Time Series Modelling with Unobserved Components: Errata

Matteo M. Pelagatti

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Chapter 1

Page 4, first equation in display The loss function should be

$$\ell(E) = \begin{cases} 19E & \text{for } E \geq 0, \\ -2E & \text{for } E < 0. \end{cases}$$

Page 4, line 6 from the bottom The line should read 'better reasons suggest the use of different loss curves.'

Page 5, line 10 of the *Proof* Substitute 'by Property 2.' with 'by Property 5.'.

Page 6, line 11 of the *Proof* Correct the last sentence in this line as 'Therefore, the expectations of $(Y|X)$ and Y are equal.'

Page 6, last line of the *Proof* After the very last equal sign, it should read $\mathbb{E}\left(\mathbb{V}\text{ar}[Y|X]\right)$ instead of just $\mathbb{V}\text{ar}[Y|X]$.

Page 8, line 5 Substitute '$n+1$' with '$m+1$'.

Page 9, point 1 in Theorem 1.4 The zero should be bold: $\mathbf{0}$, because it is a vecotor (of zeros), not a scalar.

Page 9, point 4 in Theorem 1.4 The a and b should be matrices and c should be a vector. The equation should read:

$$\mathbb{P}\left[\mathbf{A}\boldsymbol{Y} + \mathbf{B}\boldsymbol{Z} + \boldsymbol{c}\middle|\boldsymbol{X}\right] = \mathbf{A}\mathbb{P}\left[\boldsymbol{Y}\middle|\boldsymbol{X}\right] + \mathbf{B}\mathbb{P}\left[\boldsymbol{Z}\middle|\boldsymbol{X}\right] + \boldsymbol{c}$$

Page 9, point 7 in Theorem 1.4 The final part of the second line of the equation should read $(\boldsymbol{Z} - \mathbb{P}[\boldsymbol{Z}|\boldsymbol{X}])$ (P should be $\mathbb{P}$).

Page 10, line 16 The second line of the equation after *Projection on orthogonal variables* should read:

$$= \boldsymbol{\mu}_Y + \boldsymbol{\Sigma}_{YX}\boldsymbol{\Sigma}_{XX}^{-1}(\boldsymbol{X} - \boldsymbol{\mu}_X) + \boldsymbol{\Sigma}_{YZ}\boldsymbol{\Sigma}_{ZZ}^{-1}(\boldsymbol{Z} - \boldsymbol{\mu}_Y)$$

Page 11, first line In the middle of the line substitute with '$\mathbb{P}[\boldsymbol{Z}|\boldsymbol{X}, \boldsymbol{Z}] = \boldsymbol{Z}$'.

Chapter 2

Page 19, Theorem 2.2 The property name '*(Positivity of variance)*' should be substituted with '*(Nonnegativity of variance)*'.

Page 19, lines 11 and 9 from the bottom and last display In all cases the mathematical symbols '$\boldsymbol{y}$' and '$\boldsymbol{y}_m$' should be substituted with the capitalised '$\boldsymbol{Y}$'.

Page 27, 2nd line of 2nd display The linear prediction of V_t should read

$$\mathbb{P}[V_t|V_{t-1}, V_{t-2}] = \cos(\lambda)V_{t-1} + \cos(2\lambda)V_{t-2},$$

as correctly reported in the following display.

Page 29, 1st line of Section 2.3 Substitute 'demographics' with 'demographic'.

Page 36, 1st line After 'From Wold's theorem' add a reference to the theorem number like '(Theorem 2.7)'.

Page 37, last formula The formula $Y_t = \sum_{j=0}^{\infty} Z_{t-j}$ should read $Y_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j}$.

Page 38, Definition 2.14 Substitute the beginning of the definition with 'The ARMA(p, q) process (2.7) is *invertible* if there is a sequence of constants $\{\pi_j\}$ such that $\sum_{j=0}^{\infty}|\pi_j| < \infty$'.

Page 41, last line That line should read

$$\phi_1\gamma(h-1) + \ldots + \phi_p\gamma(h-p) + \mathbb{E}[Z_t Y_{t-h}].$$

Page 44, 2nd line of 3rd display The line should read

$$\Theta_Q(\mathbb{B}) = 1 + \Theta_1\mathbb{B}^s + \Theta_2\mathbb{B}^{2s} + \ldots + \Theta_Q\mathbb{B}^{Qs}.$$

Page 44, 2nd line before Section 2.5 the second word should be 'model' and not 'mode'.

Page 44, last line in display of Definition 2.15 The footer of '$\boldsymbol{Y_{t-h}}$' should not be boldface: '$\boldsymbol{Y}_{t-h}$'.

Chapter 3

Page 57, 7th line from the bottom The second equation in the display should read

$$\mu(x_{t+1}) = \mu(x_t) + \lambda_S^{-\frac{1}{2}}\sigma \int_{x_t}^{x_{t+1}} W(s)\,\mathrm{d}s,$$

Page 58, first equation It should read

$$\begin{bmatrix}\eta_t \\ \zeta_t\end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix}0 \\ 0\end{bmatrix}, \frac{\sigma^2}{\lambda_S}\begin{bmatrix}\frac{1}{3}\delta_t^3 & \frac{1}{2}\delta_t^2 \\ \frac{1}{2}\delta_t^2 & \delta_t\end{bmatrix}\right).$$

Page 59, Figure 3.5 In both plots, the numbers below the x-axis should read $(0, 1/4, 1/2, 3/4, 1)$ and not $0, \pi/4, \pi/2, 3\pi/4, \pi$.

Pgae 61, 4th line from the bottom The equation in Theorem 3.1 should read

$$\mathbb{E}\left[\boldsymbol{\psi}_{t+h}\boldsymbol{\psi}_t^\top\right] = \frac{\rho^{|h|}\sigma_\kappa^2}{1-\rho^2}\boldsymbol{R}(h\lambda),$$

Page 61, last two lines The sentence should read: 'The optimal predictor of the stochastic cycle at time $t+h$ (with $h>0$) [...] '

Page 63, 10th line from the bottom The equation display after 'For the autocovariance function' should read

$$\begin{aligned}\mathbb{E}\left[\boldsymbol{\psi}_{t+h}\boldsymbol{\psi}_t^\top\right] &= \text{[as in book]} \\ &= \mathbf{R}(h\lambda)\rho^h\boldsymbol{\Sigma}_\psi \\ &= \frac{\rho^h\sigma_\kappa^2}{1-\rho^2}\boldsymbol{R}(h\lambda).\end{aligned}$$

Page 68, Lemma 3.2 The beginning should read 'Let $f_t : \mathbb{Z} \mapsto \mathbb{R}$ be a function of period $s \in \mathbb{Z}$ (i.e., $f_{t+s} = f_t$ for all t).'

Page 81, central equation, page 82, first equation In the first line of those equations the summation starts from $j=1$ and not from $j=2$.

Chapter 5

Page 92, 4th line The first item in the list should read

- $\mathbb{E}[\boldsymbol{\alpha}_1] = \boldsymbol{a}_{1|0}$ and $\mathbb{E}\left[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})^\top\right] = \mathbf{P}_{1|0}$;

Page 93, 6th line in Example 5.1 There should be a comma after ϕ: '$\mathbf{T}_t = \phi$, $\mathbf{H}_t = 0$', furthermore '$\mathbf{Q}_t = \sigma^2$'.

Page 96, 3rd line of Example 5.4 'though' should be 'through'.

Page 97, 4th line from the bottom After the sentence 'The same process can also be written as' the matrix equation has wrong indexes in the last vector after the '+' symbol. The equation should read

$$\underbrace{\begin{bmatrix} \alpha_{1,t+1} \\ \alpha_{2,t+1} \\ \vdots \\ \alpha_{r-1,t+1} \\ \alpha_{r,t+1} \end{bmatrix}}_{\boldsymbol{\alpha}_{t+1}} = \underbrace{\begin{bmatrix} \phi_1 & 1 & \dots & 0 & 0 \\ \phi_2 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \dots & \ddots & \vdots \\ \phi_{r-1} & 0 & \dots & 0 & 1 \\ \phi_r & 0 & \dots & 0 & 0 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} \alpha_{1,t} \\ \alpha_{2,t} \\ \vdots \\ \alpha_{r-1,t} \\ \alpha_{r,t} \end{bmatrix}}_{\boldsymbol{\alpha}_t} + \underbrace{\begin{bmatrix} 1 \\ \theta_1 \\ \vdots \\ \theta_{r-2} \\ \theta_{r-1} \end{bmatrix}}_{\boldsymbol{\nu}_t} \nu_t$$

Page 98, last display in the page the transition matrix of the ARI(1,1) model should read

$$\begin{bmatrix} (1+\phi_1) & 1 \\ -\phi_1 & 0 \end{bmatrix}$$

Page 99, display in Example 5.6 The transition matrix of the ARIMA(0,1,1)(0,1,1) model should read

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Page 101, line 11 'can be dropped' should read 'is to be dropped'.

Page 108, the two matrix equations In the first and second matrix equations, there is an r instead of v in the vectors on the lhs of the equal sign. In the second matrix equation, in the vector on the rhs of the '+' sign the indexing is wrong. The two equations should read respectively as

$$\underbrace{\begin{bmatrix} \alpha_{1,t+1} \\ \alpha_{2,t+1} \\ \vdots \\ \alpha_{v-1,t+1} \\ \alpha_{v,t+1} \end{bmatrix}}_{\boldsymbol{\alpha}_{t+1}} = \underbrace{\begin{bmatrix} \delta_1 & \delta_2 & \dots & \delta_{v-1} & \delta_v \\ 1 & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \dots & \vdots & \vdots \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} \alpha_{1,t} \\ \alpha_{2,t} \\ \vdots \\ \alpha_{v-1,t} \\ \alpha_{v,t} \end{bmatrix}}_{\boldsymbol{\alpha}_t} + \underbrace{\begin{bmatrix} X_t \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\boldsymbol{d}_t}$$

and

$$\underbrace{\begin{bmatrix} \alpha_{1,t+1} \\ \alpha_{2,t+1} \\ \vdots \\ \alpha_{v-1,t+1} \\ \alpha_{v,t+1} \end{bmatrix}}_{\boldsymbol{\alpha}_{t+1}} = \underbrace{\begin{bmatrix} \delta_1 & 1 & \dots & 0 & 0 \\ \delta_2 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \dots & \ddots & \vdots \\ \delta_{v-1} & 0 & \dots & 0 & 1 \\ \delta_v & 0 & \dots & 0 & 0 \end{bmatrix}}_{\mathbf{T}} \underbrace{\begin{bmatrix} \alpha_{1,t} \\ \alpha_{2,t} \\ \vdots \\ \alpha_{v-1,t} \\ \alpha_{v,t} \end{bmatrix}}_{\boldsymbol{\alpha}_t} + \underbrace{\begin{bmatrix} \omega_0 X_t \\ \omega_1 X_t \\ \vdots \\ \omega_{v-2} X_t \\ \omega_{v-1} X_t \end{bmatrix}}_{\boldsymbol{d}_t}$$

Page 112, Equation 5.9 The first line of the equation should read

$$\hat{\boldsymbol{Y}}_{t|t-1} = \mathbb{P}[\boldsymbol{Y}_t|\mathcal{Y}_{t-1}] = \boldsymbol{Z}_t\boldsymbol{a}_{t|t-1} + \boldsymbol{c}_t$$

Page 113, lines 5-10 (first sequence of equation of the proof) Every symbol $\boldsymbol{d}_t$ should be substituted with the symbol $\boldsymbol{c}_t$.

Page 113, line 7 from the bottom 'to can compute' should read 'to compute'.

Page 113, last display The first expectation after the equal sign should read

$$\mathbb{E}[\boldsymbol{\nu}_{t-1}\boldsymbol{\iota}_{t-1}^\top]$$

Page 116, equation 5.12 In the first line of this equation there should be a subscript t just after the matrix $\mathbf{K}$:

$$\boldsymbol{a}_{t+1|t} = \mathbf{T}_t\boldsymbol{a}_{t|t-1} + \boldsymbol{d}_t + \mathbf{K}_t\boldsymbol{\iota}_t,$$

Page 118, line 3 from the bottom 'Consistent with' should read 'Consistently with'.

Page 120, first line of first display The line should begin with

$$\boldsymbol{a}_{n|n} = \boldsymbol{a}_{n|n-1}+$$

Page 121, line 2 from the bottom The symbol '$P_{t|n}$' should be substituted with '$a_{t|n}$'.

Page 126, line 1 after first display There should be a blank space between 'equation' and '(5.18)'.

Page 126, Equation 5.20 There should be no transposition after any matrix $\boldsymbol{T}$. Thus the final formula should read:

$$\mathrm{vec}(\mathbf{P}^*) = (\mathbf{I} - \mathbf{T}^* \otimes \mathbf{T}^*)^{-1}\,\mathrm{vec}(\mathbf{Q}^*).$$

Page 126, last line The formula '$\tau = \infty$,.' at the end of the line should read '$\tau = \infty$.'.

Page 128, Gaussian density in the middle of the page Inside the exponential, the transpose sign ($^\top$), should be on the first instance of $\left(\boldsymbol{y}_t - \hat{\boldsymbol{y}}_{t|t-1}\right)$ and not on the second.

Page 133, line 11 from the bottom The line should read 'and (5.25) reduces to (5.22)'.

Page 134, Table 5.1 The second column of the title row should read $(0, \infty)$, that is the interval should be open on both sides. The element on the last row of the last column (Jacobian of the inverse logit transform) should read

$$b\exp(-\psi)\left[1 + \exp(-\psi)\right]^{-2}$$

Chapter 6

Page 138-139 Here, first the dot over the symbol is used to denote the variables *before* the log-transform, but after Figure 6.1 the dot over the symbol is used to denote the variables *after* the log-transform. Invert the use of the dot before or after the figure.

Page 138, Figure 6.1 The last word in the caption should be 'left'.

Page 139, first line 'added' should read 'addend'.

Page 140, line 20 'lower' should be 'slower'.

Page 140, line 6 from the bottom 'standard deviation' should read 'standard deviations'.

Page 140, line 5 from the bottom 'for grid λ-values' should read 'for a grid of λ-values'.

Page 142, Figure 6.3 In the caption the correct values for λ are '$(\lambda = -0.25, 0.00, 1.00)$'.

Page 143, lines 9-10 from the bottom 'while it gives' should read 'while they give'.

Page 148, line 5 from the bottom 'exact algorithm' should read 'exact algorithms'.

Page 154, footnote 3 'in finite sample' should read 'in finite samples'.

Page 156, 2nd line in cycle definition (mid-page) 'for orthogonal' should read 'for the orthogonal'.

Chapter 7

Page 165, line 2 from the bottom and page 166, line 17 'r' should read 'K_μ'.

Page 166, line 5 from the bottom '$N \times 1$' should read '$K_\beta \times 1$'.

Page 168, lines 2-3 'N_μ' should read 'K_μ'.

Page 172, first line 'K_β' should read 'K_ψ'.

Page 173, first line The vector should read '[0.968 0.032 0.000 0.000]'

Chapter 8

Page 179, line 10 from the bottom 'treatment' should read 'treatments'.

Page 180, third display the condition '$0 < j < \frac{1}{2}$' is wrong and it should be substituted with '$0 < j < \frac{n}{2}$'.

Page 183, line 18 'which no not show' should read 'which does not show'.

Page 190, caption of figure 8.4 In the 2nd last line '$(2, 4)$ (long dashes)' should read '$(2, 3)$ (long dashes)'.

Page 193, line 3 from the bottom delete one 'to' in 'equal to to (2,1)'.

Page 195, line 2 from the bottom add the word 'orthogonal' after 'reciprocally'.

Chapter 9

Page 207, last formula The matrix should read

$\mathbf{P}_{1|0} =$

	μ_1	μ_2	β_1	β_2	ψ_1	ψ_2	ψ_1^*	ψ_2^*	ε_1	ε_2	$\mu_2^{(-1)}$	$\mu_2^{(-2)}$	$\psi_2^{(-1)}$	$\psi_2^{(-2)}$	$\epsilon_2^{(-1)}$	$\epsilon_2^{(-2)}$
μ_1	∞	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
μ_2	0	∞	0	0	0	0	0	0	0	0	0	0	0	0	0	0
β_1	0	0	∞	0	0	0	0	0	0	0	0	0	0	0	0	0
β_2	0	0	0	∞	0	0	0	0	0	0	0	0	0	0	0	0
ψ_1	0	0	0	0	σ_{11}^{ψ}	σ_{12}^{ψ}	0	0	0	0	0	0	$\rho_1\sigma_{12}^{\psi}$	$\rho_2\sigma_{12}^{\psi}$	0	0
ψ_2	0	0	0	0	σ_{21}^{ψ}	σ_{22}^{ψ}	0	0	0	0	0	0	$\rho_1\sigma_{22}^{\psi}$	$\rho_2\sigma_{22}^{\psi}$	0	0
ψ_1^*	0	0	0	0	0	0	σ_{11}^{ψ}	σ_{12}^{ψ}	0	0	0	0	0	0	0	0
ψ_2^*	0	0	0	0	0	0	σ_{21}^{ψ}	σ_{22}^{ψ}	0	0	0	0	0	0	0	0
ε_1	0	0	0	0	0	0	0	0	σ_{11}^{ϵ}	σ_{12}^{ϵ}	0	0	0	0	0	0
ε_2	0	0	0	0	0	0	0	0	σ_{21}^{ϵ}	σ_{22}^{ϵ}	0	0	0	0	0	0
$\mu_{2(-1)}$	0	0	0	0	0	0	0	0	0	0	∞	0	0	0	0	0
$\mu_{2(-2)}$	0	0	0	0	0	0	0	0	0	0	0	∞	0	0	0	0
$\psi_{2(-1)}$	0	0	0	0	$\rho_1\sigma_{21}^{\psi}$	$\rho_1\sigma_{22}^{\psi}$	0	0	0	0	0	0	σ_{22}^{ψ}	$\rho_1\sigma_{22}^{\psi}$	0	0
$\psi_{2(-2)}$	0	0	0	0	$\rho_2\sigma_{21}^{\psi}$	$\rho_2\sigma_{22}^{\psi}$	0	0	0	0	0	0	$\rho_1\sigma_{22}^{\psi}$	σ_{22}^{ψ}	0	0
$\epsilon_{2(-1)}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	σ_{22}^{ϵ}	0
$\epsilon_{2(-2)}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	σ_{22}^{ϵ}

Page 208, lines 1-2 Substitute with 'Furthermore, ρ_h represents the autocorrelation function of $\{\psi_t\}$, which is given by $\rho_h = \rho^h \cos(h\lambda)$, as seen in Section 3.3.'

Page 208, line 6 from the bottom 'GD ' should read 'GDP'.

Page 214, second display In the matrix $\mathbf{Q}$ the second last element on the main diagonal should be '$\sigma_\zeta^2\mathbf{I}_{32}$' and not just '$\mathbf{I}_{32}$'.

Chapter 10

Page 220, line 13 The second part of the line should read 'The trend order d ($\in \{1, 2, 3, 4\}$) is used to build a'.

Page 223, line 9 There should be a carriage return before '`std.error`'

Page 241, line 6 'modes' should read 'models'.

Page 241, line 9 'most popular softwares' should read 'most popular packages'.

Page 241, line 4 from the bottom 'treatment of the initial conditions' should read 'treatment of diffuse initial conditions'.

Page 243, line 13 form the bottom correct with 'parameters in the vector `pars` into the model `model` and returns the updated model.'.

Page 243, line 11 from the bottom correct with 'values marked as `NA` on the main diagonals of the time-invariant covariance matrices `Q` and `H`.'.

Bibliography

Page 247, 2nd item correct 'euro area' with 'Euro area'.

Page 248, 3rd item remove curly brackets from 'KPSS'.