Geometric classification of 4d rank-1 $\mathcal{N} = 2$ superconformal field theories

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20th century physics

Two revolutions at the beginning of last century

Theory of relativity (SR/GR) Quantum Mechanics (QM)

Combining these:

$$\text{SR} + \text{QM}$$
$$=$$

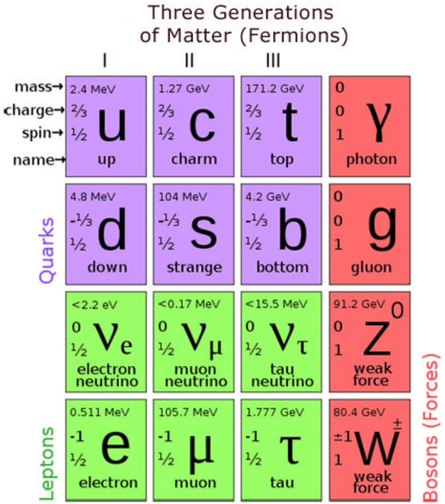
Quantum Field Theory (QFT)

Extremely fruitful framework:

- condensed matter
- particle physics

QFT and the Standard Model

The Standard Model of Particle Physics is the prime example of a QFT.

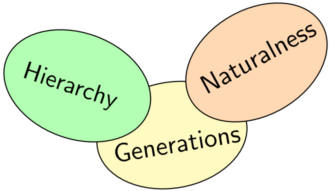


- Used in the past 50 years for all microscopic interactions;
- Successful in predictions.

Failure of weakly coupled descriptions?

Shortcomings

- No reason for structure,
- specific values of parameters,
- mass (of scalar) term can run freely;



More importantly for us,

Analytic calculations valid only in weakly coupled regimes.

Problematic comparing to (lack of) new observations.

Weak vs. strong coupling calculations

The question I want to address is:

How can we do calculations in QFTs at strong coupling in a controlled approximation?

There are numerical results (lattice gauge theories).
But, from a conceptual point of view, they don't tell us much,

Alternatives to perturbative calculations

We want to improve - go beyond the standard perturbative description of field theory models.

But what can we do?

Alternatives to perturbative calculations

We want to improve - go beyond the standard perturbative description of field theory models.

But what can we do?

1. Perform numerical simulations discretizing space-time: Lattice gauge theories;
2. Add additional symmetries constraining models: e.g. supersymmetric theories;
3. Depart from standard field theory picture: String theory scenarios.

More symmetries?

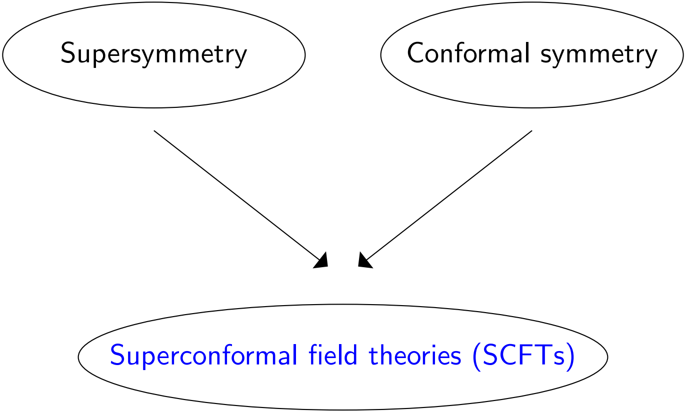
We will focus on studying field theories with additional symmetries.

Why and how do these work?

- Conformal symmetry:
  theories can be studied with sophisticated methods without writing down Lagrangians;
- Supersymmetry:
  constraints on types and structure of potential terms that can be turned on.

Example:
In SM the mass of the Higgs field m_h (or its vev v) is a free parameter, not protected $\implies$ can get arbitrary corrections.
In SUSY theories, loop corrections are protected, normally only one loop.
Certain potential terms are forbidden at all.

Superconformal field theories



Conformal invariance

Conformal theories can be seen as special QFTs at fixed points of renormalization group flows, where:

$$\beta(g) = 0$$

Because of this, there are powerful tools that let us study them.

Coupling constants don't vary with the scale.

<u>Scale invariance</u> is manifest in every aspect of the theory. This will be reflected (and exploited) in the properties of the geometries we study.

Supersymmetry

Symmetry transformations defined by anticommuting generators Q^a_α link bosonic and fermionic fields.

$$\phi \leftrightarrow \psi$$

SUSY theories are powerful because they constrain:

- Parameters: often receive up to one loop corrections;
- Potential terms: holomorphic functions (of the chiral fields).

$$\Downarrow$$

Models are more constrained and under control compared to ordinary QFTs.

Generators Q^a_α have indices; a $(= 1, \ldots, \mathcal{N})$ specifies the amount of supersymmetry ($\mathcal{N}$).

How much SUSY?

How many supersymmetries ($\mathcal{N}$) do we want?

For theories without gravity, in 4d, we can have:

$$\mathcal{N} = 1 \qquad \leftrightarrow \qquad \mathcal{N} = 4$$

The more SUSY, the more constrained the theories are.

- $\mathcal{N} = 1$: many phenomenological models, still too much freedom;
- $\mathcal{N} = 4$: only one type of multiplet, very constrained;
- $\mathcal{N} = 3$: necessarily non lagrangian theories, hard to compare to "known" cases.

$\mathcal{N} = 2$ is the sweet spot

- Extended moduli spaces of vacua with enough constraints that allow us to study exact properties of underlying theories.

How can we study $\mathcal{N} = 2$ SCFTs?

With the main ingredients, what are the approaches that we can use to study $\mathcal{N} = 2$ SCFTs?

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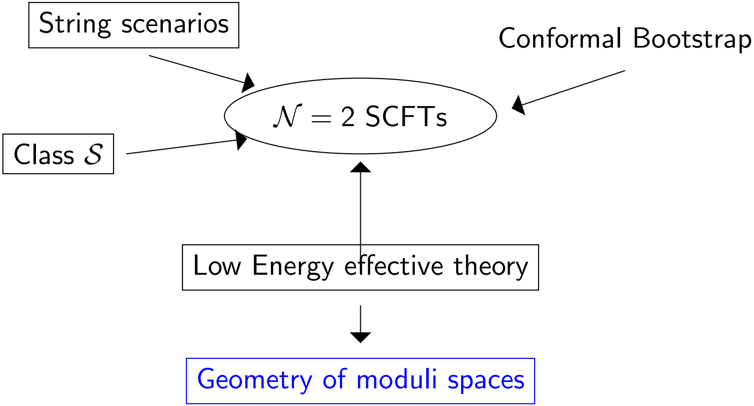

Lagrangian vs. Geometrical data

- Why do we use these methods?
  An easier approach would be to write down the $\mathcal{N} = 2$ Lagrangian, analyze its field content and properties, requiring the vanishing of the beta functions (for conformal theories).

Problem

Not all $\mathcal{N} = 2$ SCFTs have a lagrangian description!

- For $\mathcal{N} = 2$ theories we have more sophisticated techniques.

By studying the geometry of the *moduli spaces of vacua*, we aim to extract properties of the underlying SCFT.

What is a moduli space?

Moduli spaces of vacua

In field theories scalar fields can acquire vacuum expectation values (vevs). Familiar from SM:

$$\text{Higgs} \quad \phi \to \langle \phi \rangle + h = v + h$$

Vev (v) responsible for masses of matter particles.
At the same time a vev induces spontaneous symmetry breaking. In SM:

$$SU(2)_L \times U(1)_Y \to U(1)_{EM}$$

Vevs minimize potential of the theory.
In SUSY theories there is more structure since scalars have superpartners with higher spin. This prevents the occurrence of some potential terms.

$$\Downarrow$$

We can have multiple flat directions → extended moduli spaces of vacua.

The Coulomb branch

An $\mathcal{N} = 2$ moduli space of vacua has multiple components.
We will focus our attention on the Coulomb branch of the moduli space.

- A Coulomb branch is parametrized by vev(s) of the scalar in the vector multiplet (related by SUSY to the gauge bosons of the original theory). In general the gauge group is broken to

$$\mathcal{G} \to U(1)^r$$

, where $r =$ rank of $\mathcal{G}$.

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Our arena of study will be the planar rank-1 Coulomb branches, parametrized by a coordinate u.
We will exploit the residual gauge symmetry.

Rank 1 Coulomb branches

What does rank mean?

Extension of a notion from lagrangian theories. Vev on the CB has number of parameters = rank of the gauge group.

Rank = complex dimension

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Extension of a notion from lagrangian theories. Vev on the CB has number of parameters = rank of the gauge group.

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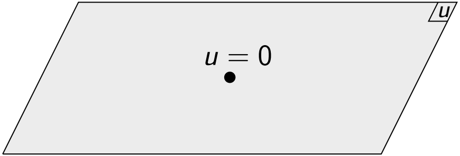
of the Coulomb branch.

A planar rank 1 CB is parametrized by one complex coordinate u.
For a conformal theory it has to be a scale invariant space:



At $u = 0$ the vev is zero, so the conformal symmetry is restored.

Our program in context

Our program and results

Let's recap what are the objects we work with and what we obtain.

- The general idea is to formulate the whole problem of classifying $\mathcal{N} = 2$ SCFTs in purely geometrical terms;

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Our program and results

Let's recap what are the objects we work with and what we obtain.

- The general idea is to formulate the whole problem of classifying $\mathcal{N} = 2$ SCFTs in purely geometrical terms;
- We constrain the structure of Coulomb branches, first in the scale invariant case, later deforming these simple geometries in order to parametrize the presence of matter fields in the original theory;
- After listing the possible deformation of each scale invariant CB, we are able to determine the global symmetry structure of a large number of SCFTs using two methods that complement each other;
- A large class of the theories we describe were not known in the literature previously. They were later found using alternative approaches.

Review of previous work

Let's put our work in historical context among the literature on the subject.

- Seiberg and Witten (SW - '94) studied $\mathcal{N} = 2$ SU(2) theories, two of them SCFTs, showing how to describe them from a geometrical point of view, through the moduli space;

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- Construction from string theory ('98-01) give a unified interpretation of the rank-1 $\mathcal{N} = 2$ known up to this stage;
- Our classification:
  1. Determine all possible deformations of CB geometries, that can correspond to SCFTs;
  2. Extend SW + MN algebraic approach to all possible deformed geometries;
  3. Generalize string construction to field theory point of view, again describing all possible CB geometries;

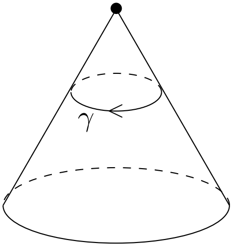
Basics of CB physics

(from SW original work: hep-th/9407087, hep-th/9408099)

Scale invariant Coulomb branches

Going back to the description of rank-1 CBs.

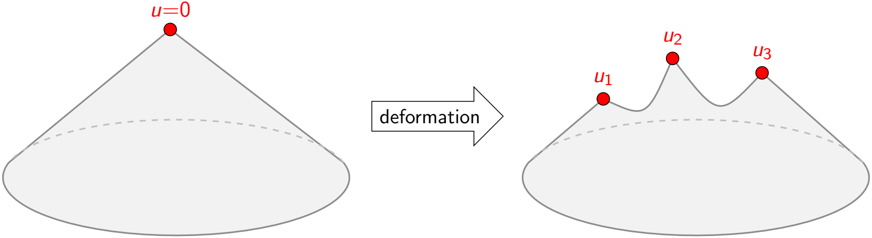
Physically, we have one free parameter, the vev of the scalar field.
This has to be proportional to the distance to the origin of the CB, the only parameter in the geometry.
This fixes the geometries to be scale invariant flat cones.



There may be non trivial features at the tip.

Generic deformation

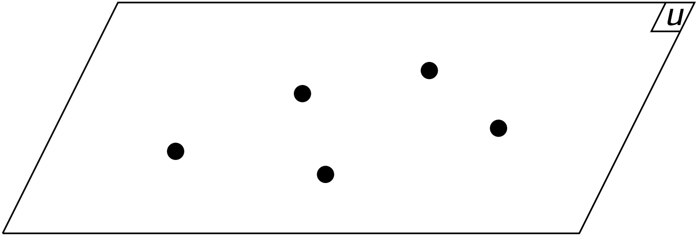
Effect of a mass term = deformation of the geometry of the CB is:



The singular point at the origin splits into a series of (lesser) singularities.

Structure of Coulomb Branches (I)

A generic rank-1 Coulomb branch geometry will then look like a complex plane with a series of punctures.



- Singular points carry the non trivial information on the geometry of the space.
- Singular point $\leftrightarrow$ failure in the low energy description of the theory.

Again, what are these punctures?

- The Coulomb branch describes the IR effective regime of a theory.
- Singularities in the space are a failure in the IR description.
- These are interpreted as matter fields becoming massless at those points. Each singular point is naturally labeled by the charges of the corresponding light field.

Structure of Coulomb Branches (II)

Again, what are these punctures?

- The Coulomb branch describes the IR effective regime of a theory.
- Singularities in the space are a failure in the IR description.
- These are interpreted as matter fields becoming massless at those points. Each singular point is naturally labeled by the charges of the corresponding light field.

The pattern of singularities on the Coulomb branch gives us information on the symmetry structure of the underlying SCFT.

Low energy gauge theory

- On a rank 1 CB, the pattern of symmetry breaking is:

$$\mathcal{G} \to U(1)$$

The low energy gauge theory is an $\mathcal{N} = 2$ supersymmetric version of QED.

- Each state is labeled by a vector of electric and magnetic charges. The gauge coupling of this theory can be written as:

$$\tau \equiv \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$$

combining the standard coupling g, with the θ parameter.
The latter appears in the presence of magnetic monopoles, providing additional features in the theory.

Electric-Magnetic duality

In the presence of electric and magnetic monopoles, there are equivalent descriptions of the same theory.

- Reversing what we call electric vs. magnetic charges and fields provides an equivalent description of the theory, where the weakly coupled states are magnetic monopoles instead of the usual electrically charged states. This is parametrized by:
$$\tau \xrightarrow{S} -\frac{1}{\tau}.$$
- A shift by 2π in the θ parameter, which is a topological quantity, is not physical. This corresponds to:
$$\tau \xrightarrow{T} \tau + 1.$$

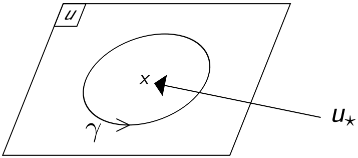
Combining these S and T transformations gives rise to the whole group $SL(2, \mathbb{Z})$.

Electric-Magnetic duality (2)

Combining the action of the S and T transformations we obtain a general $SL(2,\mathbb{Z})$ electric-magnetic duality transformation on the gauge coupling:

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad M_\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

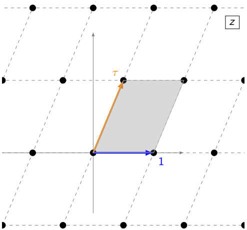
A closed loop (γ) on the Coulomb branch defines a monodromy transformation M_γ.



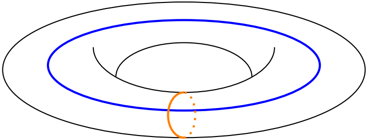
If γ encircles a singular point ($u = u_\star$), we will have a non trivial monodromy

τ as a complex torus

Because of the $SL(2,\mathbb{Z})$ transformations, the gauge coupling τ can be described in purely geometrical terms as a complex torus:



$\longleftrightarrow$



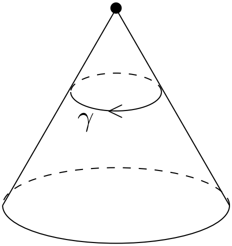
- Different lattice points are distinct EM duality frames. Going from one to another corresponds to moving on the closed loop γ;
- This changes the basis of EM charges and basis of cycles defining the torus;
- The torus, however, as a topological object, remains the same, i.e. the physics described is equivalent.

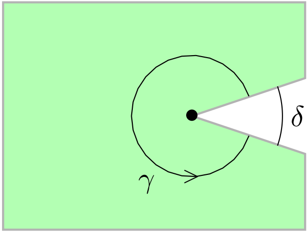
Our work

Appeared in:	arXiv:1505.04814,	JHEP 1802 (2018) 001;
	arXiv:1601.00011,	JHEP 1802 (2018) 002;
	arXiv:1609.04404,	JHEP 1802 (2018) 003
	arXiv:1602.02764,	JHEP 1605 (2016) 088
	arXiv:1806.xxxxx.	to appear

Connecting geometries and SCFTs

Recall that a SCFT has a scale invariant CB.
Imposing physical requirements - EM duality and unitarity - constrains the allowed possibilities:





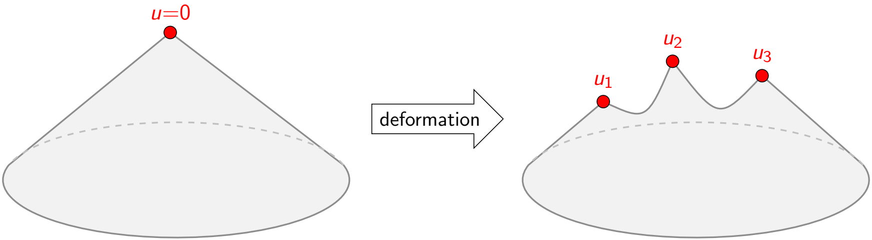
There is only a discrete set of allowed deficit angles δ.

$$\left\{ \begin{array}{c|ccccccc} \delta & 5\pi/3 & 3\pi/2 & 4\pi/3 & \pi & 2\pi/3 & \pi/2 & \pi/3 \\ \Delta(u) & 6 & 4 & 3 & 2 & 3/2 & 4/3 & 6/5 \\ K_0 & II^* & III^* & IV^* & I_0^* & IV & III & II \end{array} \right\}$$

Deformations

But we can't have only seven possibilities.

Introducing deformations of the CFT, e.g. mass terms in the superpotential, modifies the CB structure

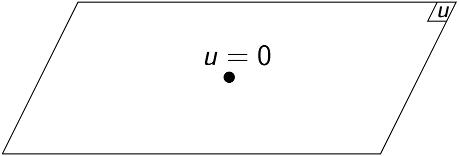


splitting the singularity at the conformal vacuum.

We explicitly deform the conformal theory to extract the properties hidden within it, i.e. in the singularity at the origin.

Rank-1 planar Coulomb Branches

A rank-1 CB appears as:



CFT ($m = 0$)

Conformal vacuum at $u = 0$
Singularity at the origin
characterized by EM
monodromy $M_0 \in SL(2, \mathbb{Z})$.

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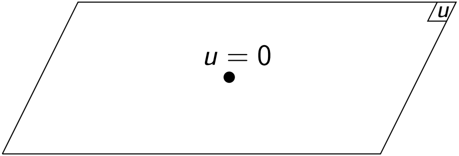
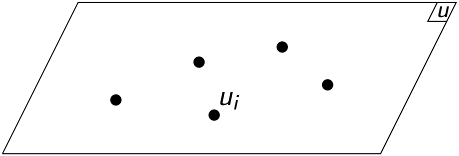


CFT ($m = 0$)

Conformal vacuum at $u = 0$
Singularity at the origin characterized by EM monodromy $M_0 \in SL(2, \mathbb{Z})$.

Turning on masses:



QFT ($m \neq 0$)

Singularity at $u = 0$ splits into multiple sings at $u = u_i(m)$, each with their EM monodromy M_i.

Allowed singular points

What are the kind of singularities and monodromies M_i's that are allowed?

Each of the split singularities have monodromies M_i in a specific conjugacy class of $SL(2,\mathbb{Z})$. There are three families:

- one of the seven scale invariant types: $(II^*, III^*, IV^*, I_0^*, IV, III, II)$;
- I_n (for $n \geq 1$) singularities $\to [T^n]$ monodromies;
- I_n^* (for $n \geq 1$) singularities $\to [-T^n]$ monodromies.

The last two are not scale invariant geometries, but have a lagrangian interpretation on their own as IR free $U(1)$ and $SU(2)$ gauge theories, respectively (n is the coefficient in the one loop beta function).

Allowed deformation patterns

Imposing physical and geometrical constraints, we obtain the set of generic allowed deformations of a scale invariant CB:

sing.	$\mathrm{ord}_0(D_x)$	deformation pattern
II^*	10	$\{{I_1}^{10}\}$, $\{{I_1}^6, I_4\}$, $\{{I_1}^2, I_4^2\}$, $\{{I_1}^4, I_0^*\}$, $\{{I_1}^3, I_1^*\}$, $\{I_3, I_1^*\}$, $\{I_1^2, I_2^*\}$, $\{I_1, I_3^*\}$, $\{I_1, III^*\}$, $\{{I_1}^2, IV^*\}$, $\{I_2, IV^*\}$
III^*	9	$\{{I_1}^9\}$, $\{{I_1}^5, I_4\}$, $\{{I_1}^3, I_0^*\}$, $\{{I_1}^2, I_1^*\}$,$\{I_2, I_1^*\}$, $\{I_1, I_2^*\}$, $\{I_1, IV^*\}$
IV^*	8	$\{{I_1}^8\}$, $\{{I_1}^4, I_4\}$, $\{I_2, I_0^*\}$, $\{I_1, I_!^*\}$
I_0^*	6	$\{{I_1}^6\}$, $\{{I_1}^2, I_4\}$, $\{{I_2}^3\}$
IV	4	$\{{I_1}^4\}$
III	3	$\{{I_1}^3\}$
II	2	$\{{I_1}^2\}$

We obtain three families of deformations: ;

- Maximal deformations ($\{I_1^\alpha\}$);
- I_4 series;
- Frozen singularities (III^*, IV^* and I_n^*).

What do we do with this ?

Directions for the classification

At this stage we have to find a map:

$$\text{Allowed CB geometries} \longrightarrow \text{Physical theories}$$

Each allowed deformation - overall CB geometry - has to be given a physical interpretation in terms of a field theory.
We can do this in two ways:

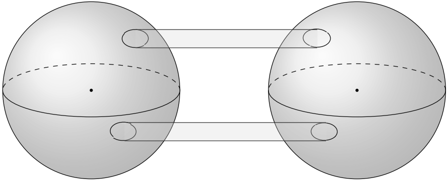
- Algebraic approach: we rewrite the torus fiber as an elliptic curve and study mass terms as part of the algebraic structure;
- Topological approach: we focus solely on the configuration of singularities on the CB and associate it to a lattice of the global symmetry algebra.of the physical theory.

The algebraic approach

How is a torus described as an elliptic curve?

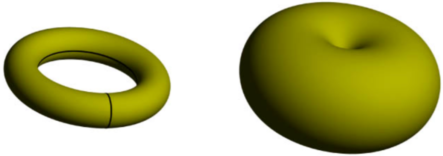
$$y^2 = x^3 + f(u)x + g(u)$$

The double sheeted x-plane is a pair of Riemann spheres connected along branch cuts, defined by the zeros of the polynomial in the RHS.



Singularities on the CB occur when two of the branch points coincide. The discriminant of the curve Δ keeps track of this.

$$\Delta = 4f^3 + 27g^2$$



Zeros of Δ are points where the torus degenerates.

Deformations in the algebraic picture

By.introducing deformations - mass terms $\{M_\alpha\}$ - to a CFT, the elliptic curve changes:

$$y^2 = x^3 + f(u)x + g(u) \quad \to \quad y^2 = x^3 + f(u, M_\alpha)x + g(u, M_\alpha)$$

A given deformation determines a factorization of the discriminant Δ. From this, we summarize our construction:

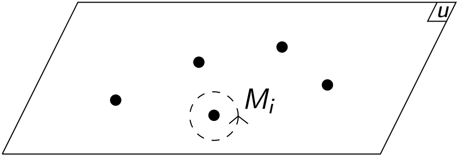
- Find a set of parameters $\{M_\alpha\}$ to give a factorization of the discriminant matching the deformation - set of singularities selected on the CB;
- Match set of deformation parameters with an assignment of symmetry structure: associate them to linear masses $\{M_\alpha = M_\alpha(m_i)\}$ and verify the transformation properties under the action of the Weyl group.

Set of deformation parameters $\leftrightarrow$ Assignment of global symmetry structure to a SCFT

Info on symmetry algebra: rank + Weyl group

The topological approach

We take the structure of the deformed CB, with its set of monodromies, which contain the physical information.



We can associate charges to singular points:
$M_i \leftrightarrow z_i = (p_i, q_i) \in \mathbb{Z}^2$.

For simple I_1 type singularities these are the EM charges of the massless states.

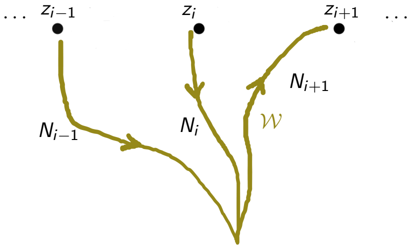
We define web prongs carrying away a corresponding "charge" from each singularity.

Lattice of neutral webs

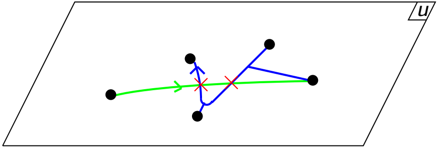
A generic web carries both global symmetry and electromagnetic charges.

We construct neutral webs stretched between singular points (no prong going off to infinity).

- They are neutral under EM;
- They can carry global "charges".



The space of these webs defines a lattice:



Lattice of neutral webs $\leftrightarrow$ Root lattice of the SCFT symmetry

Info on global symmetry algebra: rank + root lattice

Results

Let us summarize what we obtain with the two methods:

- Algebraic approach:
  we determine the number of linear mass parameters (m_i) for a given geometrical structure. This tells us the rank of the algebra.
  Acting on the linear masses we find the Weyl group action.
  The B_n and C_n algebras, corresponding to $SO(2n+1)$ and $Sp(2n)$, have the same Weyl group, so we can't distinguish them.
- Topological approach:
  the number of singularities minus the neutrality constraints tells us the rank of the lattice.
  Mapping it to a global symmetry algebra, we fix the root lattice Λ, not the root system.
  The ambiguity is at low ranks (e.g. $\Lambda(A_2) = \Lambda(G_2)$) and in general $\Lambda(C_n) = \Lambda(D_n)$, for the corresponding groups $Sp(2n)$ and $SO(2n)$.

Results combined

	Regular rank 1 N=2 SCFTs			
	Sing.	Deformation	Symmetry from curve	Lattice from webs
1.		$\{I_1{}^{10}\}$	E_8	E_8
2.		$\{I_1{}^6, I_4\}$	$\mathrm{sp}(10)$	FCC_5
3.		$\{I_1{}^2, I_4{}^2\}$	$\mathrm{sp}(4)$	CUB_2
4.		$\{I_0^*, I_1{}^4\}$	F_4	F_4
5.		$\{I_1^*, I_1{}^3\}$	$\mathrm{sp}(6)$ or $\mathrm{so}(7)$	FCC_3
6..	II^*	$\{I_1^*, I_3\}$	$\mathrm{su}(2)$	CUB_1
7.		$\{I_2^*, I_1{}^2\}$	$\mathrm{sp}(4)$	CUB_2
8.		$\{I_3^*, I_1\}$	$\mathrm{su}(2)$	CUB_1
9.		$\{I_2, IV^*\}$	$\mathrm{su}(2)$	CUB_1
10.		$\{I_1{}^2, IV^*\}$	G_2	HEX_2
11.		$\{I_1, III^*\}$	$\mathrm{su}(2)$	CUB_1
12.		$\{I_1{}^9\}$	E_7	E_7
13.		$\{I_1{}^5, I_4\}$	$\mathrm{su}(2) \oplus \mathrm{sp}(6)$	$\mathrm{CUB}_1 \oplus \mathrm{FCC}_3$
14.		$\{I_0^*, I_1{}^3\}$	$\mathrm{so}(7)$	CUB_3
15.	III^*	$\{I_1^*, I_1{}^2\}$	$\mathrm{su}(2) \oplus \mathrm{su}(2)$	$\mathrm{CUB}_1 \oplus \mathrm{CUB}_1$
16.		$\{I_1^*, I_2\}$	$\mathrm{su}(2)$	CUB_1
14.		$\{I_2^*, I_1{}^1\}$	$\mathrm{su}(2)$	CUB_1
18.		$\{I_1, IV^*\}$	$\mathrm{su}(2)$	CUB_1
19.		$\{I_1{}^8\}$	E_6	E_6
20.	IV^*	$\{I_1{}^4, I_4\}$	$\mathrm{u}(1) \oplus \mathrm{sp}(4)$	$\mathrm{CUB}_1 \oplus \mathrm{CUB}_2$
21.		$\{I_0^*, I_1^2\}$	$\mathrm{su}(3)$	HEX_2
22.		$\{I_1^*, I_1\}$	$\mathrm{u}(1)$	CUB_1
23.		$\{I_1{}^6\}$	$\mathrm{so}(8)$	FCC_4
24.	I_0^*	$\{I_2{}^3\}$	$\mathrm{sp}(2)$	CUB_1
25.		$\{I_1{}^2, I_4\}$	$\mathrm{sp}(2)$	CUB_1
26.	IV	$\{I_1{}^4\}$	$\mathrm{su}(3)$	HEX_2
27.	III	$\{I_1{}^3\}$	$\mathrm{su}(2)$	CUB_1
28.	II	$\{I_1{}^2\}$	—	$\emptyset$

Thank you!

Back-up

$\mathcal{N} = 2$ Superpotential

What are the non trivial features of a Coulomb branch?

Let's look back at lagrangian theories. For a $U(1)$ gauge theory we can write a superpotential:

$$\mathcal{W} = \sum_i Q^i(\phi + \mu_i)\tilde{Q}^i$$

governing interactions between the scalar getting a vev ϕ and the matter fields Q_is (hypermultiplets).
Upon Higgsing the theory ($\phi \to \langle\phi\rangle$), we obtain:

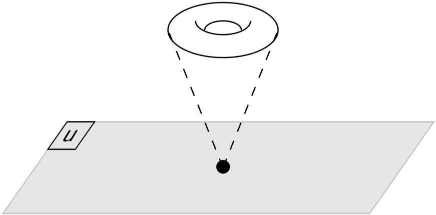
$$\mathcal{W} = \sum_i Q^i(\langle\phi\rangle + \mu_i)\tilde{Q}^i$$

- There are now special points - singularities - on the CB, where the effective mass of a matter field vanishes.
- These are points where the low energy effective description fails.

Total space of the CB

- Recall the problem we want to address is to specify the symmetry structure of a SCFT given the structure of singular points on its CB;
- The information on the singularities is encoded in the set of monodromies $\{M_\gamma\}$. These map the EM duality transformations, as a shift on a lattice.

We geometrize the EM duality transformations by introducing the total space of the CB, a T^2 fibered over each point on the base (degenerating appropriately at singularities).



Our goal is to determine the physics of the SCFTs by looking at the problem as a purely geometrical one, studying the topology of the total space.