LEARN HOW TO SELL MULTIPLE TYPES OF PRODUCTS UNDER BUDGET CONSTRAINT

ONLINE LEARNING APPLICATION PROJECT

PROJECT REQUIREMENTS

Requirement 1

Single product and stochastic environment

Requirement 2

Multiple products and stochastic environment

Requirement 3

Best-of-both-worlds algorithms with a single product

Requirement 4

Best-of-both-worlds with multiple products

Requirement 5

Slightly non-stationary environments with multiple products

SETTING

A company has to choose prices dynamically.

The goal of the company is to maximize profit in different selling scenarios with specific environment settings and according to different buyers behavior.

PARAMETERS AND INTERACTION

Given:

- A time horizon of **T** rounds
- The number of products **N**
- The set of possible prices **P**
- The production capacity **B** (expressed as the total number of products that the company can produce)
- The valuation $\boldsymbol{v_i}$ of the buyer for each type of product

At each round:

1. The company chooses the types of product to sell and set the price for each type of product
2. A buyer with a valuation for each type of product arrives
3. The buyer buys a unit of product with price smaller than his valuation
4. If the product is sold, the budget of the company is decreased

REQUIREMENT 1

1.1 - Single product and Stochastic environment without Budget constraint

1.2 - Single product and Stochastic environment with Budget constraint

ENVIRONMENT

Requirement 1.1

COMPANY

- Single product selling
- No budget constraints

BUYER

- Has a distribution over the valuation of a single product
- Modelled as a Gaussian distribution

SOLUTION

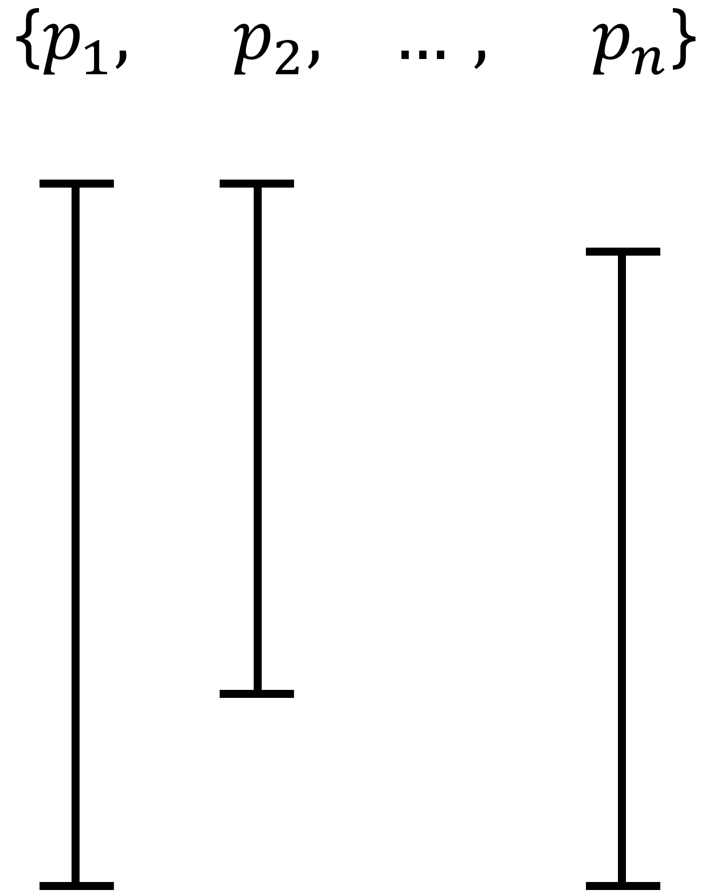
Requirement 1.1

UCB1 approach:

1. Compute UCB for all the arms (prices)
2. Choose the arm with the highest UCB
3. Update the agent

Baseline computation:

Expected rewards calculated weighting the prices vector with the conversion probability



SIMULATION

Requirement 1.1

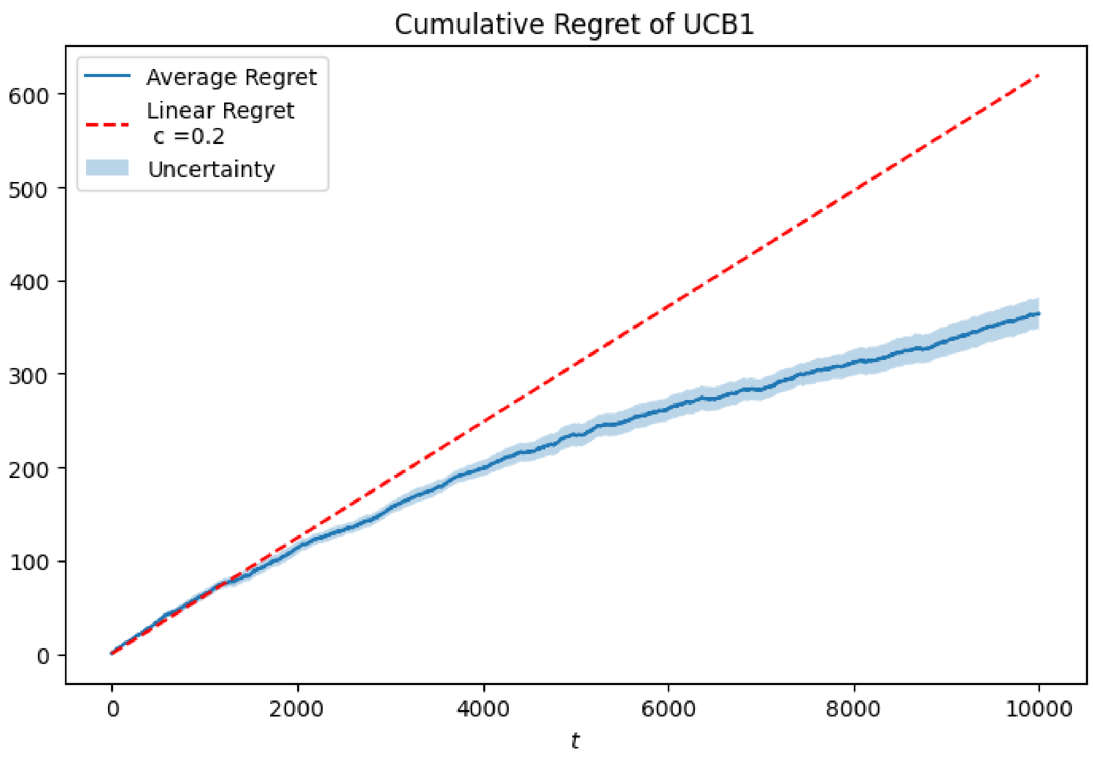
We provide results for a simulation with the following parameters:

- Time horizon: T = 10000
- Price set P on the interval [0, 1]
- Gaussian distribution [0.5, 1.0] for the buyer distribution

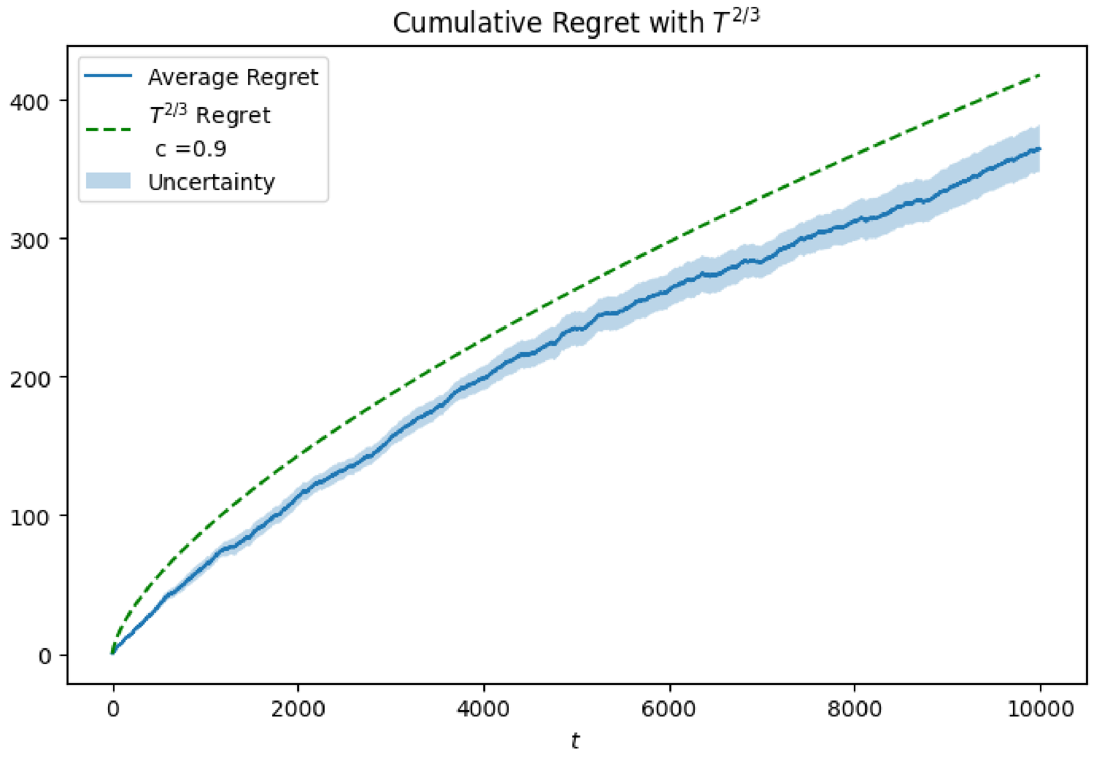
For measuring the uncertainty on the result the simulation is executed over 10 trials

RESULTS

Requirement 1.1



Cumulative Regret of UCB1



Cumulative Regret with $T^{2/3}$

ENVIRONMENT

Requirement 1.2

COMPANY

- Single product selling
- Budget constraints

BUYER

- Has a distribution over the valuation of a single product
- Modelled as a Gaussian distribution

SOLUTION

Requirement 1.2

UCB1-like approach:

1. Compute UCB for rewards and LCB for costs
2. Solve the linear program to find the optimal probabilities
3. Draw an arm from the computed distribution
4. Get the reward and the cost (unit sold)
5. Update the agent

Different baseline computation

Linear program for finding the optimal strategy **gamma**

$$OPT_t = \begin{cases} \sup\limits_{\gamma \in \Delta_{\mathcal{B}}} \bar{f}_t^{UCB}(\gamma) \\ \text{s.t. } \bar{c}_t^{LCB}(\gamma) \leq \rho \end{cases}$$

$$\Downarrow$$

$$\begin{aligned} \max_{\gamma \in \mathbb{R}^K} \quad & \sum_{i=1}^{K} \gamma_i \, \bar{f}_i^{\mathrm{UCB}} \\ \text{s.t.} \quad & \sum_{i=1}^{K} \gamma_i \, \bar{c}_i^{\mathrm{LCB}} \leq \rho, \\ & \sum_{i=1}^{K} \gamma_i = 1, \\ & 0 \leq \gamma_i \leq 1 \quad \forall i = 1, \dots, K. \end{aligned}$$

SIMULATION

Requirement 1.2

We provide results for a simulation with the following parameters:

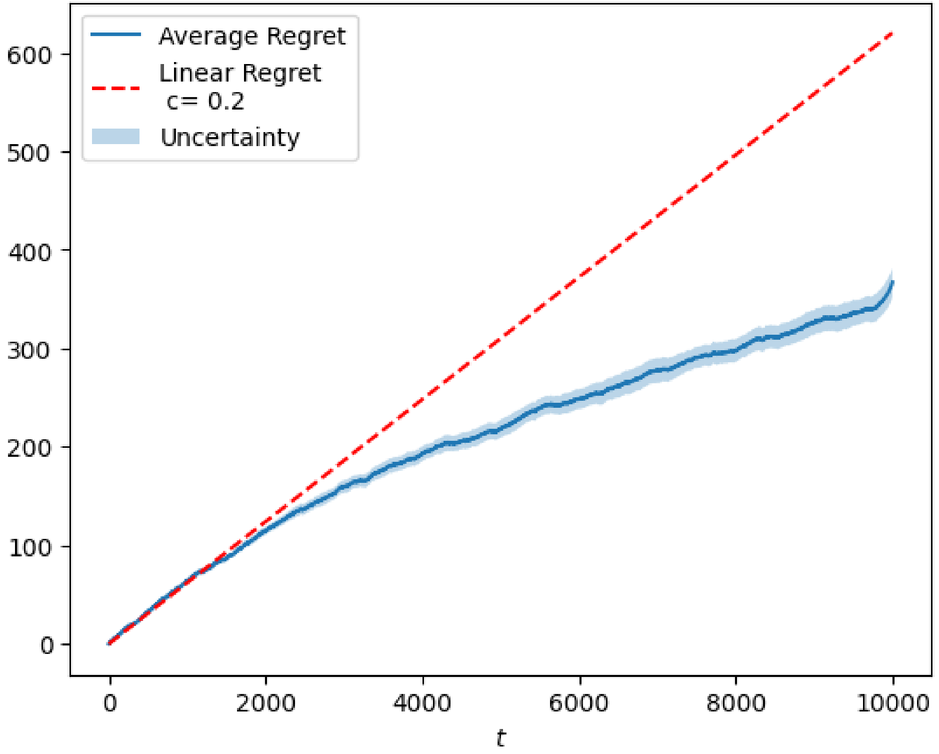
- **Time horizon:** T = 10000
- **Budget:** B = 4000
- **Price set P** on the interval [0, 1]
- **Gaussian distribution [0.5, 1.0]** for the buyer distribution

For measuring the uncertainty on the result the simulation is executed over 10 trials

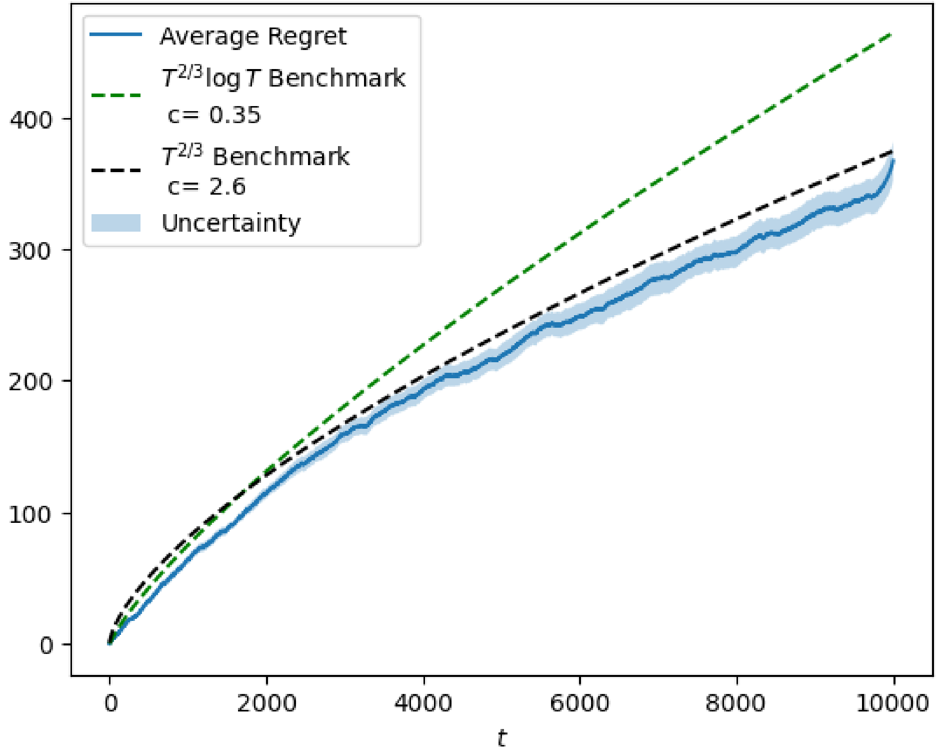
RESULTS

Requirement 1.2

Cumulative Regret of UCB1



Cumulative Regret with benchmarks



REQUIREMENT 2

Multiple products and Stochastic environment

ENVIRONMENT

Requirement 2

COMPANY

- Multiple product selling
- Budget constraints

BUYER

- Has a joint distribution over the valuation of the products
- Modelled as a Multivariate Gaussian distribution

PROPOSED SOLUTIONS

Requirement 2

APPROACH 1

Product-wise decomposition with independent UCB for each product.

Same approach as Req. 1.2 but for N > 1 products

APPROACH 2

A priori calculation of all superarms with cartesian product.

Full combinatorial optimization with linear program solving for joint pricing decisions.

APPROACH 3

Same approach as approach 2 but greedy: we don't optimize solving the linear program

Baseline Computation

Linear program for finding the optimal **gamma matrix**

SIMULATION

Requirement 2

We provide results for a simulation with the following parameters:

- **Time horizon:** T = 10000
- **Budget:** B = 16000
- **Price set P** on the interval [0, 1]
- **Number of Products:** 3
- **Multivariate Gaussian distribution** with mean vector [0.5, 0.6, 0.7] and covariance matrix [[0.1, 0.05, 0.02], [0.05, 0.1, 0.03], [0.02, 0.03, 0.1]].

For measuring the uncertainty on the result the simulation is executed over 5 trials

APPROACH 1

Requirement 2

Product-wise UCB1 approach:

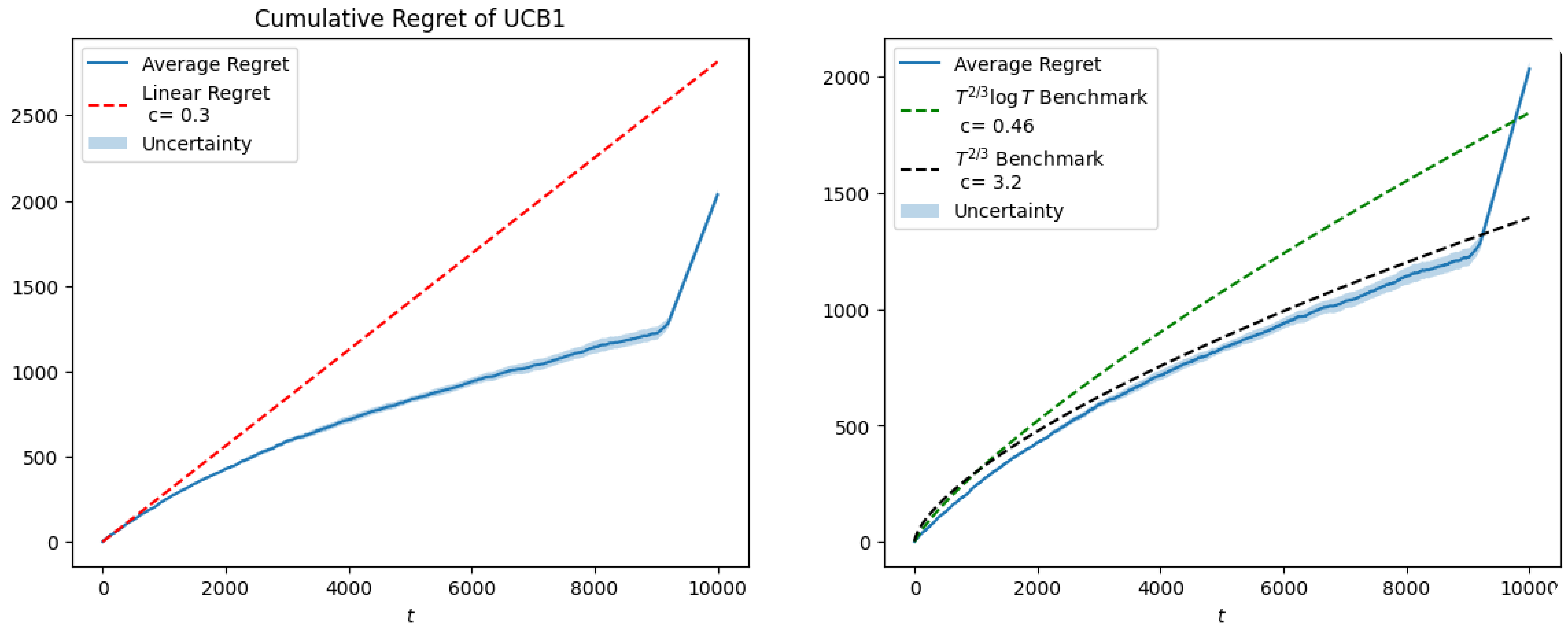
1. Compute UCB for rewards and LCB for costs for each product
2. Compute the optimal strategy gamma for each product
3. Generate and pull the superarm using the gamma matrix
4. Get prices and check for units sold
5. Update the agent

SUPERARM

PRODUCT 1	*p1*
PRODUCT 2	*p2*
PRODUCT 3	*p3*

RESULTS

Requirement 2 – Approach 1



APPROACH 2

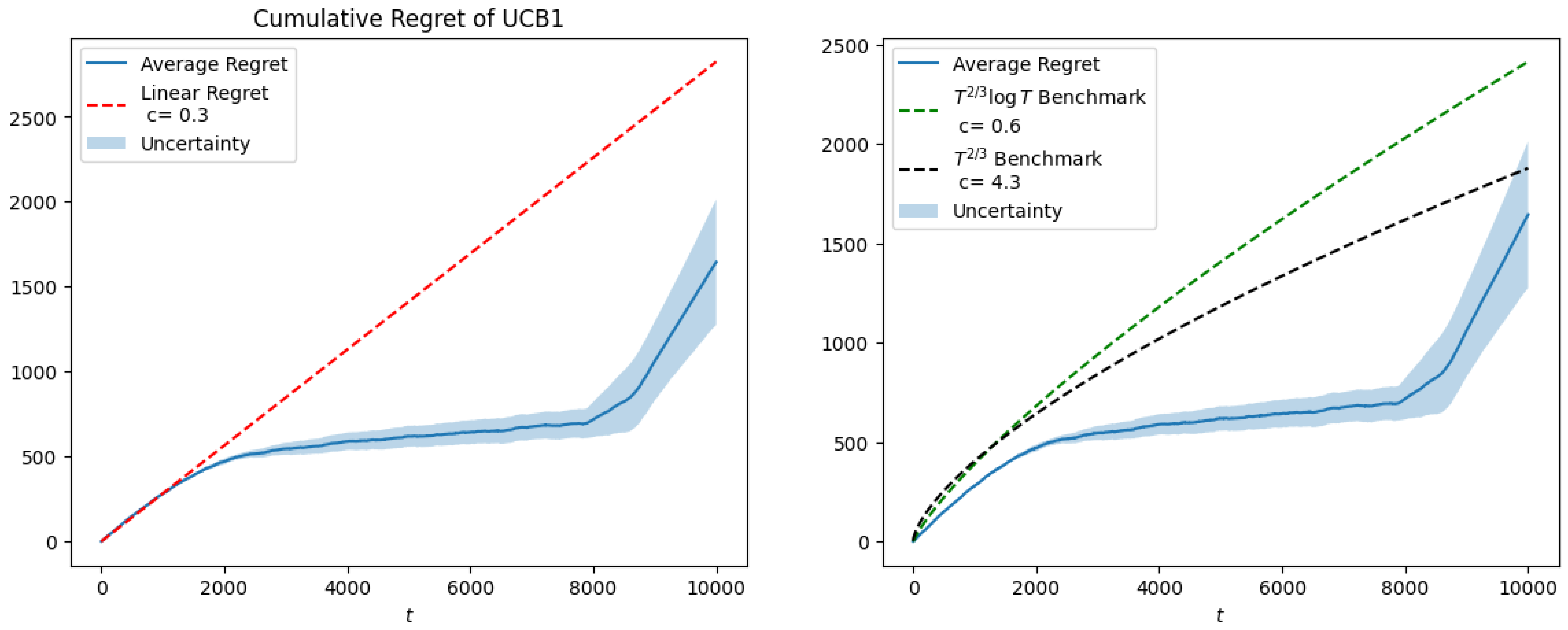
Requirement 2

Full combinatorial UCB1 approach:

1. Generate all the combination of prices (superarms) with cartesian product
2. Compute UCB for rewards and LCB for costs for each superarm
3. Solve the linear program to find the gamma
4. Pull the superarm using the gamma and get the reward and the cost (if sold)
5. Update the agent

RESULTS

Requirement 2 – Approach 2


Cumulative Regret of UCB1

APPROACH 3

Requirement 2

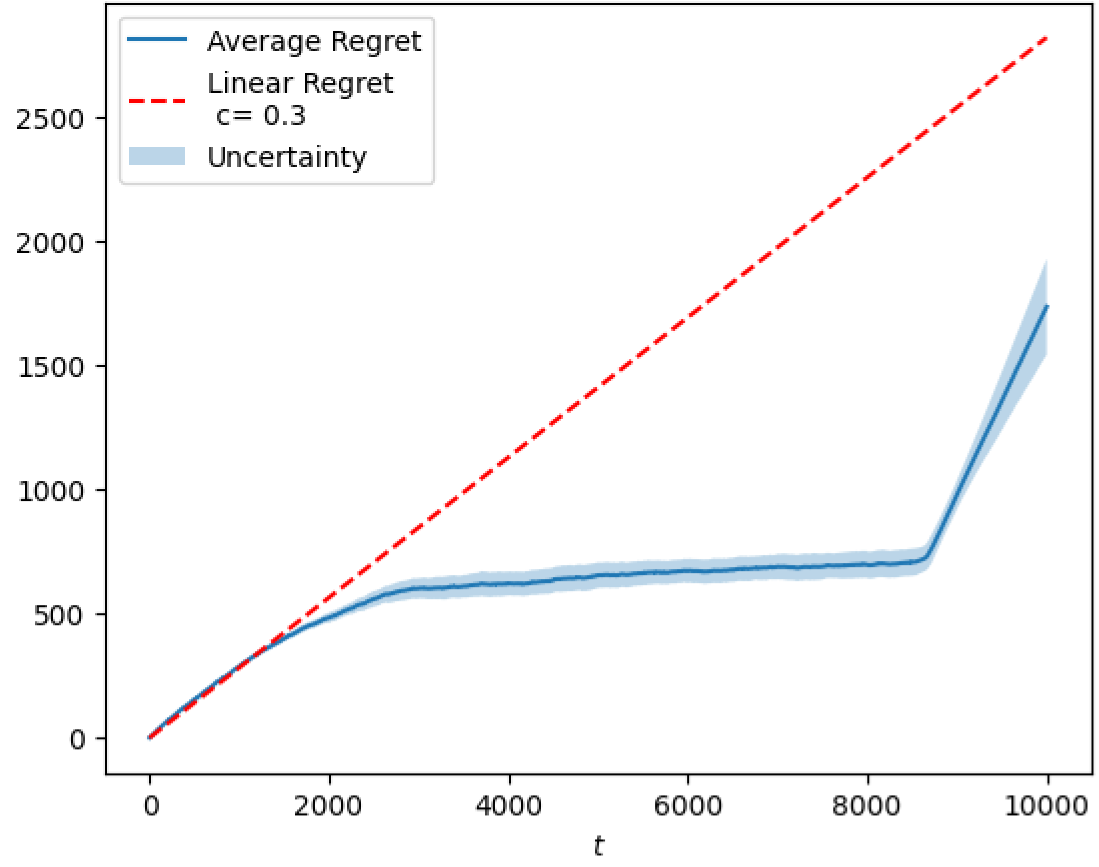
Full combinatorial UCB1 approach, with greedy:

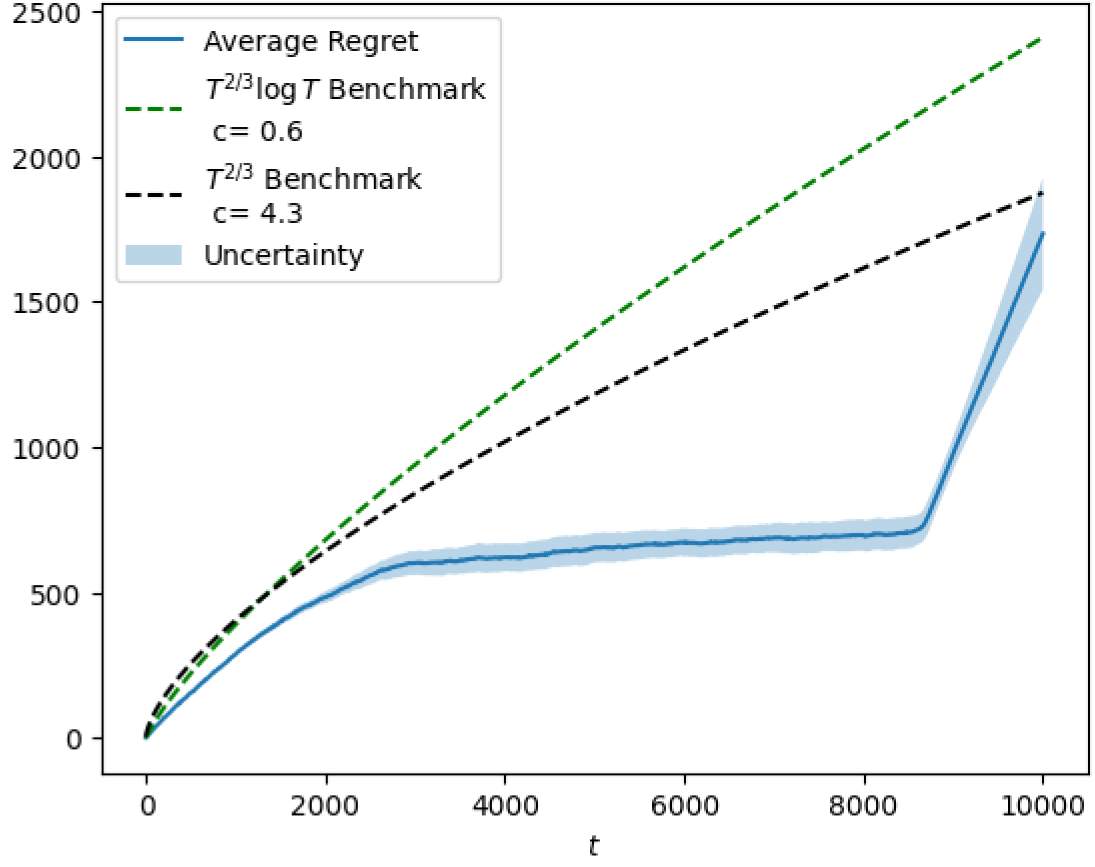
1. Generate all the combination of prices (superarms) with cartesian product
2. Compute UCB for rewards and LCB for costs for each superarm
3. Choose feasible superarm which maximize utility, without linear program optimization
4. Pull the superarm and get the reward and the cost (if sold)
5. Update the agent

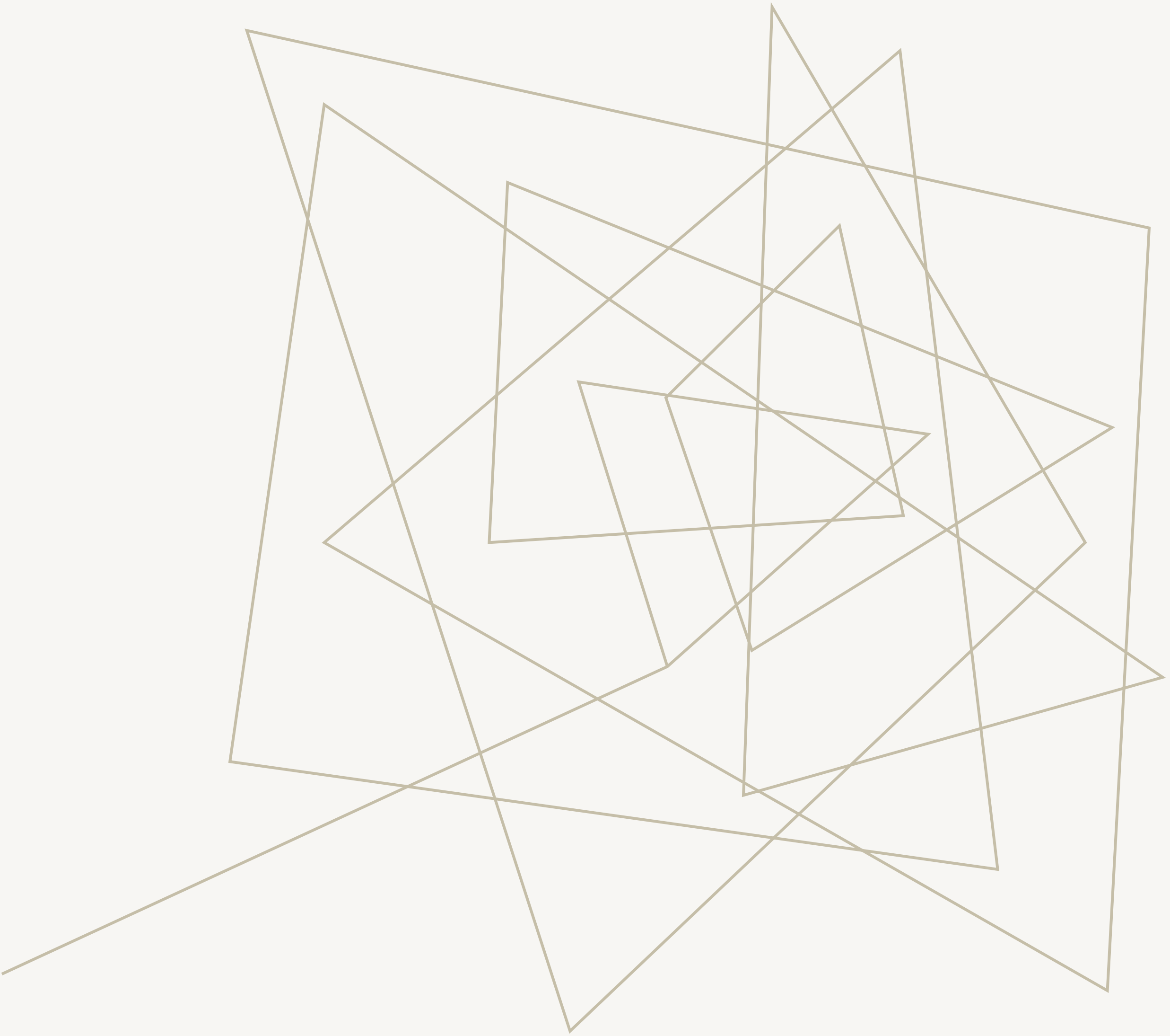
RESULTS

Requirement 2 – Approach 2

Cumulative Regret of UCB1







REQUIREMENT 3

Single product and Adversarial environment

ENVIRONMENT

Requirement 3

COMPANY

- Single product selling
- No budget constraints

BUYER

- Has a distribution over the valuation of a single product
- Modelled as a Gaussian distribution

SOLUTION

Requirement 3

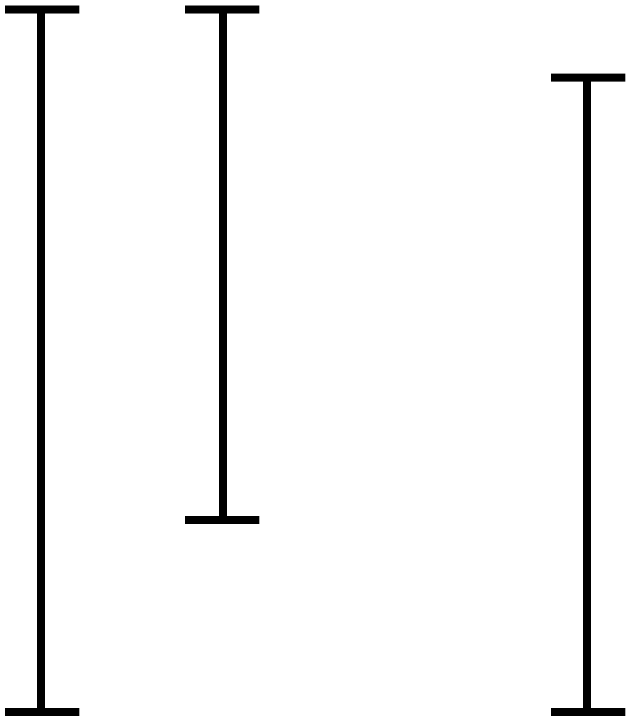
UCB1 approach:

1. Compute UCB for all the arms (prices)
2. Choose the arm with the highest UCB
3. Update the agent

Baseline computation:

Expected rewards calculated weighting the prices vector with the conversion probability

$$\{p_1, \quad p_2, \quad \dots, \quad p_n\}$$



SIMULATION

Requirement 3

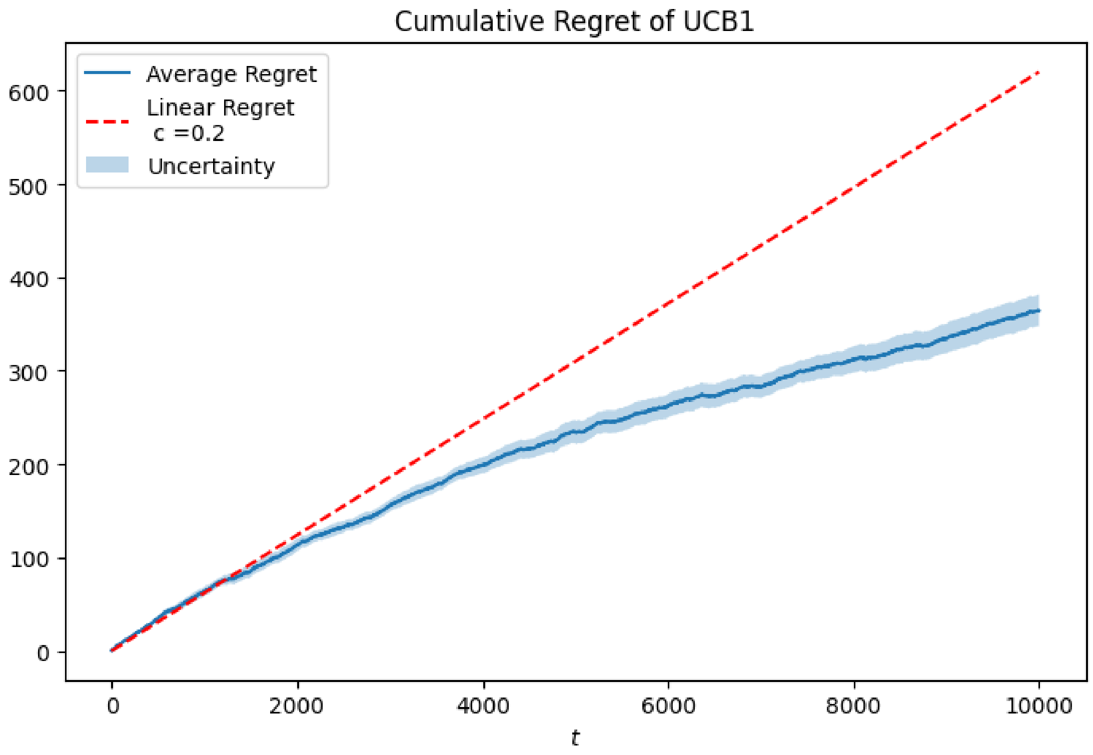
We provide results for a simulation with the following parameters:

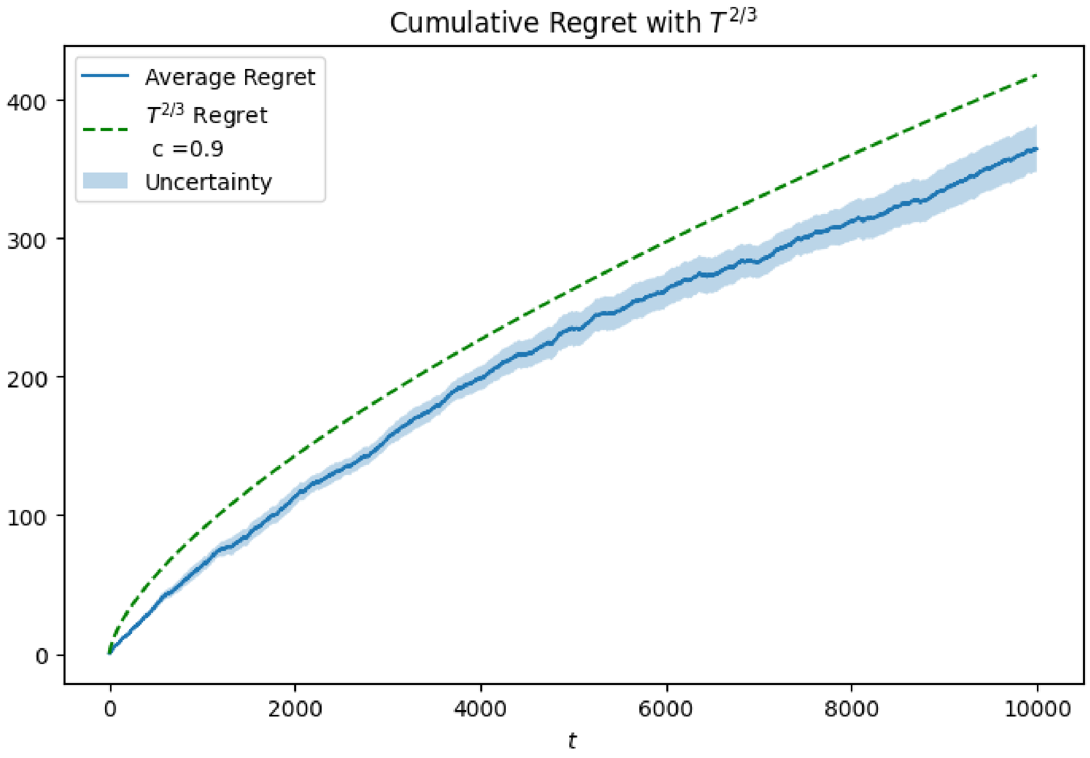
- Time horizon: T = 10000
- Price set P on the interval [0, 1]
- Gaussian distribution [0.5, 1.0] for the buyer distribution

For measuring the uncertainty on the result the simulation is executed over 10 trials

RESULTS

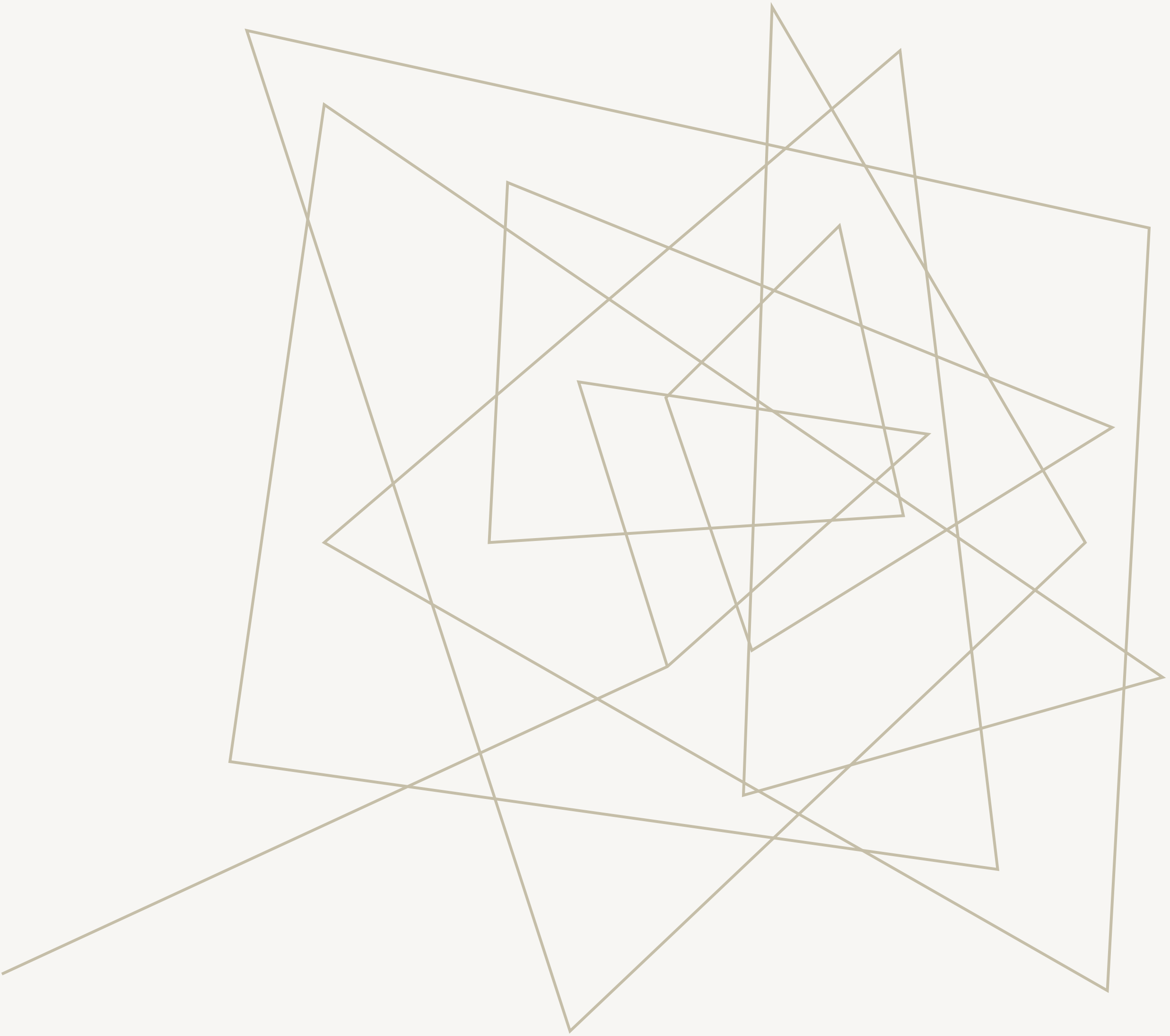
Requirement 3


Cumulative Regret of UCB1


Cumulative Regret with $T^{2/3}$

REQUIREMENT 4

Multiple products and Adversarial environment



REQUIREMENT 5

Slightly non-stationary environment