Solutions - Practical Lesson 6

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1 Solutions

1.1 Exercises

1.1.1 Exercise 6.1

Write a ForwardRateCurve (for EURIBOR/LIBOR rate curve) which doesn't compute discount factors but only interpolates forward rates; then add it to the finmarkets module.

Solution In this case it is enough to write a new `class` that has three attributes: a today date, a set of pillar_dates and the corresponding rates. There will be just a single method `forward_rate` which returns the corresponding interpolated rate.

```
In [1]: import numpy

        # an EURIBOR or LIBOR rate curve
        # doesn't calculate discount factors, only interpolates forward rates
        class ForwardRateCurve(object):

            # the special __init__ method defines how to
            # construct instances of the class
            def __init__(self, pillar_dates, rates):

                # we just store the arguments as attributes of the instance
                self.today = pillar_dates[0]
                self.rates = rates

                self.pillar_days = [
                    (pillar_date - self.today).days
                    for pillar_date in pillar_dates
                ]


            # interpolates the forward rates stored in the instance
            def forward_rate(self, d):
                d_days = (d - self.today).days
                return numpy.interp(d_days, self.pillar_days, self.rates)
```

1.1.2 Exercise 6.2

Using the function `randint` of the module `random` make a Monte Carlo simulation of rolling three dices to check the probability of getting the same values on the three of them.

From the probability theory you should expect:

$$P_{d1=d2=d3} = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot 6 = \frac{1}{36} = 0.0278$$

```
In [1]: from random import seed, randint

        seed(1)

        trials = 10000000
        success = 0
        for _ in range(trials):
          d1, d2, d3 = randint(1, 6), randint(1, 6), randint(1, 6)

          if d1 == d2 and d2 == d3:
            success += 1

        print ("The probability to get three equal dice is {:.4f}".format(success/trials))
```

```
The probability to get three equal dice is 0.0278
```

1.1.3 Exercise 6.3

Using the function `normal` of `numpy.random` simulate the price of a stock which evolves according to a log-normal stochastic process with a daily rate of return $\mu = 0.1$ and a volatility $\sigma = 0.15$ for 30 days.

Also plot the price. Try to play with μ and σ to see how the plot changes.

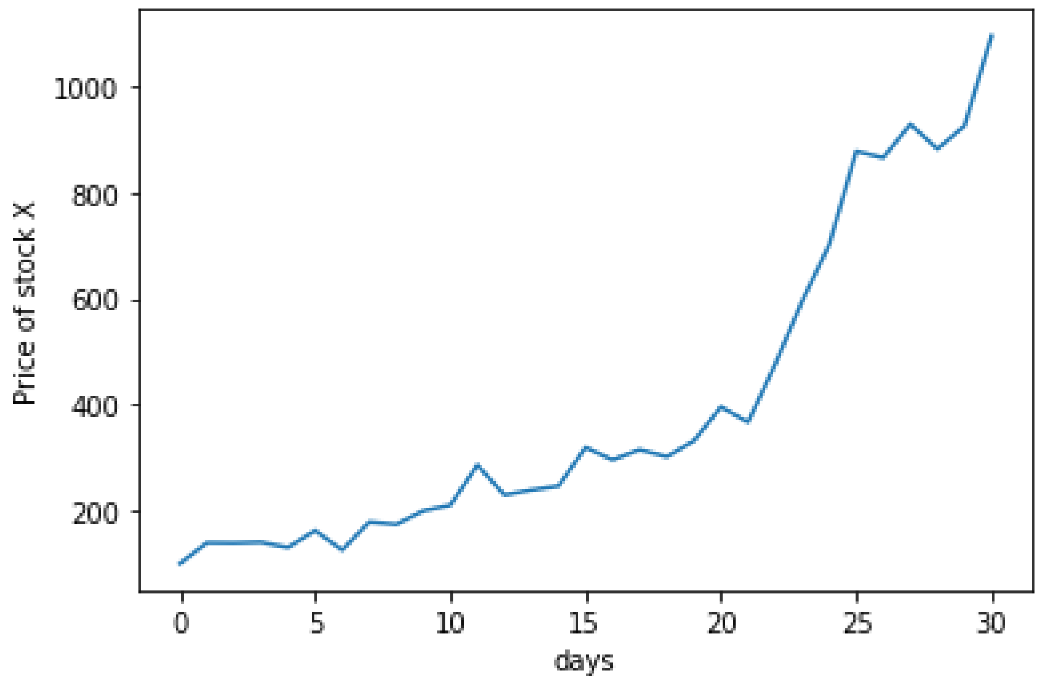
```
In [3]: from numpy.random import normal, seed
        from matplotlib import pyplot as plt
        import math

        S = 100
        mu = 0.1
        sigma = 0.15
        T = 1

        seed(1)
        historical_series = [S]
        for i in range(30):
            S = S * math.exp((mu - 0.5 * sigma * sigma) * T + sigma * math.sqrt(T) * normal())
            historical_series.append(S)

        plt.plot(range(31), historical_series)
        plt.xlabel("days")
```

```
plt.ylabel("Price of stock X")
plt.show()
```



1.1.4 Exercise 6.4

Suppouse that the Libor Forward rates are those defined *here*. Determine the value of an option to pay a fixed rate of 4% and receives LIBOR on a 5 year swap starting in 1 year. Assume the notional is 100 EUR, the exercise date is on October, 30th 2020 and the swap rate volatility is 15%.

```
In [4]: from finmarkets import InterestRateSwap
        from datetime import date
        from dateutil.relativedelta import relativedelta
        from curve_data import discount_curve, libor_curve
        from scipy.stats import norm
        import math

        pricing_date = date.today()
        start_date = date.today() + relativedelta(years=1)
        exercise_date = date(2020, 10, 30)

        irs = InterestRateSwap(start_date, 100, 0.04, 12, 5)
        sigma = 0.15

        A = irs.annuity(discount_curve)
```

```python
S = irs.swap_rate(discount_curve, libor_curve)
T = (exercise_date - pricing_date).days / 365
d1 = (math.log(S/irs.fixed_rate) + 0.5 * sigma**2 * T) / (sigma * T**0.5)
d2 = (math.log(S/irs.fixed_rate) - 0.5 * sigma**2 * T) / (sigma * T**0.5)
npv = irs.notional * A * (S * norm.cdf(d1) - irs.fixed_rate * norm.cdf(d2))

print("Swaption NPV: {:.3f} EUR".format(npv))
```

```
Swaption NPV: 13.587 EUR
```