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System Simulation  
Midterm Problem 3

$$\dot{x} = \begin{bmatrix} -4.7 & -1.55 & -0.55 \\ 0.3 & -2.75 & -0.35 \\ 1.1 & 1.85 & -2.55 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} u$$

$$y = [2 \ 1 \ 1] x$$

A) 
$$\begin{bmatrix} \lambda + 4.7 & 1.55 & 0.55 \\ -0.3 & \lambda + 2.75 & 0.35 \\ -1.1 & -1.85 & \lambda + 2.55 \end{bmatrix} \begin{bmatrix} \lambda + 4.7 \\ \lambda + 2.75 & 0.35 \\ -1.85 & \lambda + 2.55 \end{bmatrix}$$

$$-1.55 \begin{bmatrix} -0.3 & 0.35 \\ -1.1 & \lambda + 2.55 \end{bmatrix} + 0.55 \begin{bmatrix} -0.3 & \lambda + 2.75 \\ -1.1 & -1.85 \end{bmatrix}$$

algebra

$$\lambda^3 + 10\lambda^2 + 33.64\lambda + 38.56 \quad (\text{wolfram to find roots})$$

$$\lambda = -4, (-3 \pm 0.8i)$$

B) see attached plots w/ lines  
(can only plot  $\lambda = -4$  with different values of  $T > 0$ )

C) unstable + inaccurate:  $T = \frac{1}{2}$   
stable + inaccurate:  $T = \frac{1}{32}$   
stable + accurate:  $T = \frac{1}{4}$

D) closed loop simulation poles:  $\rho(z) - \lambda T \sigma(z) = \Delta(z)$

$$T = \frac{1}{2}$$

$$T = \frac{1}{2} \quad \lambda = -4 \quad \sigma(z) = \frac{14}{11}z - \frac{8}{11} \quad \rho(z) = z^2 - \frac{16}{11}z + \frac{5}{11}$$

$$z^2 - \frac{16}{11}z + \frac{5}{11} - \left(\frac{1}{2}\right)(-4)\left(\frac{14}{11}z - \frac{8}{11}\right)$$

$$= z^2 - \frac{16}{11}z + \frac{5}{11} + \frac{28}{11}z - \frac{16}{11}$$

$$= z^2 + \frac{12}{11}z - 1$$

with roots of  $\boxed{\frac{-6 \pm \sqrt{157}}{11}} = \boxed{-0.54 \pm 1.139}$

$$T = \frac{1}{32}$$

$$z^2 - \frac{16}{11}z + \frac{5}{11} - \left(\frac{1}{32}\right)(-4)\left(\frac{14}{11}z - \frac{8}{11}\right)$$

$$= z^2 - \frac{16}{11}z + \frac{5}{11} + \frac{7}{44}z - \frac{1}{11}$$

$$= z^2 - \frac{63}{44}z + \frac{4}{11}$$

with roots of

$$\boxed{\frac{57 \pm \sqrt{6065}}{88}} = \boxed{0.648 \pm 0.885}$$

$$T = \frac{1}{4}$$

$$z^2 - \frac{16}{11}z + \frac{5}{11} - \left(\frac{1}{4}\right)(-4)\left(\frac{14}{11}z - \frac{8}{11}\right)$$

$$= z^2 - \frac{16}{11}z + \frac{5}{11} + \frac{14}{11}z - \frac{8}{11}$$

$$= z^2 - \frac{2}{11}z - \frac{3}{11}$$

with roots of

$$\boxed{\frac{1 \pm \sqrt{34}}{11}} = \boxed{0.09 \pm 0.530}$$