UNIVERSITÀ
DEGLI STUDI
FIRENZE

DINFO
DIPARTIMENTO DI
INGEGNERIA
DELL'INFORMAZIONE

Corso di Dottorato in Ingegneria dell'Informazione
Curriculum: Automatica, Ottimizzazione e Sistemi Complessi

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Bayesian methods for Extended Object Tracking

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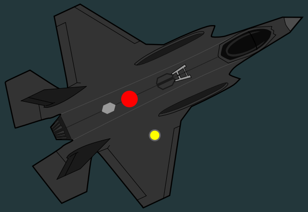
Outline

Introduction	problem definition, state of the art and motivation
Part 1	tracking for maneuvering objects
Part 2	tracking for elliptical objects
Part 3	tracking for general objects and simulation
Conclusions	summary and outlook

Introduction

Problem definition



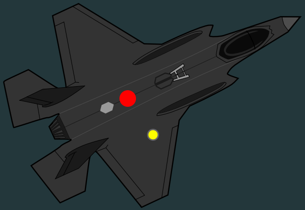
Point Object Tracking

single point ⟶ position

Extended Object Tracking

point cloud ⟶ position
point cloud ⟶ orientation
point cloud ⟶ shape

Problem definition



Point Object Tracking

single point $\longrightarrow$ position

Extended Object Tracking

point cloud $\longrightarrow$ position, orientation, shape

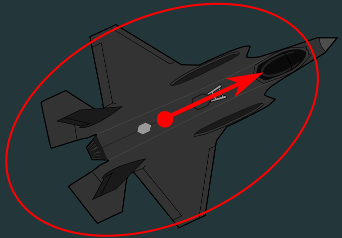
Extended Object Tracking (EOT) problem

Given the time sequence of point clouds $\mathcal{Y}_1, \ldots, \mathcal{Y}_k$, estimate in a Bayesian fashion the state x_k of the extended object (including position, orientation and shape).

Random Matrix Model [Koch, 2008]

characteristic equation

$$z(s,\theta) \triangleq p + s\,U(h) \begin{bmatrix} \ell_1 \cos\theta \\ \ell_2 \sin\theta \end{bmatrix}$$



parameters

position	$p \in \mathbb{R}^2$
heading	$h \in [-\pi, \pi)$
semi-length	$\ell_1 \in \mathbb{R}_{>0}$
semi-width	$\ell_2 \in \mathbb{R}_{>0}$

Filters

Gaussian-Inverse-Wishart [Koch et al., 2008]

MEM-EKF* [Yang et al., 2019]

Random Hypersuperface Model [Baum, 2009]

characteristic equation

$$z(s,\theta) \triangleq p + s\,U(h)\,\rho(\theta)$$

parameters

position $p \in \mathbb{R}^2$

heading $h \in [-\pi, \pi)$

radius $\rho(\cdot) \in \mathcal{C}^0([0, 2\pi])$

Filters

Random-Hypersuperfaces [Baum et al., 2014]

Gaussian-Processes [Wahlström et al., 2014]

Motivation

- **(1) Efficient statistics:** rather than process each point in $\mathcal{Y}$, use

$$\overline{y} \triangleq \frac{1}{m}\sum_{j=1}^{m} y^{(j)}$$

sample mean

$$\overline{Y} \triangleq \operatorname{vec}\left[\frac{1}{m-1}\sum_{j=1}^{m}\left(y^{(j)}-\overline{y}\right)\left(y^{(j)}-\overline{y}\right)'\right]$$

sample covariance

(vectorized)

to infer the object's position p, heading h, length $2\ell_1$, width $2\ell_2$.

PRO: cheap computational cost.

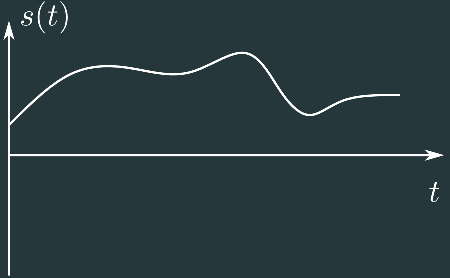
CON: loss of information.

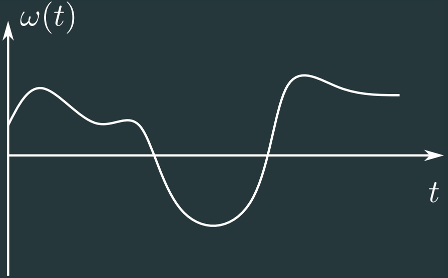
- **(2) Shape classification:** cast shape estimation as a classification problem over a known shape family (**dictionary**).
  PRO: arbitrarily complex shapes can be recognized.
  CON: only known shapes can be recognized.

Tracking for maneuvering object

Tracking for maneuvering object

An object is **maneuvering** iff its speed $s(t)$ and/or turning rate $\omega(t)$ vary in time.





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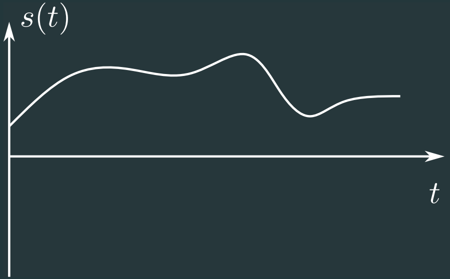

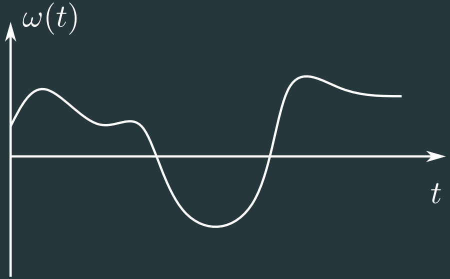

(1) such variables are useful to improve position and heading predictions.

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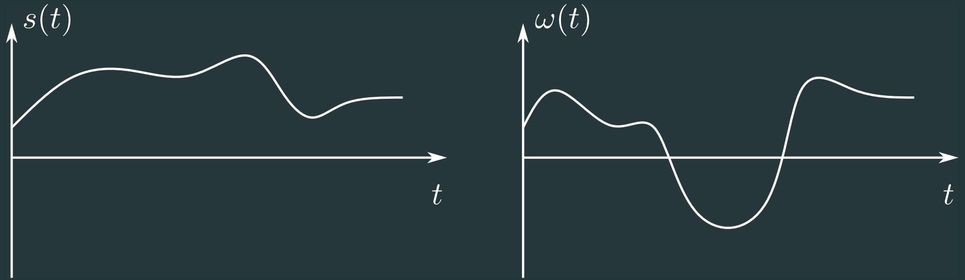

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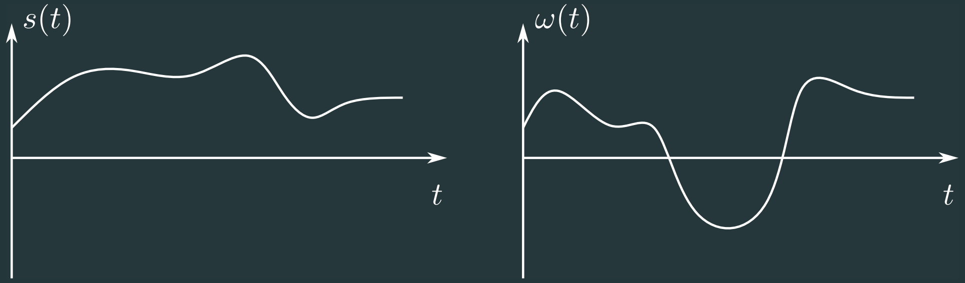

(1) such variables are useful to improve position and heading predictions.

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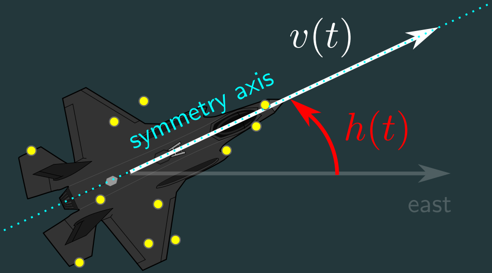
IDEA: define a prediction model to estimate s, ω and their derivatives

Tracking for maneuvering object



$$v(t) \triangleq s(t) \begin{bmatrix} \cos h(t) \\ \sin h(t) \end{bmatrix}$$

motion dynamics
(unicycle)

$$\dot{p}(t) = s(t) \begin{bmatrix} \cos h(t) & \sin h(t) \end{bmatrix}'$$
$$\dot{h}(t) = \omega(t)$$

input dynamics

$$s^{(\Lambda)}(t) \triangleq 0$$
$$\omega^{(O)}(t) \triangleq 0$$

Tracking for maneuvering object

Kinematic state

$$x \triangleq \begin{bmatrix} p' & \ell' \end{bmatrix}' \qquad \ell \triangleq \begin{bmatrix} h & s & \cdots & s^{(\Lambda-1)} & \omega & \cdots & \omega^{(O-1)} \end{bmatrix}'$$

Tracking for maneuvering object

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Dynamics discretization

$$\begin{aligned} \dot{p}(t) &= f(\ell(t)) \\ \dot{\ell}(t) &= A\ell(t) \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} p_{k+1} &= p_k + \int_{kT}^{(k+1)T} f(\ell(\tau))\, \mathsf{d}\tau \\ \ell_{k+1} &= \exp(AT)\, \ell_k \end{aligned}$$

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$\Lambda : O$ prediction model

$$\begin{aligned} p_{k+1} &= p_k + T\,\frac{f(\ell_k) + f(\ell_{k+1})}{2} + w_k^p \\ \ell_{k+1} &= \exp(AT)\,\ell_k + w_k^\ell \end{aligned} \qquad w_k \sim \mathcal{N}(0, Q)$$

[Tesori et al., 2022]

Tracking for elliptical objects

Tracking for elliptical objects

Multiplicative error model (MEM) [Baum, 2012]

$$y = p + U(h)\, D(\ell_1, \ell_2)\, q + v \qquad U(h) \triangleq \begin{bmatrix} \cos h & -\sin h \\ \sin h & \cos h \end{bmatrix}$$

$$q \sim \mathcal{N}(0, I/4)$$

$$v \sim \mathcal{N}(0, \sigma_v^2 I) \qquad D(a, b) \triangleq \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

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\end{aligned}
$$

Measurement covariance

$$
\Sigma = U(h)\,D\left(\left(\frac{\ell_1}{\sqrt{2}}\right)^2 + \sigma_v^2, \left(\frac{\ell_2}{\sqrt{2}}\right)^2 + \sigma_v^2\right)U(h)'
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IDEA: estimate h, ℓ_1, ℓ_2 directly from $\overline{Y} \approx \mathrm{vec}\,[\Sigma]$

Tracking for elliptical objects

Augmented $\Lambda : O$ state x and pseudo-measurement $\mathbb{Y}$

$$x \triangleq \begin{bmatrix} p' & h & s & \cdots & s^{(\Lambda-1)} & \omega & \cdots & \omega^{(O-1)} & \ell_1 & \ell_2 \end{bmatrix}'$$

$$\mathbb{Y} \triangleq \begin{bmatrix} \overline{y}' & \overline{Y}' \end{bmatrix}'$$

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Prediction equations (Extended Kalman Predictor)

$$x_{k|k-1} \triangleq F(x_{k-1|k-1})$$
$$P_{k|k-1} \triangleq J\, P_{k-1|k-1}\, J' + Q \qquad J \triangleq \left.\frac{\partial F}{\partial x}\right|_{x_{k-1|k-1}}$$

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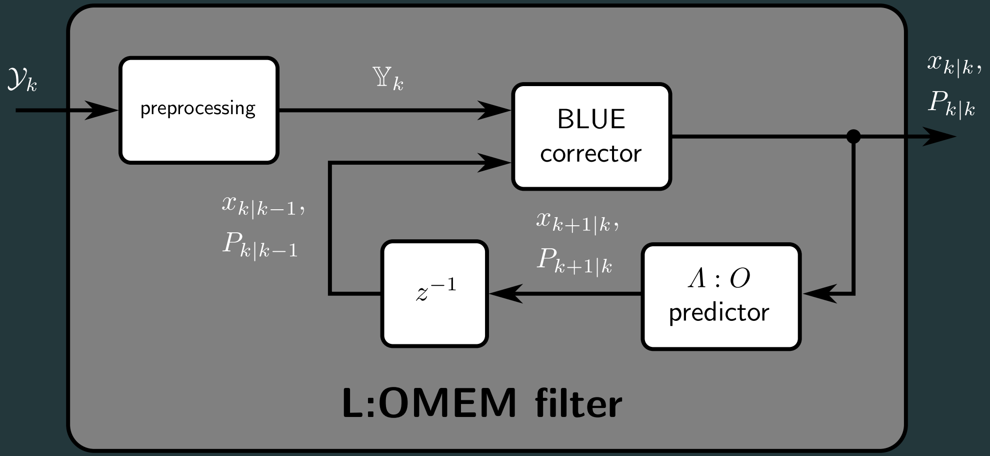
$$P_{k|k-1} \triangleq J\, P_{k-1|k-1}\, J' + Q$$

Correction equations (Best Linear Unbiased Estimator)

$$x_{k|k} \triangleq x_{k|k-1} + \Sigma_{x\mathbb{Y}} \Sigma_{\mathbb{Y}}^{-1} \left(\mathbb{Y}_k - \mathbb{Y}_{k|k-1} \right)$$
$$P_{k|k} \triangleq P_{k|k-1} - \Sigma_{x\mathbb{Y}} \Sigma_{\mathbb{Y}}^{-1} \Sigma_{x\mathbb{Y}}'$$

Tracking for elliptical objects



[Tesori et al., 2023]

Tracking for general objects

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We may know it is a car, ship, or aircraft — but not its specific manufacturing type.

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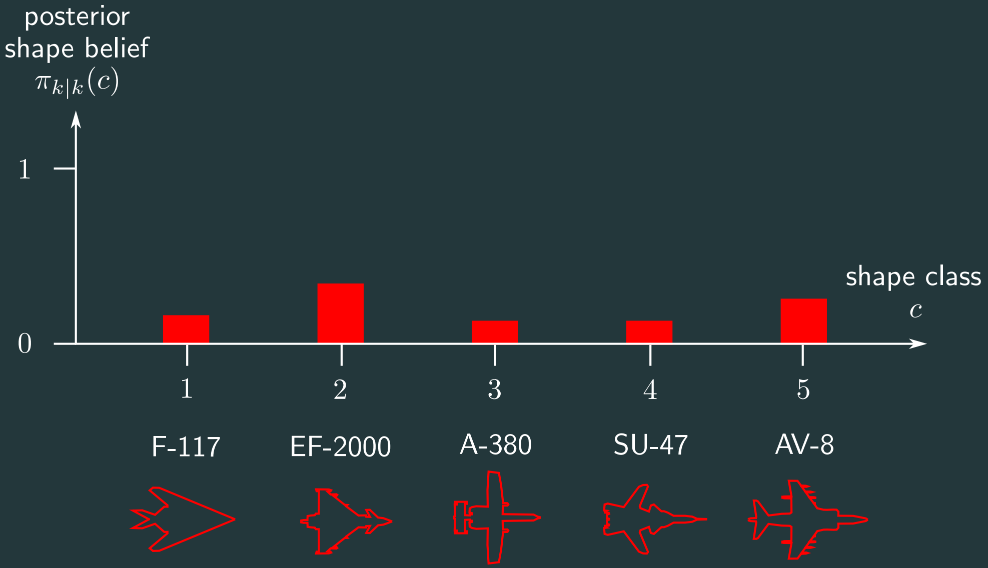
IDEA: tackle shape estimation as a classification problem

Tracking for general objects



Linear Spline Model

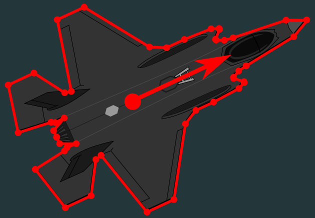
contour equation

$$z(\alpha) \triangleq p + U(h)\, B(\alpha) \widetilde{S}$$

parameters

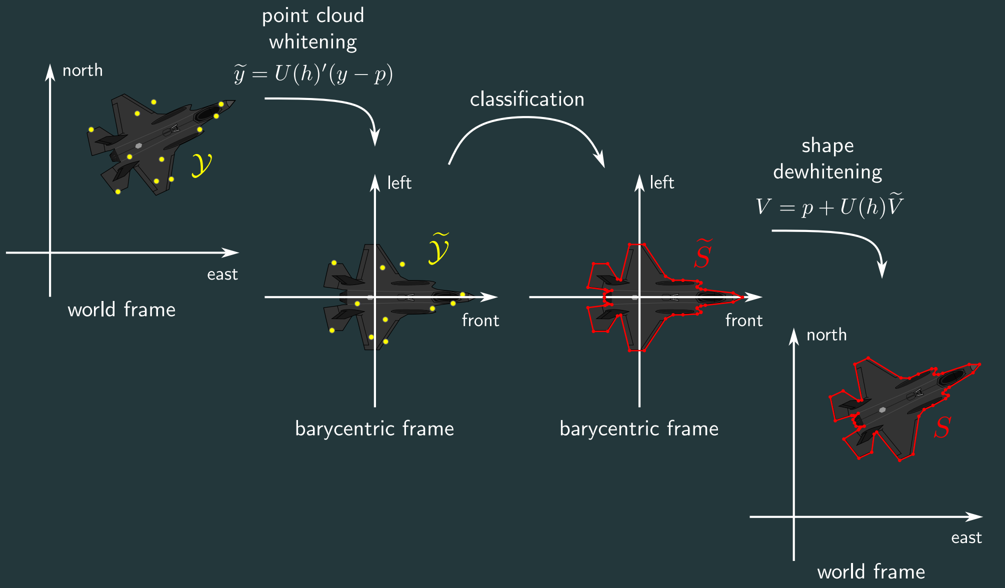
position $\quad p \in \mathbb{R}^2$

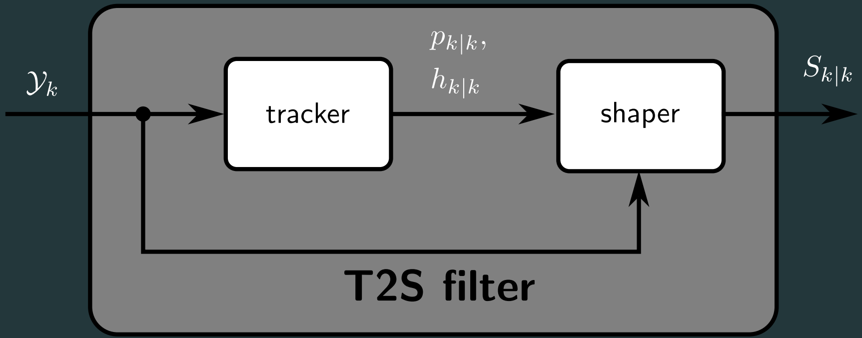
heading $\quad h \in [-\pi, \pi)$



shape vector $\quad \widetilde{S} \triangleq \begin{bmatrix} \widetilde{V}_1' & \cdots & \widetilde{V}_r' \end{bmatrix}' \in \mathbb{R}^{2r}$

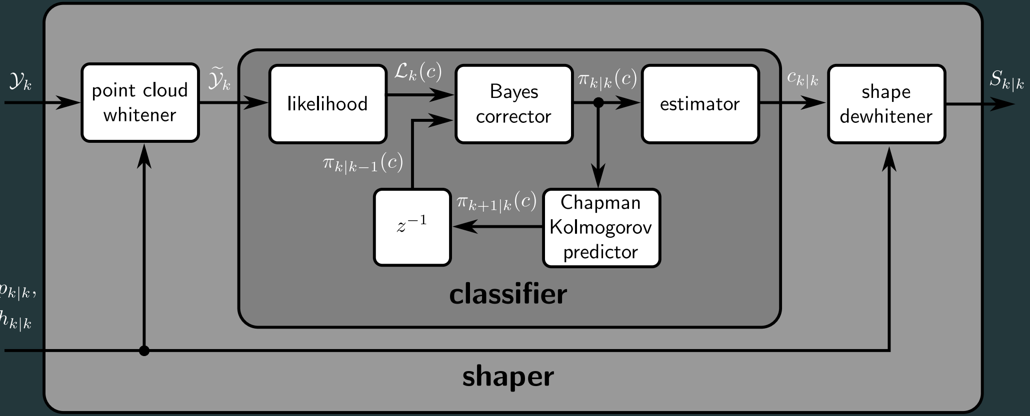
Tracking for general objects





[Tesori et al., 2024]

Tracking for general objects



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Tracking for general objects

Chapman-Kolmogorov predictor

$$\pi_{k|k-1}(c) \triangleq (1-\lambda)\,\pi_{k-1|k-1}(c) + \lambda\, u(c)$$

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Bayes corrector

$$\pi_{k|k}(c) \triangleq \frac{\mathcal{L}\left(\widetilde{\mathcal{Y}}_k | \widetilde{S}^{(c)}\right)\, \pi_{k|k-1}(c)}{\sum_\nu \mathcal{L}\left(\widetilde{\mathcal{Y}}_k | \widetilde{S}^{(\nu)}\right)\, \pi_{k|k-1}(\nu)}$$

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MAP estimator

$$c_{k|k} \triangleq \arg\max_c \pi_{k|k}(c)$$

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MAP estimator

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PROBLEM: suitably define a likelihood function $\mathcal{L}\left(\widetilde{\mathcal{Y}}|\widetilde{S}\right)$

Measurement model (surface)

$$\widetilde{y} \triangleq \widetilde{z} + v$$

$$\widetilde{z} \sim \mathcal{U}(\widetilde{\mathcal{I}}) \qquad \widetilde{\mathcal{I}} \triangleq \text{object surface}$$

$$v \sim \mathcal{N}(0, R)$$

Tracking for general objects

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Single point and point-cloud likelihoods

$$\mathcal{L}_{\mathsf{s}}(\widetilde{y}|\widetilde{S}) \triangleq \int \mathcal{N}(\widetilde{y} - \widetilde{z}; 0, R)\, \mathcal{U}(\widetilde{z}; \widetilde{\mathcal{I}})\ \mathsf{d}\widetilde{z}$$

$$\mathcal{L}_{\mathsf{s}}(\widetilde{\mathcal{Y}}|\widetilde{S}) \triangleq \prod_{j=1}^{m} \mathcal{L}_{\mathsf{s}}\left(\widetilde{y}^{(j)}|\widetilde{S}\right)$$

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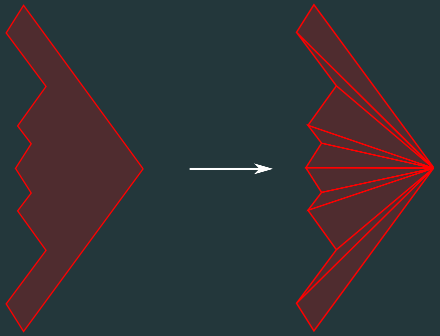
Uniform scattering distribution

$$\mathcal{U}(z; \widetilde{\mathcal{I}}) \triangleq \frac{\mathbb{1}_{\widetilde{\mathcal{I}}}(z)}{\int_{\widetilde{\mathcal{I}}}\ \mathsf{d}\zeta} = \frac{\mathbb{1}_{\widetilde{\mathcal{I}}}(z)}{\text{area}(\widetilde{\mathcal{I}})}$$

Tracking for general objects

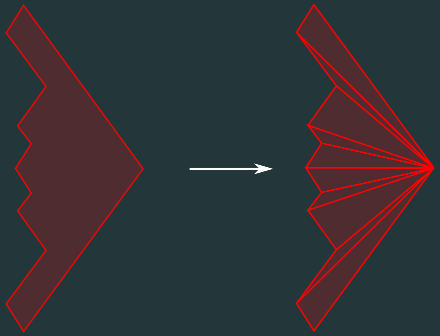


$\widetilde{\mathcal{I}}$

triangular decomposition

$\bigcup_{i=1}^{n} \widetilde{\mathcal{I}}_i$

Tracking for general objects

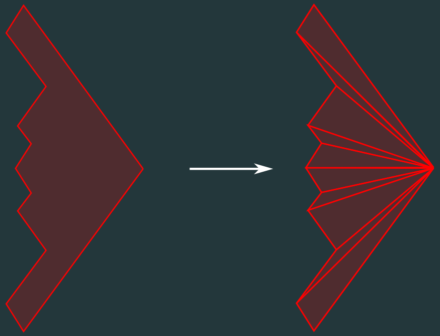


$\widetilde{\mathcal{I}}$ $\longrightarrow$ $\bigcup_{i=1}^{n} \widetilde{\mathcal{I}}_i$

triangular
decomposition

Uniform scattering distribution

$$\mathcal{U}(z;\widetilde{\mathcal{I}}) = \frac{\sum_{i=1}^{n} \mathbb{1}_{\widetilde{\mathcal{I}}_i}(z)}{\mathrm{area}(\widetilde{\mathcal{I}})} = \sum_{i=1}^{n} \underbrace{\frac{\mathrm{area}(\widetilde{\mathcal{I}}_i)}{\mathrm{area}(\widetilde{\mathcal{I}})}}_{\triangleq w_{\mathrm{s},i}} \underbrace{\frac{\mathbb{1}_{\widetilde{\mathcal{I}}_i}(z)}{\mathrm{area}(\widetilde{\mathcal{I}}_i)}}_{=\mathcal{U}(z;\widetilde{\mathcal{I}}_i)}$$



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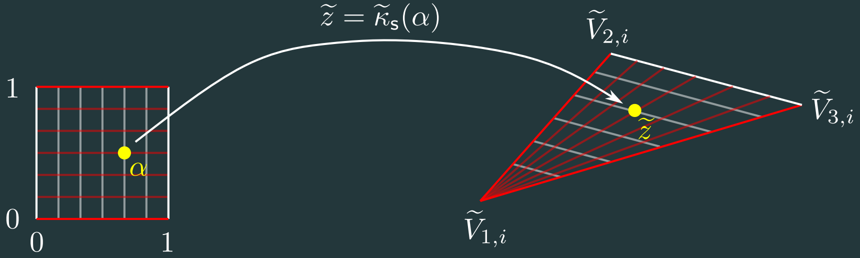
Single point likelihood

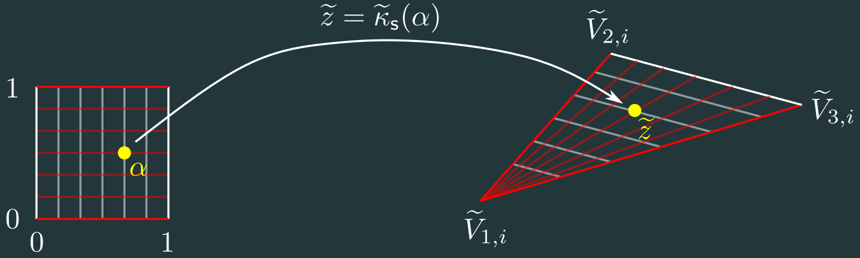
$$\mathcal{L}_{\mathsf{s}}(\widetilde{y}|\widetilde{S}) = \sum_{i=1}^{n} w_{\mathsf{s},i} \underbrace{\int \mathcal{N}(\widetilde{y} - \widetilde{z}; 0, R)\, \mathcal{U}(\widetilde{z};\widetilde{\mathcal{I}}_i)\, \mathsf{d}\widetilde{z}}_{\triangleq \mathcal{L}_{\mathsf{s},i}(\widetilde{y};R)}$$

Tracking for general objects



$$\widetilde{\kappa}_{\mathsf{s}}(\alpha) \triangleq (1-\sqrt{\alpha_1})\,\widetilde{V}_{1,i} + \sqrt{\alpha_1}(1-\alpha_2)\widetilde{V}_{2,i} + \sqrt{\alpha_1}\alpha_2\widetilde{V}_{3,i}$$

Tracking for general objects



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$$\begin{aligned}\mathcal{L}_{\mathsf{s},i}(\widetilde{y};R) &= \int_{[0,1]^2} \mathcal{N}(\widetilde{y}-\widetilde{\kappa}_{\mathsf{s}}(\alpha);0,R)\ \mathsf{d}\alpha \\ &\approx \frac{1}{N_i}\sum_{k=1}^{N_i}\mathcal{N}\left(\widetilde{y}-\widetilde{\kappa}_{\mathsf{s}}(\alpha^{(k)});0,R\right) \qquad \alpha^{(k)} \sim \mathcal{U}([0,1]^2)\end{aligned}$$

Measurement model (contour)

$$\widetilde{y} \triangleq \widetilde{z} + v$$

$$\widetilde{z} \sim \mathcal{U}(\partial\widetilde{\mathcal{I}}) \qquad \partial\widetilde{\mathcal{I}} \triangleq \text{object contour}$$

$$v \sim \mathcal{N}(0, R)$$

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Measurement model (contour)

$$\widetilde{y} \triangleq \widetilde{z} + v$$

$$\widetilde{z} \sim \mathcal{U}(\partial\widetilde{\mathcal{I}}) \qquad \partial\widetilde{\mathcal{I}} \triangleq \text{object contour}$$

$$v \sim \mathcal{N}(0, R)$$

Single point and point-cloud likelihoods

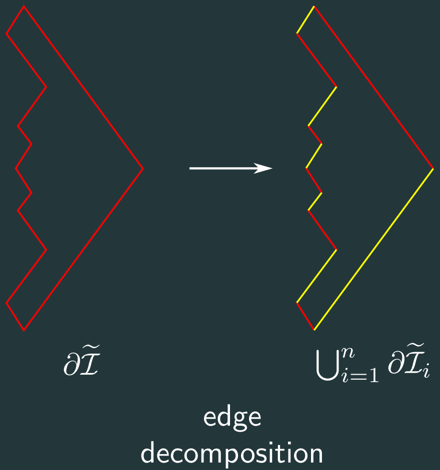
$$\mathcal{L}_\mathsf{c}(\widetilde{y}|\widetilde{S}) \triangleq \int \mathcal{N}(\widetilde{y} - \widetilde{z}; 0, R)\ \mathcal{U}(\widetilde{z}; \partial\widetilde{\mathcal{I}})\ \mathsf{d}\widetilde{z}$$

$$\mathcal{L}_\mathsf{c}(\widetilde{\mathcal{Y}}|\widetilde{S}) \triangleq \prod_{j=1}^{m} \mathcal{L}_\mathsf{c}\left(\widetilde{y}^{(j)}|\widetilde{S}\right)$$

Uniform scattering distribution

$$\mathcal{U}(z; \partial\widetilde{\mathcal{I}}) \triangleq \frac{\mathbb{1}_{\partial\widetilde{\mathcal{I}}}(z)}{\int_{\partial\widetilde{\mathcal{I}}} \mathsf{d}\zeta} \triangleq \frac{\mathbb{1}_{\partial\widetilde{\mathcal{I}}}(z)}{\mathrm{length}(\partial\widetilde{\mathcal{I}})}$$

Tracking for general objects



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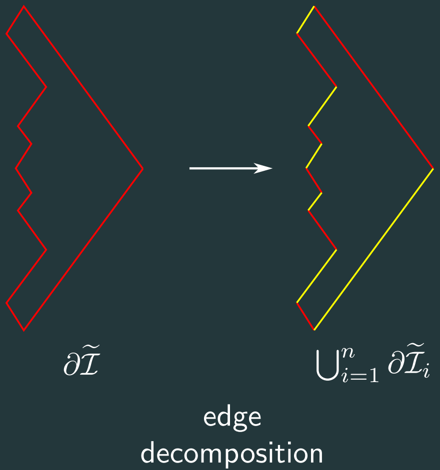

Uniform scattering distribution

$$\mathcal{U}(z;\partial\widetilde{\mathcal{I}}) = \sum_{i=1}^{n} \underbrace{\frac{\mathrm{length}(\partial\widetilde{\mathcal{I}}_i)}{\mathrm{length}(\partial\widetilde{\mathcal{I}})}}_{\triangleq w_{\mathsf{c},i}} \underbrace{\frac{\mathbb{1}_{\partial\widetilde{\mathcal{I}}_i}(z)}{\mathrm{length}(\partial\widetilde{\mathcal{I}}_i)}}_{=\mathcal{U}(z;\partial\widetilde{\mathcal{I}}_i)}$$

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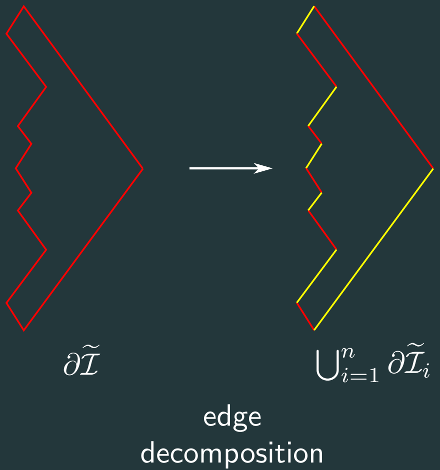

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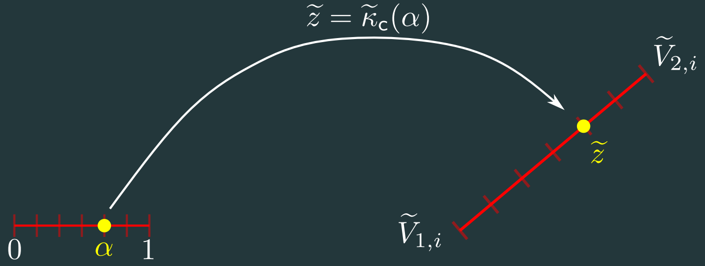
Single point likelihood

$$\mathcal{L}_{\mathsf{c}}(\widetilde{y}|\widetilde{S}) = \sum_{i=1}^{n} w_{\mathsf{c},i} \underbrace{\int \mathcal{N}(\widetilde{y}-\widetilde{z};0,R)\,\mathcal{U}(\widetilde{z};\widetilde{\mathcal{I}}_i)\;\mathsf{d}\widetilde{z}}_{\triangleq \mathcal{L}_{\mathsf{c},i}(\widetilde{y};R)}$$

Tracking for general objects



$$\widetilde{\kappa}_{\mathsf{c}}(\alpha) \triangleq (1-\alpha)\,\widetilde{V}_{1,i} + \alpha\,\widetilde{V}_{2,i}$$

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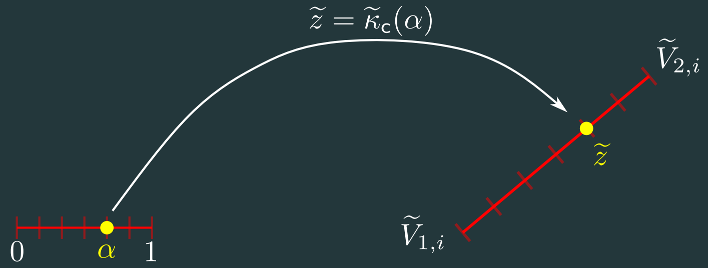

$$\widetilde{\kappa}_{\mathsf{c}}(\alpha) \triangleq (1-\alpha)\,\widetilde{V}_{1,i} + \alpha\,\widetilde{V}_{2,i}$$

$$\begin{aligned} \mathcal{L}_{\mathsf{c},i}(\widetilde{y}; R) &= \int_0^1 \mathcal{N}(\widetilde{y} - \widetilde{\kappa}_{\mathsf{c}}(\alpha); 0, R)\ \mathsf{d}\alpha \\ &= \int_0^1 \mathcal{N}(A_i\alpha + B_i; 0, R)\ \mathsf{d}\alpha \end{aligned}$$

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\end{aligned}
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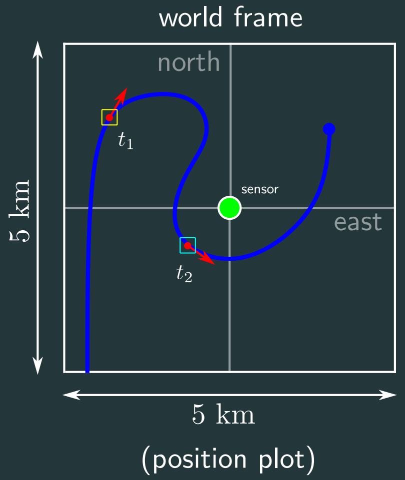
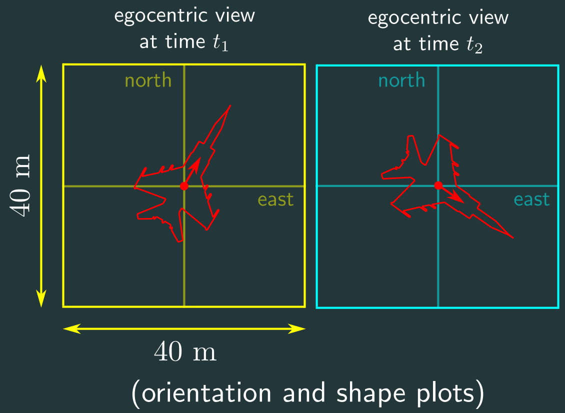
&= C_i(\widetilde{y}; R) \left[\Phi\left(\frac{1 - \mu_i(\widetilde{y}; R)}{\sigma_i(\widetilde{y}; R)}\right) - \Phi\left(-\frac{\mu_i(\widetilde{y}; R)}{\sigma_i(\widetilde{y}; R)}\right)\right]
\end{aligned}
$$

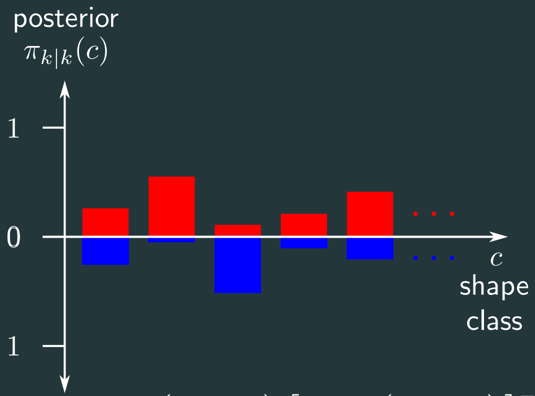
[Tesori et al., 2024]

Simulation



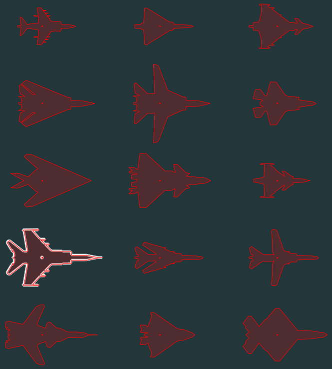


Ground truth representation



$$\ell_k(c) \triangleq \mathcal{L}\left(\widetilde{\mathcal{Y}}_k | \widetilde{S}^{(c)}\right) \left[\sum_\nu \mathcal{L}\left(\widetilde{\mathcal{Y}}_k | \widetilde{S}^{(\nu)}\right)\right]^{-1}$$

normalized likelihood



dictionary
(shapes in scale)

Shape belief representation

Simulation

T2S demo

Conclusions

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Thank you

"The first principle is that you must not fool yourself,
and you are the easiest person to fool."
— *Richard Feynman*