

Ohm:  $V=IR$  Conduction Mechanism: electronic  
 $R=\frac{\rho l}{A}$   $\sigma=\frac{1}{\rho}$  (flow of e's or holes), ionic (diffusion of charged ions, liquids+solids with temp)  
 $J=\sigma E$  Energy Band: At  $\uparrow$  separation, atoms are independent. Close atoms (solids), individual electron states may split into series of close states (E band). Extent of splitting depends on interatomic sep. distance. Valence Band: occupancy based on outermost shells of atoms (interact at close independent range). Conduction Band: range of e- energies where e- moves freely on impurity Fermi energy ( $E_f$ )>E of last (highest) +host metal filled state at 0K. Conductor if band is partially full or overlap, insulator or semiconductor if valence band is full and conduction band is empty.  
 $\sigma=n|e|\mu_e$  \*Little energy is required to promote e's to empty states (E field provides sufficient E). Frictional forces counter acceleration of e's due to E field, result of scattering e's by imperfections in lattice. (e- loses KE and changes direction, causes resistance to E current)  
 $n_i=n_0e^{\frac{-E_g}{2kT}}$   
for extrinsic replace  $E_R$ :  
n-type:  $E_D$   
p-type:  $E_A$

Turbine blades are made of single crystals for high durability & coated with ceramic with low thermal conductivity. Creep-> time-dependent tendency of solid to deform permanently when subject to high stress and temp. Can cause matls to fail below yield stress. Primary->  $\downarrow$  creep rate due to strain hardening. Secondary-> constant strain rate. Longest stage. Balance between strain hardening and recovery. Tertiary-> acceleration of strain rate leading to failure.  $\gamma'$  phase (strengthening)-> responsible for alloy's elevated strength and resistance to creep deformation.  $\gamma$  phase-solid solution with FCC lattice and random distribution of different species of atoms.  $\gamma'$  &  $\gamma$  have similar size, and therefore low interface energy where phases meet (coherent)  $Q_{solid}=3kT$   
Temperature-> avg KE associated with atomic motion.  
Thermal E-> sum of KE and PE. Heat-> thermal E in process of transfer or conversion. Dulong&Petit law fails at low temps where C is temp-dependent. At  $\downarrow$  temps use Debye,  $C=AT^3$ . Thermal Conductivity-> ability to transport heat across temp gradient (from high to low). In solid, heat is conducted by lattice vibration waves, free e's. Metals: free e's, ceramics: phonons(vibrational), polymers: vibration/rotation.

Composite matl-> hybrid consisting of 2 phases: dispersed reinforcement phase, surrounded by matrix. Majority of polymers are amorphous but semi-crystalline is possible. Type of polymer depends on length of chain (molecular weight) and degree of crystallinity. Alignment of polymer chains at yield point causes dip in stress-strain. Thermoplastics: linear/branched, soft. Thermosets: cross-linked, hard. Deformation of Semi-Crystalline begins in amorphous region, crystalline blocks rotate and segment, amorphous regions stretch. Biomaterial-> intended to interface w/biological systems to evaluate/replace etc. any part of body. Bioinert-> little/no host response. Bioactive-> active host. Bioresorbable-> absorbed & replaced in vivo. Wear/bear surface-> desirable properties are  $\uparrow$  wear resistance,  $\downarrow$  friction, no wear particle generat. Stem matl with  $\uparrow$  toughness,  $\downarrow$  Young's mod reduces stress shielding.

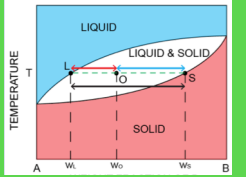
Boiling point of water is function of pressure ( $\uparrow$  pressure  $\uparrow$  bp). Melting point of water is function of composition. Phase diagrams describe conditions under stable phase. Phase-> region of substance that is uniform in chem comp., physically distinct, mechanically separable. Component-> chemically distinguishable constituent of substance. Solid solution-> solid where 2 or more components are compatible and form single phase. Solubility limit-> max concn of solute atoms that dissolve in solvent to form solid solution. Unary phase diagrams display temp vs pressure. Two types of binary phase diagrams: isomorphous (complete solubility of 3 components), eutectic (limited solubility of 2 components). Complete solubility: similar crystal structure, atomic radii, electronegativity. Iso-> 3 phase regions: Alpha, Liquid, Alpha + L. For binary system of known comp. and temp. at equilibrium, you can identify: phases, compositions of phases, relative amounts of phases. Eutectic occur when 2 elements completely dissolve in liquid state but have only limited solubility in solid. More than one solid phase exists here.

Resistivity: depends on temp, composition, degree of cold work (disloc density). \* $\uparrow$  temp  $\uparrow$  # of e's above  $E_f$  but thermal vib. &  $\uparrow$  vacancy concn. decrease  $\mu_e$  which is dominant.  
Conduction in Semicond&insulators  
No available empty states adjacent to filled valence band. Band gap energy (difference in E between 2 states)=E required to promote e to conduction band (nonelectrical source i.e. heat, light etc).  $E_g>3eV \rightarrow$  insulator. # of excited e's into conduction band depends on  $E_g$  & temp.  $E_g \uparrow = \sigma \downarrow$ . Intrinsic-> Elec behavior dependent on properties of pure material.  $E_g<2eV$ . At  $T=0$ , conductivity=0. At  $T>0$ , heat helps e's jump band gap (creation of e-hole pairs). Every e excited into conduction band, a hole is left behind (# holes=#e's) Extrinsic-> Elec behavior dictated by impurity/solute atoms (dopants). Commercial semiconds are all extrinsic. Can be n-type (excess e's) or p-type (excess holes). Electron-binding E-> E required to excite e from impurity state to state within conduction band ( $E_D$ ) -> no hole created.  $E_D<<E_g$ , (donor state) \*p-type has acceptor state above valence creating hole in valence (only charge carrier produced, free e is not created in conduction band).

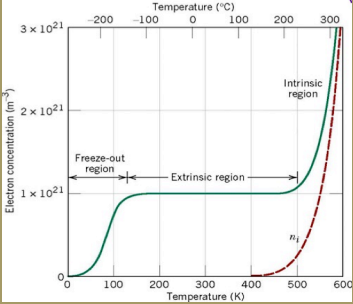
Slow Coarsening-> large precipitates grow and smaller ones shrink. Driven by minimization of total interface energy and prevalent at high temps. Part of superalloy's strength and resistance to creep is due to low  $\gamma'$  coarsening rates. Coherent  $\gamma$ -  $\gamma'$  interfaces have low interfacial energy, and low driving force for detrimental coarsening of  $\gamma'$ . Retains strength due to continued presence of small second phase  $\gamma'$  precipitates. Single Crystal Blades-> Grain boundaries act as highway for diffusion, weakening material. Can be avoided by making single crystal turbine blades w/o grain boundaries. Order Hardening-> coherent  $\gamma$ -  $\gamma'$  interface provides little resistance to passage of dislocations. If dislocation moves through lattice, Al atoms move to Ni sites (vice versa), creating Ni-Ni and Al-Al. Called anti-phase boundary (energetically unfavourable). Excess energy required to create anti-phase boundary gives rise to dislocation drag. Order hardening is dominant hardening mechanism in super alloys.  
 $C=dQ/dT$   $3R=25J/mol.K$   $\frac{dL}{L_0}=\alpha\Delta T$   $\frac{\Delta V}{V_0}=\alpha_v\Delta T$   $TSR=\frac{\sigma_f k}{E\sigma_l}$   
Glass-> made by cooling molten silica below GTT (molecular movement slowed and matl solidifies). Has to be processed. Thermal expansion effects in solids is small. Thermal expansion coefficient ( $\alpha$ )-> amount by which matl expands in response to temp change. For isotropic thermal expan.->  $\alpha_v=3\alpha$ . Thermal expansion is due to asymmetric curvature of PE trough (NOT increased atomic vibrational amplitudes w/ rising temp).

Phase transformation-> conditions of physical system (temp, pressure, composition) change, such that current phases are no longer at eqm. Solidification: form single-phase solid from liquid. Precipitation: form second-phase within single-phase solid. Both bashed on nucleation (critical) and growth. Activation energy-> E increase needed to undergo phase transformation. Nucleation-> when nucleus first forms, volume transformed from liquid to solid causes  $\downarrow$  in free E. Interface between solid and liquid forms, which  $\uparrow$  free E. Free E reduction depends on nucleus volume, and interface E on nucleus surface area. Activation energy, at a certain value of r, increase in free E is max. Radius at which this occurs is critical nuclei radius,  $r^*$ . For  $r<r^*$ , net increase in free E (nucleus dissolves back into liquid). For  $r>r^*$ , net decrease in free E (nucleus is stable, will grow). Nucleation (and solidification) occur more easily and rapidly when activation energy is smaller. Activation E for nucleation can be reduced if instead of forming spherical nuclei within liquid (homogenous), nuclei form on pre-existing solids (heterogeneous). Rapid solidification leads to finer grain size/more uniform. To  $\downarrow \Delta G^*$   $\uparrow \Delta G_v$  by  $\downarrow$  temp. Can also  $\uparrow$  rate of heterogeneous nucleation by adding additional sites where nuclei can form (inoculant particles).

Lever rule: %weight of solid phase:  $X_s=\frac{w_0-w_L}{w_s-w_L}$  (red line)



Temp. Dependence of semicond-> In intrinsic: concn of carriers  $\uparrow$  with  $\uparrow$  temp. Intrinsic+extrinsic: At low temp (freeze-out): thermal E insufficient to excite e movement from donor or into acceptor. At intermediate temp (extrinsic region): carrier concn depends on dopant concn, intrinsic excitations are negligible. At high temp (intrinsic region): intrinsic excitations dominate. In extrinsic: conductivity at intermediate temp depends on dopant concn. At high temps, in intrinsic region, dependence on dopant concn disappears.  $\downarrow$  in conductivity approaching intrinsic region. Thermal fluctuation  $\uparrow$  scattering, despite constant level of charge carriers. Solid State Devices: Diode-> p and n-type regions in contact (p-n junction). Single crystal with different dopants. Behaviour of junction depends on direction of bias voltage. Forward bias: low resistance. Electron-hole recombination occurs at interface. New electrons and holes are injected at end contacts. Reverse bias: insulator, rectifier. Electrons and holes leave region near the junction. No source to create new defects there. Depleted region behaves like intrinsic semiconductor. Transistor-> Same principles of p-n junction, applied to complicated designs. Junction transistor/MOSFET. Can amplify a signal, or act as a switch. Two states (conducting/non) allow binary info to be represented (basis for arithmetic ops). Integrated circuits-> created from Si, using photolithographic. Diffusion/ion implantation can implant elements. Modifies elec properties of diff regions. Can create n/p,  $\uparrow$  cond/resis.

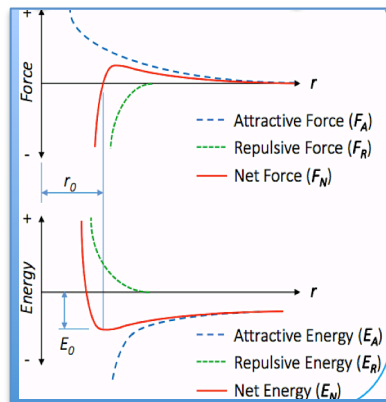


Thermal Shock Resistance-> capacity to withstand failure due to sudden change in temp. Making Glass Stronger: Tempering-> surface in compression, interior in tension. Ion Exchange Stuffing-> stuff large ions into glass surface, creating surface compressive strain and interior tension. Large K ions replace smaller Na ions

Forming 2nd phase has same free E changes as solidif. ( $\downarrow$  free E bc volume of matl changed,  $\uparrow$  free E bc  $\alpha/\beta$  interface) but also:  $\uparrow$  free E bc strain on matrix bc size mismatch

Activation E for nucleation in precipitation can be  $\downarrow$  by:  $\uparrow$  supersaturation ( $\uparrow \Delta G_v$ ) by moving further into 2-phase region,  $\downarrow$  interface or strain E (done by heterogenous nucleation on lattice defects). Homo nucleation is more common in precip.  
 $\Delta G^*=\frac{16\pi\gamma^3}{3(\Delta G_v+\Delta G_s)^2}$   
 $r^*=\frac{2\gamma}{\Delta G_v+\Delta G_s}$   
solidificat.:  $\Delta G_v=\frac{\Delta H_f(T_m-T)}{T_m}$

Lever rule: %weight of solid phase:  $X_s=\frac{w_0-w_L}{w_s-w_L}$  (red line)



**Melting point**  
 ↓ bond energy = ↓ melting temp  
 ↓ bond energy favours gas  
 ↑ bond energy favours solid  
**Thermal Expansion**  
 Curvature is broad & asymmetric = can have ↑ thermal expansion coefficient  
 ↑ temp = atoms vibrate more & average position can deviate  
**Modulus of Elasticity**  
 Steeper E Curve = ↑ Elastic Mod  
 $F_N(r) = F_A(r) + F_R(r)$   $e = 1.602 \times 10^{-19}$   
 $E_A = -\frac{A}{r}$   $A = \frac{|Z_1||Z_2|e^2}{4\pi\epsilon_0}$   
 $\epsilon_0 = 8.85 \times 10^{-12}$

$J = \frac{M}{At} = \frac{1}{A} \frac{dM}{dt} = \frac{q}{A}$   
 q = number or mass of atoms passing through material per unit time  
 $J = -D \frac{dc}{dx} = -D \frac{C_A - C_B}{x_A - x_B}$   
 $\frac{dq}{dt} = JA$   
 $D = D_0 \exp\left(-\frac{Q_d}{RT}\right)$   
 $Q_d$  = activation energy (J/mol or eV/atom)  
 Impurity Diffusion: the diffusion of one type of atomic species within a multi-component material

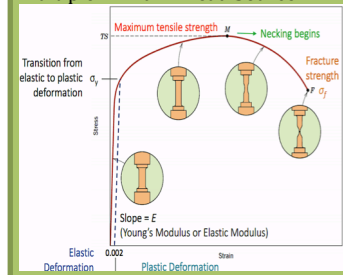
**Diffusion**  
 Depends on: 1) Diffusion mechanism (Substitutional vs. Interstitial) 2) Temperature 3) Type of crystal structure of host lattice (interstitial diffusion is easier in BCC than FCC) 4) Type of crystal imperfections:  
 • Faster along grain boundaries than elsewhere inside a crystal  
 • Faster along dislocation lines than through the bulk  
 • Excess vacancies will enhance it  
 5) Concentration of diffusing species  
 Self-diffusion: diffusion in a single component material & all atoms that exchange positions are the same type  
 • Elements diffuse faster in  $\alpha$ -Fe than in  $\gamma$ -Fe because  $\alpha$ -Fe has a more open structure ( $\gamma$ -Fe is close packed) so the activation barrier for diffusion is lower  
 • Interstitial diffusion is faster than vacancy diffusion  
 • Fick's first law: diffusion flux is proportional to concentration gradient  
 • Diffusion coefficient varies with T

$\tau_R = \sigma \cos(\lambda) \cos(\phi)$   
 min stress to initiate yielding:  
 $\sigma_y = \frac{\tau_{CRSS}}{\cos(\lambda) \cos(\phi)}$   
 max stress:  $\sigma_y = 2\tau_{CRSS}$   
 H-P:  $\sigma_y = \sigma_0 + k_y \sqrt{1/D}$   
 $D$  = avg grain diameter  
 $\sigma_y = \sigma_0 + k_s \sqrt{C}$   
 $C$  = solute conc'n  
 Adding jog:  $U_{jog} = Gb^3$   
 $\sigma_y = \sigma_0 + k_w \sqrt{\rho}$   
 $\rho$  = disloc. density  
 $\% Cold W = \frac{A_0 - A_d}{A_0} \times 100$   
 Annealing:  $U_s = Gb^2 \rho$   
 Grain growth:  $D^n - D_0^n = Kt$   
**Grain size red.** → disloc. can't glide across grain boundary  
 \*Fine-grained mats have greater grain boundary area (slows dislocation motion)  
 \*H-P eqn to determine red.  
 \*↑ toughness & strength  
**Solid-Solution**  
 → hardening metals as a result of solute atoms (i.e. alloying addition)  
 \*Substitution or Interstitial  
 \*Resistance to slip ↑  
**Particle/Precipitate** → act as obstacles and when disloc. are pinned at 2 ends by particles, multiple in Frank-Read Source

**Dislocation density ( $\rho$ ):**  
 dis. length/volume or # of dis/area  
 \*Dis. cause distortions that store energy  
 \*Lattice Strain (areas of T&C) created around dis.  
 \*If dis. exerts attractive force → annihilation (perfect crystal)  
**Deform metal:** ↑ Strain E  
 ↑ dislocation density  
 ↓ conductivity  
 ↓ corrosion resistance  
 \*Reverted by annealing  
**Single Crystals** → each shear band results from generation of lots of dislocations & their propagation in slip system with max  $\tau_R$ .  
 Poly → grain orient. is random. Some grains have low  $\cos(\lambda) \cos(\phi)$   
**Work/Strain** → ductile metal becomes stronger as it is plastically deformed bc # of disloc. ↑ and mobility ↓  
 \*Cold-Working \*Disloc. slip is stochastic  
 \*New disloc. made at grain boundaries & particle clusters  
 \*↑ yield strength ↓ ductility  
 when disloc. are pinned at 2 ends by particles, multiple in Frank-Read Source

**Slip** occurs when shear stress on slip plane exceeds critical resolved shear stress threshold  
 \*Occurs on specific planes and directions (slip system)  
 \*Most dense plane & direction w/ highest linear density  
 FCC → 111 plane, 110 direction  
 BCC → 110 plane, 111 direction  
 \*Grains elongate in direction of strain bc of deformation in poly  
 \*Resolved Shear Stress ( $\tau_R$ ) is shear component of applied tensile stress resolved along slip plane that is other than perp or parallel to stress axis  
 $\tau_{CRSS}$  is value of  $\tau_R$  at which yielding begins (prop of matl)  
**Annealing**  
 1. Recovery → dislocs organize and annihilate (relief of internal strain energy by heat)  
 \*↓ strength, ↓ # dislocs, ↑ ductility  
 \*Grains are unchanged  
 \*Electric & thermal cond. restored  
 2. Recrystallization → new strain free grains nucleate (equiaxed)  
 \*low disloc. density \*new grains (low  $\rho$ ) grow into old (high  $\rho$ )  
 \*Rapid ↓ strength \*Ductility restored \*Similar character. of pre cold-worked \*depend on temp & time \*↑ softness, ductility  
 \*driving force = stored E  
 Hot Working → plastically deform material above recrystallization temp.  
 \*annealing occurs simult. w/ deformation \*large deformations  
 $T > 0.6T_m$  = Hot Work  $T < 0.4T_m$  = Cold

3. Grain Growth → ↑ avg. grain size of polycrystalline mats (require ↑ heat)  
 \*Does not need to be preceded by step 1 & 2  
 \*large grains grow at expense of smaller  
 \*small ↓ strength \*total grain boundary area ↓  
 yielding ↓ E \*driving force = minimize grain boundary E \*Grain bound. E related to area & curvature  
 \*Boundaries migrate towards center  
 \*At room temp, fine-grained metal  
 ↑ strength, toughness than coarse-grained



V <sub>atoms</sub> = $\frac{4}{3}\pi R^3$	Simple Cubic	BCC Crystal	FCC Crystal	HCP Crystal
• Atoms/cell: 1	• Atoms/cell: 1	• Atoms/cell: 2	• Atoms/cell: 4	• Atoms/cell: 6/2
APF = $\frac{vol\ all\ atoms}{vol\ unit\ cell}$	• APF = 0.52	• APF = 0.68	• APF = 0.74	• APF = 0.74
$\rho = \frac{n \cdot A}{V_c \cdot N_A}$	• CN = 8	• CN = 8	• CN = 12	• CN = 12
A = atomic weight	• a = 2R	• a = $\frac{4R}{\sqrt{3}}$	• a = $2\sqrt{2}R$	• a = 2R
n = atoms/cell	• X close-packed	• X close-packed	• close-packed	• c/a = 1.63 (ideal)
N <sub>A</sub> = Avog's # (6.02x10 <sup>23</sup> )	Anisotropy (opposite of isotropic)			
$n\lambda = 2d \sin\theta$	Physical properties of a material differ depending on crystallographic direction in which measurement's taken (e.g. conductivity, elastic modulus, index of refraction)			
$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$	• Structural symmetry ↓ anisotropy ↑			

$N_V = N \cdot \exp\left(-\frac{Q_v}{RT}\right)$  {or kT}  
 $N = \frac{N_A \cdot p}{A}$   
 $N$  = #atomic sites/unit vol.  
 $X_{sol} = \exp\left(-\frac{Q_{sol}}{kT}\right)$   
**Planar Defect**  
 • Include external surfaces (have more dangling bonds that have excess energy), grain boundaries (atomic mismatch in a transition region where grains meet), twin boundaries (produced by applied mechanical shear forces or during annealing heat treatments)  
 • Most solids are polycrystalline (many grains separated by grain boundaries)  
**Point Defect**  
 • Include vacancies, interstitials, self-interstitials  
**Factors affecting solubility**  
 • Atomic size factor, crystal structure, electronegativity, valences  
**Line Defect**  
 • Unlike point, dislocations are thermodynamically unstable  
 • Edge: associated with lattice distortion produced in vicinity of the end of an extra half-plane of atoms (burgers vector perpendicular)  
 • Screw: associated with lattice distortion created when normally parallel planes are joined together to form helical ramp (burgers vector parallel)

**Stress:**  $\sigma = \frac{F}{A_0}$   
 Related to dF/dr  
 Slope of stress-strain line  
**Strain:**  $\epsilon = \frac{l_i - l_0}{l_0}$   
 Measures stiffness  
 Linear stress-strain for elastic deformation  
**Shear Strain:**  $\gamma = \tan\theta$   
 Depends little on alloying  
**σ = Eε** **τ = Gγ**  
 Depends weakly on temp.  
**v = -**  
 $\frac{\epsilon_x}{\epsilon_y} = -\frac{\epsilon_y}{\epsilon_z}$   
 (E<sub>melt</sub> = 0.5 E<sub>0K</sub>)  
 for iso: E = 2G(1+v)  
**U<sub>r</sub> =**  
 $\frac{\sigma_y^2}{2E}$  (linear)  
 Yielding → onset of plastic deformation  
**Ductility**  
 How much material plastically deforms before fracture  
**%EL =**  
 $\frac{l_f - l_0}{l_0} \times 100$   
**%AR =**  
 $\frac{A_0 - A_f}{A_0} \times 100$   
**Elastic Modulus**  
 Plastic deformation  
 Crystalline → slip  
 Amorphous → viscous  
 flow mechanism  
 Metals → gradual transition  
 Dislocation motion  
 Edge → B vector and disloc. line are perp. (disloc. line moves parallel to B vector)  
 Screw → opposite of edge  
**Toughness: Energy**  
 required to fracture unit volume of material (Area under stress-strain)  
 Metals are tougher than ceram. polymer  
**Resilience:**  
 Capacity of matl to absorb energy when deformed elastically & upon unloading recover the energy. Mod. of resilience (U<sub>r</sub>) → strain energy per volume (J/m<sup>3</sup>)  
 Area under stress-strain up to point of yielding

**Fracture** → crack formation & propagation bc of stress  
**Ductile Frac.** → stable (resist cont. unless ↑ stress)  
 → ↑ plastic deform, %AR & EL  
 → ↑ E absorption  
 → caused by damage accum.  
 → nucleation, growth, coalescence of microvoids  
 → particles & imperfections assist in void nucleation  
 → Fracture is rough & fibrous  
 → Shear 45° to tensile direc.  
**Brittle Frac.** → unstable  
 → ↓ plastic deform, E absorb. bc of rapid crack propagat.  
 → Amorphous: shiny, smooth  
 → Intragranular → through grains (inter follows boundaries)  
**Cleavage:**  
 crack propagation corresponding to successive & repeated breaking atomic bond along specific planes  
 Ductile → single piece, extensive deformation  
 Brittle → many pieces, little deformation  
**Static loading** → ↓ strain rate  
 → If mats are loaded and deformed rapidly (↑ strain rate), ductile materials can behave in a brittle manner  
**Impact tests** determine material response to temp. and ↑ strain rate  
 → Charpy & Izod (difference in specimen support)  
**Ductile-to-Brittle transition**  
 occurs when temp. of matl ↓  
 → BCC fail by cleavage along <100> plane (not FCC, HCP)  
 \*Measured fracture strength is always ↓ than expected bc of microscopic flaws  
 \*Applied stress is amplified at the tip of the crack  
 → dependent on crack orientation & geometry  
 → Stress Conc'n factor (K<sub>t</sub>) is measure of degree to which an external stress is amplified at tip of crack  
 Stress at tip of crack can break bonds. Crack propagates if elastic E released during fracture > surface E created  
 Crack grows if:  $\sqrt{EG_c} = \sigma \sqrt{\pi a}$   
 (K = K<sub>t</sub>) Grif Crit.  
 K = stress intensity factor, K<sub>c</sub> = critical stress factor or fracture toughness, G<sub>c</sub> = toughness  
 K<sub>c</sub> property that measures matl's resistance to brittle fracture  
 ↓ when crack is present  
 \*When thickness > crack dimension, K<sub>t</sub> becomes independent of thickness  
 \*Plane strain fracture toughness (K<sub>IC</sub>) ↓ with ↑ strain rate and ↓ temperature  
 $K_{IC} = Y \sigma_c \sqrt{\pi a}$  (units: MPa√m)  
 \*Factors affecting fracture: stress conc'n, critical crack length, fracture toughness  
 \*If max stress > yield strength in ductile, plastic deform. occurs (if in brittle matl, cleavage/fracture)