FSR-HOMEWORK 4

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Exercise 1

In the context of underwater robotics, it is important to consider the combined effects of gravity and buoyancy when studying the behaviour of rigid bodies immersed in a fluid. In contrast to the gravitational force, the buoyancy effect exerts a force that pushes the submerged rigid body upwards. This force is proportional to the density of the fluid in which the body is submerged. For this reason, the effect is negligible in aerial applications, given that the density of the air is significantly lower than that of water. The buoyancy effect also has implications for the design of the robot. This occurs because, while gravity acts on the centre of mass, the buoyancy effect acts in a different point, known as the centre of buoyancy. Given that these two forces act in opposite directions, if the two centres are not aligned on the same axis, the robot may experience unwanted moments. Therefore, a prudent design choice would be to construct the robot in a way that this circumstance does not occur.

Exercise 2

a.

False. The mass effect is not a quantity of fluid that can be added to the system in order to increase its mass. Rather, it should be regarded as the reaction forces that are equal in magnitude and opposite in direction, which are required in order to accelerate the fluid that is moved together with the robot. The additional mass effect is primarily determined by the robot's geometry, which implies that the associated mass matrix, M_a, may not be positive definite. Consequently, it cannot be considered a mass. It can be demonstrated that, under the assumption of an ideal fluid, a low speed of the UUV, and the absence of sea currents, the matrix M_a is symmetric and positive definite.

b.

True. This is also the rationale behind its neglect in other applications, such as aerial or legged robotics, given that the air density is not directly comparable to that of drones, quadrupeds, etc.

c.

True. The introduction of a positive definite term with respect to the velocity in the model dynamics serves to prove the asymptotic stability of the system via the Lyapunov theorem. In fact, the incorporation of a positive definite kinetic energy term within the Lyapunov candidate function serves to facilitate the verification that its time derivative is negative.

d.

False. It is preferable to refer to disturbances resulting from ocean currents in the world frame. This approach allows us to model disturbances as a constant and irrotational twist vector. The related effect can be later incorporated into the dynamic model of a rigid body moving in a fluid by considering the relative velocity in the body-fixed frame during the derivation of the Coriolis, centripetal and damping terms.

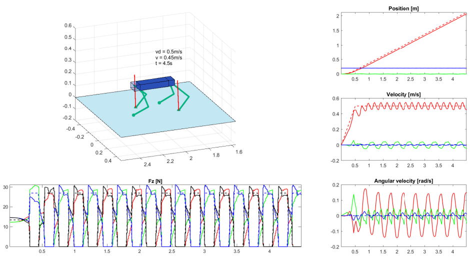
Exercise 3

a.

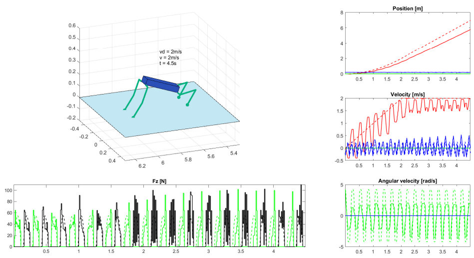
```
[zval,basic_info,adv_info] = qpSWIFT(sparse(H),g,sparse(Aeq),beq,sparse(Aineq),bineq);
```

b.

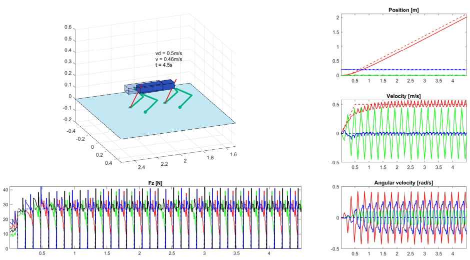
We will commence our analysis by comparing the gaits.



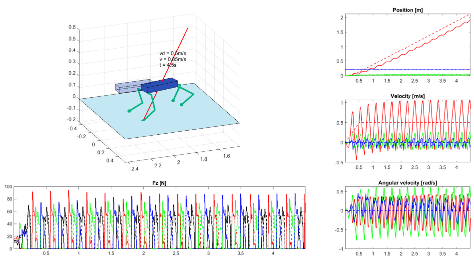
Trot



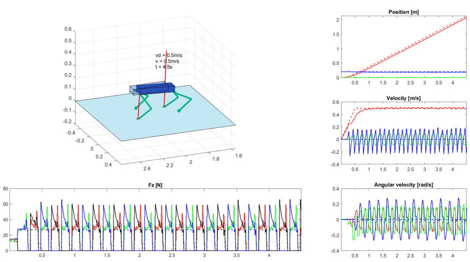
Bound



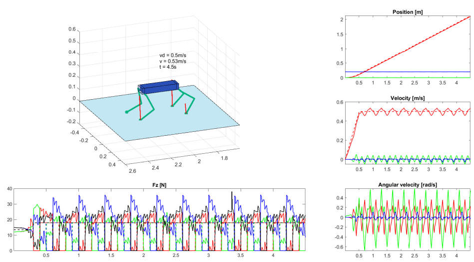
Pacing



Gallop



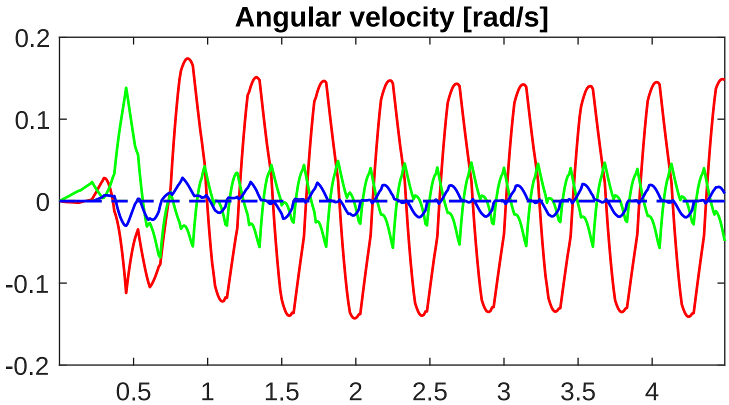
Trot run



Crawl

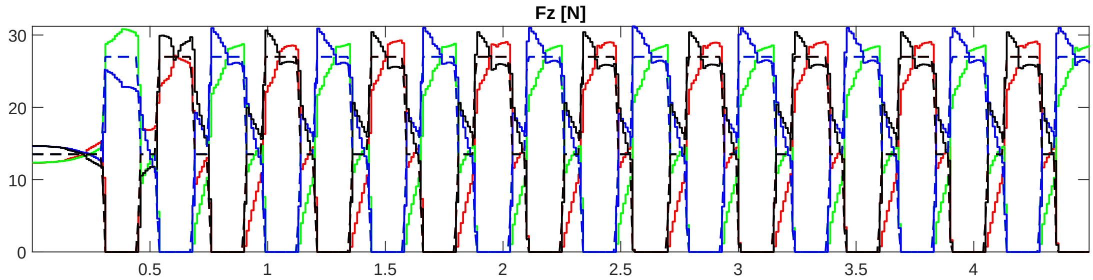
Simulations obtained with parameters given in the original file

- **Trot**: The alternating swing and stance phases of the robot's movement ensure a high degree of stability during the trajectory. During the swing phases, two feet are in contact with the ground (RF-LH or LF-RH), which results in a substantial oscillation of the centre of mass around the x-axis (roll).

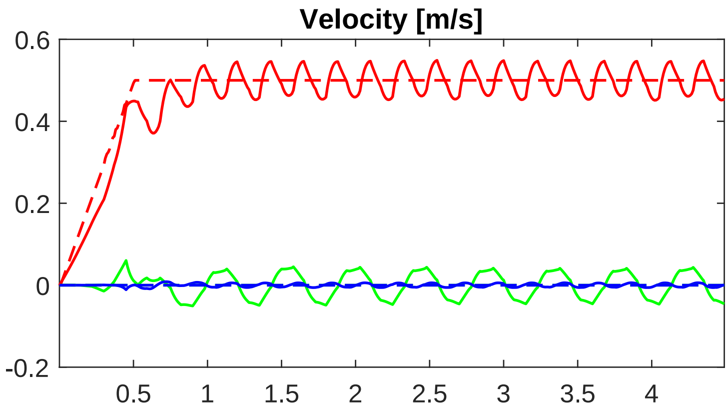


Roll velocity in red

The evolution of the GRFs in a square-wave-like fashion can be attributed to the relatively long duration of the contacts.

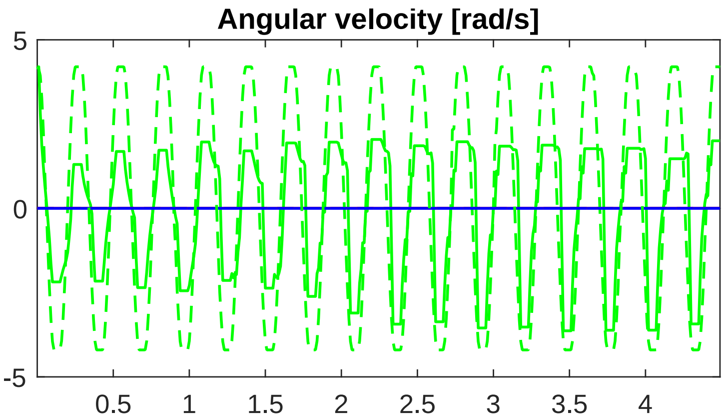


The nominal velocity on the x-axis is tracked in an oscillatory manner, with the mean value of the oscillation converging to the desired value.



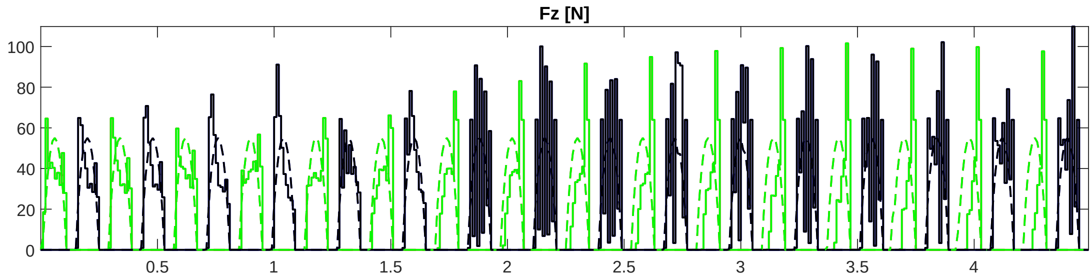
Reference velocity along x axis in red

- **Bound**: The stance phase is absent, and the reference trajectory for the centre of mass is planned with an oscillatory rotation around the y-axis (pitch) in mind (RH-LH or RF-LF).

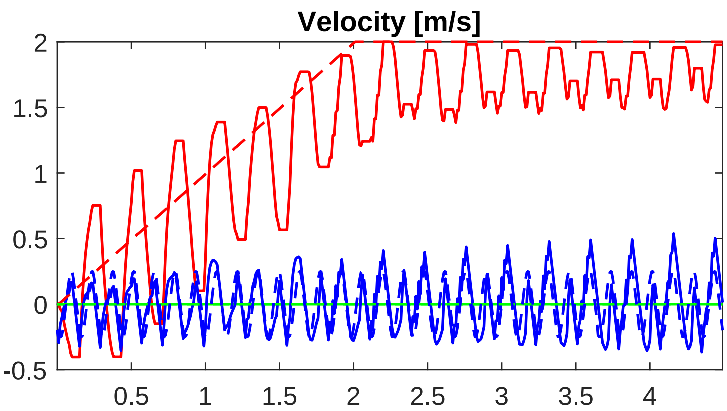


Pitch velocity in green

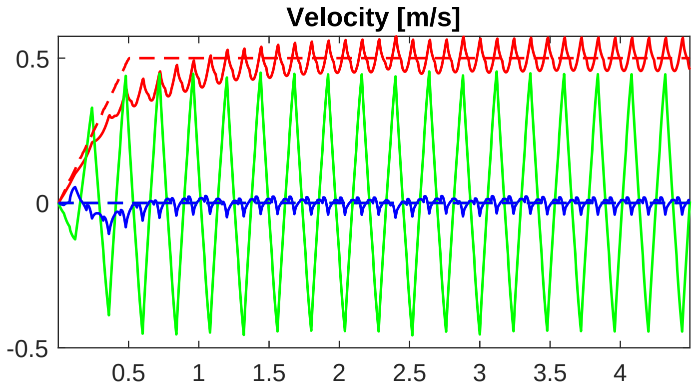
Given the brief duration of the contacts, the GRFs exhibit higher values and evolve in an impulsive manner.



This gait is automatically planned with a higher desired velocity, presumably because a reduction in the frequency of the hops would result in the robot losing its equilibrium and falling. It is possible for the robot to travel at high speeds with this gait, but it is unable to follow a continuous reference accurately due to the continuous bounce.

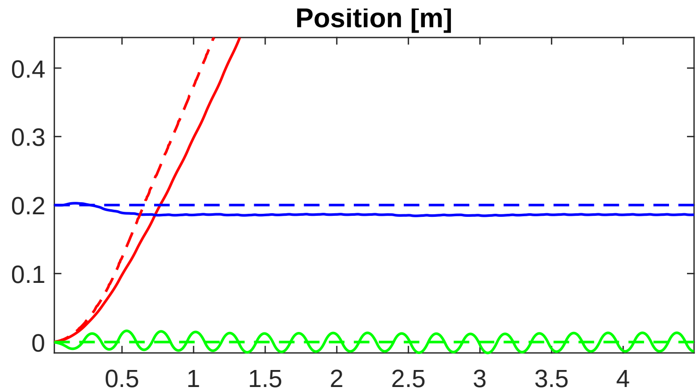


- **Pacing**: The robot does not enter a stance phase and instead moves forward while oscillating side to side (RF-RH or LF-LH). The reference trajectory for the centre of mass is planned with an oscillatory motion around the y-axis in mind.



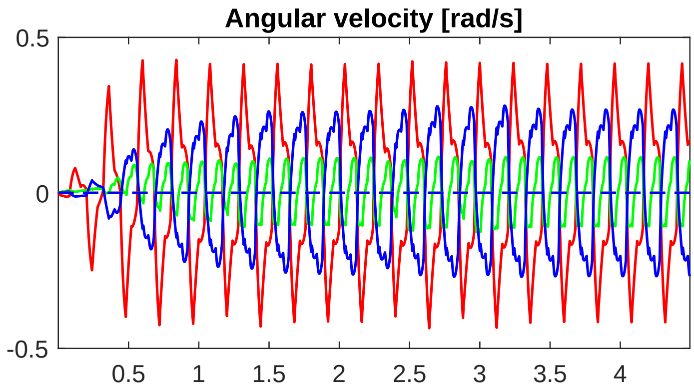
Oscillated trend on the y-axis (in green)

For the reasons previously enumerated and the fact that the robot's entire weight is borne by only two legs at a time, the system will exhibit not only a positional error along the x-axis, but also along the y- and z-axes during the trajectory.

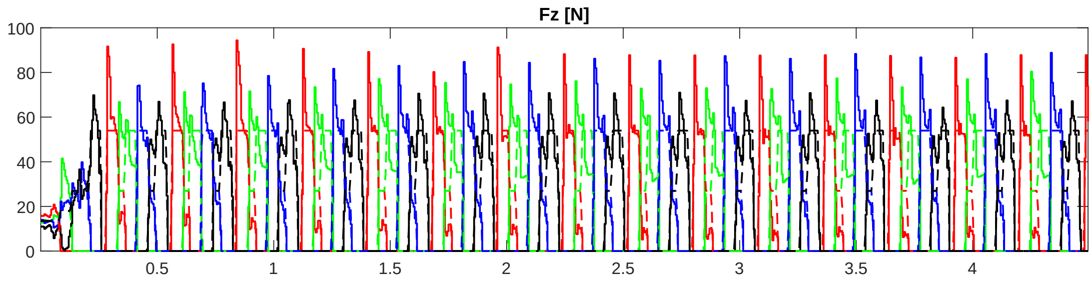


Oscillatory behaviour on y (green) and steady-state error on x (blue)

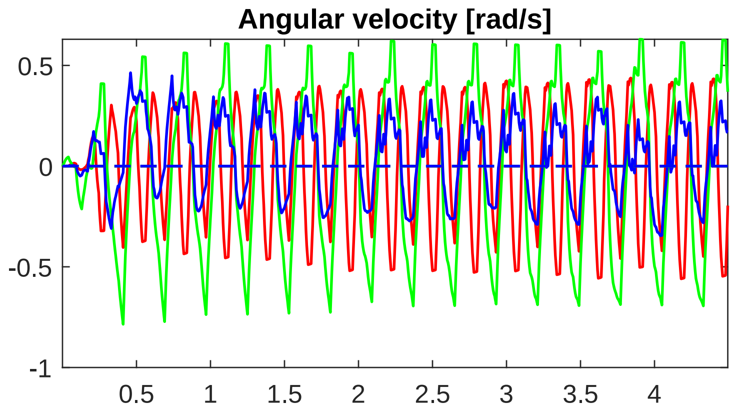
Nevertheless, the tracking of the constant speed reference is correctly performed. Additionally, side-to-side oscillation results in the robot exhibiting significant angular velocities.



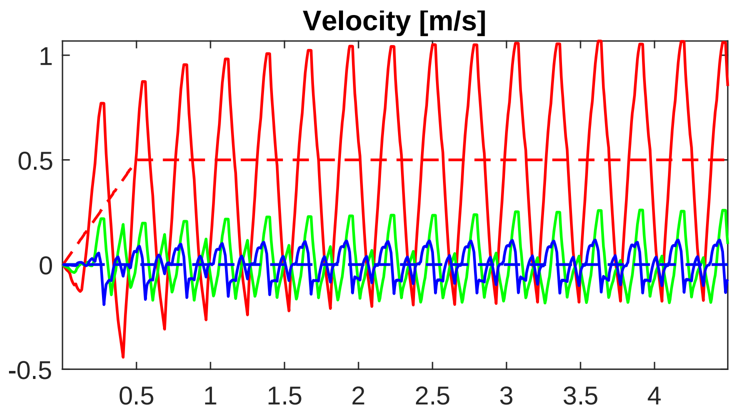
- **Gallop**: The robot does not undergo a stance phase. Its forward motion is achieved through a process of alternating bouncing on all four feet. Due to the brief duration of the contacts, the GRFs are characterised by high impulsiveness and amplitude.



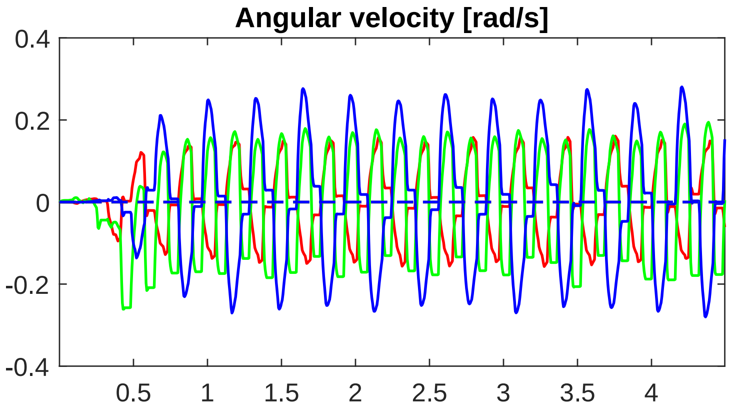
Moreover, the character of the GRFs is also responsible for the oscillatory behaviour observed around the rotation axes.



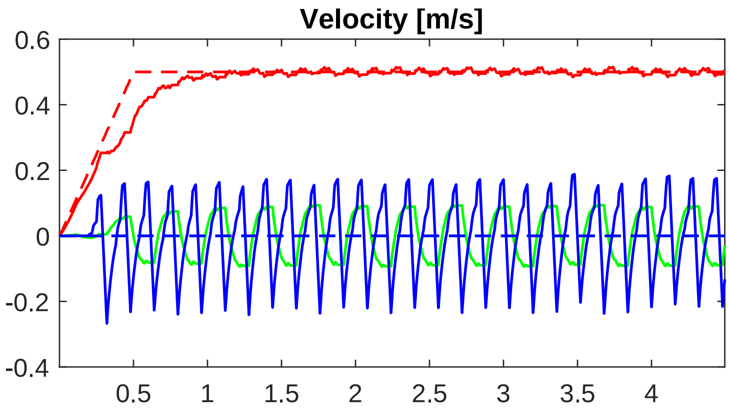
Furthermore, the final heading velocity solution exhibits pronounced oscillations in amplitude around the constant reference velocity, which can be attributed to the specific planning of the CoM reference trajectory.



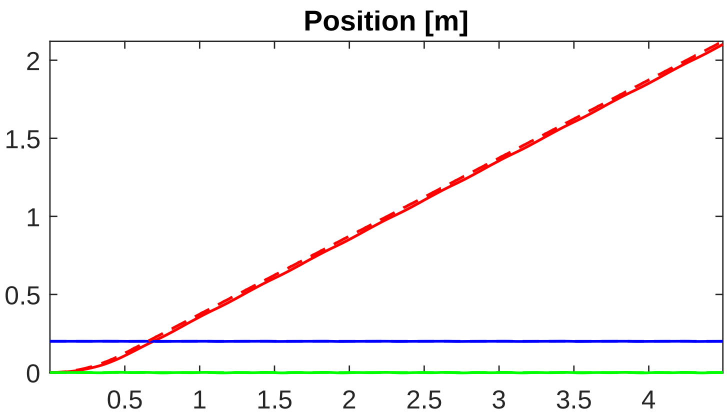
- **Trot run**: This is a variation of the *Trot* gait. The distinguishing feature is that there is no stance phase, despite the fact that the order of the contacts is planned in a similar way. As previously observed, the absence of a stance phase results in greater oscillations in the body's course.



This type of trend allows for the most accurate tracking of the heading velocity reference.

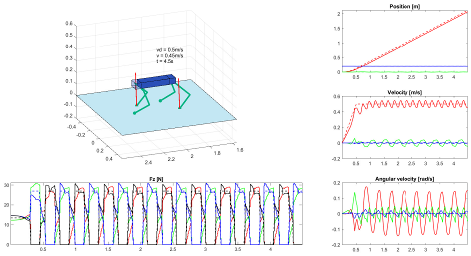


- **Crawl**: this gait does not have a stance phase. However, due to the fact that throughout the swing phase three feet are always in contact with the ground, the structure of the robot is always very stable. This results the highest degree of position tracking.

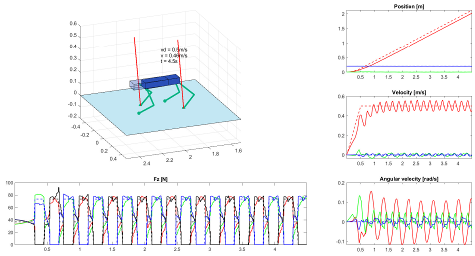


Let's analyze how changing certain parameters affect the performance of the various gaits:

- **Mass**: An increase in the mass of the robot will primarily affect the amplitude of the ground reaction forces, which in turn will influence the performance of the control system. Furthermore, gaits with less dynamically unstable swing phases will exhibit a higher error value on the z-axis.

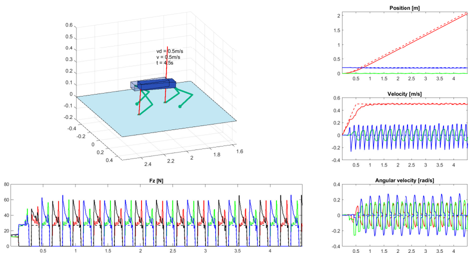


Trot gait, mass = 5.5 Kg

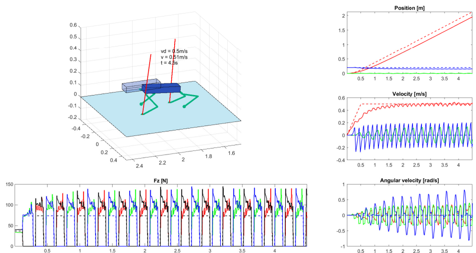


Trot gait, mass = 15 Kg

Worst tracking performances and higher values of GRFs



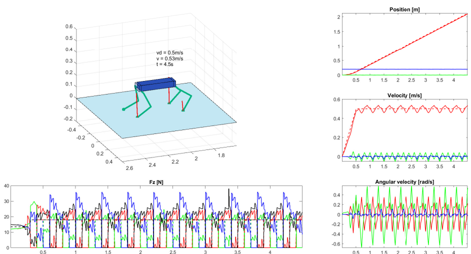
Trot run gait, mass = 5.5 Kg



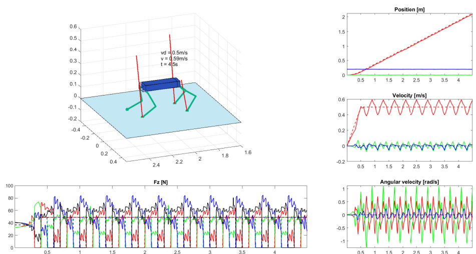
Trot run gait, mass = 15 Kg

Increased steady state error along z-axis

Furthermore, depending on the type of gait chosen, the degree of rotation velocity exhibited by the robot around its centre of mass will be higher/lower for a higher/lower value of the mass.



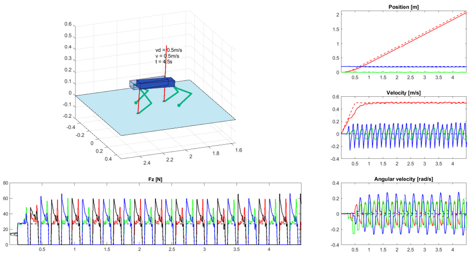
Crawl gait, mass = 5.5 Kg



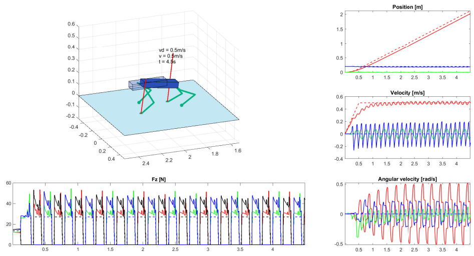
Crawl gait, mass = 15 Kg

Higher oscillations on the angular velocity

- **Friction**: A change in the friction of the ground also affects the ground reaction forces, particularly in terms of their transient and periodic duration. A reduction in friction below a certain threshold leads to a deterioration in tracking performance, an inability to maintain a desired gait (i.e video `gait_1_mu01`, where the robot fails to start moving in the first part of the trajectory), and in the most extreme cases, slippage (i.e. video `gait_3_mu01`).



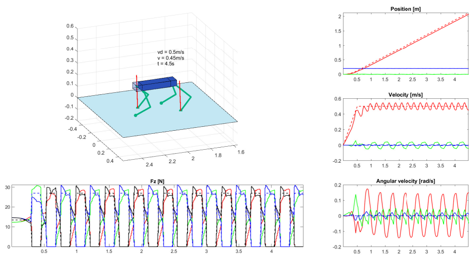
Trot run gait, $\mu = 1$



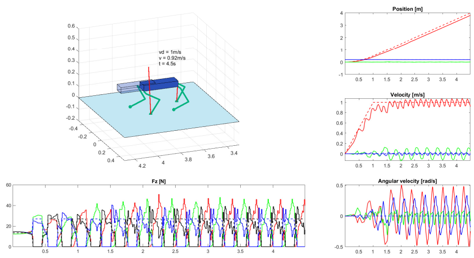
Trot run gait, $\mu = 0.1$

Change in the trend of the GRFs, more oscillation of the body, bigger error on z-axis

- **Desired Velocity**: The control system is obliged to increase the desired velocity reference, which in turn causes the values of the GRFs to rise in order to keep pace with this increase. As a result, oscillations around the centre of mass become more pronounced.



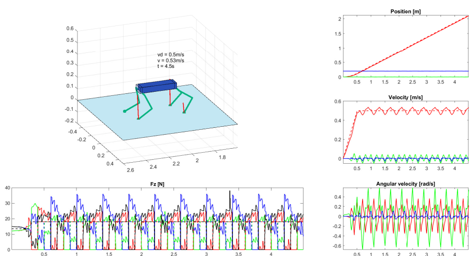
Trot gait, $v_d = [0.5; 0]$ m/s



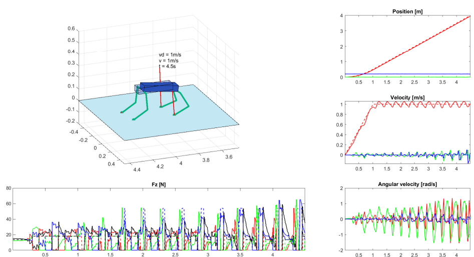
Trot gait, $v_d = [1; 0]$ m/s

Higher values of GRFs, higher values of angular velocities

  Furthermore, for certain gaits, an increase in velocity necessitates a change in the manner in which the feet are planted. To illustrate, consider the crawl gait (i.e. `gait_5_vd1`). When the reference heading velocity is augmented, the robot is no longer required to maintain a constant contact with the ground through all three feet.



Crawl gait, $v_d = [0.5; 0]$ m/s



Crawl gait, $v_d = [1; 0]$ m/s

Always three feet on the ground for lower velocities. Even a single foot in contact with the ground is sufficient for higher speeds.

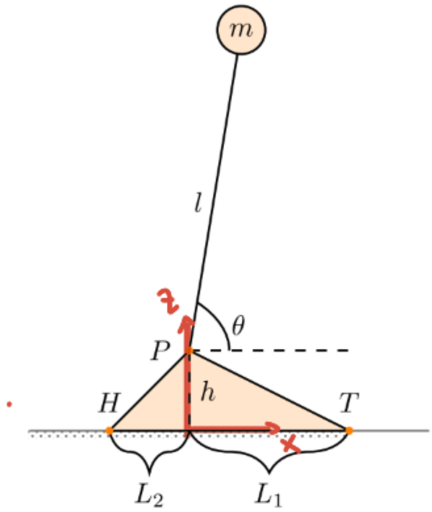
Exercise 4

a.

While the system is not actuated, it behaves like an inverted pendulum. As the coordinates have been defined, the equilibrium point at $\theta = \frac{\pi}{2}$ is unstable. Any perturbation of this state will cause the system to fall and reach another equilibrium. Therefore, for $\theta = \frac{\pi}{2} + \epsilon$, the system is not stable.

b.

The initial step is to define a reference frame for the computation. The z-axis was selected as the normal to the ground, passing through point P, and the x-axis was aligned with the ground, pointing towards point T.



In this instance, the definition of the **ZMP** on the ground is as follows:

$$ZMP_x = p_c^x - \frac{p_c^z}{\ddot{p}_c^z - g_0^z}\left(\ddot{p}_c^x - g_0^x\right) - \frac{1}{m\left(\ddot{p}_c^z - g_0^z\right)}\dot{L}^y$$

where:

- $p_c^x = l\,\cos\theta$
- $p_c^z = h + l\,\sin\theta$
- $\ddot{p}_c^z = l\,\ddot{\theta}\,\cos\theta - l\,\dot{\theta}^2\,\sin\theta$
- $\ddot{p}_c^x = -l\,\ddot{\theta}\,\sin\theta - l\,\dot{\theta}^2\,\cos\theta$
- $g_0^z = -g$
- $g_0^x = 0$
- $\dot{L}^y = m\,(l^2 + h^2)\,\ddot{\theta}$ *(Huygens-Steiner theorem)*

Explicitly:

$$ZMP_x = l\,\cos\theta + \frac{(h + l\,\sin\theta)\,(l\,\ddot{\theta}\,\sin\theta + l\,\dot{\theta}^2\,\cos\theta) - (l^2 + h^2)\,\ddot{\theta}}{l\,\ddot{\theta}\,\cos\theta - l\,\dot{\theta}^2\,\sin\theta + g}$$

c.

In the case of a single support, where the angular velocity and acceleration are both zero, the **ZMP** on the ground equation becomes (in the same reference frame as the previous point).

$$ZMP_x = p_c^x - \frac{p_c^z}{g_0^z}\, g_0^x$$

where:

- $p_c^x = l\,\cos\theta$
- $p_c^z = h + l\,\sin\theta$
- $g_0^z = -g$
- $g_0^x = 0$

Therefore:

$$ZMP_x = l\,\cos\theta$$

As is well known, the robot will fall down when the zero moment point falls outside the support polygon. In this case, the ZMP is simply the projection of the centre of mass position on the x-axis, and the support polygon is the line connecting the point H to the point T. At this point, it can be stated that the robot will not fall down until:

$$\begin{cases} \theta > \cos^{-1}\left(\frac{L_1}{l}\right) \\ \theta < \cos^{-1}\left(-\frac{L_2}{l}\right) \end{cases}$$

Therefore:

$$\theta \in \left[\cos^{-1}\left(\frac{L_1}{l}\right), \cos^{-1}\left(-\frac{L_2}{l}\right)\right]$$