Laws of biological growth?

Complex Systems

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This assignment should be returned as a commented Jupyter notebook (.ipynb file). Go to `http://jupyter.org` to install Jupyter or run it in your browser (`https://try.jupyter.org`).

The datasets are on my GitHub page, at `https://github.com/msmerlak/complex-systems-course/`

If your response happens to be inspired by online content, say it clearly and show us that you actually used your brain in the process.

Exercise 1. The dataset growth_scaling.csv contains information about the growth dynamics of 1500+ living species, both prokaryotes and eukaryotes.

1. Identify a mathematical function $m(t)$ which could plausibly represent the growth curve of a living being (i.e. its mass as a function of time from birth to death/replication) and plot it, with labelled axes, plot label, etc — make it look nice. Explain why your choice of $m(t)$ is sensible and what approximations underlies it, if any.
2. Compute within your chosen model (analytically if you can, else numerically) the time t^* when the organism grows fastest, i.e. when dm/dt reaches its maximum. Add this point on your plot of $m(t)$.
3. In the dataset, the value x_g corresponds to measured values of $m(t^*)$ (in grams) and y_g to $dm/dt(t^*)$ (in grams per year) respectively. What relationship do you expect to hold between x_g and y_g?
4. Test this assumption with a suitable regression (e.g. from scipy.optimize import least_squares).
5. Plot the data again, this time on log-log axes. Does a pattern emerge?
6. Explain how a power-law relationship of the form $y = ax^k$ looks on log-log axes.
7. Conclude: what is the relationship between growth and mass at maximum growth across the tree of life? Estimate the relevant parameters by a suitable fitting procedure.

Exercise 2. The dataset growth_curves.csv contains empirically measured growth curves $m(t)$ for 13 different animals.

1. How similar do you expect these growth curves to be, given the species chosen?
2. Plot all 13 curves (as always with labelled axes etc.). How similar are they?
3. It has been argued that the equation
$$\frac{dm}{dt} = am^{3/4} - bm$$
provides a good model of the growth dynamics of animals. Solve this differential equation. How many free parameters does the solution have?
4. Estimate these parameters for each species using a suitable fitting procedure.

5. Compute the limiting value $M = \lim_{t\to\infty} m(t)$ as a function of the parameters. Estimate M for each species.
6. Add the following two columns to the dataset: $r = (m/M)^{1/4}$ and $\tau = (at/4M^{1/4}) - \ln[1 - (m_0/M)^{1/4}]$. Plot r vs. τ for all species on a single plot and comment on the result.

Exercise 3. Do the results presented in this assignment qualify as “laws of complex systems"? Share your thoughts.