Reading Group on Stochastic Modelling

A brief overview of:

Incomplete Simultaneous Discrete Response Model with Multiple Equilibria

Tamer (2003)

Review of Economic Studies

In relation to:

A Structural Model of Dense Network Formation

Mele (2017)

Econometrica

Mele (2017) footnote n.17 p.830

"The **second part of the assumption 1 [see below]** *is an identification restriction, that guarantees the model's* **coherency** *in the sense of Tamer (2003)."*

Individual i **values his popularity** effect as much as k **values the indirect link to** j through any "bridging" individual:

$$\underbrace{w_{kj}^{\theta_v} = w(X_k, X_j|\theta_v)}_{\substack{\textbf{indirect link:} \\ \text{utility for } k\neq i,j \\ \text{for indirectly linking with } j \\ \text{through any } i\neq k,j \\ (k)\to(i)\to(j)}} = \underbrace{v(X_k, X_j|\theta_v) = v_{kj}^{\theta_v}}_{\substack{\textbf{popularity:} \\ \text{utility for any } i\neq k,j \\ \text{for bridging } k \text{ to } j \\ (k)\to(i)\to(j)}} \quad \forall k, j \in \mathcal{I} \times \mathcal{I}$$

Under assumption 1.2 any individual $i \in \mathcal{I}$ **internalizes all the externalities generated by his links**:

- The **popularity** component of $U_i(g, \mathbf{X}|\Theta)$ is **equal to** the sum, over all $j \in \mathcal{I} - i$, of the utility of **indirect links of j passing through i**, which are the indirect links that can be influenced by i's link-formation **decisions**;

Summary of Tamer (2003)

Modelling framework - **2x2 entry-game with perfect information**

two players ($i \in \{-1, 1\}$), action set ($y_i \in \{0, 1\}$) and externalities δ_i. The payoff π_i of player i is defined as:

$$\pi_i := y_i(x_i\beta_i + y_{-i}\delta_i + u_i)$$

Where:

- $\mathbf{y} = (y_{-1}, y_1)$ are response variables;
- $\mathbf{x} = (x_{-1}, x_1) \in \mathcal{R}^d$ are observable exogenous variables;
- $\mathbf{u} = (u_{-1}, u_1)$ are random variables unobservable to the econometricians;
- $\beta = (\beta_{-1}, \beta_1, \delta_{-1}, \delta_1)$ are parameters of interest;

Distinction of model identification issues - **Incoherency Vs Incompleteness**

1− **incoherent model**: hasn't a well-defined reduced form, or, is logically inconsistent. For example:
if externalities δ_{-1} and δ_1 are both negative, the above model gives
$Pr[(0,0)|x] + Pr[(0,1)|x] + Pr[(1,0)|x] + Pr[(1,1)|x] > 1$

2− **incomplete model**: the relationship from input variables (x_is and u_is) to responses (y_is) is a **correspondence** and not a function. For example:
if δ_is are both negative, $\exists$ a non-empty region of $\mathbf{u}$'s *support for which the model predicts a non-unique outcome* $(1, 0)$ OR $(0, 1)$

Contribution and findings of Tamer (2003)

- **Identifies sufficient conditions for parameter point identification (when externalities have same sign);**
- **Develops a technique for semi-parametric maximum (quasi)likelihood (SML) estimation:** by "replacing" $Pr[(y_{-1}, y_1)|x]$ for outcomes (0,1) and/or (1,0), with local approximations of the the empirical relative frequencies of these outcomes as a function of exogenous variables;

Why Assumption 1.2 relevant for identification in Mele (2017)?

1. **externalities are "paired"**: each **indirect-link** effect has a **corresponding popularity effect** with same sign, value and parameter;
2. **number of parameters of the model is reduced**: from 4 $(\theta_u, \theta_m, \theta_w, \theta_v)$ to 3 $(\theta_u, \theta_m, \theta_v)$. Condition necessary for model completeness (?);
3. guarantees that the system of **conditional linking probabilities** implied by the model **generate a proper joint distribution** of the network matrix;
4. can **use** the potential function $\mathcal{Q}$ **to construct the network as a best-response potential game**. Via **sequential** link-formation **decisions** the game converges through an **improvement path** to a **Pure Strategy Nash Equilibrium** network with $Pr = 1$;

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