Report – Homework 2
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Control a manipulator to follow a trajectory

1. Substitute the current ontrol al velocity profile with a cubic ontrol al linear trajectory
   a. Define a new KDLPlanner::trapezoidal_vel function that takes the current time t and the acceleration time tc as double arguments and returns three double variables s, $\dot{s}$ and $\ddot{s}$ that represent the curvilinear abscissa of your trajectory.

```
139  s_struct KDLPlanner::trapezoidal_vel(double time, double tc){
140
141      s_struct s_abs;
142
143      accDuration_ = tc;
144
145      double qc_ddot_ = 5/(std::pow(trajDuration_,2));
146
147      if(time <= accDuration_)
148      {
149        s_abs.s_ = 0 + 0.5*qc_ddot_*std::pow(time,2);
150        s_abs.s_dot_ = qc_ddot_*time;
151        s_abs.s_ddot_ = qc_ddot_;
152      }
153      else if(time <= trajDuration_-accDuration_)
154      {
155        s_abs.s_ = 0 + qc_ddot_*accDuration_*(time-accDuration_/2);
156        s_abs.s_dot_ = qc_ddot_*accDuration_;
157        s_abs.s_ddot_ = 0;
158      }
159      else
160      {
161        s_abs.s_ = 1 - 0.5*qc_ddot_*std::pow(trajDuration_-time,2);
162        s_abs.s_dot_ = qc_ddot_*(trajDuration_-time);
163        s_abs.s_ddot_ = -qc_ddot_;
164      }
165
166      return s_abs;
167
168  }
```

```
24   struct s_struct{
25       double s_=0;
26       double s_dot_=0;
27       double s_ddot_=0;
28   };
29
```

   b. Create a function named KDLPlanner::cubic_polinomial that creates the cubic polynomial curvilinear abscissa for your trajectory. The function takes as argument a double t representing time and returns three double s, $\dot{s}$ and $\ddot{s}$ that represent the curvilinear abscissa of your trajectory.

```
170  s_struct KDLPlanner::cubic_polinomial(double time){
171
172      s_struct s_abs;
173
174      //offline value
175      double a0 = 0;
176      double a1 = 0;
177      double a2 = 3/std::pow(trajDuration_,2);
178      double a3 = -2/std::pow(trajDuration_,3);
179
180      s_abs.s_    = a3*std::pow(time,3) + a2*std::pow(time,2) + a1*time + a0;
181      s_abs.s_dot_  = 3*a3*std::pow(time,2) + 2*a2*time + a1;
182      s_abs.s_ddot_ = 6*a3*time + 2*a2;
183
184      return s_abs;
185
186  }
187
```

2. Create circular trajectories for your robot

   a. Define a new constructor KDLPlanner::KDLPlanner that takes as arguments the time duration _trajDuration, the starting point Eigen::Vector3d _trajInit and the radius _trajRadius of your trajectory and store them in the corresponding class variables.

```
///////////////////////////////////////////////      kdl_planner.h
KDLPlanner(double _trajDuration, double _accDuration,
           Eigen::Vector3d _trajInit, Eigen::Vector3d _trajEnd);
KDLPlanner(double _trajDuration, double _trajRadius,
           Eigen::Vector3d _trajInit);
KDLPlanner(double _trajDuration, double _accDuration,
           double _trajRadius, Eigen::Vector3d _trajInit);
KDLPlanner(double _trajDuration, double _accDuration,
           double _trajRadius, Eigen::Vector3d _trajInit, Eigen::Vector3d _trajEnd);
```

```
16  KDLPlanner::KDLPlanner(double _trajDuration, double _trajRadius, Eigen::Vector3d _trajInit)
17  {
18      trajDuration_ = _trajDuration;
19      trajInit_ = _trajInit;
20      trajRadius_ = _trajRadius;
21      trajEnd_ = _trajInit;                                    kdl_planner.cpp
22  }
```

   b. Create the positional path as function of s(t) directly in the function KDLPlanner::compute_trajectory: first, call the cubic_polinomial function to retrieve s and its derivatives from t; then fill in the trajectory_point fields traj.pos, traj.vel, and traj.acc.

   c. Do the same for the linear trajectory.

```
94
95  trajectory_point KDLPlanner::compute_trajectory(double time ,char* path, char* vel_prof)
96  {
97      /* trapezoidal velocity profile with accDuration_ acceleration time period and trajDuration_ total duration.
98         time = current time
99         trajDuration_  = final time
100        accDuration_   = acceleration time
101        trajInit_  = trajectory initial point
102        trajEnd_   = trajectory final point */
103
104     trajectory_point traj;
105
106     s_struct curv_abs_;
107
108     if(strcmp(vel_prof,"cubic")==0)
109         curv_abs_ = KDLPlanner::cubic_polinomial(time);
110     else if(strcmp(vel_prof,"trapez")==0)
111         curv_abs_ = KDLPlanner::trapezoidal_vel(time,accDuration_);
112     else
113         std::cout<<"ERRORE ABS"<<std::endl;
114
115     if(strcmp(path,"circle")==0){
116         traj.pos[0] = trajInit_[0];
117         traj.pos[1] = trajInit_[1] - trajRadius_*cos(2*M_PI*curv_abs_.s_);
118         traj.pos[2] = trajInit_[2] - trajRadius_*sin(2*M_PI*curv_abs_.s_);
119
120         traj.vel[0] = 0;
121         traj.vel[1] = 2*M_PI*trajRadius_*curv_abs_.s_dot_*sin(2*M_PI*curv_abs_.s_);
122         traj.vel[2] = -2*M_PI*trajRadius_*curv_abs_.s_dot_*cos(2*M_PI*curv_abs_.s_);
123
124         traj.acc[0] = 0;
125         traj.acc[1] = 2*M_PI*trajRadius_*curv_abs_.s_ddot_*sin(2*M_PI*curv_abs_.s_)+4*M_PI*M_PI*trajRadius_*std::pow(curv_abs_.s_dot_,2)*cos(2*M_PI*curv_abs_.s_);
126         traj.acc[2] = -2*M_PI*trajRadius_*curv_abs_.s_ddot_*cos(2*M_PI*curv_abs_.s_)+4*M_PI*M_PI*trajRadius_*std::pow(curv_abs_.s_dot_,2)*sin(2*M_PI*curv_abs_.s_);
127     }
128     else if(strcmp(path,"linear")==0){
129         traj.pos = trajInit_ + (curv_abs_.s_ * (trajEnd_ - trajInit_));
130         traj.vel = curv_abs_.s_dot_ * (trajEnd_ - trajInit_);
131         traj.acc = curv_abs_.s_ddot_ * (trajEnd_ - trajInit_);
132     }else{
133         std::cout<<"ERRORE PATH"<<std::endl;
134     }
135     return traj;
136
137  }
138
```

For easier testing I decided to use only one compute_trajectory function and I modified the robot_test.cpp file so that I can decide what type of trajectory and velocity profile I want to use at runtime.

3. Test the four trajectories

    a. Modify your main file kdl_robot_test.cpp and test the four trajectories with the provided joint space inverse dynamics controller.

    b. Plot the torques sent to the manipulator and tune appropriately the control gains Kp and Kd until you reach a satisfactorily smooth behavior.
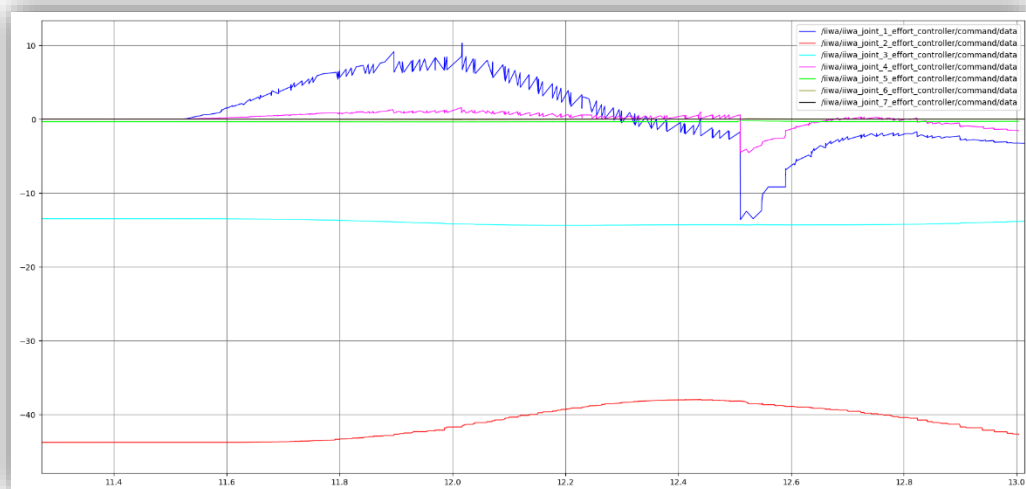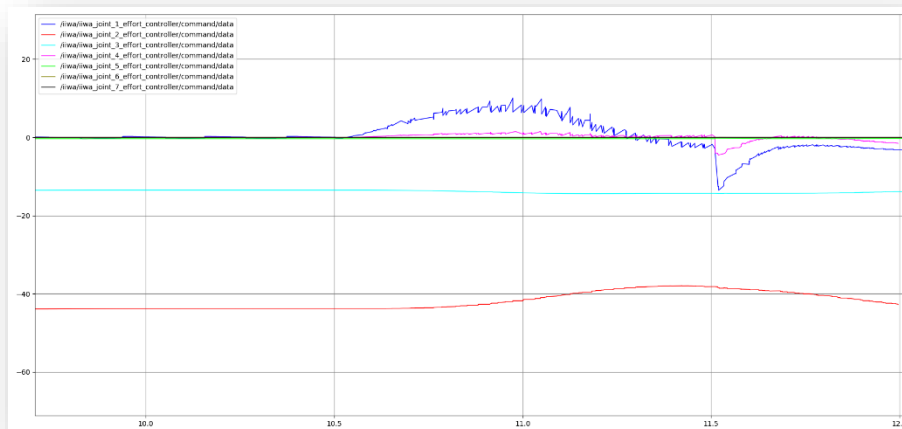
linear_trapezoidal

circular_cubic

circular_trapezoidal

Since the visualization capabilities of rqt are quite limited, we also plotted the torques using Foxglove.

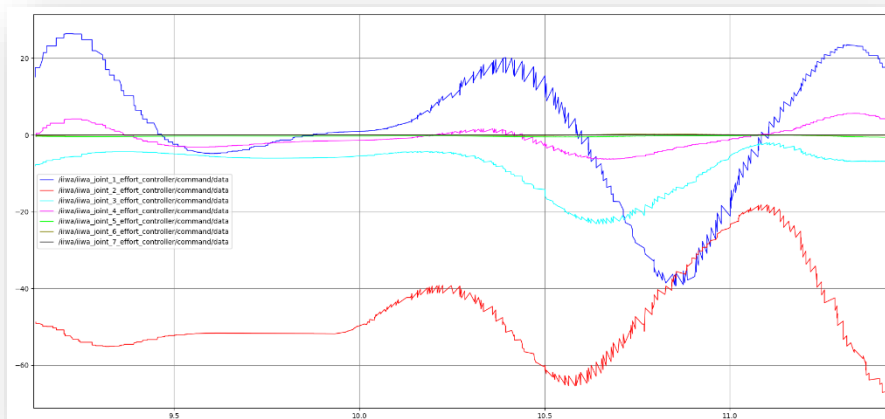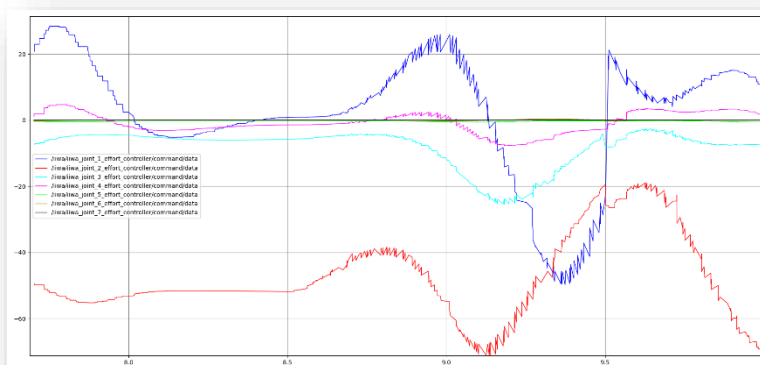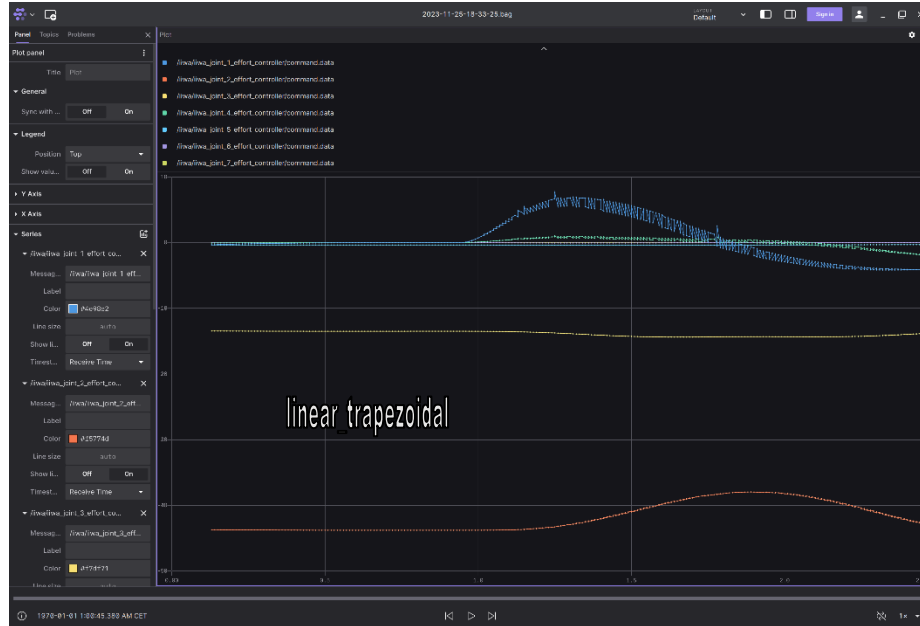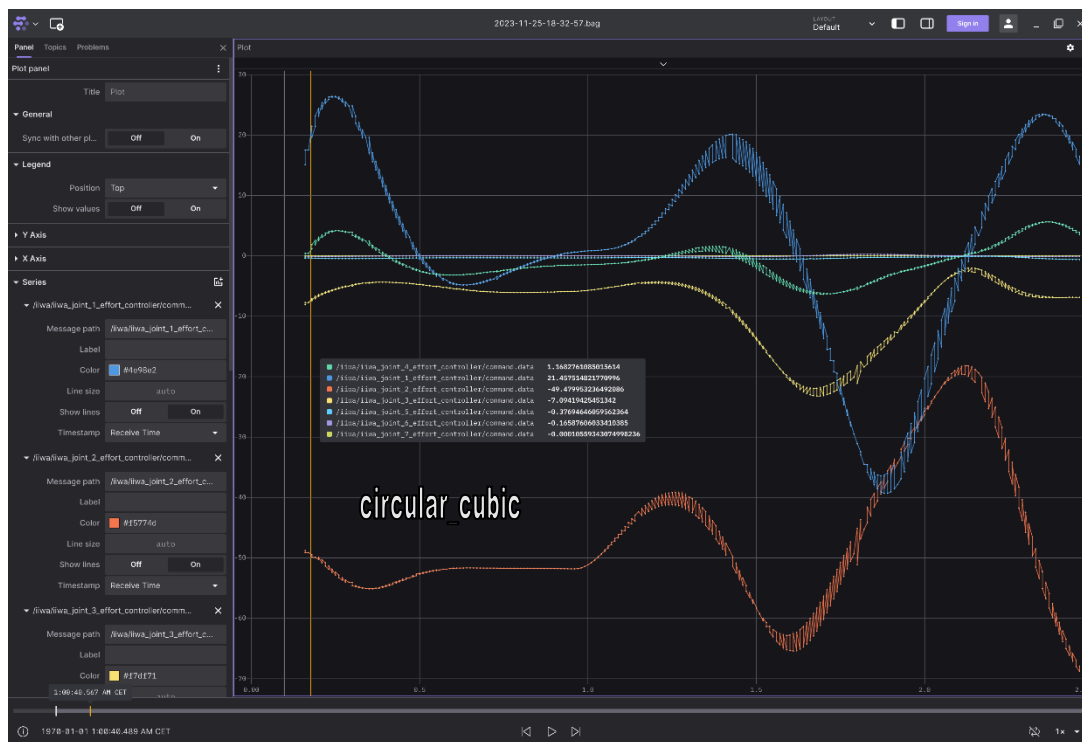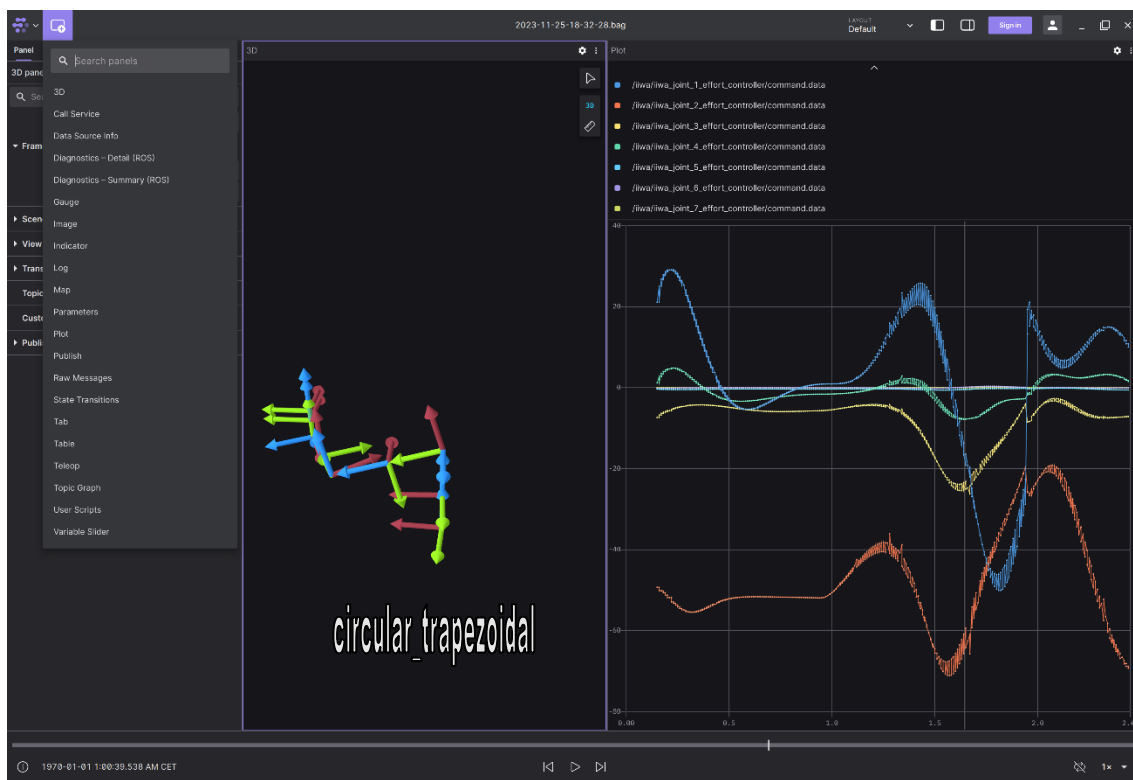circular_trapezoidal

circular_cubic

c. **Optional**: Save the joint torque command topics in a bag file and plot it using MATLAB.

```matlab
bagMsgs = rosbagreader("4.bag")

bagSelection1 = select(bagMsgs,'Topic','/iiwa/iiwa_joint_1_effort_controller/command');
bagSelection2 = select(bagMsgs,'Topic','/iiwa/iiwa_joint_2_effort_controller/command');
bagSelection3 = select(bagMsgs,'Topic','/iiwa/iiwa_joint_3_effort_controller/command');
bagSelection4 = select(bagMsgs,'Topic','/iiwa/iiwa_joint_4_effort_controller/command');
bagSelection5 = select(bagMsgs,'Topic','/iiwa/iiwa_joint_5_effort_controller/command');
bagSelection6 = select(bagMsgs,'Topic','/iiwa/iiwa_joint_6_effort_controller/command');
bagSelection7 = select(bagMsgs,'Topic','/iiwa/iiwa_joint_7_effort_controller/command');


ts1 = timeseries(bagSelection1);
ts2 = timeseries(bagSelection2);
ts3 = timeseries(bagSelection3);
ts4 = timeseries(bagSelection4);
ts5 = timeseries(bagSelection5);
ts6 = timeseries(bagSelection6);
ts7 = timeseries(bagSelection7);
```

```matlab
subplot(7,1,1)
plot(ts1)
grid on
subplot(7,1,2)
plot(ts2)
grid on
subplot(7,1,3)
plot(ts3)
grid on
subplot(7,1,4)
plot(ts4)
grid on
subplot(7,1,5)
plot(ts5)
grid on
subplot(7,1,6)
plot(ts6)
grid on
subplot(7,1,7)
plot(ts7)
grid on
```
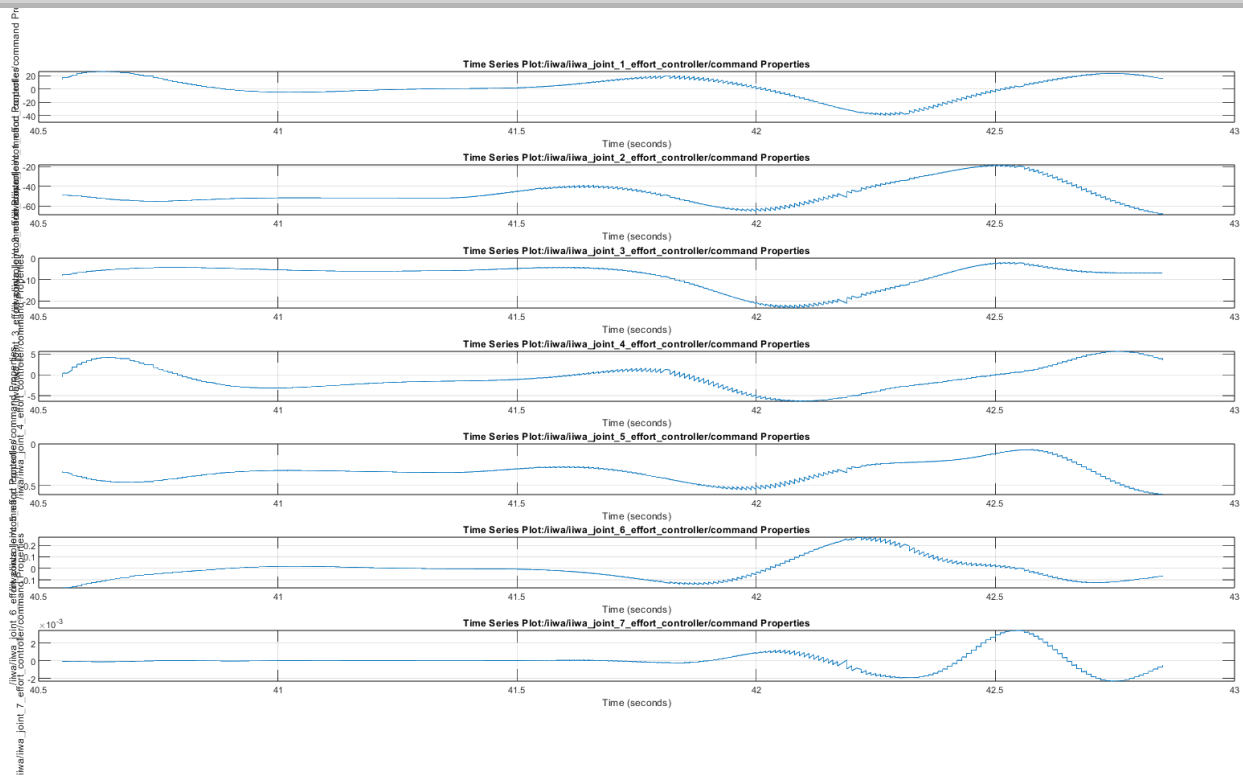
4. Develop an inverse dynamics operational space controller

   a. Into the kdl_contorl.cpp file, fill the empty overlayed KDLController::idCntr function to implement your inverse dynamics operational space controller. Differently from joint space inverse dynamics controller, the operational space controller computes the errors in Cartesian space.Thus the function takes as arguments the desired KDL::Framepose, the KDL::Twist velocity and, the KDL::Twist acceleration. Moreover, it takes four gains as arguments: _Kpp position error proportional gain, _Kdp position error derivative gain and so on for the orientation.

   b. The logic behind the implementation of your controller is sketched within the function: you must calculate the gain matrices, read the current Cartesian state of your manipulator in terms of end effector parametrized pose x, velocity $\dot{x}$, and acceleration $\ddot{x}$, retrieve the current joint space inertia matrix M and the Jacobian and its time derivative, compute the linear e_p and the angular e_o errors(some functions are provided into the include/utils.h file), finally compute your inverse dynamics control law following the equation

$$\tau = By + n, \quad y = J^{\dagger}(\ddot{x}_d + K_d\dot{\tilde{x}} + K_p\tilde{x} - \dot{J}_A\dot{q})$$

```
27  ∨  Eigen::VectorXd KDLController::idCntr(KDL::Frame &_desPos,
28                                           KDL::Twist &_desVel,
29                                           KDL::Twist &_desAcc,
30                                           double _Kpp, double _Kpo,
31                                           double _Kdp, double _Kdo)
32     {
33         // calculate gain matrices
34         Eigen::Matrix<double,6,6> Kp, Kd;
35         Kp=Eigen::MatrixXd::Zero(6,6);
36         Kd=Eigen::MatrixXd::Zero(6,6);
37         Kp.block(0,0,3,3) = _Kpp*Eigen::Matrix3d::Identity();
38         Kp.block(3,3,3,3) = _Kpo*Eigen::Matrix3d::Identity();
39         Kd.block(0,0,3,3) = _Kdp*Eigen::Matrix3d::Identity();
40         Kd.block(3,3,3,3) = _Kdo*Eigen::Matrix3d::Identity();
41
42         // read current state
43          KDL::Jacobian JEE=robot_->getEEJacobian();
44         Eigen::Matrix<double,6,7> J = toEigen(JEE);
45         Eigen::Matrix<double,7,7> I = Eigen::Matrix<double,7,7>::Identity();
46         Eigen::Matrix<double,7,7> M = robot_->getJsim();
47         Eigen::Matrix<double,7,6> Jpinv = weightedPseudoInverse(M,J);
48         //Eigen::Matrix<double,7,6> Jpinv = pseudoinverse(J);
49
50         // position
51         KDL::Frame cart_pose = robot_->getEEFrame();
52         Eigen::Vector3d p_d(_desPos.p.data);
53         Eigen::Vector3d p_e(cart_pose.p.data);
54         Eigen::Matrix<double,3,3,Eigen::RowMajor> R_d(_desPos.M.data);
55         Eigen::Matrix<double,3,3,Eigen::RowMajor> R_e(cart_pose.M.data);
56         R_d = matrixOrthonormalization(R_d);
57         R_e = matrixOrthonormalization(R_e);
58
```

```
switch (exit){
    case 1:     //lineare e trapezoidale
        Kp=100;
        Ko=100;
        break;
    case 2:     //lineare e Cubica
        Kp=100;
        Ko=100;
        break;
    case 3:     //Circolare e trapezoidale
        Kp=60;
        Ko=20;
        break;
    case 4:     //Circolare e cubica
        Kp=30;
        Ko=10;
        break;
    default:
        Kp=1000;
        Ko=1000;
        break;
}
```

```cpp
58
59          // velocity
60          KDL::Twist cart_twist = robot_->getEEVelocity();
61          Eigen::Vector3d dot_p_d(_desVel.vel.data);
62          Eigen::Vector3d dot_p_e(cart_twist.vel.data);
63          Eigen::Vector3d omega_d(_desVel.rot.data);
64          Eigen::Vector3d omega_e(cart_twist.rot.data);
65
66          // acceleration
67          Eigen::Matrix<double,6,1> dot_dot_x_d;
68          Eigen::Matrix<double,3,1> dot_dot_p_d(_desAcc.vel.data);
69          Eigen::Matrix<double,3,1> dot_dot_r_d(_desAcc.rot.data);
70
71          // compute linear errors
72          Eigen::Matrix<double,3,1> e_p = computeLinearError(p_d,p_e);
73          Eigen::Matrix<double,3,1> dot_e_p = computeLinearError(dot_p_d,dot_p_e);
74
75          // shared control
76          // Eigen::Vector3d lin_acc;
77          // lin_acc << _desAcc.vel.x(), _desAcc.vel.y(), _desAcc.vel.z(); //use desired acceleration
78          // lin_acc << dot_dot_p_d + _Kdp*(dot_e_p) + _Kpp*(e_p); // assuming no friction no loads
79          // Eigen::Matrix<double,3,3> R_sh = shCntr(lin_acc);
80
81          // compute orientation errors
82          Eigen::Matrix<double,3,1> e_o = computeOrientationError(R_d,R_e);
83          Eigen::Matrix<double,3,1> dot_e_o = computeOrientationVelocityError(omega_d,
84                                                                              omega_e,
85                                                                              R_d,
86                                                                              R_e);
87          Eigen::Matrix<double,6,1> x_tilde;
88          Eigen::Matrix<double,6,1> dot_x_tilde;
89          x_tilde << e_p, e_o;
90          dot_x_tilde << dot_e_p, -omega_e;//dot_e_o;
91          dot_dot_x_d << dot_dot_p_d, dot_dot_r_d;
```

```cpp
93      // null space control
94      double cost;
95      Eigen::VectorXd grad = gradientJointLimits(robot_->getJntValues(),robot_->getJntLimits(),cost);
96
97  //    std::cout << "----------------------" << std::endl;
98  //    std::cout << "p_d: " << std::endl << p_d << std::endl;
99  //    std::cout << "p_e: " << std::endl << p_e << std::endl;
100 //    std::cout << "dot_p_d: " << std::endl << dot_p_d << std::endl;
101 //    std::cout << "dot_p_e: " << std::endl << dot_p_e << std::endl;
102 //    std::cout << "R_sh*R_d: " << std::endl << R_d << std::endl;
103 //    std::cout << "R_e: " << std::endl << R_e << std::endl;
104 //    std::cout << "omega_d: " << std::endl << omega_d << std::endl;
105 //    std::cout << "omega_e: " << std::endl << omega_e << std::endl;
106 //    std::cout << "x_tilde: " << std::endl << x_tilde << std::endl;
107 //    std::cout << "dot_x_tilde: " << std::endl << dot_x_tilde << std::endl;
108 //    std::cout << "jacobian: " << std::endl << toEigen(robot_->getEEJacobian()) << std::endl;
109 //    std::cout << "jpinv: " << std::endl << Jpinv << std::endl;
110 //    std::cout << "jsim: " << std::endl << robot_->getJsim() << std::endl;
111 //    std::cout << "c: " << std::endl << robot_->getCoriolis().transpose() << std::endl;
112 //    std::cout << "g: " << std::endl << robot_->getGravity().transpose() << std::endl;
113 //    std::cout << "q: " << std::endl << robot_->getJntValues() << std::endl;
114 //    std::cout << "Jac Dot qDot: " << std::endl << toEigen(robot_->getEEJacDotqDot()) << std::endl;
115 //    std::cout << "qdot: " << std::endl << robot_->getJntVelocities() << std::endl;
116 //    std::cout << "Jnt lmt cost: " << std::endl << cost << std::endl;
117 //    std::cout << "Jnt lmt gradient: " << std::endl << grad.transpose() << std::endl;
118 //    std::cout << "----------------------" << std::endl;
119
120     // inverse dynamics
121     Eigen::Matrix<double,6,1> y;
122     y << dot_dot_x_d - toEigen(robot_->getEEJacDotqDot())*robot_->getJntVelocities() + Kd*dot_x_tilde + Kp*x_tilde;
123
124     return M * (Jpinv*y + (I-Jpinv*J)*(/*- 10*grad*/ - 1*robot_->getJntVelocities()))
125             + robot_->getGravity() + robot_->getCoriolis();
126 }
```

```cpp
// Eigen::VectorXd KDLRobot::getEEJacDotqDot()
// {
//     return s_J_dot_ee_.data;
// }

KDL::Jacobian KDLRobot::getEEJacDotqDot()
{
    return s_J_dot_ee_;
}
```
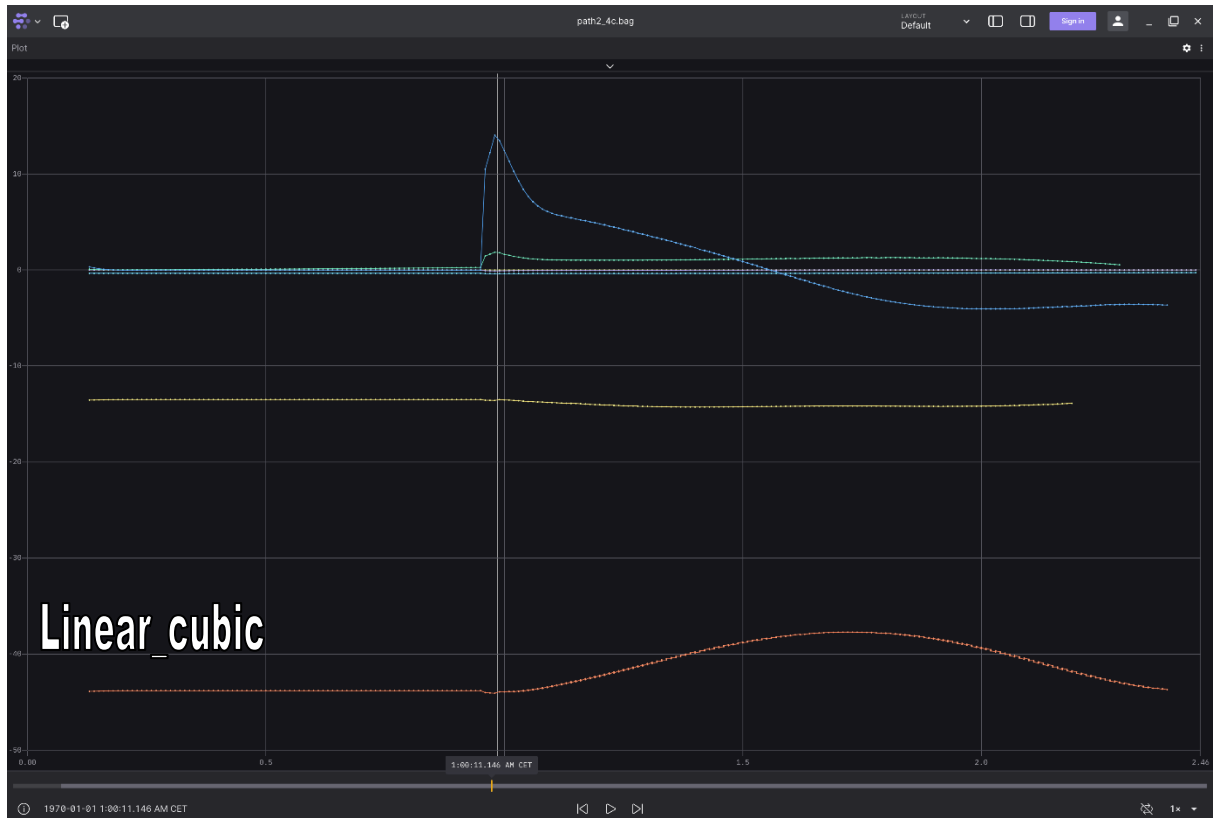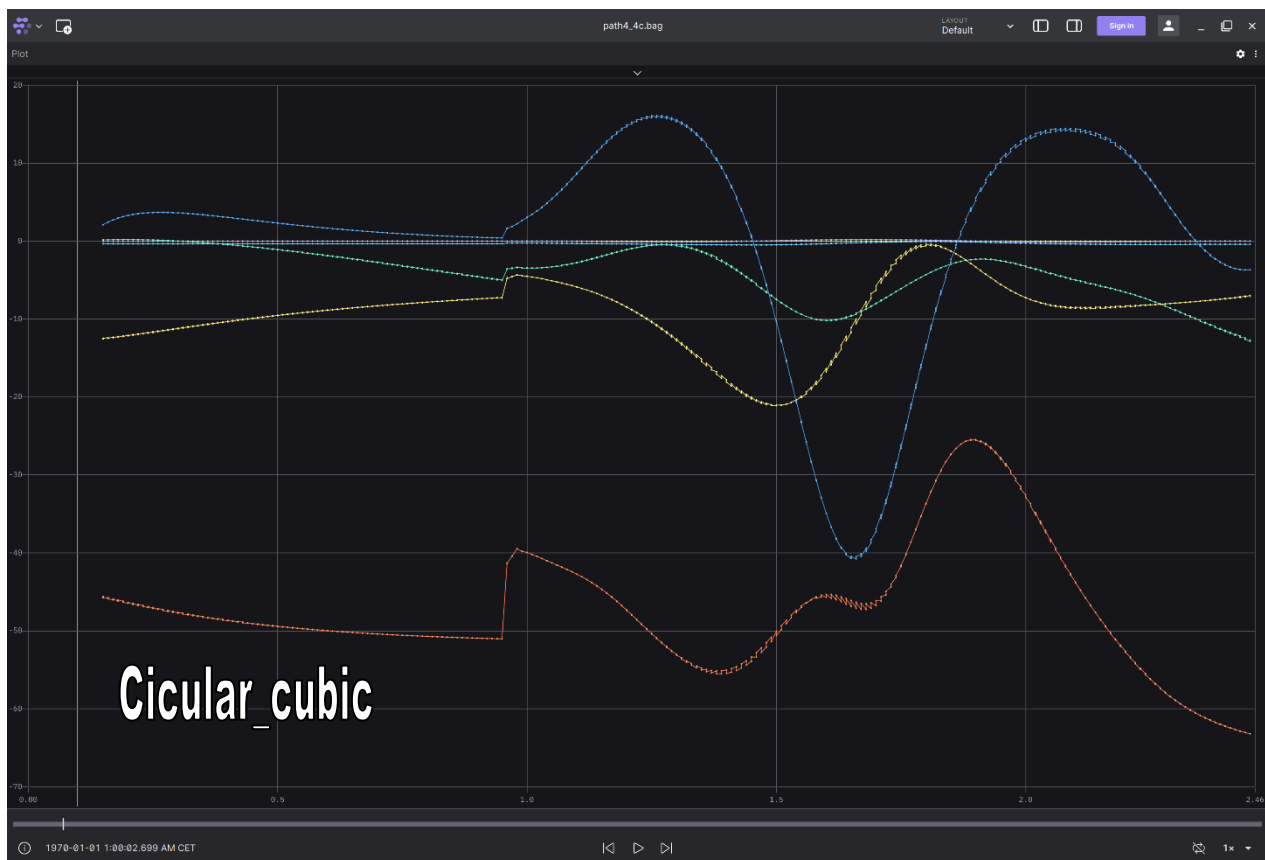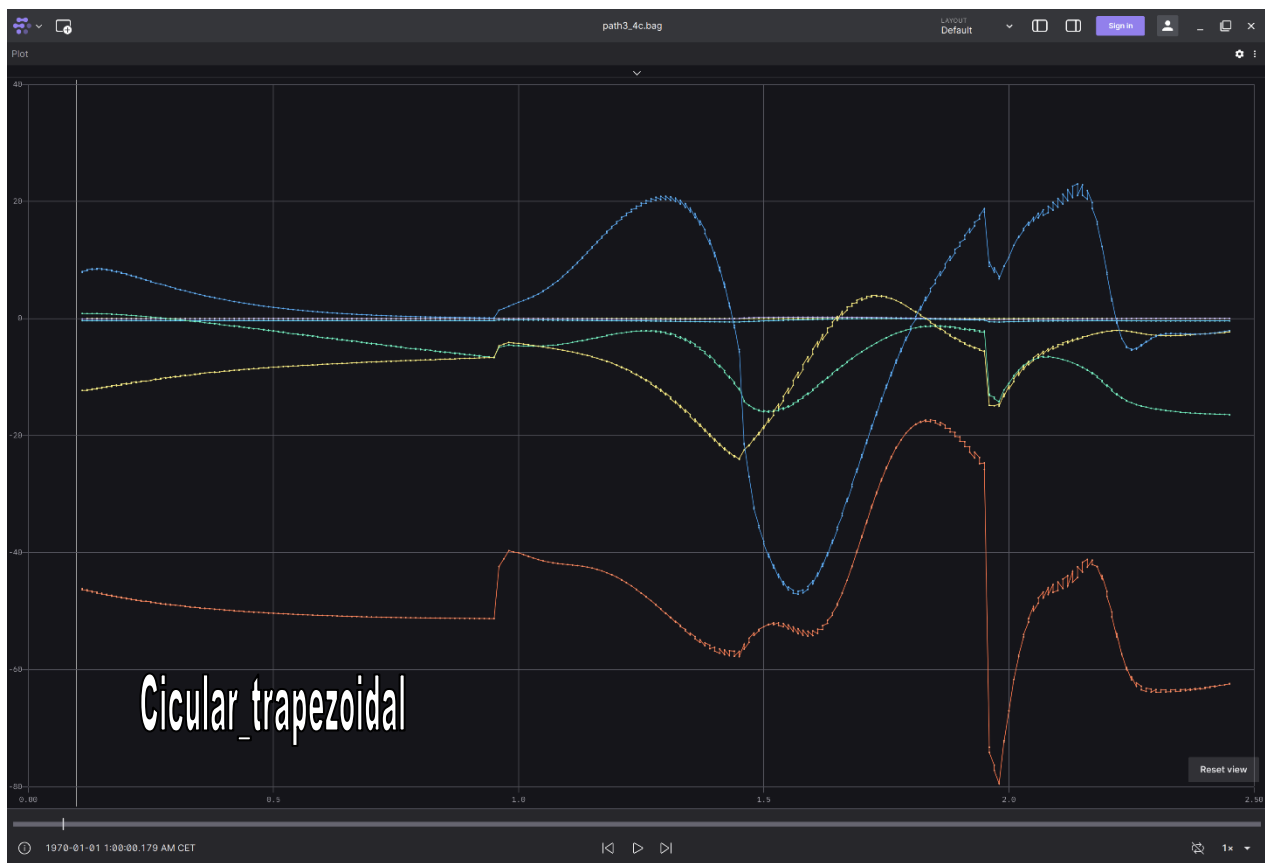
c. Test the controller along the planned trajectories and plot the corresponding joint torque commands.

Cicular_trapezoidal

Cicular_cubic

Link repository: https://github.com/matteolangella/RL-23-24-HW2.git