INSTRUCTOR: M.BREUGEM, COLLEGIO CARLO ALBERTO

OFFICIAL MARK: 30/30 WITH HONOURS

TAKE-HOME EXAMS OF *ASSET PRICING*

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Homework 1

Overture: the tree

T0	Vanilla debt	Convertible
A	$ 100.00	
B	$ 86.62	$ 87.27
Warrant	$ 4.91	$ 4.58
Stock	$ 8.48	$ 8.15

T1: up, p=0.5	Vanilla debt	Convertible
A	$ 125.00	
B	$ 97.56	$ 98.90
Warrant	$ 10.06	$ 9.39
Stock	$ 17.38	$ 16.71

T1: down, p=0.5	Vanilla debt	Convertible
A	$ 80.00	
B	$ 80.00	$ 80.00
Warrant	$ -	$ -
Stock	$ -	$ -

T2: up up, p=0.25	Vanilla debt	Convertible
A	$ 156.25	
B	$ 100.00	$ 102.75
Warrant	$ 20.63	$ 19.25
Stock	$ 35.63	$ 34.25

T2: up down, p=0.25	Vanilla debt	Convertible
A	$ 100.00	
B	$ 100.00	$ 100.00
Warrant	$ -	$ -
Stock	$ -	$ -

T2: down up, p=0.25	Vanilla debt	Convertible
A	$ 100.00	
B	$ 100.00	$ 100.00
Warrant	$ -	$ -
Stock	$ -	$ -

T2: down down, p=0.25	Vanilla debt	Convertible
A	$ 64.00	
B	$ 64.00	$ 64.00
Warrant	$ -	$ -
Stock	$ -	$ -

Act II: bondholders first mover advantage

	Bond	Warrant	Strategy
Convert, warrant exerciced	$102.75	$19.25	warrant exercised
Convert, warrant not exerciced	$ 117.19	$ -	
Not to convert, warrant exerciced	$100.00	$20.63	warrant exercised
Not convert, warrant not exercice	$100.00	$ -	
Bond value if converted	$102.75		bond converted
Bond value if not converted	$100.00		

Difference (point 2 vs point 1)

	Absolute	Relative
convertiblity option	$ 0.65	0.76%
Warrant	$ -0.33	-6.67%
Stock	$ -0.33	-3.86%

1 Act one: warrant

Let $\pi(t)$ denote the payoff of a generic node considered at stage t, with t ∈ [0,1,T=2], while $\pi(x)$ denotes the payoff of a specific node of the three. The four terminal nodes belong to X=[up up, up down, down up, down down]. Π denotes the immediate exercise payoff and R the interest rate.
The warrant is priced as a call with due regard to the capital increase at the exercise and the consequent dilution effect. Calling N the number of shares already outstanding, K the warrant (W) strike, A the asset value and B the bond, the payoff at exercise of the warrant is:

$$\Pi^W(t) = max\left(\frac{A(t)-B(t)+K}{N+1} - K, 0\right) = max\left(\frac{A(t)-B(t)+15}{2} - 15, 0\right)$$

In the only exercise case, x= up up, the value of the warrant payoff is: $\Pi^W(x=up, up) = \frac{156.25-100+15}{2} - 15.$
The drift suggests no early exercise premium for the warrant, however accounting for its American stile at each node:

$$\pi^W(t) = max\left(\frac{E^q[\pi^W(t+1)]}{R}, \Pi^W(t)\right) = max\left(\frac{p^u\pi^W(x=child.up)+p^d\pi^W(x=child.down)}{R}, max\left(\frac{A(t)-D(t)+K}{2} - K, 0\right)\right)$$

By backward induction W(0) can be recovered.
We price the bond, in order to recover the Equity as residual claim. A(x) denotes the terminal value of the assets $\forall x \in$ X;
Only the worst scenario, x=down down, is problematic:

$$B(0) = \frac{E^q[\Pi^B(x)]}{R^2} = \frac{E^q[min(A(x), FaceValue=100)]}{R^2} = \frac{100(p^u+p^up^d)+A(x=down, down)p^dp^d}{R^2}$$

Concluding with the share price S, for each node it holds[1]:

$$S(t) = A(t) - B(t) - W(t) = \frac{E^q[\pi^S(t+1)]}{R} = \frac{p^u\pi^S(x=child.up)+p^d\pi^S(x=child.down)}{R} = \frac{p^uS(x=child.up)+p^dS(x=child.down)}{R}$$

The warrant to vanilla option ratio corresponds to 1 - dilution ratio.

2 Act two: convertible bond

The bond presents a convertibility option, i.e. a European call on equity with strike the face value. The sequential bond vs warrant holder game has a unique SPNE[2] in each terminal node. In x=up up, all the players have a dominant strategy: to exercise their respective options. Equivalently, the following arbitrage-free system holds, $\forall x \in$ X:

$$\begin{cases} \Pi^W(x) = W(x) = max\left(\frac{A(x)+K}{5} - K, 0\right) = max\left(\frac{A(x)+K-B(x)}{2} - K, 0\right) \\ \Pi^B(x) = B(x) = max\left(\frac{3\,(A(x)-W(x))}{4}, min\,(A(x), 100)\right) \end{cases}$$

The share price formula is unchanged with respect to act one. Convertibility option value increases with rate and volatility.

[1] II equality holds only for non terminal nodes.
[2] Only the scenario x=up up deserves a study: warrant exercise was out of the money in all the other scenarios with vanilla debt. Similarly, we have seen in class that bond conversion was OTM for all the other scenarios: the exercise sets cannot increase increasing the dilution.

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Homework 2

1 Part One: S&P 500 as Market Portfolio

All the 19 countries with data starting from 1990 are included. Source for the risk free rate is prof.French website as for countries stock market returns: the first portfolio group is selected, as it is more general[1] (Consider the first column: market returns). S&P 500 data are from investing.com. The explanatory power of the market returns in determining the country-specific portfolio return is:

$$R_t{}^i - R_t{}^f \;=\; \alpha^i + \beta^i(R_t{}^m - R_t{}^f) + \epsilon_t \tag{1}$$

Taking expectations:

$$\mathbf{E}[R^i - R^f] \;=\; \alpha^i + \beta^i(\mathbf{E}[R^m - R^f)] \tag{2}$$

Taking $\lim_{\alpha \to 0}$ and switching to fitted excess returns:

$$\mathbf{E}[\widehat{R^i - R^f}] \;=\; \beta^i(\mathbf{E}[R^m - R^f)] \;\stackrel{?}{=}\; \mathbf{E}[R^i - R^f] \tag{3}$$

Then run:

$$\mathbf{E}[R^i - R^f] \;=\; \gamma + \lambda\beta^i \quad \text{or, rescaling the x axis by } \mathbf{E}[R^m - R^f],\;\; \mathbf{E}[R^i - R^f] \;=\; \gamma^s + \lambda^s\mathbf{E}[\widehat{R^i - R^f}] \tag{4}$$

That fits CAPM equation (4) if $\gamma = 0 \land \lambda = \mathbf{E}[R^m - R^f]$. Note the $\alpha = 0$ in almost all the regressions: is pointless to model the expected risk premium of a country with the market one if there is a systematic under/over performance affecting many of the assets. Lowest betas are for Japan and Switzerland. The Nordic countries, apart from Denmark, are at the top level of the market risk.
Regression suggests CAPM only partially holding, as $\lambda_{\text{S\&P}}$ is relatively larger than the expected market risk premium $\lambda_{\text{S\&P}}{}^s = 1.3781$, with $R^2 = 0.34$. Consistently with the model we have $\gamma_{\text{S\&P}} = 0$. CAPM not holding can be attributed either to the model not being accurate[2] or to the market portfolio not being representative. S&P is affected by country and capitalization (first 500) restrictions and by a relevant tech overweight. This prevents from accessing to the high risk, high remuneration segment of small and micro cap stocks, flattening the efficient frontier, thus diminishing the marginal remuneration of risk.

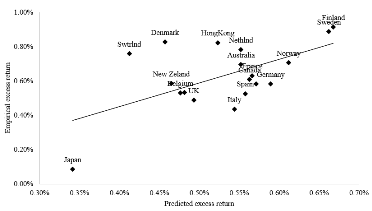
2 Part two: MSCI World and Gold

Now, consider a more global market portfolio, downloaded directly from the index provider, still affected by the capitalization bias[3]. No way will the model improve as $\mathbf{E}[R^m - R^f)] \downarrow \land \beta^m = 1$. The average absolute increase in β is 0.112, with Japan almost doubling and Canada unaffected: this means only an higher sensitivity to the market risk, not market excess return becoming the only explanation to country excess return. The share of countries with a positive $\alpha_{\text{MSCI}}{}^i$ is now $> 20\%$, which suggest a reflection before going deeper on the analysis. There is no improvement in the explanatory power of the market portfolio, the new R^2 being 0.13. In fact, λ_{MSCI} is again larger than the expected market risk premium, with $\lambda_{\text{MSCI}}{}^s = 1.581$ and $\gamma_{\text{MSCI}} = 0$. Note, $\lambda_{\text{MSCI}}{}^s > \lambda_{\text{S\&P}}{}^s > 1$.
Concerning the soundness of gold capturing some source(s) of risk, as suggested in class, the main risk hedged by gold is inflation and sometimes a more general financial instability risk. 30% of the sample shows $\beta_{\text{gold}}{}^i = 0$, while $MAX[\beta_{\text{gold}}{}^i] = 0.35$. A good starting point might be to study the negative correlation between Central Bank independence and β_{gold}, accounting for Australia and Canada, the 3^{rd} and 4^{th} global gold producers[4].

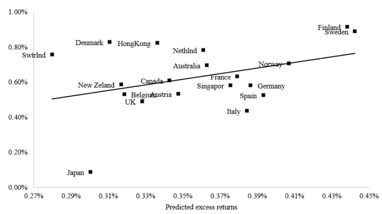
β point estimation

	Japan	Swtrlnd	Denmark	New Zeland	Belgium	Austria	UK	HongKong	Italy	Nethlnd	Australia	Spain	Canada	France	Singapor	Germany	Norway	Sweden	Finland
$\beta_{\text{S\&P}}$	0.61	0.74	0.82	0.83	0.85	0.86	0.88	0.94	0.97	0.99	0.99	1.00	1.01	1.01	1.02	1.06	1.10	1.19	1.20
β_{MSCI}	0.89	0.83	0.92	0.94	0.94	1.03	0.97	1.00	1.14	1.07	1.07	1.16	1.02	1.12	1.11	1.14	1.21	1.31	1.30
β_{Gold}	0.15	0.12	0.14	0.22	0.15	0.22	0.10	0.24	0.07	0.09	0.35	0.04	0.33	0.07	0.27	0.10	0.30	0.13	0.15

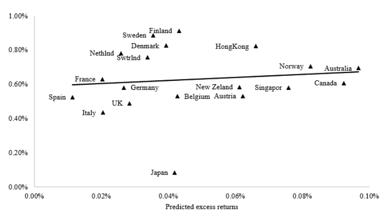
CAPM verification: $\mathbf{E}[R^i - R^f] \;=\; \gamma^s + \lambda^s\mathbf{E}[\widehat{R^i - R^f}]$



S&P 500, $\lambda^s = 1.38, R^2 = 0.34$



MSCI, $\lambda^s = 1.59, R^2 = 0.14$



Gold, $\lambda^s = 0.87, R^2 = 0.01$

[1] In fact, companies are included in the first portfolio subset even if some multiples are missing.
[2] If there is only one source of risk, why do agents need so many securities to manage their risks?
[3] MSCI World standard refers to large and mid cap.
[4] https://davideromelli.com/cbidata/

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Homework 3

1 Partial equlibrium

Denote by X the payoff matrix. The market is complete:

$$\text{Rank}(X) = N = K \tag{1}$$

The admissible consumption values:

$$D(f(C_0) * g(C_1)) = \{C_0 | C_0 \in D(f) \wedge C_1 | C_1 \in D(g)\} \Rightarrow C_0 > 0 \wedge C_1 > 0 \tag{2}$$

The intertemporal function is symmetric Cobb-Douglas: for any level of R (the interest rate) it holds:

$$U = \ln(C_0) + \ln(\mathbf{E}[C_1]) \Rightarrow {C_0}^{optimal} = \frac{W\alpha}{\alpha+\beta} = \frac{W}{2} = \theta' p = 1 \Rightarrow \text{U'}(C_0) = 1 = \frac{1}{C_0} = \phi_0 \tag{3}$$

By recovering ϕ_1, with p being the vector of prices, observe $\|\phi_1\|_\infty$ positive (arbitrage-free market):

$$\phi_1 = X^{-1}p \tag{4}$$

Equation 3 simplifies optimal consumption problem:

$$\begin{cases} \Pi_{\text{k=1}} U'(C_{1,\text{k=1}}) = \dfrac{\Pi_{\text{k=1}}}{C_{1,\text{k=1}}} = \phi_{1,1} \Rightarrow C_{1,\text{k=1}} = 2.1739 \\ \Pi_{\text{k=2}} U'(C_{1,\text{k=2}}) = \dfrac{\Pi_{\text{k=2}}}{C_{1,\text{k=2}}} = \phi_{1,2} \Rightarrow C_{1,\text{k=2}} = 3.1250 \\ \Pi_{\text{k=3}} U'(C_{1,\text{k=3}}) = \dfrac{\Pi_{\text{k=3}}}{C_{1,\text{k=3}}} = \phi_{1,3} \Rightarrow C_{1,\text{k=3}} = 1.6667 \end{cases} \tag{5}$$

The optimal portfolio allocation θ:

$$\theta = \begin{bmatrix} \theta_{stock1} \\ \theta_{stock2} \\ \theta_{bond} \end{bmatrix} = (X')^{-1} \begin{bmatrix} C_{1,k=1} \\ C_{1,k=2} \\ C_{1,k=3} \end{bmatrix}' \Rightarrow \begin{bmatrix} \theta_{stock1} = 0.5072 \\ \theta_{stock2} = -3.1703 \\ \theta_{bond} = 5.2174 \end{bmatrix} \tag{6}$$

That requires no short-selling restriction and short-selling at the risk-free rate r. These constraints will in fact impact $R \uparrow$ in the next point.

2 General equilibrium

Utility function does not allow negative saving, i.e. borrowing money from other agents. Considering the representative investor, as no one borrows, no one can lend. W being unchanged and R being irrelevant:

$$\tilde{C_0}^{optimal} = 1 \tag{7}$$

Invoke the budget constraint $\forall k \in \Omega_1$ to recover state prices, with $x_{\text{i,j}} \in$ X:

$$\begin{cases} \tilde{C}_{1,\text{k=1}} = x_{\text{Stock1,k=1}} + x_{\text{Stock2,k=1}} = 2.7 \Rightarrow \tilde{\phi}_{1,1} = \dfrac{\Pi_{\text{k=1}}}{\tilde{C}_{1,\text{k=1}}} = 0.1235 \\ \tilde{C}_{1,\text{k=2}} = x_{\text{Stock1,k=2}} + x_{\text{Stock2,k=2}} = 2.4 \Rightarrow \tilde{\phi}_{1,2} = \dfrac{\Pi_{\text{k=2}}}{\tilde{C}_{1,\text{k=2}}} = 0.1389 \\ \tilde{C}_{1,\text{k=3}} = x_{\text{Stock1,k=3}} + x_{\text{Stock2,k=3}} = 1.7 \Rightarrow \tilde{\phi}_{1,3} = \dfrac{\Pi_{\text{k=3}}}{\tilde{C}_{1,\text{k=3}}} = 0.1961 \end{cases} \tag{8}$$

By arbitrage-free pricing:

$$\begin{bmatrix} p_{stock1} \\ p_{stock2} \\ p_{bond} \end{bmatrix} = X\tilde{\phi_1}' = \begin{bmatrix} p_{stock1} = 0.4916 \\ p_{stock2} = 0.5084 \\ p_{bond} = 0.4584 \end{bmatrix} \tag{9}$$

Consistently $p_{\text{stock1}} + p_{\text{stock2}} = 1$. R must be consistent with p_{bond}, even if there is no such a market:

$$R = \sum_{k=1}^{3} \frac{1}{\tilde{\phi}_{1,\text{k}}} = 2.1814 = \frac{1}{p_{\text{bond}}} \tag{10}$$

Which represents the deterministic consumption: the larger the draw-down from that required consumption level, the larger the risk-neutral probability. This confirms risk-adversion of the investor.

$$\tilde{\pi}_1 = \begin{bmatrix} \tilde{\pi}_{1,k=1} \\ \tilde{\pi}_{1,k=2} \\ \tilde{\pi}_{1,k=3} \end{bmatrix} = \frac{\tilde{\phi}_1}{\sum_{k=1}^{3} \tilde{\phi}_{1,\text{k}}} = \frac{\begin{bmatrix} \tilde{\phi}_{1,k=1} \\ \tilde{\phi}_{1,k=2} \\ \tilde{\phi}_{1,k=3} \end{bmatrix}}{\sum_{k=1}^{3} \tilde{\phi}_{1,\text{k}}} = \begin{bmatrix} \tilde{\pi}_{1,k=1} = 0.2693 \\ \tilde{\pi}_{1,k=2} = 0.3030 \\ \tilde{\pi}_{1,k=3} = 0.4277 \end{bmatrix} \tag{11}$$

Lagrange multipliers are consistent with this analytical solution.

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Homework 4

1 Fama-French 3 factor

	Food	Beer	Smoke	Games	Books	Hshld	Clths	Hlth	Chems	Txtls	Cnstr	Steel	FabPr	ElcEq	Autos
HML	-0.032	-0.109	0.129	-0.229	0.383	-0.122	0.265	-0.350	0.260	0.601	0.311	0.896	0.527	0.451	0.304
SMB	-0.218	-0.394	-0.195	0.453	0.771	-0.270	0.245	0.390	0.290	0.575	0.591	1.187	0.462	0.664	0.548
MKT	0.596	0.671	0.640	1.181	0.886	0.711	1.153	0.821	1.112	1.001	0.930	1.299	1.282	1.241	1.297
Alfa	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.001	0.000	0.001	-0.001	0.001	0.000	0.001
Pricing error volatility	0.005	0.006	0.012	0.009	0.008	0.007	0.007	0.005	0.006	0.015	0.006	0.009	0.005	0.006	0.012
	Carry	Mines	Coal	Oil	Util	Telcm	Servs	BusEq	Paper	Trans	Whlsl	Rtail	Meals	Fin	Other
HML	0.180	0.246	1.020	0.912	-0.086	0.148	-0.548	-0.045	0.282	0.426	0.270	-0.057	-0.179	0.464	0.259
SMB	0.118	0.423	1.554	0.320	-0.291	-0.060	0.056	0.158	0.248	0.359	0.512	0.025	-0.159	0.079	-0.008
MKT	1.018	0.795	1.035	1.147	0.448	0.727	1.166	1.351	0.896	1.145	0.944	0.935	0.785	1.111	0.880
Alfa	-0.001	0.000	-0.003	-0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Pricing error volatility	0.009	0.008	0.019	0.008	0.006	0.005	0.004	0.005	0.006	0.006	0.004	0.005	0.005	0.003	0.004

2 Cumulative abnormal returns

For each industry i compute the Cumulative Abnormal returns, taking into account the 3 relevant days of march 2020 t:

$$\text{let } X_{\text{t}} = [F^{hml}{}_{\text{t}}, F^{smb}{}_{\text{t}}, F^{mkt}{}_{\text{t}}]\ , \forall t \in \text{dates} = [9.03.20, 12.03.20, 16.03.20], \forall i \in \text{industries}$$

$$CAR_{\text{i}} = \sum_{t=1}^{T} \epsilon_{\text{i,t}} = \sum_{t=1}^{T} r_{\text{i,t}} - \hat{r}_{\text{i,t}} = \sum_{t=1}^{T} r_{\text{i,t}} - \alpha_{\text{i}} - r_{\text{risk-free,t}} - \beta_{\text{i}} X_{\text{t}} \tag{1}$$

Switching to inference, assuming the point 1 volatility is a suitable estimator for the pricing error volatility in the event window:

$$z - stat_{\text{i}} = \frac{CAR_{\text{i}}}{\sigma(\epsilon_{\text{i}})\sqrt{card(\text{dates})}} = \frac{CAR_{\text{i}}}{\sigma(\epsilon_{\text{i}})\sqrt{3}} \longrightarrow \begin{matrix} H_0 : CAR_{\text{i}} = 0 \\ H_1 : CAR_{\text{i}} \neq 0 \end{matrix} \tag{2}$$

					Significant negative abnormal return							
Industry		Util ***	Meals***	Cnstr***	Beer ***	Food ***	Carry***	Oil ***	Telcm***	Smoke***	Whlsl*	
CAR		-0.17253	-0.08866	-0.09311	-0.0867	-0.04902	-0.07767	-0.04893	-0.02489	-0.05511	-0.01175	
Z.stat		-15.7708	-9.85561	-9.06807	-8.2483	-5.27785	-5.1359	-3.34593	-2.78384	-2.73636	-1.66603	
					Abnormal return not different from zero							
Industry			Games	Hshld	Hlth	Mines	Fin	Other	Txtls	Servs		
CAR			-0.01904	-0.01118	-0.0074	-0.00031	0.001396	0.005102	0.026345	0.007579		
Z.stat			-1.23632	-0.98691	-0.93043	-0.02303	0.276256	0.803452	1.028783	1.172866		
					Significant positive abnormal return							
Industry	Autos*	Clths**	Chems***	Coal ***	Steel***	Books***	Rtail***	Paper***	Trans***	ElcEq***	FabPr***	BusEq***
CAR	0.03685	0.029251	0.03047	0.133393	0.062187	0.056077	0.034955	0.053708	0.051456	0.086218	0.081991	0.083766
Z.stat	1.756258	2.303837	2.926019	4.033854	4.203432	4.311298	4.401709	4.778515	5.190479	8.149378	9.074114	9.69147

There are positive and negative abnormal returns, so the *signum* must be taken into account differently from the class exercise. The discussion will follow the table order. A concise summary: *the lower the market β, the lower the CAR, and vice versa.*

Limits of physical interaction and relevant exposure to retail consumers determine negative abnormal returns for food and beverage, civil constructions, vacation and utilities. In conjunction with the determinant mentioned above, the drawdown in oil price explains the negative abnormal return for the oil industry and carry, as aeroplanes are changed when inefficient. In this first group of negative abnormal returns, several industries present negative small minus big β, signalling their resilience and anti-cyclical behaviour. An industry with a low margin of manoeuvre for incumbents should protect investors during downturns: this did not happen.

Why high beta industries contained their losses concerning the proposed model? For the negligible exposure to retail consumers, e.g. chemicals, coals, fabricated products and machinery, busyness equipment and steel. An incremental determinant for some industries showing positive abnormal returns is their role in the new *at home* mood, e.g. book industry and retail industry. The previous distinction is not a partition: the two determinants affect the paper industry and the electrical equipment. The transportation industry has a positive abnormal return: the first-order effect is due to a decrease in oil price, as the sales compression was initially supposed to be temporary. Furthermore, the hedging between people transportation and cargo started to benefit from the limits in physical interactions.

Why did healthcare not outperform the market at the commencement of a pandemic? It is due to the structure of the US healthcare market, the decreased non-infectious healthcare activity and the forecasts of income contraction. Industries that should have decreased more decreased less and vice versa: a proof of the model's limits, at minimum, in panic selling days?