POLITECNICO

MILANO 1863

Exercise RFID

INTERNET OF THINGS

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1.1. Exercise 3.1

1.1.1. Request

Find the overall collision resolution efficiency η in the different cases in which the initial frame size is set to r1=1,2,3,4,5,6.

1.1.2. Data

- $r_1 = 1, 2, 3, 4, 5, 6$

- $L_2 = 4$

- $L_3 = 51/8$

1.1.3. Solution

To compute the η in the different cases we can use the formula:

$$\eta = \frac{N}{L_N} \tag{1.1}$$

So we need to calculate first L_4 in all the six cases required by the exercise. We can do it using the reported formula:

$$L_4^* = r_1 + P(S = 0) \cdot L_4 + P(S = 1) \cdot L_3 + P(S = 2) \cdot L_2 + P(S = 3) \cdot L_1 \tag{1.2}$$

Probabilities

In order to solve this equation we need to calculate all the probabilities in the six cases:

- $r_1 = 1$

$$P(S = 0) = 1 \tag{1.3}$$

$$P(S = 1) = P(S = 2) = P(S = 3) = P(S = 4) = 0 \tag{1.4}$$

All the 4 tags must choose the single existing slot.

- $r_1 = 2$

$$P(S = 0) = (\frac{1}{2})^4 \cdot 2 + (\frac{1}{2})^4 \cdot \binom{4}{2} \cdot \frac{2}{2} = \frac{1}{2} \tag{1.5}$$

$$P(S = 1) = (\frac{1}{2})^4 \cdot 4 \cdot 2 = \frac{1}{2} \tag{1.6}$$

$$P(S = 2) = P(S = 3) = P(S = 4) = 0 \tag{1.7}$$

- $r_1 = 3$

$$P(S = 0) = (\frac{1}{3})^4 \cdot 3 + (\frac{1}{3})^4 \cdot \binom{4}{2} \cdot \frac{2 \cdot 3}{2} = \frac{7}{27} \tag{1.8}$$

$$P(S = 1) = (\frac{1}{3})^4 \cdot \binom{4}{1} \cdot 3 \cdot 2 = \frac{8}{27} \tag{1.9}$$

$$P(S = 2) = (\frac{1}{3})^4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot \frac{1}{2} = \frac{4}{9} \tag{1.10}$$

$$P(S = 3) = P(S = 4) = 0 \tag{1.11}$$

- $r_1 = 4$

$$P(S = 0) = (\frac{1}{4})^4 \cdot 4 + (\frac{1}{4})^4 \cdot \binom{4}{2} \cdot 4 \cdot 3 \cdot \frac{1}{2} = \frac{5}{32} \tag{1.12}$$

$$P(S = 1) = (\frac{1}{4})^4 \cdot 4 \cdot 4 \cdot 3 = \frac{3}{16} \tag{1.13}$$

$$P(S = 2) = (\frac{1}{4})^4 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot \frac{1}{2} = \frac{9}{16} \tag{1.14}$$

$$P(S = 3) = 0 \tag{1.15}$$

$$P(S = 4) = (\frac{1}{4})^4 \cdot 4 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \frac{3}{32} \tag{1.16}$$

- $r_1 = 5$

$$P(S = 0) = (\frac{1}{5})^4 \cdot 5 + (\frac{1}{5})^4 \cdot \binom{4}{2} \cdot 5 \cdot 4 \cdot \frac{1}{2} = \frac{13}{125} \tag{1.17}$$

$$P(S = 1) = (\frac{1}{5})^4 \cdot 4 \cdot 5 \cdot 4 = \frac{16}{125} \tag{1.18}$$

$$P(S = 2) = (\frac{1}{5})^4 \cdot 4 \cdot 5 \cdot 3 \cdot 4 \cdot 3 \cdot \frac{1}{2} = \frac{72}{125} \tag{1.19}$$

$$P(S = 3) = 0 \tag{1.20}$$

$$P(S = 4) = (\frac{1}{5})^4 \frac{5!}{(5-4)!} = \frac{24}{125} \tag{1.21}$$

- $r_1 = 6$

$$P(S = 0) = (\frac{1}{6})^4 \cdot 6 + (\frac{1}{6})^4 \cdot \binom{4}{2} \cdot 6 \cdot 5 \cdot \frac{1}{2} = \frac{2}{27} \tag{1.22}$$

$$P(S = 1) = (\frac{1}{6})^4 \cdot 4 \cdot 6 \cdot 5 = \frac{5}{54} \tag{1.23}$$

$$P(S = 2) = (\frac{1}{6})^4 \cdot 4 \cdot 6 \cdot 3 \cdot 5 \cdot 4 \cdot \frac{1}{2} = \frac{5}{9} \tag{1.24}$$

$$P(S = 3) = 0 \tag{1.25}$$

$$P(S = 4) = (\frac{1}{6})^4 \frac{6!}{(6-4)!} = \frac{5}{18} \tag{1.26}$$

Now we can calculate the value of L_4:

$$
\begin{aligned}
L_4 &= 4 + \sum_{i=0}^{3} P(S = i) \cdot L_{4-i} = \\
&= 4 + P(S = 0) \cdot L_4 + P(S = 1) \cdot L_3 + P(S = 2) \cdot L_2 + P(S = 3) \cdot L_1 = \\
&= 4 + \frac{5}{32} \cdot L_4 + \frac{3}{16} \cdot L_3 + \frac{9}{16} \cdot L_2 + 0
\end{aligned}
\tag{1.27}
$$

$$L_4 = \frac{4 + \frac{3}{16} \cdot L_3 + \frac{9}{16} \cdot L_2}{1 - \frac{5}{32}} = \frac{953}{108}$$

Now we can calculate the values of L_4^* in the six requested cases, once computed them we can calculate the overall collision resolution efficiency η for each case.

- $r_1 = 1$

$$L_4^* = 1 + 1 \cdot L_4 = \frac{1061}{108} \simeq 9,824$$

$$\eta_1 = 0,407 \tag{1.28}$$

- $r_1 = 2$

$$L_4^* = 2 + \frac{1}{2} \cdot L_4 + \frac{1}{2} \cdot L_3 = \frac{4147}{432} \simeq 9,600 \tag{1.29}$$

$$\eta_2 = 0,417$$

- $r_1 = 3$

$$L_4^* = 3 + \frac{7}{27} \cdot L_4 + \frac{8}{27} \cdot L_3 + \frac{4}{9} \cdot L_2 \simeq 8,954 \tag{1.30}$$

$$\eta_3 = 0,447$$

- $r_1 = 4$

$$L_4^* = 4 + \frac{5}{32} \cdot L_4 + \frac{3}{16} \cdot L_3 + \frac{9}{16} \cdot L_2 = \frac{953}{108} \simeq 8,824 \tag{1.31}$$

$$\eta_4 = 0,453$$

- $r_1 = 5$

$$L_4^* = 5 + \frac{13}{125} \cdot L_4 + \frac{16}{125} \cdot L_3 + \frac{72}{125} \cdot L_2 \simeq 9,038 \tag{1.32}$$

$$\eta_5 = 0,443$$

- $r_1 = 6$

$$L_4^* = 6 + \frac{2}{27} \cdot L_4 + \frac{5}{54} \cdot L_3 + \frac{5}{9} \cdot L_2 \simeq 9,466 \tag{1.33}$$

$$\eta_6 = 0,423$$
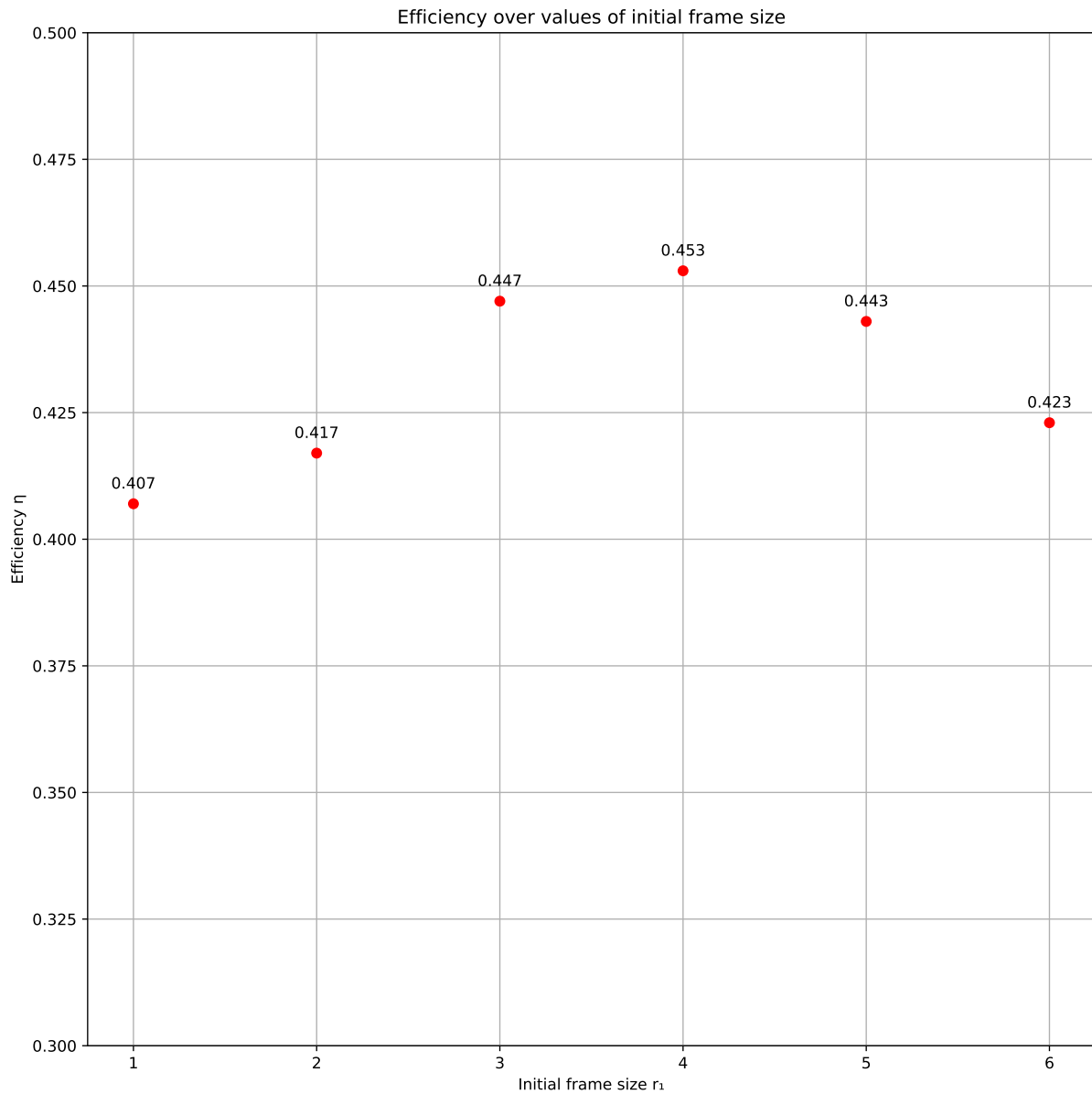
1.2.   Exercise 3.2

Figure 1.1: Exercise 3.2 plot

1.3.   Exercise 3.3

The maximum value of efficiency, denoted by η, is observed when the initial frame size is $r_1 = 4$.

This result aligns well with theoretical expectations, which predict that the highest efficiency is achieved when the frame size matches the number of tags, i.e., when $r_1 = N$.

When r_1 is smaller than N, the number of available slots is insufficient to accommodate all tags without contention, resulting in a high number of collisions and, consequently, a lower efficiency. On the other hand, when r_1 is larger than N, the frame includes more slots than necessary, leading to an increased number of empty slots. These unused slots do not contribute to successful transmissions and thus reduce the overall efficiency.

Therefore, the optimal value of r_1 represents a trade-off that minimizes collisions while avoiding excessive empty slots, which is precisely achieved when r_1 equals the number of tags N.

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