0.000       0.469       0.554
sigma2         0.9787      0.034     29.064      0.000       0.913       1.045
================================================================================
===
Ljung-Box (L1) (Q):                   0.67   Jarque-Bera (JB):
1.57
Prob(Q):                              0.41   Prob(JB):
0.46
```

```
Heteroskedasticity (H):                  1.09   Skew:
0.04
Prob(H) (two-sided):                     0.30   Kurtosis:
2.89
================================================================================
===

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-
step).
```

0.3 Point forecasts and accuracy

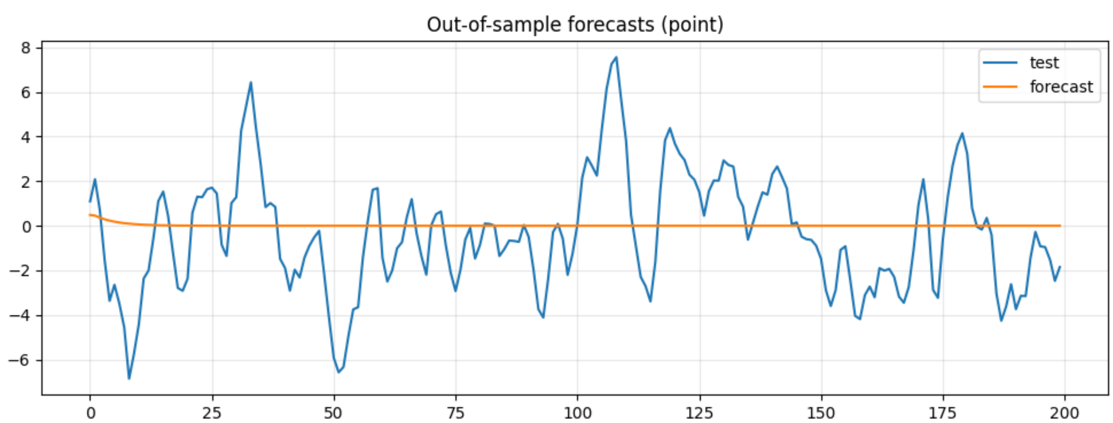
We produce H-step-ahead forecasts for the test period and evaluate accuracy using standard loss functions (MAE and RMSE).

[3]:
```python
pred = fit.get_forecast(steps=H)
y_hat = pred.predicted_mean

mae = np.mean(np.abs(y_test - y_hat))
rmse = np.sqrt(np.mean((y_test - y_hat) ** 2))
print("MAE:", mae)
print("RMSE:", rmse)

plt.figure(figsize=(12, 4))
plt.plot(y_test, label="test")
plt.plot(y_hat, label="forecast")
plt.title("Out-of-sample forecasts (point)")
plt.grid(alpha=0.3)
plt.legend()
plt.show()
```

```
MAE: 2.10458178927103
RMSE: 2.637130596618867
```



0.3.1 Rolling 1-step (se richiesto / consigliato)

[4]:
```
from statsmodels.tsa.arima.model import ARIMA

history = list(y_train)
roll_forecasts = []

for t in range(H):
    m = ARIMA(history, order=(1, 0, 2), trend="n").fit()
    roll_forecasts.append(m.forecast(steps=1)[0])
    history.append(y_test[t])

roll_forecasts = np.array(roll_forecasts)

mae_r = np.mean(np.abs(y_test - roll_forecasts))
rmse_r = np.sqrt(np.mean((y_test - roll_forecasts) ** 2))
print("Rolling MAE:", mae_r)
print("Rolling RMSE:", rmse_r)
```

```
Rolling MAE: 0.8045793501230344
Rolling RMSE: 1.0131027773912855
```

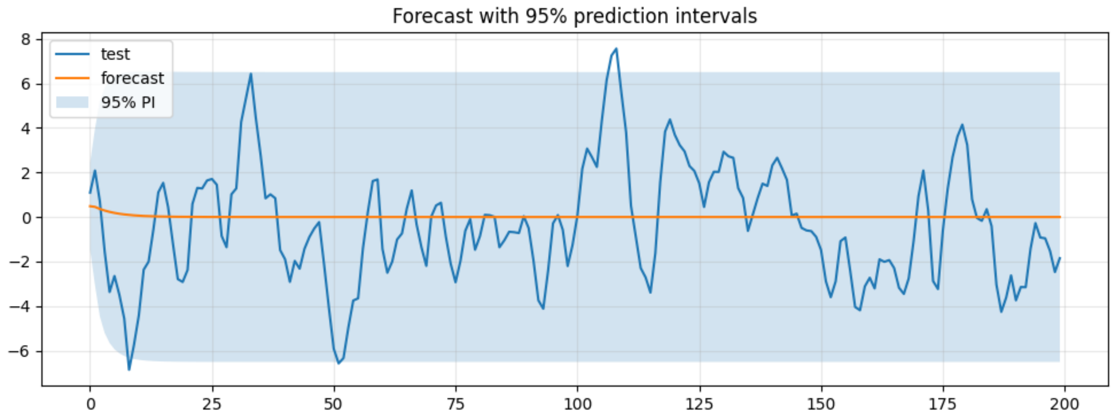
0.4 Interval forecasts

We compute 95% prediction intervals. In a correct ARMA specification, most test observations should fall within these bands, and the interval width reflects forecast uncertainty increasing with the horizon.

[5]:
```
ci = pred.conf_int(alpha=0.05)  # 95%
lower = ci[:, 0]
upper = ci[:, 1]

plt.figure(figsize=(12, 4))
plt.plot(y_test, label="test")
plt.plot(y_hat, label="forecast")
plt.fill_between(np.arange(H), lower, upper, alpha=0.2, label="95% PI")
plt.title("Forecast with 95% prediction intervals")
plt.grid(alpha=0.3)
plt.legend()
plt.show()

coverage = np.mean((y_test >= lower) & (y_test <= upper))
print("Empirical coverage:", coverage)
```




```
Empirical coverage: 0.98
```

0.5 Benchmark comparison

We compare ARMA forecasts against a simple naive benchmark: $\hat{y}_{t+1} = y_t$. Despite its simplicity, the naive model is often hard to beat in short-horizon forecasting.

[6]:
```
naive = np.r_[y_train[-1], y_test[:-1]]  # forecast at t uses previous observed␣
 ↪value
mae_n = np.mean(np.abs(y_test - naive))
rmse_n = np.sqrt(np.mean((y_test - naive) ** 2))
print("Naive MAE:", mae_n)
print("Naive RMSE:", rmse_n)
```

```
Naive MAE: 1.015481502653484
Naive RMSE: 1.2632077429018165
```