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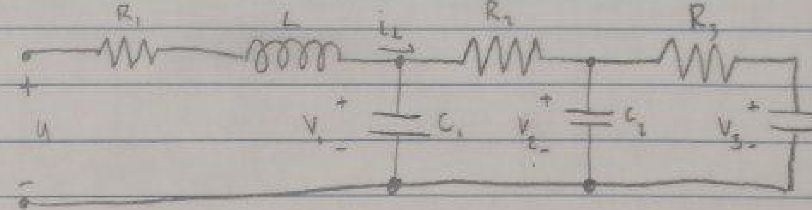
System Simulation
HW 5

(A) a) assign each C and L to a state:



$R_1 = 500\ \Omega$ $\quad C_1 = 4.7\mu F$ $\quad L = 2\ H$
$R_2 = 1k\ \Omega$ $\quad C_2 = 4.7\mu F$
$R_3 = 1k\ \Omega$ $\quad C_3 = 4.7\mu F$

assing each voltage (V_1, V_2, V_3) and ~~currentt~~ current (i_L)
$x_1(t) = V_1 \quad x_2(t) = V_2 \quad x_3(t) = V_3 \quad x_4(t) = i_L$

b) sum of current at node V_1: $-x_4 + \frac{x_1 - x_2}{R_2} + C_1 x_1' = 0$

$$x_1' = -\frac{1}{R_2C_1}x_1 + \frac{1}{R_2C_1}x_2 + \frac{1}{C_1}x_4$$

at node V_2: $\frac{x_2 - x_1}{R_2} + \frac{x_2 - x_3}{R_3} + C_2 x_2' = 0$

$$x_2' = \frac{1}{R_2C_2}x_1 - \left(\frac{1}{R_2C_2} + \frac{1}{R_3C_2}\right)x_2 + \frac{1}{R_3C_2}x_3$$

at node V_3: $\frac{x_3 - x_2}{R_3} + C_3 x_3' = 0$

$$x_3' = \frac{x_2}{R_3C_3} - \frac{x_3}{R_3C_3}$$

c) sum of voltages in loop: $u - Rx_4 + Lx_4' + x_1 = 0$

$$x_4' = -\frac{x_1}{L} - \frac{R_1x_4}{L} + \frac{u}{L}$$

d) single matrix equation $x' = Ax + Bu$

$$x' = \begin{bmatrix} -\frac{1}{R_2C_1} & \frac{1}{R_2C_1} & 0 & \frac{1}{C_1} \\ \frac{1}{R_2C_2} & -\left(\frac{1}{R_2C_2} + \frac{1}{R_3C_2}\right) & \frac{1}{R_3C_2} & 0 \\ 0 & \frac{1}{R_3C_3} & -\frac{1}{R_3C_3} & 0 \\ -\frac{1}{L} & 0 & 0 & -\frac{R_1}{L} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u$$

e) only 3 capacitors used ~~for~~ for output, so output equation looks like

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

cap. voltages

(B) with component values:

$$x' = \begin{pmatrix} -212.77 & 212.77 & 0 & 212.77 \\ 212.77 & -425.5 & 212.77 & 0 \\ 0 & 212.77 & -212.77 & 0 \\ -0.5 & 0 & 0 & -250 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.5 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x$$

(C) if only C_3 is the output, output equation would look like

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x$$

eigenvalues of A matrix (using matlab): $\lambda = -606, -216 \pm i285, -62$

using ss2tf with the A, B and C matrices,

$$H(s) = \frac{4.8159E9}{s^4 + 1.101E3\,s^3 + 4.549E5\,s^2 + 1.0186E8\,s + 4.8159E9}$$

roots of this system using Matlab are $s = -606, 216 \pm i285, -62$

same as eigenvalues for A