Properties of Continuous Functions

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Theorem 1 *Suppose* $f : X \to Y$ *is a continuous function and* $g : Y \to Z$ *is also a continuous function. Then* $g \circ f$ *is a continuous function.*

Proof
Suppose there is an open set $\omega \in Z$. Then $g^{-1}\omega$ is an open set. Then $f^{-1}(g^{-1}(\omega))$ is an open set. Thus open sets in Z under the action of $f^{-1} \circ g^{-1}$ are open in X, so $g \circ f$ is continuous.

Lets consider interiors and how they interact with continouous functions.

Theorem 2 *Let* $f : X \to Y$ *be continouous and* $L \subseteq Y$*. Then* $f^{-1}(Int(L)) \subseteq Int(f^{-1}(L))$*.*