

# Debt and GDP

November 6, 2025

## 1 Linear Regression Analysis

In this project, we simulate a simple linear relationship between public debt (as a percentage of GDP) and the logarithm of GDP across a set of synthetic countries. The purpose is to demonstrate the application of Ordinary Least Squares (OLS) regression and to visualize the fitted line with its 95% confidence interval.

### 1.1 Imports

In this cell, we import the Python libraries required for data analysis and visualization:

- **pandas**: used for creating and manipulating tabular data in the form of DataFrames
- **statsmodels.api**: provides the statistical tools needed to perform the Ordinary Least Squares (OLS) regression and obtain the model summary
- **matplotlib.pyplot**: used to produce visualizations such as scatter plots and fitted regression lines
- **numpy**: provides numerical operations and random number generation used to create synthetic data

We also apply a clean plotting style (`seaborn-whitegrid`) to improve the readability of the charts in the final report

```
[1]: import pandas as pd
import statsmodels.api as sm
import matplotlib.pyplot as plt
import numpy as np

plt.style.use("seaborn-v0_8-whitegrid")
```

### 1.2 Data generation

In this step, we create a synthetic but realistic dataset that mimics the relationship between a country's public debt (as a percentage of GDP) and its Gross Domestic Product (GDP).

We simulate data for 16 countries, each identified by a code (C1, C2, ..., C16):

- The variable `Debt_to_GDP` is generated from a uniform distribution between 40% and 140%
- The variable `log_GDP` (the natural logarithm of GDP) is modeled as a linear function of the debt ratio:

$$\log(GDP) = 26 + 0.005 \times Debt\_to\_GDP + \varepsilon$$

where  $\varepsilon$  represents random noise drawn from a normal distribution

We then exponentiate `log_GDP` to obtain the actual GDP values (`GDP_USD`).

This step produces a dataset with a controlled positive correlation between debt and GDP, suitable for demonstrating the linear regression procedure.

```
[2]: np.random.seed(42)

countries = ["C" + str(i) for i in range(1, 17)]
debt_ratio = np.random.uniform(40, 140, size=16)

log_gdp = 26 + 0.005 * debt_ratio + np.random.normal(scale=0.08, size=16)
gdp = np.exp(log_gdp)

df = pd.DataFrame(
    {
        "Country": countries,
        "Debt_to_GDP": debt_ratio,
        "GDP_USD": gdp,
        "log_GDP": log_gdp,
    }
)

print(df.head())
```

	Country	Debt_to_GDP	GDP_USD	log_GDP
0	C1	77.454012	2.939357e+11	26.406627
1	C2	135.071431	3.299790e+11	26.522295
2	C3	113.199394	3.002867e+11	26.428004
3	C4	99.865848	3.083022e+11	26.454346
4	C5	55.601864	2.383445e+11	26.196983

### 1.3 Model fitting

In this cell, we apply a simple linear regression model to estimate the relationship between the logarithm of GDP (`log_GDP`) and the public debt ratio (`Debt_to_GDP`).

The model specification is:

$$\log(GDP_i) = \beta_0 + \beta_1 \times Debt\_to\_GDP_i + \varepsilon_i$$

where:

- $\beta_0$  is the intercept (baseline level of  $\log(GDP)$  when debt = 0)
- $\beta_1$  measures the change in  $\log(GDP)$  for each additional percentage point of debt
- $\varepsilon_i$  is the random error term

We use the Ordinary Least Squares (OLS) method from the `statsmodels` library, which estimates the coefficients by minimizing the sum of squared residuals.

The resulting `summary()` table provides:

- estimated coefficients (`coef`)
- standard errors
- t-statistics and p-values (for significance testing)
- the coefficient of determination ( $R^2$ ), indicating how much of the variation in  $\log(GDP)$  is explained by the debt ratio

This output allows us to assess both the strength and the statistical significance of the relationship.

```
[3]: X = sm.add_constant(df["Debt_to_GDP"])
y = df["log_GDP"]
model = sm.OLS(y, X).fit()

print(model.summary())
```

```
OLS Regression Results
=====
Dep. Variable: log_GDP R-squared: 0.814
Model: OLS Adj. R-squared: 0.800
Method: Least Squares F-statistic: 61.10
Date: Thu, 06 Nov 2025 Prob (F-statistic): 1.79e-06
Time: 15:51:22 Log-Likelihood: 21.187
No. Observations: 16 AIC: -38.37
Df Residuals: 14 BIC: -36.83
Df Model: 1
Covariance Type: nonrobust
=====

            coef    std err          t      P>|t|      [0.025      0.975]
-----
const      26.0389   0.049    533.397      0.000     25.934     26.144
Debt_to_GDP  0.0041   0.001      7.816      0.000      0.003      0.005
=====
Omnibus: 4.221 Durbin-Watson: 2.117
Prob(Omnibus): 0.121 Jarque-Bera (JB): 2.019
Skew: 0.813 Prob(JB): 0.364
Kurtosis: 3.619 Cond. No. 265.
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

## 1.4 Visualization of the regression line and confidence interval

This cell produces a scatter plot of the data and overlays the fitted regression line, along with its 95% confidence interval.

Steps performed:

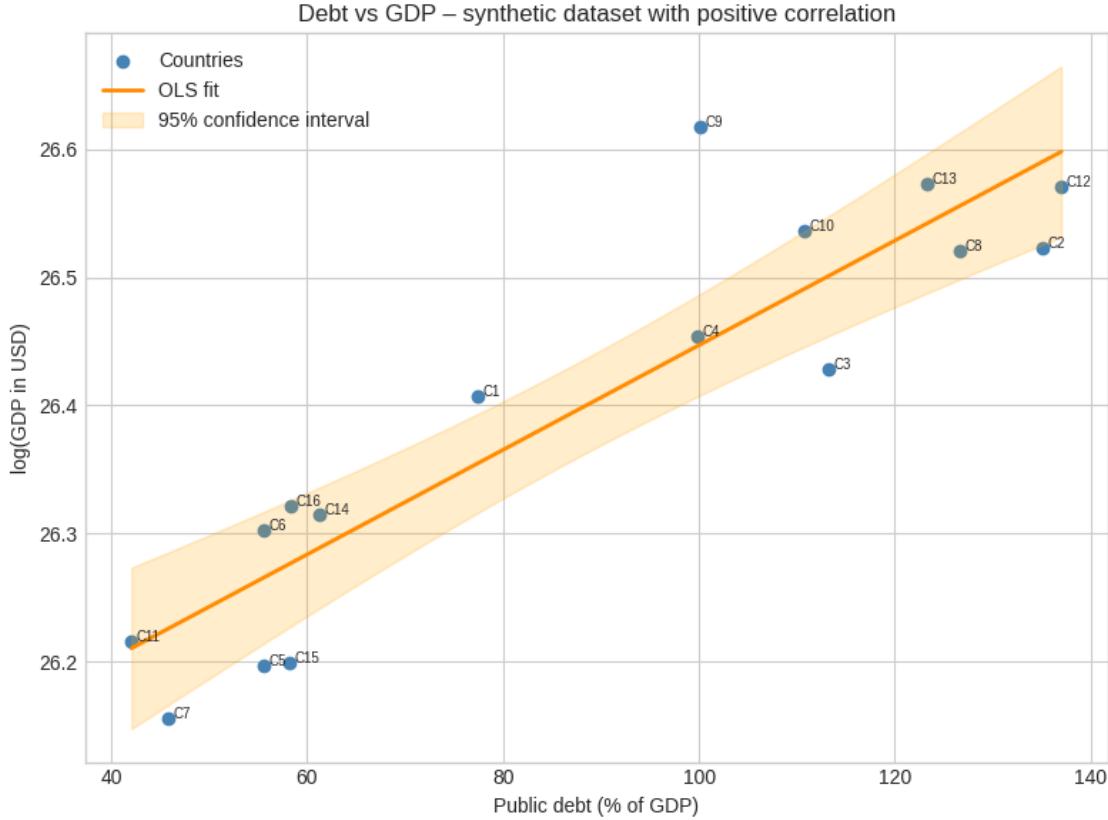
1. A smooth grid of `Debt_to_GDP` values is created using `numpy.linspace()`
2. For each value, the model predicts the corresponding `log(GDP)` and computes the confidence bounds using the `get_prediction()` method from `statsmodels`
3. The following graphical elements are then displayed:
  - **Blue points:** observed data for each country
  - **Orange line:** fitted regression line (`OLS fit`)
  - **Shaded orange area:** 95% confidence interval around the fitted line
  - **Country labels:** added next to each point for interpretability

The plot provides an intuitive visual understanding of the estimated relationship and the associated uncertainty. In this synthetic dataset, the upward-sloping line confirms a positive correlation between public debt and GDP in logarithmic scale.

```
[4]: x_line = np.linspace(df["Debt_to_GDP"].min(), df["Debt_to_GDP"].max(), 100)
X_pred = sm.add_constant(x_line)
pred = model.get_prediction(X_pred).summary_frame(alpha=0.05)

plt.figure(figsize=(8, 6))
plt.scatter(df["Debt_to_GDP"], df["log_GDP"], color="steelblue", ▾
            label="Countries")
plt.plot(x_line, pred["mean"], color="darkorange", linewidth=2, label="OLS fit")
plt.fill_between(
    x_line,
    pred["mean_ci_lower"],
    pred["mean_ci_upper"],
    color="orange",
    alpha=0.2,
    label="95% confidence interval",
)
for _, row in df.iterrows():
    plt.text(row["Debt_to_GDP"] + 0.5, row["log_GDP"], row["Country"], ▾
             fontsize=7)

plt.xlabel("Public debt (% of GDP)")
plt.ylabel("log(GDP in USD)")
plt.title("Debt vs GDP - synthetic dataset with positive correlation")
plt.legend()
plt.tight_layout()
plt.show()
```



## 2 Interpretation of results

The estimated coefficient  $\beta_1$  represents the expected change in the logarithm of GDP for a one-percentage-point increase in the debt-to-GDP ratio.

Since the dependent variable is in logarithmic form,  $\beta_1$  can be approximately interpreted as a percentage change in GDP. For example, if  $\beta_1 = 0.005$ , it means that an increase of 1 percentage point in debt/GDP is associated with an estimated 0.5% increase in GDP (in this synthetic model).

The regression summary also provides:

- the **p-value**, which indicates whether the estimated coefficient is statistically significant;
- the **R-squared**, showing the proportion of variation in  $\log(\text{GDP})$  explained by the debt ratio.

The plot visually confirms this positive linear relationship, and the 95% confidence band illustrates the uncertainty surrounding the fitted line. Overall, this example demonstrates the standard workflow of a simple linear regression: data generation, model estimation, and interpretation of statistical and graphical results.

### 2.1 Automatic textual interpretation

This optional cell extracts the estimated regression coefficient ( $\beta_1$ ) and the corresponding p-value from the fitted model and generates an automatic, human-readable interpretation.

Specifically:

- `beta1` stores the estimated slope coefficient associated with `Debt_to_GDP`;
- `pval` stores its p-value, indicating statistical significance;
- the formatted output expresses the estimated percentage change in GDP corresponding to a 1-point increase in the debt-to-GDP ratio.

This cell is useful for producing a concise textual summary that can be directly included in a written report or presentation slide.

```
[5]: beta1 = model.params["Debt_to_GDP"]
pval = model.pvalues["Debt_to_GDP"]
print(f"Estimated coefficient: {beta1:.4f}")
print(
    f"Interpretation: A 1% increase in debt/GDP corresponds to an estimated "
    f"{beta1*100:.2f}% increase in GDP."
)
print(f"p-value: {pval:.3f}")
```

```
Estimated coefficient: 0.0041
Interpretation: A 1% increase in debt/GDP corresponds to an estimated 0.41%
increase in GDP.
p-value: 0.000
```