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ESC HW 11

(11) a) $H_c(s) = \frac{1}{s^2 + s\sqrt{2} + 1}$ $\Delta t = 1\,ms$ $s \to \left(\frac{2}{\Delta t}\right)\cdot\left(\frac{z-1}{z+1}\right)$

$H_{d,tustin}(z) = 1 \Big/ \left(\left(\frac{2}{0.001}\right)\cdot\left(\frac{z-1}{z+1}\right)\right)^2 + \sqrt{2}\left(\frac{2}{.001}\right)\cdot\left(\frac{z-1}{z+1}\right) + 1$

: used wolfram alpha for the algebra

$$H_{d,tustin}(z) = \frac{(z+1)^2}{4002834.006 z^2 - 8000024.017 z + 3997178.002}$$

b) zeroes: 2 at $z = -1$

poles: 0.9974425, 1.00117

c) ~~[illegible]~~

see attached for 11c

d) $H_{d,exact}$ from matlab $= \frac{1.666E-7 z^2 + 6.662E-7 z + 1.665E-7}{z^2 - 1.999z + 0.9986}$

$H_{d,tustin}$ from matlab $= \frac{2.498E-7 z^2 + 4.996E-7 z + 2.498E-7}{z^2 - 1.999z + 0.9986}$

$$\|H_{d,exact} - H_{d,tustin}\|_{L_1[0,2\pi]} = \int_0^{2\pi} \frac{-8.322E-8 z^4 + 3.329E-7 z^3 - 4.993E-7 z^2 + 3.329E-7 z - 8.322E-8}{z^4 - 3.997z^3 + 5.992z^2 - 3.992z + 0.9972}$$

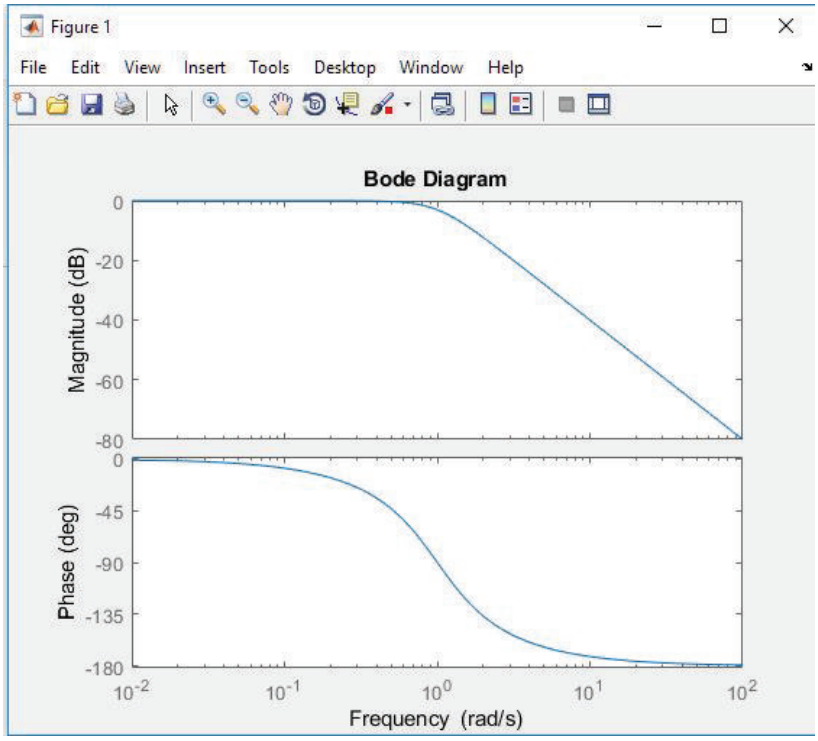
$= -4.19176E-7$

$$\|H_{d,exact} - H_{d,tustin}\|_{L_2[0,2\pi]} = \left(\int_0^{2\pi} \text{whatever is up there } dz\right)^{1/2}$$

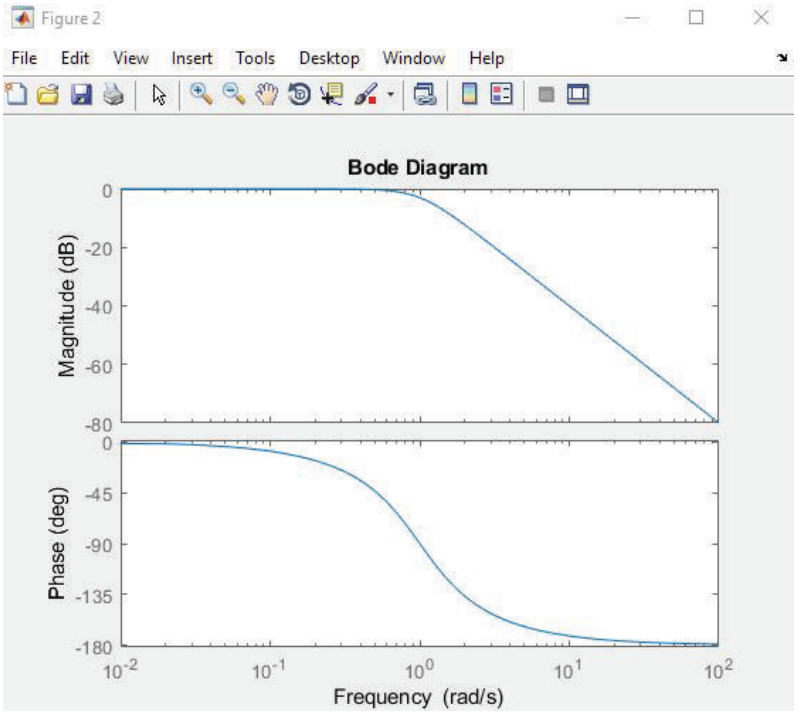
$= 6.4734E-4$

$$\|H_{d,exact} - H_{d,tustin}\|_{L_\infty[0,2\pi]} = \max = 3.38953E-7$$

↳ at $z = 0.950832$



11c plot (tustin)



11c plot (exact)