Matt McDade
System Simulation
Midterm Problem 3

$$\dot{x} = \begin{bmatrix} -4.7 & -1.55 & -0.55 \\ 0.3 & -2.75 & -0.35 \\ 1.1 & 1.85 & -2.55 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 1 & 1 \end{bmatrix} x$$

A) $\begin{bmatrix} \lambda+4.7 & 1.55 & 0.55 \\ -0.3 & \lambda+2.75 & 0.35 \\ -1.1 & -1.85 & \lambda+2.55 \end{bmatrix}$

$(\lambda+4.7)\begin{bmatrix} \lambda+2.75 & 0.35 \\ -1.85 & \lambda+2.55 \end{bmatrix} - 1.55\begin{bmatrix} -0.3 & 0.35 \\ -1.1 & \lambda+2.55 \end{bmatrix} + 0.55\begin{bmatrix} -0.3 & \lambda+2.75 \\ -1.1 & -1.85 \end{bmatrix}$

∴ algebra

$\lambda^3 + 10\lambda^2 + 33.64\lambda + 38.56$ (wolfram to find roots)

$\boxed{\lambda = -4, (-3 \pm 0.8i)}$

B) see attached plots w/ lines
(can only plot $\lambda = -4$ with different values of $T > 0$)

C) unstable + inaccurate: $T = \frac{1}{2}$
stable + inaccurate: $T = \frac{1}{32}$
stable + accurate: $T = \frac{1}{4}$

D) closed loop simulation poles: $\rho(z) - \lambda T \sigma(z) = \Delta(z)$

$T = \frac{1}{2}$: $T = \frac{1}{2}$ $\lambda = -4$ $\sigma(z) = \frac{14}{11}z - \frac{8}{11}$ $\rho(z) = z^2 - \frac{16}{11}z + \frac{5}{11}$

$z^2 - \frac{16}{11}z + \frac{5}{11} - \left(\frac{1}{2}\right)(-4)\left(\frac{14}{11}z - \frac{8}{11}\right)$

$= z^2 - \frac{16}{11}z + \frac{5}{11} + \frac{28}{11}z - \frac{16}{11}$

$= z^2 + \frac{12}{11}z - 1$ with roots of $\boxed{-\frac{6}{11} \pm \frac{\sqrt{157}}{11} = -0.54 \pm 1.139}$

$T = \frac{1}{32}$: $z^2 - \frac{16}{11}z + \frac{5}{11} - \left(\frac{1}{32}\right)(-4)\left(\frac{14}{11}z - \frac{8}{11}\right)$

$= z^2 - \frac{14}{11}z + \frac{5}{11} + \frac{7}{44}z - \frac{1}{11}$

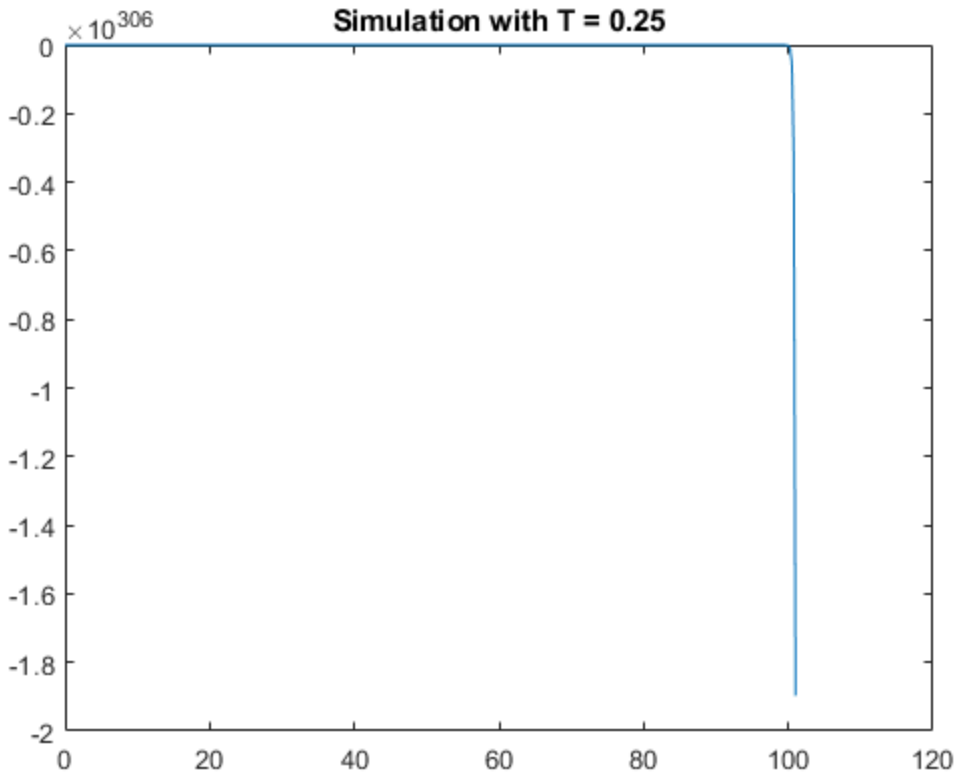
$= z^2 - \frac{57}{44}z + \frac{4}{11}$ with roots of $\boxed{\frac{57}{88} \pm \frac{\sqrt{6065}}{88} = 0.648 \pm 0.885}$

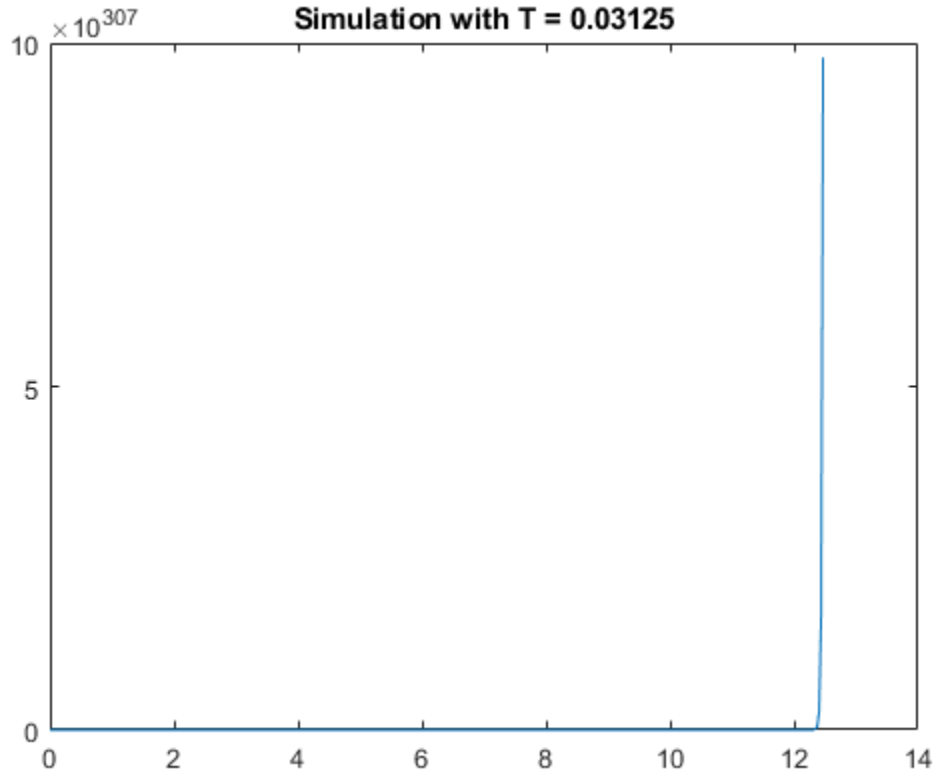
$T = \frac{1}{4}$: $z^2 - \frac{16}{11}z + \frac{5}{11} - \left(\frac{1}{4}\right)(-4)\left(\frac{14}{11}z - \frac{8}{11}\right)$

$= z^2 - \frac{16}{11}z + \frac{5}{11} + \frac{14}{11}z - \frac{8}{11}$

$= z^2 - \frac{2}{11}z - \frac{3}{11}$ with roots of $\boxed{\frac{1}{11} \pm \frac{\sqrt{34}}{11} = 0.09 \pm 0.530}$

```
% Matt McDade
% System Simulation
% Midterm Exam Problem 3E

T = [1/4; 1/32];

for i = 1:2
    t = [0:T(i):101];
    N = length(t);

    xp = zeros(3,N);
    f  = zeros(3,N);
    fp = zeros(1,N);

    p = tf([0 T(i)*14/11 T(i)*8/11],[1 -16/11 5/11]);

    % state space system
    A = [-4.7 -1.55 -0.55;
        0.3 -2.75 -0.35;
        1.1 1.85 -2.55];
    B = [1 0 -1]';
    C = [2 1 1];

    for k = 1:N-1
        f(:,k+1) = A*xp(:,k) + B;

        xp(:,k+1)=xp(:,k)+T(i)*((14/11)*f(:,k) - 8/11) - f(:,k+1) -
 (16/11)*f(:,k) + (5/11);

        fp(k+1)=C*xp(:,k+1);
    end

    figure(i);
    clear title
    plot(t,fp)
    title(['Simulation with T = ', num2str(T(i))])
end
```


Simulation with T = 0.25
×10^306
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
-2
0
20
40
60
80
100
120


Simulation with T = 0.03125
×10^307
10
5
0
0
2
4
6
8
10
12
14

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B) Lines for T = 1/2, 1/4, 1/32 respectively