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System Simulation
Midterm Problem 1

A) zero at $z = \frac{4}{7}$: $\sigma(z) = \beta_2 z^2 + \beta_1 z + \beta_0 = 0$

$\rightarrow$ explicit, $\boxed{\beta_2 = 0}$

$\rightarrow \frac{4}{7}\beta_1 + \beta_0 = 0$

need 3 lamberts equations: $C_0 = \alpha_2 + \alpha_1 + \alpha_0 = 0$

$\rightarrow \boxed{\alpha_2 = 1}$ second order

$\rightarrow \alpha_1 + \alpha_0 = -1$

$C_1: 2\alpha_2 + \alpha_1 - \beta_2 - \beta_1 - \beta_0 = 0$

$\rightarrow \alpha_1 - \beta_1 - \beta_0 = -2$

$C_2: \frac{2^2}{2!}\alpha_2 + \frac{1^2}{2!}\alpha_1 - 2\beta_2 - \beta_1 = 0$

$\rightarrow \frac{1}{2}\alpha_1 - \beta_1 = 2$

solve 4×4 system of equations with ☆ to get

$\alpha_2 = 1$, $\alpha_1 = -\frac{16}{11}$, $\alpha_0 = \frac{5}{11}$, $\beta_2 = 0$, $\beta_1 = \frac{14}{11}$, $\beta_0 = -\frac{8}{11}$

$$\boxed{H_p(z) = \frac{T\left(\frac{14}{11}z - \frac{8}{11}\right)}{z^2 - \frac{16}{11}z + \frac{5}{11}}}$$

B) same equations as above, this time not setting $\beta_2 = 0$, thus adding another lambert equation:

$C_3: \frac{8}{6} + \frac{1}{6}\alpha_1 - 2\beta_2 - \frac{1}{2}\beta_1 = 0$

solving the 5×5 similarly as before to get:

$\alpha_2 = 1$ $\alpha_1 = \frac{-236}{151}$ $\alpha_0 = \frac{85}{151}$ $\beta_2 = \frac{70}{151}$ $\beta_1 = \frac{44}{151}$ $\beta_0 = \frac{-48}{151}$

$$\boxed{H_c(z) = \frac{T\left(\frac{70}{151}z^2 + \frac{44}{151}z - \frac{48}{151}\right)}{z^2 - \frac{236}{151}z + \frac{85}{151}}}$$