Speed of Light in Air

Electromagnetic waves represent energy in the form of oscillating electric and magnetic fields which propagate through vacuum with a speed $c = 2.9979246\text{x}10^8$ m/s. Electromagnetic radiation which is visible to the human eye is called 'light'. The visible portion of the electromagnetic spectrum covers a range of wavelengths from about 700 to 400 nm (or a frequency range of about $4.3\text{x}10^{14}$ to $7.5\text{x}10^{14}$ Hz). When light travels in a medium, the electric field causes electrons in the medium to oscillate. These accelerated charges absorb energy and re-emit it in the form of electromagnetic waves which get superimposed on the original wave. If one assumes that the atomic electrons are bound by a spring-like force characterized by a natural frequency ω_0, one can show that the index of refraction in the medium for light of frequency ω is

$$n_{\text{med}}(\omega) \cong 1 + \frac{\eta_e q_e}{2\varepsilon_0 m_e (\omega_0^2 - \omega^2)} , \qquad (1)$$

where q_e = charge on an electron,
m_e = mass of an electron,
η_e = number of electrons per unit volume.
ε_0 = permittivity of free space,

Table I: Results for T = 15°C and P = 760 mmHg

λ [nm]	$(n_{\text{air}} - 1)$ [x10^{-4}]	λ [nm]	$(n_{\text{air}} - 1)$ [x10^{-4}]	λ [nm]	$(n_{\text{air}} - 1)$ [x10^{-4}]
400	2.817	500	2.781	600	2.763
410	2.812	510	2.779	610	2.762
420	2.808	520	2.777	620	2.761
430	2.803	530	2.775	630	2.760
440	2.799	540	2.773	640	2.759
450	2.796	550	2.771	650	2.758
460	2.792	560	2.769	660	2.757
470	2.789	570	2.768	670	2.756
480	2.786	580	2.766	680	2.755
490	2.784	590	2.765	690	2.754

Our goal in this experiment is to measure the index of refraction of air at a single frequency in the visible portion of the spectrum. For air, the value of ω_0 falls in the ultraviolet range, so, throughout the visible region, we always have $\omega_0^2 > \omega^2$ and Eq. (1) predicts that $n(\omega)$ should gradually increase as we move from red to blue frequencies. Experimental results agree with this prediction as evidenced by the data shown for air in Table I. (In cases where $\omega_0^2 < \omega^2$, Eq. (1) gives $n(\omega) < 1$. This implies $v > c$, which may seem incorrect. However, v is the phase velocity of the electromagnetic wave at a single frequency, i.e., it is not the velocity at which a signal travels through the medium. A signal is composed of waves covering a range of ω-values, and the speed at which a signal propagates turns out to depend on the way $n(\omega)$ changes with frequency. This velocity does not turn out to be greater than c.)

As shown by the data in Table I, the value of $n(\omega)$ is very close to unity throughout the visible region of the spectrum; that is, the speed of light in air is very close to the speed of light in vacuum (e.g., at $\lambda = 400$ nm, $n_{air} = 1.0002817$). To detect this small difference we will use a very sensitive instrument known as a Michelson interferometer whose design is representative of a fairly large group of devices used in optics research. As illustrated in Fig. 1, filtered light from an extended source (i.e., monochromatic, but not necessarily coherent) illuminates a diffusing screen located at position A. Light from a particular point S on the screen strikes a partially-silvered mirror (the 'beam-splitter') which transmits half of the original beam through a glass compensator toward mirror M_1 and reflects the other half toward mirror M_2. The beam which travels toward M_1 is called the 'reference' beam, and the one that goes toward M_2 is called the 'sample' beam. The mirrors reflect both beams back to the beam-splitter where a portion of the light gets recombined on route to a detector located at D. The compensator at C insures that the distances traveled by the reference and sample beams through glass can be made equal (i.e., the compensator is made of the same type of glass, and has exactly the same thickness and orientation, as the beam-splitter). In practice, the total path lengths of the sample and reference beams do not have be exactly equal, just comparable.

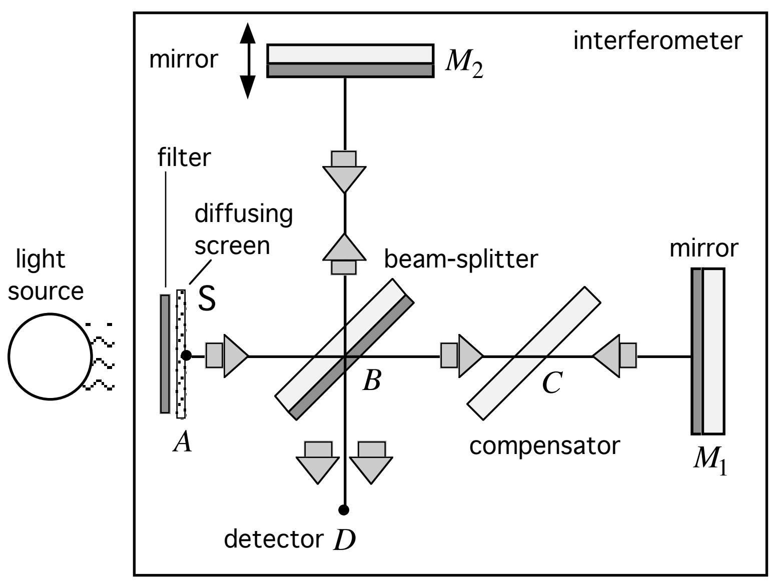

Fig. 1

Measurements are made by looking through the beam-splitter toward M_2. If the interferometer is properly aligned, an example of what one should see is illustrated in Fig. 2(a), namely, concentric bands of light representing alternating areas of constructive and destructive interference. To understand how this pattern comes about, we can utilize the rules of geometric optics which state that when an point source is viewed in a plane mirror, the reflected light is equivalent to that from a virtual point source equidistant from the plane of the mirror on the opposite side. Applying this rule to M_1, M_2, and the beam-splitter, one can show that the overall effect of the interferometer is to produce a pair of virtual images of the diffusing screen at two slightly different distances behind M_2 as depicted in Fig. 2(b). The virtual source points S_1 and S_2 are images of the point S, and P_1 and P_2 correspond likewise to the point P. Although the waves emanating from points S and P are generally not in phase with each other, waves from the pair of points S_1 and S_2 must be in phase because they are both images of the same point. This is the case for any pair of image points (e.g., P_1 and P_2). Because they travel different distances, the waves from S_1 and S_2 will generally be out of phase when they arrive at D and an observable interference will occur there. The difference in path lengths from points P_1 and P_2 to point D will not be the same as that associated with S_1 and S_2. Looking into the spectrometer from point D, the path difference will vary as one looks along different directions. A bright band of light will be seen in those directions where pairs of image points have a path difference equal to an integer number of wavelengths and so produce constructive interference. In directions where the path length difference is an integer number of half-wavelengths, destructive interference will occur and a dark band will be seen. Therefore, the field of view will resemble that shown in Fig. 2(a).

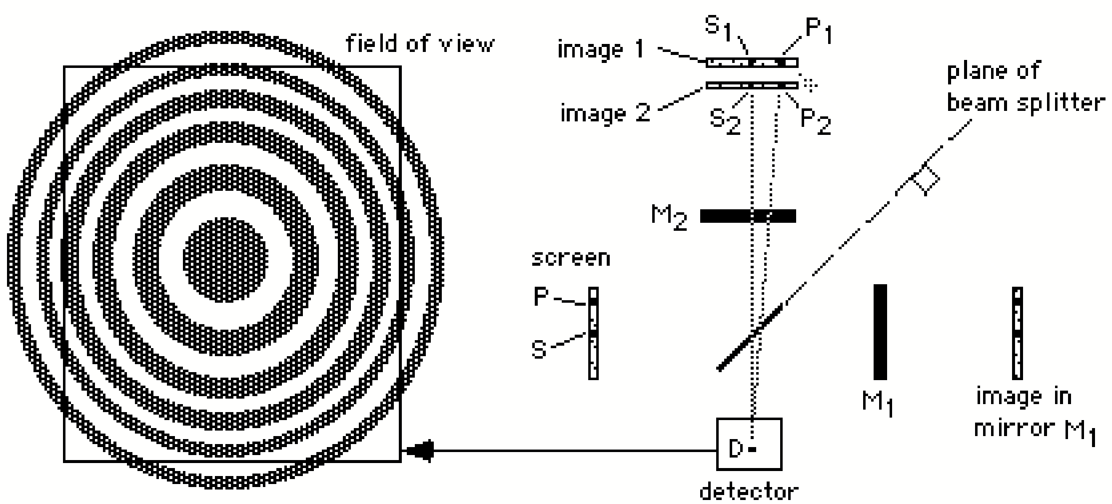

Fig. 2(a) Fig. 2(b)

What makes the Michelson interferometer so useful as a laboratory tool is the fact that the slightest change that occurs to either of the two separate beams gives rise to a well-defined alteration of the interference pattern. In effect, the instrument measures changes that occur in the sample beam by comparing it to the unaltered reference beam. The inherent sensitivity of this scheme is extremely high because the wavelength of light is so small. For example, suppose the light waves from the image points S_1 and S_2 interfere constructively at point D. If the wavelength of the monochromatic light is λ, then a displacement of mirror M_2 by as little as $\lambda/4$ causes the previous interference to become totally destructive (i.e., the total path length increases by $\lambda/2$). In practice, the Michelson interferometer can routinely measure displacements on the order of $\pm\lambda/10$. If blue light is used, this represents a spatial resolution of approximately $\pm 4\text{x}10^{-8}$ m.

Measurements

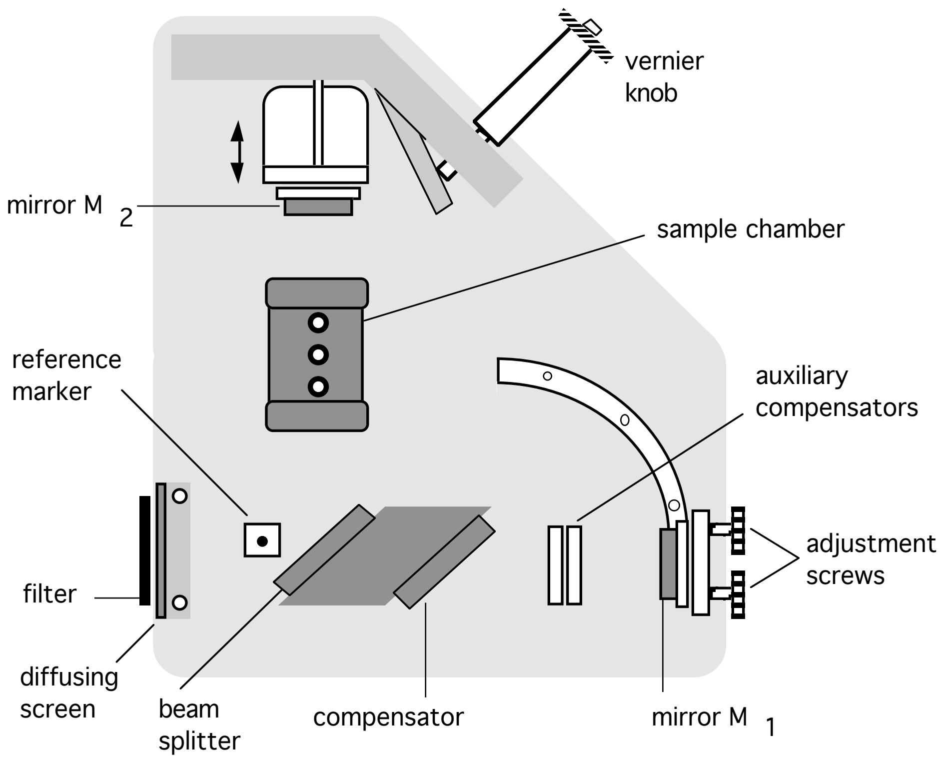

Fig. 3

A top-view of the Michelson interferometer used in this experiment is shown in Fig. 3. A green filter mounted on the front of the diffusing screen transmits only the relatively strong green line emanating from a mercury-vapor lamp, so the light from the far side of the diffusing screen is monochromatic of wavelength (in vacuum) λ_{vac} = 546.1 nm. A sample chamber of length L allows one to vary the number of air molecules along the path traveled by the sample beam. The auxiliary compensators are needed in the reference beam because of the glass windows on the vacuum chamber (i.e., to keep the optical path length comparable to that of the sample beam). The position of mirror M_2 is controlled by a vernier knob and the orientation of mirror M_1 can be varied by means of two set screws. These mirror-controls determine the general shape of the observed interference pattern and normally should not be changed from their intial settings (your instructor can fine-tune these adjustments for you, if necessary). A small vertical pointer placed in the path of the reference beam serves as a reference mark for the interference pattern.

A measurement of the index of refraction of air consists of counting the number of 'fringe shifts' which occur at the reference marker as air is slowly removed from the sample chamber.

As air is removed from the chamber, the interference pattern appears to 'drift' across the field of view. The small vertical pointer provides a fixed reference point with respect to which one can count the number of bright (or dark) bands as they move by. By aligning the marker in the middle of a bright (or dark) band at the start, and keeping one's eye in a fixed position during the counting process, the total number of fringe shifts can be readily determined to a resolution of about ±1/5 fringe.

Suppose the sample chamber is completely evacuated. How many oscillations of the light can fit inside the chamber? The sample beam passes through the chamber twice, so the total distance equals $2L$. If the sample chamber is evacuated, the number of oscillations inside the chamber is

$$N_{\text{vac}} = \frac{2L}{\lambda_{\text{vac}}} \quad . \tag{2}$$

The wavelength of the light in air is smaller than it is in vacuum (light travels more slowly in air, so there must be a corresponding decrease in wavelength in order to keep the frequency constant). We must have

$$f = \frac{c}{\lambda_{\text{vac}}} = \frac{v_{\text{air}}}{\lambda_{\text{air}}} \quad , \tag{3}$$

where v_{air} and λ_{air} are, respectively, the light's speed and wavelength in air. So, with air present, the number of wavelengths contained inside the chamber increases to

$$N_{\text{air}} = \frac{2L}{\lambda_{\text{air}}} \quad . \tag{4}$$

For every additional oscillation that fits inside the chamber, the relative phase between the sample and reference beams changes by 2π, i.e., one complete fringe shift is observed. So, if N_{obs} is the total number of fringe shifts observed, then

$$\begin{aligned} N_{\text{obs}} = N_{\text{air}} - N_{\text{vac}} &= \frac{2L}{\lambda_{\text{air}}} - \frac{2L}{\lambda_{\text{vac}}} \quad , \\ &= \frac{2L}{\lambda_{\text{vac}}} \cdot \left(\frac{\lambda_{\text{vac}}}{\lambda_{\text{air}}} - 1 \right) \quad . \end{aligned} \tag{5}$$

From Eq. (3), we see that $\lambda_{\text{vac}}/\lambda_{\text{air}} = c/v_{\text{air}} \equiv n_{\text{air}}$. Substituting this into Eq. (5) gives

$$N_{\text{obs}} = \frac{2L}{\lambda_{\text{vac}}} \cdot (n_{\text{air}} - 1) \quad . \tag{6}$$

As Eq. (1) shows, the value of $(n_{\text{med}} - 1)$ is directly proportional to η_{e} which is, in turn, proportional to the number of air molecules per unit volume. Consequently, values of $(n_{\text{air}} - 1)$ depend on the temperature and pressure of the air inside the chamber and are always quoted relative to some 'reference' temperature and pressure conditions. For the data given in Table I, these reference conditions are T_{ref} = 15°C = 288°K and P_{ref} = 760 mm Hg.

If one assumes that the air inside the sample chamber behaves like an ideal gas, it is relatively easy to find the relationship between values of $(n_{air} - 1)$ measured under arbitrary conditions and those obtained under reference conditions (here, denoted as $(n_{air} - 1)_{ref}$). If the air in the chamber behaves as an ideal gas, then

$$PV = \mu RT \quad , \tag{7}$$

where R is the universal gas constant, and μ is the number of moles in the sample volume V. If $(n_{med} - 1)$ is directly proportional to μ / V, then, for arbitrary temperature and pressure conditions, we should find that

$$(n_{air} - 1) = \kappa \cdot \left(\frac{P}{T}\right) \quad , \tag{8}$$

where κ is some unknown constant. Under reference conditions Eq. (8) yields

$$(n_{air} - 1)_{ref} = \kappa \cdot \left(\frac{P_{ref}}{T_{ref}}\right) \quad . \tag{9}$$

Dividing Eq. (8) by Eq. (9), and solving for $(n_{air} - 1)$ yields

$$(n_{air} - 1) = (n_{air} - 1)_{ref} \cdot \left(\frac{P}{T}\right) \cdot \left(\frac{T_{ref}}{P_{ref}}\right) \quad . \tag{10}$$

If we substitute this last result into Eq. (6), we obtain an expression which shows how the number of observed fringe shifts varies with temperature and pressure, i.e.,

$$N_{obs} = \frac{2L}{\lambda_{vac}} \cdot (n_{air} - 1)_{ref} \cdot \left(\frac{P}{T}\right) \cdot \left(\frac{T_{ref}}{P_{ref}}\right) \quad . \tag{11}$$

Eq. (11) shows that N_{obs} varies linearly with the pressure inside the chamber. If we record N_{obs} as a function of P, the slope of the linear data should be

$$\frac{dN_{obs}}{dP} = \frac{2L}{\lambda_{vac}} \cdot (n_{air} - 1)_{ref} \cdot \left(\frac{1}{T}\right) \cdot \left(\frac{T_{ref}}{P_{ref}}\right) \quad . \tag{12}$$

Once dN_{obs} / dP is experimentally known, Eq. (12) allows us to evaluate $(n_{air} - 1)_{ref}$. Since it is reasonable to assume that the air inside the sample chamber is in thermal equilibrium with its surroundings, the value of T can be taken to be the ambient room temperature.

Experimental Procedure

To minimize distortion, the mirrors used in the interferometer are all 'front-surface' optical elements (i.e., the reflective coatings are on the front surface of the glass instead of the rear surface). This means that simply touching the mirrors can damage or destroy them. When working with the interferometer observe the general rule -- NEVER TOUCH THE ACTIVE

SURFACES OF OPTICAL ELEMENTS. If your interferometer needs adjustment, ask your instructor for assistance.

The canisters which house the mercury bulbs become very hot after a short time, so, if you have to reposition the lamps, do so by moving their support-bases. These bulbs have an important operational characteristic: after the bulbs warm up, <u>if power is turned off, they will not turn on again until they cool down</u> (a process which can take 10-15 min). To avoid this delay, make sure your data is complete before you turn off your lamp.

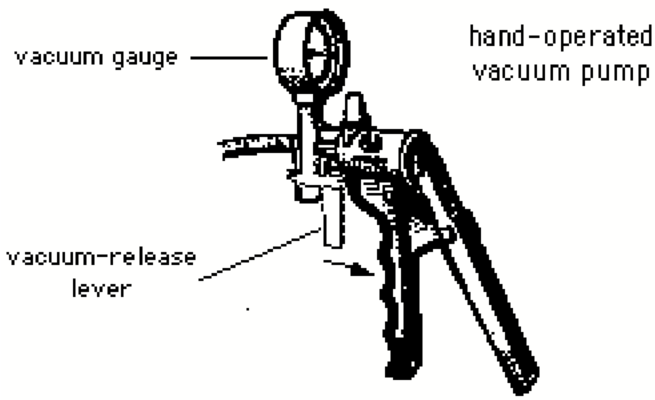

Fig. 4

A hand-operated vacuum pump (see Fig. 4) will be used to evacuate the sample chamber. The reading of the vacuum gauge (which we will denote as G) equals the ambient atmospheric pressure minus the pressure inside the chamber (when the pressure inside the chamber decreases, the gauge reading increases). We can ignore the the fact that $dP = -dG$ and set $dN_{obs}/dP = dN_{obs}/dG$ because we only concerned with the magnitude of dN_{obs}/dP (no polarity is being assigned to the direction of the fringe shifts). Therefore, we can derive a value of $(n_{air} - 1)_{ref}$ from Eq. (12) by experimentally evaluating

$$(n_{air} - 1)_{ref} = \frac{\lambda_{vac}}{2L} \cdot P_{ref} \cdot \left(\frac{T}{T_{ref}}\right) \cdot \frac{dN_{obs}}{dG} , \qquad (13)$$

To avoid having to disassemble the sample chamber, value of L has been measured for you; this length is found to be

$$L \pm \Delta L = 5.00 \pm 0.05 \text{ cm} .$$

You should use this result in the analysis of your data.

Each lab group will sign out a vacuum pump and (0-50°C) thermometer. Each group should use the thermometer to measure room temperature, then post their value on the blackboard. These results can be used to calculate the best-estimate for T, i.e., $\overline{T} \pm \sigma_T$.

1. Turn on the mercury light source and check to see that a suitable interference pattern is visible in the field of view of your interferometer. If not, have your instructor make the necessary adjustments.
2. Attach the vacuum pump to the hose of the sample chamber and make sure that the connection is leak tight. Do this by squeezing the handle of the pump a few times to create a partial vacuum in the chamber, then watch the fringe pattern (for 30 to 60 sec) to make sure that it does not slowly drift across the field of view (if it does, check the hose connection or ask your TA for assistance). Break the vacuum using the pump's vacuum-release lever (see Fig. 4).
3. Measure N_{obs} vs. G by having one lab partner operate the vacuum pump and count the fringe shifts, while the other lab partner reads the gauge and records the data:
   a) Adjust the position of your eye so that the reference marker lines up with a bright (or dark) interference band.
   b) Without moving your eye, reduce the pressure slightly so that only 2 to 3 fringes pass by the marker. Carefully estimate the number of fringe shifts to within an accuracy of ±1/5 of a fringe and record this value as $N(1)$. Record the gauge's reading as $G(1)$.
   c) If necessary, reposition your eye so that the marker again lines up with a bright (or dark) interference band, and repeat step 3(b). Record the number of fringe shifts as $N(2)$ and the gauge's reading as $G(2)$.
   d) Repeat step 3(b) until you have about 10 values of $N(k)$ and $G(k)$ (i.e., k = 1 to 10).
   e) For each value of $G(k)$, calculate the total number of fringe shifts, $N_{obs}(k)$; that is, $N_{obs}(1) = N(1)$, $N_{obs}(2) = N(1) + N(2)$, $N_{obs}(3) = N(1) + N(2) + N(3)$, etc.
   f) Plot $N_{obs}(k)$ vs. $G(k)$ and determine the linear best-fit slope, $S_1 \pm \Delta S_1$.
4. The members of the lab group should switch roles and repeat the measurements described in step 3. Call the linear best-fit slope associated with this second trial $S_2 \pm \Delta S_2$. Evaluate dN_{obs}/dG as the average of $S_1 \pm \Delta S_1$ and $S_2 \pm \Delta S_2$, i.e.,

$$(\bar{S} \pm \sigma_S) = (S_1 + S_2)/2 \pm \sqrt{(\Delta S_1)^2 + (\Delta S_2)^2}\,/2 \quad .$$

5. Using error propagation, calculate your experimental result for $(n_{\text{air}} - 1)_{\text{ref}}$, i.e., evaluate

$$(n_{\text{air}} - 1)_{\text{ref}} \pm \sigma \quad = \quad \frac{\lambda_{\text{vac}}}{2(L \pm \Delta L)} \cdot P_{\text{ref}} \cdot \left(\frac{\bar{T} \pm \sigma_T}{T_{\text{ref}}} \right) \cdot (\bar{S} \pm \sigma_S) \quad .$$

   Based on this result, what is your experimental value of $\nu_{\text{air}} \pm \Delta\nu_{\text{air}}$?
6. Given the data in Table I use linear interpolation to find a handbook value for $(n_{\text{air}} - 1)_{ref}^{table}$ at $\lambda = 546.1$ nm. Compare this result to the value of $(n_{\text{air}} - 1)_{\text{ref}} \pm \sigma$ obtained above. Does your experimental result agree with the handbook value?

Questions

1. Suppose your sample chamber has a small leak in it. Would the presence of this leak influence the results from this experiment? Describe a method by which you make a quantitative estimate of the leak rate (say, in units of moles/sec).
2. Explain how one might use a Michelson interferometer to measure the coefficient of linear expansion of certain type of material.