TWO STEP PROTOCOL FOR REFUGEE RESETTLEMENT

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The refugee crisis has hit the heart of the European Union in recent years, with an exponential increase in asylum requests from 2008 to 2015 and averaging a total of around 700,000 requests per year thereafter. The quest for a fair and efficient redistribution protocol is, therefore, very important. We propose a new game-theoretically informed protocol whose objective is twofold: on one hand, guaranteeing states to be paid fairly for their service while respecting their budget and on the other permitting the refugee families to be assigned to the states they mostly prefer up to the states budgets and based on a list of priorities that creates no conflict among refugee families. The method we employ is based on two successive steps, whereby the first is an auction where each state communicates to EU Refugee Committee their unitary annual cost per refugee family and a budget on how much money they are willing to spend to handle asylum requests. Secondly, refugee families engage in a Top-Trading Cycle algorithm where they exchange their spots in states according to their preference list. We prove that our two-step mechanism possesses the following desirable properties: no state would like to exchange their allocation and payment with those of another state; we say that the mechanism is envy-free for states. Secondly, refugee families have no incentive to lie about their preference lists and are assigned collectively to the optimal set of states; we say that the mechanism is truthful and Pareto-optimal for refugees. Thirdly, the EU Refugee Committee, spends just marginally more than they would spend for an optimal protocol which completes the assignment in exponential time in the worst case. Its main drawback is that that not all refugee families are guaranteed to be assigned to some state. We, hence, defer such a "market-clearing" allocation protocol to future work. Although the presented mechanism is far from conclusive, it constitutes one of the new possible ways of addressing this issue, and policies of similar importance, more systematically and from a mathematical standpoint.

NOTA BENE

The proofs hereby presented have not been carefully double-checked by a second reader. If you are interested in the content of the report and you notice any eventual imprecision in the proofs, please do not hesitate to contact me at `marusso@student.ethz.ch`. I will be very happy to correct any mistake I might have made.

ORGANIZATION

The road map of the report will be the following: in section 1 we introduce the reader to the problem and motivate the need for creation of new algorithms to solve refugee resettlement. In section 2, we will discuss the previous approaches to the problem and the assumptions they make use of and we would like to challenge. In section A of the appendix, we outline the problems that arise from trying to solve the problem through too simple of a model. There, we present an incomplete and unsuccessful attempt to solve the problem. It will take into account only binary preferences of refugees and will completely disregard states' possible budgets. Nevertheless, this attempt will still help build up some intuition about the main model: for the interested reader, please take a look at this section of the appendix to have more of a mathematical justification for the subsequent model. Indeed the main model, in section 3, encompasses as a first step a budget-constrained multi-unit auction which will treat refugee families as identical units to be contended in an auction where bids are negative: each refugee family represents a cost to the state and states need to be paid by the EU Refugee Committee (the auctioneer) to participate to the protocol. This auction will give us robust guarantees on the total amount spent by the EU Refugee Committee to reward states and will make sure that no state will be "envious" of some other state's allocation and payment. Nevertheless, if the model were to be confined to this type of auction, then, it would violate any bare minimum conception of human rights: as a matter of fact, within this second failed attempt to model the problem, refugee families never express their preferences and have no word on the outcome of the protocol. This is the reason why, our main model will then, in the second step, make refugee families trade (with no money involved) their places within the states they have been assigned to, if at all convenient to them. This way, each refugee is the happiest they could be with respect to their declared preference list over states. This money-free trading mechanism will be carried out through a generalization of the celebrated Top-Trading Cycle Algorithm [14]. Uncoupling the bankruptcy and the assignment question makes us achieve the best of both worlds, with each state being satisfied with their allocation and each refugee being the happiest they could have been had they not participated. We, thus, conclude in section 4 with possible directions and future work which could be lead by fundamental ethical questions.

1 INTRODUCTION AND MOTIVATION

Since its formation, the European Union, as we know it today, has always been seen as a land of prosperity, opportunity as well as multicultural and multilateral respect. For this very reason and for geographical proximity, individuals that are forced to flee their home country, whether located in the Middle-East, Maghreb or Sub-Saharian Africa, often decide to come to the EU and seek for asylum. The EU with the Dublin Convention, originally signed in 1990 and then replaced by the Dublin II and III Regulations of 2003 and 2013 respectively [10–12], has tried to legislate on the refugee settlement question rather unsuccessfully: in brief, the state whose soil has been first stepped into by refugees is responsible for the their asylum. Geographically, this rule induces an evident discrepancy between Southern and Northern European States as the former would inevitably have to handle many more asylum requests. The fallacious design of such a protocol has lead to an unfair and ineffective protection of the refugees, as argued both by the ECRE and the UNHCR [8]: the Lipa crisis in Bosnia-Herzegovina going on as we write is a clear and tragic example of this [3]. Another unfortunate consequence has been the rise of European far-right populist movements, which have gained ground at an exponential rate in the recent past. This, in turn, yielded states to view refugees as a cost rather than as an asset. The need of a new, mathematically informed redistribution protocol is of the uttermost importance and priority. In recent years, the refugee resettlement policy question has been revisited in a much more mathematical flavour than ever before, with clear positive signals. In order to understand how the approaches may vary, it is fundamental to realize that there are two simultaneous problems we are trying to solve and that should be kept in mind when proposing new approaches for the Refugee Resettlement Problem. On one hand, the **assignment** question, i.e. where specific refugees get relocated. On the other hand, the **bankruptcy** question, i.e. whether or not states' capacities constraints are satisfied. The models hereby proposed will try to solve the two problems at the same time and, in both of them, let us use the following notation.

Notation. Let R denote the set of refugee families and let S denote the set of member states with $n := |R|$, $m := |S|$, and $n \gg m$. Furthermore, let us denote by $\mathcal{A}$ the EU Refugee Committee, a *super partes* referee, solely interested to the "common good".

2 RELATED WORK

One of the most renowned approaches to solve the assignment question of above is the *Aggregate-Divide* or *Divide-Aggregate* method [6], a centralised scheme where, based on some indicators, directly or indirectly related to the refugee redistribution problem itself (this may be quality of life, average salary, etc.), the central governmental entity decides which state receives which "bundle of refugees" without a real consultation with the member states (this has shown a certain degree of success, nevertheless). On the other hand, regarding refugees as inhomogeneous, one may use *Roth's assignment Markets Protocols* as in [2, 7, 15], a promising as well far more decentralised approach to the question at hand. Nonetheless, assignment methods present a few negative nuances that need to be addressed, such as the fact that, despite their stability, they are optimal for the proposing party and pessimal for the receiving one. Their main drawback however relies on the modeling assumptions. In fact, supposed the two interacting parties are refugee families and EU member states, on one hand, it is true that refugee families have a list of preferences over states: this may be a consequence of their native language, of the fact that some state possesses already a vast community of people from the same country or several other reasons. On the other hand, nonetheless, states do not really have a full preference list over refugee families for two main reasons: first and foremost, states cannot express preferences over individuals as this would urge an immediate call for antidiscrimination litigations. Secondly, from a more practical point of view, it would be unreasonable to think about a state agency profiling each and every of the tens of thousands of refugee families whilst, contextually, compiling a full preference list based on some obscure indicators. None of the cited papers, and, to the best of our knowledge, none of the existing work, have ever addressed the possibility that the European Refugee Commission could incentivise states to want to host refugee families via payments while giving refugees their best available choice among European states.

3 MAIN MODEL: BID-N-TRADE

3.1 Preliminaries

Problem 1, 2, 3 outlined in A.3 make us recognise what the pitfalls in the warm-up model really are. Hence, we enhance the model to face the refugee resettlement problem with a novel idea, that of using two Algorithmic Game Theory algorithms in two successive steps to solve the bankruptcy and the assignment problems separately. In the first step, as mentioned, we run a Budget-constrained multi-unit auction so to obtain an assignment of units to bidders (refugees to states). In the second step, for all refugees that now belong to a state, we make them trade their spot across different states so that they collectively (Pareto-optimally) get their preferred state and the number of units (refugees) per state does not change, i.e. the budget is still respected.

MODEL 1 (BID-N-TRADE). For the three parties involved:

Refugees. For each refugee family $r \in R$, r reports to the EU Refugee Committee $\mathcal{A}$ a ranking preference list over states of the form $\rho_r : s^{(1)} \succ s^{(2)} \succ \cdots \succ s^{(n)}$. Refugee family r may misreport it true preference which is, in fact, π_r.

States. For each state $s \in S$, s reports to the EU Refugee Committee $\mathcal{A}$ a tuple $(c_s, B_s) \in \mathbb{R}^2_+$ composed of a unitary (annual) cost per refugee family and a total annual budget. State s may misreport the true cost as well as the true budget. Their utility will be modeled as follows:

$$u_s(w_s, P_s) = \begin{cases} P_s - c_s \cdot w_s, & \text{if } c_s \cdot w_s \leq B_s \\ -\infty, & \text{otherwise} \end{cases}$$

EU. EU Refugee Committee $\mathcal{A}$ designs a two-step mechanism described in **Step 1** and **Step 2** below so that, for payment scheme $P_s \in \mathbb{R}_+$ and assignment $\mu : R \to S$ of refugees to states, the following five properties are satisfied:

- **(P1)** Each refugee r is matched exactly to one state, i.e. $\sum_{s \in S} \mu_{rs} = 1$, where μ_{rs} is the variable indicating whether assignment μ has assigned refugee r to state s.
- **(P2)** Each refugee r is incentivised to report truthfully, i.e. $\rho_r = \pi_r$.
- **(P3)** The allocation and payment scheme resulting from the auction are "envy-free", i.e. for all pair of states $s, s' \in S$, $P_s - c_s \cdot w_s \geq P_{s'} - c_s \cdot w_{s'}$. This means that no state thinks they could have gotten a better allocation of refugee families or a better payment if they compare themselves to others.
- **(P4)** Each state s voluntarily participates, i.e. $u_s \geq 0$.
- **(P5)** The number of assigned refugee families across states is equal to the total number of refugee families, i.e. $\sum_{s \in S} w_s = \sum_{r \in R} \sum_{s \in S} \mu_{rs} = n$.

Note that we do not insist in states reporting truthfully, insomuch as this requirement would yield to unusable impossibility results [5].

3.2 Step 1: Budget-constrained multi-unit auction

3.2.1 Introduction. Hereby, we will closely follow the approach explained in [4], a truly interesting paper which concerns budgeted bidders and finds its vast applications in Internet AdWords auctions and several other ones. We thought their approach and modeling could be very naturally applied in this context as we will formally show below. Since the auction in [4] is interested in allocating units to bidders that are interested in buying them, whereas, in our case, states need to be paid, then, we could regard tuple $(c_s, B_s) \in \mathbb{R}^2_+$ as tuple $(v_s, b_s) \in \mathbb{R}^2_-$, where states report a negative value (a cost) and a negative budget (how much they are maximally willing to spend, negatively, to obtain that item). The reason why we do not insist on truthfulness is that, a proved in [5], there is no multi-unit auction where both bids and budgets are private such that it is both incentive compatible (truthful) and Pareto-optimal. Moreover, as discussed in [4] as well as in [5], truthfulness in multi-unit auctions produces unfair pricing: the same unit of a good gets sold to two distinct bidders for different prices. As a side note, when budgets are publicly known, then an adaptive version of Ausubel's Clinching Auction allocates all the units truthfully and Pareto-optimally as shown in [5]. However, in a pessimistic view, we assume budgets to be also private and not publicly known, even though it would not be so far from reality to assume that budgets are, in fact, publicly known.

3.2.2 Bid for Good (`B4G`*) Mechanism.* Let us now start with some definitions and notation to then introduce the multi-unit auction we are using for **Step 1** in 3.2.

Definition 1. In particular, we will define the concept of demand of a state and candidate payments, both key ingredients to the `B4G` mechanism. For convenience, assume unit costs are sorted as in $c_1 \leq \cdots \leq c_n$.

(Demand) Let demand $D_s(p)$ be the number of refugee families state s is wishing to be assigned for unitary item payment p. Formally,

$$D_s(p) = \begin{cases} 0, \text{ if } p < c_s \\ \min\left\{m, \left\lfloor \frac{B_s}{c_s} \right\rfloor\right\}, \text{ otherwise} \end{cases}$$

(Payments) Denote by G_s the set of states with unitary costs below state s's one. Formally,

$$G_s = \{s' \in S | c_{s'} < c_s\}$$

The candidate payments belonging to set $\overline{p} = \left\{\overline{p_1}, \ldots, \overline{p_n}\right\}$ are defined as

$$\overline{p_s} = \begin{cases} \max\left\{c_s, \frac{\varepsilon \cdot \sum_{t \in G_s} B_t}{n^2}\right\}, \text{ if } \frac{\varepsilon \cdot \sum_{t \in G_s} B_t}{n^2} < c_{s-1} \\ c_s, \text{ otherwise} \end{cases}$$

States as bidders and refugee families as units to be assigned to states under a payment will participate to the following mechanism. Its main overarching idea is that of scanning through the candidate payments and decide which one suits best.

<u>**Mechanism**: `B4G`</u> *(for a complete description please refer to section 4.1 of [4])*

INIT `COST` $\leftarrow 0$
FOR $p \in \overline{p}$:
  IF $\sum_{s \in S} D_s(p) < m$:
    $w_s = D_s(p), \forall s \in S$
  ELSE:
    COMPUTE $u_s, \forall s \in S$
    $w_s = \min\{u_s, D_s(p)\}, \forall s \in S$
  `COST` $\leftarrow \min\{\texttt{COST}, p \cdot \sum_{s \in S} w_s\}$
RETURN `COST`

Describing the above informally, for each candidate payment $p \in \overline{p}$, we perform the following series of operations: we check the cumulative demand $\sum_{s \in S} D_s(p)$ with respect to supply m. When smaller, all states obtain their desired demand of refugee families, i.e. $w_s = D_s(p)$. Otherwise, we compute an upper bound u_s on the number of refugee families a state can obtain while keeping the allocation "envy-free" and a state is assigned the smaller between the computed upper bound and their actual demand, i.e. $w_s = \min\{u_s, D_s(p)\}$. After having performed this allocation, we compute the corresponding total cost $p \cdot \sum_{s \in S} w_s$ and update it if smaller than the one found so far.

Theorem 1. *In the case of $m \geq n^2/\varepsilon$, mechanism* `B4G` *induces an "envy-free" allocation and is a polynomial time $(1 + \varepsilon)$-approximation to the optimal cost-minimizing (negative revenue-maximizing) "envy-free" mechanism.*

Proof. Please refer to theorem 4, and all the preceding lemmas, contained in section 4.1 of [4] for a full and detailed proof. □

What shown above essentially means that no state is willing to swap their outcome with that of another state. Moreover, the EU Refugee Committee $\mathcal{A}$ spends a vanishingly small fraction ε more than they would have spent by running a non-polynomial exact "envy-free" mechanism. Formally put, $\mathcal{A}$ achieves a cost

$$\texttt{COST}^* := p^* \cdot \sum_{s \in S} w_s^* \leq (1+\varepsilon) \cdot \mathrm{OPT}$$

3.2.3 Limitations. **Problems 2, 3** outlined in section A.3 are now completely overcome through **Step 1** in 3.2 since `B4G` outputs an "envy-free" allocation where states utilities are always non-negative, based on both unitary reported costs and reported budgets. Nevertheless, mechanism `B4G` in 3.2.2 may not result into "market-clearing" allocation, i.e. an allocation to states that saturates the supply of refugee families. In other words, the designed mechanism may not allocate all the refugee families to states, and some of them will be left unassigned: this is, as a matter of fact, an important problem that needs to be solved even increasing the total cost EU Refugee Committee $\mathcal{A}$ will incur.

3.3 Step 2: Top Trading Cycle Multiple Occupancy Allocation

3.3.1 Introduction. We now would like to address **Problem 1** in section A.3 of the warm-up model we outlined. Indeed, as mentioned, each refugee family has a preference list over states which induces a strict ordering among them.

3.3.2 Top-Trading State Cycle (TTSC) Algorithm. For the `TTSC`, we have a change in perspective: rather than looking at the assignment from the states' viewpoint, we will analyze it from the side of the refugee families. The overarching idea of this algorithm is that states are now inanimate objects called "houses" where there are multiple "tenants", the refugee families, who are exchanging their spots in the house according to their preference list. So, if for example, a refugee family prefers another state to the one they had been assigned to and another refugee family has the exact opposite preference, then they will swap. Indeed, `TTSC` starts from the initial allocation of refugee families to the state they had been assigned to via `B4G` in **Step 1** of section 3.2. It is usually not true that refugee families will be assigned to their preferred state initially. Thus, it is fair to think of a money-free protocol that makes them exchange their spots so long as they are willing to. The algorithm is really simple to describe informally: starting from the allocation of `B4G`, let each refugee family point to their favourite state. This procedure will induce a directed graph where each node is represented by a refugee family and each outgoing edge expresses the top preference over the states (note that one refugee family might point to the state where they already reside). We now reallocate according to all the cycles that have formed and update the graph by removing the refugee families (along with their outgoing edges) and reducing the spots in the affected states accordingly. Refugee families that were not included in a cycle in the previous round and that used to point to states with no more available spots, will now need to update their preference lists by removing their current top state. We now repeat the algorithm and run it until completion. Let us note that a state here could be interpreted as a collection of nodes (refugee families) that are progressively removed from the graph.

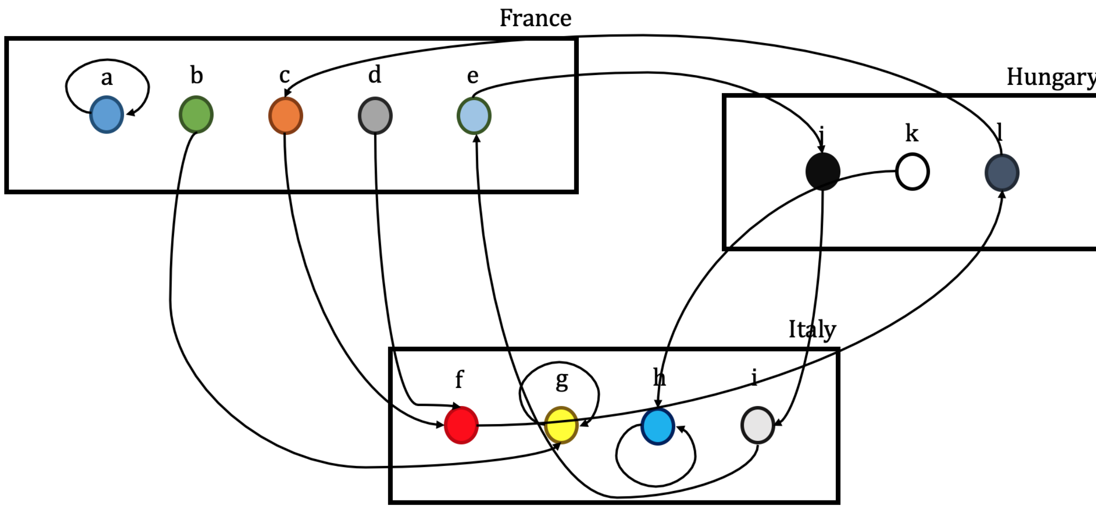
Algorithm: `TTSC`

INPUT ρ_r, for all $r \in R$
INIT $G \leftarrow (R, S, \emptyset)$, `ASSIGNMENT` $\leftarrow$ assignment(`B4G`(R, S))
WHILE $R \neq \emptyset$
  FOR $r \in R$: ◃ Update the graph with remaining refugee families and spots in states
    $e \leftarrow (r, \mathrm{TOP}(\rho_r))$ ◃ Make refugee families point to their favourite remaining spots in states
    $E \leftarrow E \cup \{e\}$
  $G \leftarrow (R, S, E)$
  REMOVE all directed cycles in G and SWAP spots accordingly
  UPDATE $R, S,$ `ASSIGNMENT` accordingly
RETURN `ASSIGNMENT`

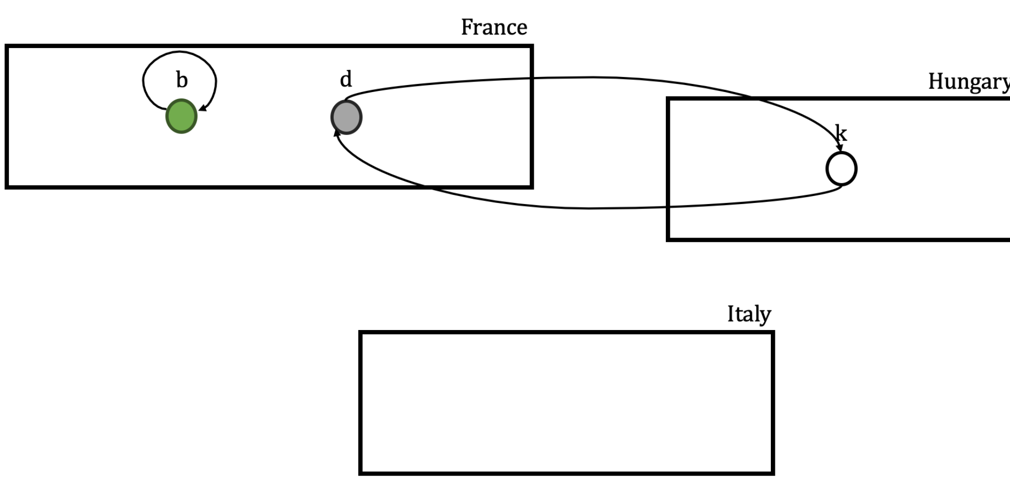
Let us now consider the following toy example with 12 refugee families and 3 states right after the B4G assignment and let us observe how the algorithm would behave under the following preference lists:

ρ_a : France ≻ Italy ≻ Hungary ρ_b : Italy ≻ France ≻ Hungary ρ_c : Italy ≻ France ≻ Hungary
ρ_d : Italy ≻ Hungary ≻ France ρ_e : Hungary ≻ Italy ≻ France ρ_f : Hungary ≻ France ≻ Italy
ρ_g : Italy ≻ France ≻ Hungary ρ_h : Italy ≻ Hungary ≻ France ρ_i : France ≻ Hungary ≻ Italy
ρ_j : Italy ≻ Hungary ≻ France ρ_k : Italy ≻ France ≻ Hungary ρ_ℓ : France ≻ Hungary ≻ Italy

During phase 1, we will have



After eliminating cycles and updating the graphs as well as spots available in states, we will have



Once again, after eliminating cycles, TTSC terminates.

Theorem 2. *Algorithm* TTSC *returns an assignment that has the following desirable properties: termination, weakly improved allocation, incentive compatibility (truthfulness), budget-compliance and unique core allocation (collusion-resistance).*

Proof. Please refer to section B of the appendix for a full and detailed proof. □

Remark 1. Algorithm TTSC does not always return a "fair" allocation as in the following sense. Consider the case where refugees families' lists are

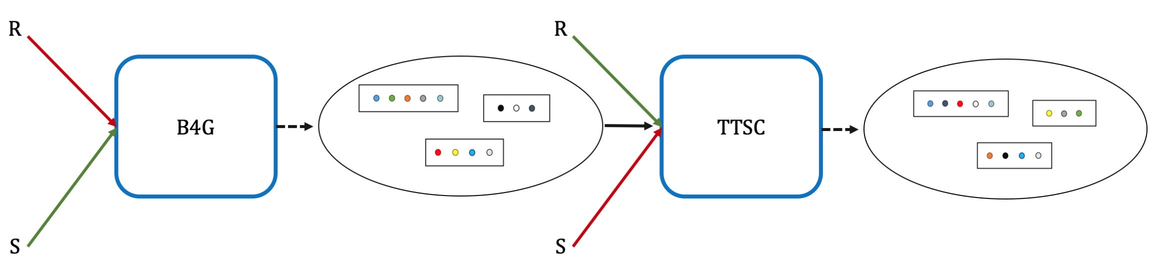
$$\rho_a : \text{Italy} \succ \text{France} \qquad \rho_b : \text{France} \succ \text{Italy} \qquad \rho_c : \text{France} \succ \text{Italy}$$

The question now becomes: who should we choose between b and c to exchange their spot in Italy with a? Our algorithm could be slightly modified and choosing uniformly at random in such cases, thus, guaranteeing ex-ante "fairness" but never ex-post "envy-freeness": if b is chosen by the coin toss, then b will exchange with a and c will be envious.

3.3.3 Limitations. **Problem 1** outlined in section A.3 is also now completely solved given that our model is based on ranking preferences and that, by theorem 2, refugee families are collectively guaranteed the best possible outcome for themselves and, moreover, no refugee family has any incentive not to respect the protocol to obtain an individually better outcome. If they did, then they would need to make another family worse off but they also might make themselves worse off, which implies that it is not convenient to them. In addition, states, who are passive participants here, still have their budgets respected and, thus, have no incentive to lie during **Step 1** as per section 3.2. However, as per remark 1, fairness, as in "who chooses" and "who gets chosen", may be at odds with the desirable properties we have derived for TTSC but perhaps this is another interesting impossibility result that we could defer to future work.

4 SUMMARY AND FUTURE WORK

After having presented a failed attempt to the solution of the refugee resettlement problem, in section 3 we propose a new method, BID-N-TRADE composed of two parts: the first is B4G, i.e. a multi-unit (negative bid) auction where "units" are refugee families and states are the bidders "purchasing" them with unitary annual costs and budgets. Once the auction has allocated the refugee families envy-freely, so that no state will envy the allocation and payment of another state, refugee families take an active part in the protocol: they exchange their spots within states as it is convenient, so that states budgets are still respected and refugee families are collectively the happiest they could be. This is best summarized in the following diagram: let us observe that before the B4G box, only states are active participants (green), while after only refugee families are. Also, let us note change in colour in the allocations before and after TTSC is applied: it means that refugee families have exchanged their spots within TTSC.



The above approaches raise a myriad of ethical questions. It would be ideal to formulate them as mathematical questions and try to answer them either with constructive proofs or with impossibility results. The first seems to be a major hope whilst the latter two are still very interesting questions but apparently less challenging:

(1) B4G needs to be improved in that, beyond the properties it already possesses, it should be capable to "clear the market". That means that each refugee should be assigned to exactly one state and cannot be left out as this is neither ethically nor politically acceptable.
(2) TTSC needs to be generalized to a context where preferences are not strict, and refugee families may be indifferent between multiple states: this would also be more of a realistic assumption. Indeed, the results could extend to non-strict ordering simply by applying ideas as in [1].
(3) We should be able to state what the meaning of fairness in the BID-N-TRADE procedure is and what it entails for both states and refugees.

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APPENDIX

A WARM-UP: BINARY ONE-PARAMETER ASSIGNMENT

A.1 Preliminaries

This represents the first attempt to step aside from the classical assumptions of two-sided assignments which is inapplicable in this case as argued in the preceding section. If on one hand, we do capture the fact that states are indifferent to which particular refugee family is assigned to them, on the other, this simplified model introduces a series of new issues that cannot be overlooked. Let us formalize the model below.

MODEL 2 (BINARY ONE-PARAMETER). For the three parties involved:

Refugees. For each refugee family $r \in R$, r reports to the EU Refugee Committee $\mathcal{A}$ a binary preference vector over states $\rho_r \in \{0,1\}^m$ such that the i^{th} entry is 1 whenever r declares to be "compatible" with state s_i and 0 otherwise. Refugee family r may misreport it true binary preference which is, in fact, $\pi_r \in \{0,1\}^m$.

States. For each state $s \in S$, s reports to the EU Refugee Committee $\mathcal{A}$ a unitary (annual) cost per refugee family $c_s \in \mathbb{R}_+$. State s may misreport it true cost which is, in fact, $t_s \in \mathbb{R}_+$.

EU. EU Refugee Committee $\mathcal{A}$ designs a payment scheme for each state $P_s \in \mathbb{R}_+$ and a assignment $\mu : R \longrightarrow S$ of refugees to states such that the following five properties are satisfied:

(P1) Each refugee r is matched exactly to one state, i.e. $\sum_{s \in S} \mu_{rs} = 1$, where μ_{rs} is the variable indicating whether assignment μ has assigned refugee r to state s.

(P2) Each refugee r is incentivised to report truthfully, i.e. $\rho_r = \pi_r$.

(P3) Each state s is incentivised to report truthfully, i.e. $c_s = t_s$.

(P4) Each state s voluntarily participates, i.e. $u_s := P_s - t_s \cdot w_s \geq 0$, where $w_s \leq n$ is the number of refugee families assigned to state s.

(P5) The number of assigned refugee families across states is equal to the total number of refugee families, i.e. $\sum_{s \in S} w_s = \sum_{r \in R} \sum_{s \in S} \mu_{rs} = n$.

Let us now turn our semi-formal model into a formal optimization problem. $\mathcal{A}$, hence, solves the following Integer Linear Program, then relaxed to a Linear Program.

ILP-RRB1P

$$
\begin{aligned}
\textbf{Objective function:} \quad & \min \sum_{r \in R} \sum_{s \in S} \mu_{rs} t_s \\
\textbf{Subject to:} \quad & \sum_{s \in S} \mu_{rs} = 1, \forall\, r \in R \\
& \sum_{r \in R} \sum_{s \in S} \mu_{rs} = n \\
& \mu_{rs} \in \{0,1\}, \forall\, r \in R, s \in S
\end{aligned}
$$

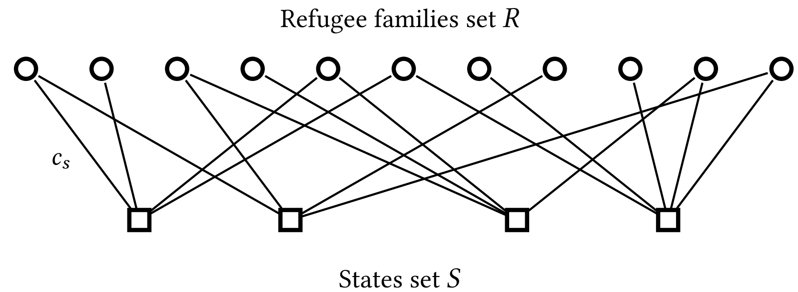
LP-RRB1P

$$
\begin{aligned}
\textbf{Objective function:} \quad & \min \sum_{r \in R} \sum_{s \in S} \tilde{\mu}_{rs} t_s \\
\textbf{Subject to:} \quad & \sum_{s \in S} \tilde{\mu}_{rs} = 1, \forall\, r \in R \\
& \sum_{r \in R} \sum_{s \in S} \tilde{\mu}_{rs} = n \\
& \tilde{\mu}_{rs} \in [0,1], \forall\, r \in R, s \in S
\end{aligned}
$$

Hereby, we are giving for granted that $\mathcal{A}$ will indeed receive $\rho_r = \pi_r$ for all refugees r and $c_s = t_s$ for all states s. However, $\mathcal{A}$ needs to ensure that via a suitable payment scheme P_s.

A.2 Refugee Resettlement Binary One Parameter (RRB1P) Mechanism

Before delving into the payment scheme construction, let us translate pictorially the optimization problem framed above into the following diagram.



Hereby, the diagram illustrates a bipartite graph $G = (R \cup S, E)$ where the top nodes (circles) represent refugee families and bottom nodes (squares) states. Each edge (or lack of thereof) connecting a refugee r to state s is simply $\rho_{rs} = \pi_{rs}$, equal to 1 when an edge appears and to 0 otherwise. Each incident edge to a given state s is connoted by a weight equal to $c_s = t_s$. This illustration is based on the assumption that $\mathcal{A}$ elicits the true reports from refugees and states. Indeed, we will prove this is the case. Let us, henceforth, consider the following simple mechanism.

Mechanism: $\mathtt{RRB1P}$

Algorithm: $\mathtt{ALG}(G, \rho, c)$

INIT $\mathtt{COST} \leftarrow 0$, $\mu \leftarrow \emptyset$
FOR $r \in R$:
PICK edge $e \in E$ such that $c_e = \min_{s \text{ incident to } r} c_s$
UPDATE $\mathtt{COST} \leftarrow \mathtt{COST} + c_e$, $\mu \leftarrow \mu \cup \{e\}$
RETURN $\mathtt{COST}, \mu$

Payment: P We choose payments $P_s(\mathtt{ALG}, c)$ as per Myerson's Lemma [9], namely

$$P_s(\mathtt{ALG}, c) = c_s \cdot w_s\left(\mathtt{ALG}\left(c_s, c_{-s}\right)\right) + \int_{c_s}^{\infty} w_s\left(\mathtt{ALG}\left(x, c_{-s}\right)\right) dx, \text{ if integral finite}$$

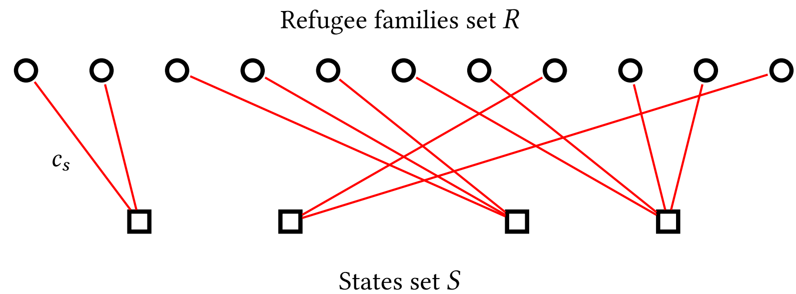
$$P_s(\mathtt{ALG}, c) = Q_s(c_{-s}) + c_s \cdot w_s\left(\mathtt{ALG}\left(c_s, c_{-s}\right)\right) - \int_{0}^{c_s} w_s\left(\mathtt{ALG}\left(x, c_{-s}\right)\right) dx, \text{ if integral not finite}$$

Hereby, we define Q_s as the cost of the cheapest assignment μ_{-s} if edge set E_s, incident to state $s \in S$, were not to be used. Formally, for $E_s \subseteq E$ being the set of edges without state s being considered, $\mathcal{M}_{-s} = \{\mu \in \mathcal{M} | E_s \cap E(\mu) = \emptyset\}$ being the set of all possible assignments without state s and $\mu_{-s} \in \arg\min_{\mu \in \mathcal{M}_{-s}} \text{cost}(\mu, c_{-s})$ being the cheapest assignment without state s,

$$Q_s(c_{-s}) = \text{cost}\left(\mu_{-s}, c_{-s}\right)$$

Clearly, this is a function that does not depend on c_s.

In order for us to have a visual understanding of how an optimal (minimum cost) assignment may look like, let us take a close look at the following diagram.



Hereby, the red edges represent the ones included in the optimal assignment by ALG. We could check that this is indeed a assignment as each and every circle node is mapped to a square node.

Theorem 3. *Mechanism* RRB1P *(Refugee Resettlement Binary One Parameter), through payment scheme P, induces truthful reporting on refugee families, under binary preferences assumption, and on states, under the assumption that their cost and utility depends solely on the unitary annual cost. Moreover, through algorithm* ALG, RRB1P *solves the optimization problem of above optimally.*

Proof. We will prove optimality and monotonicity of ALG jointly below and truthfulness, for both refugee families and states separately.

Optimality and Monotonicity. ALG's optimality is easy to show. Since each refugee family has to be assigned to exactly one state, the globally optimal set of edges is simply the union of the locally optimal sets of edges. Hereby, local refers to the edges that are incident to a node representing a refugee. Since for each $r \in R$, ALG chooses the incident edge e of minimum cost, local and, thus, global optimality is achieved. For what concerns monotonicity, ALG is monotone if the following function is monotone non-increasing for all $s \in S$ and for all c_{-s}:

$$w_s\left(\mathsf{ALG}\left(c_s, c_{-s}\right)\right)$$

This is indeed the case for ALG insofar as, fixed a state $s \in S$ and fixed the reports of the other states c_{-s}, if s reports a cost $c_s' > c_s$, then, for sure a minimization algorithm such as ALG will assign less units to that state.

Refugees Truthfulness. Under the assumption that refugees have binary preferences over states, then, it is weakly dominant to report edges of "compatibility" truthfully. In fact, omitting edges would yield to an assignment to which refugee r is indifferent with respect to the truthful one. Clearly, we are hereby assuming that each and every refugee will be guaranteed an assignment to exactly one state. If refugees have ranking preferences over states (which is indeed usually the case), then, the above analysis is no longer true.

States Truthfulness. The EU Refugee Committee $\mathcal{A}$ needs to pay states so that they do not misreport their true unit cost t_s to some other c_s. Given that states only rely onto a single parameter c_s, this is a one-parameter problem. By Myerson's Lemma [9], so long as ALG is monotone, then RRB1P payments P imply truthfulness for single-parameter agents, i.e. states. □

A.3 Limitations

Mechanism RRB1P in A.2 seems to good to be true, and indeed is. We hereby present the problems that arise by implementing it, both from a theoretical perspective and from a practical standpoint.

(Problem 1) Refugee families usually have ranking preferences over states and not just binary ones. Indeed, a refugee family might prefer a state to another and they are not indifferent among "compatible" states:

in other words, some states are more "compatible" than others. This, of course, contradicts the binary preference assumption. Once the assumption no longer models the problem appropriately, and ranking preferences arise, then, if RRB1P is run, a refugee family might be incentivised to omit all the edges that do not connect them to their top choice in their preference list, as they are guaranteed a place.

(Problem 2) For states, the crucial thing to analyse is the integral in the payment expression above:

- When the integral is finite, i.e. when a refugee is not only compatible to the considered state but also to some other state, then not only does the payment scheme guarantee truthfulness but also voluntary participation of the given state. The latter means that, by participating to the mechanism, the state cannot lose money and will have a non-negative utility, as per **(P4)** in the model above.
- However, when the integral is infinite, in order to guarantee truthfulness, a state's utility might be negative , thereby violating **(P4)**. This effect is even more amplified if **Problem 1** holds true as more and more refugees will be incentivised to report only their top choice edge.

(Problem 3) In reality, state $s \in S$ might have a limited number of places available, which could also be seen as a monetary budget B_s trespassed which their cost will become exorbitant.

Since none of these three central issues are solved by the first incomplete attempt, let us now turn our attention to the second incomplete attempt.

B PROOF OF THEOREM 3

Proof. We will prove the desirable properties of TTSC separately below. This proof is inspired by the one provided in [13].

Termination. The algorithm terminates since at each step of TTSC, we remove at least one spot from at least one of the states. By monotonicity, there will be a time step where all the spots available will be exhausted and the algorithm will have converged. Note that this argument also shows that TTSC converges in a number of steps that is polynomial in at most the number of refugee families: indeed, since not all families are necessarily assigned to some state by B4G, then the total number of spots in states is at most the cardinality of set R. Therefore, TTSC runs in $\text{poly}(|R|) = \text{poly}(n)$ time.

Weakly Improved Allocation. When TTSC terminates, each refugee family is at least as satisfied with their final allocation as with the initial one. This is due to the fact that after cycles (and, hence, spots) have been removed, all remaining refugee families keep their spot in the original state (and then possibly point to some other state), whereas refugee families that got a different spot in some other state indeed had pointed to that spot and, thus, preferred it.

Truthfulness. Let $\mathcal{N}_t$ be the set of refugee families who have been assigned a spot in step t of the algorithm. By induction, refugee families that belong to N_1 are best off as they obtain their globally top choice and, therefore, have no incentive to lie about their preference. Now, let $r_j \notin \mathcal{N}_t$ be a refugee family that did not get reallocated in step t and that preferred a spot among the ones reallocated in $\mathcal{N}_t$. However, even by changing their outgoing edge (thus, lying), this would not change their own outcome given that refugee families that do belong to $\mathcal{N}_t$ do not point back to r_j as this would make them strictly worse off.

Budget compliance. By how the algorithm is designed, the budget for each state $s \in S$ will be respected since it is impossible for a refugee family $r \in R$ to obtain a spot in that state without having swapped their own spot with a refugee family $r' \in R$ residing in s.

Before continuing with the proof, it is crucial to observe one fact. Despite the above four properties are highly desirable, they are also trivial to achieve: if we assigned every refugee family their initial spot, the we would have trivially satisfied all the above properties. However, if two two refugee families $r, r' \in R$ knew about each other's preferences (suppose they have preferences such that they would like to exactly exchange their spot), then they could collude, pull out of this algorithm and exchange their spots "off-the-market", i.e.

behind the scenes. Thus, we also need to show that the algorithm is also collusion resistant, in jargon, that `TTSC` produces a unique core allocation.

Unique Core Allocation. Assume there exists a subset of refugee families $C \subseteq R$, blocking coalition, such that there is a way for the members of C to reallocate their spots in a way that makes at least one of them strictly better off and none of the rest strictly worse. Let us consider the smallest such t for which $\mathcal{N}_t \cap C \neq \emptyset$, that is the first iteration of the algorithm where a member of the coalition $r \in C$ gets a spot in some state $s \in S$. Since no other member $r' \in C$ may belong to $\bigcup_{h \in [t-1]} \mathcal{N}_h$, as otherwise r would have not been the first, then, no reallocation within C could make r better off. Uniqueness of the core follows from the fact that all refugee families in $\mathcal{N}_1$ get their top choice. Hence, these refugee families must get their top choices also in any core allocation as otherwise they would contain a blocking coalition of refugee families who did not get their top choice. The same holds inductively with refugee families in $\mathcal{N}_2, \mathcal{N}_3, \ldots$, of course with the set of remaining choices available. □