THE BISPECTRUM: INTRODUCTION, STATUS AND LOOKING FORWARD

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BLACK HOLES AND ACCRETION

- Helps us understand accretion physics.
- Dynamic on a number of (accessible) timescales

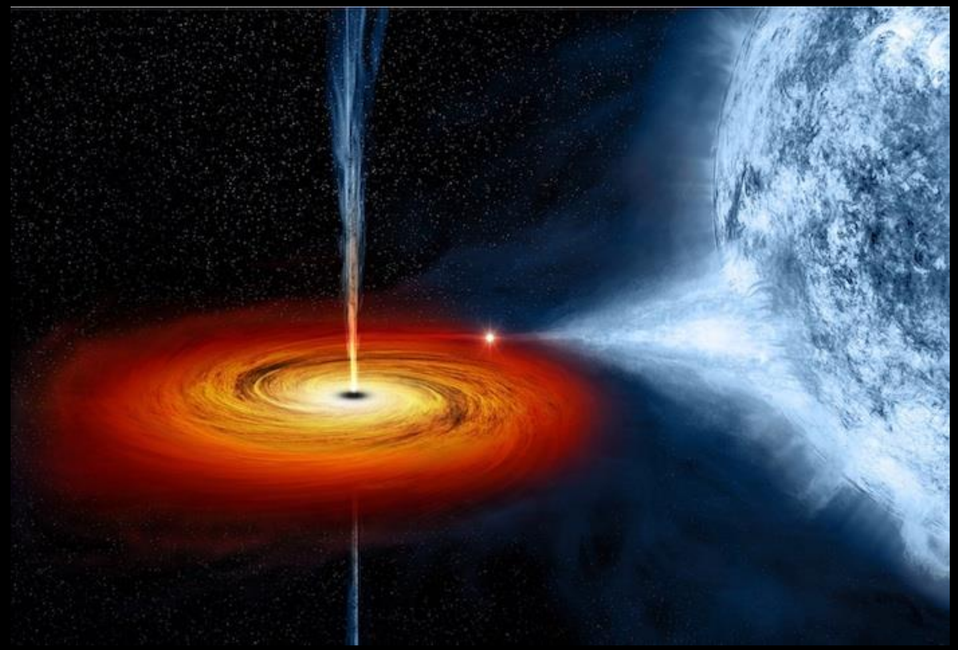


Image: ESA

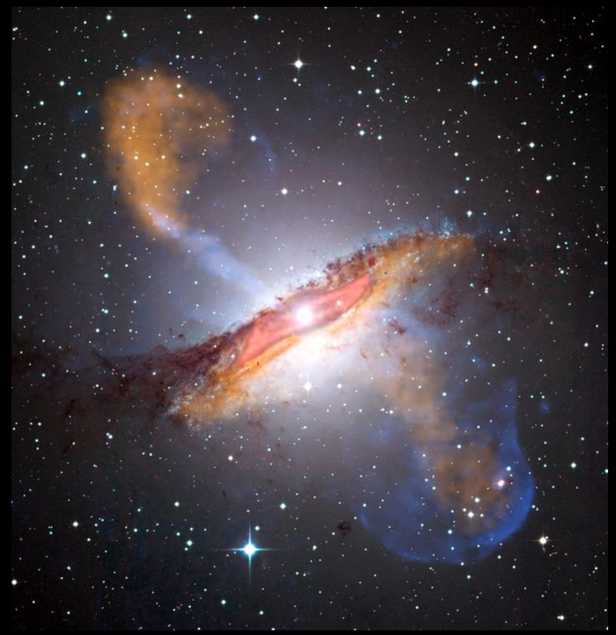


Image: NASA

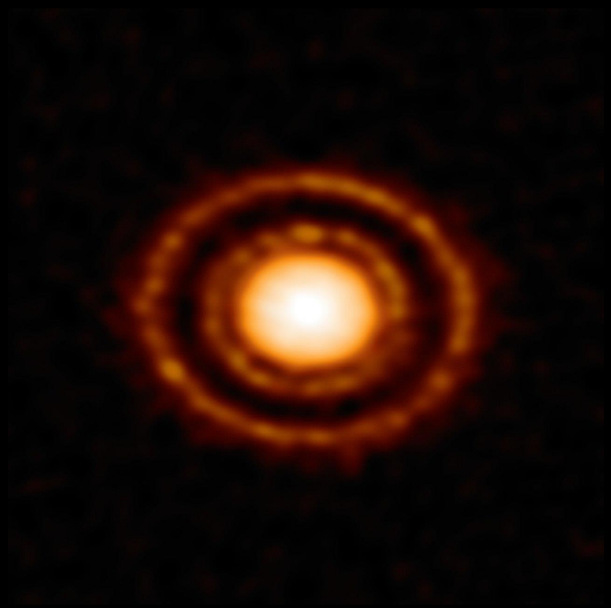


Image: ALMA

TYPES OF (FAST) VARIABILITY

Periodic

Pulsations

Quasi-Periodic

Low frequency QPOs

High Frequency QPOs

Burst Oscillations

mHz QPOs

Aperiodic

Broad band noise

B I S P E C T R U M

- The bispectrum is a higher order time series analysis technique.
- Fourier equivalent of 3 point correlation function
- Retains phase information!

BISPECTRUM IN OTHER FIELDS

- Earliest applications in oceanography - wave breaking, energy transfer between wave components.

Image: Surfer Today

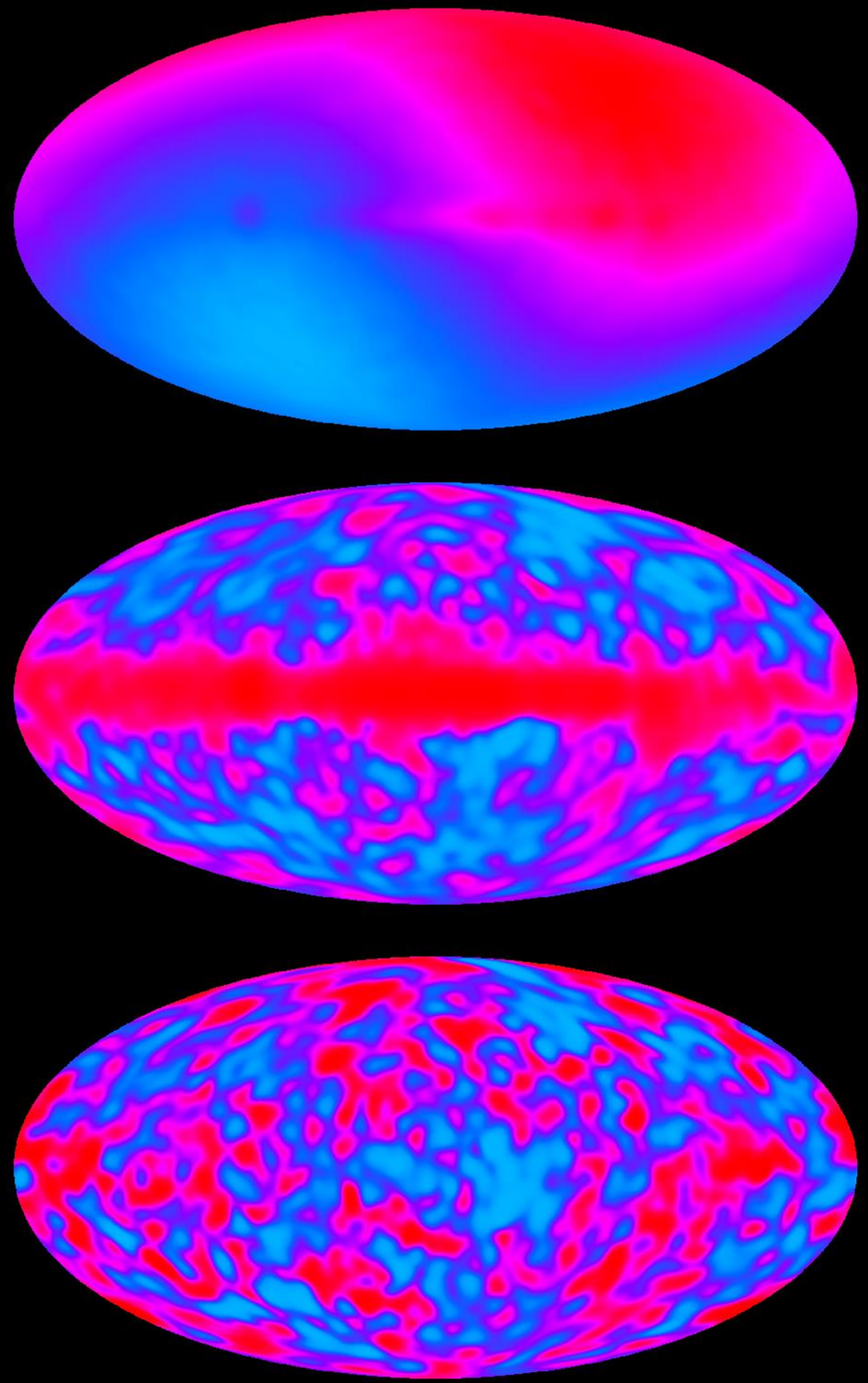
BISPECTRUM IN OTHER FIELDS

- Earliest applications in oceanography - wave breaking, energy transfer between wave components.
- Speech recognition
- Brain wave activity - Monitoring of anaesthesia, emotion recognition
- ECGs - identifying types of cardiac arrhythmia, diagnosis of sleep apnea

Image: Tech Crunch

BISPECTRUM IN OTHER FIELDS

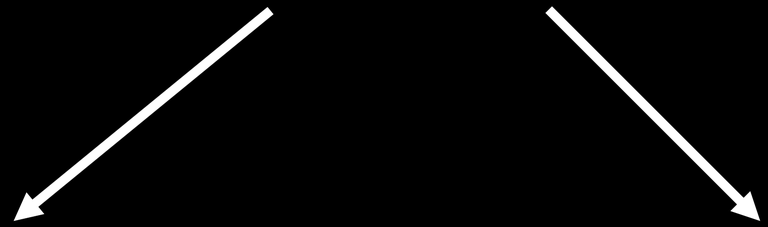
- Earliest applications in oceanography - wave breaking, energy transfer between wave components.
- Speech recognition
- Brain wave activity - Monitoring of anaesthesia, emotion recognition
- ECGs - identifying types of cardiac arrhythmia, diagnosis of sleep apnea
- Astronomy - Cosmic Microwave Background



RELATED QUANTITIES

$$B(f_1, f_2) = \frac{1}{K} \sum_{i=0}^{K-1} X_i(f_1) X_i(f_2) X_i^*(f_1 + f_2)$$

Bispectrum

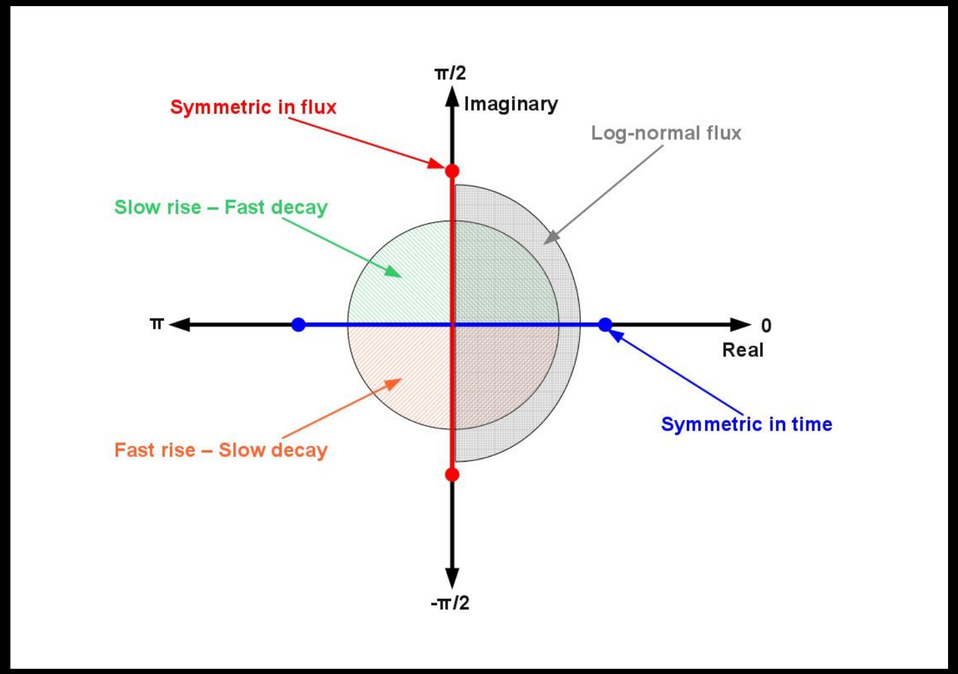


Biphase

Bicoherence

BIPHASE

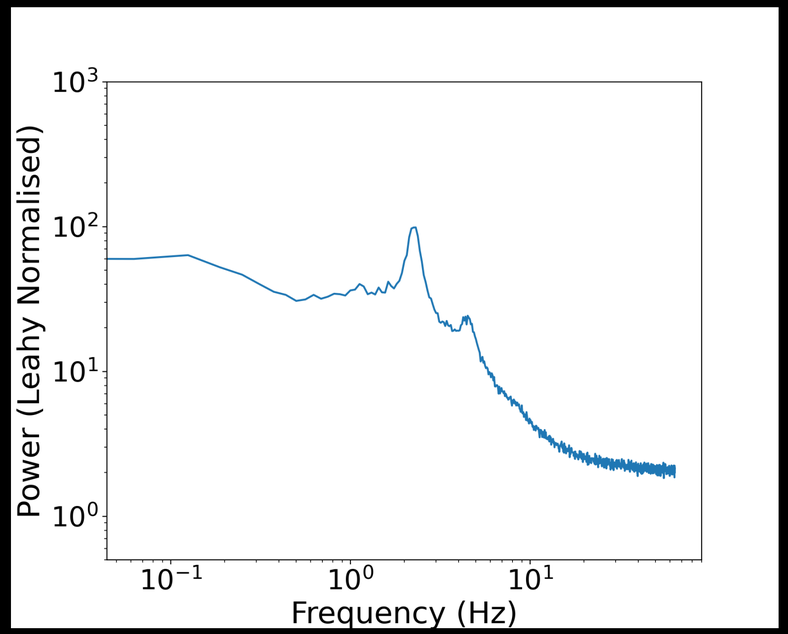
- Phase of the bispectrum
- Defined over the 2pi interval
- Time reversibility and Flux asymmetry
- Phase wrapping



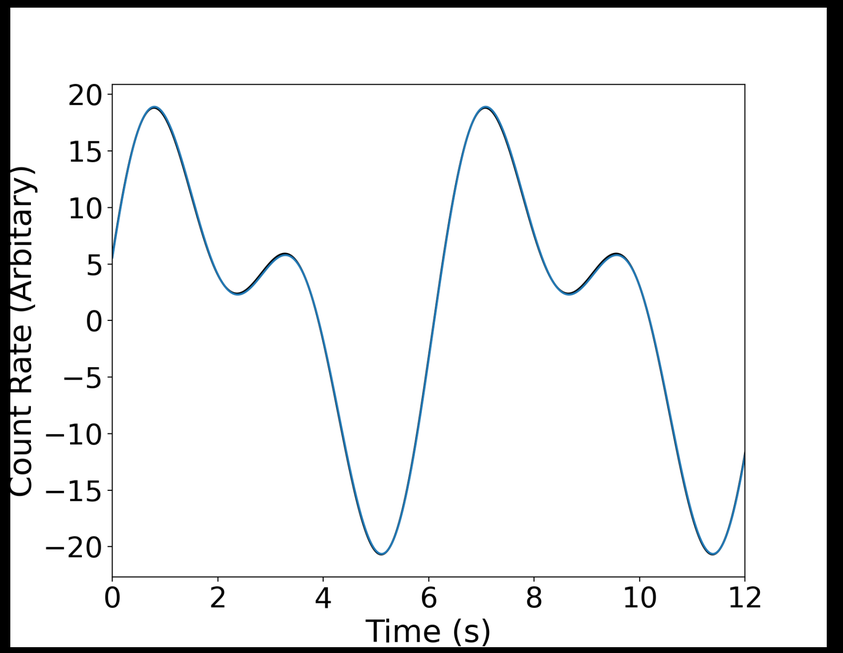
IC: Simone Scaringi

The biphase holds important information about the underlying waveform

WAVEFORM RECONSTRUCTION

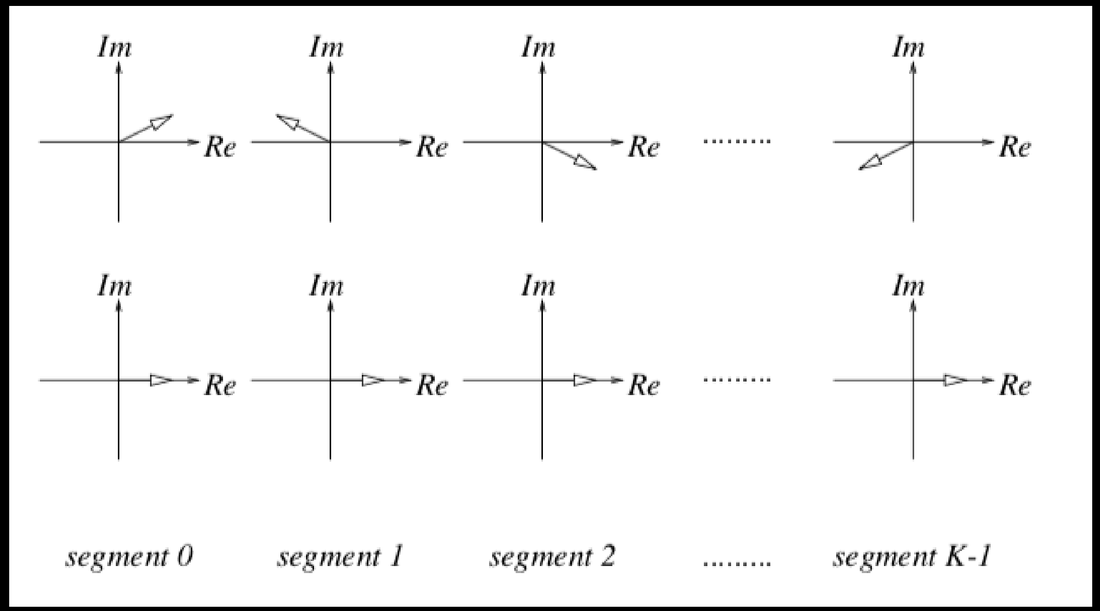


- Amplitude from power spectrum
- Phase difference from biphase
- Obtain average waveform!
- Higher harmonics easy to add.



BICOHERENCE

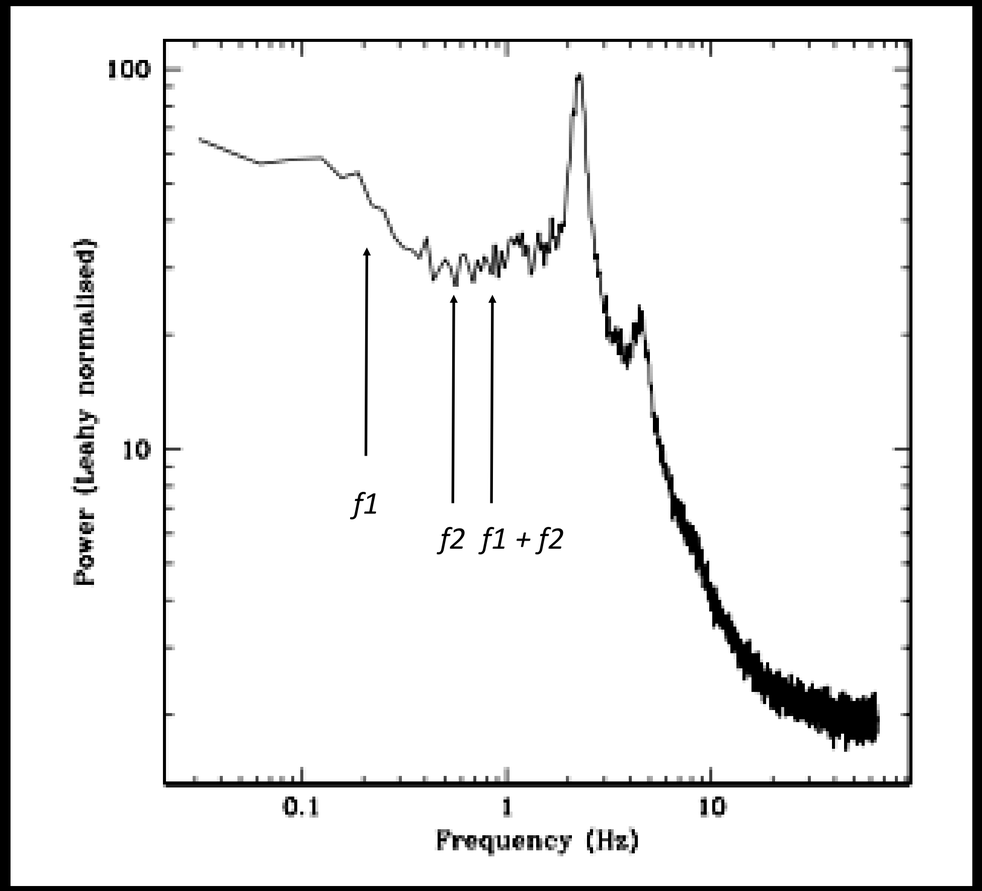
- Measure of how consistent biphase is between measurements
- Linear process, phases are random. Non-linear process, phases are correlated
- Bicoherence measures how correlated



BICOHERENCE

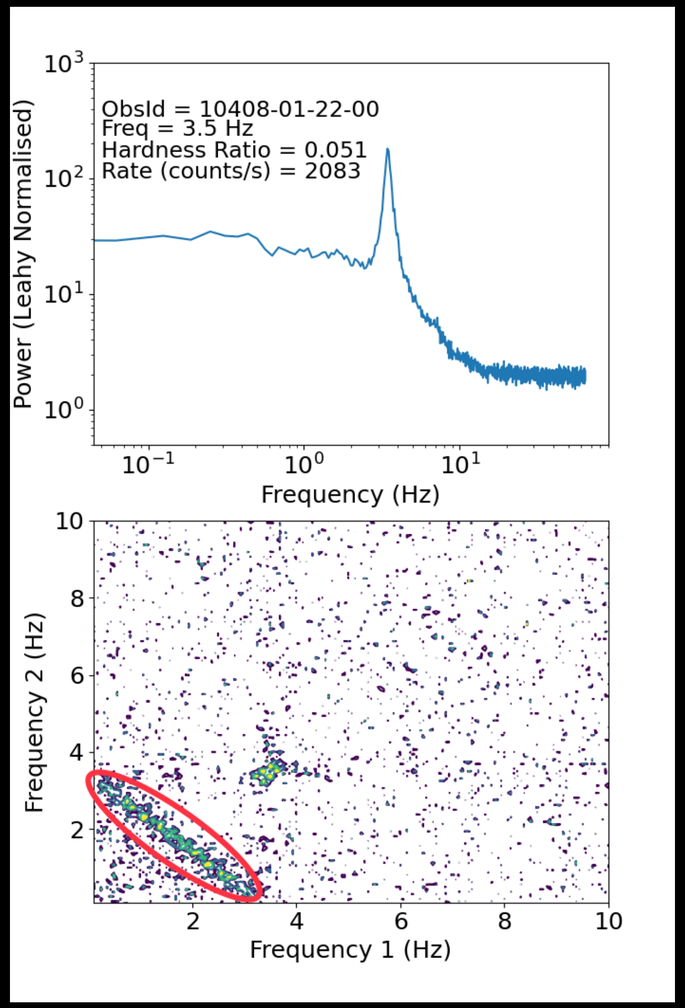
Bicoherence is a measure of interaction between variability at different timescales.

- Consider f1, f2, and f1+f2
- Interaction between parts of disk

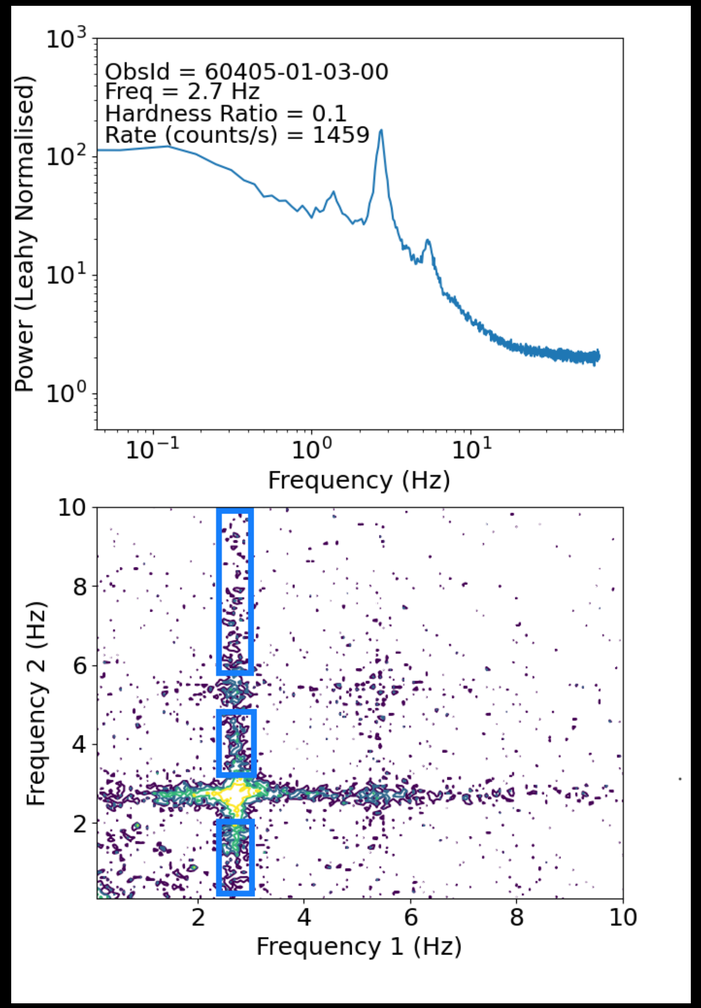


3 BICOHERENCE PATTERNS

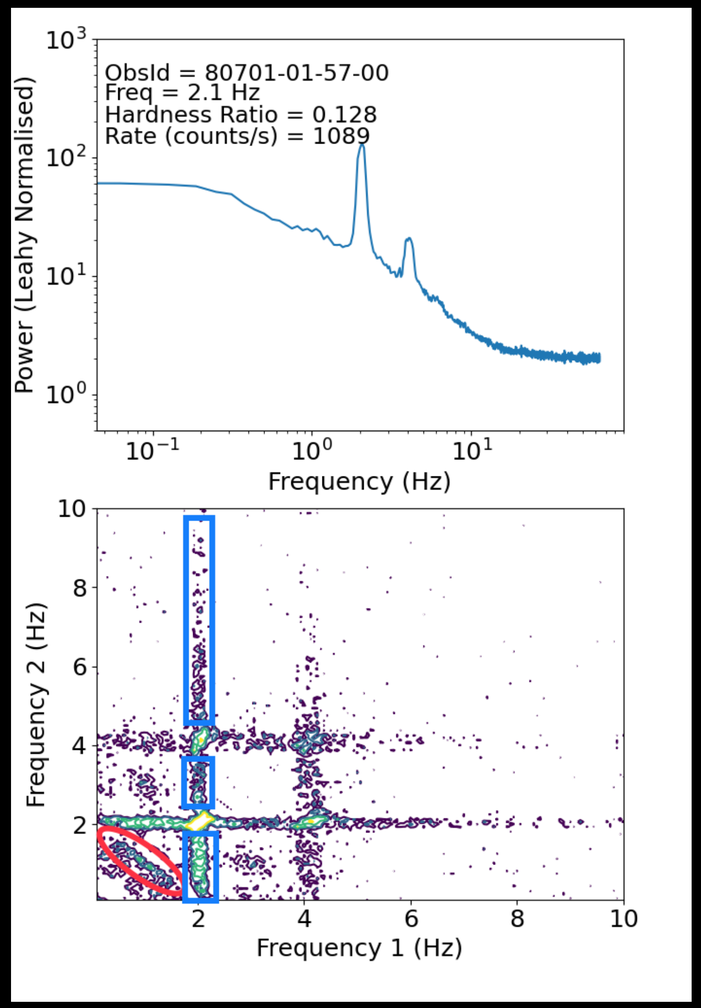
Hypotenuse



Cross



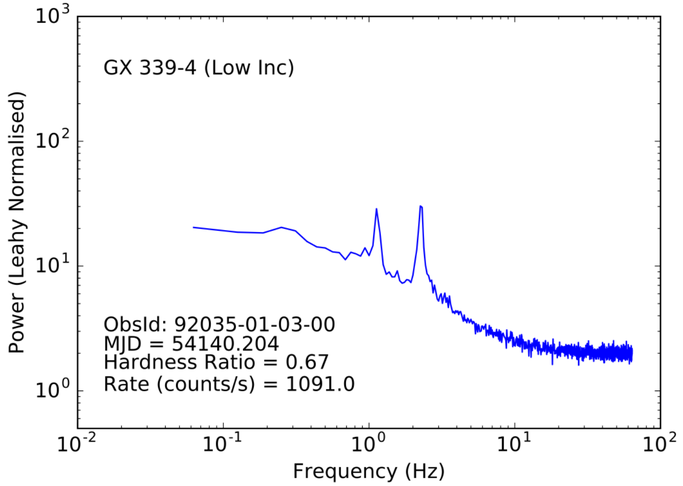
Web

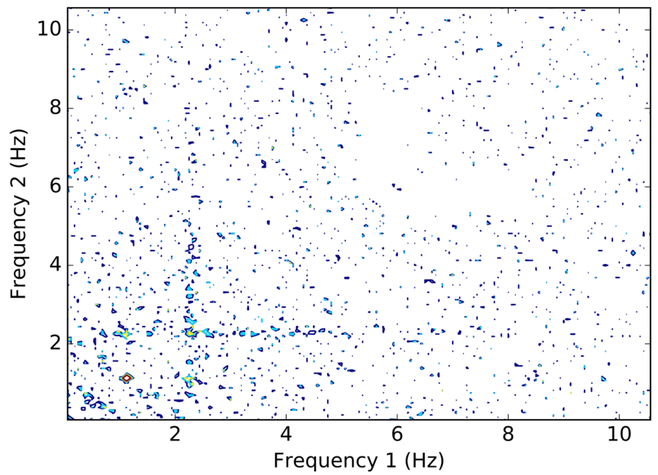


Adapted from Maccarone et al. 2011

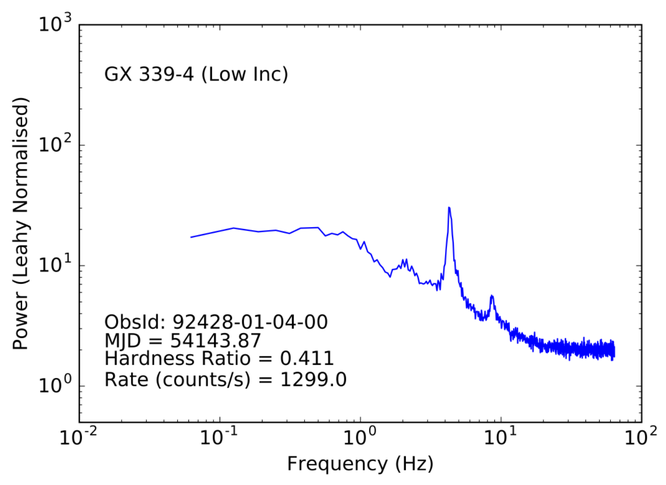
1. GRADUAL CHANGE IN BICOHERENCE

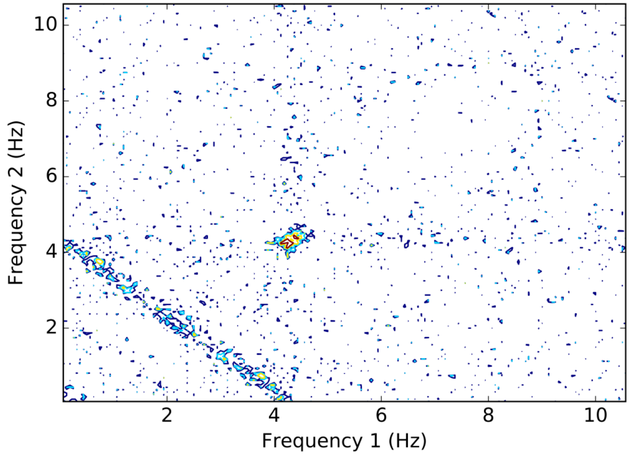
Web





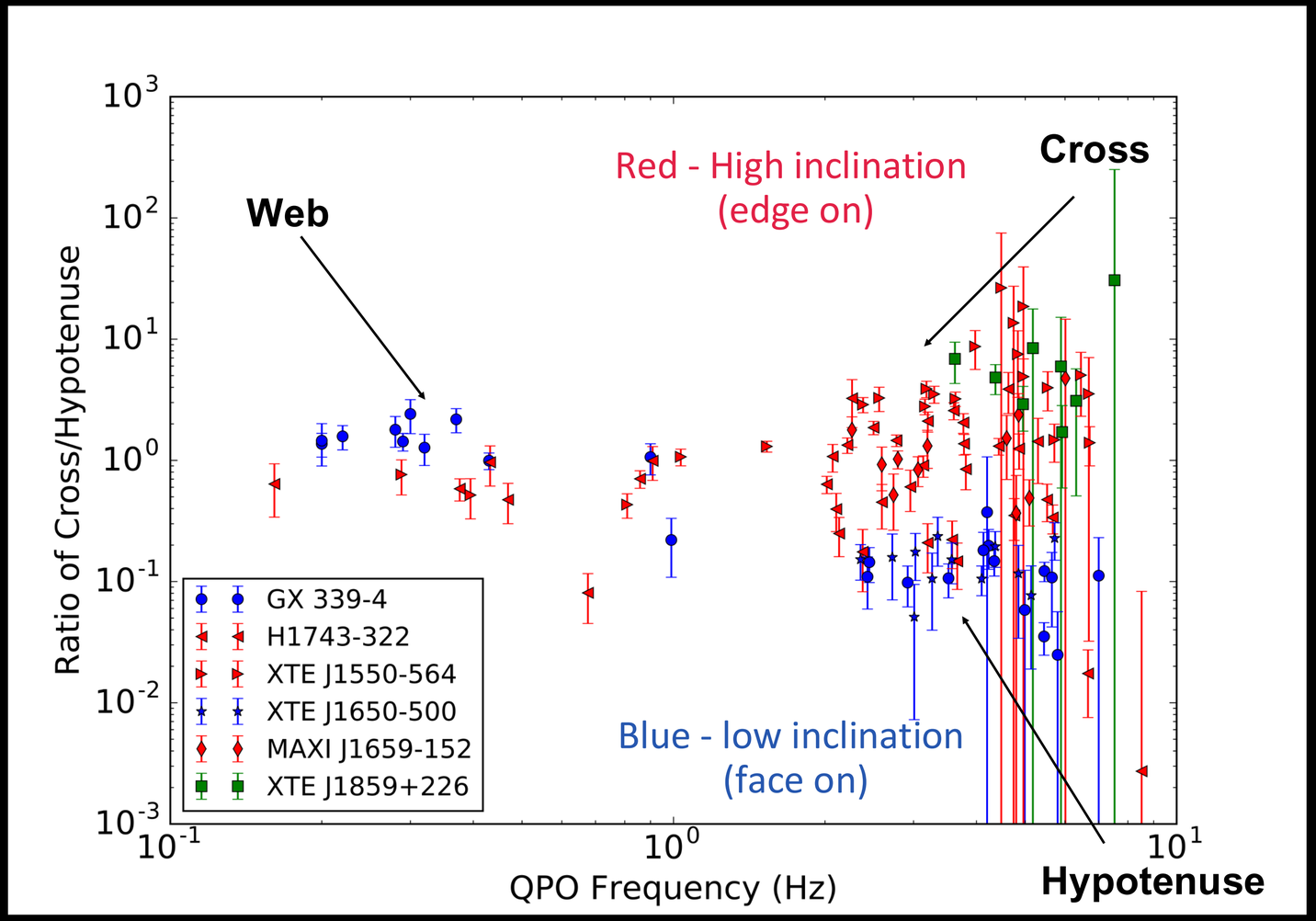
Hypotenuse





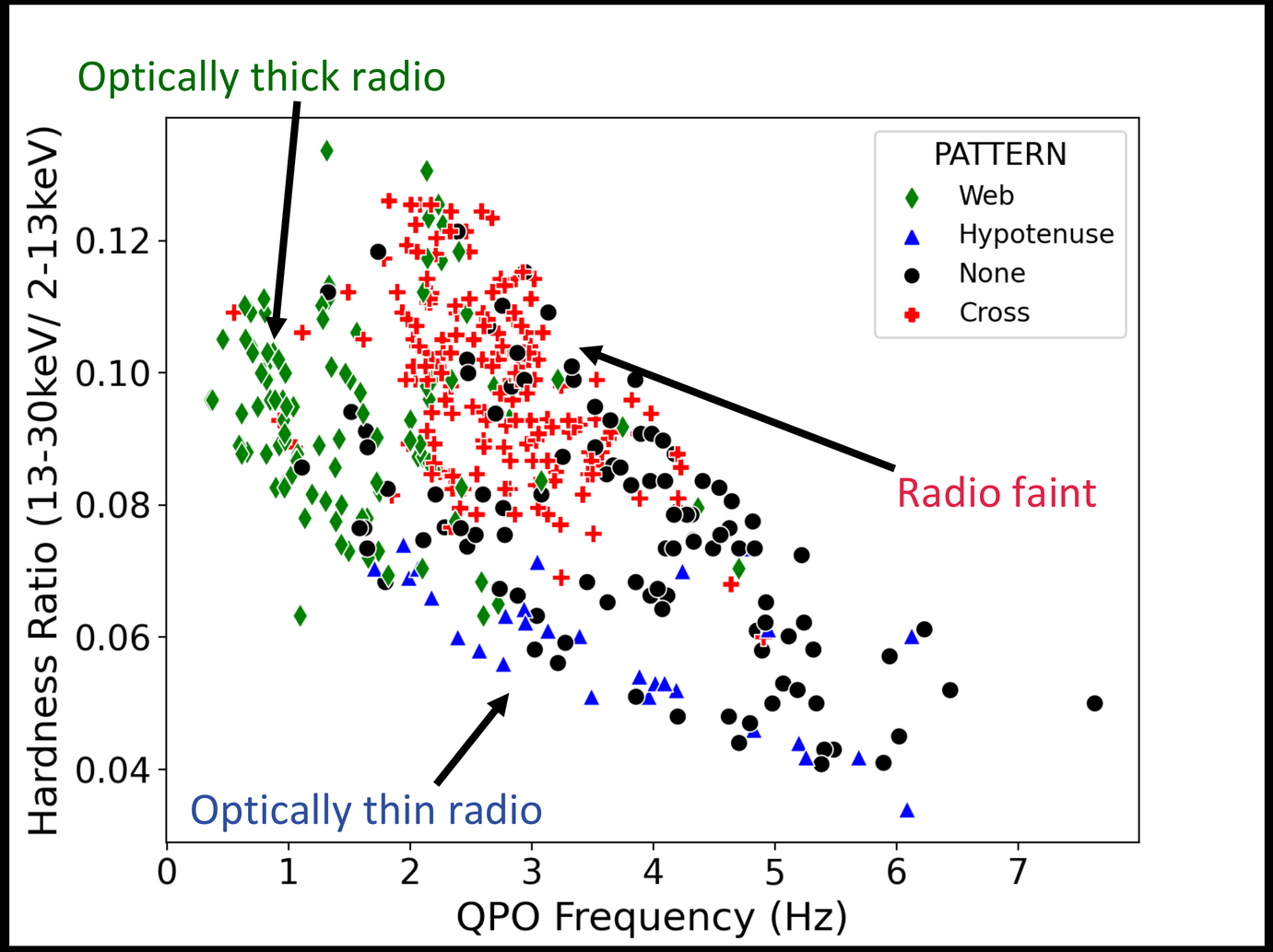
Arur & Maccarone 2019

2. GRADUAL CHANGE IS INCLINATION DEPENDENT



Arur & Maccarone 2020

3.CORRELATION WITH RADIO PROPERTIES IN GRS 1915+105

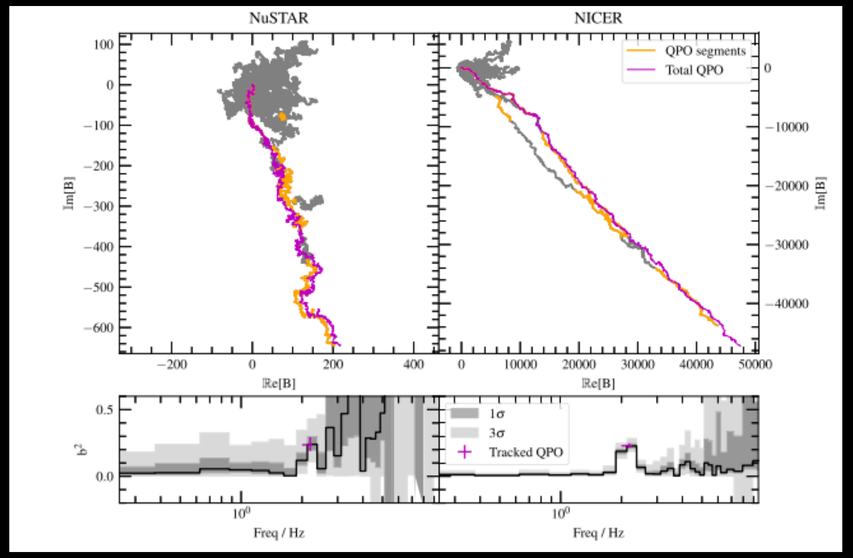


Arur & Maccarone 2022

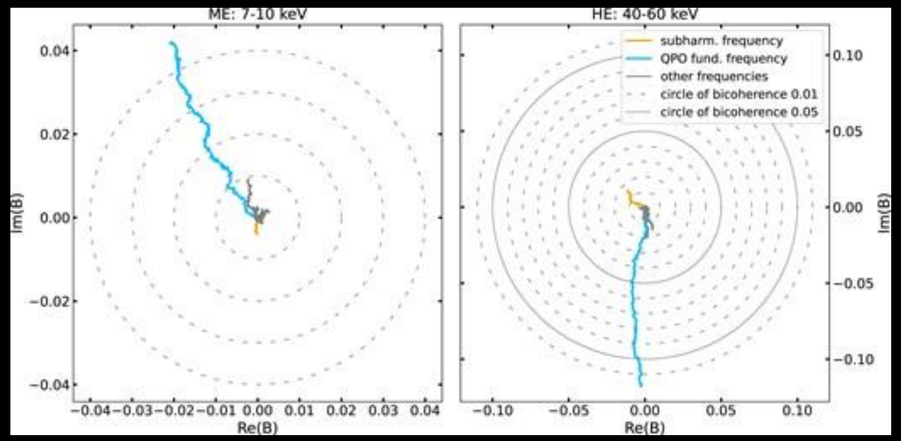
3 KEY QPO BICOHERENCE RESULTS

- Gradual evolution in bicoherence - probe of non-linearity
- Gradual change is inclination dependent - indicates geometric effect
- Correlation with radio properties - probe disk/jet interactions

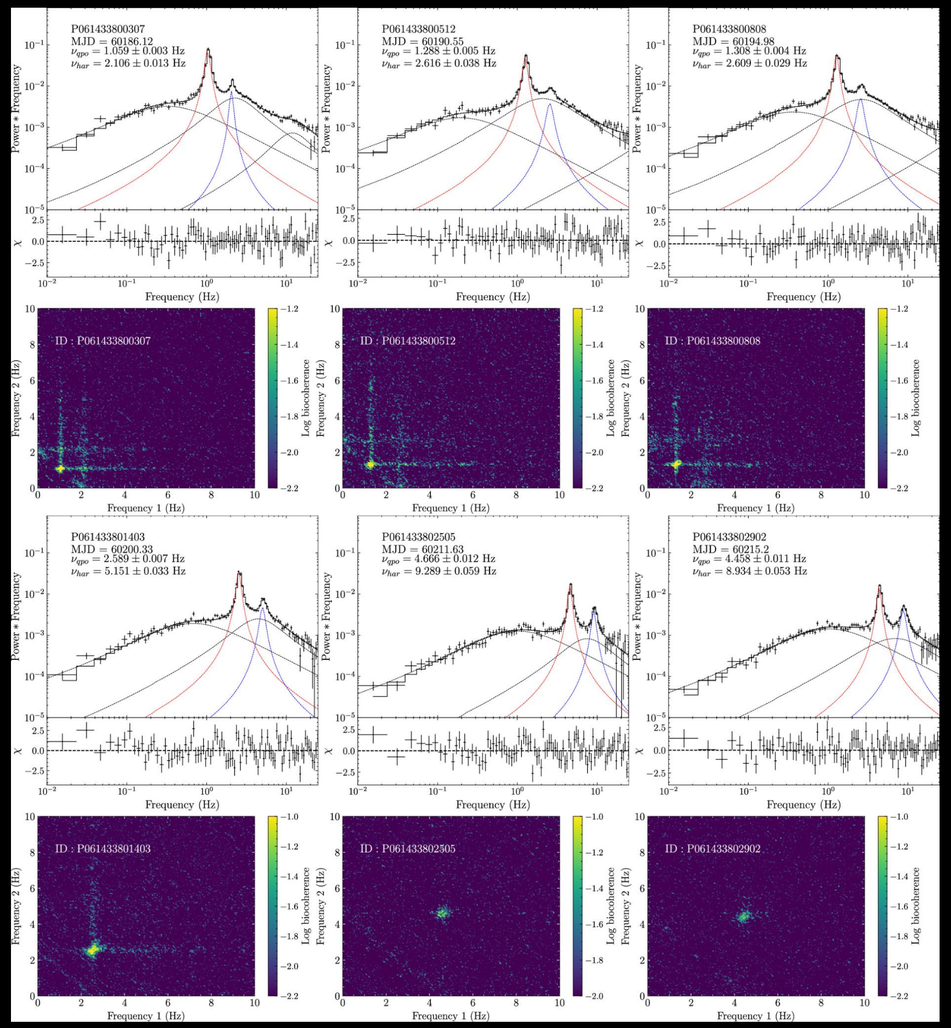
USAGE OF THE BISPECTRUM IS SPREADING!



Nathan et al. 2022, for GRS 1915+105



Bollemeijer et al. 2024: shows strong energy dependence for biphase



Zhu et al. 2024: likely low-medium inclination for Swift 1728-1613

INTERPRETING RESULTS USING THE BISPECTRUM

Direct Comparison

- Use light curves from simulations
- Calculate bicoherence
- Computationally intensive

Model predictions

- Qualitative picture of what bicoherence looks like
- Quantitative calculations
- Need more models!

Non-linear oscillator

- Time series of mathematically well defined systems
- Calculate bicoherence
- Develop intuition for underlying physics

BICOHERENCE SNR

$$\text{SNR} \sim \frac{Nb^2}{1-b^2}$$

QPOs

- Squared bicoherence of ~ 0.1 (10%)
- For SNR ~ 50, need ~500 segments
- For long RXTE observations, we can reach this

Aperiodic Variability

- Squared. bicoherence of ~ 0.01 (1%)
- For SNR ~ 50, need ~5000 segments
- Can average over a number of (low) frequencies to increase SNR

MODEL PREDICTIONS

- Some predictions exist for aperiodic variability for BHs (but not other systems)
- Some work on QPOs in simulations, but the predictions are not always explicitly stated

More work needed here!

MATHEMATICAL MODELS

- Damped driven oscillator shows promise, but more work is needed in exploring parameter space
- Other models also need to be explored (e.g Hindmarsh-Rose)
- Possible way to understand subharmonics - Ask me more in the discussion session!

SUMMARY & CONCLUSION

- Bispectrum is a valuable tool for understanding X-ray variability!
- The biphase holds important information about the shape of the underlying waveform
- Bicoherence is a measure of interaction between variability at different timescales.
- The field needs more modelling and theoretical work to fully unlock its potential!