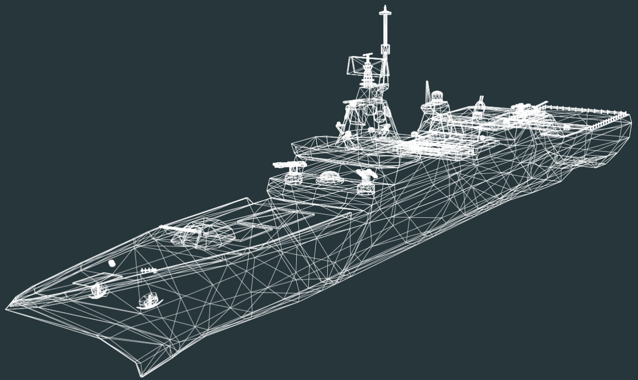

Bayesian methods for Extended Object Tracking

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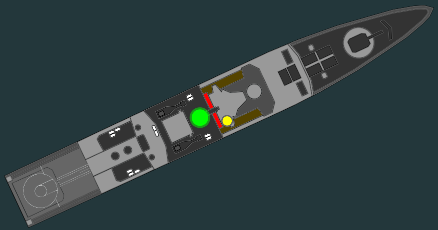
Outline

Introduction	whoami,problem definition, state of the art and motivation
Part 1	tracking for maneuvering objects
Part 2	tracking for elliptical objects
Part 3	tracking for general objects
Conclusions	future research directions

Introduction

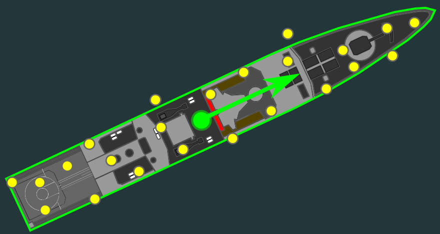
Whomai

Problem definition



Point Object Tracking

single point $\longrightarrow$ position



Extended Object Tracking

point cloud → position, orientation, shape

Extended Object Tracking (EOT) problem

Given the time sequence of point clouds $\mathcal{Y}_1, \ldots, \mathcal{Y}_k$, estimate in a Bayesian fashion the state x_k of the extended object (including position, orientation and shape).

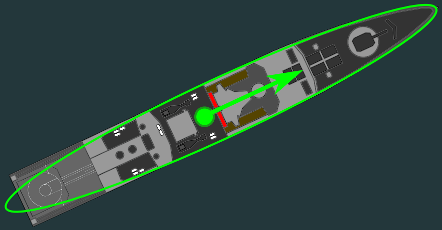
Random Matrix Model [Koch]

characteristic equation

$$z(s, \theta) \triangleq p + s\,U(h) \begin{bmatrix} a \cos\theta \\ b \sin\theta \end{bmatrix}$$

parameters

position	$p \in \mathbb{R}^2$
heading	$h \in [-\pi, \pi)$
semi-length	$a \in \mathbb{R}_{>0}$
semi-width	$b \in \mathbb{R}_{>0}$



Filters

Gaussian-Inverse-Wishart [Koch]

MEM-EKF* [Baum]

State of the art

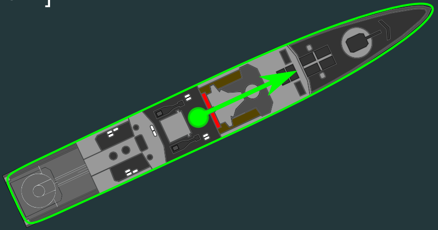
Random Hypersuperface Model [Baum]

characteristic equation

$$z(s,\theta) \triangleq p + s\,U(h)\,\rho(\theta)$$

parameters

position	$p \in \mathbb{R}^2$
heading	$h \in [-\pi, \pi)$
radius	$\rho(\cdot) \in \mathcal{C}^0([0, 2\pi])$



Filters

Fourier UKF [Baum]

Radial Gaussian Processes [Wahlstrom]

Motivation

- **Maneuvering objects:** take advantage of the heading information to improve turning rate estimation in coordinated turn models (and their generalizations).
  PRO: discard interacting multiple models in prediction.
  CON: fragile to measurement noise.

Motivation

- **Efficient statistics:** rather than process each point in $\mathcal{Y} = \{y_1, \ldots, y_m\}$, process

$$\overline{y} \triangleq \frac{1}{m}\sum_{j=1}^{m} y^{(j)}$$

sample mean

$$\overline{Y} \triangleq \frac{1}{m-1}\sum_{j=1}^{m} \left(y^{(j)} - \overline{y}\right)\left(y^{(j)} - \overline{y}\right)'$$

sample covariance

to infer the object's position p, heading h, length $2\ell_1$, width $2\ell_2$.

PRO: cheap computational cost.

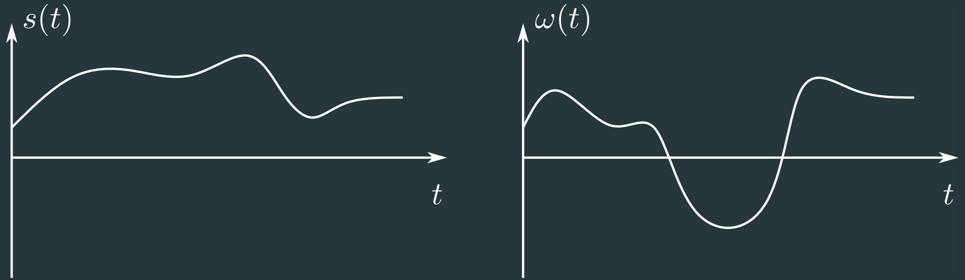
CON: loss of information.

- **Shape classification:** cast shape estimation as a classification problem over a known shape family (**shape library**).
  PRO: arbitrarily complex shapes can be recognized.
  PRO: robustness to occlusion.
  CON: only known shapes can be handled.

Tracking for maneuvering objects

Tracking for maneuvering objects

An object is **maneuvering** iff its speed $s(t)$ and/or turning rate $\omega(t)$ vary in time.



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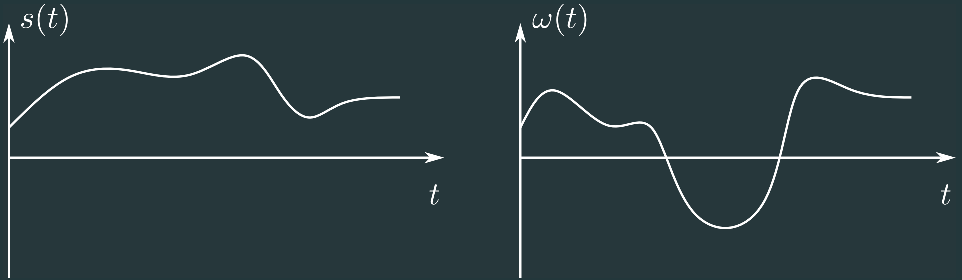

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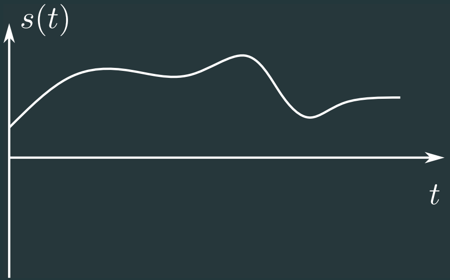

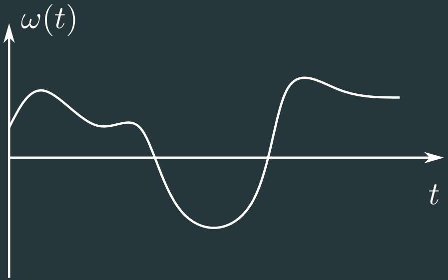

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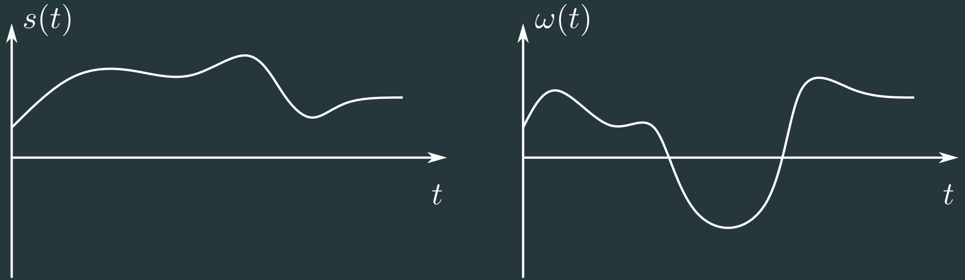

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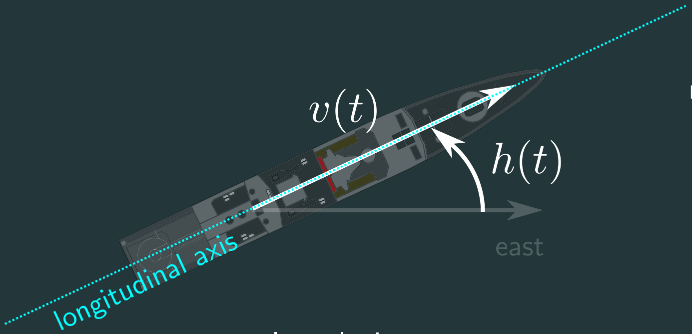
IDEA: define a prediction model to estimate s, ω and their derivatives

Tracking for maneuvering objects



polar velocity

$$v(t) \triangleq s(t) \begin{bmatrix} \cos h(t) \\ \sin h(t) \end{bmatrix}$$

motion dynamics

(unicycle)

$$\dot{p}(t) = v(t)$$

$$\dot{h}(t) = \omega(t)$$

input dynamics

$$s^{(\Lambda)}(t) \triangleq 0$$

$$\omega^{(O)}(t) \triangleq 0$$

Tracking for maneuvering objects

Kinematic state

$$x \triangleq \begin{bmatrix} p' & \ell' \end{bmatrix}' \qquad \ell \triangleq \begin{bmatrix} h & s & \cdots & s^{(\Lambda-1)} & \omega & \cdots & \omega^{(O-1)} \end{bmatrix}'$$

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Dynamics discretization

$$\begin{aligned} \dot{p}(t) &= f(\ell(t)) \\ \dot{\ell}(t) &= A\ell(t) \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} p_k &= p_{k-1} + \int_{(k-1)T}^{kT} v(\ell(\tau))\,\mathsf{d}\tau \\ \ell_k &= \exp(AT)\,\ell_{k-1} \end{aligned}$$

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$\mathit{\Lambda} : O$ prediction model

$$\begin{aligned} p_k &= p_{k-1} + T\, \frac{v(\ell_{k-1}) + v(\ell_k)}{2} + w_k^p \\ \ell_k &= \exp(AT)\, \ell_{k-1} + w_k^\ell \end{aligned} \qquad w_k \sim \mathcal{N}(0, Q)$$

$\Lambda : O$ predictor

$$x_{k|k-1} = \overline{f}_{k|k-1}$$
$$P_{k|k-1} = F_{k|k-1} + Q$$

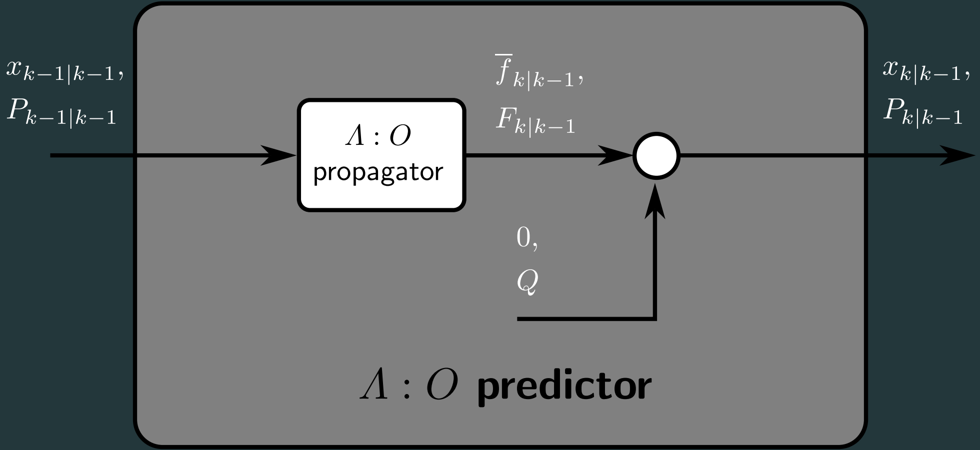
where

$$f(x) \triangleq \left[\; p + T\frac{v(\ell)+v(\exp(AT)\ell)}{2}; \quad \exp(AT)\ell \;\right]$$
$$\overline{f}_{k|k-1} \triangleq \int f(x)\,\mathcal{N}(x; x_{k-1|k-1}, P_{k-1|k-1})\;\mathrm{d}x$$
$$F_{k|k-1} \triangleq \int \left(f(x) - \overline{f}_{k|k-1}\right)\left(f(x) - \overline{f}_{k|k-1}\right)' \mathcal{N}(x; x_{k-1|k-1}, P_{k-1|k-1})\;\mathrm{d}x$$

and the integrals can be computed via:

- linearization (EKF);
- Gaussian quadrature (e.g., UKF, CKF);
- or any other integration method (Grid integration, Importance Sampling, etc...).

Tracking for maneuvering objects



Tracking for elliptical objects

Tracking for elliptical objects

Multiplicative error model (MEM) [Baum]

$$
\begin{aligned}
y &= p + U(h)\, D(e)\, q + v \\
q &\sim \mathcal{N}(0, I/k) \\
v &\sim \mathcal{N}(0, \sigma_v^2 I)
\end{aligned}
\qquad
\begin{aligned}
U(h) &\triangleq \begin{bmatrix} \cos h & -\sin h \\ \sin h & \cos h \end{bmatrix} \\
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Measurement distribution (given p, h, ℓ)

$$y \sim \mathcal{N}\left(p, \quad U(h)\, D\left(\left[\frac{a^2}{k} + \sigma_v^2; \qquad \frac{b^2}{k} + \sigma_v^2\right]\right) U(h)'\right)$$

- $k = 2$ if we model a point cloud distributed over the object contour;
- $k = 4$ if we model a point cloud distributed over the object surface.

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- $k = 4$ if we model a point cloud distributed over the object surface.

IDEA: estimate p, h, ℓ *directly* from $\overline{y} \approx p$, $\overline{Y} \approx \Sigma_y$

QUESTION: what does it mean *directly*?

ANSWER: we define the pseudo-measurement, called *static estimate*, as

$$\mathbb{Y} \triangleq \begin{bmatrix} \hat{p}' & \hat{h} & \hat{e}' \end{bmatrix}'$$

and its covariance

$$\Sigma_{\mathbb{Y}} \triangleq \begin{bmatrix} \Sigma_{\hat{p}} & \Sigma_{\hat{p}\hat{h}} & \Sigma_{\hat{p}\hat{e}} \\ * & \Sigma_{\hat{h}} & \Sigma_{\hat{h}\hat{e}} \\ * & * & \Sigma_{\hat{e}} \end{bmatrix}$$

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Then we perform a *one-shot* (!!!) Kalman correction based on $\overline{Y}$.

NOTE: mixed terms $\Sigma_{\hat{p}\hat{h}}$, $\Sigma_{\hat{p}\hat{e}}$, $\Sigma_{\hat{h}\hat{e}}$ are neglected for simplicity.

Tracking for elliptical objects

How do we compute the static estimates?

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$$\hat{p} \triangleq \overline{y} \qquad \Sigma_{\hat{p}} = \frac{1}{m}\,\overline{Y}$$

$$\hat{h} \triangleq \frac{1}{2}\,\mathrm{atan2}\left(2\overline{Y}_{12}, \overline{Y}_1 - \overline{Y}_2\right) \qquad \Sigma_{\hat{h}} = \frac{1}{m-1}\frac{\lambda_1\,\lambda_2}{(\lambda_1-\lambda_2)^2}$$

$$\hat{e} \triangleq \sqrt{k}\begin{bmatrix}\sqrt{\lambda_1-\sigma_v^2}\\ \sqrt{\lambda_2-\sigma_v^2}\end{bmatrix} \qquad \Sigma_{\hat{e}} = \frac{1}{m-1}\frac{k}{2}\begin{bmatrix}\frac{\lambda_1^2}{\lambda_1-\sigma_v^2} & \frac{\lambda_1\lambda_2}{2\sqrt{(\lambda_1-\sigma_v^2)(\lambda_2-\sigma_v^2)}}\\ * & \frac{\lambda_2^2}{\lambda_2-\sigma_v^2}\end{bmatrix}$$

where λ_1, λ_2 are the eigenvalues of $\overline{Y} = [\overline{Y}_1, \overline{Y}_{12}; *, \overline{Y}_2]$ and m is the cloud cardinality. $\Sigma_{\hat{h}}$, $\Sigma_{\hat{e}}$ are obtained via first-order error propagation, i.e.

$$\Sigma_\chi = \frac{\partial\chi}{\partial\mathrm{vec}\overline{Y}}\,\Sigma_{\mathrm{vec}\overline{Y}}\left(\frac{\partial\chi}{\partial\mathrm{vec}\overline{Y}}\right)' \qquad \chi = \hat{h}, \hat{e}$$

Tracking for elliptical objects

Recall

$$\Sigma_{\hat{h}} = \frac{1}{m-1}\frac{\lambda_1\,\lambda_2}{(\lambda_1-\lambda_2)^2} \qquad \Sigma_{\hat{\ell}} = \frac{1}{m-1}\frac{k}{2}\begin{bmatrix} \frac{\lambda_1^2}{\lambda_1-\sigma_v^2} & \frac{\lambda_1\lambda_2}{2\sqrt{(\lambda_1-\sigma_v^2)(\lambda_2-\sigma_v^2)}} \\ * & \frac{\lambda_2^2}{\lambda_2-\sigma_v^2} \end{bmatrix}$$

Tracking for elliptical objects

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Implicit assumptions in Extended Object Tracking:

$$\lambda_1 \gg \lambda_2 \gg \sigma_v^2$$

We can consider the margins $\lambda_1-\lambda_2$, $|\lambda_2-\sigma_v^2|$ as **quality indicators** of the Signal-to-Noise Ratio (SNR) characterizing the point cloud.

Tracking for elliptical objects

Augmented $\mathit{\Lambda} : O$ state x and static estimate $\mathbb{Y}$

$$x \triangleq \begin{bmatrix} p' & \ell' \end{bmatrix}' \qquad \ell \triangleq \begin{bmatrix} h & s & \cdots & s^{(\Lambda-1)} & \omega & \cdots & \omega^{(O-1)} & e' \end{bmatrix}'$$

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Prediction equations (random walk for e)

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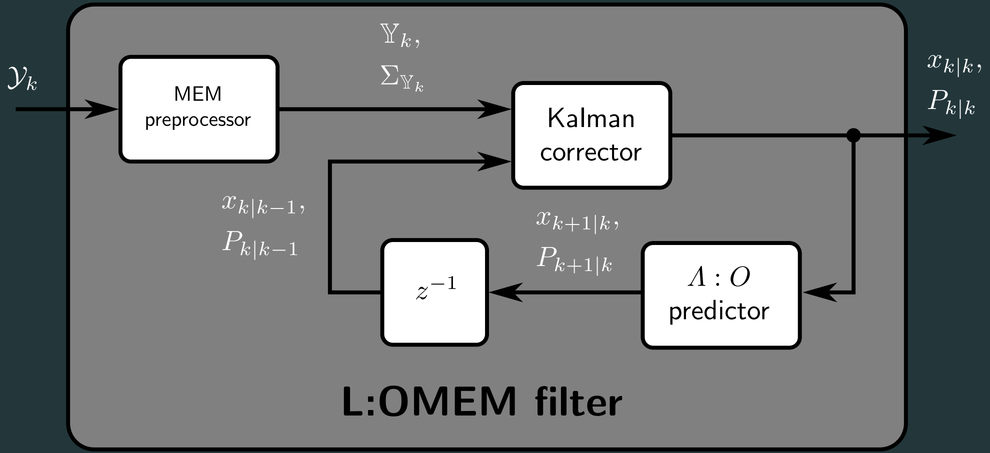
Correction equations

$$L_k = P_{k|k-1} H' \left(H P_{k|k-1} H' + \Sigma_{\mathbb{Y}_k} \right)^{-1}$$

$$x_{k|k} = (I - L_k H) x_{k|k-1} + L_k \, \mathbb{Y}_k$$

$$P_{k|k} = (I - L_k H) P_{k|k-1}$$

Tracking for elliptical objects



Tracking for elliptical objects

Preprocessing has 2 main advantages over conventioal approaches:

- **computational efficiency:** instead of processing m points sequentially ($\mathcal{O}(m)$) or processing a single stack of m points ($\mathcal{O}(m^3)$), preprocessing reduces the correction to $\mathcal{O}(1)$;

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- **computational efficiency:** instead of processing m points sequentially ($\mathcal{O}(m)$) or processing a single stack of m points ($\mathcal{O}(m^3)$), preprocessing reduces the correction to $\mathcal{O}(1)$;
- **white box correction:** the static estimate $\mathbb{Y}_k$ is a subset of the object state x_k and not a a nonlinear function $h(x)$ of it (as in RMM and RHM). Hence, we have "maximum correlation" $\Sigma_{x\mathbb{Y}}$ between observation $\mathbb{Y}_k$ and state x_k.

Tracking for general objects

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Elliptic models are great for several reasons:

- Easy to implement and, more importantly, computationally cheap;
- Allows for closed-form Bayesian updates
  (+ simple multi-object, multi-sensor extensions);
- They can classify objects with well-distinguished extensions.

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- Easy to implement and, more importantly, computationally cheap;
- Allows for closed-form Bayesian updates
  (+ simple multi-object, multi-sensor extensions);
- They can classify objects with well-distinguished extensions.

However, in some scenarios they are deemed to fail:

- When we have to distinguish objects with similar extensions;
- When we have to deal with **occlusions**.

Tracking for general objects

TODO: Elliptic fails picture

Tracking for general objects

QUESTION: how do we overcome the limitations of elliptic models?

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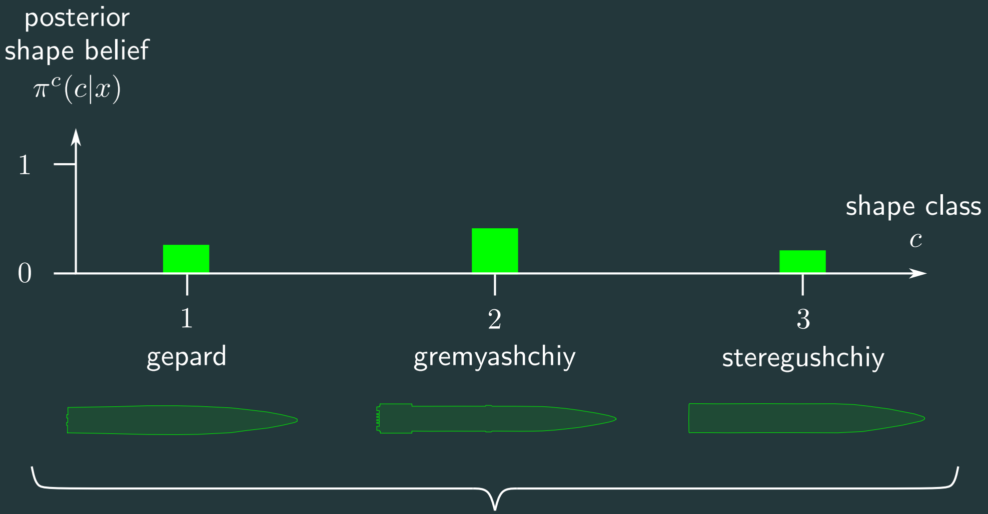
Assumption: we have at disposal a **shape library** of C known "shapes" $c = 1, .., C$.

Tracking for general objects



shape library (e.g. Russian navy from Wikipedia and blueprints)

Tracking for general objects

Why not using Random Hypersurface Models?

Tracking for general objects

Why not using Random Hypersurface Models?

- RHMs handle only star-convex shapes.
- RHM-based filters employ Kalman filters (EKFs, UKFs) including in the state vector n Fourier coefficients, or n radius points, or n vertex positions.

$$\text{RHM regression:} \quad \mathcal{O}(n^3)$$

- Typically, we use the estimated shape by RHM filters to classify tracked objects. Why not directly perform classification?

Tracking for general objects

Hybrid L:OMEM state

$$\boldsymbol{x} \triangleq \begin{bmatrix} x' & c \end{bmatrix}' \qquad x \triangleq \begin{bmatrix} p' & h & s & \cdots & s^{(\Lambda-1)} & \omega & \cdots & \omega^{(O-1)} & e' \end{bmatrix}'$$

$$c \in \{1, \ldots, C\}$$

Joint tracking and classification belief

$$\pi(\boldsymbol{x}) \triangleq \pi(x, c) = \underbrace{\pi^x(x)}_{\text{kinematic belief}} \underbrace{\pi^c(c|x)}_{\text{shape belief}}$$

[1] not necessarily L:OMEM

Hybrid L:OMEM state

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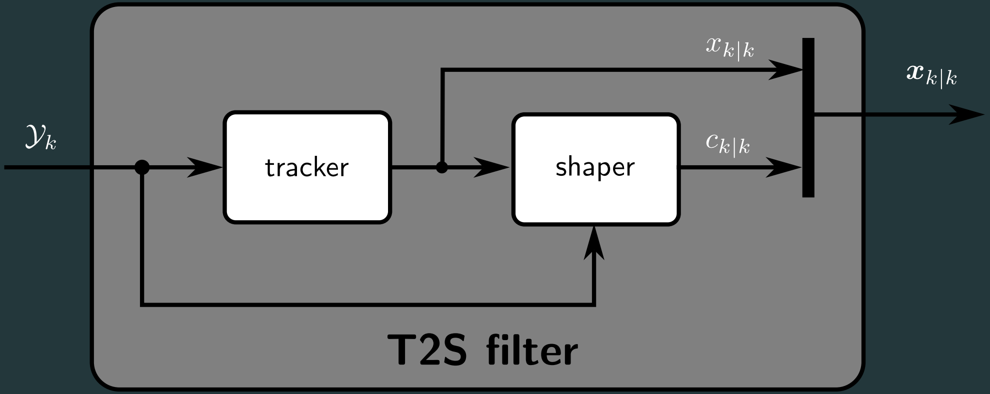
Track-to-Shape (T2S) filter

- employs a *tracker*[1] to update $\pi^x(x)$ according to data;
- employs a *shaper* to update $\pi^c(c|x)$ according to data .

[1] not necessarily L:OMEM

Tracking for general objects



Tracking for general objects

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A reasonable definition of "shape" should uniquely depend on the object geometry.

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A reasonable definition of "shape" should uniquely depend on the object geometry. We look for a definition that is:

- invariant to translation;
- invariant to rotation;
- invariant to scale.

and generalizes the elliptic model and the RHM model.

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We look for a definition that is:

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and generalizes the elliptic model and the RHM model.

Accordingly, we define the *object shape* as a closed and non self-intersecting polygon contained in the unit square $[-0.5, +0.5]^2$.
Such polygon is defined by a **shape vector** $\widetilde{S}$ stacking the vertex coordinates.

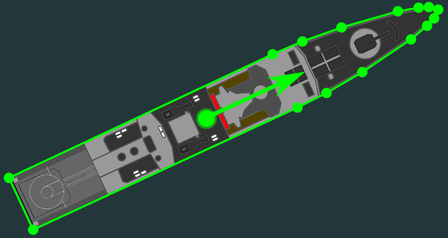
Linear Spline Model

contour equation

$$z(\alpha) \triangleq p + U(h)\, D(e)\, B(\alpha)\widetilde{S}$$

parameters

position	$p \in \mathbb{R}^2$
heading	$h \in [-\pi, \pi)$
semi-length	$a \in \mathbb{R}_{>0}$
semi-width	$b \in \mathbb{R}_{>0}$



Filters

T2S (Track-to-Shape)

TNS (Track-and-Shape)

shape vector $\widetilde{S} \triangleq \begin{bmatrix} \widetilde{V}_1' & \cdots & \widetilde{V}_r' \end{bmatrix}' \in \mathbb{R}^{2r}$

Tracking for general objects

Since the shape library $\{\widetilde{S}^{(c)}\}_{c=1}^{C}$ is defined over the unit square $[-0.5, +0.5]^2$, we need to **whiten** the measurements before feeding them to the shaper.

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This is an operation based on the output of the tracker

$$\widetilde{\mathcal{Y}} \triangleq \{\widetilde{y}^{(j)}\}_{j=1}^{m} \qquad \widetilde{y}^{(j)} \triangleq \left(U(\hat{h})\, D(\hat{e})\right)^{-1} \left(y^{(j)} - \hat{p}\right)$$

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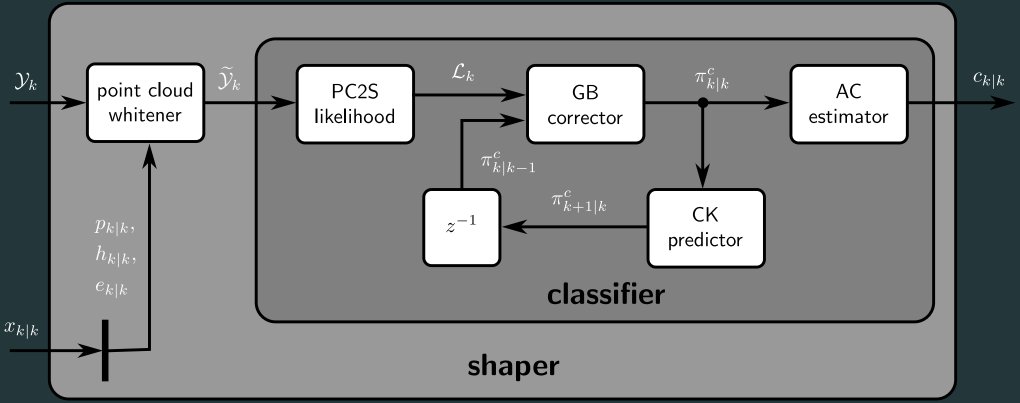
Once whitened, the pointcloud can be compared to the shapes in the library via a **Bayesian classifier**, composed of:

- **(1)** an **Anti-Chattering** (AC) estimator.
- **(2)** a **Chapman-Kolmogorov** (CK) prediction step based on some suitable transition matrix;
- **(3)** a **Generalized Bayesian** (GB) correction step based on some suitable Pointcloud-to-Shape (PC2S) likelihood function;

Tracking for general objects



Tracking for general objects

Notations

$$\pi^c \triangleq \begin{bmatrix} \pi^c(1|x) & \cdots & \pi^c(C|x) \end{bmatrix}'$$

$$\mathcal{L} \triangleq \mathrm{diag}\left(\mathcal{L}(\widetilde{\mathcal{Y}}|\widetilde{S}^{(1)}), \ldots, \mathcal{L}(\widetilde{\mathcal{Y}}|\widetilde{S}^{(C)})\right)$$

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Chapman-Kolmogorov prediction

$$\pi^c_{k|k-1} = \mathcal{T}\, \pi^c_{k-1|k-1}$$

for a suitable transition matrix $\mathcal{T}$.

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Generalized Bayesian correction

$$\pi^c_{k|k} \propto \mathcal{L}_k^{\frac{1}{\tau}}\,\pi^c_{k|k-1}$$

for a suitable temperature parameter $\tau > 0$ and a suitable PC2S likelihood matrix $\mathcal{L}$.

(1) AC estimator

$$\pi^{c}_{k|k} = \mathcal{A}\,\pi^{c}_{k|k-1}$$

this estimator smooths out frequent changes in the Maximum A Posteriori (MAP) class estimate.

(2) Transition matrix

$$\mathcal{T} \triangleq (1-\lambda)\,\mathcal{S} + \lambda\,\mathcal{R}$$

where $\lambda \in (0,1)$ is a forgetting factor and:

- **similarity matrix**

$$[\mathcal{S}]_{ij} \triangleq \mathrm{sim}\left(\widetilde{S}^{(i)}, \widetilde{S}^{(j)}\right)$$

  This term makes the classifier robust against geometric ambiguities between similar shapes. Examples of similarity metrics are complementary Hausdorff distance, chamfer distance, earth mover distance, etc...

- **regularization matrix**

$$[\mathcal{R}]_{ij} \triangleq \frac{1}{C}$$

  This term makes the classifier robust against underflow issues.

(3) PC2S likelihood

$$\mathcal{L}\left(\widetilde{\mathcal{Y}} \,\middle|\, \widetilde{S}^{(c)}\right) \triangleq \mathcal{L}^{\mathrm{C}}\left(|\widetilde{\mathcal{Y}}| \,\middle|\, \widetilde{S}^{(c)}\right) \prod_{\widetilde{y} \in \widetilde{\mathcal{Y}}} \mathcal{L}^{\mathrm{S}}\left(\widetilde{y} \,\middle|\, \widetilde{S}^{(c)}\right)$$

where:

- $\mathcal{L}^{\mathrm{C}}\left(|\widetilde{\mathcal{Y}}| \,\middle|\, \widetilde{S}^{(c)}\right)$ is the **cardinality likelihood**.
  It provides a cheap pre-screening of unlikely shapes based on the number of points in the cloud;
- $\mathcal{L}^{\mathrm{S}}\left(\widetilde{y} \,\middle|\, \widetilde{S}^{(c)}\right)$ is the **spatial likelihood**.
  It provides a deep analysis of the compatibility between each point in the cloud and the shape under test.

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  It provides a deep analysis of the compatibility between each point in the cloud and the shape under test.

Accordingly, the pointcloud $\widetilde{\mathcal{Y}}$ is modeled as an **Independent and Identically Distributed Cluster (IIDC)** Random Finite Set.

Measurement model (surface)

$$\widetilde{y} \triangleq \widetilde{z} + v$$

$$\widetilde{z} \sim \mathcal{U}(\widetilde{\mathcal{I}}) \qquad \widetilde{\mathcal{I}} \triangleq \text{object surface}$$

$$v \sim \mathcal{N}(0, R)$$

Tracking for general objects

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Single point and point-cloud likelihoods

$$\mathcal{L}_{\mathsf{s}}(\widetilde{y}|\widetilde{S}) \triangleq \int \mathcal{N}(\widetilde{y} - \widetilde{z}; 0, R)\, \mathcal{U}(\widetilde{z}; \widetilde{\mathcal{I}})\ \mathsf{d}\widetilde{z}$$

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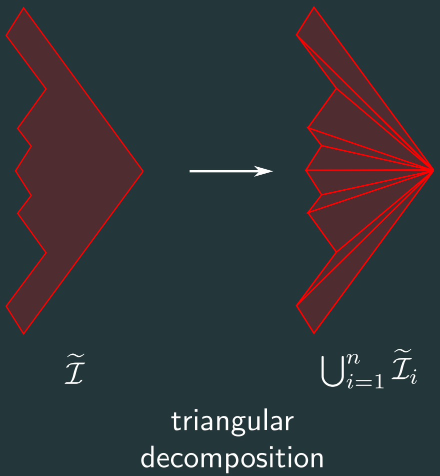
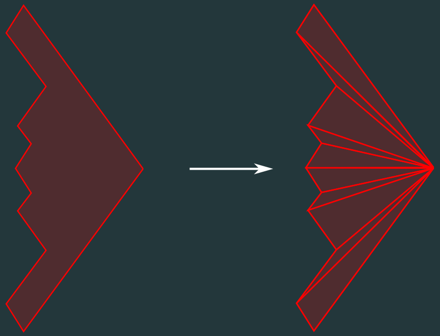
Uniform scattering distribution

$$\mathcal{U}(z; \widetilde{\mathcal{I}}) \triangleq \frac{\mathbb{1}_{\widetilde{\mathcal{I}}}(z)}{\int_{\widetilde{\mathcal{I}}} \;\mathsf{d}\zeta} = \frac{\mathbb{1}_{\widetilde{\mathcal{I}}}(z)}{\mathrm{area}(\widetilde{\mathcal{I}})}$$

Tracking for general objects



Tracking for general objects



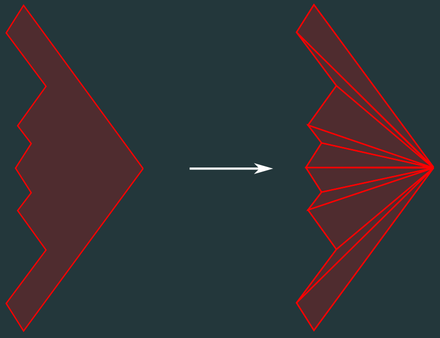
$\widetilde{\mathcal{I}}$ $\qquad$ $\bigcup_{i=1}^{n} \widetilde{\mathcal{I}}_i$

triangular
decomposition

Uniform scattering distribution

$$\mathcal{U}(z;\widetilde{\mathcal{I}}) = \frac{\sum_{i=1}^{n} \mathbb{1}_{\widetilde{\mathcal{I}}_i}(z)}{\mathrm{area}(\widetilde{\mathcal{I}})} = \sum_{i=1}^{n} \underbrace{\frac{\mathrm{area}(\widetilde{\mathcal{I}}_i)}{\mathrm{area}(\widetilde{\mathcal{I}})}}_{\triangleq w_{\mathrm{s},i}} \underbrace{\frac{\mathbb{1}_{\widetilde{\mathcal{I}}_i}(z)}{\mathrm{area}(\widetilde{\mathcal{I}}_i)}}_{=\mathcal{U}(z;\widetilde{\mathcal{I}}_i)}$$

Tracking for general objects



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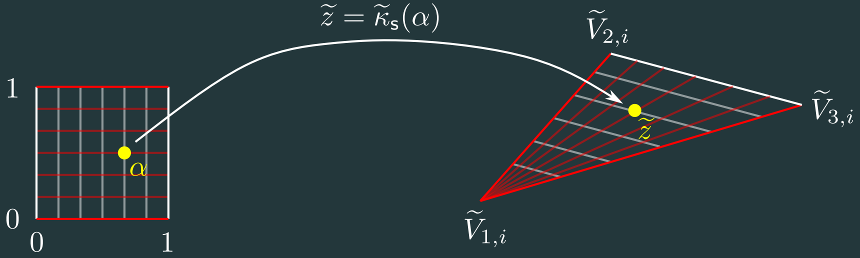
Single point likelihood

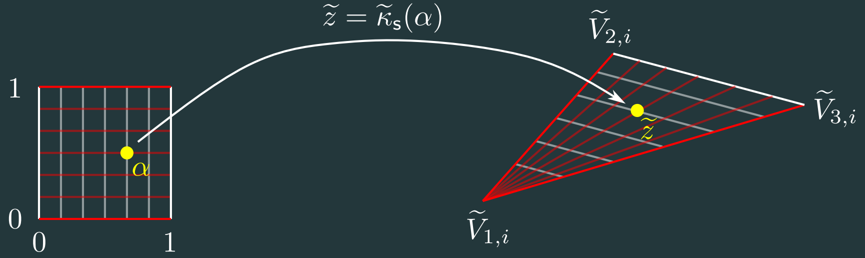
$$\mathcal{L}_{\mathsf{s}}(\widetilde{y}|\widetilde{S}) = \sum_{i=1}^{n} w_{\mathsf{s},i} \underbrace{\int \mathcal{N}(\widetilde{y} - \widetilde{z}; 0, R)\, \mathcal{U}(\widetilde{z}; \widetilde{\mathcal{I}}_i)\ \mathsf{d}\widetilde{z}}_{\triangleq \mathcal{L}_{\mathsf{s},i}(\widetilde{y};R)}$$

Tracking for general objects



$$\widetilde{\kappa}_{\mathsf{s}}(\alpha) \triangleq (1-\sqrt{\alpha_1})\,\widetilde{V}_{1,i} + \sqrt{\alpha_1}(1-\alpha_2)\widetilde{V}_{2,i} + \sqrt{\alpha_1}\alpha_2\widetilde{V}_{3,i}$$

Tracking for general objects



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$$\begin{aligned}\mathcal{L}_{\mathsf{s},i}(\widetilde{y};R) &= \int_{[0,1]^2} \mathcal{N}(\widetilde{y}-\widetilde{\kappa}_{\mathsf{s}}(\alpha);0,R)\ \mathsf{d}\alpha \\ &\approx \frac{1}{N_i}\sum_{k=1}^{N_i}\mathcal{N}\left(\widetilde{y}-\widetilde{\kappa}_{\mathsf{s}}(\alpha^{(k)});0,R\right) \qquad \alpha^{(k)} \sim \mathcal{U}([0,1]^2)\end{aligned}$$

Measurement model (contour)

$$\widetilde{y} \triangleq \widetilde{z} + v$$

$$\widetilde{z} \sim \mathcal{U}(\partial\widetilde{\mathcal{I}}) \qquad \partial\widetilde{\mathcal{I}} \triangleq \text{object contour}$$

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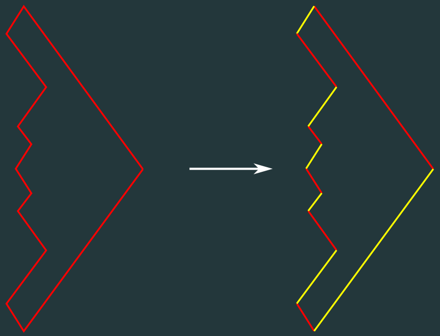
Tracking for general objects

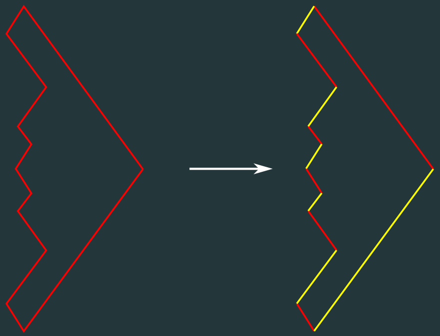


$\partial\widetilde{\mathcal{I}}$

edge decomposition

$\bigcup_{i=1}^{n} \partial\widetilde{\mathcal{I}}_i$

Tracking for general objects



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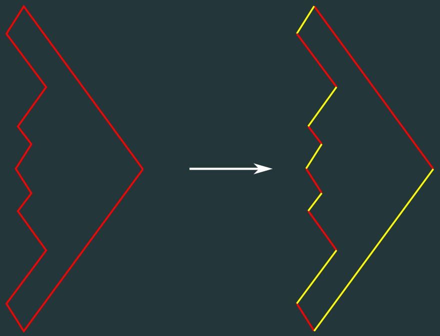
edge decomposition

Uniform scattering distribution

$$\mathcal{U}(z;\partial\widetilde{\mathcal{I}}) = \sum_{i=1}^{n} \underbrace{\frac{\mathrm{length}(\partial\widetilde{\mathcal{I}}_i)}{\mathrm{length}(\partial\widetilde{\mathcal{I}})}}_{\triangleq w_{\mathsf{c},i}} \underbrace{\frac{\mathbb{1}_{\partial\widetilde{\mathcal{I}}_i}(z)}{\mathrm{length}(\partial\widetilde{\mathcal{I}}_i)}}_{=\mathcal{U}(z;\partial\widetilde{\mathcal{I}}_i)}$$

Tracking for general objects



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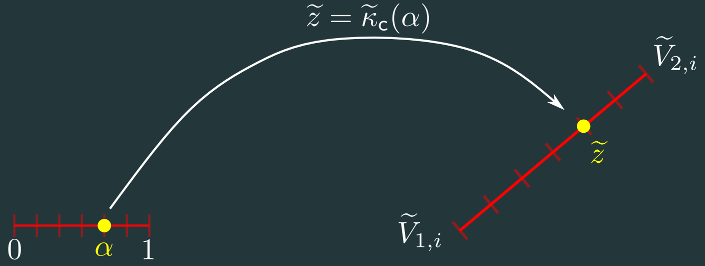

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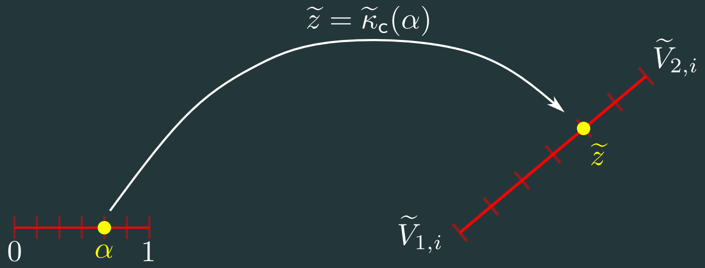
Tracking for general objects



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$$\begin{aligned}\mathcal{L}_{\mathsf{c},i}(\widetilde{y}; R) &= \int_0^1 \mathcal{N}(\widetilde{y} - \widetilde{\kappa}_{\mathsf{c}}(\alpha); 0, R)\ \mathsf{d}\alpha \\ &= \int_0^1 \mathcal{N}(A_i\alpha + B_i; 0, R)\ \mathsf{d}\alpha\end{aligned}$$

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Tracking for general objects

$$
\begin{aligned}
\mathcal{L}_{\mathsf{c},i}(\widetilde{y}; R) &= \int_0^1 \mathcal{N}(A_i\alpha + B_i; 0, R)\ \mathrm{d}\alpha \\
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&= C_i(\widetilde{y}; R) \int_0^1 \mathcal{N}\left(\alpha; \mu_i(\widetilde{y}; R), \sigma_i^2(\widetilde{y}; R)\right)\ \mathrm{d}\alpha
\end{aligned}
$$

Tracking for general objects

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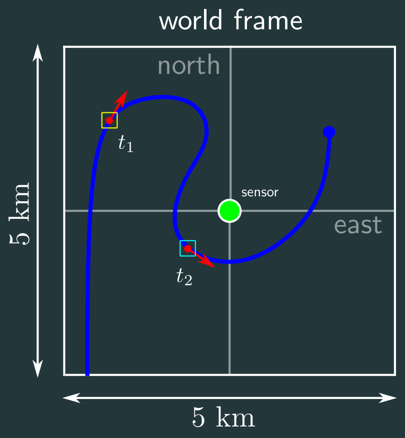
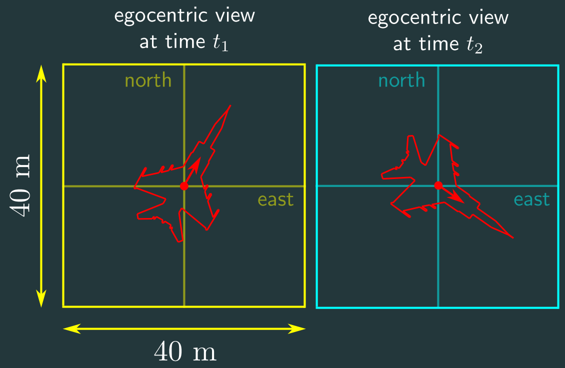
&= C_i(\widetilde{y}; R) \left[\Phi\left(\frac{1 - \mu_i(\widetilde{y}; R)}{\sigma_i(\widetilde{y}; R)}\right) - \Phi\left(-\frac{\mu_i(\widetilde{y}; R)}{\sigma_i(\widetilde{y}; R)}\right)\right]
\end{aligned}
$$

[Tesori et al., 2024]

Simulation

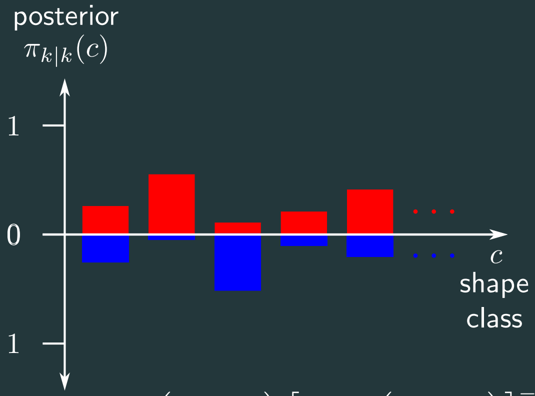


(position plot)



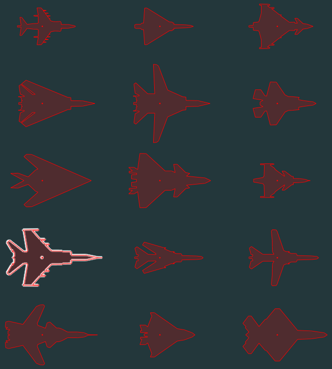
(orientation and shape plots)

Ground truth representation



$$\ell_k(c) \triangleq \mathcal{L}\left(\widetilde{\mathcal{Y}}_k | \widetilde{S}^{(c)}\right) \left[\sum_\nu \mathcal{L}\left(\widetilde{\mathcal{Y}}_k | \widetilde{S}^{(\nu)}\right)\right]^{-1}$$

normalized likelihood



dictionary
(shapes in scale)

Shape belief representation

Simulation

T2S demo

Conclusions

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- accuracy significantly drops with non-uniform point clouds;
- high computational cost;
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- **Direction 4:** agnostic shaping via MAP optimization and deep learning
- **Direction 5:** 3-dimensional EOT via computer vision models

Thank you

"The first principle is that you must not fool yourself,
and you are the easiest person to fool."
— *Richard Feynman*