Python for Finance

Exercises

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Contents

Chapter 1

Introduction to python

Exercise 1.1

What is the built-in function that python uses to iterate over a number sequence ? Write an example that uses it.

Solution 1.1

The built-in function used to iterate over a sequence of numbers is range. It returns a sequence of numbers taking three parameters that represents respectively the lower boundary of the sequence, the upper boundary of the sequence and the step. If just one parameter is passed the default lower boundary is 0 and the step is 1. Note that the upper boundary is excluded from the sequence.

```
for i in range(10, 20, 2):
    print (i)

10
12
14
16
18
```

```
for i in range(5):
    print (i)

0
1
2
3
4
```

Exercise 1.2

What is a string in python ? Declare one string variable and try to manipulate it (concatenate, make uppercase, capitalize, replace characters, split...).

Solution 1.2

A string is simply a sequence of characters.

```
aString = "this is a string"

aString = aString + ", just an example"
print (aString)
this is a string, just an example

print (aString.upper())
'THIS IS A STRING, JUST AN EXAMPLE'

print (aString.capitalize())
'This is a string, just an example'

print (aString.replace("just an", "for"))
'this is a string, for example'

print (aString.split(","))
['this is a string', ' just an example']

if (aString.endswith("example")):
    print ("This string is really an example.")
else:
    print ("This string is not an example")
This string is really an example.
```

Exercise 1.3

What does the continue do in python ? Show an example of its usage printing all the odd numbers between 0 and 10.

Solution 1.3

continue is used to skip cycles in for loops. Note that % is the module operator, it returns the reminder of a division.

```
for i in range(10):
    if i%2 == 0:
        continue
    else:
        print (i)

1
3
5
7
9
```

Exercise 1.4

When should you use the break in python ? Show an example of its usage.

Solution 1.4

`break` is the command used to interrupt a while loop even if the while condition is still satisfied.

```
i = 0
while i < 11:
    if (i/2 > 3):
        break
    print (i)
    i = i + 1

0
1
2
3
4
5
6
```

Exercise 1.5

Which `python` function will you use to convert a number to a string ? Show an example.

Solution 1.5

`str()` is the correct function to use in order to cast a number to a string.

```
x = 2.34
print ("This {} is of type {}".format(x, type(x)))
print ("This {} is of type {}".format(str(x), type(str(x))))

This 2.34 is of type <class 'float'>
This 2.34 is of type <class 'str'>
```

Exercise 1.6

Import the math module and compute the logarithm of 2.09, the exponential of 1.57 and the area of a circle of radius 6 cm (circle area = $\pi \cdot r^2$).

Solution 1.6

```
import math

print ("log(2.09) = {}".format(math.log(2.09)))
print ("exp(1.57) = {}".format(math.exp(1.57)))
print ("area of circle (R=6cm) is about {:.2f} cm2".format(math.pi*6**2))

log(2.09) = 0.7371640659767196
exp(1.57) = 4.806648193775178
area of circle (R=6cm) is about 113.10 cm2
```

Exercise 1.7

Given the following variables

```
S_t = 800.0 # spot price of the underlying
K = 600.0 # strike price
vol = 0.25 # volatility
r = 0.01 # interest rate
ttm = 0.5 # time to maturity, in years
```

write out the Black Scholes formula and save the value of a call in a variable named 'call_price' and the value of a put in a variable named 'put_price'.
Hint: remember that there are many modules available in python that let you save a lot of time. In this case we need the cumulative distribution function of the standard normal distribution which can be found in `scipy.stats` module, the name of the function is `norm`.

Solution 1.7

The BS equation for the price of a call is:

$$C(S,t) = S_t N(d_1) - Ke^{-r(T-t)}N(d_2)$$

where

- S_t is the spot price of the underlying
- K is the strike price
- r is the risk-free interest rate (expressed in terms of continuous compounding)
- $N(\cdot)$ is the cumulative distribution function of the standard normal distribution
- $T-t$ is the time to maturity
- σ is the volatility of the underlying

$$d_1 = \frac{\ln(\frac{S_t}{K}) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

```
from math import log, exp, sqrt
# You'll need the Gaussian cumulative distribution function
from scipy.stats import norm

S_t = 800.0
ttm = 0.5
K = 600.0
vol = 0.25
r = 0.01

d1_num = (log(S_t/K)+(r+0.5*pow(vol, 2))*ttm)
d1_den = vol*sqrt(ttm)
d1 = d1_num /d1_den
d2 = d1 - d1_den

call_price = S_t * norm.cdf(d1) - K * exp(-r*ttm)*norm.cdf(d2)
put_price = - S_t * norm.cdf(-d1) + K * exp(-r*ttm)*norm.cdf(-d2)

print ("{:.3f} {:.3f}".format(call_price, put_price))

205.472 2.480
```

Chapter 2

Data Containers

Exercise 2.1

What is a dictionary in `python` programming ? Create a dictionary, modify it and then print all its items.

Solution 2.1

A dictionary is a container that maps a key (any object) to a value (any object), contrary to lists which map an integer (the index) to a value (any object).

```
dictionary = {"calculus":28, "physics":30, "chemistry":25}

dictionary["laboratory"] = 27
dictionary["chemistry"] = 24

print ("Exam\t\tVote")
for k, v in dictionary.items():
    print ("{}:\t{}".format(k, v))
```

```
Exam            Vote
calculus:       28
physics:        30
chemistry:      24
laboratory:     27
```

Exercise 2.2

Write code which, given the following list

```
input_list = [3, 5, 2, 1, 13, 5, 5, 1, 3, 4]
```

prints out the indices of every occurrence of

```
y = 5
```

Solution 2.2

```
l = [3, 5, 2, 1, 13, 5, 5, 1, 3, 4]

for i in range(len(l)):
    if l[i] == 5:
        print (i)

1
5
6
```

Note that lists already have a way to get the occurrences of an item: `l.count(5)` would have done the job.

Exercise 2.3

Write a `python` program to convert a list of tuples into a dictionary where the keys are the first elements of each tuples and the values the second. Input:

```
l = [("x", 1), ("x", 2), ("x", 3), ("y", 1), ("y", 2), ("z", 1)]
```

Solution 2.3

```
l = [("x", 1), ("x", 2), ("x", 3), ("y", 1), ("y", 2), ("z", 1)]

d = {}
for item in l:
    d[item[0]] = item[1]

print (d)

{'x': 3, 'y': 2, 'z': 1}
```

Note that there is just one occurrence of the key x and y because keys has to be unique and setting the same key to a different value simply overwrite the existing entry.

Exercise 2.4

Write a `python` program to replace the last value of each tuples in a list. Input:

```
l = [(10, 20, 40), (40, 50, 60), (70, 80, 90)]
```

Solution 2.4

```
l = [(10, 20, 40), (40, 50, 60), (70, 80, 90)]

for i in range(len(l)):
    new_tuple = l[i][0:2] + (l[i][2] + 10,)
    l[i] = new_tuple

print (l)

[(10, 20, 50), (40, 50, 70), (70, 80, 100)]
```

Exercise 2.5

Write a python program to count the elements in a list until an element is a tuple. Input:

```
{[1, 5, 'a', (1,2), {'test':1}]}
```

Solution 2.5

```
l = [1, 5, "a", (1,2), {"test":1}]

number_of_items = 0
for item in l:
    if type(item) != tuple:
        number_of_items = number_of_items + 1
    else:
        break

print ("There are {} items before a tuple.".format(number_of_items))

There are 3 items before a tuple.
```

Exercise 2.6

Write a python script to concatenate following dictionaries to create a new single one. Input:

```
dic1={1:10, 2:20}
dic2={3:30, 4:40}
dic3={5:50, 6:60}
```

Solution 2.6

```
dic1={1:10, 2:20}
dic2={3:30, 4:40}
dic3={5:50,6:60}

dic_tot = dict()
dic_tot.update(dic1)
dic_tot.update(dic2)
dic_tot.update(dic3)

print (dic_tot)

{1: 10, 2: 20, 3: 30, 4: 40, 5: 50, 6: 60}
```

Exercise 2.7

Write a python script to check whether a given key already exists in a dictionary.

Solution 2.7

```
dic = {"a":1, "b":2, "c":3}

print ("z" in dic)
print ("a" in dic)

False
True
```

Exercise 2.8

Write a python program to combine two dictionary adding values for common keys. Input:

```
d1 = {'a': 100, 'b': 200, 'c':300}
d2 = {'a': 300, 'b': 200, 'd':400}
```

Solution 2.8

```
d1 = {'a': 100, 'b': 200, 'c':300}
d2 = {'a': 300, 'b': 200, 'd':400}

d = {}
d.update(d1)

for k in d2.keys():
    if k in d:
        d[k] = d[k] + d2[k]
    else:
        d[k] = d2[k]

print (d)

{'a': 400, 'b': 400, 'c': 300, 'd': 400}
```

Exercise 2.9

Given the following dictionary mapping currencies to 2-year zero coupon bond prices, build another dictionary mapping the same currencies to the corresponding annualized interest rates.

```
d = {
'EUR': 0.98,
'CHF': 1.005,
'USD': 0.985,
'GBP': 0.97
}
```

Solution 2.9

The price of a n-years zero coupon bond is:

$$P = \frac{M}{(1+r)^n} = M \cdot D$$

where M is the value of the bond at the maturity, r is the risk-free rate and n is the number of years until maturity.
Hence:

$$D = \frac{1}{(1+r)^n} \implies r = \left(\frac{1}{D}\right)^{\frac{1}{n}} - 1$$

```
from math import exp

# initialize an empty dictionary in which to store result
rates = {}

maturity = 2
discount_factors = {
    'EUR': 0.98,
    'CHF': 1.005,
    'USD': 0.985,
    'GBP': 0.97
}

# loop over the input dictionary to get the currencies
for currency, df in discount_factors.items():
    # calculate the rate and store it in the output dictionary
    rates[currency] = pow(1/df, 1/maturity) - 1

for r in rates.items():
    print (r)

('EUR', 0.010152544552210818)
('CHF', -0.002490663892367073)
('USD', 0.007585443719756668)
('GBP', 0.015346165133619083)
```

Chapter 3

Date and Time

Exercise 3.1

Write code that:

- print the day of the week of your birthday
- print the weekday of your birthdays for the next 120 years

Solution 3.1

```
import datetime

birthday = datetime.date(1974, 10, 20)
print (birthday.weekday()) # remember it starts form 0

6
```

```
from dateutil.relativedelta import relativedelta

for i in range(120):
    print ((birthday + relativedelta(years=i)).weekday())

6
0
2
3
4
5
0
1
2
3
4
...
```

Exercise 3.2

Write code to determine whether a given year is a leap year and test it with 1800, 1987 and 2020. **Hint:** a leap year is divisible by 4, by 100 and by 400.

Solution 3.2

```
years = [1800, 1987, 2020]

for y in years:
    if y % 400 == 0:
        print ("{} is a leap year ".format(y))
    elif y % 100 == 0:
        print ("{} is NOT a leap year ".format(y))
    elif y % 4 == 0:
        print ("{} is a leap year ".format(y))
    else:
        print ("{} is NOT a leap year ".format(y))

1800 is NOT a leap year
1987 is NOT a leap year
2020 is a leap year
```

Exercise 3.3

Write code to print next five days starting from today.

Solution 3.3

```
d = datetime.date.today()
for i in range(1, 6):
    print (d + relativedelta(days=i))

2020-08-04
2020-08-05
2020-08-06
2020-08-07
2020-08-08
```

Exercise 3.4

Build again dates as in Exercise 3.1 (i.e. the weekday of your birthdays for the next 120 years) and count how many of your birthdays is a Monday, Tuesday, ... , Sunday until 120 years of age. Print out the result using a dictionary. (expected output something like: `{6: 10, 0: 10, 2: 9, 3: 10, 4: 10, 5: 10, 1: 9}`)

Solution 3.4

```
import datetime
from dateutil.relativedelta import relativedelta
birthday = datetime.date(1974, 10, 20)

d = {}
for i in range(120):
    wd = ((birthday + relativedelta(years=i)).weekday())
    if wd in d.keys():
        d[wd] = d[wd] + 1
    else:
        d[wd] = 1

print (d)

{6: 17, 0: 18, 2: 17, 3: 17, 4: 17, 5: 17, 1: 17}
```

Exercise 3.5 (Date Generator)

In the next lessons we will create many contracts (e.g. swaps) which take in input lists of dates like for example the payment dates. Since it would be very boring to write long list of dates for each of these contracts, the goal of this exercise is to write code which given a start date and a number of months, returns a list of dates of **annual** frequency from the start date to the ending of the period after the specified number of months.
For example

- 2019-11-10 start date 12 months → 2019-11-10, 2020-11-10
- 2019-11-10 start date 24 months → 2019-11-10, 2020-11-10, 2021-11-10

Note that if the number of months is not a multiple of 12, the last period should simply be shorter than 12 months. For example:

- 2019-11-10 start date 9 months → 2019-11-10, 2020-08-10
- 2019-11-10 start date 15 months → 2019-11-10, 2020-11-10, 2021-02-10

Once you have done save this code in a file called `finmarkets.py`, this will become our financial library and will be extended and used later on.

Solution 3.5

```
from finmarkets import generate_swap_dates
from datetime import date
from dateutil.relativedelta import relativedelta

start_dates = date(2019, 11, 10)
n_months = 15
dates = []
for i in range(0, n_months, 12):
    dates.append(start_date + relativedelta(months=i))
dates.append(start_date + relativedelta(months=n_months))

print(dates)

[date(2019, 11, 10),
 date(2020, 11, 10),
 date(2021, 2, 10)]
```

Chapter 4

Function and Classes

Exercise 4.1

Take the code for the Black-Scholes formula from Exercise 1.7 and wrap it in a function. Then, use this function to calculate the prices of calls with various strikes, using the following data.

```
s = 800
# strikes expressed as % of spot price
moneyness = [ 0.5, 0.75, 0.825, 1.0, 1.125, 1.25, 1.5 ]
vol = 0.3
ttm = 0.75
r = 0.005
```

The output should be a dictionary mapping strikes to call prices.

Solution 4.1

```
from math import log, exp, sqrt
from scipy.stats import norm

def d1(S_t, K, r, vol, ttm):
    num = log(S_t/K) + (r + 0.5*pow(vol, 2)) * ttm
    den = vol * sqrt(ttm)
    return num/den

def d2(S_t, K, r, vol, ttm):
    return d1(S_t, K, r, vol, ttm) - vol * sqrt(ttm)

def call(S_t, K, r, vol, ttm):
    return S_t * norm.cdf(d1(S_t, K, r, vol, ttm)) \
       - K * exp(-r * ttm) * norm.cdf(d2(S_t, K, r, vol, ttm))

s = 800
# strikes expressed as % of spot price
moneyness = [ 0.5, 0.75, 0.825, \
              1.0, 1.125, 1.25, 1.5 ]
vol = 0.3
ttm = 0.75
r = 0.005

result = {}
for m in moneyness:
    result[s*m] = call(s, m*s, r, vol, ttm)
print(result)

{400.0: 401.66074527896365,
  600.0: 213.9883852521275,
  660.0: 166.85957363897393,
  800.0: 84.03697017660357,
  900.0: 47.61880394696229,
  1000.0: 25.632722952585738,
  1200.0: 6.655275227771156}
```

Exercise 4.2

Write two classes, `Circle` and `Rectangle` that given the radius and height, width respectively allow to compute area and perimeter of the two shapes. Test them with the following:

```
a_circle = Circle(5)
print ("My circle has an area of {} m**2".format(a_circle.area()))

a_rectangle = Rectangle(3, 6)
print ("My rectangle has a perimeter of {} m and an area of {} m**2" \
```

```
    .format(a_rectangle.perimeter(), a_rectangle.area()))
```

Solution 4.2

```
from math import pi
class Circle:
    def __init__(self, radius):
        self.radius = radius

    def area(self):
        return pi*self.radius**2

class Rectangle:
    def __init__(self, width, height):
        self.height = height
        self.width = width

    def area(self):
        return self.width*self.height

    def perimeter(self):
        return self.width*2 + self.height*2

circle = Circle(5)
print ("My circle area is {:.1f} m**2".format(circle.area()))

rect = Rectangle(3, 6)
print ("My rect area is {:.1f} m**2 and the "\
       "perimeter is {} m".format(rect.area(), rect.perimeter()))

My circle area is 78.5 m**2
My rect area is 18.0 m**2 and the perimeter is 18 m
```

Exercise 4.3

Define a class `Songs`, its `__init__` should take as input a dictionary (`lyrics` that contains lyrics line by line). Define a method, `sing_me_a_song` that prints each element of the lyrics in his own line. Also test it with the follwing input.

```
lyrics = {"Wonderwall":["Today is gonna be the day",
                        "That they're gonna throw it back to you",
                        "By now you should've somehow", "..."],
          "Wish you were here":  ["So, so you think you can tell",
                                  "Heaven from hell",
                                   "Blue skies from pain", "..."]}
```

Solution 4.3

```
class Songs:
        def __init__(self, lyrics):
            self.lyrics = lyrics

        def sing_me_a_song(self, title):
            song = self.lyrics[title]
            print ("Title: {}".format(title))
            print ("********************")
            for line in song:
                print (line)

lyrics = {"Wonderwall":["Today is gonna be the day",
                        "That they're gonna throw it back to you",
                        "By now you should've somehow", "..."],
          "Vado al massimo":  ["Voglio veder come va a finire",
                        "Andando al massimo senza frenare"
                        "Voglio vedere se davvero poi",
                        "Si va a finir male", "..."]}

songs = Songs(lyrics)
songs.sing_me_a_song("Wonderwall")

Title: Wonderwall
********************
Today is gonna be the day
That they're gonna throw it back to you
By now you should've somehow
...
```

Exercise 4.4

Define a Point2D class that represent a point in a plane. Its `__init__` method should accept the point coordinates `x` and `y`. Write a method `distanceTo` that compute the distance of the point to another passed as input. Test the class by printing the distance of the point $P = (4, 5)$ to the origin $P = (0, 0)$ and to $P = (3, 4)$.
Hint: in the Cartesian plane the distance between two points is: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Solution 4.4

```
from math import sqrt
class Point2D:
    def __init__(self, x, y):
        self.x = x
        self.y = y

    def distanceTo(self, x, y):
        dist = sqrt((self.x-x)**2 + (self.y - y)**2)
        return dist

    def distanceTo_v2(self, p):
        dist = sqrt((self.x-p[0])**2 + (self.y - p[1])**2)
        return dist

    def distanceTo_v3(self, p):
        dist = sqrt((self.x-p.x)**2 + (self.y - p.y)**2)
        return dist

point = Point2D(4, 5)
p0 = (0, 0)
point0 = Point2D(0, 0)
print ("distance to p0: {:.2f}".format(point.distanceTo(p0[0], p0[1])))
print ("distance_v2 to p0: {:.2f}".format(point.distanceTo_v2(p0)))
print ("distance_v3 to p0: {:.2f}".format(point.distanceTo_v3(point0)))

p1 = (3, 4)
point1 = Point2D(3, 4)
print ("distance to p1: {:.2f}".format(point.distanceTo(p1[0], p1[1])))
print ("distance_v2 to p1: {:.2f}".format(point.distanceTo_v2(p1)))
print ("distance_v3 to p1: {:.2f}".format(point.distanceTo_v3(point1)))

distance to p0: 6.40
distance_v2 to p0: 6.40
distance_v3 to p0: 6.40
distance to p1: 1.41
distance_v2 to p1: 1.41
distance_v3 to p1: 1.41
```

Exercise 4.5

Write a class `Student` which inherits from `Person` defined during Lesson 6. This new class should have two new attributes: `grade` which keeps the type of school and `votes` a dictionary which will record the student's votes and the corresponding course. Then add two methods, one to add votes and another to compute the average vote. Instantiate a "student" add some votes and show how good it has been.

Hint: this is the `Person` class already developed.

```
class Person:
    def __init__(self, name, birthday):
        self.name = name
        self.birthday = birthday
        self.employment = None

    def age(self, d=date.today()):
        age = (d - self.birthday).days/365
        print ("{} is {:.0f} years old".format(self.name, age))

    def mainOccupation(self, occupation):
        self.employment = occupation
        print ("{}'s main occupation is: {}".format(self.name, self.employment))
```

Solution 4.5

```
class Student(Person):
    def __init__(self, name, birthday, school):
        Person.__init__(self, name, birthday)
        self.grade = school
        self.votes = {}

    def addVote(self, subject, vote):
        self.votes[subject] = vote

    def average(self):
        print ("List of votes")
        print ("-------------")
        for k, v in self.votes.items():
            print ("{}: {}".format(k, v))

        avg = sum(self.votes.values())/len(self.votes)
        print ("-------------")
        print ("Avg: {:.1f}".format(avg))

student = Student("Mario", date(1980, 5, 6), "Liceo Scientifico G. Galilei")
student.addVote("Calculus", 8)
student.addVote("Literature", 5.5)
student.addVote("Latin", 6.5)
student.average()
```

```
List of votes
-------------
Calculus: 8
Literature: 5.5
Latin: 6.5
-------------
Avg: 6.7
```

Chapter 5

Data Manipulation and Its Representation

Exercise 5.1

Using `pandas` import data stored in `stock_market.xlsx` (click on the name to see and download it). With the resulting dataframe determine:

1. remove duplicates and missing data (how many rows are left ?)
2. stocks with positive variation;
3. the first five stocks with the lowest price.

Solution 5.1

1. First load the excel file into a dataframe and look at data structure.

```
import pandas as pd

df = pd.read_excel("stock_market.xlsx")

print (len(df))
df.head()

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```

	Symbol	Name	Price	Change	Change%	Volume (M)	Avg Volume (M)	Market Cap (B)
0	GE	General Electric Company	6.07	-0.19	-0.0304	142.732	102.268	53.132
1	NOK	Nokia Corporation	4.78	0.33	0.0742	117.960	31.296	27.083
2	F	Ford Motor Company	6.61	-0.13	-0.0193	115.394	87.719	26.288
3	PINS	Pinterest, Inc.	34.29	9.10	0.3613	111.864	15.550	20.110
4	AAPL	Apple Inc.	425.04	40.28	0.1047	93.574	35.035	1821.000

As usual if we are not sure that our data is *clean* we should check for duplicates and NaN and take care of them. The `duplicated` method returns the status of each row (duplicate or not, True or False). If we would like just to see the duplicated entries we could combine the `duplicated` method with the selection syntax like this:

```
df[df.duplicated() == True]
```

	Symbol	Name	Price	Change	Change%	Volume (M)	Avg Volume (M)	Market Cap (B)
40	RUN	Sunrun Inc.	36.69	0.02	0.0005	20.113	3.604	4.489

So it looks like we have just one duplicate and we can remove it:

```
print ("Before duplicates removal: {}".format(len(df)))
df = df.drop_duplicates()
print ("After duplicates removal: {}".format(len(df)))

Before duplicates removal: 50
After duplicates removal: 50
```

Then we need to take care of the NaN, again if we want to check the rows with NaN we can select (here the syntax is a little bit more complicated since we need to use `any` to look for Nan

in every column):

```
df[df.isna().any(axis=1)]

   Symbol                                  Name  Price  Change  Change%  \
23   NCLH  Norwegian Cruise Line Holdings Ltd.  13.64   -0.53  -0.0374
47    NBL                   Noble Energy, Inc.    NaN   -0.23  -0.0225

    Volume (M)  Avg Volume (M)  Market Cap (B)
23      28.402          64.895             NaN
47      18.462          13.535           4.795
```

Since we don't want to artificially modify our sample we just drop rows with NaN:

```
print ("Before NaN removal: {}".format(len(df)))
df = df.dropna()
print ("After NaN removal: {}".format(len(df)))

Before NaN removal: 51
After NaN removal: 49
```

2. The second point asks to determine the companies with a daily positive variation. Clearly we have to apply to the dataframe a selection on the "Change" (or "Change%") column requiring positive values.

```
pos_var = df[df.loc[:, "Change"] > 0]

print (len(pos_var))
pos_var.head() # just printing the first 5 rows

16

  Symbol                          Name   Price  Change  Change%  Volume (M)  \
1    NOK            Nokia Corporation    4.78    0.33   0.0742     117.960
3   PINS              Pinterest, Inc.   34.29    9.10   0.3613     111.864
4   AAPL                   Apple Inc.  425.04   40.28   0.1047      93.574
6    BAC  Bank of America Corporation   24.88    0.04   0.0016      62.039
8     FB               Facebook, Inc.  253.67   19.17   0.0817      53.030

   Avg Volume (M)  Market Cap (B)
1          31.296          27.083
3          15.550          20.110
4          35.035        1821.000
6          72.793         215.562
8          24.521         723.726
```

So in origin we had 48 stocks and just 16 have a positive variation of its price.

3. The last question requires to print the first 5 stocks with the lowest prices. In this case it is enough to sort by price the dataframe (ascending) and then just select the first 5 entries.

```
highest_price = df.sort_values(by=['Price'], ascending=True)[:5]

highest_price
```

```
   Symbol                       Name  Price  Change  Change%  Volume (M)  \
25   ABEV                Ambev S. A.   2.68   -0.16  -0.0563      26.136
32    BBD      Banco Bradesco S. A.   4.22   -0.32  -0.0705      22.129
1     NOK         Nokia Corporation   4.78    0.33   0.0742     117.960
15    OPK         OPKO Health, Inc.   5.15   -0.76  -0.1286      35.762
33    MRO  Marathon Oil Corporation   5.49   -0.02  -0.0036      21.249

    Avg Volume (M)  Market Cap (B)
25          36.654          41.999
32          22.046          36.739
1           31.296          27.083
15          17.792           3.450
33          34.098           4.339
```

Exercise 5.2

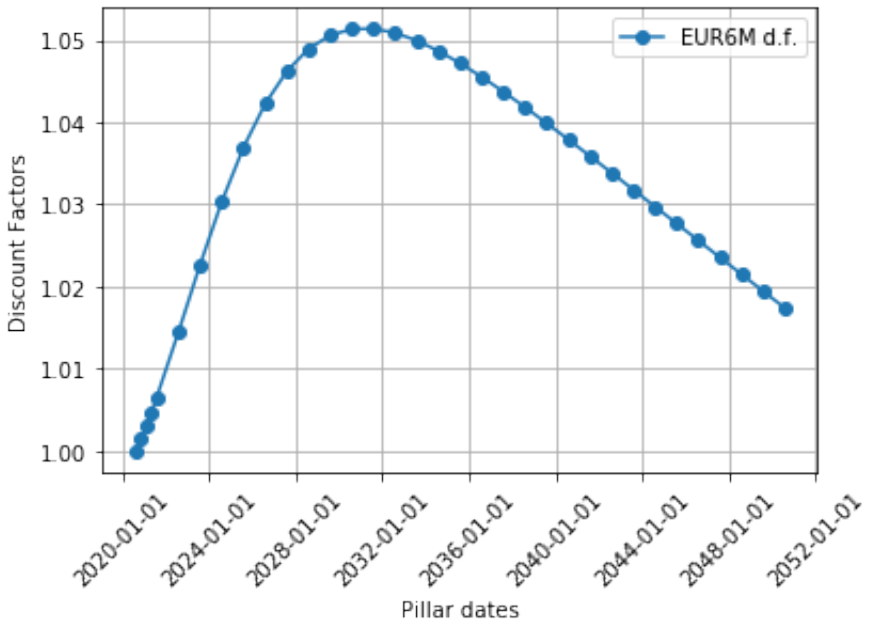
Given the following discount factors plot the resulting discount curve, possibly adding axis labels and legend.

```
dfs = [1.0, 1.0014907894567657, 1.0031038833235129, 1.0047764800189012,
       1.0065986105304596, 1.014496095021891, 1.022687560553011,
       1.0303585751965112, 1.0369440287181253, 1.0422287558021188,
       1.0461834022163963, 1.0489228953047331, 1.0505725627906783,
       1.0513323539753632, 1.0513777790851995, 1.0508768750534248,
       1.049935905228433, 1.0486741093761602, 1.047175413484517,
       1.0455115431993336, 1.0437147446170034, 1.0418294960952215,
       1.0398823957504923, 1.0378979499878478, 1.0358789099539805,
       1.0338409767365169, 1.031791178324756, 1.0297378455884902,
       1.0276772747965244, 1.0256154380560942, 1.0235543974485939,
       1.0214974135391857, 1.0194401540150835, 1.0173862951028778]

pillars = [datetime.date(2020, 8, 3), datetime.date(2020, 11, 3),
           datetime.date(2021, 2, 3), datetime.date(2021, 5, 3),
           datetime.date(2021, 8, 3), datetime.date(2022, 8, 3),
           datetime.date(2023, 8, 3), datetime.date(2024, 8, 3),
           datetime.date(2025, 8, 3), datetime.date(2026, 8, 3),
           datetime.date(2027, 8, 3), datetime.date(2028, 8, 3),
           datetime.date(2029, 8, 3), datetime.date(2030, 8, 3),
           datetime.date(2031, 8, 3), datetime.date(2032, 8, 3),
           datetime.date(2033, 8, 3), datetime.date(2034, 8, 3),
           datetime.date(2035, 8, 3), datetime.date(2036, 8, 3),
           datetime.date(2037, 8, 3), datetime.date(2038, 8, 3),
           datetime.date(2039, 8, 3), datetime.date(2040, 8, 3),
           datetime.date(2041, 8, 3), datetime.date(2042, 8, 3),
```

```
        datetime.date(2043, 8, 3), datetime.date(2044, 8, 3),
        datetime.date(2045, 8, 3), datetime.date(2046, 8, 3),
        datetime.date(2047, 8, 3), datetime.date(2048, 8, 3),
        datetime.date(2049, 8, 3), datetime.date(2050, 8, 3)]
```

Solution 5.2

```
import datetime

dfs = [1.0, 1.0014907894567657, 1.0031038833235129, 1.0047764800189012, ...]

pillars = [datetime.date(2020, 8, 3), datetime.date(2020, 11, 3), ...]

from matplotlib import pyplot as plt
import matplotlib.dates as mdates

plt.plot(pillars, dfs, marker="o", label="EUR6M d.f.")

plt.gca().xaxis.set_major_formatter(mdates.DateFormatter('%Y-%m-%d'))
# this one instead rotate labels to avoid superimposition
plt.xticks(rotation=45)
plt.xlabel("Pillar dates")
plt.ylabel("Discount Factors")
plt.grid(True)
plt.legend()
plt.show()
```



Chapter 6

Interpolation, Discount Factors and Forward Rates

Exercise 6.1 (Assertion)

Python has a useful command called `assert` which can be used for checking that a given condition is satisfied, and raising an error if the condition is not satisfied.
The following line does not cause an error, in fact it does nothing since 1 is lower than 2, hence the condition is met.

```
assert 1 < 2
```

This causes an error (the condition is evaluated to false.

```
assert 1 > 2
```

`assert` can take a second argument with a message to display in case of failure.

```
assert 1 > 2, "Two is greater than one"
```

Now takes the `df` function from Chapter 6 of the Lecture Notes and modify it by adding some assertions to check that:

- the pillar date list contains at least 2 elements;
- the pillar date list has the same length as the discount factor one;
- the first pillar date is equal to the today's date;
- the value date (first argument `d`) is greater or equal to the first pillar date and also less than or equal to the last pillar date.

Then try using the function with some invalid data to make sure that your as sertions are correctly checking the desired conditions

Solution 6.1

```
# import modules and objects that we need
from datetime import date
import numpy
import math

today_date = date(2017, 10, 1)
pillar_dates = [date(2017, 10, 1),
                date(2018, 10, 1),
                date(2019, 10, 1)]
discount_factors = [1.0, 0.95, 0.8]

def df(d, observation_date, pillar_dates, discount_factors):
    ############### CHECKS ################
    assert len(pillar_dates) >= 2, " need at least 2 pillar dates"

    assert len(pillar_dates) == len(discount_factors), \
        "number of pillar dates should be equal to \
        the number of pillar discount factors"

    assert observation_date == pillar_dates[0], \
        "first pillar date should be the observation date"

    assert pillar_dates[0] <= d <= pillar_dates[-1], \
        "Invalid value date %s" % (d)
    ############### END OF CHECKS ################

    log_discount_factors = []
    for discount_factor in discount_factors:
        log_discount_factors.append(math.log(discount_factor))

    pillar_days = []
    for pillar_date in pillar_dates:
        pillar_days.append((pillar_date - observation_date).days)

    d_days = (d - observation_date).days

    interpolated_log_discount_factor = \
        numpy.interp(d_days, pillar_days, log_discount_factors)

    return math.exp(interpolated_log_discount_factor)

df(date(2019, 1, 1), today_date, pillar_dates, discount_factors)

0.9097285910181567
```

Exercise 6.2 (Black-Scholes Again)

Copy into the file `finmarkets.py` the function used to compute Black Scholes formula used in

Ex. 4.1. This is another utility for our financial library. Then repeat Ex. 4.1 now using the version of the Black and Scholes formula in the `finmarkets` module.

Solution 6.2

```
import finmarkets

s = 800
# strikes expressed as % of spot price
moneyness = [ 0.5, 0.75, 0.825, \
              1.0, 1.125, 1.25, 1.5 ]
vol = 0.3
ttm = 0.75
r = 0.005

result = {}
for m in moneyness:
    result[s*m] = call(s, m*s, r, vol, ttm)
result

{400.0: 401.66074527896365,
  600.0: 213.9883852521275,
  660.0: 166.85957363897393,
  800.0: 84.03697017660357,
  900.0: 47.61880394696229,
  1000.0: 25.632722952585738,
  1200.0: 6.655275227771156}
```

Exercise 6.3 (Discount Curves)

Following the steps outlined in Chapter 6 of the Lecture Notes, implement a `DiscountCurve` class and add it to `finmarkets` module. The class should have as attributes the pillar dates and the corresponding discount factors and two methods, one to interpolate discount factors and another to calculate forward rates. Finally using that class compute the forward 6M LIBOR coupon using the curves given below in pre and post 2008 crisis way.

Input:

```
observation_date = date (2020, 1, 1)
t1 = date(2020,4, 1)
t2 = date(2020, 10, 1)

# for EONIA
pillar_dates_eonia = [date(2020 , 1 ,1),
                      date(2021, 1, 1),
                      date(2022, 10 ,1)]
discount_factors_eonia = [1.0, 0.97, 0.72]

# for LIBOR 6M
pillar_dates_libor = [date(2020, 1 ,1),
```

```
                      date(2020, 6, 1),
                      date(2020, 12 ,1)]
discount_factors_libor = [1.0, 0.95, 0.90]
```

Solution 6.3

```
import math
import numpy
from datetime import date

class DiscountCurve:

    def __init__(self, today, pillar_dates, discount_factors):
        self.today = today
        self.pillar_dates = pillar_dates
        self.discount_factors = discount_factors

    def df(self, d):
        log_discount_factors = \
          [math.log(discount_factor)
           for discount_factor in self.discount_factors]
        pillar_days = [(pillar_date - self.today).days
                       for pillar_date in self.pillar_dates]
        d_days = (d - self.today).days
        interpolated_log_discount_factor = \
            numpy.interp(d_days, pillar_days, log_discount_factors)
        return math.exp(interpolated_log_discount_factor)

    def forward_rate(self, d1, d2):
        return (self.df(d1) / self.df(d2) - 1.0) * \
                (365.0 / ((d2 - d1).days))
```

```
from finmarkets import DiscountCurve

observation_date = date (2020, 1, 1)
t1 = date(2020,4, 1)
t2 = date(2020, 10, 1)

# for EONIA
pillar_dates_eonia = [date(2020 , 1 ,1),
                      date(2021, 1, 1),
                      date(2022, 10 ,1)]
discount_factors_eonia = [1.0, 0.97, 0.72]

# for LIBOR 6M
pillar_dates_libor = [date(2020, 1 ,1),
                      date(2020, 6, 1),
                      date(2020, 12 ,1)]
discount_factors_libor = [1.0, 0.95, 0.90]


eonia_curve = DiscountCurve(observation_date,
                            pillar_dates_eonia,
                            discount_factors_eonia)
libor_curve = DiscountCurve(observation_date,
                            pillar_dates_libor,
                            discount_factors_libor)


npv = eonia_curve.df(t1) * libor_curve.forward_rate(t1, t2)

# Compute it in the pre-2008 way
npv_pre2008 = libor_curve.df(t1) * libor_curve.forward_rate(t1, t2)

print ("NPV post 2008:", npv)
print ("NPV pre 2008:", npv_pre2008)

NPV post 2008: 0.11533243116069992
NPV pre 2008: 0.11269481011359303
```

Exercise 6.4 (Forward Rate Curve)

Write a ForwardRateCurve class (for EURIBOR/LIBOR rate curve) which doesn't compute discount factors but only interplatates forward rates; then add it to the `finmarkets` module (this function is used to define the LIBOR curve needed throughout future lessons).

Solution 6.4

In this case it is enough to write a new `class` that has three attributes: a today date, a set of pillar_dates and the corresponding rates. There will be just a single method `forward_rate` which returns the corresponding interpolated rate.

```
import numpy

# an EURIBOR or LIBOR rate curve
# doesn't calculate discount factors, only interpolates forward rates
class ForwardRateCurve(object):

   # the special __init__ method defines how to
   # construct instances of the class
   def __init__(self, pillar_dates, rates):

       # we just store the arguments as attributes of the instance
       self.today = pillar_dates[0]
       self.rates = rates

       self.pillar_days = [
           (pillar_date - self.today).days
           for pillar_date in pillar_dates
       ]


   # interpolates the forward rates stored in the instance
   def forward_rate(self, d):
       d_days = (d - self.today).days
       return numpy.interp(d_days, self.pillar_days, self.rates)
```