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Enhanced pension: an Italian way to long term care?

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Abstract: Long term care is an emerging field of life insurance: according to the U.S. Department of Human Services, 70% of adults 65 and over will require some type of long-term care during their lives and that is why this scope is devoted one-third of the Medicaid programme. To quantify the problem, we report some consideration on the economic and social-psychological cost of caregiving. The efficiency of the private market on underwriting this risk is highly discussed by N. Barr (2010), for questions of asymmetric information and (genetic) adverse selection. The main difference with acute health insurance is the whole life length of the contract for long term care, instead of the year over year payment. Among the different long-term contracts, we consider a possible option for converting the savings of Italian pension funds: instead of buying an ordinary annuity, it is possible to buy a contract that pays a doubled amount if the individuals lose his autonomy. To fairly price this enhanced pension, we introduce the role of natural hedging in insurance and explain how it is possible to profit from this in life insurance. We then recover mathematical set-up through the instruments of Markov chains to reach closed formulae (only Markov) and algorithmic pricing with Montecarlo simulation. Before reaching this, we will discuss how to estimate transition probabilities for the transition matrix, according to the approach proposed by Levantesi and Menzietti (2016). Results of our experiment are relevant: the benefit from hedging coming from the anticorrelation between life expectancy and morbidity (invalidity) is mainly attributable to males but is substantially used to decrease the premium load on females' contracts. Males and females could be seen as diversified assets of an insurance portfolio and the equilibrium level might be reached only with enough incentives. We even investigate additional guarantees, a lumpsum at losing autonomy and a death case one, for their relevant role in risk reduction and in absorbing insurance needs.

Enhanced pension: an Italian way to long term care?

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1) Introduction

Enhanced pension: an Italian way to long term care?

The main problem of a growing old population is the need for long term care aid. We will first address the question of how to fund it, discussing if long term care (LTC) should still be a suitable case for public insurance, as depicted in Barr 2010, with little room for manoeuvre for private actors. The author will keep an eye on the problem of informal care and the features of private insurance contracts within different countries, in terms of the structure of the products and their implications. Concerning the Italian market, note the diffusion of an enhanced pension product, where the insured receives an annuity that doubles once the insured loses his autonomy. We will discuss the main advantages and the main limits of this form of insurance: we will examine his role concerning more "traditional" insurance products, where the authors pay periodical premiums for a specific coverage that lasts until his death and a lump-sum amount once he entered LTC costs. We set a semi-Markov model to study the whole-life dynamic of the insured (if he is alive and healthy if he/she is dead if he/she is LTC), focusing on transition probabilities. The choice between a Markov model and a more generalized one has been correctly weighted: but only in this last framework, it is possible to consider the duration of LTC status in estimating LTC mortality. We then realized a numerical simulation discussing results: first, we do calculate fair premia (in terms of conversion coefficients), then we split into pure annuity share and LTC insurance share. Then a relevant sensitivity analysis has considered the above-mentioned variables and the interest rate. We have extended our research to more sophisticated products enhanced pensions with a death guarantee (of five or ten years) to decrease the overall risk of the contract, and enhanced pensions with a lump sum payment once to insured enters LTC status, to account even for fixed and not only variable costs of LTC. Results are clear enough: there is still a certain distance between observed prices of the different companies and between observed prices and our numerically recovered results. Consider that still there is not a standard definition of what loss of autonomy is in the different companies, and this leads to differences in price. Concerning our numerical results, we observe that the market price is significantly lower than our recovered price, while results are substantially in like for women. This is mainly due to the relevant impact of semi-Markov models (that are substantially different between the two genders) and to the need for an equilibrium between males and females for an insurance company. Furthermore, since females are less risky than males, there is a load repartition that is increasing in risk: the higher the risk, the higher the load in relative terms. Our result is, in conclusion, fully depicting an underdeveloped and still opaque market. In chapter II, we introduce the economics of long-term care contracts, explaining differences between acute healthcare and different needs, secondly, we cope with the question of measuring the loss of autonomy with a comparison with the most used scales. Furthermore, we cope with the indirect costs of caregiving, on social relationships, mental health, and labour supply: from this, we can observe that there are for sure positive externalities in ensuring this risk. Unfortunately, as observed by Barr there are important limits for efficient private insurance: in the conclusion of this chapter, we cope with the specific contract of the Italian market, the enhanced pension. In chapter III, we introduce relevant stochastics instruments to price this product: the *Rota way* is fully adopted for Markov processes. We introduce the concepts of absorbing and recurrent and transient states. We then provide formulas for pricing of different components of the contract (healthy case, death case, LTC case) and a brief introduction on how to recover transition probabilities between the different states of the semi-Markov chain, that are completely suitable for numerical pricing. We discuss and propose a correction for the mortality of LTC individuals according to the time they spent in this status: in the first years of loss of autonomy there is, in fact, an accumulation of mortality. In the very last chapter (IV), we produce the results of our Montecarlo simulation, starting from the estimation of fair premium levels and Sejour times for different ages of conversion and genders. In conclusion, we observe that the contract is sold at discount to females and premium to males. We provide an adequate explanation for this. We apologize for the sections not being partitions, in the sense that intersections between economics, actuarial mathematics and algorithm creation show relevant cardinality.

2) The economics of long-term care contracts

2.1) Against an acute-based healthcare

In the last two years, we have experienced the *debacle* of many systems of preventive medicine and public health. This, was, highly probably, not due to the lack of ability of physicians but, more in truth, to a decreased funding allocation in the last decades. Overall considered, however, the health expenditure in main countries has not diminished. What happened is, at least stopping to a first order approximation, crystal clear enough: since the Long-term care expenditures increased sharply over the last years, other expenditures (mainly preventive medicine) needed to be reduced proportionally to follow (existing, unfortunately) financial constraints. What matter in economics is not the level of a variable but is the growth rate over a certain number of years: not only long-term care expenditure (formally and informally defined in that way[1]) increased in the last years, but is projected, as will be referenced after, as to increase even more. This is one of the biggest problems of the growing share of the older population in Europe and all developed countries is the higher share of people needing different forms of aid: this aid is not necessarily a medical one, but it could be a social or informal one. One of the most relevant articles on this topic Colombo, F. “Help wanted? providing and paying for long-term care.” (2011) shows that the share of the population involved in long term care (LTC) is on average 2.3% of the population in (Organisation for Economic Co-operation and Development) OECD countries, with a peak of 5% in Austria and northern countries and a lower level in Mediterranean nations, such as Italy, Spain, and Slovak. The number of elderlies in the US aged over eighty will double in relative terms concerning the population, within 2050: as LTC usage is extremely sensitive to age, this will increase LTC. By contrast, 40% of LTC are nonelderly individuals, where needs come from injuries or developmental disabilities or mental illnesses: this is going to decrease as important medical improvements happened for cancer & cardiovascular pathologies, stricter regulations improvements to prevent injuries and the early diagnosis of genetical illnesses leads to early and improved medications.

The main problem of the existing long-term care insurance systems is that public expenditures are growing rapidly, and yet, at the same time, individuals still have uncovered expenses. These twin concerns have led policymakers in search of solutions that rely on ensuring a greater share of this risk through private markets; further interesting to investigate share of institutional care use vs homecare use: the ratio is 30% institutional care and 70% home care, but there are remarkable exceptions of the prevalence of institutional care, like Ireland, Iceland and Canada and opposed situations, like the UK, where there is a huge prevalence of Homecare. A common element to all the developed countries is that the share of pocket expenditure for LTC is higher than the share of Out-of-pocket expenditure for health insurance: this element was the main driver for Medicaid in the US, as LTC accounts for 33% of overall expenditures. The high share of cash expenses is remarkable: from a theoretical point of view in LTC risk insurance dominates self-insurance, but in front of the inefficiency of markets, individuals prefer partially self-insurance. As the event becomes more probable for the whole cohort and the individual has no relative risk concerning the rest of the insured population the befits of insurance decrease, reaching asymptotically equivalence between insurance and cash support. Again, for older individuals the most uncertain variable is not if they will become LTC once they will be older than eighty-five, but if they will reach this age.

The average expenditure for OECD countries is 1.5% of GDP: not necessarily a lower amount is an indicator of bad social security services, it could be correlated with a younger or a healthier population. Private expenditure is less than 20% of the overall consumption, apart from the remarkable exception of Switzerland, where more than 60% of the expenditure is private and the

1 If a diabetic or cardiopathic accesses first aid for illness-related complications or takes an above-average sejour after a trauma, this is computed as acute expenditures in full, in truth, is should be accounted, for the incremental part as long term care expenditure. We can call this phenomenon as shadow long term care expenditure

contrary case of Norway and Sweden, the top two countries for overall LTC expenditure, where the expenditure is set up substantially in full through public money. As for all forms of insurances, it is possible, a priori, to rely both on a public and private insurance system[2]. Consider that, focusing on the prevalence of LTC on the overall population, The higher the mean age of the country, the higher the share of LTC users, the higher the GDP per capita, the higher the share of LTC users3. Surely there are negative externalities from the public market onto a private one. Removing public market inefficiencies is a necessary, but not sufficient condition for the deployment of a private one. What if after the necessary reforms in public insurance the private market does not develop? This means that the private market of LTC is inefficient per se, and not dependently on externalities. There will be the same private supply, but less than the public one: this will result in a loss of welfare. This is not an unimaginable scenario, considering how poorly LTC insurance could face a dramatic, sharp rise of the share of insured presenting loss of autonomy.

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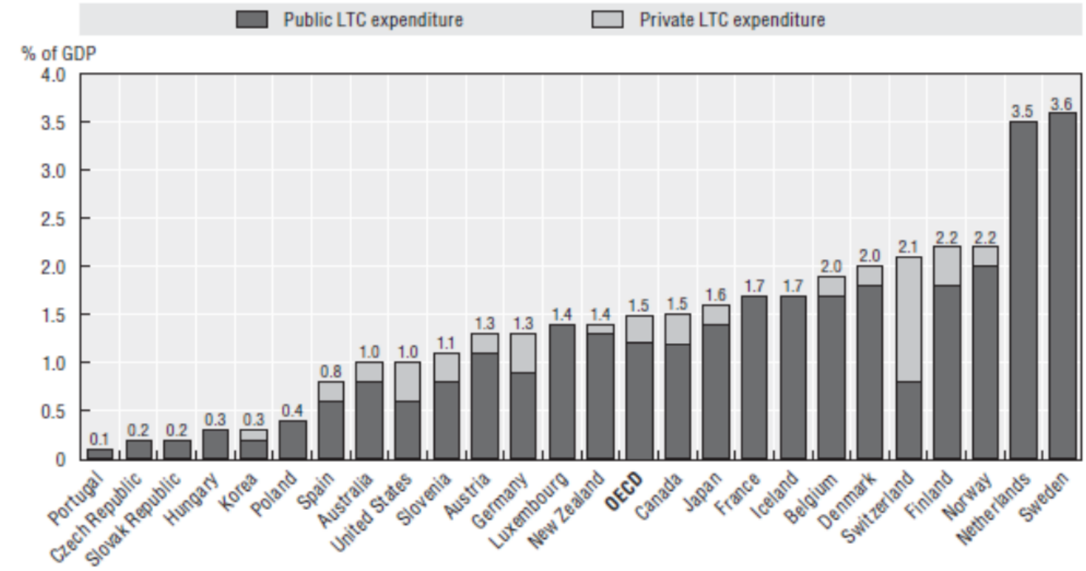
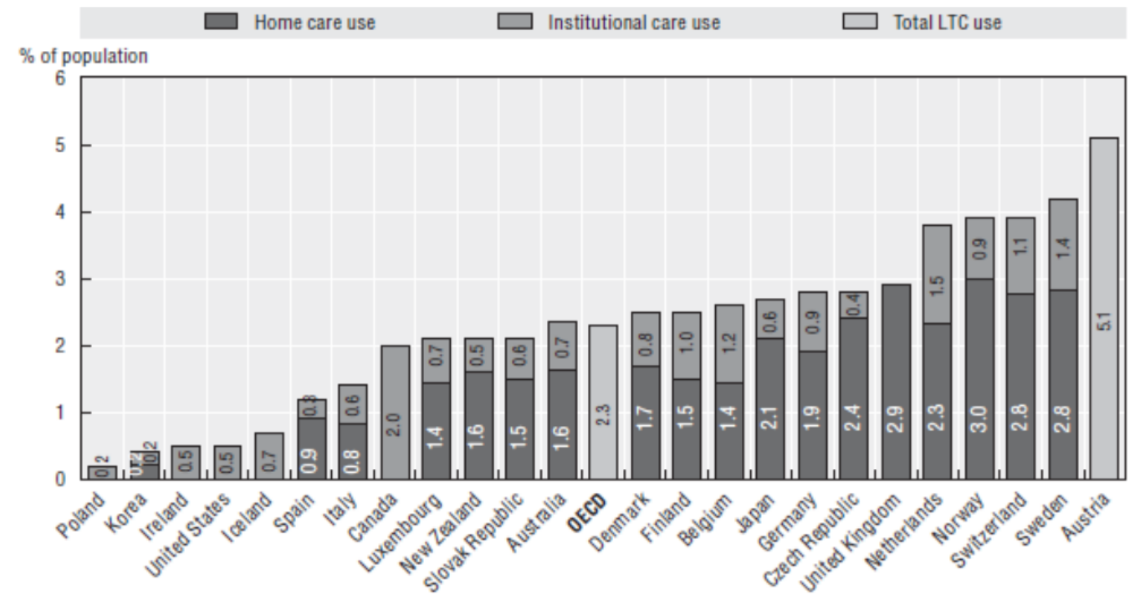
2 Long term care expenditure as a percentage of GDP, with distinction for private and public expenditure. Data refer to years 2010-2011. Source of imagines: Colombo, F., et al. (2011), Help Wanted? Providing and paying for Long-Term Care, OECD Health Policy Studies, OECD Publishing, Paris.



SOURCE : Colombo, F. et al. (2011), Fig. 1.8.

3 Incidence of long-term care users, with distinction between home care and Institutional care. Data refer to years 2010-2011.



SOURCE : Colombo, F. et al. (2011), Fig. 1.2.

The market for long term care consists of individual rather than group contracts, this is even due to the differed expected insurance claim (and so, a not-so-forward-looking employer will be interested in providing health insurance for the active life, but not long-term care one once the employed is retired): this consideration holds even in countries where the fiscal treatment of work-related and individual products is the same. In Italy, the context is complicated by some fiscal differences: employer-provided contracts are tax-free while individual insurance presents a 19% tax credit up to 1200 € of premium, with no opportunity to carrying forward. Diversification and risk pooling is more effective with a group contract than with individual ones. This particularly under diversifies the risk for genetic illnesses in the insurance portfolio, increasing sharply the left tail risk for the insurance company: this is riskier when the insured amount in case of loss of autonomy is particularly higher or there are no limits. People that know being at-risk for genetic or quasi-genetical illnesses not only insure more than the average in number, but the amount insured is higher than the average. Others said the underwriter should check the ratio insured amount/income or insured amount/wealth and in case of (right) outlier's act by consequence. Consider from the study of Brown and Finkelstein (2011) [4] the determinants for owning a Long-Term Care plan: first, the income, with a deeper dependence pat for women, and then the marital status. Concerning age, we observe a reversed-U-shaped intensity: we start with low levels in class 60-64, we observe the maximum ownership rate between 65 to 79 and in conclusion, we have rebates to lower level even in individuals from 80 years[5.] The main explanatory variable for the difference between single and married is due to the higher liquidity in general set aside by single and with the lower interest in protecting the mate from the psychological costs and material costs of healthcare. Moreover, if sons know that they will have to take care of only one parent, the "value at risk" might be lower. The main limit in developing a private market solution for this need is the lack of information and pricing structures for developing our abused supply and a low interest by the workers in ensuring their autonomy in the future.

In Germany from the early 90s, there is a fully funded compulsory long-term care insurance. Consider that older individuals dispose of an income that is less than the average income in the country, and these decreases are more relevant for older individuals, on average, and for people of countries with

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4 Brown, Jeffrey R., and Amy Finkelstein. 2011. "Insuring Long-Term Care in the United States." Journal of Economic Perspectives, 25 (4): 119-42, and the appendix https://assets.aeaweb.org/asset-server/articles-attachments/jep/app/2504_Brown_app.pdf

5 Private long term care ownership rate for the US, selected by social and economic variables.

		By wealth quintile				
	Whole sample	*Top*	*Fourth*	*Third*	*Second*	*Bottom*
Whole sample	13.8%	26.9%	19.0%	10.7%	6.6%	4.1%
By gender						
Men	13.6%	25.5%	17.1%	10.0%	4.8%	5.5%
Women	13.9%	28.4%	20.7%	11.2%	7.8%	3.3%
By marital status						
Married	16.3%	28.0%	19.2%	10.3%	5.9%	5.5%
Single	10.4%	23.5%	18.8%	11.2%	7.3%	3.6%
By age group						
60–64	12.7%	24.1%	18.7%	9.3%	5.8%	4.7%
65–69	14.7%	29.6%	19.4%	8.8%	5.9%	5.5%
70–74	15.0%	29.6%	16.8%	14.8%	6.6%	3.5%
75–79	14.7%	28.2%	21.1%	10.5%	8.6%	2.6%
80–84	13.9%	25.0%	20.8%	12.5%	6.9%	5.0%
85+	10.9%	22.1%	19.2%	8.7%	7.6%	1.6%

Source : Brown and Finkelstein (2011), Tab. 1, p. 124.

poor publicly financed LTC insurances: this is particularly relevant as we have seen that income is one of the most relevant determinants in buying an LTC annuity. That is why taking part in the scheme is mandatory for all the workers in the active phase. The premium is 2% of the row salary, independently from the actuarial risk and there are three (four) levels of needs:

- Care level I: a need for assistance for at least 90 min per day with basic care needs of at least 45 min per day.
- Care level II: a need for assistance must be at least 180 min per day with basic care needs of at least 120 min per day.
- Care level III: a need for assistance is required for at least 300 min per day with basic care needs of at least 240 min per day.
- Case of hardship: the care fund can provide more services in line with care benefits and inpatient care.

The allowances linked to these three states increase more than proportionally. Introducing levels increases the complexity of the contract, but the acquired flexibility and tailor-made ability to absorb needs show undoubtedly the benefits for a level based LTC insurance. By contrast, in the generality of the countries, there is a fewer level of public intervention and higher space for a private one: in this term, it is important to remark the key role played by France, with a well-established private insurance market, with a wide and reliable offer of products, sold through the bancassurance channel: also, the French system supplies two distinct levels of care. Consider that the significant success of *bancassurance* in selling LTC insurances in France (60% of the whole contracts[6]) should be adequately scrutinized: is this, possibly, signalling agency problems? Is it possible, roughly speaking, that potential insured prefer to buy a contract deeply conditioned by their lifestyle, their attitudes and by their past's health issues by an agent that is generally less aware of their status than the *family insurer*? We should consider that generally, insurance has a wider portfolio of sensitive pieces of information that could badly affect the underwriting process. We mention, in conclusion, a detailed study on transition probabilities for long term care insurances for Switzerland[7]: first of all, the authors show the impact of the duration of the status on the permanence and transition probabilities (in the same status, or to a deeper level of care or the death) and with a remarkable level of originality the propose a (semi-) Markov model with the following states: [healthy, helped at home, helped in an institution, death]. This approach might be particularly helpful in a better fitting of mortality curves and a better response to the insurance needs by newly defined contracts: the realistic assumption on which the author is relying is that institutional care is harder than domiciliary one and that, consistently on what we will consider, there is no reversion from institutional care to domiciliary one. Once an insured enters the institutional care state, the LTC contract substantially transforms into an ordinary annuity: the insured could only either continue staying in this state or pass away.

2.2) Measuring insured needs

Another question is how to define the loss of independence: the first definition is Katz scale of activities of daily living (ADL)[8], which consists of six abilities: bathing, dressing, toileting, transferring, continence and feeding. The lower the stand-alone capabilities in a specific need, the higher the points received. LTC starts only after a certain floor of points accumulated. Not necessarily all the abilities are equally weighted. Successively, insurers set up an instrumental scale of activities

[6] Etienne Dupourqué, Long Term Care in the United States (2019). In actuarial aspects of long term care, springer actuarial, 2019 p.50

7 Michel Fuino, Joël Wagner, Long-term care models and dependence probability tables by acuity level: New empirical evidence from Switzerland, Insurance: Mathematics and Economics, Volume 81,2018, Pages 51-70.

8 Katz S, Downs TD, Cash HR, Grotz RC. Progress in development of the index of ADL. Gerontologist. 1970 Spring;10(1):20-30. doi: 10.1093/geront/10.1_part_1.20. PMID: 5420677.

of daily living (iADL)[9]: the five abilities are the ability to use the telephone, shopping, food preparation, housekeeping, laundry, mode of transportation, responsibility for own medication and ability to manage finances. Functioning and screening for LTC is like the already mentioned procedure and is frequent to use a combination of iADL and ADL measures, decreasing the age bias risk. Therefore, the dimension of the LTC problem appears not to be a physical dimension, where the individual is not able to deal with himself and an instrumental one, where the main difficulties are in the relationship with external instruments. To this point, the French scale AGGIR seems to tackle the point, as it favours the creation of group iso-resources, i.e., clusters with the same difficulties and the same abilities. This scale is suitable for public insurance, to allow the creation of a cluster of insured with the same problems our group of problems: the idea was to top up, their monetary benefit with service and residence provides, however little few apart from the APA monthly benefit had been put in place. There is also an operational benefit linked to this scale, that is the opportunity of building facilities and services directly tackling the GIR group, going from one (complete loss of autonomy) to six (substantial autonomy). GIR levels are suitable for allowing to tackle the insured needs more directly and more efficiently both in terms of adding quite adequate monetary allowance and supplying the right package of services, decreasing the basis risks of insurance the covers only partially need but decreasing monetary outflows.

The main limit off the AGGIR scale is the number of levels (6, where private based are on 3 or 2 levels) that might be too hard to manage for a private actor. By contrast, this will be an advantage when studying the transition between various levels of care, as it is a full need-based index. The two levels of care of the France private insurance are equivalent to levels 1-2 and levels 3-4 of the public scale. Consider in conclusion the key role the AGGIR scale, originally perceived as a public one, might play for the private insurance market, allowing the transition from intensity-based insurance to a need-based one. We notice that even in defining long term care, especially for Mediterranean countries, we still cope with pre-existing public insurances definitions.

This scale served as a first whole-country estimation of transition probability which suggested the following results[10]: the transition from independent living to disability (GIR 1-4) is more probable for individuals younger than 75 and, in general, more probable for men than for women. Education and health status play a protective role, while a relatively high number of pregnancies >=2 exposes

9 Lawton MP. The functional assessment of elderly people. J Am Geriatr Soc. 1971 Jun;19(6):465-81. doi: 10.1111/j.1532-5415. 1971.tb01206. x. PMID: 5094650.

10 Transition probabilities from healthy status to GIR 1-4 and from low dependence to full one. Logistic regression. Source: Duée at al., La dépendance des personnes âgées: une projection à long terme,2004. For males, the odds of transition to a more severe state (*Très dependant*) doubles for lower waiting time in the state *Moyenment dépendant.*

		Variables explicatives			
Y = Probabilité de transition	Cst.	Santé (-)	Etudes	2-3 enfants	Durée = 1
Non dépendant → disability (GIR 1-4)					
Homme, âge <= 75	1.109	1.445	0.484	-	-
Homme, âge >= 75	0.878	1.416	1.095	-	-
Femme, âge <= 80	0.845	1.070	-	0.455	-
Femme, âge >= 80	0.650	1.166	-	-	-
Dépendant (GIR 1-4) → Très dépendants (GIR 1-2)					
Homme, âge <= 75		Probabilité moyenne : 17.4 %			
Homme, âge >= 75		Probabilité moyenne : 32.5 %			
Femme, âge <= 80		Probabilité moyenne : 21.1 %			
Femme, âge >= 80		Probabilité moyenne : 27.5 %			
Moy. dépendant (GIR 3-4) → Très dépendants (GIR 1-2)					
Homme	-0.277	0.512	-	-	0.731
Femme	0.570	0.579	-	-	-

Source : Duée et al. (2005)

to a higher risk's women under 80s. Concerning the model for transition probability: i.e., from the lower level of ADL impairment to a higher one (from GIR 1-2 to GIR 3-4), the relevant variables apart from gender and earlier health status is the duration of the earlier LTC status. Roughly speaking, once an individual becomes GIR 1-2 it is more probable than the average that in the first year he will pejorate to GIR 3-4. This is a relevant point, the explanatory variable of transition is not only gender, age, health status and studies but in a relevant manner the duration of the earlier status. This analysis holds in estimating the mortality conditioned to GIR levels. Consider that the same intensity might be associate with different monetary needs, sometimes the correlation is not enough strict, and the risk of underfunding situations is particularly high, especially if the scale adopted under-weights functional abilities: also, hybrids scale ADL and iADL are in favour of the first group. Concerning the Italian context, we do see an absence of level classifications: Italian insurances are not splitting LTC risk in different severities: as a result, the definition of what is LTC becomes at the same moment truly relevant and complex: note that this definition will influence the transition probability from health to LTC and even the mortality of LTC in the semi-Markov chain we will use for pricing. Insurance is working once help is for at minimum two ADL out of six or, more flexibly twenty points out of sixty, where each ability accounts for 10 points and partial values as 2,5 and 7 are applicable. As the reader may imagine this is a big limit of the Italian system that allows recognising only severe and widespread states thus not introducing any intermediate gradation. In front of uncertainty, in front of lack of forecast, the response should rely on graduation between different states and flexibility of the contract, something that is not at all present in the Italian insurance market. Moreover, the inflation in healthcare risk is not insured so the insured remains exposed to a rise in future healthcare cost, or even to the risk of over-insuring in case of a decrease. LTC nursing is labour intensive, more labour intensive than general healthcare and the inflation in LTC will beat inflation and inflation in healthcare costs. As a result, the inflation in LTC is well approximated by the real wages growth rate, considering that LTC workers, especially for higher intensity of care, are highly skilled. In Italy, in conclusion, virtually all the policies are in nominal terms, exposed to a wide specific and generic inflation risk: the problem affects all OECD countries unless Germany compulsory LTC covering general inflation risk. By contrast, we must consider that is even possible that in a medium-term scenario (10-20 years) the costs of LTC will decrease, up to a significant paradigm shift in our society (already taking place, in some metropolitan and sub-metropolitan areas) and to technology improvements (the massive use of telemedicine). Furthermore, even if the number of assisted is going to increase, is expectable that the number of caregivers relying on domiciliary care will increase, as an effect of the "sharing economy". This probably will mean that LTC might become more affordable so that the incremental annuity in case of loss of autonomy should be lower. However, no data are supporting the author's view.

The relative growth of long-term care is increasing concerning age faster than acute healthcare costs; as a result, as the population is rapidly ageing especially in developed countries, expecting that the share of LTC expenditure will quickly rebalance the share in acute care expenditure. The other point is that facing severe diseases family care will not be enough and specialist intervention will be relevant: when asking for a complex intervention individual and their family look over financial risk: they want suggestions, they need information and support for managing this complex service; moreover, this reduces more than hazard risk for the insurance. This happens in Germany where the insured has the full choice between monetary benefits and direct service providing: the hardest the loss of independence (level 2 or level 3), the higher the advantage in asking for a centralized providing, another important determinant of the choice it is the level of income: the higher the income the higher the probability of asking for services. This problem was present in the early French insurance market, which is today the deepest and the most widespread, turning to gradation (partial loss of auto sufficiency) and service/care providing. This shows why American residential living communities can offer long term care insurance, even if strictly speaking they are not insurance companies. Thanks to this practice, there will be a wider offer for services providing and the insurance intermediation is so increasing welfare, as the payments (generally lump sum) is directly made at the begging of the retirement period to the residential living community: for this reason, higher reserves are requested.

2.3) The social cost of caregiving

Long-term care cost is not only health costs but also social costs: the question is that informal care is central for the impacts on the Labour supply of women and their mental health. The share of LTC covered by informal care is quite relevant, even if difficult to estimate. Low-end estimates suggest that informal care accounts for 60% of the formal one, while others believe their economic impact is statistically equivalent, as informal one has no financial manifestation. The median forecast of the impact of informal care on mental health disease is 25%, that is the probability of experiencing mental health problems increases by 25% for an individual supplying informal care, but there is the extreme scenario where the increase is 80% in Greece.

The increase is particularly relevant, as roughly 30% of the active population is providing informal care, with a peak of 40% in Denmark, mostly with instrumental activities. There are, on this side, privacy concerns that policyholders must consider in terms of access to bank accounts, healthcare and legal status, messages, and correspondence by the caregiver: this means that the insured might prefer an external professional caregiver and not a relative, that might carry a conflict of interests. This extreme scenario is present in countries with poor LTC infrastructures, where the caregiver is supposed to take into consideration a wider portfolio of situations that are, by contrast, relegated to specialized nursing services in more developed ones.

Since the less severe elements of insurance social problems are as important as healthcare questions, informal care might be enough in the first level of the German scale and the first the true level of the French one. From sequent, providing monetary benefits for caregivers will decrease labour supply and this impact will differ countries with the highest viscosity in the Labour market as the social insurance allowance will be perceived as a fair substitute off labour income: German experience clearly shows that in the lowest income states where labour market viscosity is a real problem there is the greatest percentage of insured asking for the monetary benefit and, consequently, informal care. Note: an increase of 1% in intensive caregivers decreases by 0.36% the employment rate. Germany, the only country where there is a full set up of a public system to deal with long term care problems presents a positive bias: given this level of intensive caregiver, we expect 10% less on employment[11]. Consider that countries of southern or central Europe present a higher rate of exit from the labour market of the caregiver. The higher exit rate is from a male. In the United Kingdom, the highest

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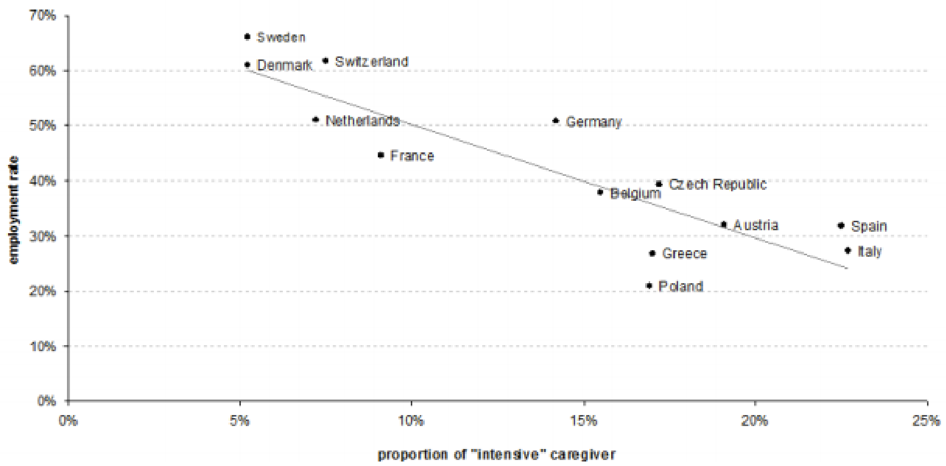
11 Informal caring impact on labour supply



SOURCE : SHARE survey, OECD (2011)

shocks for male compared to the one for a woman. Incrementally, we would like to consider the marginal effect of long hours of care (i.e., more than 8 hours per day) that are associated with an incremental reduction of the participation rate of 0.1. Again, we see that in the Anglo-Saxon countries the variable long hours of care are producing more effects on males than females: by contrast in central Europe the variable seems not to affect males and in southern Europe is equivalent between males and females. The highest overall effects are, in the end, in Australia and southern Europe.

Consider that the share of the active population, where formal and informal caregivers will significantly decrease in the next years, in Italy the share of the active population was 65% in 2010 and will plough down to 60% in 2030 and 50% to 2050. This will mean that it will become more difficult, for the next LTC generations, to find adequate support for informal care and that prices of professional nursing will rise. In terms of the demographic trend, the share of the world population between 65-79 will increase from 10% to 17.5% in OECD, and from 15% to 24% in Europe. Looking at the very old-age dependency ratio (i.e., the share of the population over 80) the tendency is more relevant, with an expected increase from 4 to 15% in EU countries and from 2.5% to 13% in OECD countries. This confirms the positive correlation between the old-age dependency ratio and expenditure in LTC in EU countries. A solution comes from the increased ratio of couples living together (i.e., the higher survival rate of males): In Italy, the ratio of male/female will go from 60% to 85% in 2050: the share of women living still married that will be able to take care of the husband will increase from 40% to 50%. This is not the end of the story, as the share of couples with both ADL deficits will increase from 8% to 12% in France and from 5% to 8% in Italy, with an average increase of 3% in the EU-Countries. We should consider that intensive carers are more likely to be old and disadvantaged, in northern and central Europe, where the ratio of intensive care provided by the spouse is more than ten times: intensive cares are greater than 50%. i.e., most carers provide intensive services, with a ratio higher than one, so more than one intensive care insured than non-intensive ones: these are particularly hard in Central and Southern Europe, and the United Kingdom and the United States. Unfortunately, it seems that both private and public insurance systems are not taking the questions of couples needing both LTC aid enough into consideration, as the private EU system and the public interventions measures completely design on the individual. The possibility of a wife to take care of the husband (or vice versa, less frequently) should not be outweighed and overestimated, for two reasons: the impact on an old caregiver, with a strict relationship with the insured, are higher than usual, and in numerical terms, this benefit will not be able to compensate for the decrease in the number of caregivers, with a net loss of roughly 40% and higher skills will be required. If this single probability of transition to loss of independence will significantly decrease in the next years, thanks to medical progress and thanks to technological progress that will call their summer responsibilities today not technologically supervised, as the share of population older than sixty will increase their overall number ask not independence insured will dramatically more than double in France, in Italy and overall speaking in the western world. The higher uncertainty is concerning the average duration of LTC status[12] : Most authors believe in the future it will decrease,

12 Trends in total life expectancy (TLE) and Disability-free life expectancy (DLFE) according to the different theories: A) compression theory, B) equilibrium theory and C) pandemic Theory. Consider that, and this is not a negligible solution, equilibrium theory could even be reached as the average between two components: males in the pandemic theory state and females in the compression theory state. Source: Pitacco, Actuarial Models of LTC, Ch.6,2014

as the disability-free life expectancy will increase, others believe that it will stay constant or that it will increase because of the mortality drop in LTC. In face of this point, informal healthcare is not negative at all: there are important advantages that should lead to a more aware regulation and management of home informal healthcare. This is possible thanks to cash benefit to elderly to buy formal or informal care at home, which seems, especially in Mediterranean cultures as a preferred place: the benefit should also be for the government, as direct costs are lower. This last point is under debate, and it is due to the missing externalities and depends on the severity of the situation[13] . A final advantage is that it provides the elderly with the opportunity to stay at home longer, for their entire life, decreasing the pressure on specialized structures that can, by contrast, follow the fewer "level 3" cases with a better quality of the services, lower stress on operators and better-perceived quality for the insured. Another important, behavioural point, is that in so doing it is possible to avoid to easiest situations to meet hard contexts, thus improving their mental health and overall welfare, decreasing the probability of transition to the hardest levels of care.

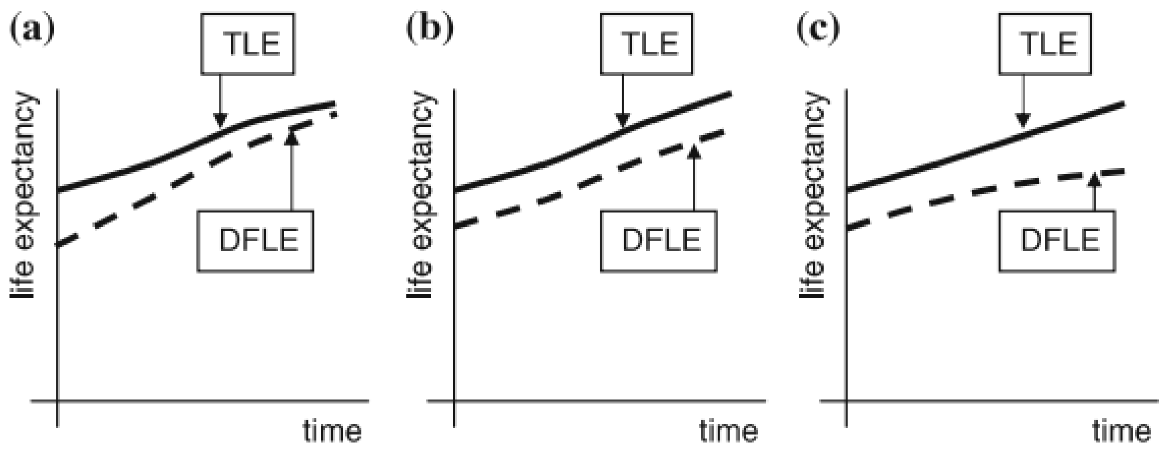
2.4) The limits of a private Market

We would like to summarize the main obstacles to the development of an efficient insurance market: from the supply side, the main limitation is the inability to measure risk, according to a fundamental work of N. Barr (2010) on this specific topic[14], while from the demand side the main topics are still uncertainty about the contract and future. There is no prevalence concerning the most problematic côté. This paper has been shown as a deep "economical introduction" after a careful review of possible peers and alternatives: the author is particularly glad to prof. Mazzaferro for the help in this relevant task. We produce an introductory and not exhaustive list, of the main limits of the actual private insurance market to ease the following argumentations:

Limits of private LTC insurance according to Barr, 2010

Problems for the insurers (supply side)

- X Relevant common shocks on lasting and transition to LTC status
- X Agency problems, lack of information on health status
- X Adverse selection and moral hazard
- X The high variance of relevant actuarial and biometric distributions



13 This measure increases the benefit, but, in truth, also the cost (Carlson, B.L., Foster, L., Dale, S.B. and Brown, R. (2007) "Effects of cash and counselling on personal care and well-being", Health Service Res. 42(1), 467–487.).
14 Barr, Nicholas (2010) Long-term care: a suitable case for social insurance. Social policy & administration, 44 (4). pp. 359-374. ISSN 0144-5596 DOI: 10.1111/j.1467-9515.2010. 00718.x

Problems for the insured (demand side)

- X Long time horizon (intertemporal myopia[15])
- X Incertitude and complexity of the contract
- X Risk of the solvency of the insurer, difficult to price
- X Risk of decrease/increase in public coverage

Solutions proposed by Barr

- ✓ Stricter regulation (than it was in 2010) and the author suggest, considering EU, a higher degree of harmonization to allow the distribution of pan European products, as for vanilla pension funds (i.e., regulation 2019/1238). A possible explanation for the lower degree of EU-wide harmonization of life insurance if compared to banking and capital markets union is the following: it is easier to harmonize different national standards when the system of rules is protecting the seller (Banks, financial institutions in general) more than the buyer. Should the system of the rules be more protective for the buyer than for the seller (as in life insurance, and insurances in general) reaching a sufficient level of harmonization between national standards becomes harder. This is explained this way: if there is more protection for the seller the NPV of harmonization (and so of a bigger market) is generally higher for the company.
- ✓ Public intervention (*nomen est omen*): *A suitable case for social insurance.*

Barr firstly introduces the insurer problems in contract with insured one, the first one is very basic and general to all insurance, but particularly costly in Long Term care contracts: *insurance requires that the probabilities in the pricing equation are substantially independent, that is, that there are a predictable number of winners and losers. Actuarial insurance cannot cope with common shocks, i.e., with shocks that concern the whole insured population as the ability to diversify the risk by insuring individuals with a low-risk profile will exhaust and the social welfare created by the insurance will significantly decrease*[16]. This happens also when the risk-aversion in front of the specific risk covered by the insurance is decreasing. Thus, the probability must be lower than one, if not it is certain that the insured will lose welfare, and the insurance premium will exceed the insured loss. There is no possibility of spreading risks, hence no gain from joining a risk pool. This problem arises, according to Barr, *in two different ways with medical insurance*[17]. The first way is old age: the probability of elderly people requiring medical care is high. Especially, the probability of coping with a chronic disease that requires constant treatments, attention, and expenditures. The second way of the problem arising, still according to Barr, is pre-existing medical conditions: *actuarial insurance can cover potential problems, but not actual problems, that is, medical problems which the individual already has at the time that he/she applies for insurance*[18]. The threshold between what is insurable and what is not insurable is more discretionary than what seems. Individuals do not show the symptoms of the pathology today but are already ill or will be ill in a brief time: in this case, especially for lower insured amount, a medical/diagnostic examination could be antieconomic. This results, especially if applied for large numbers, in a potential shadow liability.

The two conditions just discussed relate to the fundamental nature of insurance as a device for sharing risk, while, the remaining conditions, still proceeding with Barr in ch.4 of his seminal paper (see note 12), *reflect information problems in insurance markets*. There are problems linked to the nature of the specific insurance contract, and with the need for estimating different probabilities: where the insured event is rare, estimates of the probability will have a large variance, as in particular where the insured event has a long-time horizon. The longer the time horizon, the higher the

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16 Barr, p.361

17 Barr, p.361

18 Barr, p.361

uncertainty of the pricing. Nothing comes for free, however: if the long-time horizon will significantly decrease the probability of the event occurrence this might be beneficial both for the insured, reducing the premium, and the insurer, reducing the present value of claim size. This explains why it might be convenient, even if it increases the variance of the claim's distribution, to buy this insurance through periodical premia in the age range [30,60]: on this point, the high starting age set in the Italian insurance market increases the ratio premium/income. If individuals will start to buy insurance at a higher age, even considering lower ages of conversion (so of effective coverage) the variability of the outflow of the insured will increase, but the overall underwritten risk will decrease. At that point an adequate equilibrium, depending on country and gender at a minimum must be reached: too early underwriting will increase variance, too late underwriting will increase the overall insured risk. These considerations hold considering both the single premium case the instalment one, where the insured starts to buy insurance in his 30s-40s. As observed by Barr, *Efficient insurance requires that individuals pay a premium based on a high probability of loss. Adverse choice arises when the buyer can conceal from the insurer the fact that he is a bad risk and is thus an insurance-market manifestation of 'lemons'*[19]*. The problem is not that people differ in their riskiness, but that the insurer is less informed than the buyer about the applicant's riskiness. The individual knows he is a 'lemon' (i.e., a bad risk), but can conceal the fact from the insurer, hence the description of adverse choice as 'hidden knowledge*[20]*.'*

The problem can arise, as reported by Barr *if healthcare is an important part of employer benefits: firms with the best healthcare packages will tend to attract workers with health problems or with a higher probability of developing this kind of issue, thus reducing the firm's competitiveness*[21]. Sloan and Norton, observed Barr, (1997) have shown[22] that adverse choice, both in terms of effective and perceived welfare, is altering the functioning of the market.

The second class of asymmetric information, moral hazard, arises where the insured person can influence the insurer's expected loss without the insurer's knowledge. The problem arises in two ways, concerning the probability of the event and the individual loss, with the well-known third-party payment problem[23]. For the probability of the event, it is the classical moral hazard problem, well known in injury and health insurance: the possibility is in follow up and medical trials before underwriting and periodically during the insurance period to adjust the premium and/or adopt early intervention measures, that sometimes the insured does not consider enough relevant. For the hidden action, the first, suboptimal solution is setting payments independently by the needs once LTC status is on as actually is in the Italian insurance market: the alternative is careful reviews with doctors acting in the interest of the company. This ensures full coverage but prevents excessive inefficient spending. A useful solution is cost sharing via deductibles (i.e., the individuals pay for the first X euros of damage). It could be in percentage terms, or in conjunction with a floor/cap mitigation.

This result, in a decreased ability to offer this product at a fair price and to adapt a product to individual needs without losing actuarial fairness and without losing control of the risk level of the insurance. This results in a load increasing concerning age: from 15% in 60 to 20% in 65 and 25% in 70, in front of such higher part of the premium paid for remunerating the insurer there is less willing to buy an insurance product. High loads are not enough to explain the so bad behaviour of the LTC private insurance market. Ideally, we can expect with a constant load of 15% the share of insured will

19 Akerlof, G. A. (1970). The Market for "Lemons": Qualitative Uncertainty and the Market Mechanism. The Quarterly Journal of Economics, 84, 488-500
20 Barr, p.362
21 Barr, p.362
22 Sloan, F., & Norton, E. (1997): 'Adverse selection, bequests, crowding out, and private demand for insurance: evidence from the LTC insurance market', Journal of Risk and Uncertainty, 15, 201-219.
23 Barr, pp.362-363

double[24]. From the demand side, by contrast, the main obstacle is a behavioural one. The explanation is quite intuitive: despite being averse to the risk, they are not willing to consider this kind of insurance and prefer to defer the choice whether to insure themselves or not. Individuals are in trouble with intertemporal myopia[25] and are not mentally ready to face the what-if long term care scenario. Alongside these supply-side problems are problems from the perspective of individuals. Insurance policies for long-term care are both long-term and complex. As a result, consumers face problems now widely recognized from the economics of information and behavioural economics. The following questions illustrate the problems individuals face in choosing an insurance policy in a competitive system. What type of care includes? Does the policy cover only residential care, or domiciliary care; is a person entitled to residential care based on general infirmity or only if he or she has clearly defined specific ailments? How will the answers to these questions change with advances over the years in medical technology? On what financial basis is care provided? Can the insurer increase premiums if a person becomes riskier; is there a ceiling on the monthly cost of care; is there a maximum duration over which benefit is payable? Will those figures change over time in line with changes in prices, changes in wages, or changes in the cost of care? How well specified is the contract? Can insurers change the basis of cover; does the wording make clear the circumstances in which an individual can make choices; what arrangements deal with any disagreements between the policyholder and the insurer?

Complications arise, as further observed by Barr, *because people may do not know the level of coverage, they have*[26], or they might be unable to face a probable increase in premiums levels or they do not want to undergo a periodical medical examination that might lead to an increased premium. In conclusion, consider that the main obstacle by the demand side is of behavioural order: limited knowledge, insufficient or excessive reliance on information provided by the broker/seller, taboos linked with loss of autonomy and involvement of relatives in ensuring choice and difficulties linked to underwriting process: some individuals might not be willing to undertake a medical examination or a set of trials to subscribe an insurance contract, even for privacy concerns.

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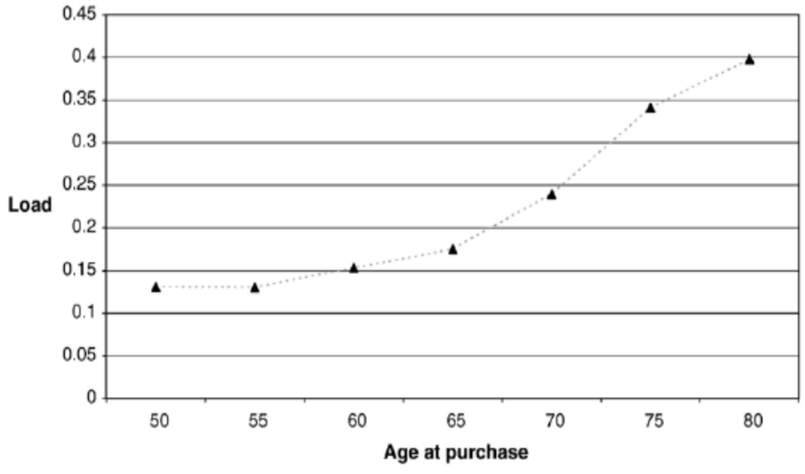
24 Loads by age of purchase: the higher the load, the lower the residual welfare for the insured. Source: Jeffrey R. Brown, Amy Finkelstein, why is the market for long-term care insurance so small? Journal of Public Economics, Volume 91, Issue 10,2007, Pages 1967-1991.



SOURCE : Brown and Finkelstein (2007), Fig. 1, p. 1981.

25 As a good ad panel-based introduction to the problem of intertemporal myopia we suggest Fornero, Elsa and Lo Prete, Anna. (2019). Voting in the aftermath of a pension reform: The role of financial literacy. Journal of Pension Economics and Finance. The authors show the correlation between myopia of the electors and their financial illiteracy. Conclusion is crystal clear: the lower the financial literacy, the higher the cost of pensions (social security) reforms.
26 Barr, pp.367

These described fallacies are not "perceived" by the insurable population, they are a real and objective lack of information and transparency. In the face of such complexities, Burkhardt, and Hills[27] found that their academic study could not unearth the data necessary for proper assessment of policies, calling seriously into question the ability of individuals to make informed choices. Of course, the problem is for lower aware individuals. At a minimum, there is a need for regulation to ensure that all policies cover at least a basic package. If public funding becomes more generous, people with extensive private insurance end up with an inefficiently large amount of cover. Conversely, cuts in public funding may leave people under-insured; and if such under-insurance occurs late in life, the added private cover is expensive. All the reasons already explained show clearly with the private insurance market is not enough deep and not enough liquid variety and quantity of products[28]. The average load on LTC products is double that of ordinary healthcare ones and is more sex dependent than LTC: it is 32% on average, 13% for women and 55% for men. This is due to the abovementioned difference in natural hedging: if natural hedging decreases the value of the fair premium more for men than for women, if the insurance company has to adequate reserve against enhancements in this beneficial anticorrelation, in higher safety loads for men than for women. In conclusion, natural hedging supplies benefits in terms of fair pricing: this benefit presents an effect on the insurance premium, but the insurer does not know the dynamic of these correlations. As the market for actuarial risk is still an incomplete market, with consequences in terms of efficiency and liquidity, the benefit from natural hedging[29] is significantly higher than in other forms of operation hedging where efficient

27 Burkhardt, Tania, and Hills, John (1997), Private Welfare Insurance and Social Security: Pushing the Boundaries (York: Joseph Rowntree Foundation).

28 To summarize, consider also Glenzer, F., Achou, B. Annuities, long-term care insurance, and insurer solvency. Geneva Pap Risk Insur Issues Pract 44, 252–276 (2019). https://doi.org/10.1057/s41288-019-00125-x: *To this end (explaining little private LTC market), we model a shareholder value maximising insurance company that is subject to solvency regulation. Because liabilities from LTC insurance (which depend on future morbidity and mortality) are more volatile than liabilities from annuities (which only depend on future mortality), capital provisions to ensure compliance with regulatory solvency requirements are higher if an insurance company offers LTC insurance instead of annuities. At the same time, a higher volatility in the LTC insurance segment also implies a higher expected payoff to the insurance company's shareholders. To quantify which effect prevails and which product policy is optimal, we conduct an empirically calibrated simulation study with stochastic mortality and LTC needs. Our results show that offering LTC insurance increases the upside potential to shareholders, but that effect is more than offset by a higher need for external capital. Consequently, if shareholders are to accept an LTC insurance segment, holders of an LTC insurance policy need to pay considerable markups. The more LTC insurance contracts the insurer has sold, the higher the markups.* This explains why selling stand-alone LTC products is sub-optimal and more expensive for the insured.

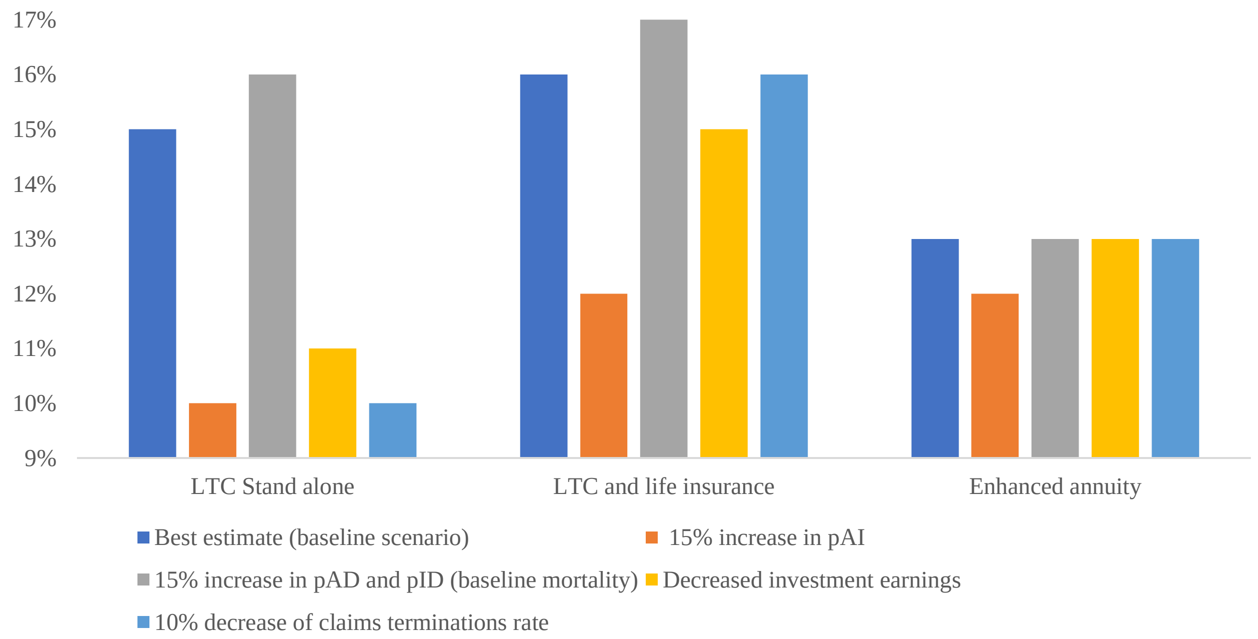
29 Concerning LTC risk there are also different ways of introducing natural hedging: for example, instead of a combination between LTC and Active states of the insured, are sold combined products that decrease their value for each LTC insurance payments. Practically speaking, if the face value of the life insurance is 100.000$ after 10 payments for LTC insurance each of 1.000$, the face value of the life insurance will be 90.000$. Once the life insurance face value is ended, LTC payments stops. The effectiveness of this form of hedging, unused in Italy, is however significantly lower than the enhanced pension one: by contrast, we must observe that there is still a risk reduction for the insurer with respect to the basic stand-alone coverage and that the idea of adding to the enhanced pension a part of death benefit is still of interest, too. An extensive comparison between the hedging effectiveness of enhanced pensions and LTC+life had been conducted by Chow, Friedrich and Helwig in 2012. In fact, the combination "LTC and life" presents a higher average return and higher variability (confirming even in this context the importance of the risk-reward relationship). The two hardest scenarios for the LTC and life insurance combination, are clearly a spike in invalidity transition probability and in a decrease of the investment rate. By contrast, in the enhanced pension contract, all the five scenarios present the same rate of return, apart from a low decrease in the second one (115% of disability probability). This means, by consequence, that the insurer does not "pay" in case of worse conditions, but even that it does not profit from positive shifts, like the increase's active mortality. From a certain point of view, however, the solution LTC and life could be more appreciated, as it shows a significantly higher return (+400 bps about) on baseline scenario, even higher of the stand-alone LTC, and an adequate level of risk mitigation, especially for decreased investment earnings and higher lower levels of claim termination rate than a stand-alone long-term coverage that is, whenever lower rewarding. Furthermore, shifting from an individual- perspective to a couple perspective, the most suitable and less risky contract for a couple is the LTC and life combination, as the whole insured capital in case of death might be used for both the individuals. This explains why there is still an important prevalence, especially in some countries, of the II product and not of the III one as the main risk-diversification way.

risk transfer markets exist. While for nonbiometric risks, like forex risk, interest rate risk and others operational hedging are complementary and substitutes at the same time and we can equate their marginal costs to find an equilibrium, here we are in a corner solution where the marginal benefit of natural hedging is substantially greater than the financial one.

Considerations should lead to examining the crowding-out effect of the private expenditure on the public one, as it is for other forms of services as healthcare and retirement. In front of an increased supply of LTC aid by the public (state, federal level, municipality) for a substitution effect, the individual is less likely to buy an LTC or purchase an LTC insurance with lower coverage. It is arguable the behaviour of using means-testing:" For example, individuals may be able to hide assets from the means-test by various means, including giving cash gifts to children or grandchildren, setting up trusts, or spending the money on assets that are excluded from the means evaluate. An efficient means-testing could be based, for Italy, on the amount of first pillar annuity: this in the long rung might influence workers to early retirement. It is important to consider that public insurances are supplying the same amount independently from the gender: this means that the net actuarial value of the insurance is different for people and the willingness to contribute, too as women are expecting higher LTC expenditures, but do pay the same premium. LTC is a complex, deeply interconnected scenario nor the public neither the private way is stand-alone considered able to supply decent protection for a large amount of population. The problem of early-stage insurance arose in terms of the distinction between actuarial fairness and solvency. Solvency granted with reserves is aiming at financial equilibrium, while actuarial fairness relates to patrimonial and economical equilibrium over time. Actuarial fairness does not at all ensure solvency, in life insurance. If the market is in an early

Shock (rows) vs expected drawdown (columns)	LTC Stand alone	LTC and life	Enhanced pension
115% p^{AI}	5%	4%	1%
115% p^{AD} (this increases, here, also ^{pID})	-1%	-1%	0%
Decreased investment earnings	4%	1%	0%
90% of claims terminations rate	5%	0%	0%
Average return/std (personal elaborations)	396%	738%	2639%

Enhanced annuity beats LTC+life insurance for natural hedging.
Source: Quantification of the Natural Hedge Characteristics of Combination Life or Annuity Products Linked to Long Term Care Insurance. By Chow, Friderich and Helwing for Milliman, 2012



stage, there will be a first stage where the business receives only payment and then only after a long time (20 years for traditional stand-alone insurance) claims become significant and the equilibrium makes actuarial meaning. For a still immature market, as in main countries, this should, theoretically speaking work as a further incentive for early subscribers, as with low probability for the first claimants there will be liquidity problems or insurance default.

The concept could reverse: what happens if I am not in the first and the solvency provisions were inadequate? Alternatively said, what if premia collected were not adequate and the insurance defaults? Individuals tend to rely on insurance companies for a small period but today, especially in mainland Europe, still not on private insurance for the largest time intervals. On this point, an obvious advantage could be for the enhanced pension solution, as the time horizon of the whole contract halved. An aspect that merits examination is how the supply of data in transforming life insurance and the advancements in diagnostic sciences that allow an early diagnosis, due to genetic testing: it is increasing both the accuracy of these tests and the spectrum of diseases under-diagnosis. For illnesses related to LTC insurance, consider ALS, Alzheimer disease (only half of the cases) and Huntington disease[30] (HD). The unique genetic and clinical features of HD may limit its generalizability; nevertheless, we feel that these results are likely to be informative about the behaviour of individuals with more common diseases: Taylor et al., before the appreciable study on HD, give a discussion of similar issues, focusing on the case of Alzheimer disease, with the obvious difference that Alzheimer is not deterministic and shows more treatment perspectives, at least in the medium run. As individuals know they carry a higher risk the probability of increasing the coverage of the insurance contract increase by 131%[31], as their probability of needing LTC increased by 48% in the central estimate. The authors find a group that should be less risky: the idea is that once you know you hold a minimal risk of demanding disease you decrease your LTC coverage. Up to now, there is no empirical evidence on this point.

The first evidence is that individual at risk insures themselves more than the general population and individuals evaluated for carrying the positive mutation ensure more, according to the adverse choice

30 Huntington disease, being based on Mendelian genetic, allows us to show higher effects on welfare, since the probability is roughly 0.5 for males and this maximises the variance of the associated Bernoulli distribution.

31 Associations between an Alzheimer risk estimator (APOE genotype) and insurance policy modification. Taylor DH Jr, Cook-Deegan RM, Hiraki S, Roberts JS, Blazer DG, Green RC. Genetic testing for Alzheimer's and long-term care insurance. Health Aff (Millwood). 2010 Jan-Feb;29(1):102-8. The most common situation is presenting two e3 traits, and by consequence the insured has a risk equivalent to the one of the general population: by contrast, having at least one e2 traits is protecting from developing Alzheimer, while having at least one e4 significantly increases the risk. Changing long-term care insurance includes both phenomena of repricing in increase and decrease, and even cases of increased coverage. Once individuals know being at a risk that is different from the general population one, they act consequently: either they increase the coverage, if they are high risk, either they show the result of the tests for a premium reduction or decrease the amount insured, if they are low risk. We remember in the US there is no general prohibition in using genetic tests for long term care insurance.

Associations Between APOE Genotype And Outcomes Of Interest

	Piedmont Health Survey of the Elderly		Rotterdam Study	REVEAL II	
	Odds ratio of nursing home admission [95% CI][a]	Probability[b] of nursing home admission	Odds ratio of developing Alzeimer's disease [95% CI][c]	Odds ratio of changing long-term care insurance [95% CI]	Probability[d] of changing long-term care insurance
At least one e2 trait and no e4 trait	0.80 [0.52–1.22]	0.082	0.5 [0.0–5.4]	1.55 [0.43–5.60]	0.149
Two e3 traits	1.00	0.101	1.00	1.00	0.087
At least one e4 trait	1.48 [1.09–2.01]	0.127	4.6 [1.3–16.1]	2.31 [1.11–4.81]	0.237

driving factor. This higher underwriting percentage is due to the higher actuarial value: a 40-year individual with a 50% HD risk takes back 240% of the premium paid and it is 400% for an individual at elevated risk. This is due to the different probability of becoming LTC: individuals with an expressed genetic mutation are twice as risky for HD as those who only have genealogical elements. The critical point is how much "stable" is this asymmetrical information, and for how long: healthy individuals, regardless of their HD risk, do pay the same premium: in underwriters do not consider genetical history individuals that are at risk for HD have with a worthy financial instrument. By contrast, is early symptoms of HD, or similar-related pathologies, this veil is likely to end by a broker in a direct or phone interview this might end in insurance denied. Waiting till the manifestation of early symptoms for taking an insurance product is not best: these "common depicted" procedures are not fraud if there is no provision to communicate genetic risk. These illnesses are costly for LTC insurance as the average life and the intensity of care needed is higher than the average of the insured. What happens if a testing system for this disease is available on large scale and the population has the "option" to make public their genetic patrimony? From a legal perspective, there are relevant differences between the US and Europe. In the US genetic testing cannot be part of underwriting decisions and rate settlement for health insurance (Genetic Information Non-discrimination Act, 2008), but it should be an instrument when considering reserving even in this product. genetic testing is disposable for LTC and Life insurance, both in underwriting, rating, and reserving decisions unless further restrictions are present at the state level (Florida, in particular). Concerning Europe, the Oviedo Convention of the council of Europe prohibits widely usage of genetic testing in the insurance market.[32]However, speaking about Oviedo Convention *de facto* means returning to the *vexata quaestio about the reception of Council of Europe* regulation in the EU, a task still fully managed by the national states.

To deny regulatory arbitrage, we would like to expect a homogeneity of the legal point of view at least in the western countries. What if the insurance asks for these tests, for the larger amount insured to avoid fat tails, as it was in Canada till 2017? First, McCarty and Mitchell show that more risk-averse individuals own more life insurance and that in general, they are less likely to die early. These should apply also to LTC insurance: the probability of losing autonomy for insured is systemically lower than in the general population than introducing a basis risk concerning the "theoretical" business of insurance, i.e., a difference between the risk-in population and the risk in a portfolio.

New data analysis techniques should allow controlling or at least mitigating this risk. The first scenario refers to the situation where the test information, from now on "the information" is private[33].

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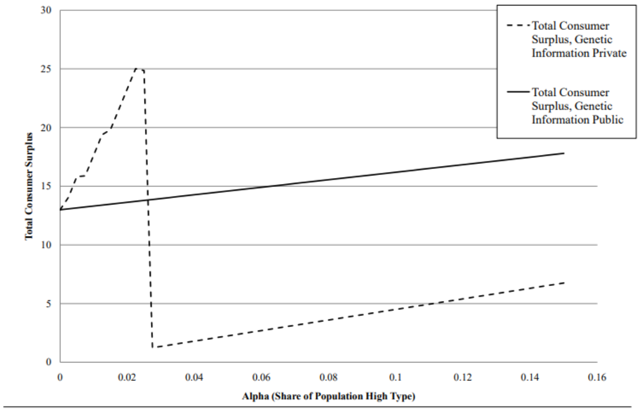
32 "*L'assureur ne peut opposer au candidat à une assurance le résultat de tests génétiques prédictifs ayant pour objet la recherche d'une maladie qui n'est pas' encore déclarée ni, par voie de conséquence, lui reprocher de ne pas dévoiler, au moment de son adhésion, une telle prédisposition dès lors que la maladie ne s'est pas encore manifestée, selon la décision. Quelles que soient les prédispositions génétiques de l'assuré, elles ne remettent pas en cause le caractère aléatoire du contrat*", (TGI Nanterre, *affaire M.me Clemence Gayral C/ société AVIVA LIFE*, n° RG 19/06316, see also *L'Argus de l'assurance*, 25/10/2019). The heterogeneity of the contracts in the whole EU shows how far away is the Union from the Life Insurance levelled playing field within different countries (and so, national companies) if compared to Capital Market union and banking union.

33 Consumer surplus under varying information regimes, in function of the share of the population with high type. Source: Oster, Emily & Shoulson, Ira & Quaid, Kimberly & Dorsey, E., Genetic adverse selection: evidence from long-term care insurance and Huntington disease (2009).

The relevant variable is the share of the population that shows an elevated risk of developing the illnesses. Note that this scenario is best only if the share of individuals affected is lower than a share X. The monopolist will significantly increase the price and minimal risk individuals will not any longer ensure, so the overall insured population is lower than x. Now is Pareto-improving to show the test results: this does not improve the utility of the individual that is showing it, as it will allow the monopoly to practise price discrimination. A solution might be partial price discrimination, considering that the aggregate benefit could compensate the few individuals that lose very much, especially if the share is low. This suggests that legislation prohibiting insurers from seeing genetic information may be a mixed blessing; in the short run, this will increase consumer surplus, but overall, it may reduce it. A simple summary of available genetic testing suggests that – even if everyone eligible for genetic testing for relevant diseases got it – the share of the "high type" would only be around 0.1%. Only 10% of the risk exposed population reported predictive testing[34]. Testing for HD or ALS or Alzheimer is a weighty decision, typically following a counselling period, this is due to the absence of care and improvement opportunities. This shows that certainly in the current state and for the immediate future, consumer surplus will be higher with confidential information than without. As the share of the population that will suffer from the diseases will increase up to a critical threshold, that depends on the incremental costs for the insured linked to the pathology, this will put up the insurance price up to a point where also a share of the high-risk individuals will not insure themselves.

This is since the probability of becoming LTC is increasing and so the marginal benefit of insurance vs self-insurance is going to zero. The higher the incremental costs linked to the pathology, the lower the required threshold for a complete failure of the insurance market. As tests become more widespread and pathologies being tests becomes more common, as the newer test might not be *strictu sensu* genetical, the probability of incurring this failure becomes higher. The market is not properly functioning as a large part of the insured (the whole expressing the low type) will not insure himself. In the public information consumer surplus is a more stable pattern, increasing as the share of the population increasing, with no peaks nor decreasing patterns, so, all the drivers mentioned negatively before here are weight positively. And, by definition, consumer welfare is higher than the welfare in confidential information solutions while it is lower before the threshold. The maximum welfare in confidential information solutions is at once before the critical threshold before mentioned. After that, an immediate drop and then a new increasing pattern: from now on, the social welfare in the confidential information is one-third of the social welfare with the confidential information. By contrast, the pattern here is increasing faster than the share of confidential information.

We remark that these genetic tests are not already widespread in the EU insurance system and if asked, are only for the relevant insured amount. It is unlawful to ensure the same risk with a different insurer, to obtain the same insurance but with a lower underwriting due diligence: this prohibition should still apply to LTC insurance? Could insurance force undergo genetic testing to cover a risk? Are these the right trade-off between freedom of business and freedom not only to privacy but to



Notes: This figure shows consumer surplus under private information versus public (i.e. observable to the insurer) information.

34 Meyers, RH, "Huntington's Disease Genetics," NeuroRx, 2004, 1, 255–262.

refuse early diagnosis decreasing quality of life? Both these rights are constitutional, but the limits for freedom of business are stricter and allowing for higher protection of personal freedom, considering that principles said in the first twelve articles are more important. An important theme is a distinction between tests that are diagnosing future almost-certain illnesses for tests showing a high probability or, more laxly, a probability higher than the general population one of developing a disease or a group of diseases. There are important positive aspects, as without tests strict relatives of people affected by the abovementioned disease would hardly be insurable, while with a positive result of a test they will end up having the same premium class of the general population, without negative influence by the genealogical genetic chain.

2.5) The Italian market: stand-alone products or enhanced pensions?

The scenario described in earlier paragraphs will be a reality in less than 15 years, according to the forecast trend of the number of individuals needing some forms of help in the western world, demanding by consequences a review of the public expenditure on this topic and the sincere, genuine effort of private market actors. The majority of LTC insurances is occupational-related and few companies are offering individual LTC plans with a lack of flexibility in terms of premium payment and coverages: this will result detrimental in future ages where the majority of today workers will not have enough means to afford the payment of such expensive insurance policies unless converting there your pension fund capital into an enhanced pension, and thus finally relying on public assistance in the countries where compulsory social insurance is not present. If this allowance could be considered sufficient for a lower context where informal care will be enough in spite we might observe that in the last weeks of life the cost often insured for the national sanitary service is dramatically high and one more is dramatically increasing over years thanks to medical and technological advanced the question is the social welfare of the developed countries will be able to face a what in terms of impact on the national sanitary service could be considered a constant pandemic better said this mechanism if fully able to overcharge of the healthcare system.

The distinction between these three scenarios is fully relying on the “parameters” of the country: for developed countries, we can consider as a dependable proxy of the future development, the situation where the incidence rate of LTC decreases, but if the highest cohort is becoming older, the overall number of LTC insured (so the overall expenditure) for LTC will be significantly increasing. Looking at the forecast for France, from 2010 to 2060 supplies a decrease of 20% at the real end of the distribution and an expected decrease of 10% in the middle of the distribution (after 85 years). By contrast, at the same time-space, the overall number of LTC-needing will more than double in France. Consider the ratio between male and female insured, which is going to decrease in developed countries: the number of females needing LTC will increase more than the number of men: this is due to the differences in expected mortality, in favour of women. This will for sure resulting in decreased opportunity to take adequate care of acute states does decrease the level of perceived social welfare the best solution will be to introduce a form of social insurance for long term care so bad with the other wait funding not only the actual workers will not experience a dramatic situation once we will understand how few they have set aside for a long time. In the Italian context of LTC, we would like to consider and make a distinction, according to the real possibility of distinction, between the different kinds of coverages available: a first distention is between occupational-related insurances and free-market coverages. The majority of stand-alone LTC product is part of occupational-related insurances, that are, generally speaking, more complete and are sold as a part of a bundle (with life insurance and with health components), while the free market coverage could be ensured in two ways: there are stand-alone LTC that provides an annuity of a fixed amount in case of loss of independence where the accumulation phase is paid thanks to the periodical premium paid during the life of the worker and/or the conversion of the capital accumulated through pension funds in an enhanced pension (i.e. an annuity that pays X if the individual is alive and healthy and pays the double if the individual is in the long term care status).[35]

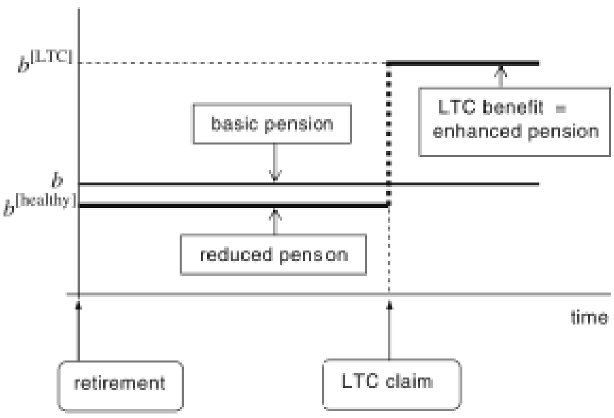
This last product is country specifical as Italy shows the highest reliance on this opportunity. This is the instrument we want to price, i.e., we want to find conversion coefficients for capital into this form of an annuity. Why is our country particularly relying on this product? Which are his specificities? His pros and cons? The main advantage is that the coverage is working once the individual retires[36] as in the Italian pension scheme is not possible the conversion into an annuity before reaching requirements for first pillar retirement (60-67 years). This point is relevant: first, the share of risk insured is lower, as the coverage is bought in an elderly state of life: this of course results in lower premia, but even in a lower share of the risk insured. With the traditional stand-alone latch coverage, the insured period starts 10-15 years before retirement, in a period where the transition from Healy to LTC starts to be not negligible. It means, roughly speaking, that an individual relying on the conversion of his pension funds capital into an LTC annuity is uninsured in the period from 50 years to the conversion. This is not, a priori, negative, considering that there are three main sources of LTC needs: injury, illnesses, and elderly dementia: while the last two are independent of the lifestyle of the individual, the injury insurance could be a major source of moral hazard. Of course, the highest share of injury leading to LTC happens when the individual is active (shifting from 30% in the active stage to 5% in the retirement phase).

An individual that does not take a significant injury risk (that drivers carefully, that does not play extreme sports) might find it useful to insure himself against LTC only once the share of risk associated with the injury decreased. Coming to the advantages for the insurer of the enhanced pension, looking at the impact of the choice of offering this bundle (ensuring longevity and independence) through this fully loaded contract instead of managing two separate contracts (an annuity and LTC insurance). If there is no significant arbitrage between life and non-life rules, if the company wants to reach the same level of profits on all the insurances he is providing and if there is a significant negative correlation between the expected life on autonomous individuals and the expected life of LTC individuals

$$\mathbf{E}\text{ (life | Healthy)}>\mathbf{E}\text{ (life | LTC)}$$

there is a greater actuarial advantage in providing these instruments, thanks to operational hedging (natural hedging) put in place by the bundle between life and LTC insurance. Said in another way, the cash flow of the company in an enhanced pension is less variable than the sum of the variability of life insurance and an LTC one. As variance is the main source of actuarial risk and is a determinant

35 Benefits provided by enhanced pension: instead of a whole-life constant benefit the enhanced pension pays so to the health status, so we have a couple of increasing benefits [B_{healty}, B_{ltc}] with $B_{healty}<B<B_{ltc}$. Source: Pitacco, Health Insurance: Basic Actuarial Models, 2014. B-Bhealty implicitly quotes the probability and the cost of LTC event. In our study we set, according to the Italian market $B_{ltc}=2B_{healty}$. Source of the imagine: Pitacco, Health Insurance, Basic actuarial models, Springer Actuarial 2014.



36 or after if he or she decides to postpone the conversion for getting a higher annuity

of the premium, this means that a company offering the bundle will experience a comparative advantage in face of the companies selling only stand-alone positions, ad this will lead to a higher profit (or a lower premium level). This made the insured profit: if an individual is long on two assets zig and zag, and the insured is short on both the assets zig and zag the negative correlation will increase the welfare of both.

This form of operative hedging is in actuarial terms Natural hedging: the classical form of natural hedging in life insurance is the ensured annuity, that for the first X years (5 or 10 years) pays both in case of life and of death of the insured. This is more effective as there is perfect anticorrelation: the insured is alive or dead: the higher the negative correlation, the more effective the edging. By contrast, reversibility of annuity is not a well-considered operation hedging as the variability of the outflows is not significantly decreasing: the longevity risk that is underwritten increases. A form of operation hedging does cumulate in a beneficial way, which is what we will consider at the end of our work, by pricing a counter-ensured annuity in the first X years and that pays a double amount if the insured is in LTC status? The insured is so covering both his longevity and healthcare status risks and is even ensuring his beneficiaries in case of premature death in the first 5, 10 years. The main assumption, here, is the negative correlation between LTC probability and life expectancy. this assumption today still holds, more for a man than for women and more for age 55-70 than elderly stage >75. Medical and technological progress is relaxing this assumption and there are already today mortality curves of LTC indissociable from the curves of healthy people.

The author believes that the truth stays in the middle: even if in the future the correlation will lose intensity, there will still be a certain degree of hedging, especially for younger subscribers (55-65 years) thanks to the negative correlation. For assessing the level of natural hedging of the insurance portfolio age and sex of the insured must count and the correlation is significantly elastic to this determinant. Natural hedging is, as already said, beneficial even to the insured: that Is why the average coverage of LTC insurance is 72% for men and 61% for women[37]. This spread, relevant as it is saying that there is a gender gap in coverage of needs once the insured becomes LTC, which is due to a higher correlation with extra mortality and LTC in males. If the LTC plan is paid on the periodic premium, generally for a long time, 5-20 years, the probability of being, at a certain point, unable to afford these expenses is not negligible, especially for a lower-income group of the population: this enhances as the insures can increase premium every 5 years in general and that stopping to pay the premium might lead to the anticipated ending of the LTC coverage. Once the insured becomes unable to take care of himself/herself, apart from monthly or regular costs there are fixed costs linked to entry into LTC status (the higher the damage severity, the higher the fixed costs): adapting the home, deposits for the residential living community (paying in advance 2 or 3 months) early costs of diagnosis and psychological follow-up for the insured, for the caregivers and relatives. The ratio between fixed and variable costs is higher for traumas and cardiological conditions, while it is lower for oncological and neurological ones (in general).

We believe that the best insurance should not only cover variable costs through an LTC annuity (or an enhanced pension one) but even fixed costs of loss of autonomy through a lump-sum payment once the triggered events happen. Furthermore, the possibility of asking for this immediate and effective coverage of fixed cost might probably result in a lower insured amount by the main guarantee (the LTC annuity) which may show portfolio improvement for the insurance company. A couple of insurances companies are offering this opportunity in the stand-alone LTC, while no opportunity of this kind is available in the enhanced pension: the amount is five times the monthly annuity and is in the interval [10 000€, 50 000€]. The fixed costs of entering LTC status grow rapidly concerning the level of impairment and this cost is deeply country-specifical, with Mediterranean

37 Brown, J. and Finkelstein, A. (2011) “Insuring long-term care in the United States”, Journal of Economic Perspectives 25 (4), 119–141.

countries having the highest values in responding to the shock, both for psychological questions and the contractual conditions of old age residences. Another point is that there are limits on options applicable to the annuity are the insurer's business policy discretion, options for an LTC annuity means to renounce to a counter-insurance on the first 5-10 years of the annuity in most Italian companies and so pros and cons of this hedging policy will go under scrutiny. In the Lump-Sum added insurance the duration of the LTC status has no impact on premium calculation not in the profitability: the only relevant variable becomes the probability of becoming LTC. By contrast, it seems that the transition between different LTC levels is not associated with relevant fixed costs unless situations where there are new purely physical barriers. Linked to this, remember that today life insurance policy can manage the longevity risk of the overall population thanks to longevity bonds & longevity swaps, but the same does not apply for the risk of transition from health to LTC. Concerning the mortality of LTC people, it holds that if there exists an efficient linear form for describing LTC mortality ($q_{ltc}=\alpha+\beta q_{generalpop}$) thanks to general population one, at that point, the best hedge ratio will be exactly β, with $\beta >1$ leading to a useful over hedging. The reference variable becomes the ratio between the two mortalities and if the ratio is higher than expected (so LTC mortality is higher than forecasted) loss on naked position will not cover swaps losses, by contrast, if the ratio is lower than expected (so LTC mortality is lower than forecasted) losses on futures will not cover naked position profits.

The benefits of an enhanced pension are not only concerning diversification: economies of scale and scope do account in comparison concerning other forms of coverage. Managing one insurance contract is better than managing two separate contracts, which are not necessarily with the same company and might belong to life and non-life divisions. This is important especially as this contract if entire life and, so, once the individual is older the benefits of having only one contract are more immediate appreciable. Concerning the flexibility of the contract, the enhanced pension sold in Italy, where in the case of LTC is two times, for sure experiences a lack of flexibility and adaptability to the needs of the insured; by contrast, the ratio proposed is quite immediate and allows, still considering the old age of pensioners to an immediate comprehension of the contract and management once LTC becomes a reality. The level of insurance supplied is very variable, in the function of the original annuity insured. Roughly speaking, if the original pension is between 1000€ and 2000€ we can consider these instruments as reliable insurance also for the most severe situations if the original annuity is lower than 1000€, the insurance is supplying adequate coverage only for a lower level of care intensity, and if the original pension is higher than>2000€ the risk is over-insured. Solution for over insurance is immediate, as it will be enough to partially convert into LTC annuity and partially into plain vanilla insurance; by contrast, considering the average annuity of actual subscribers of pension funds, 600€, we should repose the following question: enhanced pension & stand-alone LTC coverage: complements or substitutes?

Hopefully, competitive co-operators: relying on the conversion of pension funds capital into LTC annuity might still bear the risk of early LTC, mainly because of injury. As a result, for an individual able to buy an LTC (double) annuity lower than 1000€, the best solution may be buying both the enhanced pension and a stand-alone LTC insurance only for the highest losses of independence (the third level of German scale), completed the benefit of lump-sum payment as coverage of fixed cost for LTC, allowing for a lower premium or a higher amount for severe and delicate situations and providing enough liquidity for the early-stage expenses. This combination with pension funds will lead to a competitive effort ameliorant stand-alone LTC product. Consider that the fiscal treatment of these two products is different, causing relevant distortions. Premiums paid for stand-alone LTC coverage (even for the lump-sum payment) counts as tax expenditure, by contrast, there is no tax advantage for workers converting their pension funds' assets into an enhanced pension: a possible, solution is a reduction of the original tax rate from 15% to 12% for individuals converting their capital into an enhanced pension. Tax expenditures linked to LTC, as for other life insurances is in terms of premium/year and is disrespectful of different risks for men and women: practically speaking a man could ensure himself a lower tax exempted LTC insurance and/or life insurance than a woman if they make the same stream of payment, with an average advantage of 20% or more for life insurance.

The policymaker considers a public interest to postpone the conversion of pension once the insured satisfies the requirement for first pillar pension, again this means leaving the worker uninsured against loss of independence due to injury and illnesses (cancer & cardiovascular) happening before he converts into an enhanced pension. As the retirement age is rising sharply, a solution should be to allow early conversion into an annuity (i.e., before retiring) for workers converting into an LTC annuity: this will work as an incentive for a market-reliant solution for ensuring both longevity and LTC risk. A solution is in the expansion of enhanced pension available, a set of ratios B_{ltc}/B_{healt} is suggestable, both greater and lower than the actual one[38], according to the formula:

$$\Pi_{x0}(t)(B) = \Pi_{x0}(B_{health}(t), B_{ltc}(t))$$

the premium of enhanced pension paying B_{ltc} and B_{heal}t must be equivalent to the premium of a vanilla annuity paying B: this structured product is the linear sum of an annuity of amount B_{healt} paid whenever the insured is alive and an amount B_{ltc} only if the insured loses his autonomy, in actuarial notation:

$$B_{health} a_{x0}^{aa} + B_{ltc} a_{x0}^{ai}.$$

Thus, if we follow the assumption that individuals with a higher annuity will live more in the LTC state and they will ask for a ratio<2, this will decrease the risk. By contrast, an individual with lower income and life expectancy will demand higher ratios, decreasing the risk for the insurance company. Increasing the possible conversion ratios results in risk diversification and a narrower target, which indirectly increases risk diversification. By contrast, we do believe that setting levels of the LTC in monetary terms could lead to a higher risk for the insurer. Even if 1000€ is not extreme it could be for contracts with a modest amount of base annuity. If this were the case, the risk of accidental discards in the portfolio would increase, in the sense that the total payment by the insurer would significantly depend on who is making the LTC claim, and not just on the numerosity. This is a good risk as it reduces through risk pooling.

38 E.g., the vector= [1 1.25 1.5 1.75 2 2.25 2.5 2.75 3]

3) The actuarial model for long term care contracts

3.1) Markov and semi-Markov models

The interest of our research is to price the following contract[39]: an annuity that pays K yearly if the insured is alive and 2K if the insured is in the LTC status. We want to find the conversion coefficient Π that allows converting capital into an annuity so that our year payment in health status is equal to

$$K= \frac{Capital}{\pi} \text{ and so, we have that Capital=K}\pi$$

In case of unit payment in the healthy status we have that the following equality holds:

$$\text{premium=Capital=K}$$

and, in general, apart from the specific value of K that depends on the amount that the insured wants to invest in this contract, it still holds Capital=Kπ. We can refer to the coefficient of conversion and premium substantially as synonyms, apart from a constant.

We will first tackle yearly annuity premium, the extension to shorter payment terms is, in a sense, trivial, and follows the usual rules for life insurance. The most common payment frequencies are yearly, half-yearly, quarterly, and monthly base. This depends even on the insured amount: if the annuity value is particularly low (i.e., less than 3000€/year) it is hardly possible to ask for a payment frequency less than the yearly one.

To do this, we consider a three-stage model, and the following notation holds: x denotes the policyholder age at capital conversion, that is the moment when the insurance contracts start through the payment of a lump-sum premium. Capital conversion in this case coincides with the LTC and life policy issue, with the premium being fully paid at the same moment. Henceforth, x denotes the policyholder's age at policy issue, and we assume there is a finite ultimate age $\omega<\infty$, in our case 120-x (since a conclusion, in a numerical method, must be reached for efficient programming). The time t measures time since capital conversion and thus corresponds to contract seniority. The policyholder's history is described by the stochastic process $\{X_t, t > 0\}$, with X_t in $\{alive, invalid, death\}$={1,2,0}. We denote alive with the letter A, invalid with the letter I and death with the letter D. We can set a one-to-one correspondence as there is a bijective function linking the status and the payment. *Obiter dictum*, an ordinary annuity is a stochastic process where $\{alive, invalid, death\}$={1,1,0}, while in case we want to pay an amount $\gamma>1$ in I status it will be enough to put $\{alive, invalid, death\}=\{1,\gamma, 0\}$, The state "alive" represents the ordinary annuity, diminished by the opportunity cost of the additional coverage, while the state invalid is the effective LTC coverage. The only transitions allowed are from A (active state-healthy) to I (invalid state) and D (dead state) and from I to D: it is not possible to reverse from I to A. This choice is a realistic assumption that simplifies calculations, as the dynamic becomes hierarchical and can be described in terms of fewer variables (transition probabilities).

At that point, we need to introduce, in general, a Markov process and to fit this concept into our model[40]. Let X_0, X_1, $X_{2\ldots}$ be a sequence of independent random variables with $X_t \in T$. We think the

39 This kind of contract is a particular form of enhanced pension, mainly used in Italy as a conversion option for pension funds. We mention E. Pittaco, ERM and QRM in life insurance: an actuarial primer ch.8. Springer Actuarial, 2020 and E. Pitacco Health insurance, Basic actuarial models Springer Actuarial, 2014 as the main sources for the product definition. It is offered by about 10 pension funds a possible conversion form.

40Refer to *Kenneth Baclawski, Mauro Cerasoli, Gian-Carlo Rota, Introduzione alla probabilità, seconda edizione, Unione Matematica Italiana, 1990 - 391 pagine.* An English version had never been published, since the sudden premature death of G-C. Rota happened in Cambridge (MA) in 1999. The author even considered, in a values of X_n's as being the state of the Markov chain. Thus, if (X_n=i) we say that the process is in the state I at time n. Moreover, if (X_n=i) and if (X_{n+1}=j), then we say that there has been a transition from state i to state j. The conditional probability P (X_{n+1} = j | X_n = i) is called the transition probability from state i to state j at time n. By the law of alternatives, the probability distribution of X_{n+1} is determined by the transition probabilities and the probability distribution of X_n:

$$P(X_{n+1} = j) = \sum_i P(X_{n+1} = j \mid X_n = i)P(X_n = i)$$

As a result, we see that all the probability distributions of the X_n's as well as all their joint distributions are determined by the distribution of X_0 and the transition probabilities. The main property of the Markov chain is the weak Markov property: A sequence X_0, X_1.... of integer random variables forms a Markov chain if for any integers i_0, $i_1 \dots i_n$

$$P(X_n = i_n \mid (X_0 = i_0) \cap (X_1 = i_1) \cap \dots (X_{n-1} = i_{n-1})) = P(X_n = i_n \mid (X_{n-1} = i_{n-1})$$

In other words, the future states of the Markov chain are dependent only on the present state and not on how the Markov chain reached the present state. We call this condition the Markov property.

In truth, this property will not be respected in our formalization: the cost of sticking to this rule is not to use the impact of the length of sejour onto the mortality probability of invalid people. But a model that does not use the above-mentioned property to model LTC contract is purely a Markov-sticking model. They increase their complexity in different ways: with more levels of LTC or with different aetiologies of LTC that will present, in both cases different and adequate mortality structures still sticking to the Markov property. It is not true, in general, that a semi-Markov model is more accurate than a Markov one, in our context: the choice must be made even considering the target (public insurance, private insurance, professional-related one) and the data available[41].

A Markov chain is said to be homogeneous if all the transition probabilities P (X_{n+1} = j | X_n = i) do not depend on n. By contrast, if the transition probabilities depend not only on the states i and j but also on n, in such a case our process is continually changing or inhomogeneous. Furthermore, inhomogeneous processes are generally led by a vectorial dimension of transition probabilities and could be referred easily or hardly to homogeneous ones[42]: in our case, unfortunately, the transition probabilities evolve along the main diagonal of the age x year matrix.

When we write the transition probabilities as a matrix, we get a matrix M called the transition probability matrix of the Markov chain. The rows represent the starting states and the columns represent the ending states, during each unit of time. The transition probability matrix determines the Markov chain except for the probability distribution of X_0. The entries of the matrix must be between 0 and 1, and the sum of the entries of each row is 1. On the other hand, we can say nothing about the columns. In our case there are boundaries, as the movement is single directional: from healthy to invalid or death and from invalid to death, with obviously all the death remaining that (absorbing

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complementary way, something more up to date, in particular chapter VI of Le Gall, *Mouvement brownien, martingales et calcul stochastique* (springer,2013) and, in English, Rick Durrett. Probability: theory and examples. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge, fourth edition, 2010

41 Quentin Guibert, Frédéric Planchet. Non-Parametric Inference of Transition Probabilities Based on Aalen-Johansen Integral Estimators for Acyclic Multi-State Models: Application to LTC Insurance. 2018. ffhal-01183542v2

42 As observed by G.C. Rota (1979) referring to the process of sampling without replacement: many apparently inhomogeneous Markov chains can be reinterpreted as homogeneous Markov chains, so that this concept is not special as it might first appear.

barrier) and with the impossibility to recover from invalid to healthy. As a result, our chain should be represented by a directed graph (digraph), in particular a graph where for each pair of vertexes there is at most one arch. Moreover, since our states can be topologically sorted in a way that all the edges follow the same orientation, is a directed acyclic graph (DAG). The transition matrix being a triangular one is a sufficient but not necessary condition for this happening. We should consider that classification of the states is generally designed for homogeneous non-triangular processes, and consequently, we have to adequate the general theory to the *pathologic* case: this could be done with some slight differences. We say that a state is recurrent if, given the fact that the insured is in that state, he will for sure return to visit that state.

$$f_i = P\ (\text{ever re-enter } i | X_t = i)$$

If f_i=1 we say the state is recurrent, if not it is called transient. A recurrent state could even be an absorbing barrier in case not only f_i=1 but P (ever exit $i|X_t = i$) =0 a.s. The death state D works in that way. An absorbing barrier is characterized by the following row vector:

AB= [a_{ij}=1 if $a_{ij} \in$ diagonal, else a_{ij}=0], that in our case being i=3 is: [0 0 1]

as a part of the transition matrix[43]. If the element being one of the vector AB does not belong to the main diagonal, we speak about a reverting barrier RB.

RB= [else a_{ij}=1 for the reflection level from i to j, else 0], e.g., with i=3 RB= [1 0 0], [0 1 0]

The exit force from an absorbing barrier is 0, while the exit force from a reflecting barrier is 1, the exit force from a transient state is in between. By contrast, the states I and A are transient, since it is possible to re-enter that state only never exiting from that state, and this occurs with a probability p^{aa}<1 and p^{ii}<1, defined after as the permanence force μ^{aa} And μ^{ii} that is complementary to 1 of the respective exit forces $\mu^{a\cdot}$ and $\mu^{i\cdot}$. By consequence, our transition matrix is an upper-triangular one, defined as M(t), as the process is right-censored:

$$\text{M(t)}=\begin{vmatrix} p^{aa}(t) & p^{ai}(t) & p^{ad}(t) \\ 0 & p^{ii}(t) & p^{id}(t) \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1-\ p^{ai}(t)-\ p^{ad}(t) & p^{ai}(t) & p^{ad}(t) \\ 0 & 1-\ p^{id}(t) & p^{id}(t) \\ 0 & 0 & 1 \end{vmatrix}$$

As all the Markov transition probabilities matrices is a row-stochastic matrix, i.e.,

$$a_{ij} \geq 0\ for\ every\ i,j\ and \sum_j a_{ij} = 1\ for\ every\ i\ in\ [1,2,3] = [A, I, D].$$

We observe that unless we assume a pathologic context were for some $\tilde{t}$ we observe

$$p^{ad}(\tilde{t}) = 0\ and\ \ p^{id}(\tilde{t}) = 0$$

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43 G.C. Rota (1979) introduced the absorbing barrier as the termination of one gamblers ruin problem formalized as a Markov Chain. Rota found two absorbing barriers in this game: either the wealth goes to 0 or surpasses a predefined amount K, the game is terminated. It seems by consequence, that we have two concepts: a positive absorbing barrier and a negative one. In our "game", the LTC insurance we have only the negative one. In truth, it is arguable the idea of an upper absorbing barrier, as the amount is fictitious, and this second termination is easily avoidable by saying that the player wants to play until he gets ruined if he gets. So, the only *natural* absorbing barrier is the lower one, representing the ruin: in our case the ruin for the insured is the death.

The matrix cannot be doubly stochastic. We say that a state i is accessible from the state j there is some $\tilde{t}$ for which $p^{ij}>0$. I is accessible from A and D is accessible from I and A, while I is not accessible from D and A is not accessible from I or D. This means that there are no communicating states and, as a result, three classes are representing the states and the matrix is not irreducible. We cannot cope with a doubly stochastic matrix: in our case, the only admissible doubly stochastic matrix is the permutation matrix I_3. Consider that, by definition, the probability of observing two or more transitions in a single interval (we have only one admissible single step-double transition: from A to I and then from I to D) Δt is equal to:

$$p^{aid\ in\ a\ single\ step} = \underset{\Delta t \to 1}{o}(\,p^{ai}\,p^{id}\Delta t) = \underset{\Delta t \to 1}{o}(\Delta t)$$

where Δt is the time interval selected for the representation of Markov-chain, in our case 1 year or subinterval or year, depending on the contract (half-yearly, quarterly, monthly).

While, if we remove the assumption that it is impossible to recover from the LTC status (only *reformatio in peius*) and so we admit a probability of *reformatio in melius* $p^{ia}>0$, we have a more general nontriangular matrix, defined as $M_{General}$:

$$M(t) = \begin{vmatrix} p^{aa}(t) & p^{ai}(t) & p^{ad}(t) \\ p^{ia}(t) & p^{ii}(t) & p^{id}(t) \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 - p^{ai}(t) - p^{ad}(t) & p^{ai}(t) & p^{ad}(t) \\ p^{ia}(t) & 1 - p^{id}(t) - p^{ia}(t) & p^{id}(t) \\ 0 & 0 & 1 \end{vmatrix}$$

Concerning LTC mortality we believe that the semi-Markov models are a suitable integration of pure Markov ones, as the *Sejour* time plays a relevant role: this had already been confirmed by different papers[44]. We define *Sejour*[45] time in the state I, for each X belonging to I (the invalidity situation):

$$Z_t = max\{z \leq t \mid X_t = X_{t-h}\ for\ all\ 0 \leq h \leq z\}$$

Concerning the transition probabilities from healthy status, we consider an ordinary Markov model, because age and *Sejour* time coincide: if I am healthy today, I have whenever been Healthy and so there is perfect collinearity and no additional explanatory power from the *Sejour*, so no benefit in considering a semi-Markov model. The innovation is with the transition from LTC to death, where we introduce the new variable Z_t, i.e., the couple $\{(X_t, Z_t), t > 0\}$ is still a non-homogeneous Markov process, as both components depend only on the position in the previous year (and the anagraphic age, which explains the non-homogeneity component[46]). The couple $\{(X_t, Z_t), t > 0\}$ depends only on the situation at time t-1: the state of the insured depends only on the last occupied state, apart from *Sejour* time, and *Sejour* time (considered only for invalid individuals) depend only on the last year. A complementary solution to a three-states (A, I, D) semi-Markov model would be a pure non-homogeneous Markov model where the invalidity state has been partitioned in two-three clusters, not based on the intensity, but on the organic cause of disability, in a way, that transition between different

44 Murtaugh, Christopher M., Brenda C. Spillman, and Mark J. Warshawsky. "In Sickness and in Health: An Annuity Approach to Financing Long-Term Care and Retirement Income." The Journal of Risk and Insurance 68, no. 2 (2001): 225-53. Or even Rickayzen, B. D., and D. E. P. Walsh. "a multi-state model of disability for the United Kingdom: implications for future need for long-term care for the elderly." British Actuarial Journal, vol. 8, no. 2, 2002, pp. 341–393. JSTOR, www.jstor.org/stable/41141545.

45 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham. p.132

46In homogeneous Markov Chains, the transition probabilities $p_{ij}=P(X_{n+1}=j|X_n=i)$, do not depend on n. Thus, throughout the evolution of the process through time, the transitions among states follow the same probability rules. In non-homogeneous chains, transition probabilities can vary across time.

invalidity state would not be possible[47]. The policyholders must be healthy at capital conversion, so we are conditioning concerning the event $X_0 = A$, that is the starting point of our stochastic process. We now need to introduce the following notation, referring to all transition probabilities[48]

- $_{u}p^{ai}(x+t) = P[X_{t+u} = I | X_t = A]$
probability for an individual in A at time *t* of being in the state I at time t+u
- $_{u}p^{ad}(x+t) = P[X_{t+u} = D | X_t = A]$
probability for an individual in state A at time *t* of being in state D at time t+u
- $_{u}p^{id}(x+t; z) = P[X_{t+u} = D \mid X_t = I, Z_t = z]$
probability for an individual in I at time *t* since t-z of being in state D at t+u
- $_{u}p^{aa}(x+t) = P[X_{t+u} = A | X_t = A]$
probability for an individual in state A at time *t* of being in state A at time t+u
- $_{u}p^{ii}(x+t; z) = P[X_{t+u} = I \mid X_t = I, Z_t = z]$
probability for an individual in I at time *t* since t-z of being in the I at time t+u

And if u=1 for all the already presented probabilities we have:

- $p^{ai}(x+t) = P[X_{t+1} = I | X_t = A]$
probability for an individual in A at time *t* of being in the state I at time t+1
- $p^{ad}(x+t) = P[X_{t+1} = D | X_t = A]$
probability for an individual in state A at time *t* of being in state D at time t+1
- $p^{id}(x+t; z) = P[X_{t+1} = D \mid X_t = I, Z_t = z]$
probability for an individual in the state I at time *t* since time t-z of being in state D at time t+1
- $p^{aa}(x+t) = P[X_{t+1} = A | X_t = A]$
probability for an individual in state A at time *t* of being in state A at time t+1
- $p^{ii}(x+t; z) = P[X_{t+1} = I \mid X_t = I, Z_t = z]$
probability for an individual in I at time *t* since t-z of being in I at time t+1

The risk of making a given transition, depending on the state currently occupied is captured by transition rates (or transition intensities). This concept is defined based on the key concept of life insurance, the force of mortality, which allows a continue-state representation of annuities and life insurance contract in general, extending this property to a more general multistate model describing health insurance products, including the enhanced pension. Transition intensities are represented by the right-side limit of incremental ratio for the given transition probability.

Given the three probabilities of transition, we do obtain the following forces[49]:

- $\mu^{ai}(x+t) = \lim_{h \to 0} \frac{_{h}p^{ai}(x+t)}{h}$
- $\mu^{ad}(x+t) = \lim_{h \to 0} \frac{_{h}p^{ad}(x+t)}{h}$
- $\mu^{id}(x+t; z) = \lim_{h \to 0} \frac{_{h}p^{ai}(x+t; z)}{h}$, *sub* z<t

As an insured can occupy only one status, once he makes a shift, he empties the earlier status where it was placed: the exit force from D is null (obviously), the exit force from Invalid status is equivalent to the force of mortality conditioned to invalid status and the exit force from healthy status is the force of mortality from healthy status plus the force of invalidity from healthy status. Note the progression:

47 Guillaume Biessy. Continuous time semi-Markov inference of biometric laws associated with a Long-Term Care Insurance portfolio. 2015. Ch 2.
48 Christiansen, M.C. Multistate models in health insurance. AStA Adv Stat Anal 96, 155–186 (2012). https://doi.org/10.1007/s10182-012-0189-2
49 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham. p.133

in the “last” state 0, then is one element, and then it becomes the result of two contributions. As there is only one possibility of exit from the I status we have,

$$\mu^{i\cdot}=\mu^{id}$$

while the exit force from healthy status is[50]:

$$\mu^{a\cdot}=\mu^{ai}+\mu^{ad}$$

By last, the existing force from the last status D (death) is null: it works as an absorbing barrier of the stochastic process. These barriers have the property that once one of them occurs, the subsequent states of the Markov chain are all this same state. So, we have:

$$\mu^{d\cdot}=0 \text{ for every } \omega \text{ belonging to } \boldsymbol{\Omega}$$

3.2) Recovering the fair premium level.

We are now ready to recover the fair premium level through the expected value principle as in general life insurance principles.[51]

As we have an immediate payment, we have only to account for the benefit leg, so Π=**E**(Benefits)

this principle used to compute life insurance premiums extends to all health insurance products. It states that at policy issue, the expected present value of the benefits paid to the policyholder is equal to the premium paid to the insurer. The discount factor v (0, t) is the present value today of a unit payment made at time t, with v (0,0) =1. As there are no deferred premia there is no need of recovering forward rate structures through implied price theorem, the discount factors we need are only the actual ZCB spot curve. Where, apart from the elements previously defined, we now redefine in mathematical terms the structure of the benefit B (X_t) at time t, and the anagraphic age X. This is, considering how to evolve states A, I and D a càdlàg process.

- $\mathrm{B}(X_t) = 2$ if $X_t = \mathrm{I}$
- $\mathrm{B}(X_t) = 1$ if $X_t = \mathrm{A}$
- $\mathrm{B}(X_t) = 0$ if $X_t = \mathrm{D}$

We furthermore consider the impact of the assumption of $p^{ia}(t)$=0 for every t. In particular, this implies that the process that the function that associates to every step t the specifical state $\{alive, invalid, death\}$=$\{1,2,0\}$ could be decomposed in the product of an indicator function of the existence in the life of the insured and a function that associates to the living insured a state limited to $\{alive, invalid\}$=$\{1,2\}$, we call this function B(X_t|alive) and it is a càdlàg non-decreasing process. Consequently, B (X_t) is càdlàg non-decreasing function until the death.

$$\mathrm{B}\,(X_t) = \mathrm{B}(X_t|\mathrm{alive})\ 1_{\mathrm{Alive}}\,(\mathrm{t})$$

Furthermore, if we denote of being today at the contract entry, the net wealth from the contract (in a currency unit of the year of the entry into the contract) as the following variable W(t), defining by $\bar{\pi}$ the fair premium, not the one obtained in the specific repetition of the simulation:

$$\mathrm{W(t)}= \sum_0^t \frac{\mathrm{B}(X_t)}{(1+i(0,t))^t} - \bar{\pi}$$

50 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham. p.134

51 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham.pp.135-136

Where i (0, t) is the spot risk-free interest rate from today (the day of the entry into the contract, so of the premium payment) for t years. We observe W(t) still being a càdlàg non decreasing process and a submartingale.

So, in terms of a general LTC contract priced in continuous time[52], with specific benefit levels B_{health} and B_{LTC} in continuos time:

$$\Pi = \int_0^w B_{health}(t)\,{}_tp_x^{aa}v(0,t)dt+\int_0^w {}_tp_x^{aa}\,\mu_{x+t}^{ai}(\int_0^{w-t} {}_zp_{x+t;0}^{ii}B_{LTC}(t,z)v(0,t+z)dz)dt$$

If B_{healt} and B_{LTC} are constant, as in our case, we can remove the function reference.

$$B_{health}\int_0^w {}_tp_x^{aa}v(0,t)dt+B_{LTC}\int_0^w {}_tp_x^{aa}\,\mu_{x+t}^{ai}(\int_0^{w-t} {}_zp_{x+t;0}^{ii}v(0,t+z)dz)dt$$

Where $1_A + 1_I = 1 - 1_D$ and $1_{A\cap I} = 0$, as the three states are a partition of Ω. In case the contract is based on different premium payments, and to set the premium policy, it will be needed to have the forward rate term structure for the whole line of deferred payments. So, shifting from continuous time to discrete time:

$$\Pi=\mathbf{E}\,(\sum_{t=1}^{w} B(X_t)v(0,t))=\mathbf{E}(\sum_{t=1}^{w} 1_A v(0,t) + 21_I v(0,t)) =$$

$$= \mathbf{E}(\sum_{t=1}^{w} 1_A(t)v(0,t) + \; 2\sum_{t=1}^{w} v(0,t)1_I(t)\,)$$

$$\Pi=\sum_{t=1}^{w} P(X_t = \mathrm{A})v(0,t) + 2v(0,\mathrm{t})\mathrm{P}(X_t = \mathrm{I})$$

The benefits values, shifting back in continuous time[53], are:

- $\mathbf{E}\,(\int_0^w 1_A(t)v(0,t)dt) = \int_0^w {}_tp_x^{aa}v(0,t)dt=\bar{a}_x^{aa}$

When transition intensities are piecewise constant (and so, concerning the force of interest rate this means v (s, t) = exp (− ∂ (t – s))):

$$\bar{a}_x^{aa} = \int_0^w {}_tp_x^{aa}v(0,t)dt=\int_0^w \exp\,(-\int_0^t u_{x+s}^{a\cdot}ds)\exp\,(-\partial \mathrm{t})\,dt=$$

$$\sum_{n=1}^{w}\int_{\mathrm{n}-1}^{n} \exp\,(-\int_0^t u_{x+s}^{a\cdot}ds)\exp\,(-\partial \mathrm{t})\,dt, \text{ Where } u_{x+s}^{a\cdot}(s) = \sum_{n=0}^{w} u_{x+s}^{a\cdot}1_{[n,n+1]}$$

We now compute $\int_{\mathrm{n}-1}^{n} \exp\,(-\int_0^t u_{x+s}^{a\cdot}ds)\exp\,(-\partial \mathrm{t})\,dt$, where n-1≤t≤n

$$\int_0^t u_{x+s}^{a\cdot}ds = \int_0^t \sum_{k=0}^{w} u_{x+k}^{a\cdot}1_{[k,k+1]}(s)\,ds = \sum_{k=0}^{w}\int_0^t \; u_{x+k}^{a\cdot}1_{[k,k+1]}(s)\,ds$$
$$= \sum_{k=0}^{w} u_{x+k}^{a\cdot}\int_0^t 1_{[k,k+1]}(s)ds$$

If k+1<t, then we integrate the indicator function (which is 1). This happens for k=0,1..n-2. The integration interval is [n-1,t] and has length t-(n-1). In conclusion, we have:

$$\int_0^t u_{x+s}^{a\cdot}\,ds = \sum_{k=0}^{n-2} u_{x+k}^{a\cdot} + \; u_{x+n-1}^{a\cdot}(t-(n-1))$$

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52 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham. p.137

53 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham. p.137 and 138.

Plugging the result back in our original integral:

$$\sum_{n=1}^{w}\int_{\mathrm{n}-1}^{n}\exp(-\sum_{k=0}^{n-2} u_{x+k}^{a.} + u_{x+n-1}^{a.}(t-(n-1))-t\partial)\,dt,\text{ that leads to:}$$

$$\int_0^1 \exp(-\mathrm{t}\, u_x^{a\cdot} - t\partial)\,dt+$$

$$\exp(-u_x^{a\cdot})\int_1^2 \exp(-(\mathrm{t}-1)\, u_{x+1}^{a.} - t\partial)\,dt+$$

$$\exp\,(-u_x^{a\cdot} - u_{x+1}^{a.})\int_2^3 \exp(-(\mathrm{t}-2)\, u_{x+2}^{a.} - t\partial)\,dt + \cdots + \int_{\mathrm{w}-1}^{w}$$

As each of these integrals now admits an analytical expression, we have:

$$\frac{1-\exp\,(-\partial-\mu^{a\cdot})}{\partial+\mu^{a\cdot}} + \sum_{j=1}^{w-1}\exp\,(-\sum_{k=0}^{j-1}\mu\,{}_{x+t}^{a.} - j\partial)\,\frac{1-\exp\,(-\partial-\mu\,{}_{x+j}{}^{a\cdot})}{\partial+\mu^{a\cdot}}$$

that is the premium paid to ensure the disability-free part of the contract, linked to the average disability-free life expectancy.

- $\mathbf{E}\,(\int_0^w 1_I(t)v(0,t)dt) = \int_0^w {}_tp_x^{aa}\,\mu_{x+t}^{ai}(\int_0^{w-t}{}_zp_{x+t;0}^{ii}v(0,t+z)dz)dt{=}\bar{a}_x^{ai}$

that is the premium paid to ensure the cum disability life expectancy of an individual aged t and linked to the average time an individual healthy at time t will spend in disability before death.

And we furthermore denote the integral

$$\int_0^{w-t}{}_zp_{x+t;0}^{ii}v(t,t+z)dz{=}\bar{a}_{x+t;0}^{ii},$$

This expression, as transition intensities are piecewise constant and denoting with $\tilde{u}(\mathrm{x},\mathrm{m})$ denoting the force of mortality of LTC individuals given their anagraphic age x of entry into LTC status and sejour already spent m, simplifies in:

$$\text{Let } \mu_{y+\varepsilon;z}^{id} = \tilde{u}(\mathrm{y}+[\varepsilon-z],[\mathrm{z}])$$

$$\frac{1-\exp\,(-\partial-\tilde{u}(\mathrm{x},0))}{\partial+\tilde{u}(\mathrm{x},0)} + \sum_{j=1}^{w-1}\exp\,(-\sum_{k=0}^{j-1}\tilde{u}(\mathrm{x},\mathrm{k}) - j\partial)\,\frac{1-\exp\,(-\partial-\tilde{u}(\mathrm{x},\mathrm{j}))}{\partial+\tilde{u}(\mathrm{x},\mathrm{j})}$$

As the average residual expectancy of life of an individual entering disability at time t, so it represents the average *Sejour* in disability (in-state I) and this *Sejour* is still a Markov Process. Note the particularity that this variable, the key variable of our model, is a determinant of the outcome of the semi-Markov insurance process and is itself a Markov process.

We have that the present value of this quantity, as

v (0, t) v (t, t+z) =v (0, t+z)

$$\bar{a}_{x+t;0}^{ii}v(0,t) = \int_0^{w-t}{}_zp_{x+t;0}^{ii}v(0,t+z)dz$$

We now rewrite:

$$\bar{a}_x^{ai} = \int_0^w {}_tp_x^{aa}\,\mu_{x+t}^{ai}v(0,t)\bar{a}_{x+t;0}^{ii}dt$$

$$i.e.\,\bar{a}_x^{ai} = \int_0^w \exp\left(-\partial t - \int_0^t u_{x+s}^{a\cdot} ds\right) \mu_{x+t}^{ai} v(0,t) \bar{a}_{x+t;0}^{ii} dt =$$

$$\mu_x^{ai} \bar{a}_{x;0}^{ii} \frac{1-\exp\left(-\partial - \mu_x^{a\cdot}\right)}{\partial + \mu_x^{a\cdot}} + \sum_{j=1}^{w-1} \mu_{x+j}^{ai} \bar{a}_{x+j;0}^{ii} \exp\left(-\sum_{k=0}^{j-1} \mu_{\ x+t}^{a\cdot} - j\partial\right) \frac{1-\exp\left(-\partial - \mu_{\ x+j}{}^{a\cdot}\right)}{\partial + \mu^{a\cdot}}$$

Making explicit the relevance of the *Sejour* time in determining the average time spent in disability by an individual healthy at the premium payment, we define,

$$\bar{a}_x^{a} = \mathbf{E}\left(\int_0^w 1_{A\,U\,I}(t)\, v(0,t) dt\right) = \mathbf{E}\left(\int_0^w 1_A\ (t) v(0,t) dt\right) + \mathbf{E}\left(\int_0^w 1_I(t)\, v(0,t) dt\right)$$

and, by consequence, $\bar{a}_x^{a} = \bar{a}_x^{aa} + \bar{a}_x^{ai}$. This last equality says that the total premium for a long-term care contract is equal to the sum of the ex-disability premium plus the cum-disability premium. This is linked to the following equality: the life expectancy (both in health and disability) of an individual today health is equal to the sum of the expected disability-free life and life with a disability.

Finally, our fair premium is:

Π(x)= $\bar{a}_x^{aa} + 2\bar{a}_x^{ai}$=$\bar{a}_x^{a} + \bar{a}_x^{ai}$, where it holds that $\bar{a}_x^{a} = \bar{a}_x^{aa} + \bar{a}_x^{ai}$

More in general: Π(x)= $B_{healt}\bar{a}_x^{aa} + B_{LTC}\bar{a}_x^{ai}$=$B_{healt}\bar{a}_x^{a} + (B_{healt} - B_{ltc})\bar{a}_x^{ai}$

The long-term care coverage in our case doubles the amount, and the cost of this coverage could be linearly added to the standard annuity. However, in pricing through a numerical scheme, we will use the first expression.

In conclusion, we consider as additional coverages a lump-sum payment once the individual enters the LTC state. This is generally not sold as a stand-alone product, but possibly as an additional guarantee. The general value of the lump sum is 5 times the monthly payment. Still considering continuous-time and continuous force of mortality, we have that the fair premium level of a lump-sum unit payment is a function of interest rate, permanence probability p_x^{aa} in the alive state and mortality force $\mu_{\ x+t}^{ai}$[54]:

$$\overline{A}_x^{a;a-i} = \int_0^w v(0,s)\, {}_z p_{x+t;0}^{aa} \mu_{\ x+t+s}^{ai}\, ds, \text{ where } {}_z p_{x+t;0}^{aa}$$

Is the probability that an individual that is healthy at the origin of the contract is still in the healthy state after an arbitrary z interval of time: i.e., that, from the origin of the contract no transition had taken place. Note that this contract is not an annuity but the lumps-sum component of our quasi-annuity contract: it represents the only non-periodic component of the benefit leg.

When transition intensities are piecewise constant the following simplifications holds, recovering ∂ as the constant force of interest rate (and so, concerning the force of interest rate this means v (s, t) = exp (− ∂ (t – s)))

$$\overline{A}_x^{a;a\xrightarrow{\text{lumpsum}}i} = \int_0^w v(0,t)\, p_x^{aa} \mu_{\ x+t}^{ai}\, dt =$$

$$\int_0^w \exp\left(\int_0^t u_{x+s}^{a\cdot} ds\right) \exp(-\partial t) \mu_{\ x+t}^{ai}\ dt =$$

54 $\overline{A}_x^{a;a-i}$ denotes a lump-sum payment made once the individual enters in the LTC state. 54 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham. p.139. (In truth, this case is not considered in calculations by the authors).

$$\mu_{x}^{ai}\int_0^1 \exp(-\mathrm{t}\, u_x^{a\cdot} - t\partial)\, dt +$$

$$\mu_{x+1}^{ai}\exp(-u_x^{a\cdot})\int_1^2 \exp(-(\mathrm{t}-1)\, u_{x+1}^{a\cdot} - t\partial)\, dt +$$

$$\mu_{x+2}^{ai}\exp\left(-u_x^{a\cdot} - u_{x+1}^{a\cdot}\right)\int_2^3 \exp(-(\mathrm{t}-2)\, u_{x+2}^{a\cdot} - t\partial)\, dt + \cdots$$

As each of these integrals now admits an analytical expression, we have:

$$\mu_{x}^{ai}\frac{1-\exp\left(-\partial-\mu^{a\cdot}\right)}{\partial+\mu^{a\cdot}} + \sum_{j=1}^{w-1}\mu_{x+j}^{ai}\exp\left(-\sum_{k=0}^{j-1}\mu_{x+t}^{a\cdot} - j\partial\right)\frac{1-\exp\left(-\partial-\mu_{x+j}^{a\cdot}\right)}{\partial+\mu^{a\cdot}}$$

This premium is to be summed up linearly with the premium for the vanilla enhanced pension, so we have, in our case, reporting the usual market values and assuming annual fractioning of the benefit:

$$\Pi(\mathrm{x}) = \bar{a}_x^{aa} + 2\bar{a}_x^{ai} + 5/12\overline{A}_x^{a;a-i} = \bar{a}_x^{a} + \bar{a}_x^{ai} + 5/12\overline{A}_x^{a;a-i}$$

We want even to consider a counter-ensured annuity, so an annuity that pays the "basic amount" (i.e., 1) in the first m years whether the insured is healthy or died and pays 2 if he is not healthy and that after this year's start to pays as an ordinary LTC annuity (so to pay 1 if the insured is healthy and 2 if the insured is invalid)[55]. We define this contract as $\bar{a}_x^{a\,CI}$ and in this case, we have that $\bar{a}_{x:m}^{ad}$ is the value of death insurance covering the first m years is the insured today is alive, while a_m is a contract paying surely (independently from every condition) for the first m years and ${}_{m|}\bar{a}^{\mathrm{a}}{}_{x+m}$ is a vanilla annuity (without any LTC coverage) deferred m years from the entry into the contract? Allowing us to shift to discrete pricing for the shake of compactness, we have:

$$\Pi(\mathrm{x}) = \bar{a}_x^{aa} + 2\bar{a}_x^{ai} + \bar{a}_{x:m}^{ad} =$$

$$\bar{a}_x^{a} + \bar{a}_x^{ai} + \bar{a}_{x:m}^{ad} = \mathrm{a}_m + \bar{a}_x^{ai} + {}_{m|}\bar{a}^{\mathrm{a}}{}_{x+m} =$$

$$= \sum_{t=1}^{m} v(0,t)1_{A\,U\,D}(t) + \sum_{t=1}^{w} 2v(0,t)1_I(t) + \sum_{t=\mathrm{m}}^{w} 1_A(t)v(0,t) =$$

$$= \sum_{t=1}^{m} 1_D(t)v(0,t) + \sum_{t=1}^{w} 2v(0,t)1_I(t) + \sum_{t=1}^{w} 1_A(t)v(0,t) = \sum_{t=1}^{m} 1_D(t)v(0,t) + \Pi_{\mathrm{vanilla}}(\mathrm{x})$$

Note that, by definition, if $1_D(t) = 1\ then\ 1_D(t+j) = 1\ for\ all\ j\ in\ [1, t+j=w]$. And we then have, denoting by $\bar{a}_x^{a\,CI(m)}$ an enhanced pension covering even the death guarantee in the first m years.

$$\bar{a}_x^{a\,CI(m)} = \sum_{t=1}^{m} v(0,t) + \sum_{t=1}^{w} v(0,t)1_I(t) + {}_{m|}\bar{a}^{a}{}_{x+m}$$

So, a counter-insured LTC annuity with an additional guarantee of m years is equal to the sum of the first m zero-coupon bond plus a deferred LTC annuity for an individual aged x+m deferred of m years plus a stand-alone LTC coverage in the first m years: obviously in the first addendum there is no *alea*,

55
The payoff of a counter ensured LTC annuity. Note that, even if the insured hit the LTC barrier when he is alive, in the death guarantee the payment will return to be 1. This is no longer, differently from the vanilla contract a quasi-càdlàg non decreasing process (a cadlag multiplied by an indicator), but only a càdlàg. Same considerations hold even for the lump sum accessory guarantee.

STATE	t<=m	t>m
D	1	0
A	1	1
I	2	2

We can split a counter-ensured annuity even as a certain annuity for m years plus a stand-alone LTC coverage and a deferred vanilla annuity of m years. So, the overall variability of cashflows decreases: In this case, part of the premium is going to cover death case, part invalidity risk and part longevity one. While the costs of mortality risks increase as counter-insurance lasts longer, the effective LTC cover decrease as the annuity amount decreases there is a trade-off and consequently, in a contract with the first ten-years counter-ensured, the effectiveness of coverage against invalidity could be significantly decreased. The positive and negative aspects of this option (adding the death guarantee) must be fairly and adequately considered.

3.3) The estimation of transition probabilities[56]

The main question is now how robust and reliable data are for designing an Italian table for LTC products: exactly as there are mortality and non-life tables an insurer, to underwrite a risk needs to quote it. We believe, with Levantesi and Menzietti[57], that the INPS loss of autonomy allowance is a robust enough proxy to estimate life expectancy for LTC individuals once an adequate shift is made. Barr was in a certain sense true: LTC is such a *suitable case for social insurance* that we need to recover from here the data we are looking for. Note that in designing a two level-LTC insurance we would be in serious difficulty as the Italian public disability allowance *(indennità di accompagnamento)*, differently from the AGGIR scale for the French system (or even in the German one) does not quote the intensity of the phenomena. The disability allowances are particularly suitable as few non-clinical conditions must be satisfied. Apart from showing the necessary psycho-physical difficulties, there is no necessary condition in terms of age and income (no means testing: this last point is a bit critical, as we expect that at the beginning only higher strata of the population are taking part in a voluntary LTC insurance). From the social security databases, we know every year the number of allowances being paid, the number of new allowances and ceased allowances (it might be for death, for reacquired independence or shifting to a more convenient contract): these data allowed Levantesi and Menzietti to calculate mortality of LTC individuals and the probability, conditioned to the age and sex, of entering into LTC status[58]: these tables are to be read in conjunction with the database of the Italian population of the corresponding years: (2005-2019). We are overall satisfied with the adequacy of our proxy for the transition probabilities even if some discrepancies between public and private sector pertains in term of different situations covered and wider eligibility criteria for the public one. Individuals affected by a genetic disability (like Down Syndrome) are not eligible for private coverage while they are ensured by the public one. Obviously, in the next steps, as the Levantesi and Menzietti table is mainly suited for public institutions, relevant arrangements had been performed.

Concerning life expectancy of LTC individuals, we notice that the Sejour time plays a relevant role, in conjunction with the pathology that generated the loss of autonomy: we can even say that once we

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56 The whole section is based on Baione F., De Angelis P., Levantesi S., Menzietti M., Tripodi A. (2016). Modelli attuariali per la stima di basi tecniche relative ad assicurazioni di persone. In: De Angelis P. di Falco L. Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche, althought calculations were not so in-deep and adequate contextualisation of some choices was missing.

57 Baione F., De Angelis P., Levantesi S., Menzietti M., Tripodi A. (2016). Modelli attuariali per la stima di basi tecniche relative ad assicurazioni di persone. In: De Angelis P. di Falco L. Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche p.124 and following.

58 The dataset used by Levantesi and Menzietti consist of the following information: for every year, from 2011 to 2014, it is reported how many of individuals that where perceiving the disability allowance are still alive (the survivors' group) a how many are died. Is even associated, for every insured from how many years the allowance is perceived (the sejour time): by this way it is possible to recover the probability of entering I status given a certain age and the average sejour duration. All the dataset is gender-differentiated. Is even reported, by difference, the share of the population in A status (healthy, without disability allowance) and the number of healthy individuals that passed to I status (in the same year) or directly to the D status (so that died without passing by LTC status).

know which pathology is responsible for the impairment we can even, theoretically, derive a reversely selected mortality curve for the pathology. The problem is that in cancer & cardiovascular illnesses data are robust and reliable, not so for the whole rest (dementia, Alzheimer, Parkinson, and other disabilities). Furthermore, in so proceeding we will be asked to plot the distribution of all the pathologies conditioning to age. As everything we are doing is at all linear, we expect the same results as the one provided by the shorter way (so not conditioning to the kind of disability). Furthermore, we have that in the first year after the pathology, on average, mortality is higher than in the later one and that, starting from the second year it starts to stabilize. to the expected level without Sejour (so to the transition probability that is used in the pure Markov model). This is consistent with other empirical and theoretical checks, as for many pathologies we might expect a concentration of mortality immediately after the loss of autonomy: these results, however, depends very deeply on the undergoing illness and consequently by the age of the insured, not because the age plays a direct role in the term structure of the mortality conditioned to the specific pathology but because the ages influence the probability of manifesting the specific pathology. For both males and females, in fact, up to 75-80 years the mortality for a short Sejour (equal to 2 years or less) is higher than for long Sejour times (with a higher spread form male), after this age we observe a substantial equivalence between the two levels. This is because the prevalence of tumours decreases while the importance of cardiovascular pathologies increases as the one of dementia: these two illnesses, in fact, and more even elderly dementia, present a different term structure of conditioned mortality, really flattering: it is highly improbable to die within 2-3 years from the starting of severe dementia. Furthermore, because of the contrast with tumoral diseases, there is no plateau in the ending of the term structure, which continues to increase sharply. These data are consistent with the one observed in other similar countries[59].

These specifications are made; we are now ready to explain how these probabilities are derived. Let D_{ij} (with $i \neq j$) the number of transitions from the state i to state j happened in one year for the insureds of the referring population in the year t, having age x (in the year t), while Li (x, t), the number of the subject of the referring population of age x in the state I, generally assumed by other tables.

So, we have, (with $i \neq j$), the transition probability for the age x on the year t[60], p_{ij}

$$p_{ij}(x, t) = \frac{D_{ij}(x,t)}{L_i(x,t)},$$

while permanence probabilities are $p^{ii}(x, t)$ and are defined in the same way as the previous one i.e., $\frac{D_{ii}(x,t)}{L_i(x,t)}$. If different years of observation are available, these tables are suitable for building a dynamic projection. It holds that:

$$\sum_{j, with\ j \neq i} D_{ij} + D_{ii} = L_i,$$

For example, if both the transitions are possible, we have $D_{AD}+D_{AI}+D_{AA}=L_A$ for every x, t and that D_{ii} is equal to L_i (x+1, t+1), so the very last formula could be rewritten as:

$$p^{II}(x, t) = \frac{L_i(x+1,t+1)}{L_i(x,t)}$$

These probabilities recovered from the observed data are suitable for estimating the table of mortality and loss of autonomy. In presence of a time series of observations is possible to build a projection of the dynamic in the future starting from the observed time interval. Some models, of the family of Lee-Carter study, instead of the transition probability consider the central transition rate, defined as

59 *Costruzione delle basi biometriche per l'assicurazione Long Term care*, ScorinForm, November 2012. Author: Laure de Montesquieu

60 Baione F., De Angelis P., Levantesi S., Menzietti M., Tripodi A. (2016). Modelli attuariali per la stima di basi tecniche relative ad assicurazioni di persone. In: De Angelis P. di Falco L. Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche pp 146-148.

the ratio between the observed number of transitions and the number of exposed to transition risk. The difference between $L_i(x,t)$ and $E_i(x,t)$ is the only difference between m_{ij} (x, t) and p_{ij} (x, t): $L_i(x,t)$ is recovered from a table, while $E_i(x,t)$ is observed in the population having already underwritten the insurance. If there are no relevant sample problems (the higher the wealth, the lower the mortality, if there are no geographical cluster like using an Italian table for insuring only people living in Emilia-Romagna, or if there is no professional bias), generally the difference is negligible and we have $L_i(x,t) \approx E_i(x,t)$, *simili modo* $p_{ij}(x,t) \approx m_{ij}(x,t)$,

$$m_{ij}(x, t) = \frac{D_{ij}(x,t)}{E_i(x,t)}$$

Assuming that the central rate of transition is the same within the whole integer age observed, we do observe the following relationship between the probabilities and the central transition rate of the different states of the model[61].

$$p^{AA}(x, t) = e^{-(m_{AD}(x,t)+m_{AI}(x,t))}$$

$$p^{AI}(x, t) = \frac{m_{AI}(x,t)(P_{II}(x,t)-P_{AA}(x,t))}{m_{AD}(x,t)-m_{ID}(x,t)+m_{AI}(x,t)}$$

$$p^{ID}(x, t) = 1-e^{-m_{ID}(x,t)}$$

$$p^{AD}(x, t) = 1- p^{ID}(x, t)- p^{AI}(x, t)$$

To represent the dynamics of transition probabilities between the states of the adopted model, it has been decided to implement stochastic models of projection largely used for mortality, following the idea depicted by Levantesi and Menzietti. The advantage of such a model is that they allow us to obtain a series of simulated trajectories and so we can rely both on the best estimate (mean/median scenario) and confidence interval: this will allow us to carry out sensitivity analysis on biometric risk. In the main text, we consider the adapted version of the Lee-Carter Model[62] adapted to LTC contracts by Brouhns in 2002[63], while note 37 is explained the adaptation of another famous mortality model Cairns-Blake-Dowd original model to long term care fitting. We believe it is relevant to report the outcome of the major comparison led on these two models for modelling the mortality (the mortality of general population only, not other biometrical variables) on the Italian population (this comparison is carried out in Maccheroni and Nocito, 2017, on data up to 2015. The author believes is still holding today): *As far as the CBD model is concerned, we find that projections are not reliable for describing mortality at ages before x = 75. For this reason, LC projections are preferable for describing Italian mortality in this framework of years and ages. Finally, we would like to make clear that we examined the models in their original form, so we cannot rule out the possibility that some extensions of the models might resolve the evidenced issues*[64]. Note that the improvements of the CBD model[65] of the years 2007-2009 are insufficient to change the result mentioned (that is why the author, even if available in 2017, still referred to the first version of the CBD model (the original one)) and extremely

61 Baione F., De Angelis P., Levantesi S., Menzietti M., Tripodi A. (2016). Modelli attuariali per la stima di basi tecniche relative ad assicurazioni di persone. In: De Angelis P. di Falco L. Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche p.147.

62 Ronald D. Lee and Lawrence Carter in 1992 with the article "Modelling and Forecasting the Time Series of U.S. Mortality," (Journal of the American Statistical Association 87 (September): 659–671).

63 Adapted to our context by Brouhns, Natacha & Denuit, Michel & Vermunt, Jeroen K., 2002. "A Poisson log-bilinear regression approach to the construction of projected lifetables," Insurance: Mathematics and Economics, Elsevier, vol. 31(3), pages 373-393, December.

64 Maccheroni, Carlo, and Samuel Nocito. 2017. "Backtesting the Lee–Carter and the Cairns–Blake–Dowd Stochastic Mortality Models on Italian Death Rates" Risks 5, no. 3: 34. https://doi.org/10.3390/risks5030034

65 Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A., and Balevich, I. (2009) A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. North American Actuarial Journal 13(1): 1-35.

costly in terms of overfitting. This cost of overfitting will for sure be unsustainable as our model must be able to work properly not only with mortality but also with other biometric variables (transition to long term care= probability of becoming invalid)

We then describe the LTC- adapted version of the Lee-Carter model, as it is applied for the estimate of the technical basis of LTC insurances. The key variable, for all the transition possibilities, is the level of the central transition rate from generic state i to generic state j, with j≠i.

So, we have[66]:

$$\log (m_{ij} (x, t)) = \alpha^{ij}_{x} + \beta^{ij}_{x} K^{ij}_{t},$$

$$\text{sub} \sum_x \beta_x^{ij} = 1 \, , \sum_x K_x^{ij} = 0$$

where α^{ij} is the behaviour of the transition from the state i to state j concerning the age, β^{ij}, is the transition from the state i to state j reacting to index K. The number of an individual that shifts from state i to state j follows a Poisson law by parameter $\lambda= E_i (x, t)\, m_{ij} (x, t)$ yearly. Note in this context the importance of Poisson reproducibility. Parameters K_{ij} estimated on the historical dataset are time series that can be forecasted and projected using ARIMA (0,1,0), i.e., a random walk with drift.

We have

$$K_{s+1} = K_s + \mu + CZ_{s+1}$$

where K_s is the vector 3x1 of the three parameters at the discrete time s, where s indicates the time.

- μ is the vector 3x1 of the derives of the vector of the three processes.
- C is the upper triangular matrix 3x3 constant so that CC’ is the covariance matrix of the three parameters K^s, obtained by CC’ by Cholesky decomposition.
- Z^s is a vector of 3x1 random variables normal standard mutually independent.

Concerning the correlation between K_{AD}, K_{ID} and K_{AI} we assume that they are mutually and completely uncorrelated: it means that to our purpose CC’ will be a diagonal matrix where the ii elements will be the variance of the K_i parameter. The consequent Cholesky decomposition is a diagonal matrix where the ii elements are the standard deviation of each K_i element. This is correlated with the short time interval we are considering, which makes it difficult to catch the interrelations between underlying phenomena: it will be needed to put in place even some medicals and sociological studies to fully understand the question. In particular[67],

$$CC' = \begin{vmatrix} \sigma(K)_{AD}^2 & 0 & 0 \\ 0 & \sigma(K)_{AI}^2 & 0 \\ 0 & 0 & \sigma(K)_{ID}^2 \end{vmatrix}, \text{we have a trivial Cholesky decomposition so that}$$

$$C = \sqrt[2]{CC'} = \begin{vmatrix} \sigma(K)_{AD} & 0 & 0 \\ 0 & \sigma(K)_{AI} & 0 \\ 0 & 0 & \sigma(K)_{ID} \end{vmatrix},$$

Between the different options for parameter estimations, we have the maximisation of the log-likelihood of the Poisson distribution, defined as:

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66 Baione F., De Angelis P., Levantesi S., Menzietti M., Tripodi A. (2016). Modelli attuariali per la stima di basi tecniche relative ad assicurazioni di persone. In: De Angelis P. di Falco L. Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche p.148.

67 If C is a diagonal 3x3 matrix C=diag(σ_1^2,σ_2^2, σ_3^2) then some of its square roots are diagonal matrices diag(t_1 ,t_2 , t_3), where $t_i=\pm\sqrt[2]{\sigma_i^2}$. If the diagonal elements of C are real and non-negative then it is positive semidefinite, and if the square roots are taken with non-negative sign, the resulting matrix is the principal root of C.

$$l(\lambda, X) = \sum_j \{X(j)\ln[\lambda] - \lambda - \ln[X(j)!]\}$$

But in a panel context, the observed values $X_1, X_2..X_n$ develop themselves in two-dimension instead than in one, so we have that to every X_j correspond a vector $[D(\bar{x},t_1), D(\bar{x},t_2)..D(\bar{x},t_n)]$ and so we have, to link all the unidimensional X(j), with all the possible D(x,t), to consider how the bidimensional patter affects summations:

$$\sum_j X(j) = \sum_x \sum_t D(x,t)$$

And we consequently report this summation into the log-likelihood function, obtaining from l (λ, X) to l (λ, D):

$$l(\lambda, D) = \sum_{x,t}\{D(x,t)\ln\lambda - \lambda - \ln[D(x,t)!]\}$$

As we further remember that our λ depends on E (x, t) and γ, as we have[68]

$$\lambda = E(x,t)m(x,t,\gamma)$$

$$l(\gamma, D, E) = \sum_{x,t}\{D(x,t)\ln[E(x,t)m(x,t,\gamma)] - E(x,t)m(x,t,\gamma) - \ln[D(x,t)!]\}$$

Where γ is the set of parameters that needs to be estimated and from which depends on transition rates $m(x,t,\gamma)$.

This model had been selected after a comparison with the other options, the abovementioned Cairns-Blake-Dowd[69] based on *probit* logistical model: The main difference between the LC and CBD

68 This is the only formula provided by Baione F., De Angelis P., Levantesi S., Menzietti M., Tripodi A. (2016). Modelli attuariali per la stima di basi tecniche relative ad assicurazioni di persone. In: De Angelis P. di Falco L. Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche at p.150: all the previous calculations are solely responsibility of the author.

69 Cairns, A.J.G., Blake, D., Dowd, K., et al. (2009). A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. North American Actuarial Journal, 13(1), 1-35. The authors compared quantitatively eight stochastic models of mortality explaining improvements in mortality rates in England and Wales and in the United States. Basing their observations on the Bayes Information Criterion (BIC), the investigators find that, for higher ages, an extension of the Cairns-Blake-Dowd (CBD) model that incorporates a cohort effect fits the England and Wales males' data best, while for U.S. males' data, the Renshaw and Haberman (RH) extension to the Lee and Carter model that also allows for a cohort effect provides the best fit. However, the main problem of this model identifies with the robustness of parameter estimates under the RH model, calling into question its suitability for forecasting. A different extension to the CBD model that allows not only for a cohort effect, but also for a quadratic age effect, while ranking below the other models in terms of the BIC, exhibits parameter stability across different time periods for both datasets. This model also shows, for both datasets, that there have been approximately linear improvements over time in mortality rates at all ages, but that the improvements have been greater at lower ages than at higher ages, and that there are significant cohort effects. The original model hypothesized that the mortality probability q (x, t) is governed by the following rule:

$$\text{logit}(q(x,t)) = k_t^{(1)} + k_t^{(2)}(x - \bar{x})$$

$Where\ k_t^{(1)}, k_t^{(2)}$ are two stochastic processes building a bivariate time series and governing the projection of mortality probabilities. This model does not pose problem of identifications of parameters: therefore, differently from the Lee-Carter model, there is no need in formulating constraints on the parameters. To recover the projection of the future level of mortality rate, it is possible to use a multivariate ARIMA model, that allows to enter a correlation between the two parameters $k_t^{(1)}\ and\ k_t^{(2)}$. Usually, even for this model are suitable simple ARIMA (1,1,0) model and so we have, conserving the drift:

$$Ks+1 = Ks + \Theta\,(Ks-2- Ks-1) + \mu + CZt+1$$

Alternatively, it possible, according to the KISS (keep it sophisticatedly simple) principle, to use a simpler ARIMA (0,1,0) model, in which the autoregressive component (+ Θ (Ks-2- Ks-1)) had been omitted, obtaining.

$$K_{s+1} = K_s + \mu + CZ_{t+1}$$

Where K is the vector 2x1 of the parameters $k_t^{(1)}\ and\ k_t^{(2)}$at time S.The time S indicates the time, Θ is the vector of the autoregressive component of the model, if applicable, μ is the vector 2x1 of the derives of the processes of the parameters, C is the upper triangular matrix s.t. CC' is the covariance matrix, Z is a vector of standardized normal variables. Intuitively, de difference between CBD and Lee-Carter is that in the first one the mortality is regress model, in their standard version respectively is that LC is a regression of the centrale rate of transition versus the year effect K(t), with the two regression parameters α and β depending on the anagraphic age, while CBD is a regression on the logit of the mortality ratio on the anagraphic age, with the parameters α and β of the regression models represented by $k_t^{(1)},\ k_t^{(2)}$: these two last are depending on the year where the mortality is observed. The comparison had been carried out in terms of Bayes information criterion for sex and ages. The main difference between the LC and CBD model, in their standard version respectively is that LC is a regression of the centrale rate of transition versus the year effect K(t), with the two regression parameters α and β depending on the anagraphic age, while CBD is a regression on the logit of the mortality ratio on the anagraphic age, with the parameters α and β of the regression models represented by $k_t^{(1)},\ k_t^{(2)}$: these two last are depending on the year where the mortality is observed. This criterion is well-known and acknowledged in the literature as a valid discriminant for the choice of extrapolative models in the function of their fitting to observed data. This information criterium introduces a penalty for the (high) number of parameters in the different models: as a result, between two models with the same explicative power the simplest one will be preferred. This discrimination is suitable and relevant in the proposed work: overfitting (also known as over-parametrization) will be particularly expensive as different probabilities for both males and females must be estimated with the same procedure. The Bayes information criterion, defined with $l\ (\hat{\gamma})$ the estimate of maximum log-likelihood of the vectors of parameters γ, N the number of observations and K the numbers of parameters between the different models, is[70]:

$$BIC= l\ (\hat{\gamma})\ -0.5K\ln(N)$$

Between the two (in this case two, the choice could be between m models) different models N is the same, while $\hat{\gamma}$ and K is model-specific. In our case, the CBD has several parameters that are double of the Lee-Carter one (6 vs 3) and as a result, it will be substantially penalized. Data confirm this, as the BIC discriminant shows a clear dominance by the Lee-Carter model in terms of explicative power and fitting and this dominance concerns all the transition probabilities and both the genders.

The fact that we select the same model for all the ages is not at all penalizing as it is characterized by age-specific parameters and prom suitable corrections for older ages: by consequence, there is no penalization for younger ages. However, once the fitting procedure is carried out, there are important irregularities (discontinuities) in fitting the parameters α, β, k (the second one has relevant hard shifts between ages for both the sex) and this is inconsistent with the meaning of this parameter with the model, that is describing for each age how the transition from state-to-state j depends on the value of K. As this, it has seen carried out a smoothing of these values with spline functions: this ensures value that is biologically consistent and the trend preservation. For the projection of values K_{AD} and K_{ID} it

against the age with regression parameters varying according to the calendar year, while in Lee-Carter model the mortality is regressed against calendar year with parameters depending from the anagraphic age, that is why in the first model there are two series of length K and a vector of ages, while in Lee Carter there is one vector of calendar year and two vector of length equal to the number of anagraphic ages considered in the specific fitting.

Lee-Carter VS Cairns-Blake-Dowd for fitting transition probabilities

Source: Levantesi and Menzietti, 2016

	Males			Females		
Model/probability	**PAD**	**PID**	**PAI**	**PAD**	**PID**	**PAI**
Lee-Carter	-12.034	-4.654	-5.972	-20.246	-4.969	-10.087
Cairns-Blake-Dowd	**-25.499**	**-9.605**	**-16.292**	**-45.762**	**-27.000**	**-60.604**

70 Baione F., De Angelis P., Levantesi S., Menzietti M., Tripodi A. (2016). Modelli attuariali per la stima di basi tecniche relative ad assicurazioni di persone. In: De Angelis P. di Falco L. Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche p.152.

has been used an ARIMA (0,1,0) model, because from observed data we had evidence of a substantially constant trend and, at least before 2010, this was true even for mortality of individuals LTC. Considering K_{AI} first impression was about an ARIMA (0,1,1) model, but to weigh for the administrative adjustment performed to the public allowance from which the dataset had been extrapolated, the author still opted for an ARIMA (0,1,0).

It is known that speaking about technical bases for LTC insurances we should have gone more deeply after 2050: however, considering the characteristics of the model and the availability of data a longer time frame would have resulted in unreliable estimates. What we notice in conclusion, is that the transition probability A to D (the mortality of healthy) shows a significant diminution, even in more contained at higher ages and differences between the two sexes. p_{ID} by contrast is defined by a significant increment in the whole considered interval, while the transition to disability is mostly constant, apart from a really limited decrease for the very senior ages, due to decreasing component of elderly dementia. The trajected transition probabilities are referred to as the interval 20-95, for older ages is used a graduation procedure for the transition probabilities of the ages 20-95 for each calendar year (2013-2043), using functions of the kind P-spline. This allows obtaining an extrapolation and an equation of probabilities up to 120 years, the anagraphic year of closing for our table.[71]

In conclusion, the table refers to the following transition probabilities, thanks to the obtained projection model:

- The mortality of healthy people p^{AD} (x, t)
- The mortality of invalid people p^{ID} (x, t)
- The transition to LTC status p^{AI} (x, t)

The following permanence probabilities (i.e., transition to the same state in the next year), useful in designing actuarial values for insurance products could be deduced *per viam remotionis:*

- The permanence in healthy status: p^{AA} (x, t) $^{=}$1- p^{AI} (x, t)-p^{AD} (x, t)
- The permanence in LTC status: p^{AA} (x, t) $^{=}$1- p^{ID} (x, t)

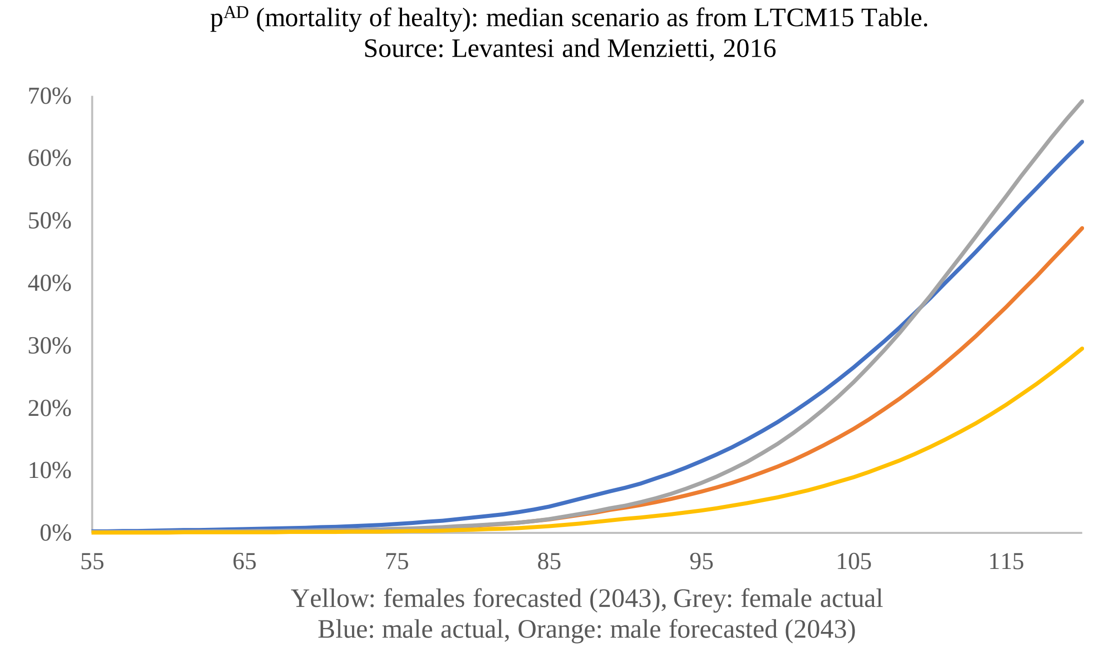
More in general, we underline that this technical basis had been developed under forecast that, even if robust, do not completely remove uncertainty and lack adequate measures of comparisons. When anomalies or abnormalities have been adequately detected, enough measures to cope with have been enforced. Model risk is even important: different models may produce different results, but the comparatively best one had been selected, in terms of fitting and adequateness. It is possible, however, that in the future the best model selectable will change, and that a different model will better represent reality. This explains also why central scenarios are whenever accompanied by upper and lower estimates (first and third quartile: $q_{0.25}$ and $q_{0.75}$).

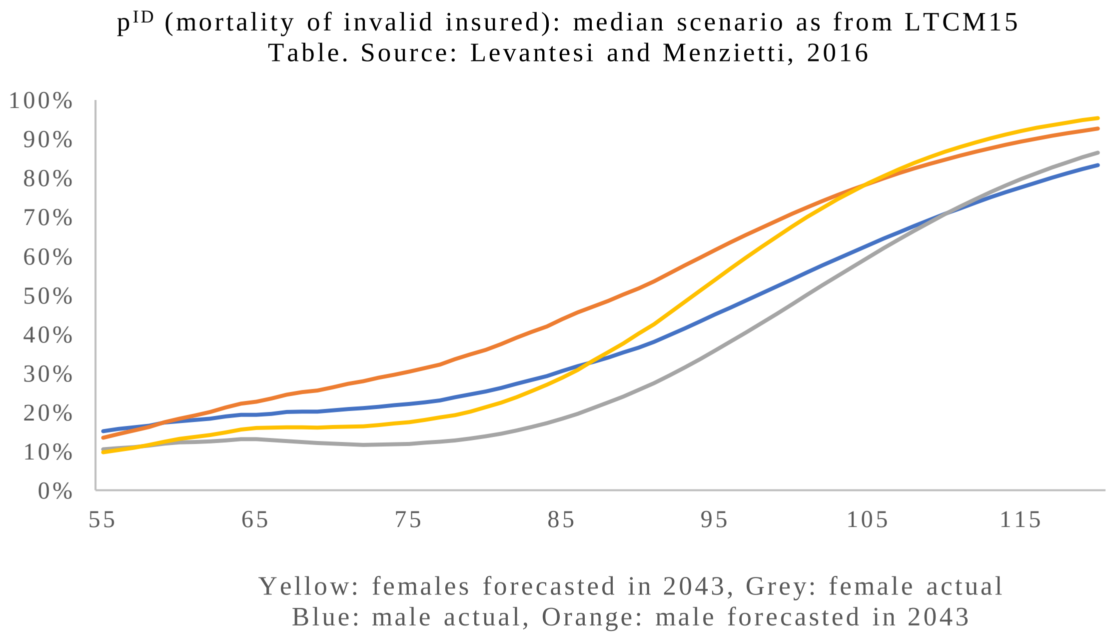
3.4) The transition probabilities and their pattern & correlations.

We now represent the outcome of the previous paragraph, i.e., the transition probabilities (p^{AD}, p^{AI}, p^{ID}) that resulted in the first Italian long term care table, the LTC(M)15 Table (it shows the three transition probability, distinguished by sex and age, their forecast in the future and table referring to other complementary products, such as driver accident insurance, professional and extraprofessional

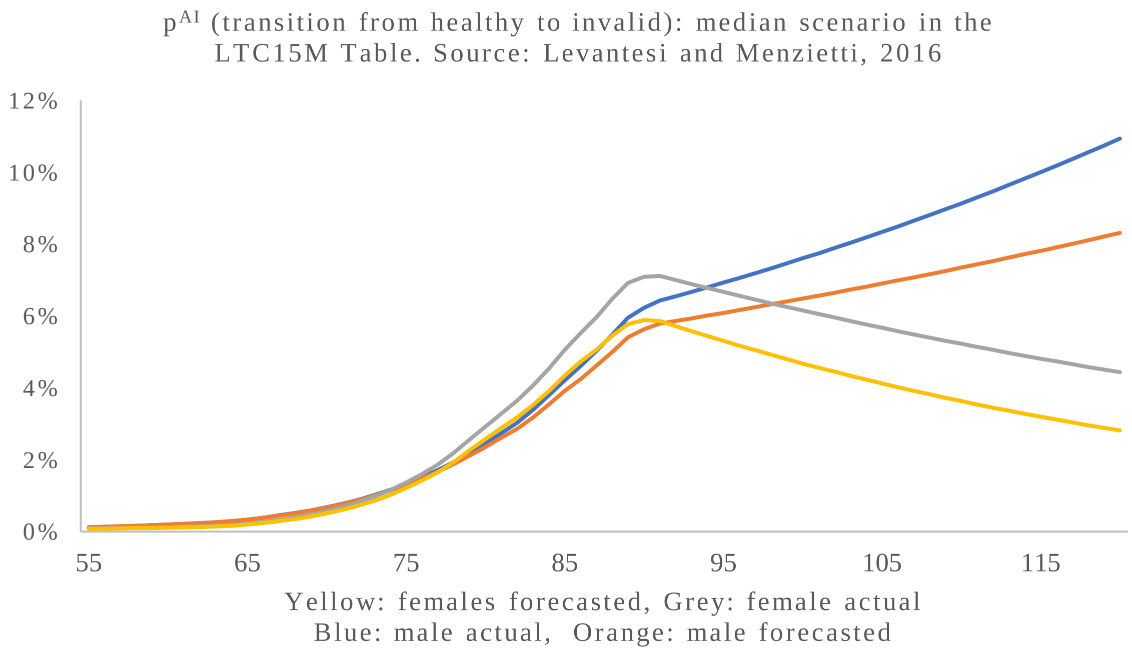
71 Richards, S., & Currie, I. (2009). Longevity Risk and Annuity Pricing with the Lee-Carter Model. British Actuarial Journal, 15(2), 317-343. And Currie ID, Durban M, Eilers PH. Smoothing, and forecasting mortality rates. Statistical Modelling. 2004;4(4):279-298

insurance), produced by Levantesi and Menzietti[72] in 2016 and mainly used by insurance companies to price specific contracts. We represent the median scenario for all the probabilities: forecasts range from 2020 to 2043, we represent the actual and the 2043 forecasted level, for both males and females. By contrast, concerning extra mortality of invalid individuals no forecast had been performed, but simply observed data has been collected and elaborated in the forms of table/graphs.





72 Baione F., De Angelis P., Levantesi S., Menzietti M., Tripodi A. (2016). Modelli attuariali per la stima di basi tecniche relative ad assicurazioni di persone. In: De Angelis P. di Falco L. Assicurazioni sulla salute: caratteristiche, modelli attuariali e basi tecniche pp. 123 and following.



Concerning the median scenario, we observe that for the mortality of healthy there will be a significant decrease in the next 20 years for both males and females. By contrast, the mortality of invalids shows an important increase: for males, however, the gap is wider and concerns younger ages, while for females there is no relevant spike in mortality of elderly invalid insured. The transition probability from healthy to an invalid will significantly decrease for both sexes, however, the typical pattern will not change females will still present a peak in areas 89-91, while males will continue to exhibit an increasing pattern up to 120 years. The transition probability will increase more sharply for the younger ages (77-87) in females than in males, which will instead present a spike in old age dementia. This is probably due to the increased life expectancy for females, for gender-specific cancers. An increase in life expectancy may result in a loss of autonomy due to the consequences of treatments. This is what seems forecastable.[73]

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73 The relevance of sejour in determining mortality in conjunction with the age is confirmed by ANOVA tests on mortality tables for males and females, dataset has been produced by Levantesi as a courtesy for study reasons:

- 1F ANOVA- MALES 55-100. INPS dataset on invalidity allowance

Groups	Count	Sum	Mean	Variance
Sejour<=1	46	7,5858	0,164909	0,000522
Sejour=2	46	10,21539	0,222074	0,000735
Sejour>2	46	10,0655	0,218815	0,004965

ANOVA TABLE

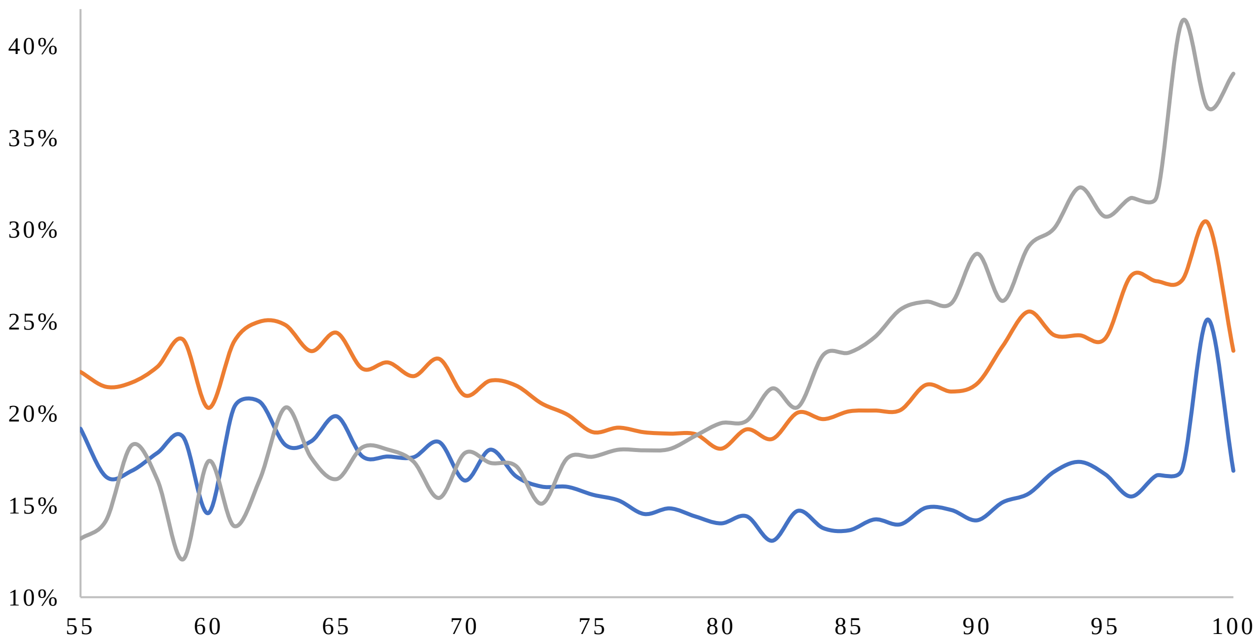
Source of the variation	SQ	Dof	MQ	F	P. Value	F crit
Between groups	0,094827	2	0,047413	22,86328	2,81E-09	3,063204
Within group	0,27996	135	0,002074			
Total variance	0,374787	137				

- 1F ANOVA- FEMALES 55-100. INPS dataset on invalidity allowance

Groups	Count	Sum	Mean	Variance
Sejour<=1	46	4,344301495	0,094441337	0,000623944
Sejour=2	46	6,345917484	0,137954728	0,001236232
Sejour>2	46	6,657930162	0,144737612	0,002682744

ANOVA TABLE

Mortality of LTC individuals (p^{ID}) by sejour time (one year, two years or more) for males. Source: personal elaborations on INPS dataset (2008-2011), datas courtesy of Levantesi



Red line: sejour>2
blue line: sejour whitin 1 and 2 years
green line: sejour lower than one year

Concerning mortality conditioned to *sejour* times, we start from males: in the early stages, the mortality over two years is lower than the mortality before two years, of the mortality 1-2 years. In contrast, the mortality of longer *sejour* becomes greater than mortality of younger individuals after 80 years. This means that the portfolio of illnesses changes in favour of elderly dementia and that there is a decrease in cancer and cardiovascular pathologies. The lower *sejour* mortality decreases in

Source of the variation	SQ	Dof	MQ	F	P. Value	F crit
Between groups	0,068526783	2	0,034263392	22,63	3,35E-09	3,063204
Within group	0,204431401	135	0,001514307			
Total variance	0,272958185	137				

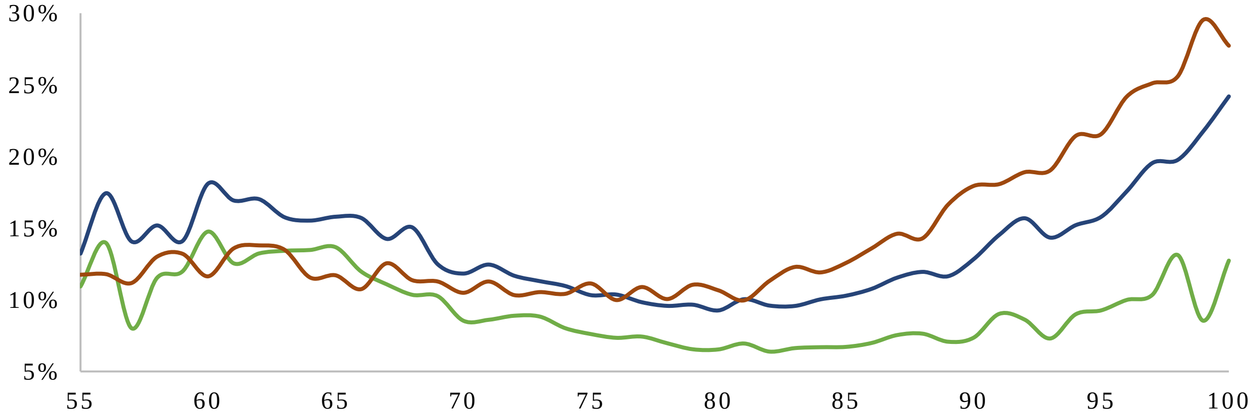
We analyse the anagraphic correlation (i.e., instead of time we consider as time variable the anagraphic age) of the three mortalities of LTC individuals: we find a persistent correlation between sejour=1 and sejour=2 and between sejour=2 and sejour>2, while a lower one between sejour=1 and sejour>2. Furthermore, we see that correlation are less strong for females, and this is relevant to have some effects on sejour distribution and fair premium estimates. In particular, the correlation between sejour=1 and sejour=2 is roughly 15% lower, while between sejour=1 and sejour>3 even of 20%. By contrast, in case of correlation between sejour=2 and sejour>2 females present a relatively 5% higher intensity.

Anagraphic correlation matrix between sejour mortalities, genders and duration.
Data courtesy of Levantesi e Menzietti, 2016.

	MALES				FEMALES		
	sejojur=1	*sejour=2*	*sejour >2*		*sejojur=1*	*sejour=2*	*sejour >2*
sejour=1	1			sejour=1	1		
sejour=2	0.9224333	1		sejour=2	0.820505	1	
sejour >2	0.477	0.704294	1	sejour >2	0.384732	0.75589	1

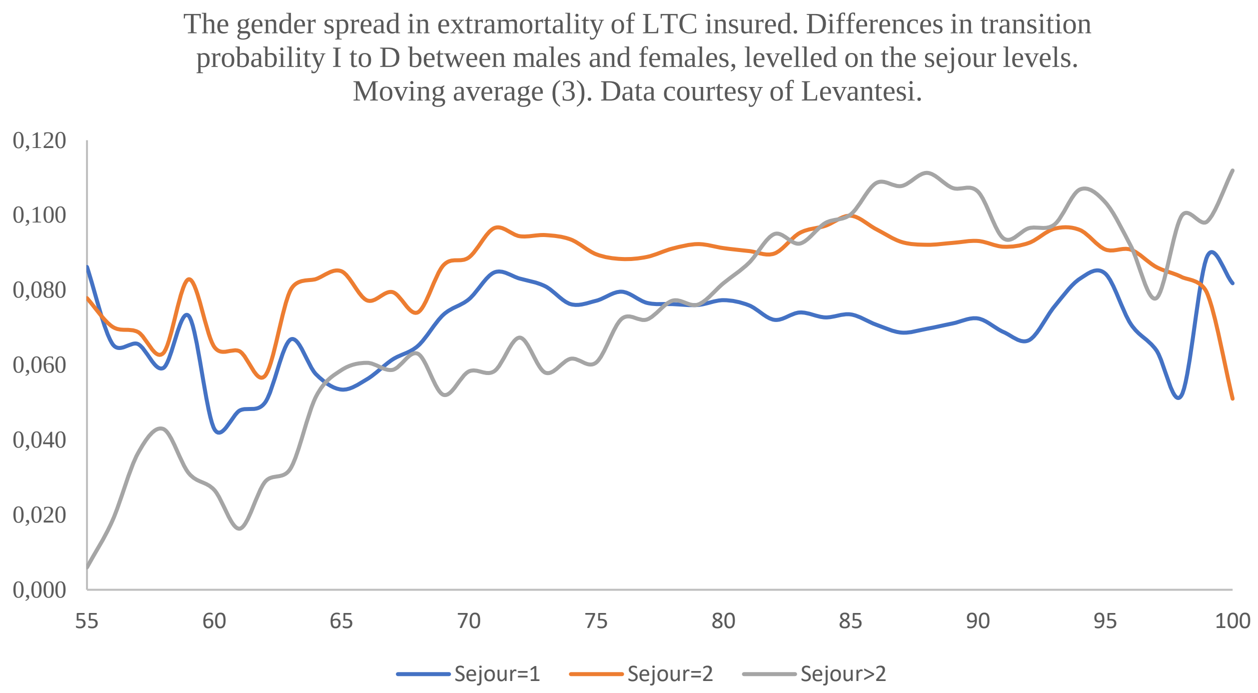
the interval 65-80 of about 5% and remains almost constant, while the longer *sejour* mortality shifts from 12 to 30%, with a constant pattern.

Mortality of LTC individuals (p^{ID}) by sejour time (one year, two years or more) for females. Source: personal elaborations on INPS dataset (2008-2011), data courtesy of Levantesi



Red line: sejour>2
blue line: sejour whitin 1 and 2 years
green line: sejour lower than one year

Concerning females, by contrast, we notice that the cardinality of the tree mortality curves is the same (at the beginning the lower sejour presents higher mortality than the higher one, while in conclusion, the mortality of higher sejour overcomes the mortality of younger sejour individuals, with a plateau in the area 75-80), but the decrease in mortality of shorter sejour is more relevant: this is since elderly dementia is less frequent and start to threaten autonomy to generally older ages. That is why the mortality compression is 10% instead of the 5% of males. By contrast, the mortality of longer sejour increases from 10% to 30%. We should even notice that the longer sejour mortality for females start to increase only from 80 years, while from males from 72.5 years: we can overall say that the pattern is similar for males and females, but it is shifted right for females of 7.5 years and even the mortality level is whenever lower for females when compared to males.



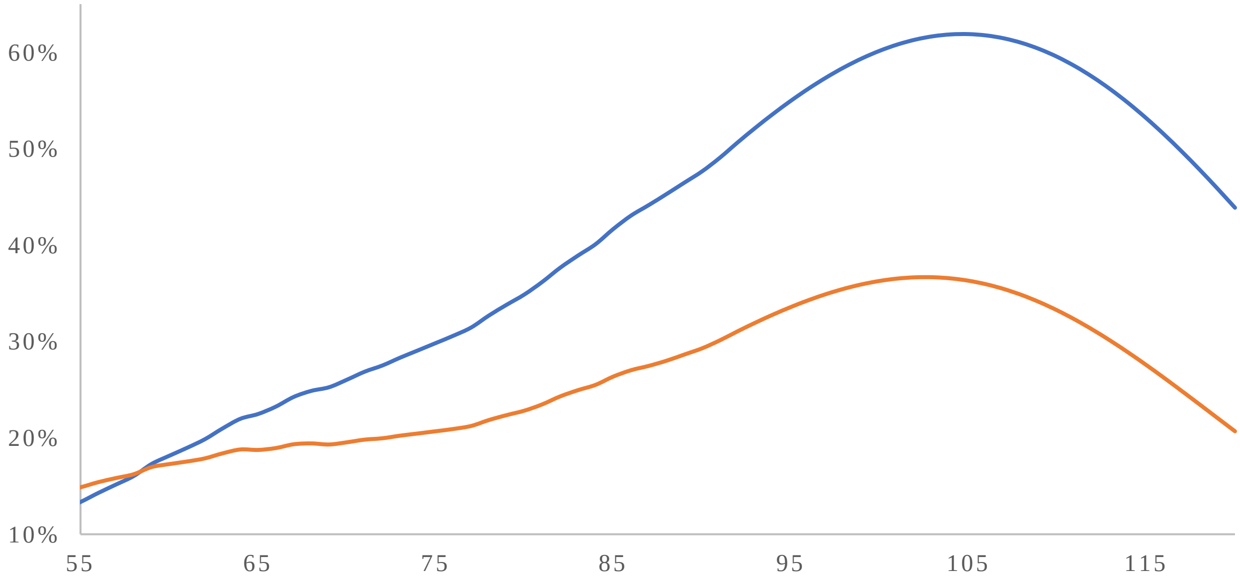
We now define, concerning all the three transition probabilities p_{ij} observed, the gender spread between males and females as

$$p_{ij}^{gender\ spread} = p_{ij}\ (\text{males}, x) - p_{ij}\ (\text{females}, x),\ \forall\ x \text{ in age during the contract} = [55,56\ldots100]$$

Concerning the spread between the mortality of males and females, we consider the 5years moving average to avoid accounting for insignificant variations. We notice that for the shortest *sejour* time (<1 year) there is no significant pattern: we start from a spread of 8% and we end with the same level and there is no significant increase or decrease through the whole period. By contrast, in the *sejour* between 1<q<2 we notice a relevant increase in the spread after the 70s: the probability of mortality conditioned to the sex increases by 2%. We notice that the mortality conditioned to the longest *sejour* time (>2) is the one that presents the higher spread dynamic as in the 55s the spread is null, or even in favour of males (this depends on gender-specific cancers) and after the 80 becomes the *sejour* time with the highest spread 10%. In conclusion, we can summarize that the longer the *sejour* time, the higher the dynamic of the spread between males and female's mortality: for longer *sejour* in fact, we have a relevant increased during the more advanced ages of the gender conditioned probability of death. This is to be considered when carrying out sensitivity analysis and even in differences in the hedging between males and females, as an increase in extra mortality of LTC will decrease the average observed *sejour* time in which the insured perceives a double amount.

In conclusion, considering the benefits of operational hedging, we need to consider the pattern of extra mortality of LTC individuals: this is defined as the difference between mortality of LTC individuals minus mortality of healthy ones. The higher the extra mortality, the higher the benefits of operational hedging in an enhanced pension.

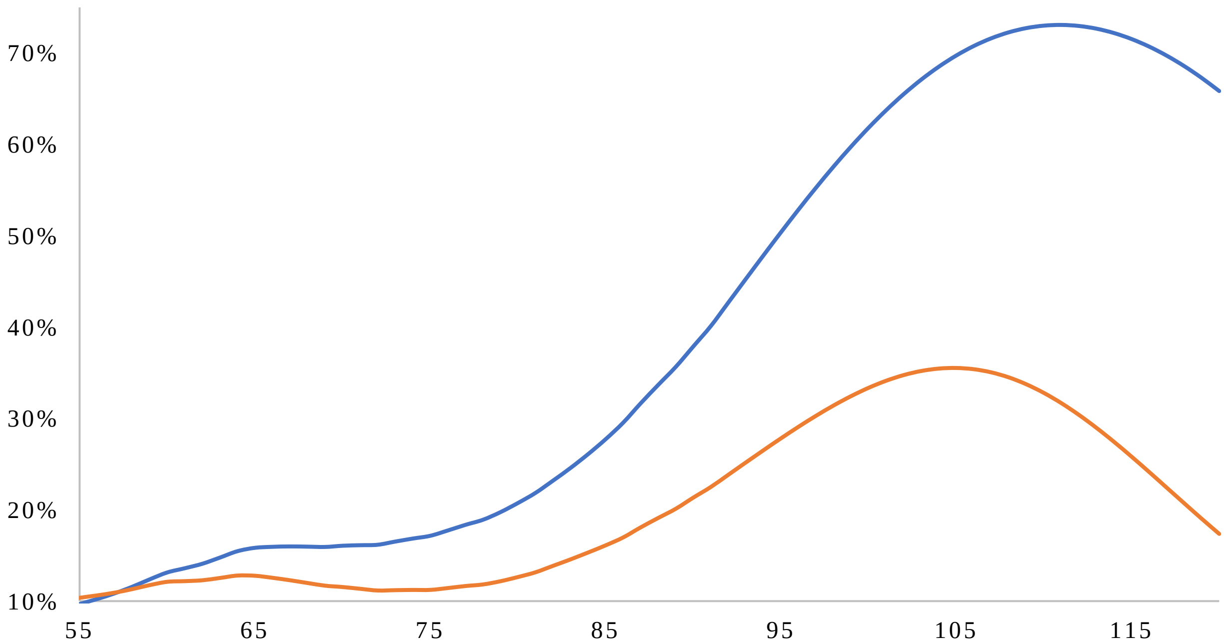
Extramortality of Males (mortality of invalid minus mortality of healthy: p^{ID}-p^{AD}), actual and forecasts according to LTC15M Table. Source: Levantesi and Menzietti, 2016



Red line: males actual, blue line: males forecast (2043)

For ages lower than 55 the extra mortality is forecasted to decrease in a significant way, but after the age of 55-56, for males, we will observe a spike in extra mortality that will follow the same pattern of the actual one but with a more pronounced peak. Note that the extra mortality will double for ages greater than 90: this will be mainly due to the decrease in healthy mortality.

Extramortality of females (mortality of invalid minus mortality of healthy: p^{ID}-p^{AD}): actual and forecasts according to LTC15M Table. Source: Levantesi and Menzietti, 2016.



Red line: females actual, blue line: females forecast

Concerning the pattern of female extra mortality, we notice that the trend is substantially like the one manifested by males, but the intensity of the increase in the ages older than 90 is even higher: if for the males the maximum extra mortality forecasted in 2043 is 200% of the actual one, for females the increase is even more relevant, and the extra mortality forecasted in 2043 is 300% of the actual one. This consideration may lead to the wrong conclusion that operation hedging will work almost surely even in the future and the degree of risk underwritten by the insurer will even decrease referring to an enhanced pension. The results presented are for sure a good starting point, but in terms of the effectiveness of operational hedging even the transition probability should be considered and a sensible decrease in the probability of transition of the whole portfolio (something probable to happen, according to our forecasts) might threaten the effectiveness of the risk reduction. What we can state is that starting from this data, operational hedging will still be in place in the next 20years and even if now the risk reduction thanks to extra mortality is more pronounced for males, we will observe an appreciable increase in risk diversification thanks to an LTC insurance contract even for females, thanks to the increase in extra mortality. This rebalances of risk-reduction between males and females will be particularly beneficial in terms of gender parity, as will allow increasing the benefit for women of an enhanced pension if compared with stand-alone insurance. Concerning these topics, we believe in the interest of the correlation matrix of transition probabilities that are reported in the note[74]: with a note that relevant collinearity, in the function of the ages, between transition probability for males: however, the collinearity between healthy mortality and LTC transition is expected to decrease by 7%, this means that in the future there will be an increase in LTC transition less supported by an increase in healthy mortality, the same holds for the correlations between the two mortalities. As an increase in extra mortality is expected, there will be lower collinearity (i.e., correlations) between the mortalities of healthy and the mortality of LTC individuals, with important consequences in terms of hedging policy: here the decrease is even more relevant being of 9%. There is no change in the correlation between transition probability and mortality of LTC: an increase in one will also influence, in the same way, the other variable. This is beneficial in terms of forecasting of the hedging policy: i.e., it is possible to base the operation hedging policy on the actual path between transition probability to LTC and extra mortality. Concerning females, we must notice that the correlation between LTC transition and healthy mortality is significantly lower (is 54% instead of 89% in 2021): the same applies to the correlation between LTC transition and extra mortality (from 97% to 74%), while no significant difference is shown between the two extra mortalities. By contrast, the trend forecasted of the correlation matrix is the same in males and females: here, we will observe a relevant decrease in the correlation between LTC transition and Healthy mortality (decrease of 9%) and a decrease in the correlation between the two extra mortalities (from 94% to 88%, so 6%). As already observed for the correlation matrix of males, there is no significant change in the correlation between LTC transition and mortality of LTC individuals, with important benefits in terms of risk

74 We now want to go deeper, in an autonomous way, in the dependence study. We present the anagraphic correlation matrix between the three transition probabilities in function of the age of the insured, distinguished by sex and age of forecasting. If for each probability P we assume that it mainly depending on the age x, and so we associate to the vector of ages a vector for each transition probability we can by consequence consider the correlations between the transition probabilities with respect to the age of the insured, still in a vanilla Markov world.

Anagraphic Correlation Males 2021

	p^{AD}	p^{AI}	p^{ID}
p^{AD}	1	0,8909	0,920731
p^{AI}		1	0,970039
p^{ID}			1

Anagraphic Correlation females 2021

	p^{AD}	p^{AI}	p^{ID}
p^{AD}	1	0,538578	0,943444
p^{AI}		1	0,738096
p^{ID}			1

Anagraphic Correlation Males 2043

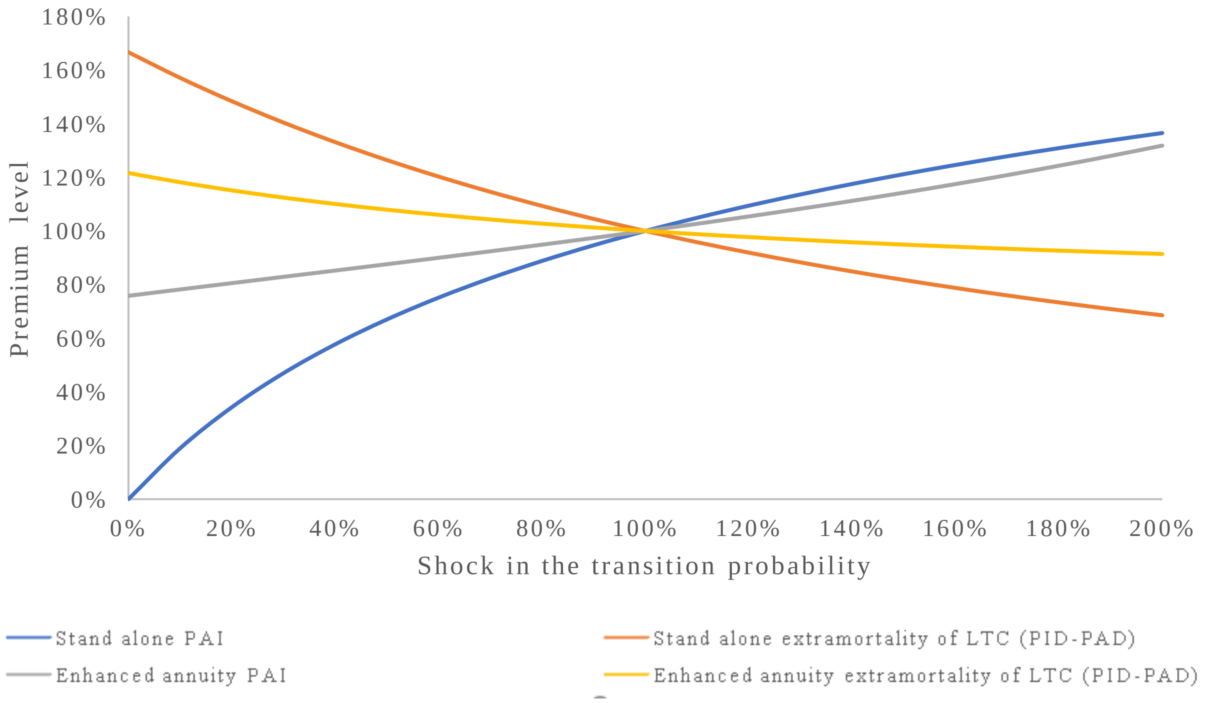
	p^{AD}	p^{AI}	p^{ID}
p^{AD}	1	0,816824	0,832765
p^{AI}		1	0,975483
p^{ID}			1

Anagraphic Correlation females 2043

	p^{AD}	p^{AI}	p^{ID}
p^{AD}	1	0,44886	0,878748
p^{AI}		1	0,755098
p^{ID}			1

management. In conclusion, we have a look at a comparison between the correlation matrix of females and males in 2043, so at the end of our forecasts: the correlation between LTC transition and healthy mortality for females is 37% lower than males (82% vs 45%), while the correlation between the transition to LTC and invalid mortality is 25% lower for females (76% vs 98%): by contrast, the correlation between the two mortalities is higher for females by 5% (the same inequality holds even in a couple of the present correlation matrix). In conclusion, we can state that, for females, transition probabilities are less correlated than for males and this is not going to change during our forecast. As to summarize, the trend is similar for both males and female's correlation matrix, with important benefits in terms of hedging policy: however, the level of the correlation is significantly different between males and females with a more pronounced dependence between the mentioned probabilities for males. The only probability that shows an increasing pattern in correlation is the dependence between the two mortalities[75].

The lower degree of risk of an enchanched annuity: sensitivity analisis with respect to a stand-alone LTC insurance. Source of data: Pitacco, ERM and QRM in Life Insurance An Actuarial Primer, (ch.8 on LTC contracts). Springer actuarial 2020.



In conclusion, if we carry out some sensitivity analysis concerning the relevant transition probability (in particular, the disability probability and the extra mortality, i.e., p^{ID}-p^{AD}: we assume here an individual of 65 years, male) we have confirmation about the lower degree of risk of an enhanced pension if compared with a stand-alone LTC product: in case of extra mortality the premium level increases up to 140% for a standalone product and decreases up to 60%: by contrast, in an enhanced pension the fork is 120% in case of zero extra mortality and 80% in case of doubled extra mortality. The same, even if reversed, could be observed for invalidity probability, where the premium level

75 Pattern of the correlation matrix, for both males and females, 2021-2043.

Pattern of the correlation matrix 2021 2043, both for females and males

	p^{AD}	p^{AI}	p^{ID}
p^{AD}		Decrease	Decrease
p^{AI}			Increase
p^{ID}			

shifts from the theoretical case where there is no invalidity (0% premium level) for the stand-alone to 130% in case of a doubled disability rate: in the enhanced pension, in case of disability equal to 0, we have a premium level of the 80%, while in case of double disability probability over the whole term structure we reach the 120% of the premium. Furthermore, the trend in the sensitivity seems more than linear for stand-alone insurance, while seeming flattered in the enhanced pension case. This confirms that although disability probability and extra mortality are mutually ensuring in both the contrasts, the higher degree of risk reduction is reached by the enhanced pension, as the pattern is, everything considered, more stable. Trajectories can marginally change by modifying the demographic assumption, but the core results are to be preserved.

3.5) Adjusting for Sejour impact on mortality of LTC individuals.

We are now ready to consider the impact of sejour on the extra mortality of individuals in the I state, so on the transition probability p^{ID}, that is crucial to determine the premium level and the sejour duration distribution. Now we tackle the key point: we need to explain how to adjust the probabilities of the Levantesi and Menzietti table (LTCM15 Table) for a semi-Markov model, i.e., considering the Sejour time when the insured is in the I state (invalid). Still, Levantesi produced an interesting empirical observation, on the period 2008-2011, with an annual frequency of sampling of the mortality of invalid individuals linked to the sejour time. This has been obtained by conditioning the mortality, in the function of the anagraphic age to three different levels: Sejour lower or equal than one year, Sejour between one and two years, Sejour greater than two years. Defined the transition probability as p^{ID} (x, z) we define the mortality spread at age x $\Delta(x)$ as:

$$\Delta(x,z) = p^{ID}(x) - p^{ID}(x,z).$$

And consequently, we will adjust the transition probabilities reported in the Levantesi and Menzietti table with the Semi-Markov component of the Sejour in the following method:

$$p(x,t)_{\text{Semi-Markov}} = \Delta(x,z) + p_{\text{Markov}}(x)$$

The author even tried to adjust for sejour in relative terms, by quoting the ratio on which the Markov mortality adapts to *Sejour* time: results are substantially in line with the previously proposed correction. Formally, by calling this relative correction factor $\vartheta(x,z)$

$$p(x,t)_{\text{Semi-Markov}} = \vartheta(x,z)\, p_{\text{Markov}}(x)$$

However, for older ages, this second correction shows high unreliability and excessive variability in results, due to the lack of data and to the bad interaction between the dynamic forecast and the correction. That is why the author decided to use the first equation and not the one being now described. The relevant difference is that for the pure Markov component of the transition probability are available projections up to 2043, according to the LTCM Table by Levantesi and Menzietti, by contrast, on the mortality spread $\Delta(x,t)$ no evidence of trend had been obtained, even due to the short interval 2008-2011 considered. To avoid an excess of specification of the model that will be detrimental, we assume that there is no dynamic in the mortality spread during the years. In conclusion, our transition probability will be the projection of the sum of two components: the purely Markov part (with projection), and the semi-Markov component (without projection), this second parts presents, for each value of x (the anagraphic age), three elements on the domain (x0, z<=1), (x0,1<z<=2), (x0, z>2), characterized by different sejour times. The decision of avoiding the extrapolation of the dynamic for extra mortality is linked to the lack of empirical data (four-yearly observations are few to test and formulate a dynamic), to strong theoretical support and the evidence of the stationarity of panels of extra mortality, for all the genders and the sejour times[76].

76 Here we report the stationarity test conducted on extra mortality concerning different sejour times and gender. Results are generally constant even among different tests and panels. The only dyscrasia between Harris-Tzavalis and Breitung

3.6) Some specific conditions

Several policy additional elements can be included in the insurance product, and LTC due to their specificness. In general, we observe that these conditions are not contract-specific: they do not depend too much on stand-alone or LTC. In this section we mainly address the duration-related condition,

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unit root test is for extra mortality of females in the first year, where according to Breitung test we have enough evidence to accept the hypothesis of unit root. All these tests have been performed on STATA®/SE.16. Since our panel is only of 4 periods, we must critically assess our results, that are however considerable since the number of panels is high enough, since power of the test (β) depends on the product (the formula depends on the specific test). Furthermore, problems are generally correlated to tests with trend specifications or with tests on the differences instead of the level.

- Harris-Tzavalis unit-root test for $\Delta(x,1)$ $males$

H0: Panels contain unit roots   Number of panels = 46
Ha: Panels are stationary   Number of periods = 4

Statistic		z	p-value
rho	0.0209	-4.1982	0.0000

- Breitung unit-root test for $\Delta(x,1)$ $males$

H0: Panels contain unit roots   Number of panels = 46
Ha: Panels are stationary   Number of periods = 4

Statistic	p-value	
lambda	-2.6776	0.0037

- Harris-Tzavalis unit-root test for $\Delta(x,2)$ $males$

Statistic		z	p-value
rho	-0.4267	-9.1540	0.0000

- Breitung unit-root test for $\Delta(x,2)$ $males$

Statistic	p-value	
lambda	-5.0849	0.0000

- Harris-Tzavalis unit-root test for $\Delta(x,z>2)$ $males$

Statistic		z	p-value
rho	-0.4599	-9.5217	0.0000

- Breitung unit-root test for $\Delta(x,z>2)$ $males$

Statistic	p-value	
lambda	-3.6511	0.0001

- Harris-Tzavalis unit-root test for $\Delta(x,1)$ $females$

Statistic		z	p-value
rho	-0.3597	-8.9106	0.0000

- Breitung unit-root test for $\Delta(x,1)$ $females$

Statistic	p-value	
lambda	-1.2777	0.1007

- Harris-Tzavalis unit-root test for $\Delta(x,2)$ $females$

Statistic		z	p-value
rho	-0.1326	-6.2466	0.0000

- Breitung unit-root test for $\Delta(x,z>2)$ $females$

Statistic	p-value	
lambda	-5.7987	0.0000

- Harris-Tzavalis unit-root test for $\Delta(x,z>2)$ $females$

Statistic		z	p-value
rho	-0.4270	-9.7001	0.0000

- Breitung unit-root test for $\Delta(x,z>2)$ $females$

Statistic	p-value	
lambda	-4.5313	0.0000

Breitung, J. 2000. The local power of some unit root tests for panel data. Advances in Econometrics, Volume 15: Nonstationary Panels, Panel Cointegration, and Dynamic Panels, ed. B. H. Baltagi, 161–178. Amsterdam: JAY Press.
Breitung, J., and S. Das. 2005.Panel unit root tests under cross-sectional dependence. Statistica Neerlandica 59: 414–433.
Harris, R. D. F., and E. Tzavalis. 1999.Inference for unit roots in dynamic panels where the time dimension is fixed. Journal of Econometrics 91: 201–226.

i.e., terms that either define the coverage period or the benefit payment period following the claim, that is, the starting of the LTC need. The insured period (or "coverage" period) is the time interval during which the insurance cover operates, in the sense that a benefit is payable only if the claim time belongs to this interval. In principle, the insured period begins at the policy issue and ends at policy termination. In LTC policies, given the purpose of the benefits, it is reasonable to assume a lifelong insured period. However, some restrictions to the insured period may follow from specific policy conditions, that generally perceive three aims: reduce adverse selection, decrease the premium level, avoid fat tails loss distribution. The waiting period (or "elimination" period) is the period following the capital conversion during which the LTC insurance cover is not yet operating to partially align the insured and insurer interests. Moreover, it is frequent that different waiting periods can be applied according to the category of sickness. The waiting period aims at limiting the effects of adverse selection, in particular, because of any pre-existing health conditions of the insured. Let us denote with m this waiting period, and assume it applies to all the causes of invalidity covered by the contract (no distinction between the kind of illnesses/trauma that generated the invalidity (cancer, cardiovascular, dementia, accident). Defined m as the waiting period, for the vanilla enhanced pension, we have, in discrete time:

$$\pi^{m}(\mathrm{x}) = \sum_{t=\mathrm{m}+1}^{w} B(t) 1_{A\,U\,I}(t)\, v(0,t) + \sum_{t=1}^{m} 1_{A\,U\,I}\,(t) v(0,t)$$

Where $\sum_{t=1}^{m} 1_{A\,U\,I}\,(t) v(0,t) = \sum_{t=1}^{w} 1_{A}(t) v(0,t) + \sum_{t=\mathrm{m}+1}^{w} v(0,t) 1_{L}(t)$

That is whenever lower than the ordinary premium, and the decrease is more relevant as m becomes higher. However, generally elimination period is no longer than two years. In practice, the usual provisions are the following:

- Loss of autonomy due to somatic ailment: 1 year
- Loss of autonomy due to cognitive ailment, such as Alzheimer's: 3 years
- Loss of autonomy due to an accident: 0 year

For Total Dependence, these clauses do not appear to be determinant on rates.

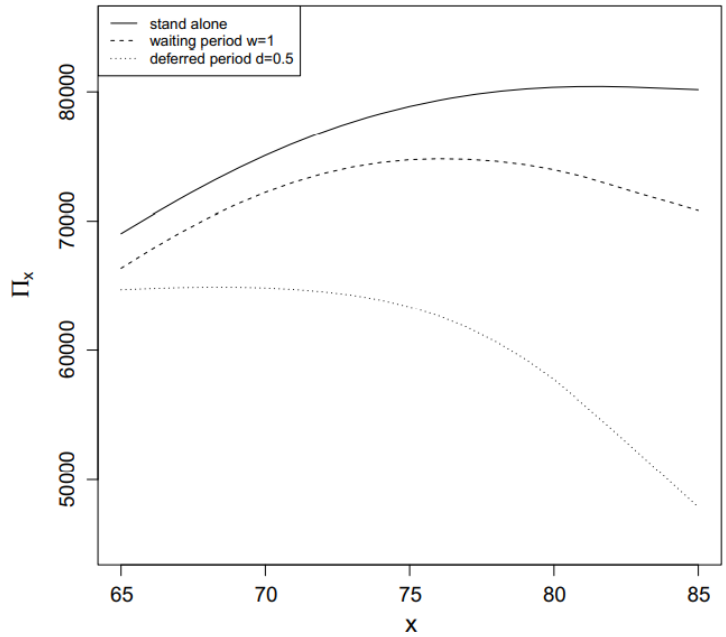


Figure 1- The change in the premium dynamic in the function of the age for different contract specifications: waiting period of 1 year and deferred period of half a year. Denuit M., Lucas N., Pitacco E. (2019) Pricing and

Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham.

In many policies, the benefit is not payable until the LTC need has lasted a certain minimum period called the deferred period. This policy condition has a two-fold purpose:

- To reduce the cost and hence the premium of the LTC insurance product; premium reduction can be particularly significant because of the high mortality immediately following the loss of autonomy.
- To ascertain the permanent character of the disease which implies the LTC need (provided that LTC benefits are only paid in the case of permanent disability, as assumed in our model). This is the main distinction between an LTC and invalidity insurance: the first one is assumed to be from now on till the death, while the second could be even temporary.

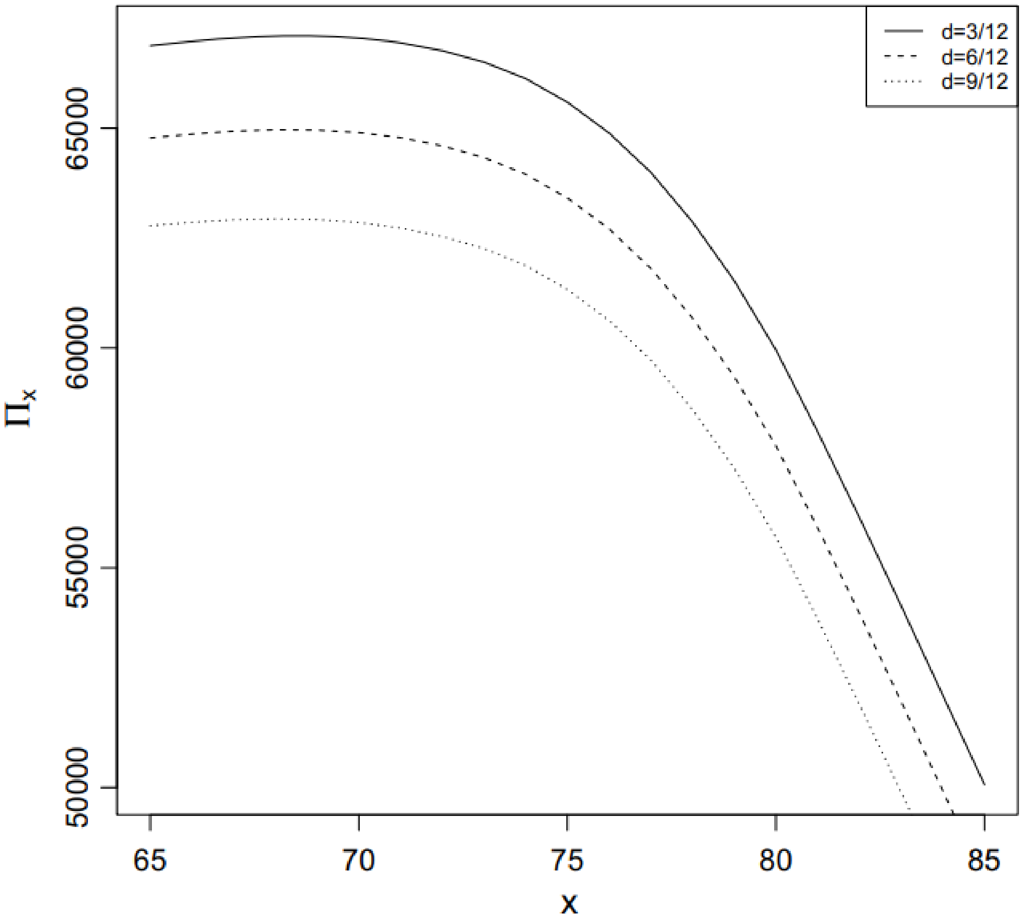


Figure 2- The effects of an elimination period on the premium dynamic concerning the age: from an elimination period of 3 months to an elimination period of 9 months. Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham.

3.7) Reserving

LTC contracts are generally lifelong with a level premium fixed at contract initiation so that the annual paid amount does not vary during the contract. This constant premium or level premium depends on the underwriting age x. As can be seen in fig.4, the annual risk premium (i.e., annual expected claim amount for an active individual) is an increasing function of age except at very advanced ages. Therefore, surpluses are constituted in the first part of the contract as level premiums exceed annual risk premiums. This surplus is called reserve and is kept aside to meet future needs. In the case when a single premium is paid at policy issue, a reserve must immediately be kept aside, and then "used" throughout the whole policy duration to meet the insurer's expected costs. By status, the insurer should have available a reserve at any time. It is defined prospectively as the actuarial value of future benefits less the actuarial value of future premiums (and, in the case of a single premium, is simply defined as the actuarial value of future benefits). Therefore, it depends on the state occupied by the policyholder at the date of calculation. In LTC insurance, we distinguish a reserve in state A at time t, henceforth denoted as $V^A(t)$, and a reserve in the state I at time t, with autonomy lost at time t − z, henceforth denoted as V^I (t, z). Of course, there is no need to define a reserve in state D as the policyholder's death automatically terminates the contract, unless the policy provides a death-guarantee annuity (counter-insurance) for the first m years. In this very last case, reserving will include a certain payment for the first m years and then the reserving for a deferred LTC insurance, that is the deferred LTC reserve levels.

The equivalence principle states that, at policy issue, the expected present value Π of the premiums paid by the policyholder matches the expected present value B of the benefits included in the contract, i.e., the actuarial net value $V(0)^a = 0$. This equivalence no longer holds during the contract. The reserve is the amount needed to restore financial equilibrium at any time t > 0. Denoting by B the general benefit (depending on the state X) and by π the capital invested in the contract, we have for the state a and i[77]:

- $V(t)^a = B(t)^a - \pi(t)^a$ for the state a
- $V(t,z)^i = B(t,z)^i - \pi(t,z)^i$ for the sate i
- No reserve is needed for the state d, unless in the counter-ensured annuity.

But as we are speaking about a single-premium anticipated contract, it holds that π(t)=0 for every state A, I, D, And so[78],

- $V(t)^a = B(t)^a$ for the state a
- $V(t,z)^i = B(t,z)^i$ for the sate i

As the premium π is fully paid in advance there are generally few reserves problem, and they tend to sort out in old ages, when the cumulated payment might overcome the original premium, even considering interest income.

The reserve at time t for a healthy (A) individual is equal to

$$V(t)^a = \bar{a}^{aa}_{x+t} + 2\bar{a}^{ai}_{x+t}$$

Considering an individual who is in the LTC state at time t, who entered that state at time t − z, the reserve is given by.

$$V(t,z)^i = 2\bar{a}^{ii}_{x+t;z},$$

Consider that the lower variability of outflows that leads to the competitive advantage of the enhanced pension fully concerns also reserving, with a lower number structural break and a calmer pattern if compared with a stand-alone long term care insurance. The reserve for an autonomous individual is represented only as a function of t, for an initial age x. The amount of reserve increases until a very high age (about 100), before falling to 0 due to the high mortality risk. Figure 3 displays the function $V(80,z)^i$ of reserving in invalid status, after 15 years from the issue, for a policyholder aged 65 at policy issue (so that the age at reserve calculation is 80). We see that $V(80,z)^i$ first increases until

77 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham. pp.152-154.

78 Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham. pp.155-156.

the end of the first year spent in LTC (i.e., for $z \leq 1$) and then decreases. This results from the high mortality during the year following the entry into the LTC state. In conclusion consider that, as reported by Nematrian financial consultancy service[79], *the standard formula for Solvency Capital Requirement (i.e. the total amount of funds that insurance and reinsurance companies in the European Union (EU) are required to hold) set out in the Solvency II[80] Delegated Act involves a capital requirement for morbidity/disability risk that is based on the change in net asset value (assets minus liabilities) arising from the combination of an increase of 35% in morbidity/disability inception rates for the first year followed by an increase of 25% for all subsequent years, and a permanent decrease of 20% in morbidity/disability recovery rates.*

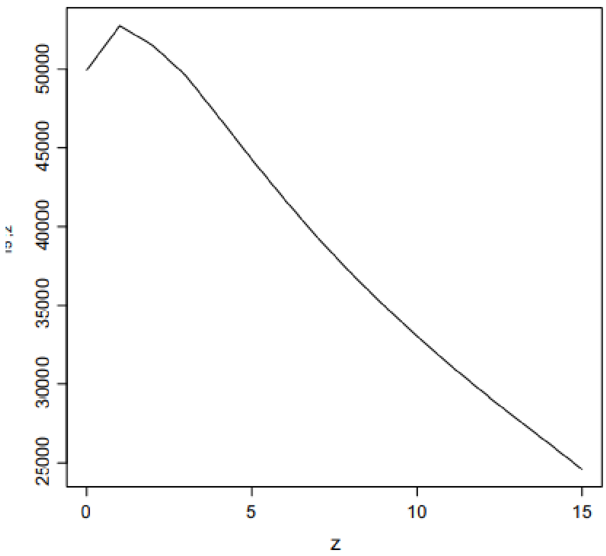


Figure 3- The level of reserving for an invalid insured aged 80, who just entered LTC state, by sejour time: we notice that the peak of reserve if after one year and that after two years the decrease rate of reserving level is even more pronounced. Source: Denuit M., Lucas N., Pitacco E. (2019) Pricing and Reserving in LTC Insurance. In: Dupourqué E., Planchet F., Sator N. (eds) Actuarial Aspects of Long-Term Care. Springer Actuarial. Springer, Cham.

79 See the relevant section on Solvency II regulation for disability insurance: on Nematrian consulting company website http://www.nematrian.com/SolvencyII_LifeDisability consulted 18 august 2021

80 European Union (2014c) Solvency II Directive Delegated Regulation (i.e., Commission delegated regulation (EU) 2015/35)

4)Numerical simulation

4.1) Introduction

To have a clear and concrete representation of the group of products that we have discussed we present the results of the numerical simulation. These simulations aim at calculating the fair value conversion factor in the function of the ages [55,70] and the sex of an LTC annuity. We import the following variables: the transition probabilities (mortality of healthy individuals, mortality of invalid individuals and transition from healthy to invalid) in conjunction with the mortality spread. Substantially, we strictly follow the semi-Markov World described in the previous chapter. We want to examine as an outcome of our experiment, the fair premium level, his scenario distribution (over 1 million repetitions), and his time-series distribution (i.e., to split the premium in the cost of the different years of life and LTC coverage). The following is the procedure: first, we import, and we check the consistency of the transition probabilities (p^{AI}, p^{AD} and p^{ID}): these transition probabilities are represented in matrix form, for a total of six matrices of data input (3 transition probabilities for both sexes). On the row of this matrix, we can read the anagraphic age of our individual (55,56,57…120), while on the columns we have the forecast for this vector from 2020 (today) up to 2085: as the forecast are reliable only up to 2043-2045, the last forecasted vector (2043) is repeated up to 2085. We then create some vector for saving the scenario, the sejour distribution and the average outflow level for the year: some vectors are built with restrictions on the length (as the length is known), while the sejour vector is only defined in his existence, as we do not know ex-ante how many individuals will end up being LTC before they die. The scenario vector has length N (number of simulations), while the average has length 119-ages of the entry: the lower the age of entry, the longer the age. The number of repetitions is generally 100.000: it is hard to increase the number of repetitions because for each simulation we must repeat the procedure for 16x2 ages of the entry of the contract (from 55 to 70, for two genders): results are however absolutely satisfying. For every one of the simulations, we have the following stages: at the starting, we set the two flags death and LTC to false, as the contract could be sold only to healthy individuals. Then for each year, starting from the age of the entry into the contract we start by checking these two flags (first death, then LTC): if both are false, we consider first the probability of dying and then the probability of ending up being long term care. We then consider in which state is the individual after the transition (still a, or i or d) and we pass this data to the next year: all the flags were off at the entry, if we end up in LTC the corresponding flag will be enabled, the same applies for Death. In case the individual becomes invalid, we consider the probability of transition to the death status: we take the transition probability of the purely Markov model, but we adjust for the length of sejour, according to the anagraphic age and the duration of sejour: first year, second year, of more than two years. If the individuals are death the cycle breaks. We then discount each cashflow by the proper deterministic interest rate (our curve models the whole curve up to 30 years, thanks to ECB data, after this the interest rate curve is constant) obtaining the fair premium level; furthermore, we discount the benefit level of each repetition of the Montecarlo experiment to recover the scenario distribution and by so the measure of the variability of the premium level, defined as:

$$\sigma^{simulation} = \sqrt{\frac{\sigma^2_{scenario}}{n-1}}$$

In the case of death guarantee, once the individual dies it is checked in both cases (from I to D and from A to D) if the additional guarantee is still working (if less than K years are passed from the entry into the contract): in this case, the residuals payments originating from the death guarantee are reported into the process trajectory, after that the simulation terminates. If the guarantee has already expired, the code terminates immediately. If the additional lump sum guarantee is present, the amount of the first LTC payment is majored by an amount that is five monthly payments.

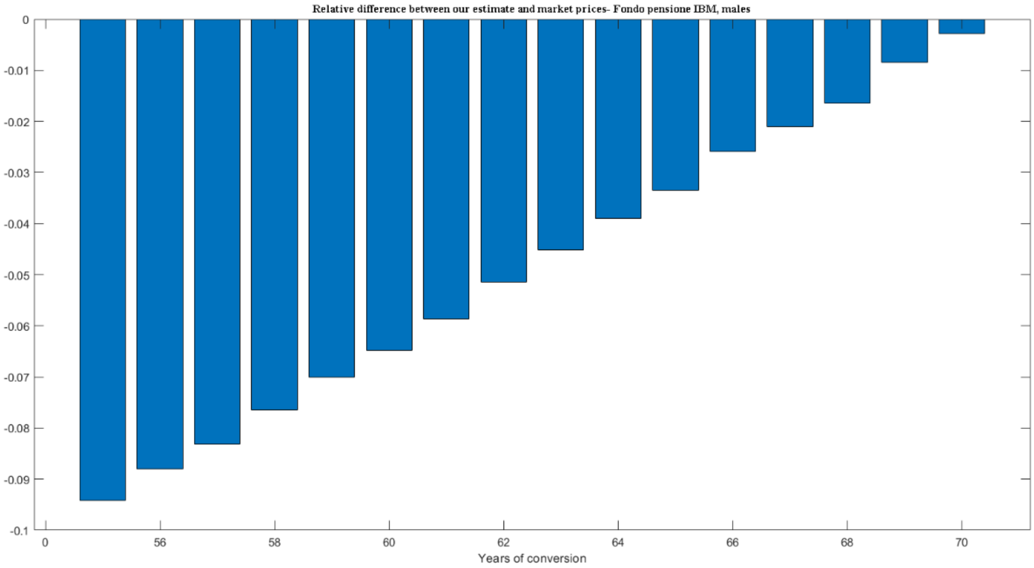
We then consider if this transition (from LTC to death) happens or not and we reconsider the output for the next years. The LTC flag was already on if the death happens the death flag will be enabled. Furthermore, we will consider the scenario distribution of sejour times (i.e., how much and how many sejour levels we will find in the distribution) and the standard error (both the overall standard error of the scenario distribution and the standard error of each year). The main object of our analysis will be the level and the pattern of the vectors above reported; we will even play some sensitivity analysis on the input variables to consider how biometric and financial assumptions will affect our results and if these results are coherent with the one reported in previous studies. In conclusion, we will even consider the case where a lump-sum payment of five monthly payments is provided once the LTC barrier is hit and the case of five and ten years of counter insurance. The interest rate we used in the simulation is the euro area government bonds spot curve (all ratings) according to ECB data of beginning June 2021, this is consistent with our assumption: using a higher interest rate will lead to an undervaluation of the liabilities of the insurance company. However, we must consider that the product we are pricing is country-specific and Italian insurance companies (even branches of foreign companies) generally hold a relevant number of Italian treasuries. As a result, we cannot price our product using the solely AAA-government bond curve as this will underestimate the average return on assets of our representative insurance business. We rember that, in life insurance context, the higher the intrest rate, the less prudent the valuation, since we are pricing liabilities and not assets.

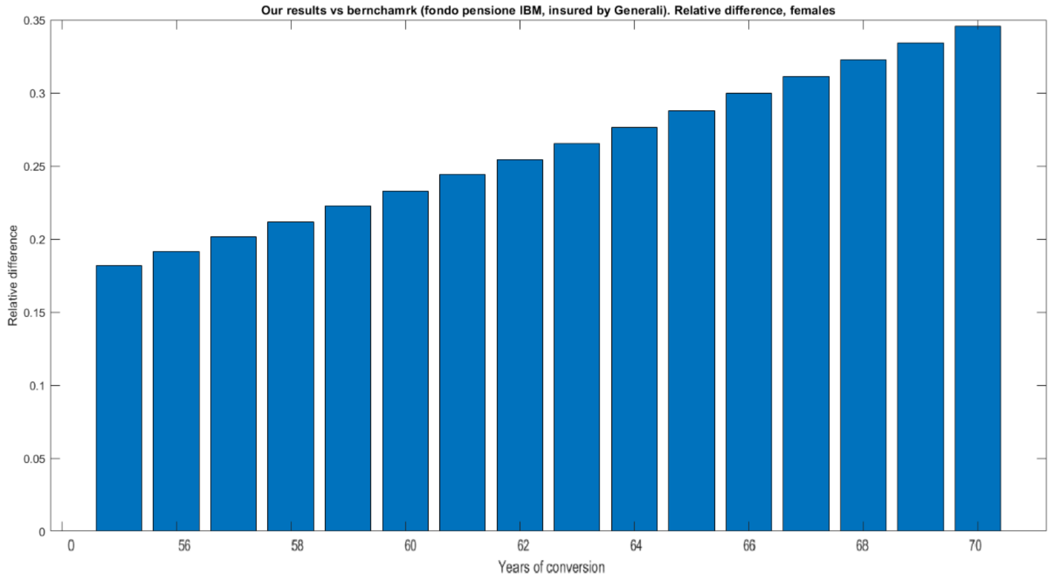
The results are compared with a market benchmark, the one that is most up to date to the date of the simulation (April to July 2021). This benchmark is found in the conversion table of the Italian pension fund of IBM "*Fondo pensione IBM*"[81], one of the oldest in the Italian panorama (1986). Usual disclaimers to such comparisons do apply: the assumptions made by the insurance are a little different from our assumptions and the definition of loss of autonomy is an enough restrictive one. Even more, the target of our market benchmark is a subset of the Italian population, for geographic and socio-economical stratifications, so that the morbidity and mortality probabilities could be relatively different from the one derived in the previous chapter. In truth, we consider the strange additional case of Credit Agricole vita (Amundi secondaPensione)[82], which sells the same product, with the same price for males and females. This is compulsory (prohibition of gender discrimination) in the first pillar insurance, but some companies, for business and social reasons (gender equality) started to do that even in the second and third pillar. The disclaimer made for the first benchmark is applicable even here, but in this case, the target population coincides with one of our models.

Coming back to the main market benchmark, IBM pension fund, with different prices for males and females, The conversion coefficients (our prices) are close for men: we would say too much close, as the market prices should include even a safety loading, while for females we observe a substantial under-pricing: this product is offered to a price that is from 18% (55 years) to 30% (70 years) lower of our fair premium level estimates. This is possible as the number of insured asking for the enhanced pension is a relatively small share and there are commercial opportunities in addition to actuarial results. As already stated in the previous points, the advantage of natural hedging might lead to a sufficient level of profit even in a context where the premium level seems insufficient to cover the outflows. This means that to ease the adhesion to this kind of annuity, that is in the interest of both the insured and the insurer, the insurer might ask for a lower premium level than the unconsolidated actuarial fair one.

81 https://www.fondopensioneibm.it/rendite.htm, pages 27-30. Are reported the conversion premium levels and the main features of the contract, in terms of definition of the LTC status.

82 Here, at page 17, is reported the relevant table https://www.secondapensione.it/ezjscore/call/ezjscamundibuzz::sfForwardFront::paramsList=service=ProxyGedApi&routeId=_dl_ZTlmOWFlYzQ5NjllZDJlNGI3YzhkNDViZGQzMGZlYjE_inline





As a first result, we observe that the cost of the LTC guarantee (that is, the innovation of our contract concerning an ordinary annuity) in our contract (obtained as $\pi_{ltc} = \frac{\pi_{enchanced\ annuity} - \pi_{vanilla\ annuity}}{\pi_{vanilla\ annuity}}$) is strictly increasing in the age of capital conversion both for females and males, for females it goes from 5.4% of the previous premium level for 55 years to 8.62% of the previous premium level for 70 years. For males, it goes from 6% at 55 years to 9.85% at 70 years.

We want to introduce and summarize the main differences between males and females in the premium level distribution and the *sejour* one. Results generally refer, if not differently specified, to the whole vectors of premium/sejour distribution.

Differences in the premium distribution

- Males are significantly more variable than females. The standard deviation for males is 8.2, while for females is 4.43. Results hold even in coefficient of variations (relative terms), being the average of males strictly lower than the one of females.
- The skewness of scenario distribution is significantly negative for males at the beginning, but the distribution is substantially symmetric at the end. By contrast, the distribution of females is significantly more negatively skewed for all ages, even if whenever shows a decreasing pattern.
- The distributions of males show at the beginning an excess of kurtosis that after disappears for older ages (at 65-66 years of entry into the contract). In the last years, even notice a defect of

kurtosis. This does not apply to females, that whenever show an excess of kurtosis, even if still decreasing in the entry age.
- In the appendix are reported VaR and expected shortfalls, adequately adjusted to context.

Differences in the sejour distribution

- The LTC- barrier hitting probability is significantly higher for males than for females (54% vs 25%), but in both cases, it shows a slowly decreasing pattern (less than 2% percentage points in both cases)
- The average sejour is significantly higher for females than for males. (2.35 vs 3.7) and does not show a relevant trend for males, while there is a minimum decreasing (0.2 years from 55 to 70 years as entry age) pattern for females. Only in case we account even for sejour equal to 0, so if we consider the average sejour of the whole sample space, even of insured that does not enter the LTC state, we have a decrease at the increase of the entry age. This holds both for females and males and is due to the decrease of the expectation of the indicator function of barrier hitting, not at the decrease of the average exit time.
- The variability of sejour distribution is significantly higher for females than for males, and in both cases shows a decreasing pattern concerning the entry age.
- Sejour distributions, differently from fair premium level distributions, show significantly positive skewness (2.5-3). The skewness of females is greater than the one of males and both show a decreasing pattern at the increase of the entry age.
- Further results are presented when we consider the share of experiments that presents a sejour greater than a certain number of years k (the share is considered on the whole sample space, so on the whole simulations, not only for the insured experiencing sejour). The higher the k, the more decreasing the pattern concerning the entry age: this pattern is however different between males and females, as males show an initial increasing trend that females do not show.

Differences in the cost of the guarantee

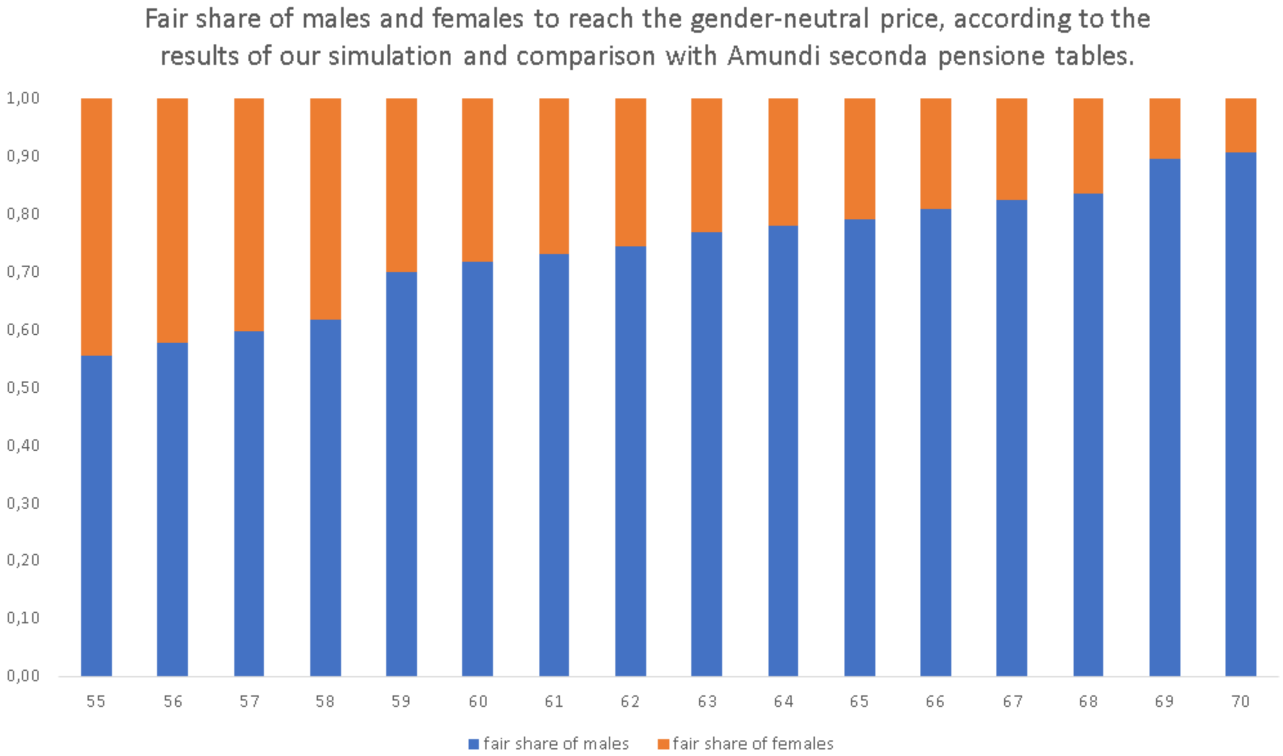
- The expected cost of an LTC guarantee is higher for males than for females if derived under the expected value principle. This is due to the lower unconditioned expected sejour and to the higher time passing between the entry contract and the barrier hitting.
- The higher the entry age, the lower the cost of the annuity. However, this is less true for enhanced pension than for vanilla annuities (the price is less elastic to the entry age). And this holds for both males and females.

Concerning the gender-neutral fund of CA vita, which asks the same premium for males and females, we must consider that they declare that premia are obtained under the convention of the share 40% females and 60%, constant for all the ages. Note that they assume the same share between males and females for all the contracts, even normal annuities and counter-ensured ones, there is no specific share between males and females for the specific annuity: all the possible forms of conversion of capital are priced with the same assumption about the share of females and males. For all the contracts in question, not only for the LTC annuities, but males also cost less than females, presenting a lower degree of premium. However, to preserve a certain margin (load), the insurance company must overweight the share with a higher premium, and so the higher risk. Starting from fair prices we recovered, we compute the share of males and females that, applied to our prices, lead to the gender-neutral premium of Amundi. We find that the real share of females leading to the fair premium level is significantly lower than the one declared, especially for older years. We notice that if we consider, as a measure of load, the difference between the share of females declared by the insurance company (40%) and the share of females consistent with a fair price policy, we obtain a load structure increasing in time. It is even possible that, in truth, knowing that females general buy earlier the annuity the assumption of 40% females and 60% males had not at all been respected.

We obtain the fair share of females $\theta(t)$, for each age t in [55, 70] as it follows:

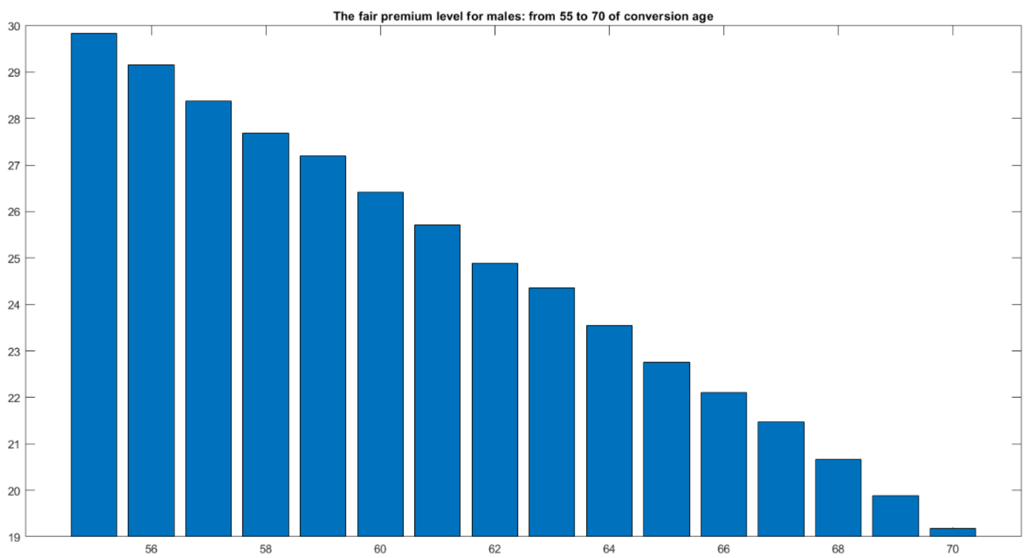
$$\pi_{genderneutral}(t) = [\theta(t) \quad 1 - \theta(t)] \begin{bmatrix} \pi_{females}(t) \\ \pi_{males}(t) \end{bmatrix}$$

At that point, we compute a proxy of the load as the difference between the shared declared and the fair share $\Delta_{females}(t) = 0.4 - \theta(t)$.

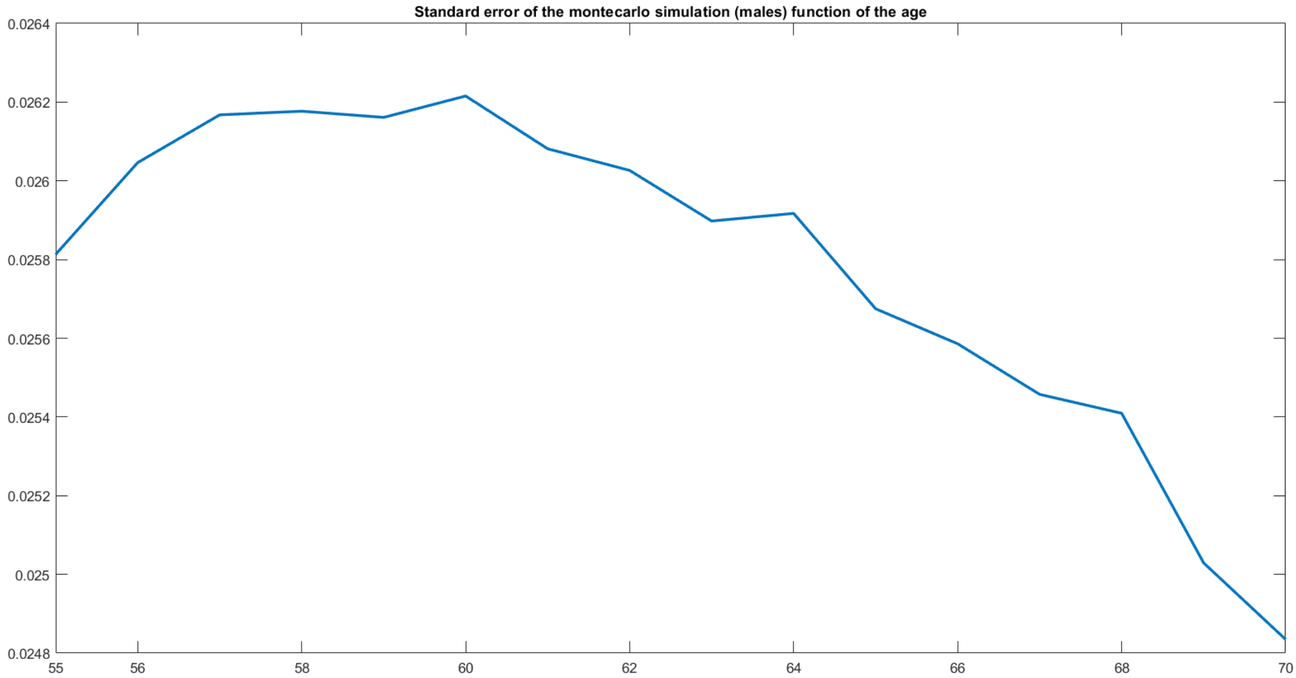


4.2) The results for males, the vanilla contract

We want first to comment on the fair premium level, the price of our instrument:

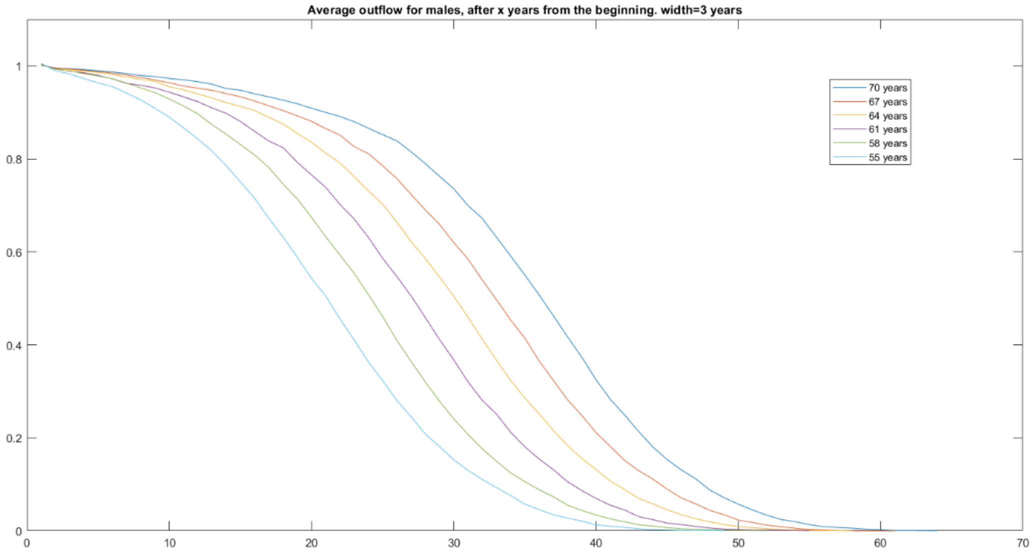


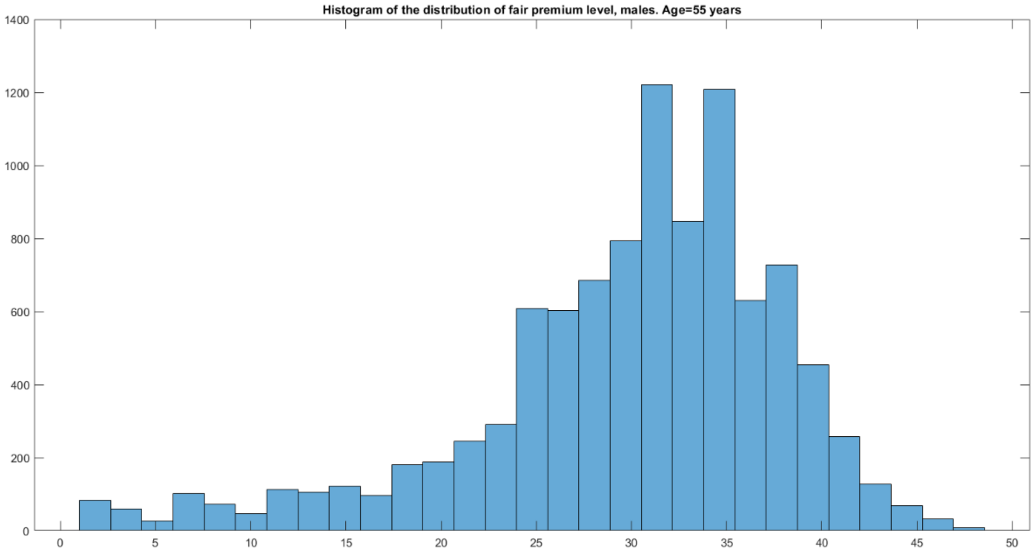
We notice that for males the fair premium level shifts from 29.87 at 55 years to 19.33 when the insured decides to convert at 70 years. That is over the years the fair premium level decreases by 35%. Please consider that the standard premium level for an annuity (males) is about 27.68 in Italy at 55 years and 15.03 at 70 years if we consider reliable market prices[83]. The decrease in the vanilla annuity is about 45%, while in the LTC it is less. This is explained by the fact that an insured that converts his capital at 70 is almost sure to activate the LTC guarantee, while for an individual that converts at 55 there is a significant probability of death before activating the guarantee. Concerning the standard error, for males, we notice the U-shaped pattern: the higher variability (and so, the higher expected safety loading) is for individuals aged between 58 and 60, while after the variability tends to decrease faster for males older than 65. The variability is mainly due to two components: the variability of life expectancy and the variability of LTC events. The first one decrease for older ages, while the second one increase, thus resulting in a U-reversed variability.[84]



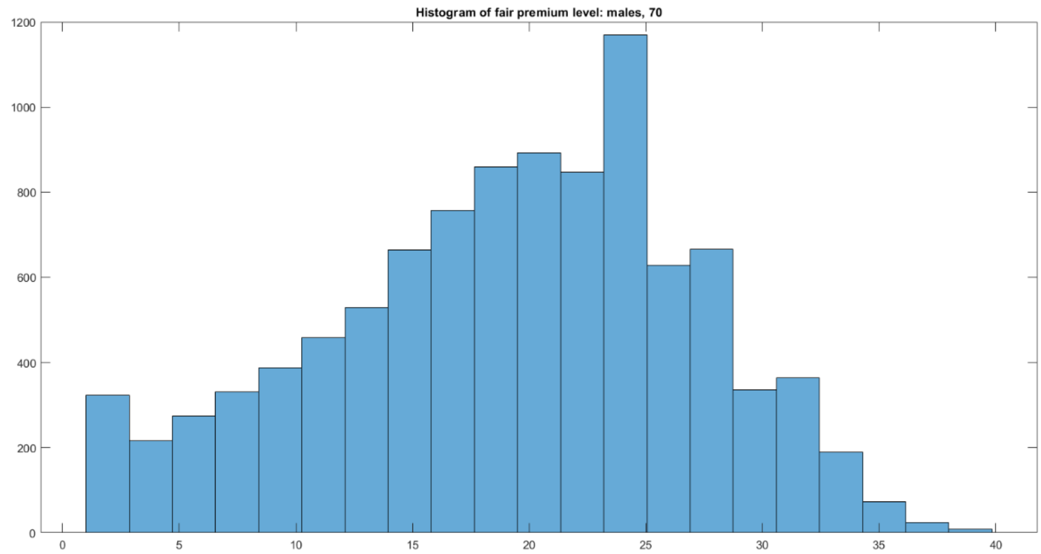
Concerning the time distribution of the outflow (the payment made every year by the insurance, not discounted), we have substantially a confirmation of what is expected: the decay x years after the entry into the contract is faster for younger individuals. For example, the average outflow of 50% of the initial one is reached after 20 years for a contract stipulated at 70 years, and after 38 years for a contract stipulated at 55 years. The slope of the curve is significantly higher going to the left, i.e., for contracts stipulated with older people.

83 We have tried even to price a vanilla annuity with our algorithm, prices are compatible with the market ones. However, this is highly inefficient and computationally expensive as it unnecessarily increases the numerical and statistical error.
84 Consider that if, instead of the standard error we use the pure-number coefficient of variation (standard error/average), we have a completely different patter. In this case, in fact, our coefficient (that measures risks in relative terms) is strictly increasing from 0.8510 to 1.310. This results however, is in our context of thin interest. The reasons are the following: the insurance company measures the loss on the contract in monetary terms, not in relative ones.





Considering the histogram of the distribution of the 100.000 replications of the Montecarlo experiment we notice a substantially two-peak distribution to the levels 32 and 35 and the high skew of the distribution with a substantial left tail (skewness -1.015). This is for sure to be considered as positive and as a part of the risk-reduction benefit due to the diversification of mortality risk. We cannot report the histogram for the whole spectrum of the ages, obviously, in the appendix, the evolution year-over-year could be refined, but we must consider that the skewness and the multimodality of the distribution slightly disappear as the age increases up to a unimodal distribution with a significantly lower level of skewness for 70 years insured (-0.36 for 70years versus -1.015 for the 55years). We notice that the distribution shows a significant decrease in kurtosis from a value relevantly higher than 3 to a value significantly lower. This means that the probability of finding outliers decreases for older ages, as already shown for the variability: this is extremely relevant as the probability of outliers (kurtosis) plays a key role in defining the risk for the insurance company on the insured position and the overall valuation of the premium safety load. This means that an insurance company is preferable to defer the purchase of an LTC insurance, exactly as for ordinary life insurance: however, this is not optimal for the insured, that shows a significant increase of the uncovered risk in the ages where he is not insured but the probability of hitting the LTC barrier is not negligible (all the ages after the 50 years up to the insurance entry). In conclusion, the share of individuals hitting the LTC is substantially stable when distinguished by the age of conversion (about 54% of the individuals, from 55% to 52% in older ages). The constant probability of hitting the barrier, and even length of sejour might seem strange, but we must consider that the events “the individual has the opportunity to buy an enhanced pension at age x" show a decreasing probability since is harder to reach 70 years still in an autonomous state, that allows entering the contract. If an individual has lived healthy up to that point, it is more probable he will live for a consistent number of additional years, a fact that increases, *ceteris paribus*, the possibility of entering a sejour and lasting in that state enough, considering the slow mortality pattern of elderly dementia.



We will now focus on the key variable of our study, i.e., the sejour time. This is the variable that we have changed significantly in modelling if compared with previously published studies, that however are not referred to these very specific products and use a different methodology in the general model[85]. The average sejour, provided that is happening is 2.35 years, with a median of 2 years of sejour: on this point, there is no significant change concerning the age of the entry into the contract (55,56…70). This independence of the distribution of the sejour and the entry into the contract, however, concerns only the first moment of the sejour distribution. We notice that the distribution of sejour times is exponentially decreasing, so we have, in general for all the ages of conversion that

$$P\ (z=0;\ \text{no barrier hitting})>P(z=1)>P(z=2)>P(z=3)\ \ldots\ \text{with } z \in N.$$

This is consistent with what has been observed by previous studies and with the theoretical development of our research. Note that in our analysis, when we consider the sejour distribution we consider the effective sejour cases, so only situations where an LTC coverage started; other said, we are discussing the average lasting of the sejour even once this event is certain and not on the probability of this happening. Even considering males, the different capital conversion ages could play a significant role and altering, but not substantially, our results. By contrast, we do consider how does change the standard deviation of the above-expressed distribution (that is specifically linked to the age of conversion) depending on the age at capital conversion. Firstly, we must observe that the mean is not showing a significant trend, while the standard deviation is strictly decreasing in the ages. As a result, we can consider that as the age of capital conversion increases, the volatility of the above plotted distribution (the distribution of sejour) significantly decreases. This means that as the individual becomes older if he enters the sejour state, the magnitude of this interval becomes far and far more forecastable. Regarding the magnitude of the sejour duration, we still infer some relevant elements from the cumulative distribution of sejour times conditioned to the age. Once the LTC barrier is triggered, the probability of experiencing a short sejour ($1 \leq z \leq 4$) is higher for younger entry ages. This mainly depends on the extra mortality pattern that is significantly more spiking in the first years. So, we observe that

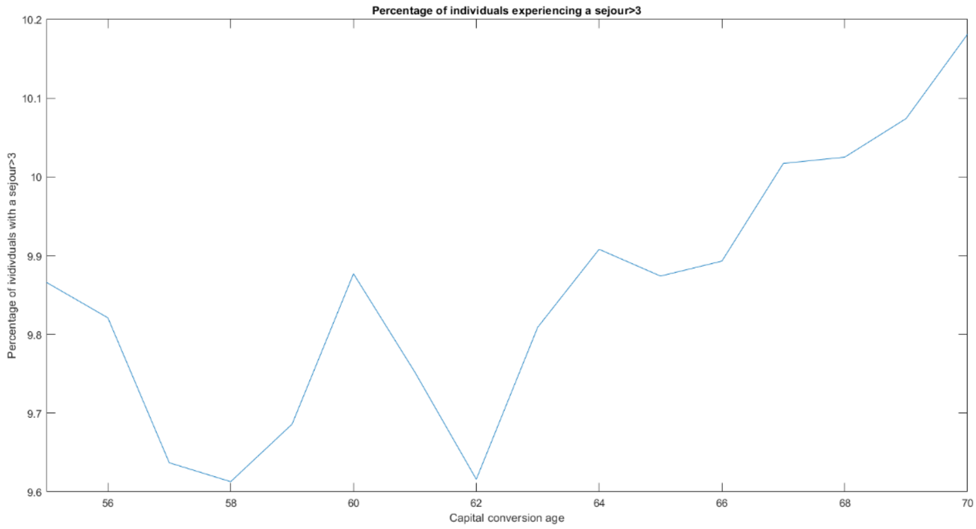
85 See Pitacco, ERM and QRM in life insurance, 2020

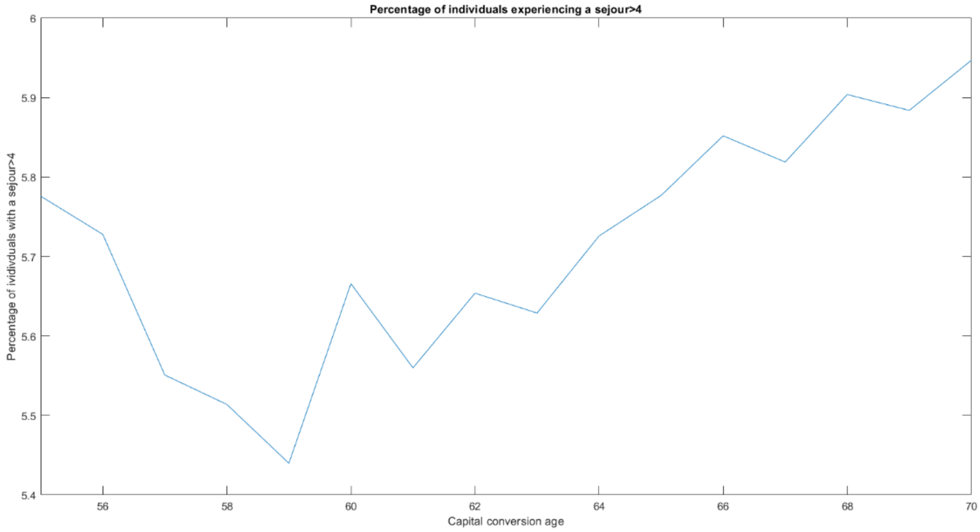
For every k in [2,3,4] and x, y denoting the age of the contract entry, with $55 \leq x < y \leq 70$ we have $P(1 \leq z \leq k \mid x) > P(1 \leq z \leq k \mid y)$.

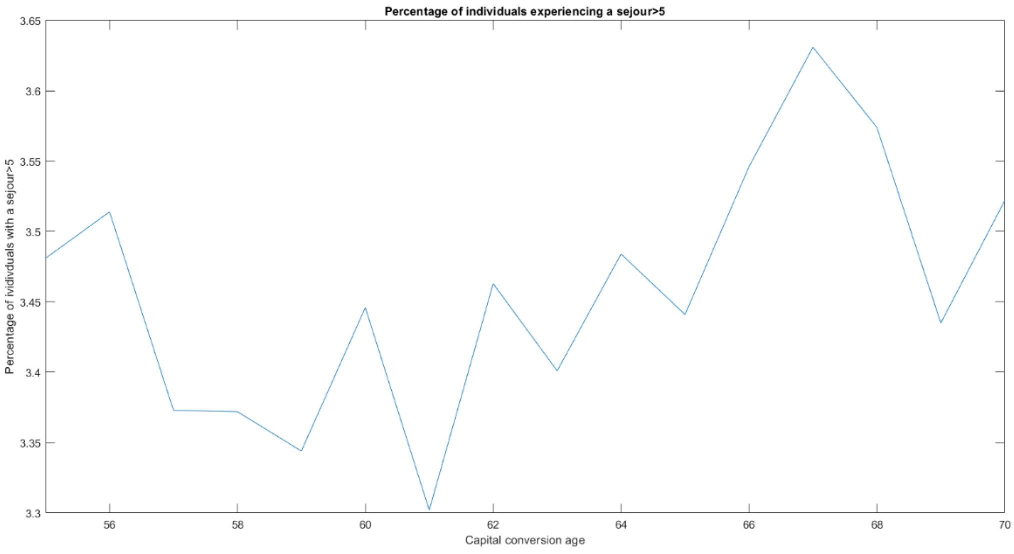
No difference is shown by the cumulative distribution function (at least significant) after this age.

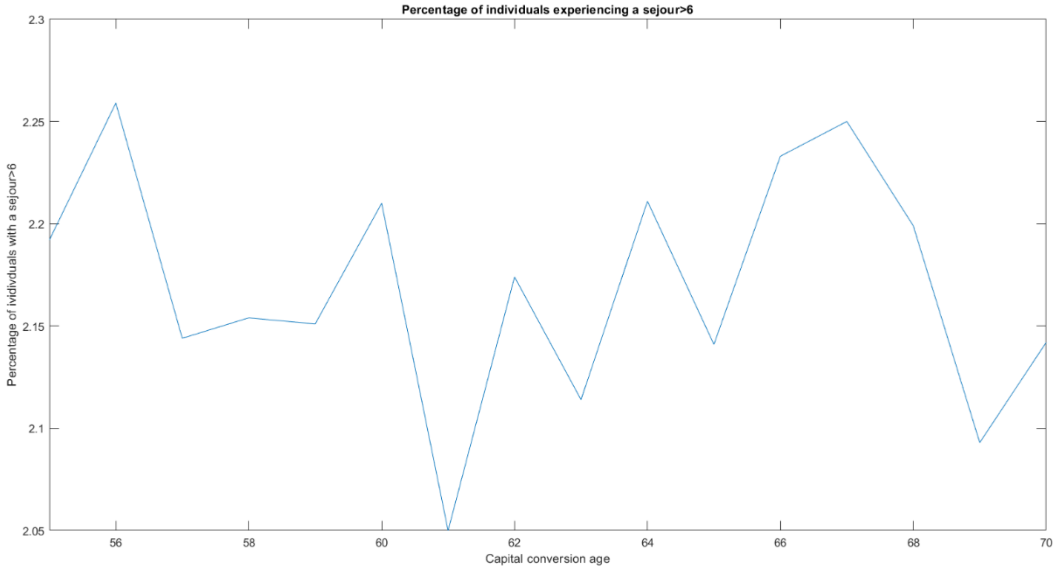
Now, we are ready to consider the following question, as a conclusion of our *excursus* on sejour times: considered that the LTC barrier has been triggered, which is the probability that the sejour lasts more than k years? The interest in this graph is the conditioning to the age of the entry of the contract: as a result, the probability could be formalized as $P(z \geq k \mid x)$ where x is the age at capital conversion. The probability of experiencing a sejour greater than 2 years is equal to roughly 10% for all the ages of capital conversion, with a slightly increasing trend, that applies however only to the ages greater than 63. We can say the share of sejour greater than 3 years is constant and equal to 10% for ages less than 63 (at entry into the contract), while it increases about 0.2% each year. Concerning the sejour longer than 4 years, we observe a pattern like the one described for the sejour longer than three years, apart from a lower general level (5.7% with a peak of 6% for individuals converting at 70 years). The increase in the share of individuals presenting a sejour longer than 4 years concerns all the conversion ages after the 60 and the spike of this increase is 0.5% in 10 years. For the share of sejour longer than 5 or 6 years, we do not find any trend concerning the age of capital conversion, this means that the entry into the contract is inelastic to the probability of finding a sejour longer than 5 of 6 years. Even tests for the absence of any linear form of the trend confirmed this.

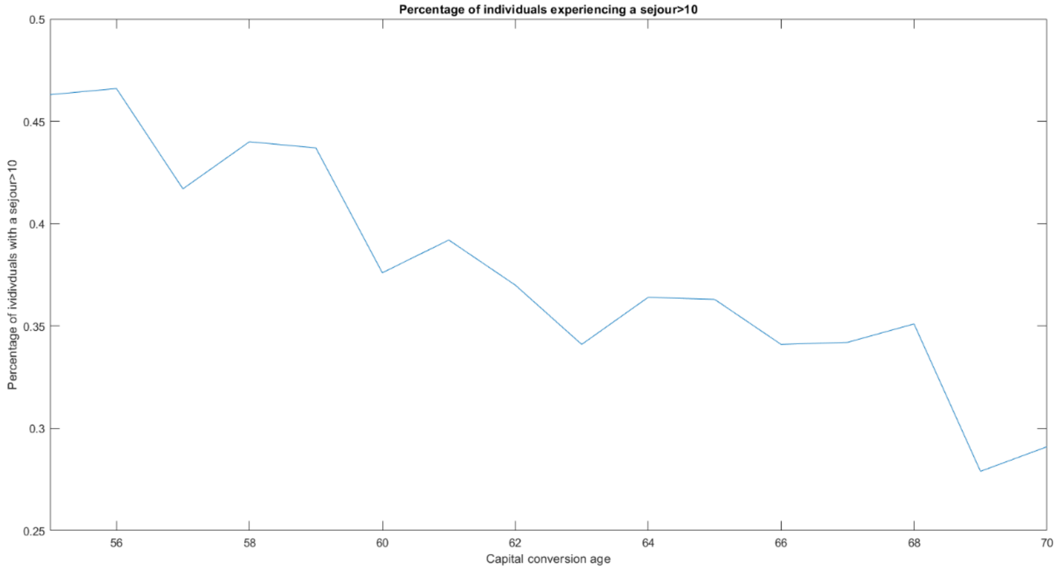
The trend completely changes once we consider the probability of finding a long sejour, i.e., a sejour longer than 10 or 15 years: at that point, the trend becomes sharply decreasing concerning the age of entry into the contract. This is since the probability of experiencing an LTC sejour is positively linked to the (cum-disability) life expectancy: the higher the residual life expectancy, the higher the probability of finding an “extreme” sejour in LTC status. For the probability of a sejour longer than 10 years, we shift from 0.45% at 55 years to 0.30% at 70 years, while when considering the probability of a sejour longer than 15 years we have a logarithmic decrease, from 0.1% to 0.01%.

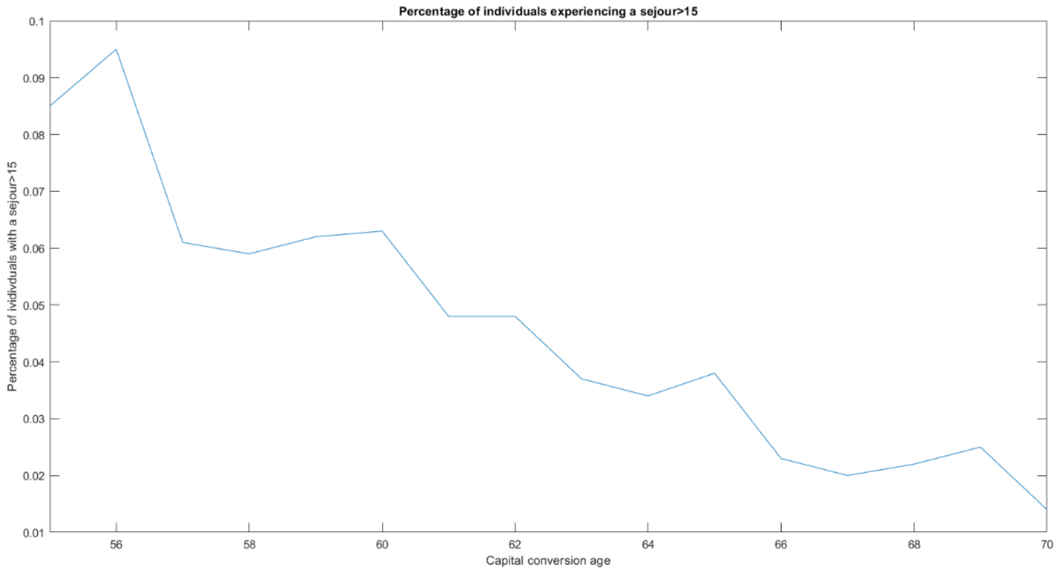

Percentage of individuals experiencing a sejour>3
Percentage of ividivduals with a sejour>3
Capital conversion age


Percentage of individuals experiencing a sejour>4
Percentage of ividivduals with a sejour>4
Capital conversion age

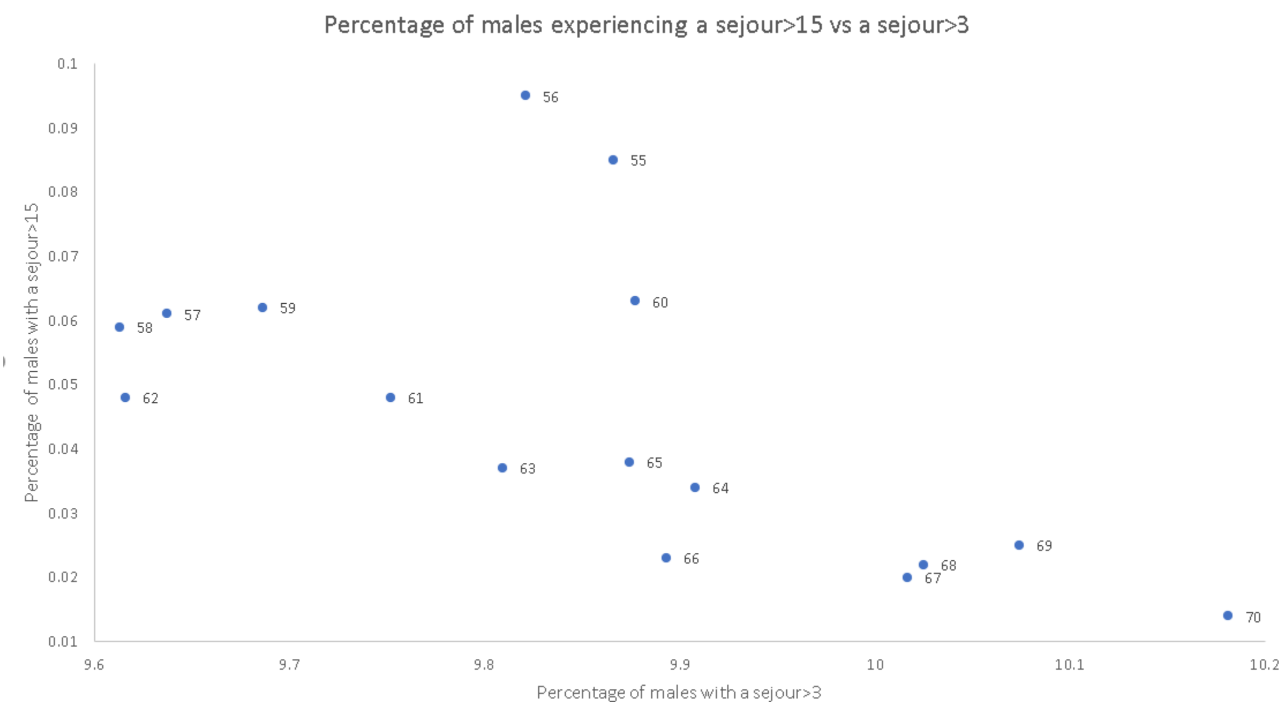

Percentage of individuals experiencing a sejour>5
Percentage of ividivduals with a sejour>5
Capital conversion age



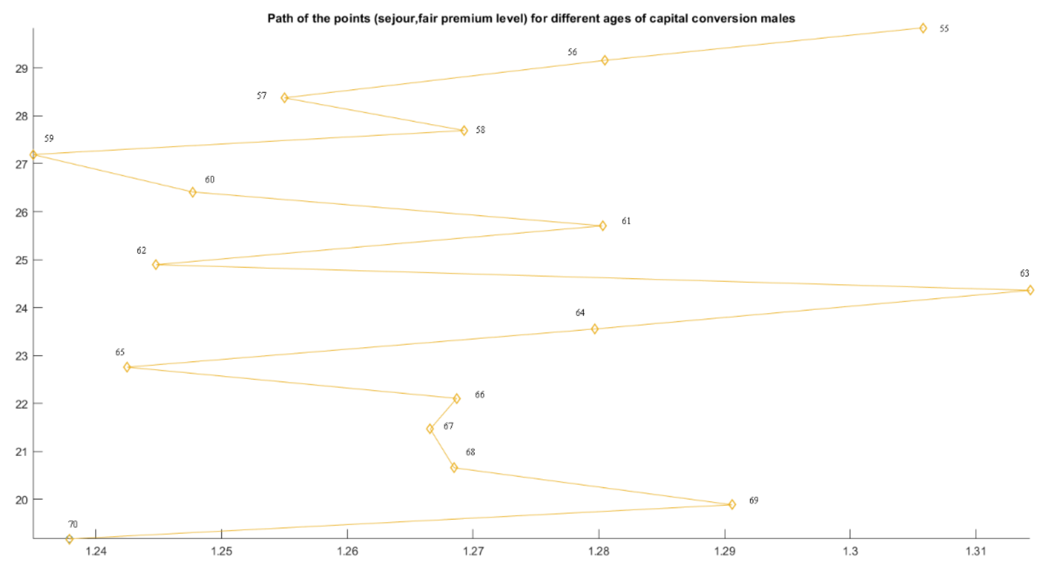




The anticorrelation between probabilities of short and short and long sejour is even confirmed by this scatterplot.



We now want to consider the relationship between sejour and fair premium level, this is done through a comet graph. A comet graph can show the dynamic of the 16 couples (corresponding to the 16 possible ages of conversion) concerning sejour duration and fair premium level, the main consideration is that an increase in the age at subscription decrease the fair premium level and decreases the expected duration of the sejour. We have adjusted for this scope the definition of sejour, considering even sejour shorter than one year, that is completely irrelevant if the annuity is paid yearly, but that becomes relevant if the annuity payment frequency is monthly, or quarterly, or half-yearly. Consequently, the average is lower because is performed on all the simulations, and the fact of having more than 50% of simulation equal to zero decreases the average. This explains why the average reported on the x-axis might be different (lower, in particular) from the one previously reported. But by contrast, we must say that while for the fair premium the pattern is strictly decreasing in the age, for the sejour level we do not observe a clear decreasing pattern over the ages of capital conversion.



We tried to approximate the distribution of sejour duration with a geometric distribution, defined in terms of failures before having the first success: this doing, the support of the distributions coincides with our admissible sejour. We observe that as the age of entry into the contract increases, running over the interval [55,56…70] the parameter p remains substantially stable over the age of entry into the contract since the MLE estimator is:

$$\hat{p} = \arg\max\ (L\,(p; z))\ \text{sub } 0 \leq p \leq 1 \text{ and } z \in N+$$

$$\hat{p} = \arg\max\ \left[\, N\ln(p) + \ln(1-p)\textstyle\sum_{i=1}^{N}(x_i - 1)\,\right] \text{ sub } 0 \leq p \leq 1 \text{ and } z \in N+$$

$$\text{FOC evalued at } \hat{p}: \frac{\partial L(p;z)}{\partial p} = \frac{N}{\hat{p}} - \frac{N}{1-\hat{p}}\textstyle\sum_{i=1}^{N}(x_i - 1) = 0$$

$$\frac{1-\hat{p}}{\hat{p}} = \frac{1}{N}\sum_{i=1}^{N}(x_i - 1) \xrightarrow{\text{yields}} \frac{1}{\hat{p}} = \frac{1}{N}\sum_{i=1}^{N}(x_i) \xrightarrow{\text{yields}} \hat{p} = \frac{1}{\mu(Z|z>0)}$$

$$\text{SOC evaluated at } \hat{p}: \frac{\partial^2 L(p;z)}{\partial^2 p} = -\frac{N}{\hat{p}^2} - \left(\frac{1}{1-\hat{p}}\right)^2 \textstyle\sum_{i=1}^{N}(x_i - 1), \text{ but since } \hat{p} = \frac{1}{\mu(Z|z>0)} = \frac{1}{\bar{x}}$$

$$\sum_{i=1}^{N}(x_i - 1) = N\bar{x} - N = N\left(\frac{1}{\hat{p}} - 1\right) = N\left(\frac{1-\hat{p}}{\hat{p}}\right)$$

$$\text{And so the SOC simplifies in}: \frac{\partial^2 L(p;z)}{\partial^2 p} = -\frac{N}{\hat{p}^2} - \left(\frac{1}{1-\hat{p}}\right)^2 N\left(\frac{1-\hat{p}}{\hat{p}}\right) = -\frac{N}{\hat{p}^2} - N\left(\frac{1}{(1-\hat{p})\hat{p}}\right) < 0$$

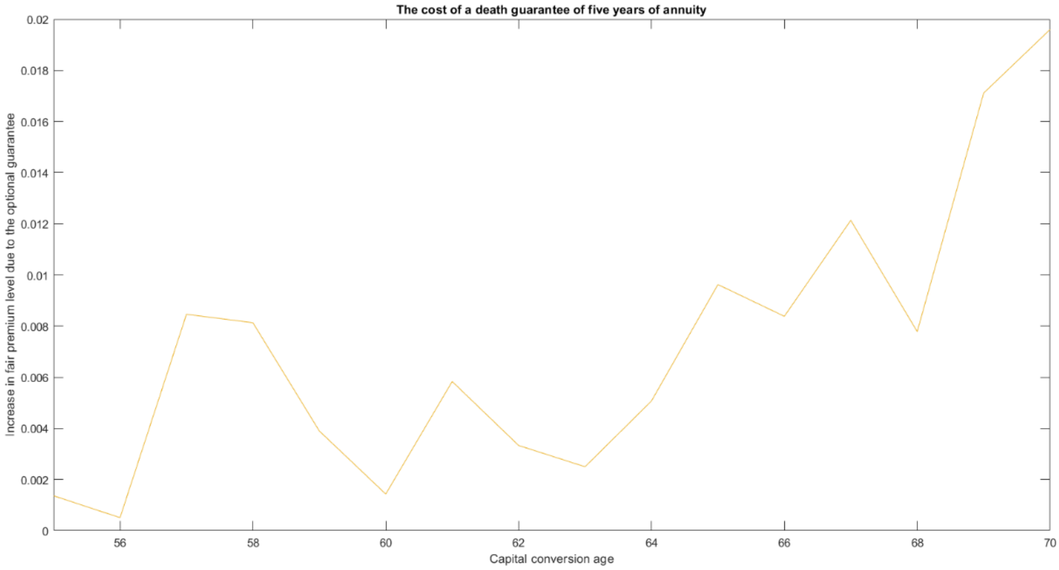
With $\hat{p} \approx 0.42$ and $\hat{p}$ being substantially inelastic to the entry age. In truth, however, there are differences between *sejour* distribution of different entry ages, but a single-parameter distribution cannot capture this. However, we must consider that since a semi-Markov correction has been applied to a non-homogeneous chain, the memoryless property of waiting times does not any longer apply, since *sejour* is affecting memory of the distribution. We tried even to generalize the geometric distribution to a Pascal one: results were insufficient in terms of bias but were showing an increasing pattern, concerning the entry age, for both n and p, which keeps expectation almost constant.

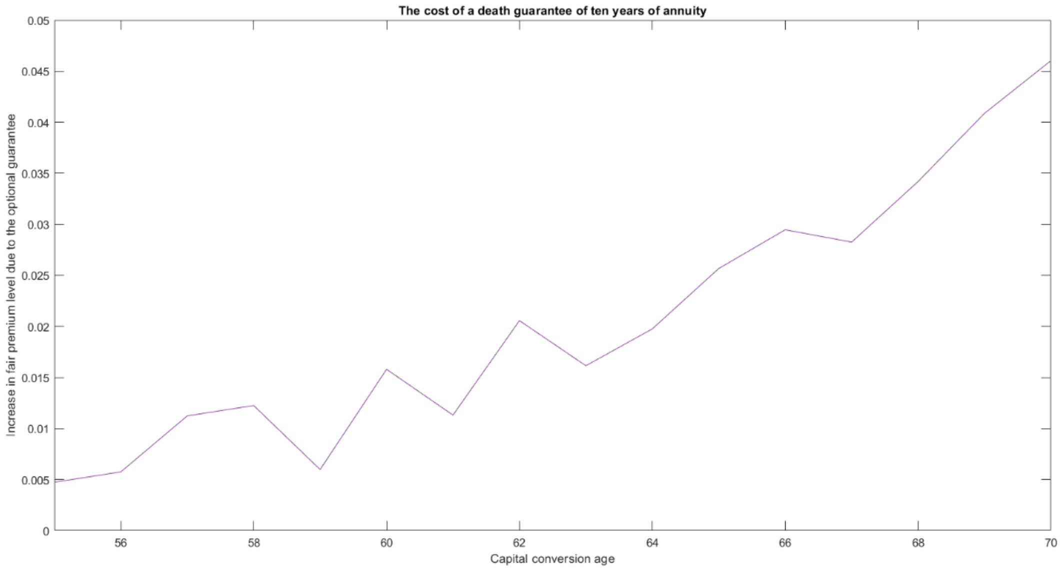
4.3) The additional guarantees for males

We now consider topping up the vanilla contract (that pays X if the insured is alive and autonomous, 2X if the insured is alive and lost his autonomy) with the set of additional guarantees already described. The code allows to put every possible level of additional guarantee (a death guarantee that lasts 6 years or 50 or 49.12 years; a lump-sum payment at the LTC hitting that is five times the monthly payment or that is 3 yearly payments), but we use the best practice of the Italian market. Note that is not anticipation, is an additional benefit. Note that for the death guarantee we are using an assumption, i.e., a simplification, that is that we will find certainly an heir: by truth, for longer periods like twenty years, it becomes less unrealistic that all the survivors of the insured are death. By consequence, for a higher interval of death guarantee (i.e., greater than 10 years) the output is not the real fair price, but his upper bound (the error is however generally small). When additional guarantees are added to a basis contract, their costs are generally quoted as the increase in percentage point of the fair premium level. We will follow this standard. Even for the increase in percentage point, we will have a vector, referring to the different age t of the entry into the contract with t ∈ to S= [55,56, 57...70].

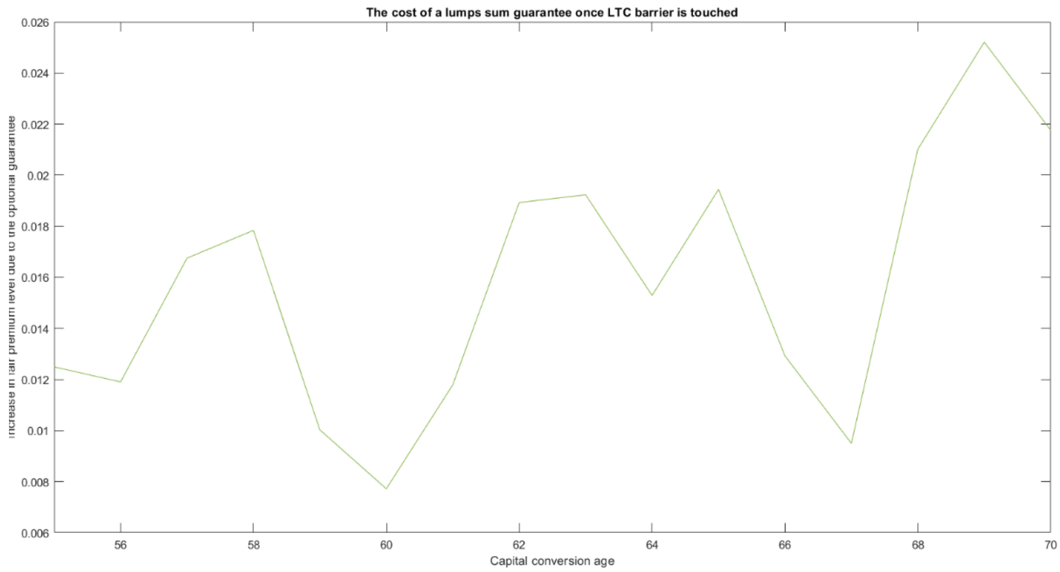
Concerning the additional guarantee of a death payment for the next 5 years from the death of the insured (to the advantage of the survivors to the insured) we have a contained cost for the lower ages, but a significant increase up to 2% of the premium level for older insured. We can observe that the costs increase rapidly through age, and this is expectable. Note that the cost of this additional guarantee in LTC is significantly lower, in percentage terms, of vanilla annuities.

Concerning the ten-year death guarantee, the cost is higher and goes from 0.5% to 5%, we do not notice a change in the amplitude of the trend: the ratio between the lower and maximum cost is still about 10 times. Death guarantees longer than ten years of age are generally rarely sold, the two standard contracts are five and ten years. It is possible, however, that in the future due to social changes a longer guarantee might become marketable.



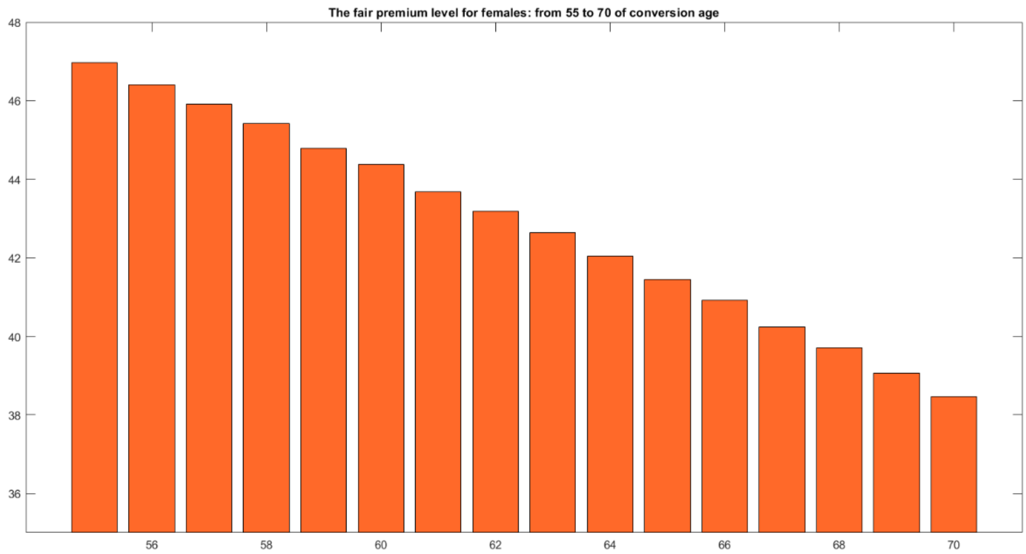


In conclusion, we focus on the lump-sum guarantee once the LTC barrier is hit. We have already explained in chapter II, paragraph five the benefits and the suitability of this additional guarantee, which has a flatter cost if compared to the previous two. The cost goes from 0.8% to 2.4% still in a constantly increasing pattern through the age of contract start. The increasing pattern is not since the probability of hitting the barrier increases, but as the barrier is hit in a shorter time interval, on average. The cost of this additional guarantee is by far one of the most sensitive components of our product to financial (i.e., interest rate) risk.

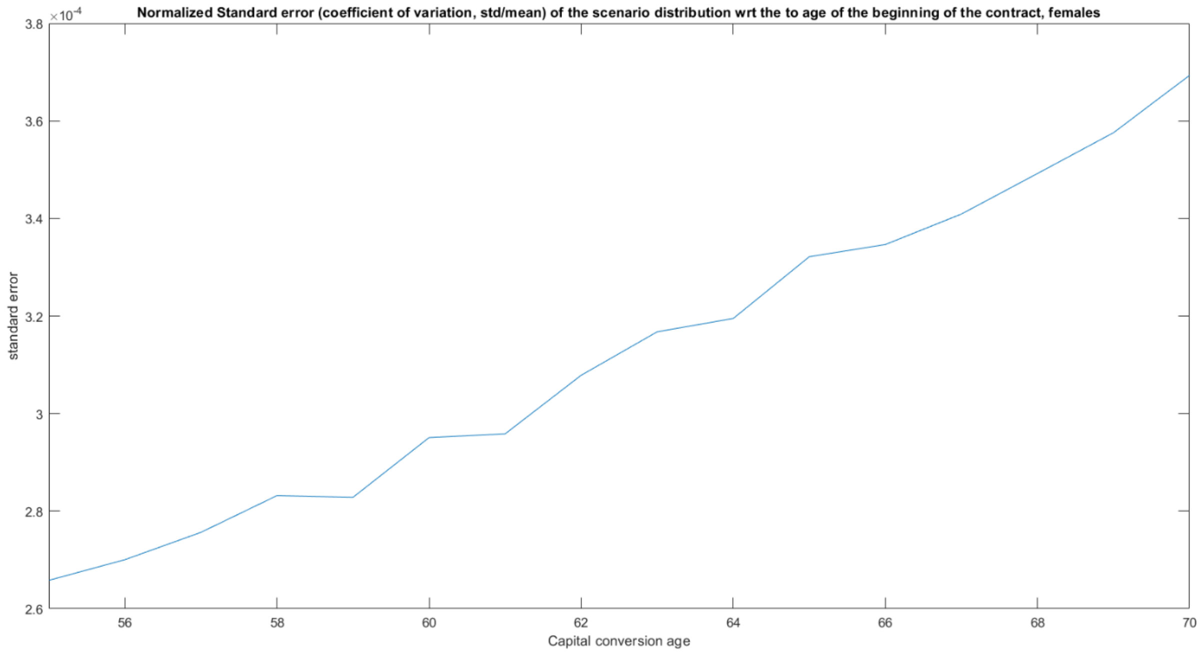


4.4) The results for females

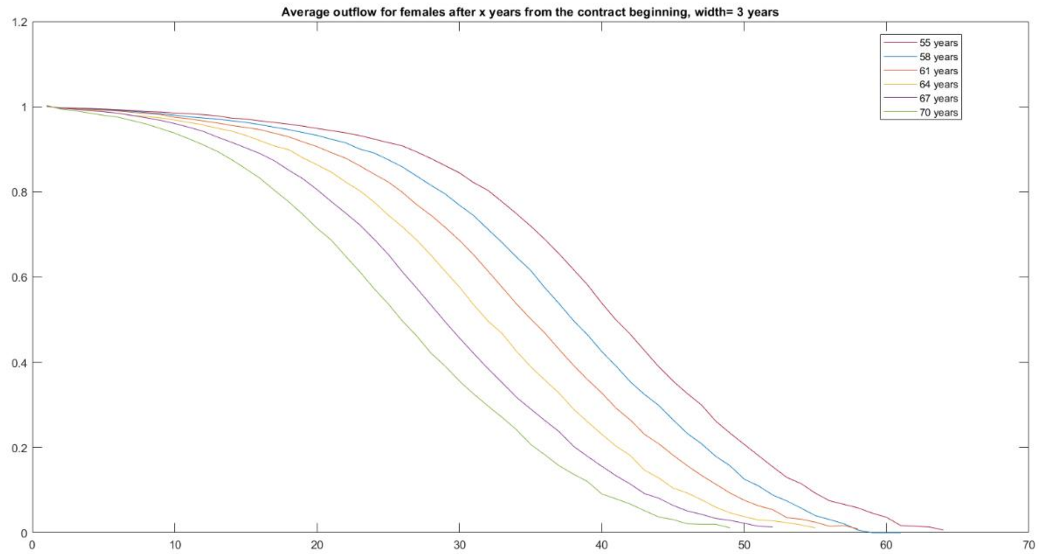
We want first to comment on the fair premium level, the price of our instrument, which is significantly more expensive (ceteris paribus) for women than for men. The gender gap is more relevant in LTC than in ordinary annuities, as males are more exposed to an LTC risk (both in terms of duration and probability).

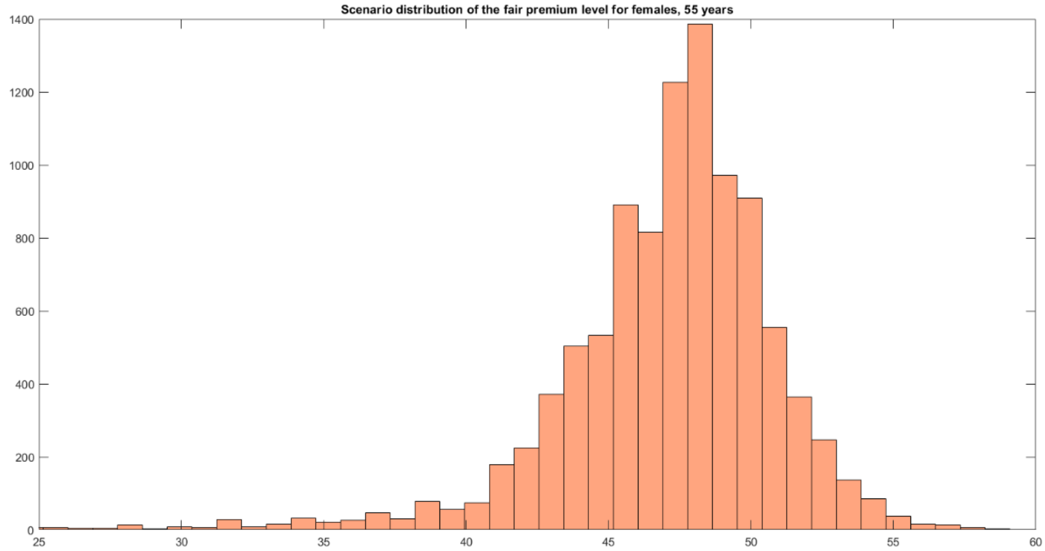


We notice that for females the fair premium level shifts from 47 at 55 years to 38 when the insured decides to convert at 70 years, this means 50% more than males at 55 years and a fair premium level substantially doubled at 70 years. Over the years the fair premium level decreases by 20%. Please consider that the standard premium level for an annuity (females) is about 27.68 in Italy at 55 years and 15.03 at 70 years, this means that 1/3 of the premium is going to cover long term care needs. The decrease in the vanilla annuity is about 60%, while in the LTC it is less- only 20% circa. Concerning the standard error, for females, we notice an increasing pattern: the higher variability (and so, the higher expected safety loading) is for older individuals, even if the range of the standard error is strict, less than 0.02. By truth, if instead of the standard error of the fair premium level we consider the pure number of the variability of the conversion coefficient (standard error/average, that we report) we have an increasing pattern, absolutely in line with the previous results for males. The comparison between males and females should be done observing these results, as the fair premium level might affect the relative relevance of the standard error. The relative variability (so the risk) of females is significantly lower than the one of males.

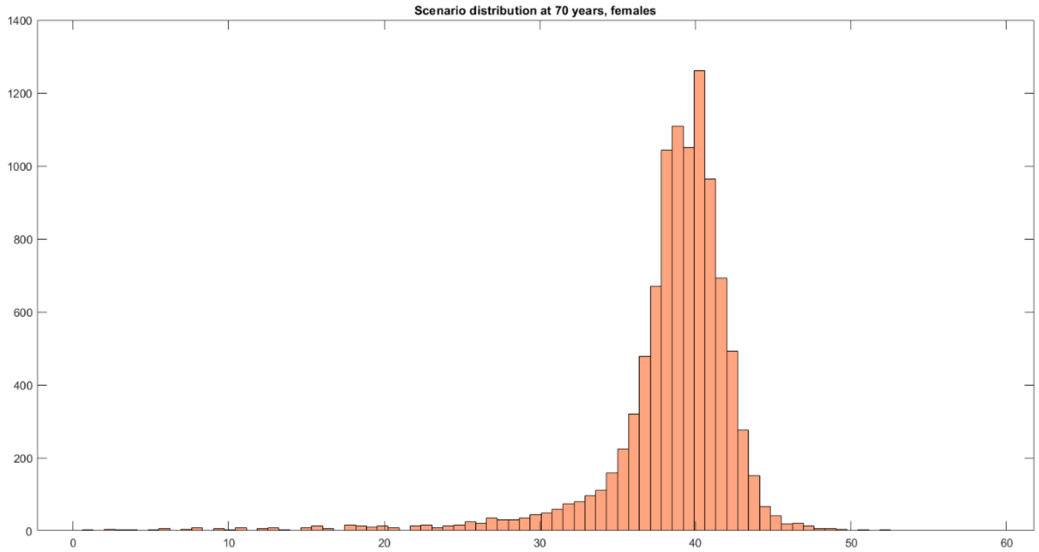


Concerning the time distribution of the outflow (the payment received every year by the insurance, not discounted), we have different results for females than for males: the decay x years after the starting of the contract is faster for older individuals (it was the opposite for males). For example, the average outflow of 50% of the initial one is reached after 40 years for a contract stipulated at 70 years, and after 60 years for a contract stipulated at 55 years. The difference in the rank of the curves between the males and females (note that the rank is completely inverted) is due to the flatter mortality for females in the years 55-70, while males experience a significant spike in mortality in these years.





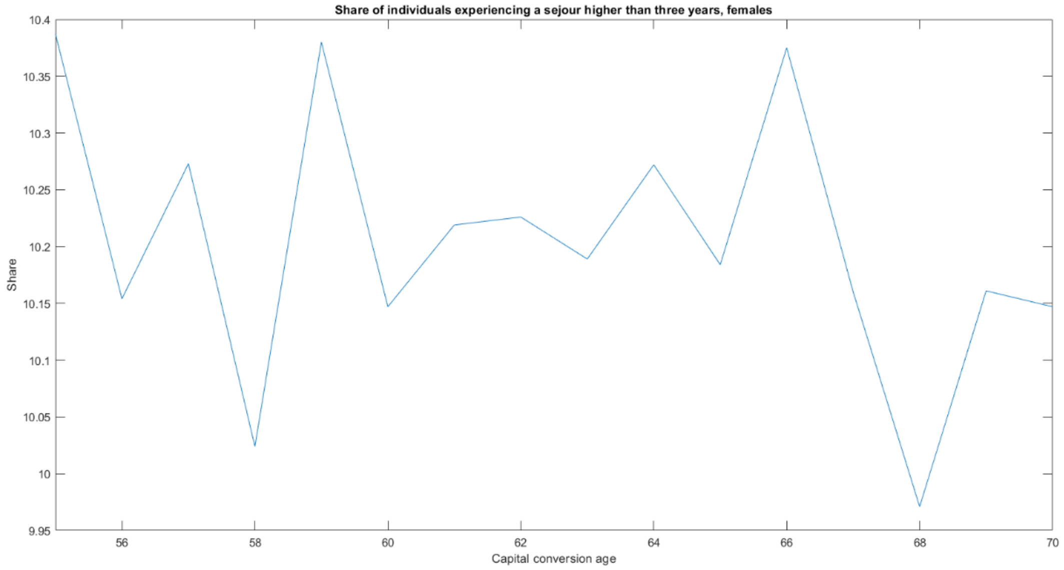
Considering the histogram of the distribution of the 100.000 replications of the Montecarlo experiment we notice a substantially one-peak distribution to the levels 47-49 and the high skew of the distribution with a substantial left tail (skewness -1.015). By contrast, we observe that for older ages, at 70, the peak of the distribution is in the area 38-40, with a still very significant left tail, while there are substantially no values after 50. We observe that, differently from males, the bimodality seems to be less pronounced at all ages. We cannot report the histogram for the whole spectrum of the ages, but we must consider that the skewness of the distribution slightly decreases as the age increases: we go from -4.5 to -3.9 from 55 to 70 years, the pattern is like one of the males, but the levels are significantly more negative. Differently from what happens for older males, older females still present a negatively skewed distribution. As for males, the share of individuals hitting the LTC is substantially stable when distinguished by the age of conversion (about 25% of the individuals), ranging from 26.2% to 25.8% with a flat performance. However, for males the share of individuals hitting the LTC barrier was 10% higher, this explains a large part of the dyscrasia between the results.

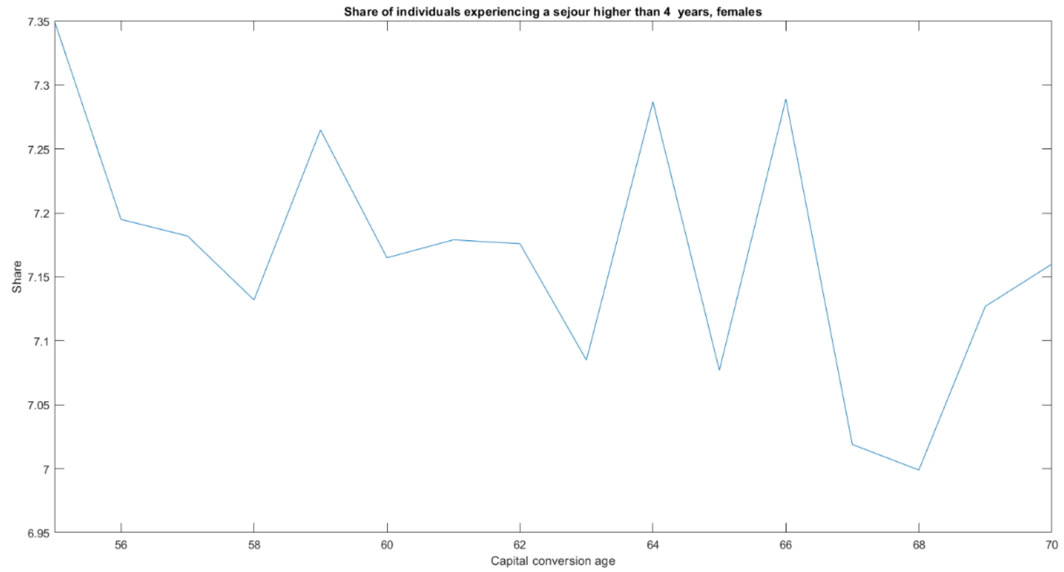


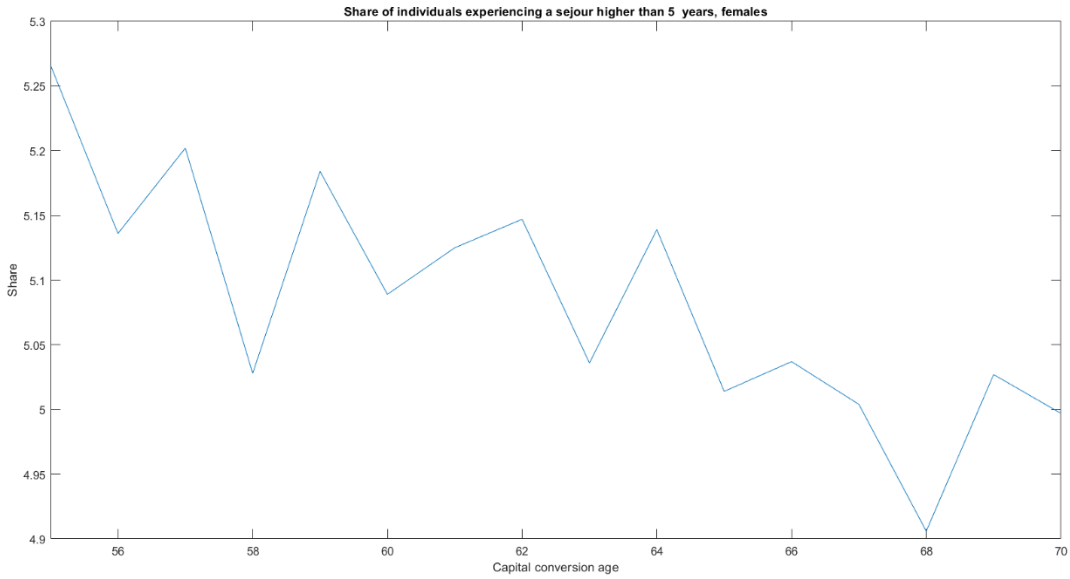
The average sejour, if it happens is 3.7 years, with a median of 3 years of sejour: on the median, there are no significant changes concerning the age of the starting of the contract S= [55,56…70], while for the mean we observe a decreasing trend from 3.8 years to 3.6 years that was unobserved for men. The independence of the distribution of the sejour and the entry into the contract, that was partially observed for males is even less applicable for females, where even the first moment is conditioned to the abovementioned variable. We still have that, as, for males, the distribution of sejour times is exponentially decreasing, so we have, in general for all the ages of conversion that P (z=0; no barrier hitting)>P(z=1)>P(z=2)>P(z=3) …. with $z \in N$.

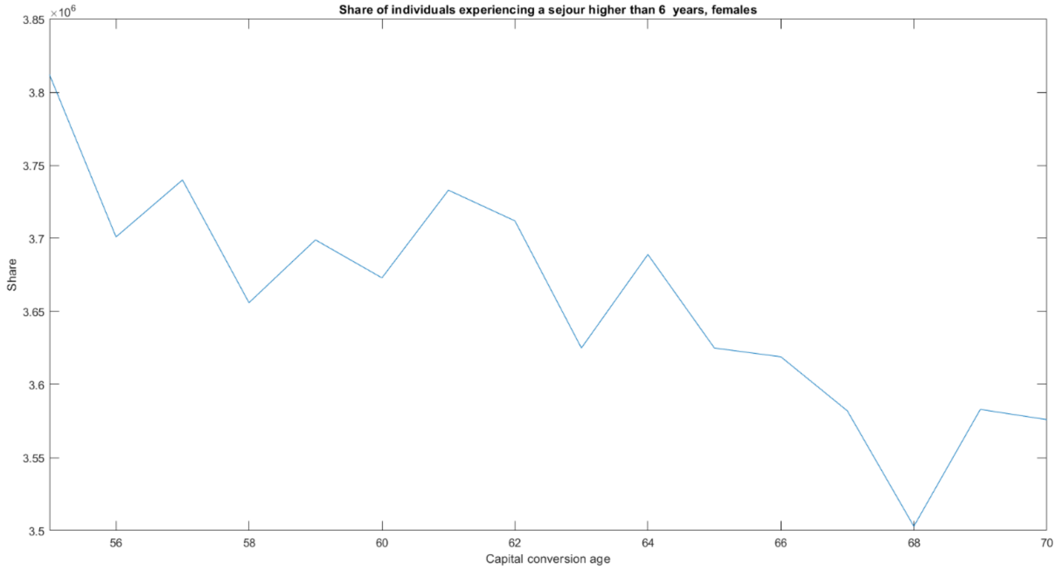
As for males, we do consider how does change the standard deviation of the above-expressed distribution (that is specifically linked to the age of conversion) depending on the age at capital conversion: we have that the standard error sharply decreases from 4.1 to 3.3, showing a trend really like one of the males. By contrast, the standard error for females is substantially double that of males (4.1 vs 2.1 and 3.3 vs 1.9). This means that as the individual becomes older if he enters the sejour state, the magnitude of this interval becomes far and far more forecastable, but the forecasting will be harder for females. The positive skewness is significantly decreasing through the ages, from 3.5 to 2.7, while for males it goes from 2.7 to 2.1. Exactly as for the skewness of the scenario (but in this case the skewness is positive instead of negative), the trend for both sexes are equal, but the skewness of females remains more pronounced. This is due to the lower impact of morbidity on life expectancy for females if compared to males.

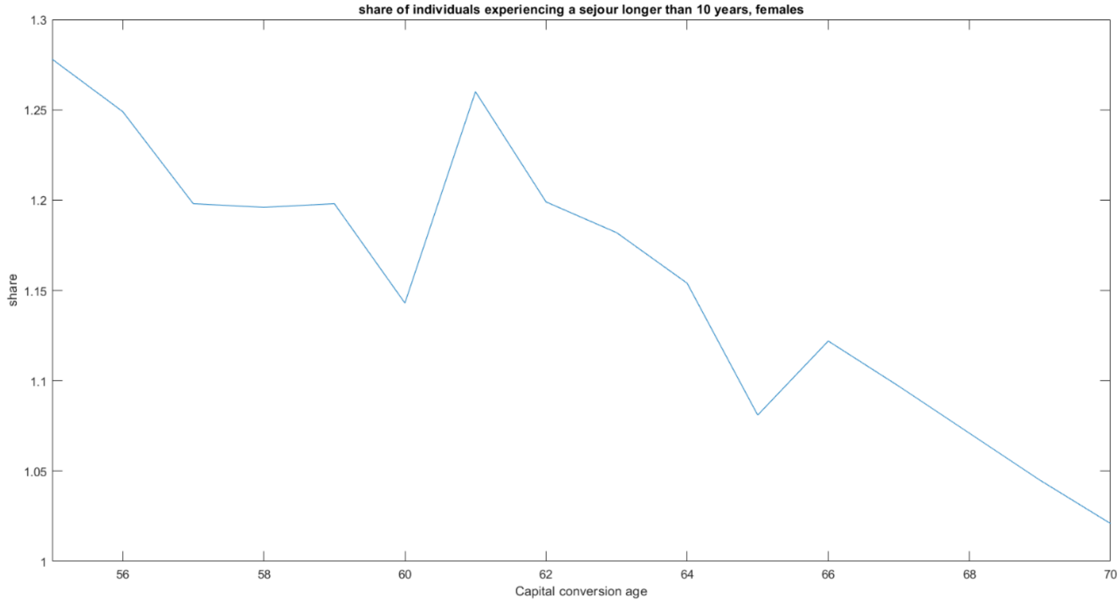
The share of individuals experiencing a sejour greater or equal of three years fluctuates around 10% without any form of trend, as for males. Concerning the sejour longer than 4 years, we observe a pattern like the one described for the sejour longer than three years, apart from a lower general level (7.15%, with a flat trend). We remember the reader that transition to I status could be the consequence of different pathologies (cancer, cardiovascular) or elderly dementia or even of traumas due to accidents. This means that what we call uniquely, loss of autonomy could follow very different patterns between ages and genders, that explains different behaviours on sejour. Up to this point, when enough data will be available, we hope to be able to carry out a principal component analysis investigation. The share of females experiencing a sejour greater or equal than four years is higher than the one of males by 1.5%. Starting from sejour longer or equal to five years, we observe a decreasing pattern for the probability of incurring in such a longer sejour. This trend continues even for a longer timespan (10 years,15 years) and is a remarkable element of distinction between the two sexes. In fact, for females, we do not have an increasing trend for shorter sejour and already from the interval 5-6 years we have an increasing pattern: the higher the age of entry into the contract, the higher the probability of incurring in a sejour longer than k years, with $k>4 \in N$

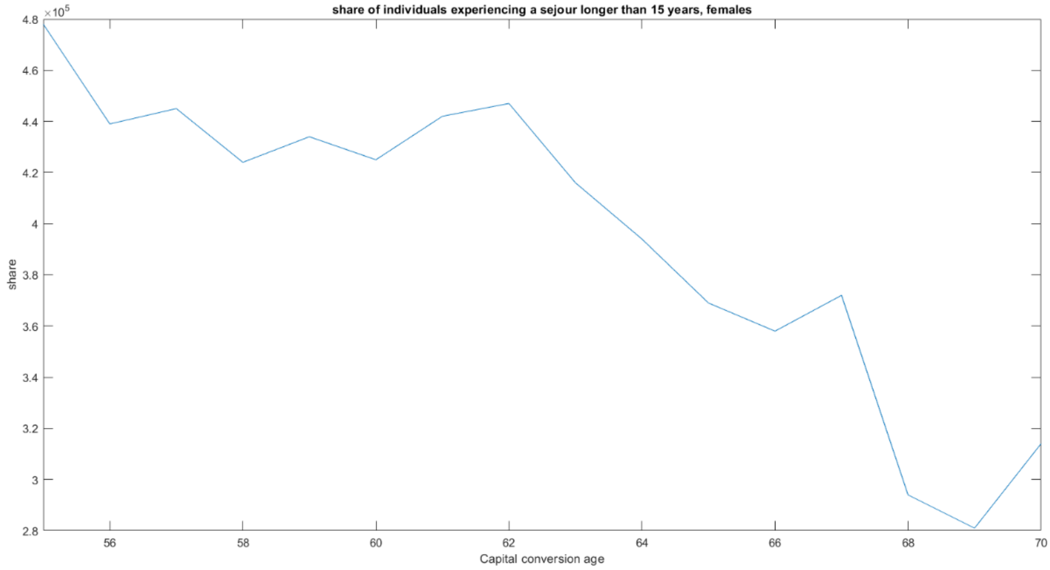

Share of individuals experiencing a sejour higher than three years, females
Share
Capital conversion age


Share of individuals experiencing a sejour higher than 4 years, females
Share
Capital conversion age

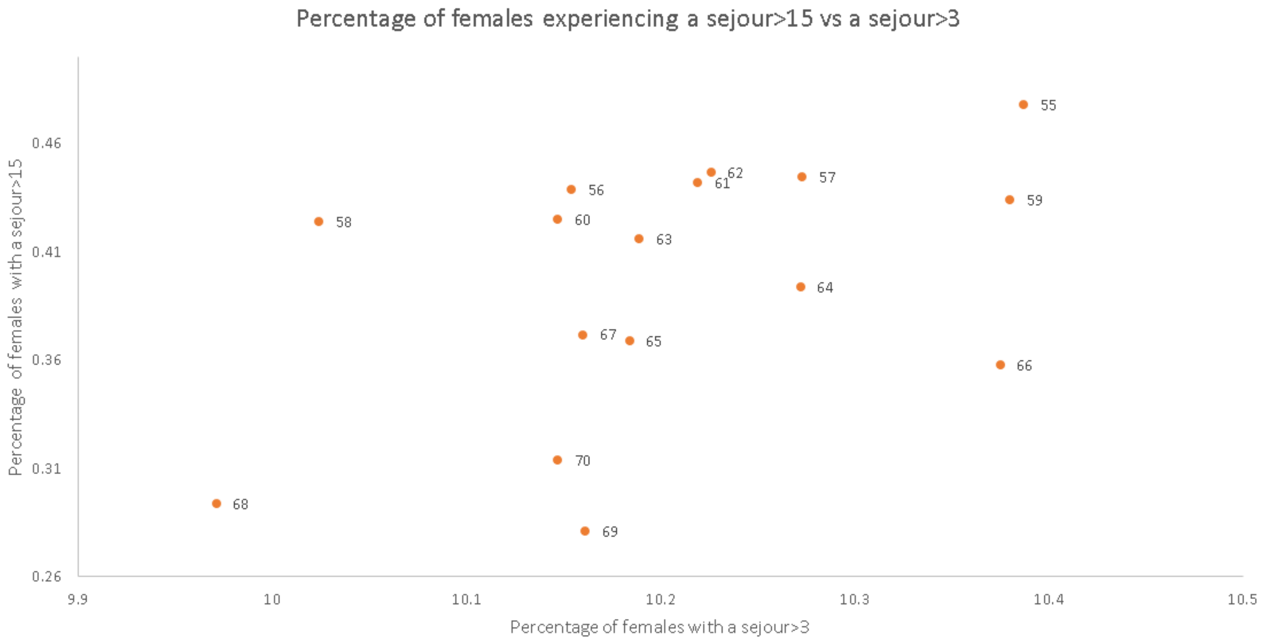

Share of individuals experiencing a sejour higher than 5 years, females
Share
Capital conversion age



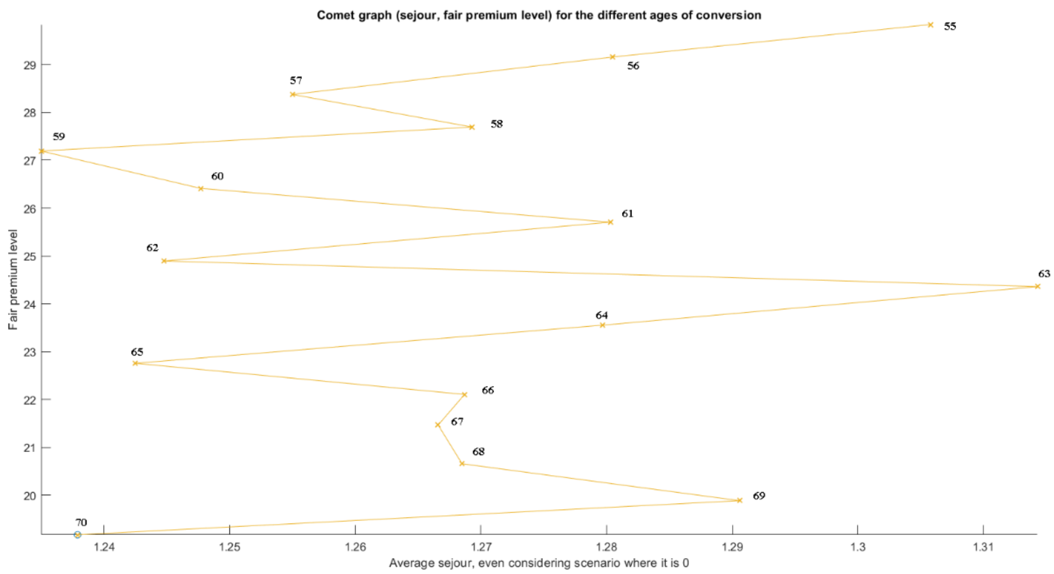




To summarize, we observe as for males the scatterplot of the share of sejour greater or equal than three years versus the share of sejour greater or equal than 15 years: while for males we observed an anticorrelation between the two variables, here we have a positive relationship, confirming that there is no trend reversal is the probability of finding a sejour longer than k years in function of the age of entry into the contract.



Again, as for males, we consider the comet graph. The results are similar: increasing the age of conversion decreases the fair premium (and the relation is strictly decreasing for every year of increase the premium is lower) and decreases the average sejour. Concerning the sejour, however, the decrease is not at all monotonous, but only on average. Consequently, on this point what has already been stated for males holds.

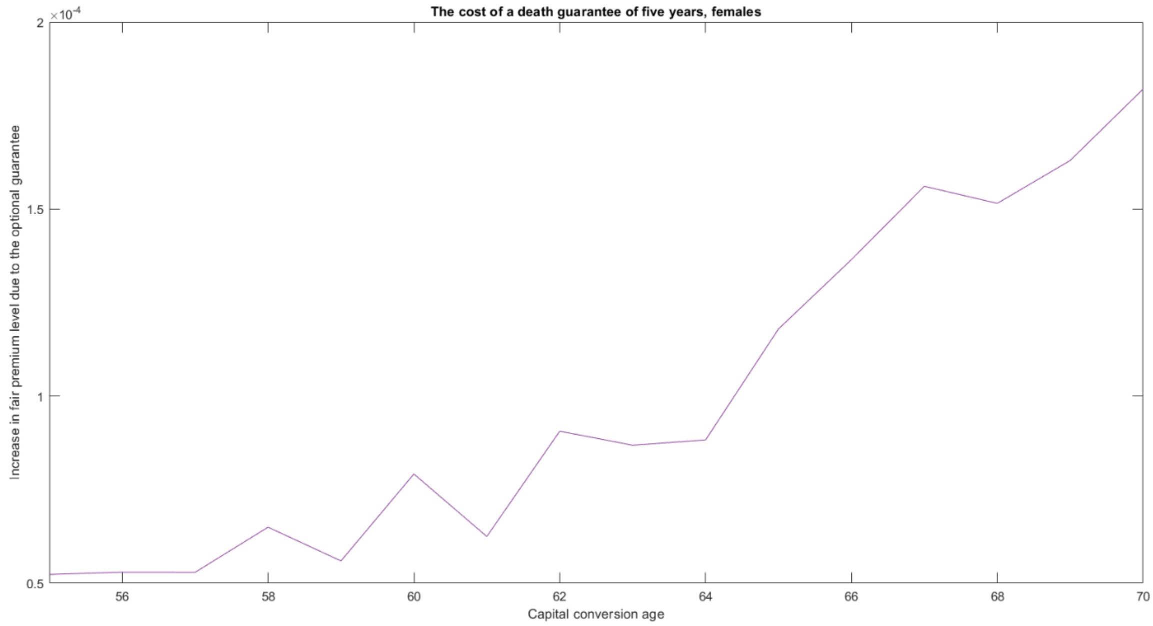


We tried to approximate the distribution of sejour duration with a geometric distribution, exactly as for males. Still, we have a constant parameter (more or less) for the whole vector of different entry ages, equal to $\hat{p} \approx 0.21$

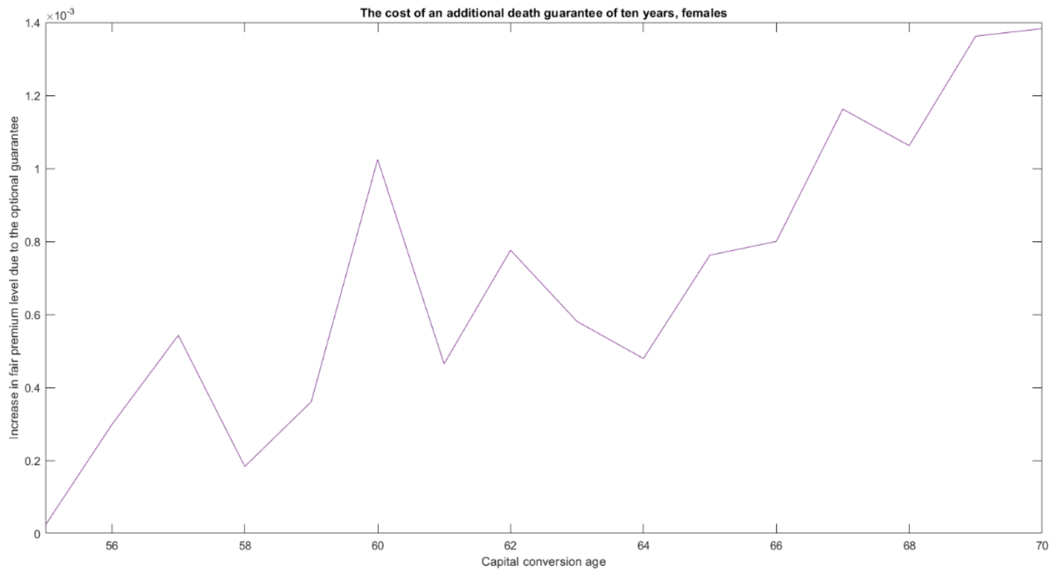
Geometric fitting- different ages and sex	p
Males-55	0.4245
Males-70	0.4216
Females-55	0.2089
Females-70	0.2153

4.5) Additional guarantees for females

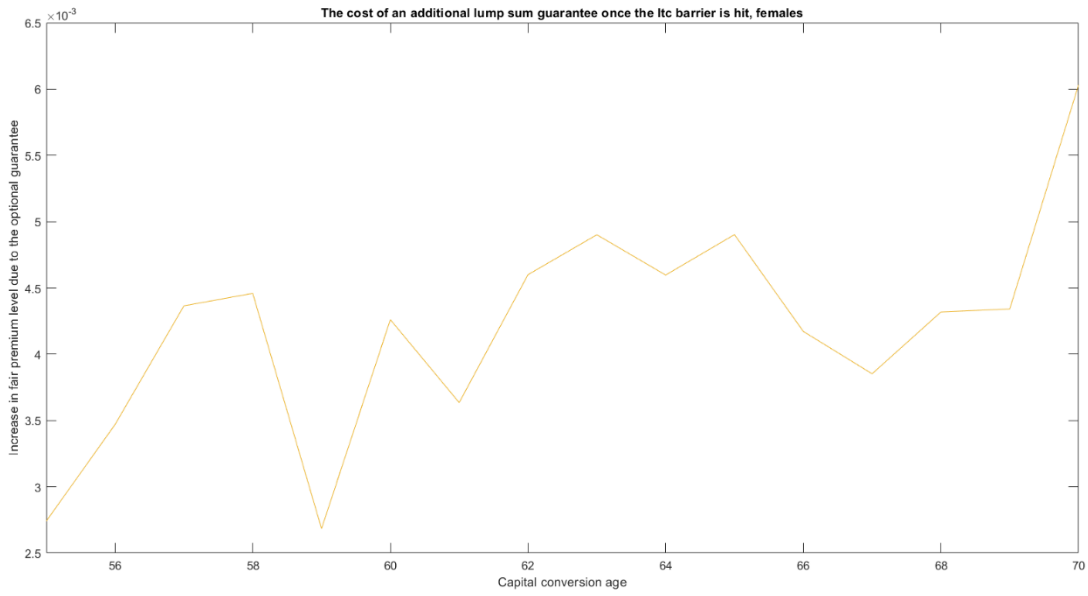
Even for females, we consider the already stated additional guarantees, that are in all three cases less expensive than the one corresponding for males. The cost of the death guarantee of five years is less than 0.01% of the fair premium level for females younger than 62 years and it increases up to 0.02% for older ages. The costs of a death guarantee of ten years are still negligible up to 62 years when it becomes about 1% and ends up being 0.15% for individuals of older ages (70). We observe that the cost in percentage point of the additional guarantee is significantly lower for females, and this is due to the higher level of the fair premium, which decreases the impact in percentage terms of the monetary outflow. Furthermore, it must be considered that the benefit of the diversification in adding a death guarantee to females is higher than for males: as there is a lower correlation between LTC condition and extra mortality in females the natural hedging effect between LTC and vanilla annuity (life case) is less beneficial: as a result, there is a greater margin of for this additional guarantee to decrease the overall variability of the contract. This also holds for males, the greater anticorrelation between LTC and life case allows for a more effective natural hedging in the ordinary enhanced pension: as a result, the marginal reduction of risk that is attributable to a death guarantee (especially a shorter one) is moderate. The lower the benefit, the higher the costs. This is what we observed and is confirmed even by the graphs on the "gender gap". Concerning the last additional guarantee, i.e., the lump sum payment once the LTC barrier is hit, we observe again a lower cost of the coverage in percentage points, for the following reasons: first, a greater fair premium level that decreases the impact in relative terms, second a lower probability of transition and third, the average age of transition is older for females. This explains why these additional guaranteed costs 1% to males at 55 years but only 0.25% to females of the same cohort, and for individuals of 70 years, the cost is 2% for males and 0.65% for females. The graphs for the difference of the cost for males and females exactly reproduce what is already found in the above-expressed result. The interesting is that for all the additional coverages, the difference in cost (we can say, the "spread") between the cost for males and the cost for females (with the first one being whenever higher) increases in the age of conversion: this is mainly due to the different structure of transition probability and to the absence of extra mortality for older females, that leads to a decrease in the benefit of the additional guarantee and so in a spike of the costs.



The cost of a death guarantee of five years, females
Increase in fair premium level due to the optional guarantee
Capital conversion age



The cost of an additional death guarantee of ten years, females
Increase in fair premium level due to the optional guarantee
Capital conversion age

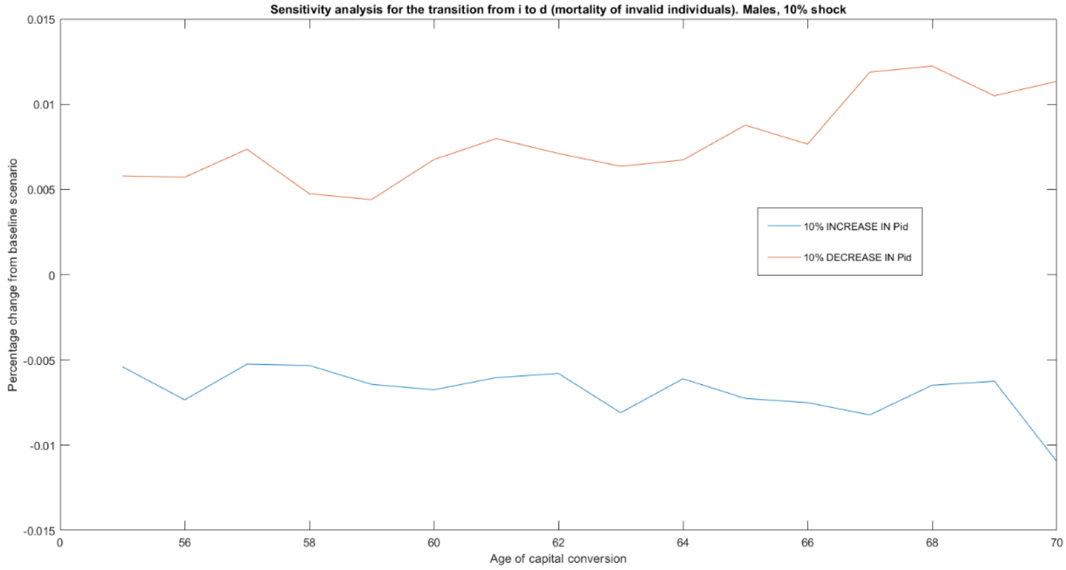


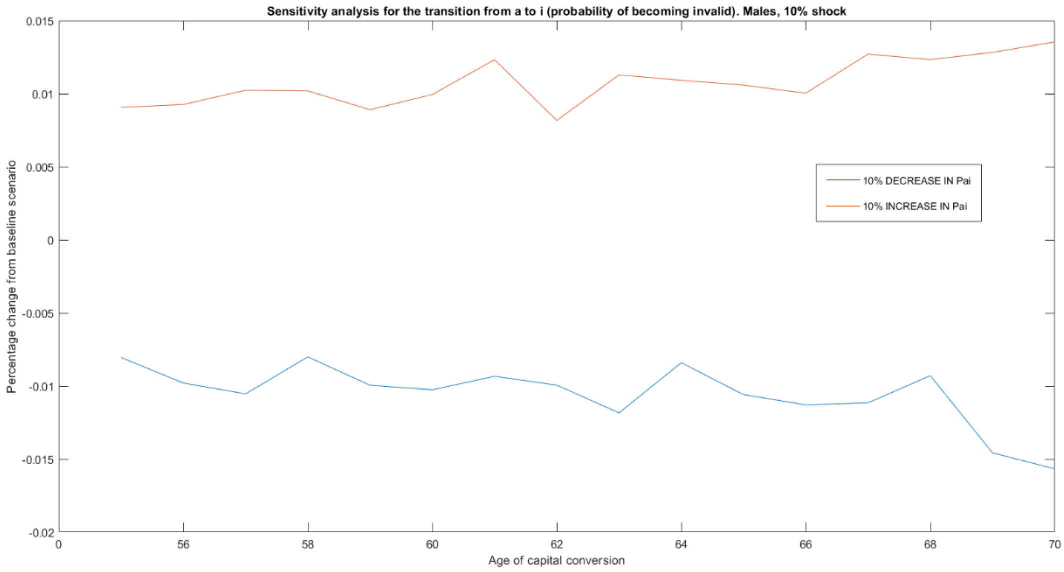
The cost of an additional lump sum guarantee once the ltc barrier is hit, females
Increase in fair premium level due to the optional guarantee
Capital conversion age

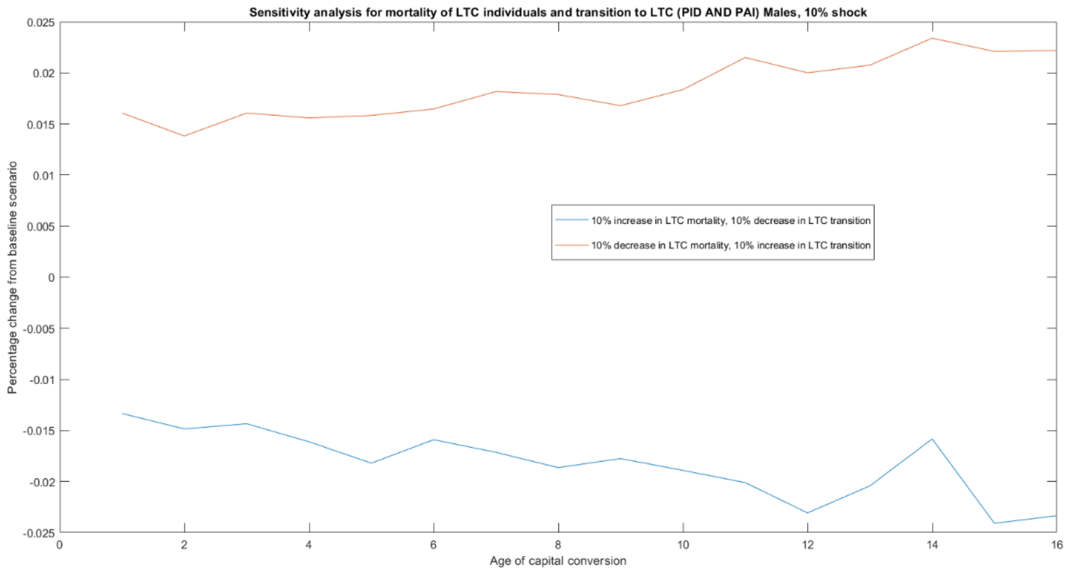
4.6) Sensitivity analysis and duration

Concerning the sensitivity analysis, we must, as usual, split the risk into two components: the market risk, which could be efficiently diversified and hedged with ordinary financial instruments, and the more difficult to hedge idiosyncratic one. Obviously, in the diversifiable risk, we will include financial one (i.e., the risk on the return of the premia re-invested in the market) and the longevity risk of the healthy population, as there are efficient derivatives market (OTC, mainly) consisting of mortality swaps and survivors' bonds. A life insurance contract is fully able to manage the longevity risk of the whole population and, as the mortality of the healthy insured is a good approximation of the mortality of the whole population, even the shifts in the transition probability p^{AD} (the mortality of health insureds). On these two mentioned variables *nihil novi*: we will focus only on the biometric variables linked to LTC: mortality of invalid insured, transition probability from healthy to LTC (so, p^{AI} and p^{AD}) and, marginally, the extramortality in the first years of sejour. Results are in line with other studies Pitacco, 2020 chapter VIII.

We start by males: we produce a shift of 10% in transition probability p^{AI} (the probability of losing the autonomy): usurpingly, the coefficient of conversion (the fair premium level) increases as the probability of becoming invalid increases, while it decreases as the mortality of unhealthy people increases. We furthermore try a conjoint shock of ±10% in mortality of LTC (only in the Markov component) and ∓ 10% in transition probabilities p^{AI} (the probability of losing the autonomy) to depict a confidence interval for our fair premium level. When we have an increase of 10% of p^{AI} the fair premium level increases by about 1%: 0.8% at 55 years to 1.6% to 70 years, with an increasing trend of the impact. When with the same probability p^{AI} the direction of the impact is decreasing, the fair premium level decreases, from 0.9% at 55 years to 1.35% at 70 years, still with an increasing absolute value pattern. An increase of 10% of the mortality of LTC individuals (only in the Markov component) leads to a decrease on the fair premium level: from -0.5% at 55 years to -1.1% at 70 years of capital conversion. A decrease of 10% in the same variable is responsible for a similar reaction but in the opposite direction: from an increase of 0.6 at 55 years to an increase of 1.1% at 70 years. We observe that for all the shocks the following relationship holds: the older the age of conversion, the higher impact of the shock. Considering in conclusion the joint shock, to produce a larger and wider confidence interval, we observe that at 55 years the interval is about ±1.5%: as forecastable, the amplitude of this interval increases up to ±2.5% at 70 years.





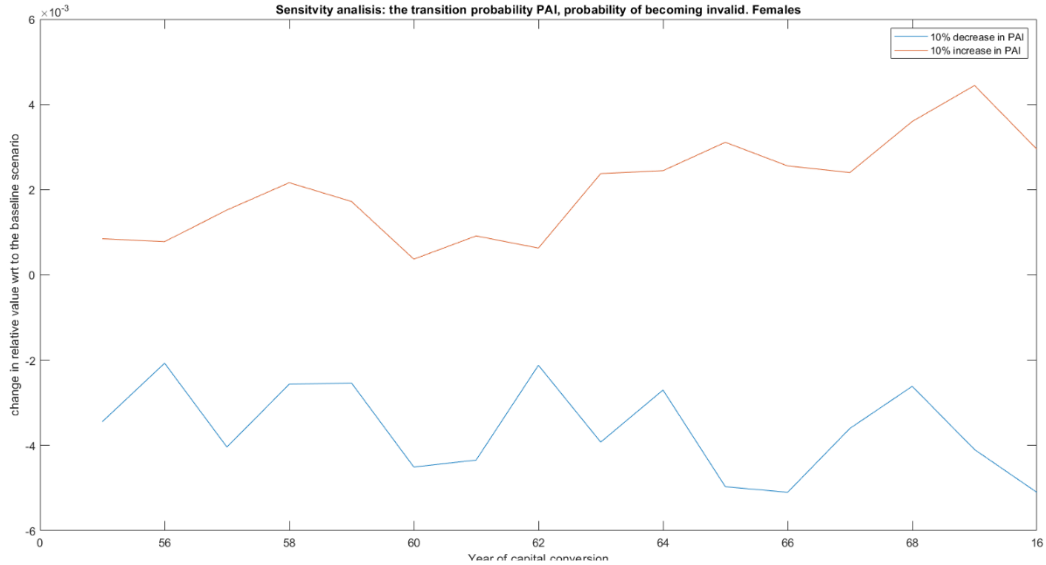


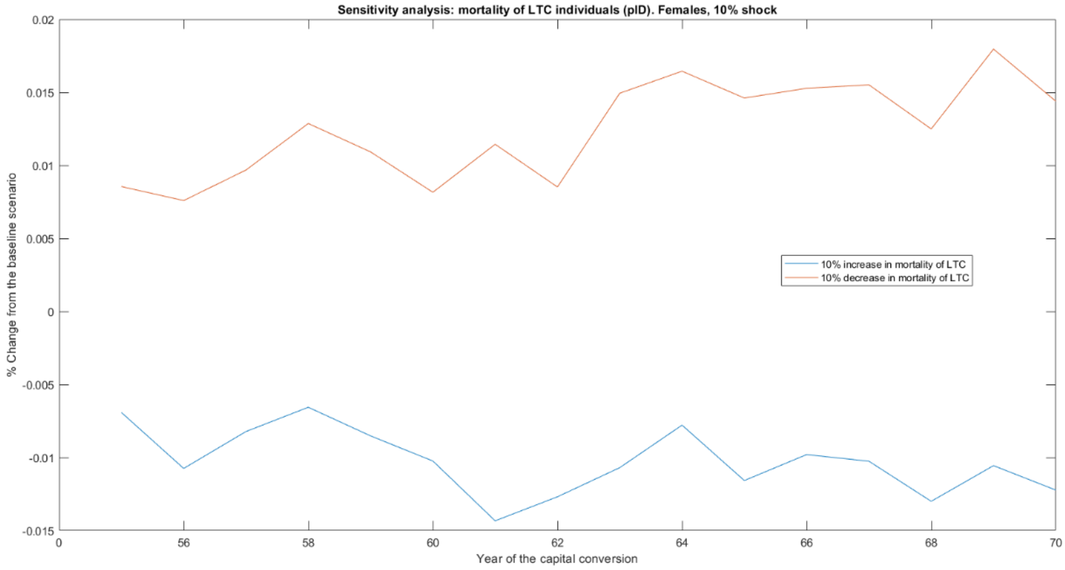
Results for females are substantially similar in the direction, but the intensity of the change is generally less relevant. First, we examine a ±10% shift in the mortality of LTC individuals (p^{ID}). As forecastable, the increase in the mortality of LTC individuals produces a decrease in the fair premium level: the decrease is 0.5% at 55 years and becomes 1.5% at 70 years, depending on the age of capital conversion, with results substantially in line with males. For the 10% decrease in mortality, we still observe results substantially in line with males, from +0.6% for the lower ages of conversion up to +1.5%. The direction is positive in this case: the lower the extra mortality of LTC individuals, the higher the fair premium level. Now we consider the other transition probability of our interest, i.e., p^{AI}, still with a ±10% shift. As for males, an increase of this probability increases the fair premium level and a decrease of this probability decreases the fair premium level, however, the effect for females seems less correlated with the age of capital conversion, while the age of capital conversion was severely affecting the intensity of the relative change for males. We can still observe, in conclusion, even for this probability a trend of the following form: "the higher the age of capital conversion, the more acute the impact of the shock", but the correlation is for sure less robust for females than it was for males. Even more, also the relative change is reduced for females: it amounts at maximum at 0.6% in both positive and negative directions, while it was 1.5% for males: the impact of the transition probability p^{AI} decreased by 60% from males to females. In conclusion, we consider a conjoint shock of ±10% in mortality of LTC (only in the Markov component) and ∓ 10% in transition probabilities from A to I (p^{AI}) (the probability of losing the autonomy) do depict a confidence interval for our fair premium level. Results are in line with what emerged for males, apart from the lower general discharge: upper bound for the worst scenario is 1% in case of conversion at 55 years and 1.7% for conversion at 70 years. By contrast, the lower bound for the better scenario is -1.6% if the conversion happens at 55 years and -2.5% if the conversion happens at 70 years: again, the increase in the anagraphic age of conversion generally increase the impact of the stressed variable.

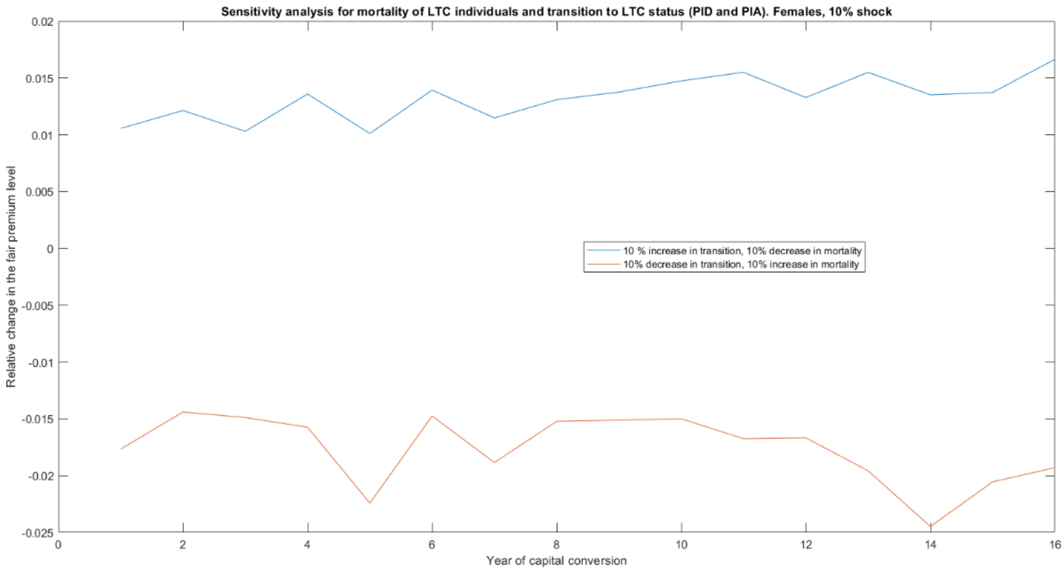
By considering, in conclusion of our work, the duration of the vanilla product (the enhanced pension), under the baseline scenario assumptions, we would like to make some considerations. The duration computed is the traditional one (not Macaulay or other modified version), defined as the centre of mass of the payments and we observe the following table:

Duration, centre of mass formula	yrs
Males55	17,64
Males70	11,83
Females55	27,91
Females70	22,00

We notice that the duration of males is lower than the one of females, with the difference between the duration of two sexes is ten years. This is due to the longer life expectancy of females and the higher morbidity of males. The decreasing pattern at the increase of entry ages is relevant for both males and females, with an average decrease of five years. All these confirm the interest for insurance companies in deferring the entry into the contract, while for sure, males are less risky than females in terms of interest rate shifts, in particular decrease (this is the risk, in life insurance).


Sensitvity analisis: the transition probability PAI, probability of becoming invalid. Females
10% decrease in PAI
10% increase in PAI
change in relative value wrt to the baseline scenario
Year of capital conversion


Sensitivity analysis: mortality of LTC individuals (pID). Females, 10% shock
10% increase in mortality of LTC
10% decrease in mortality of LTC
% Change from the baseline scenario
Year of the capital conversion


Sensitivity analysis for mortality of LTC individuals and transition to LTC status (PID and PIA). Females, 10% shock
10 % increase in transition, 10% decrease in mortality
10% decrease in transition, 10% increase in mortality
Relative change in the fair premium level
Year of capital conversion

5) Conclusion

After explaining which are the main features of LTC contracts, the pros and cons connected to this emerging industry, we carefully considered which are the marginal benefits that a contract such as an enhanced pension could bring. In conclusion, we have observed the benefits relevant to the operation hedging, even if differently spread between males and females according to the respective biometric structure. We introduced the semi-Markov models and studied the way Levantesi and Menzietti tried to estimate transition probabilities, even reporting the limits of this study. In the third and last part, we priced an Enhanced pension based on the previously formulated assumptions and our proposed sejour correction. The relevant difference observed for females (but not for males) testifies the actual viscosity of this annuity market, linked to important worries from the insurer on the future duration of LTC status more than on the transition probability from A to I. Others said, insurers selling Long Term care are worried about a decrease in extra mortality of LTC individuals is this that, brutally said, justifies the increase in the coupon. We even considered how this contract could be improved with an additional guarantee, that sometimes contributes to a higher risk reduction and sometimes not, but are, from a microeconomic point of view, of absolute need. We now came to the main quantitative result of our conclusion: females should pay 30% more and men 5% less if prices were to be computed by our fair tables. This has several explanations: first, the need of keeping a gender equilibrium in the portfolio (since females and males are two different biometric assets, there are benefits from insuring both, exactly as their benefits from insuring individuals of different ages), secondly, the need to keep under strict control the level of long-term care risk. We want to consider that we are speaking abound “portfolio weights” and, since males generally insure (at least today, and at least in Italy) for higher amounts, the importance of attracting a higher number of women becomes crystal clear, in terms of portfolio risk-reward optimization. Coming, in conclusion, to a short analysis of the load policy we observe load charge, increasing, in relative terms, concerning the level of risk underwritten that is why the difference between market and fair premium decreases in the age of conversion for males and, in the same way, females receive a discount increasing in the age of conversion (the higher the age of conversion, the higher the discount). Disclaimers referring to the differences between modelling assumptions, the definition of long term care, and differences in the mortality and morbidity assumptions still hold, being IBM workers a subset specific to the Italian population.

We conclude by observing, even based on our result, stressing the importance for a stronger harmonization of life insurance products in the whole economic area (especially non-vanilla one): this will increase the dimension of the risk pool, and this will allow exploitation of better economies of scale leading to less opaque and more efficient markets with benefits for investors and savers (workers if we speak about pension plans). Our research on long term care, however, is fair for being completed: we will in the future investigate a different way of “accounting” for sejour times, possibly non-linear and multilevel modelling of long-term care status: even if the payment received by individuals in the state I is the same, this does not mean that there are no differences in mortalities and/or in other transition probabilities. This multilevel modelling could be a partitioned one, where the individual can enter one-and-only one- long term care status during his life and an augmented one, where it is possible to move from lower mortality status to higher mortality status, still accounting (possibly) for the sejour. The innovation of sejour, however, should even be extended to vanilla annuities: the author thinks in particular, to annuities that start to pay after the death of a strict companion (wife/husband, generally for married couples): here not only the mortality of the insured should rely on his anagraphic age, but even from the year passed from the shock of losing such strict support: we rely on finding an accumulation of mortality on the years immediately following the (first) death. Furthermore, our code and our results, in general, are suitable also to price a vanilla long term care contract (that pays only if the insured is not autonomous): differences in load policy between our contract and the riskier one should be adequately investigated (in the future) since is in strict correlation with hedging policy. To conclude, semi-Markov corrections have wider applications: since in these weeks a relevant reform of criminal code is being under discussion and approbation in Italy (Rifroma Cartabia), we must observe that the probability of re-committing criminal conduct strongly depends on the time spent in prison. In this cyclic directed graph, the probability of re-accessing state *Jail* from state *Freedom* depends on the previous *sejour* in *Jail* state, and even, unfortunately, on the number (count process) of the previous barrier hitting. Note that this last variable is missing in our model since our graph is acyclic.

6) Bibliography

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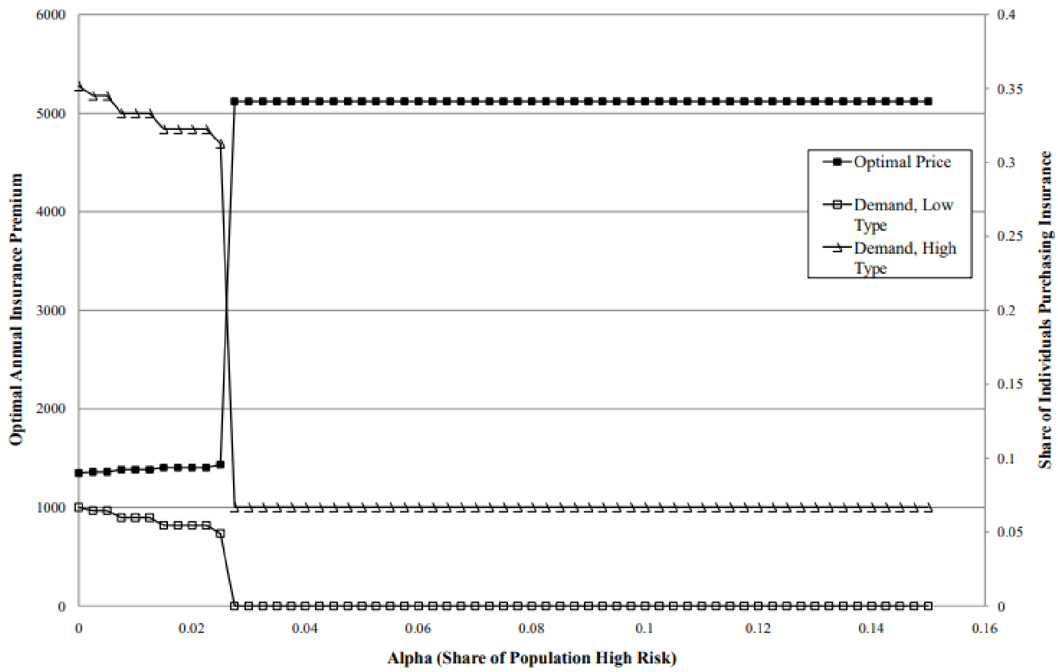
7) Appendix: relevant figures, gallery of the Montecarlo experiment and MATLAB code.

Appendix Figure 2:
Optimal Insurance Pricing and Demand as High Risk Share Varies



Notes: This figure shows variation in optimal insurance pricing and demand by the two types as the high type share of the population varies.

Figure 4- The effect of the prevalence of the higher risk individuals in the social welfare. Consequently, we can consider how this graph could affect the decision of showing the group of belonging (high risk or low one). Source: Oster, Emily & Shoulson, Ira & Quaid, Kimberly & Dorsey, E. (2009). Genetic Adverse Selection: Evidence from Long-Term Care Insurance and Huntington Disease. Journal of Public Economics. 94. 1041-1050.

A. Ability to Use Telephone		**E. Laundry**	
1. Operates telephone on own initiative-looks up and dials numbers, etc.	1	1. Does personal laundry completely	1
2. Dials a few well-known numbers	1	2. Launders small items-rinses stockings, etc.	1
3. Answers telephone but does not dial	1	3. All laundry must be done by others	0
4. Does not use telephone at all	0		
B. Shopping		**F. Mode of Transportation**	
1. Takes care of all shopping needs independently	1	1. Travels independently on public transportation or drives own car	1
2. Shops independently for small purchases	0	2. Arranges own travel via taxi, but does not otherwise use public transportation	1
3. Needs to be accompanied on any shopping trip	0	3. Travels on public transportation when accompanied by another	1
4. Completely unable to shop	0	4. Travel limited to taxi or automobile with assistance of another	0
		5. Does not travel at all	0
C. Food Preparation		**G. Responsibility for Own Medications**	
1. Plans, prepares and serves adequate meals independently	1	1. Is responsible for taking medication in correct dosages at correct time	1
2. Prepares adequate meals if supplied with ingredients	0	2. Takes responsibility if medication is prepared in advance in separate dosage	0
3. Heats, serves and prepares meals, or prepares meals, or prepares meals but does not maintain adequate diet	0	3. Is not capable of dispensing own medication	0
4. Needs to have meals prepared and served	0		
D. Housekeeping		**H. Ability to Handle Finances**	
1. Maintains house alone or with occasional assistance (e.g. "heavy work domestic help")	1	1. Manages financial matters independently (budgets, writes checks, pays rent, bills, goes to bank), collects and keeps track of income	1
2. Performs light daily tasks such as dish washing, bed making	1	2. Manages day-to-day purchases, but needs help with banking, major purchases, etc.	1
3. Performs light daily tasks but cannot maintain acceptable level of cleanliness	1	3. Incapable of handling money	0
4. Needs help with all home maintenance tasks	1		
5. Does not participate in any housekeeping tasks	0		

Figure 5-The Lawton Instrumental Activities of Daily Living Scale (IADL) is an appropriate instrument to assess independent living skills (Lawton & Brody, 1969). However, the final classification still depends on the insurance company, that associates the score to the status of healthy or long term care (even with different levels). Source: A.Bozio, slides of Paris School of Economics spring course on social security, 2019.

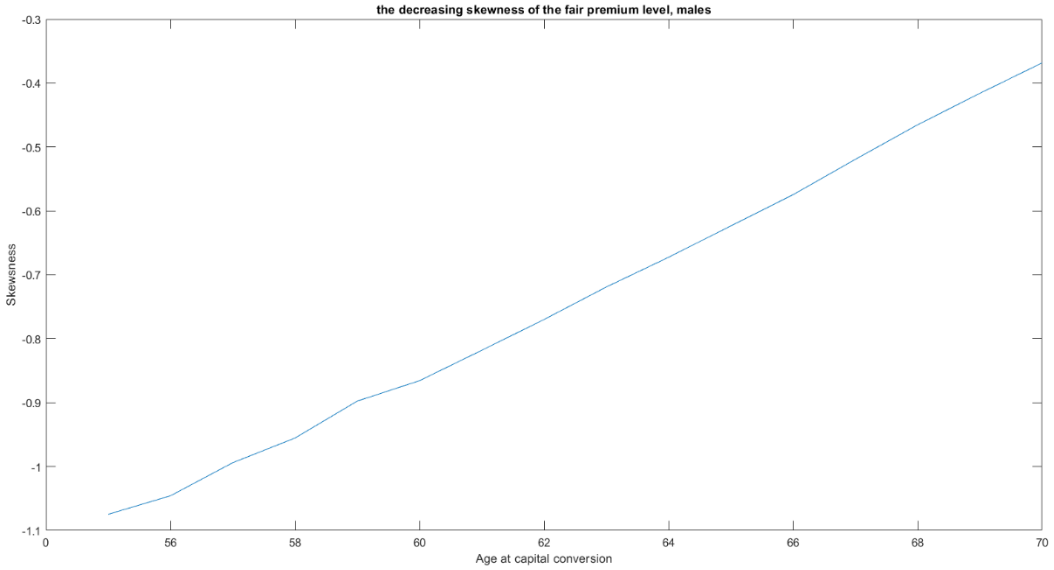
BATHING Points: __________	**(1 POINT)** Bathes self completely or needs help in bathing only a single part of the body such as the back, genital area or disabled extremity.	**(0 POINTS)** Need help with bathing more than one part of the body, getting in or out of the tub or shower. Requires total bathing
DRESSING Points: __________	**(1 POINT)** Get clothes from closets and drawers and puts on clothes and outer garments complete with fasteners. May have help tying shoes.	**(0 POINTS)** Needs help with dressing self or needs to be completely dressed.
TOILETING Points: __________	**(1 POINT)** Goes to toilet, gets on and off, arranges clothes, cleans genital area without help.	**(0 POINTS)** Needs help transferring to the toilet, cleaning self or uses bedpan or commode.
TRANSFERRING Points: __________	**(1 POINT)** Moves in and out of bed or chair unassisted. Mechanical transfer aids are acceptable	**(0 POINTS)** Needs help in moving from bed to chair or requires a complete transfer.
CONTINENCE Points: __________	**(1 POINT)** Exercises complete self control over urination and defecation.	**(0 POINTS)** Is partially or totally incontinent of bowel or bladder
FEEDING Points: __________	**(1 POINT)** Gets food from plate into mouth without help. Preparation of food may be done by another person.	**(0 POINTS)** Needs partial or total help with feeding or requires parenteral feeding.

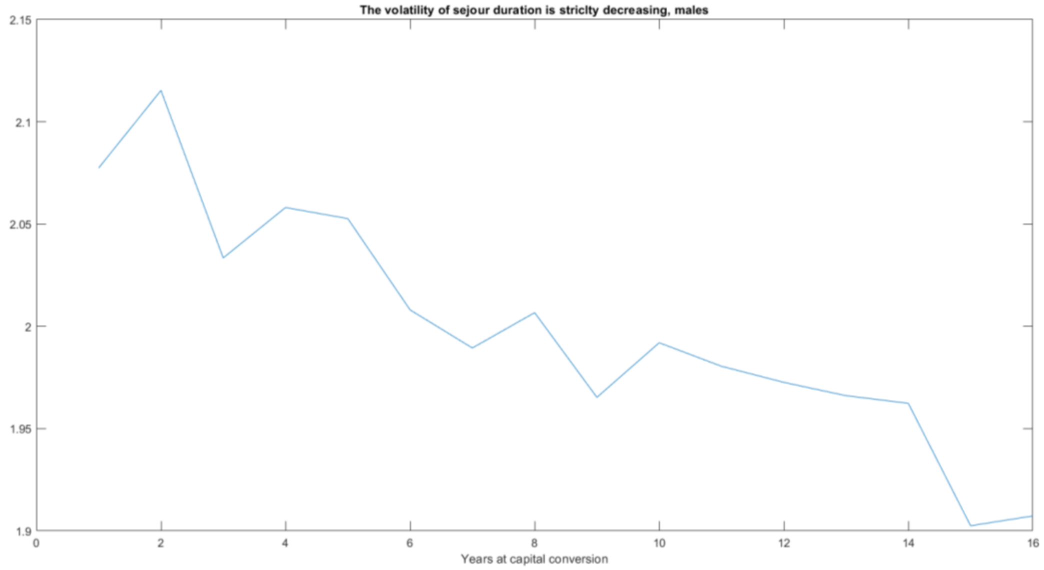
Figure 6-The Katz scale Activities of Daily Living Scale (ADL). See Katz, S., Down, T.D., Cash, H.R., & Grotz, R.C. (1970) Progress in the development of the index of ADL. The Gerontologist, 10(1), 20-30. And Katz, S. (1983). Assessing self-maintenance: Activities of daily living, mobility, and instrumental activities of daily living. JAGS, 31(12), 721-726.). Source: A.Bozio, slides of Paris School of Economics spring course on social security, 2019.

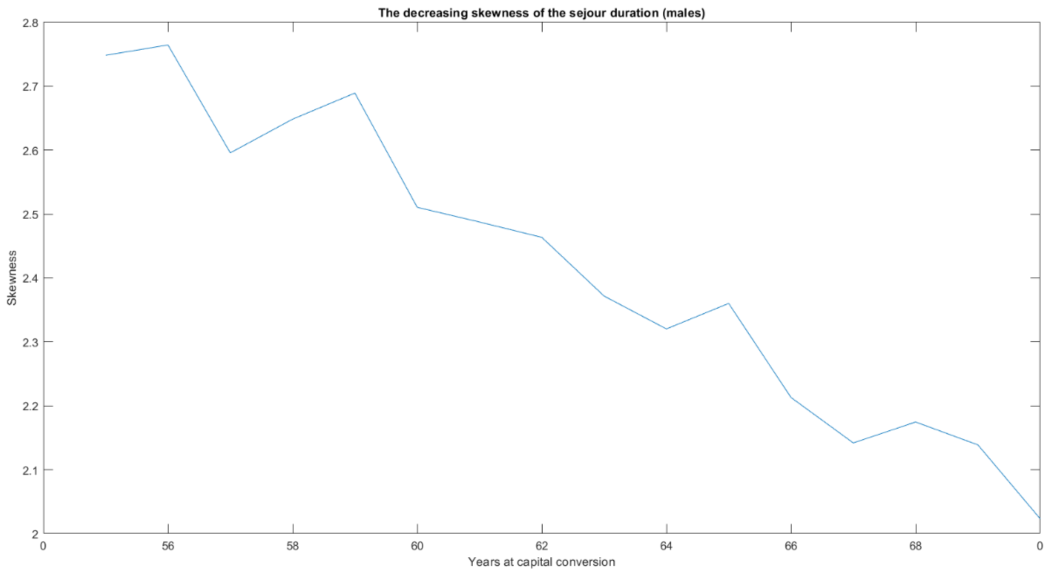
GIR	Description
GIR 1	Person who cannot move out of bed with severely altered mental health Person in end of life
GIR 2	Person who cannot move out of bed but with mental functions not fully impaired Person who can move alone but with severely altered mental impairments
GIR 3	Person with functioning mental health, partially impaired for moving around who needs daily care, multiple times a day
GIR 4	Person who needs help to stand up and bath, but can move around in autonomy
GIR 5	Person who needs help with bath, food preparation and house cleaning
GIR 6	Person autonomous in her daily activities

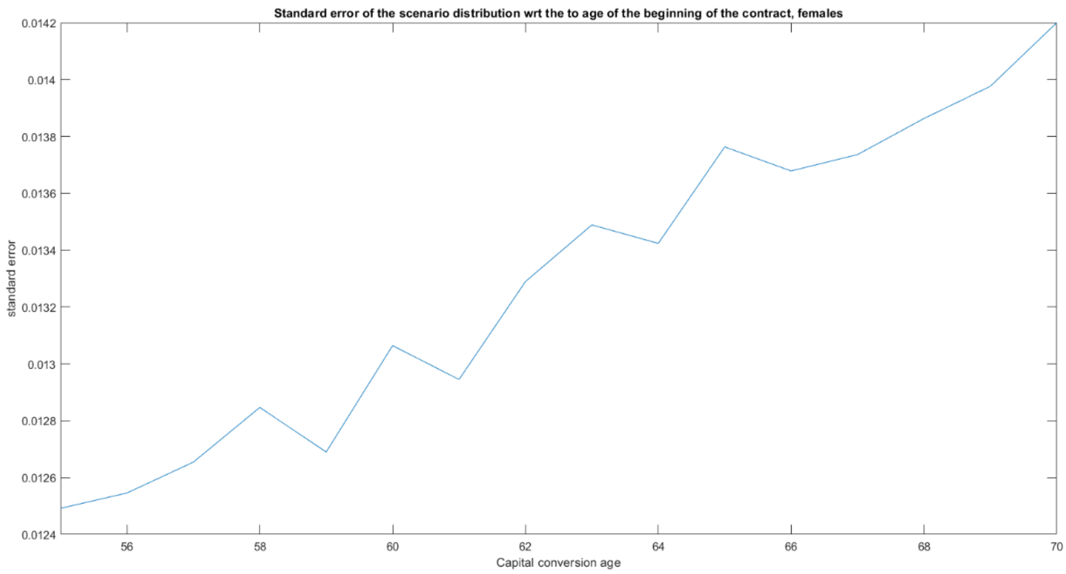
Figure 7- The French AGGIR scale. See Aguilova L, Sauzéon H, Balland É, Consel C, N'Kaoua B. Grille AGGIR et aide à la spécification des besoins des personnes âgées en perte d'autonomie [AGGIR scale: a contribution to specifying the needs of disabled elders]. Rev Neurol (Paris). 2014 Mar;170(3):216-21. French. Source: A. Bozio, slides of Paris School of Economics spring course on social security, 2019. Please also consider M. Roudier, M.J. Al-Aloucy, Analyse en composantes principales de la grille AGGIR chez les patients âgés déments,Revue Neurologique, Volume 160, Issue 5, Part 1,2004,Pages 555-558,ISSN 0035-3787. AGGIR grid is the national standardized instrument determining aimed at the dependency of old people in France living in institutions as well as in the community. The present study aimed to test the reliability of this grid to evaluate the degree of dependency in demented elderly people. Factorial validation of the AGGIR grid was performed by principal components analysis (PCA). This analysis showed a 5-factor solution: factor 1 named the property factor (27 % of the variance), factor 2 named the dynamic factor (21 %), factor 3 named the cognitive factor (20 %), factor 4 named the external mobility factor (11 %) and factor 5 named the communication factor (11 %). The result showed that the AGGIR grid takes physical dependency more into account than psychological and behavioural dependency. This result suggests a need for the readjustment of the AGGIR grid for demented patients by adding new variables considering psychosocial and behavioural disorders.

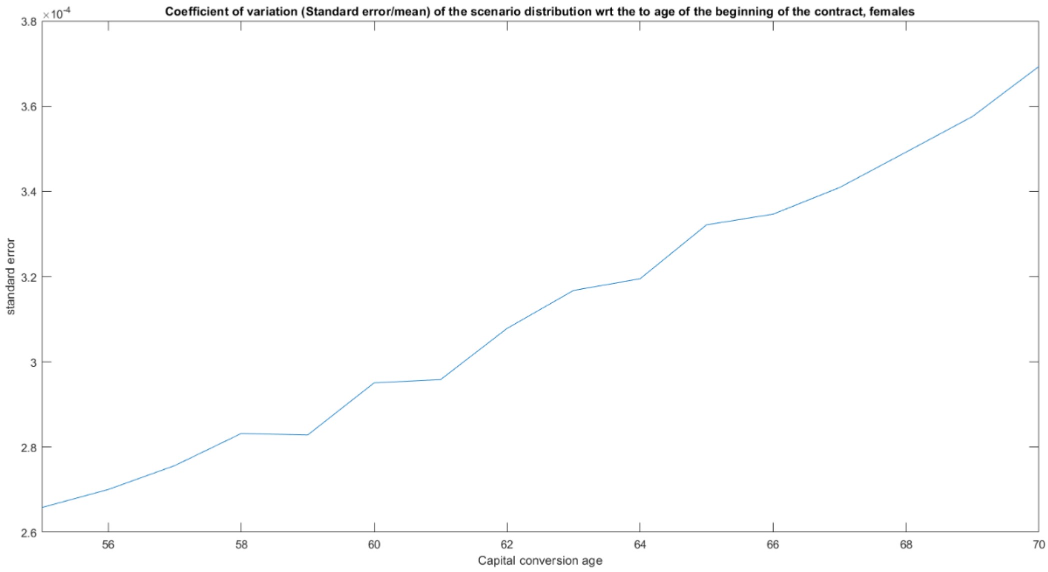
Figure 8- Different outcomes of the Montecarlo simulation. Continues for the next four pages

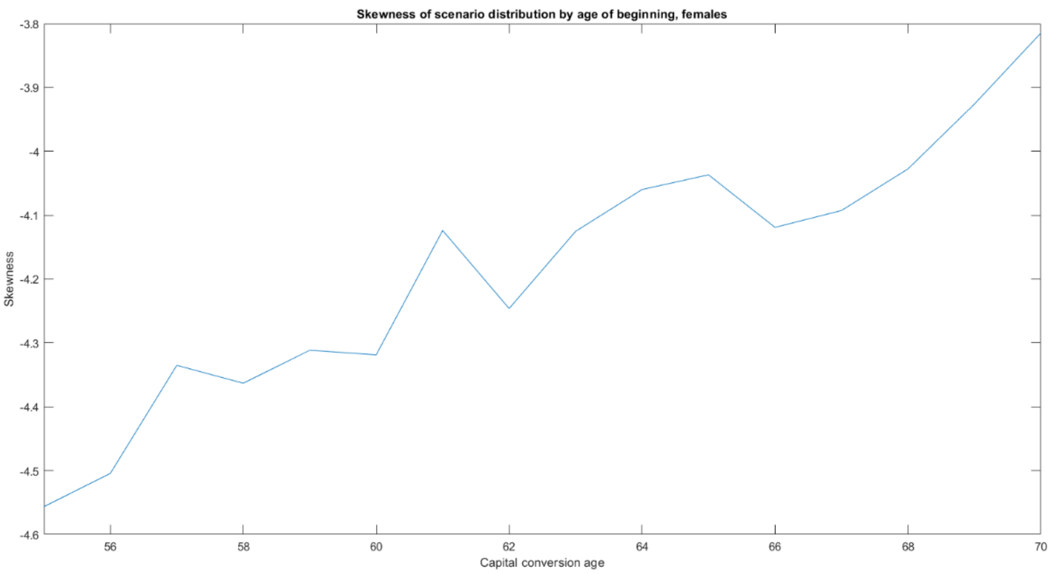


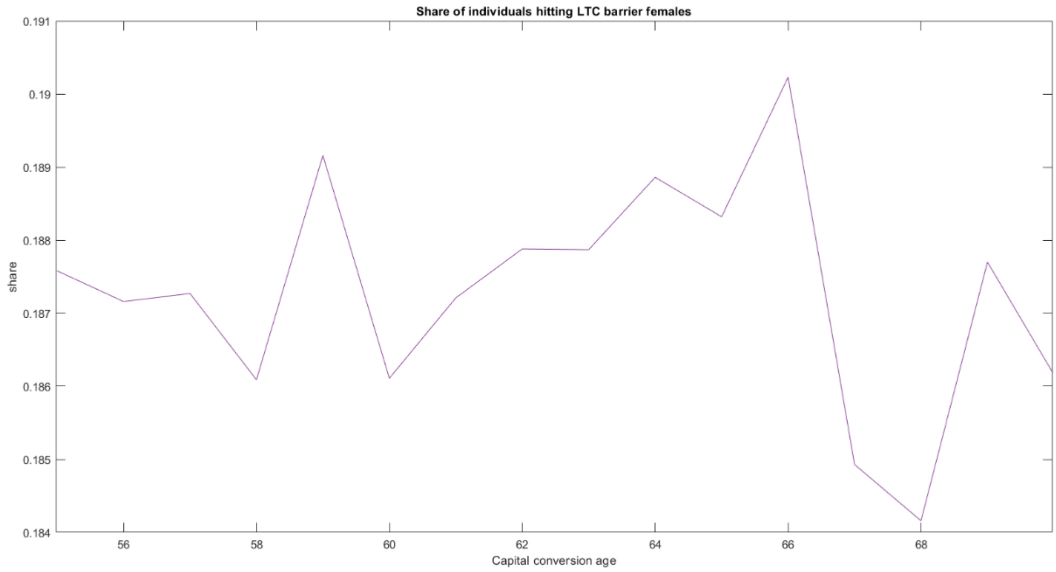


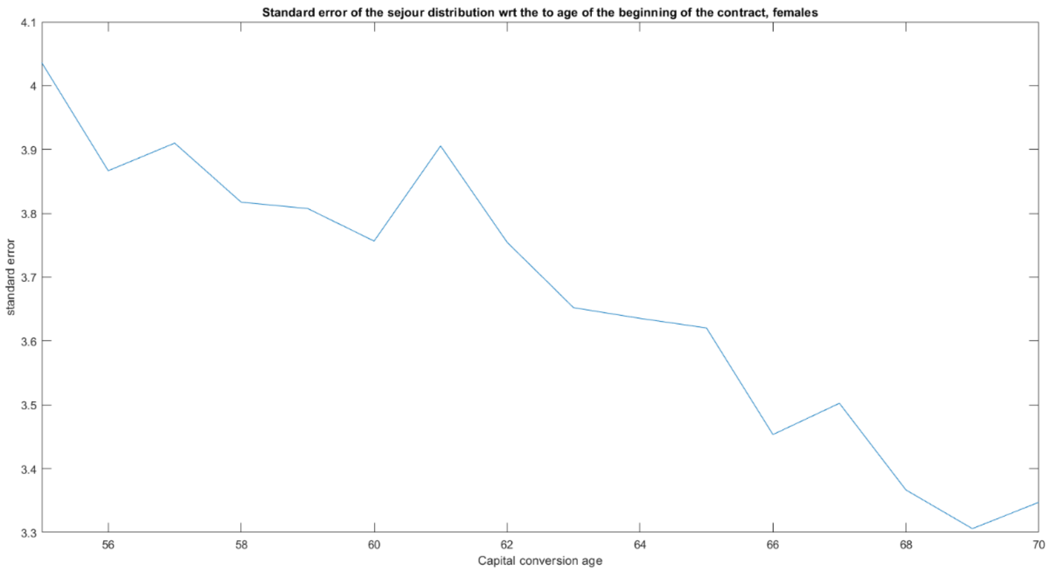


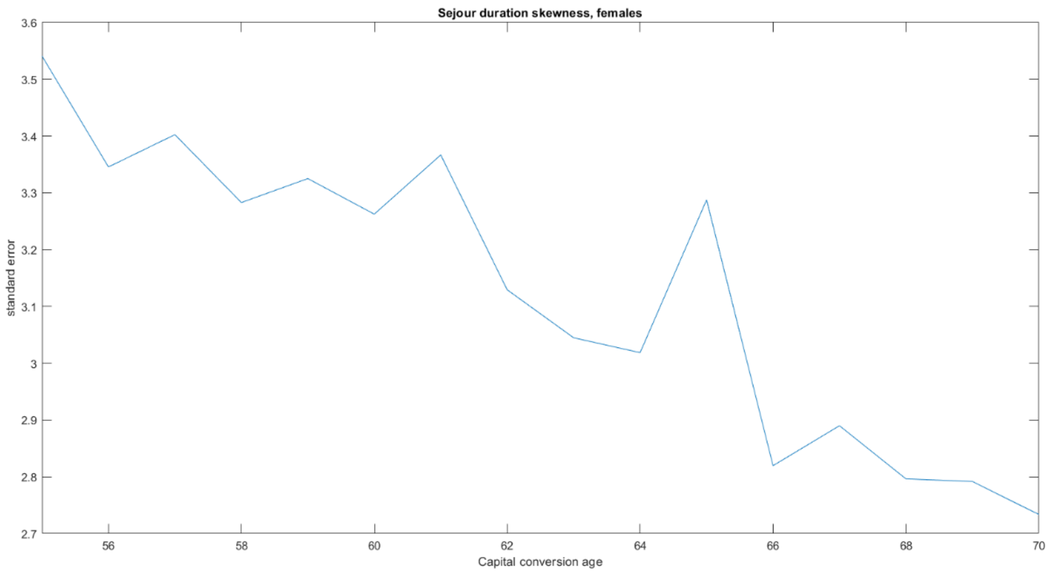

Standard error of the scenario distribution wrt the to age of the beginning of the contract, females
standard error
Capital conversion age
0.0142
0.014
0.0138
0.0136
0.0134
0.0132
0.013
0.0128
0.0126
0.0124
56
58
60
62
64
66
68
70

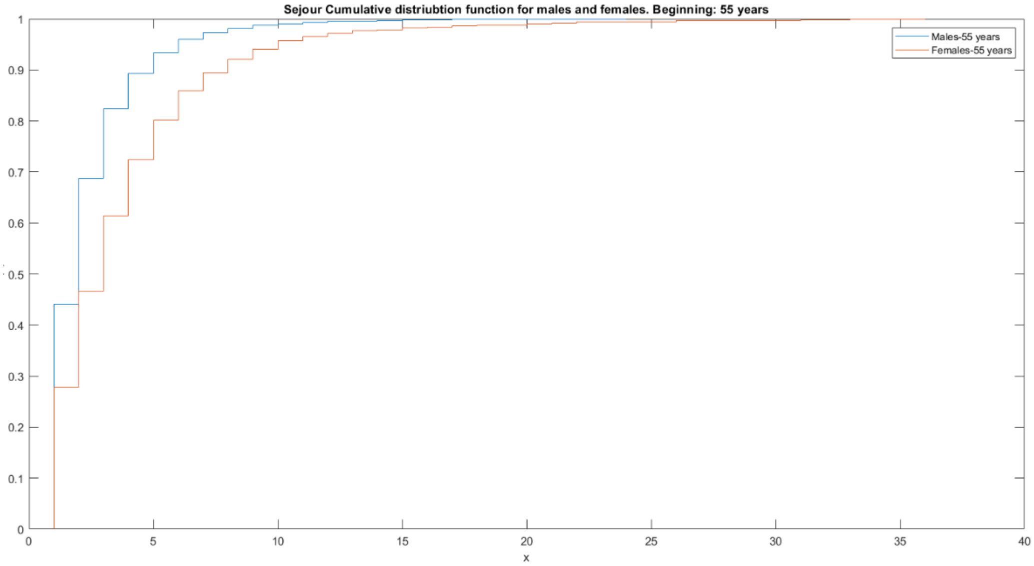

Coefficient of variation (Standard error/mean) of the scenario distribution wrt the to age of the beginning of the contract, females
×10^-4
standard error
Capital conversion age
3.8
3.6
3.4
3.2
3
2.8
2.6
56
58
60
62
64
66
68
70

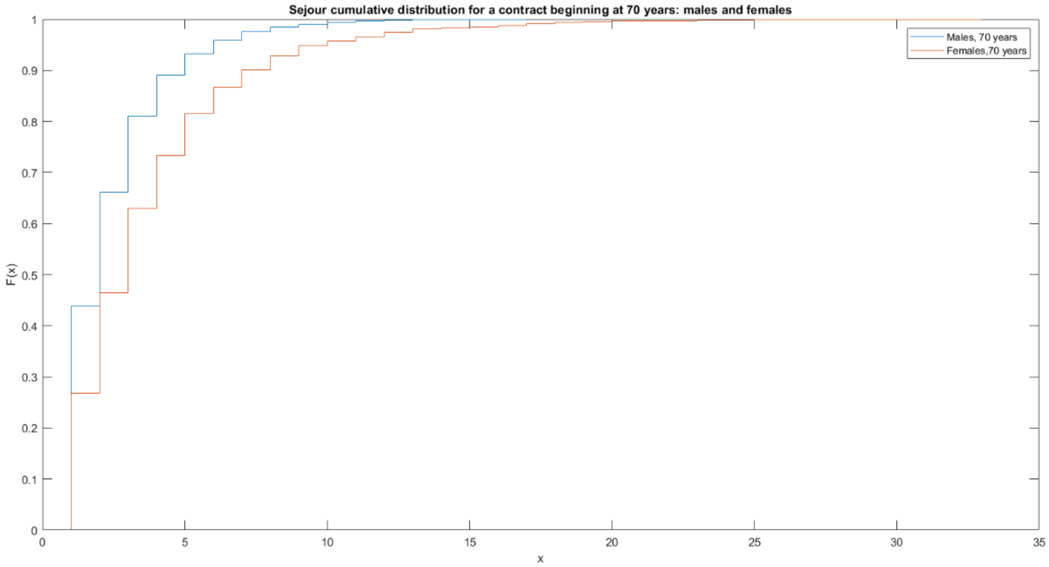

Skewness of scenario distribution by age of beginning, females
Skewness
Capital conversion age
-3.8
-3.9
-4
-4.1
-4.2
-4.3
-4.4
-4.5
-4.6
56
58
60
62
64
66
68
70

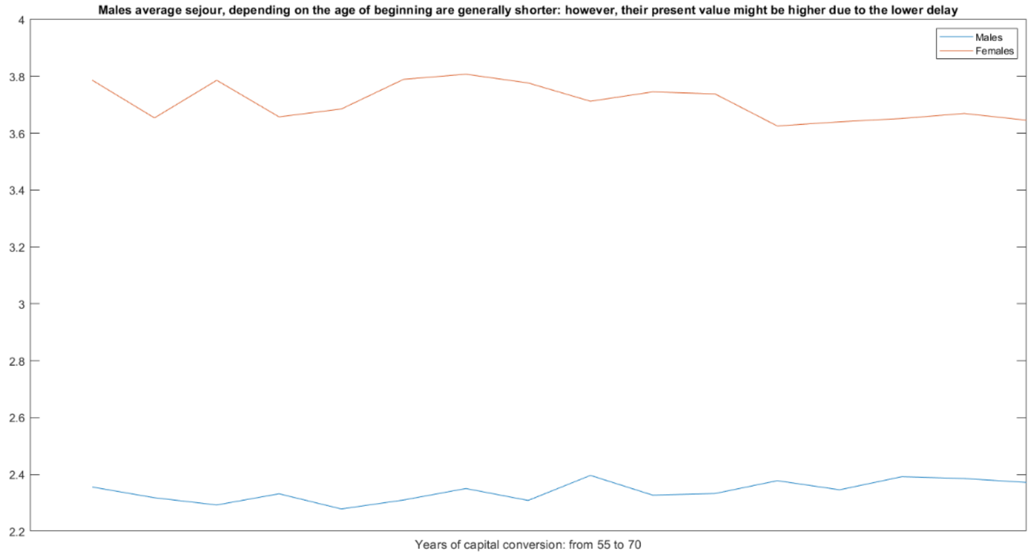

Share of individuals hitting LTC barrier females
share
Capital conversion age
0.191
0.19
0.189
0.188
0.187
0.186
0.185
0.184
56
58
60
62
64
66
68

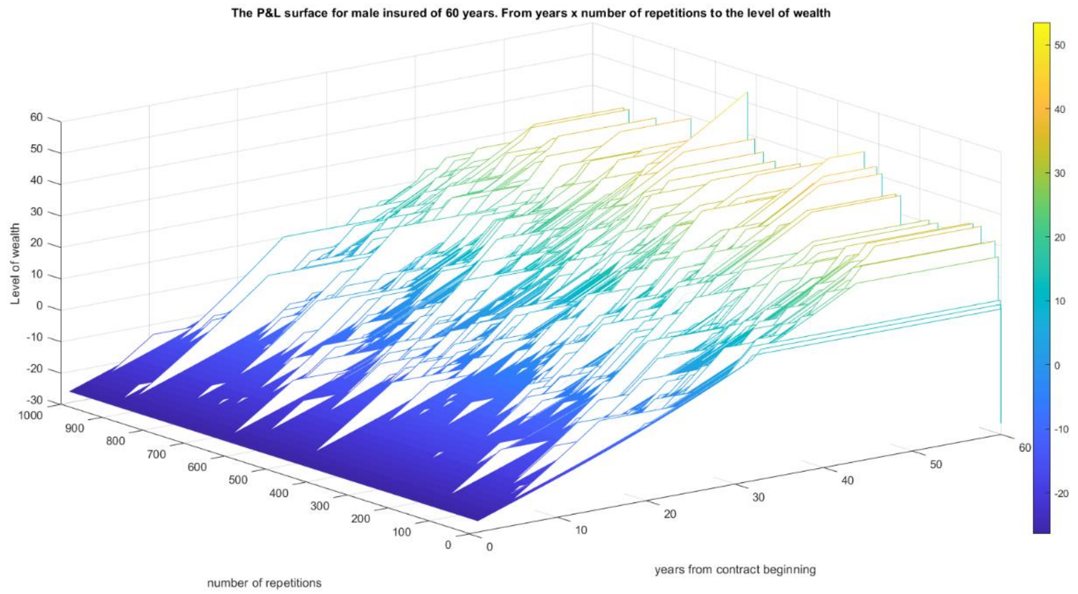

Standard error of the sejour distribution wrt the to age of the beginning of the contract, females
standard error
Capital conversion age
4.1
4
3.9
3.8
3.7
3.6
3.5
3.4
3.3
56
58
60
62
64
66
68
70

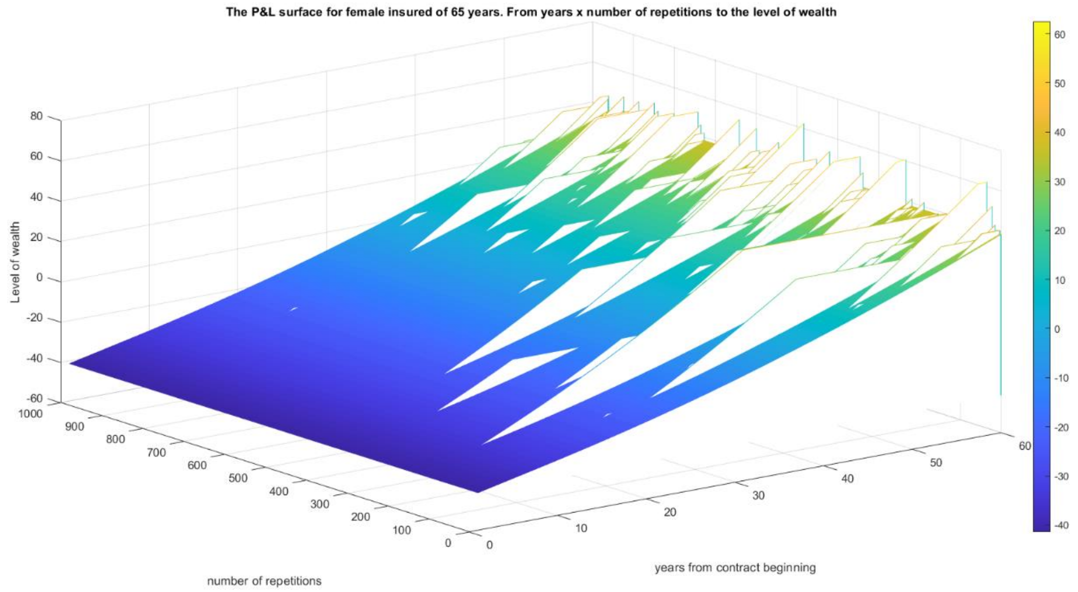

Sejour duration skewness, females
standard error
Capital conversion age
3.6
3.5
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
56
58
60
62
64
66
68
70


Sejour Cumulative distriubtion function for males and females. Beginning: 55 years
Males-55 years
Females-55 years
x


Sejour cumulative distribution for a contract beginning at 70 years: males and females
Males, 70 years
Females,70 years
F(x)
x

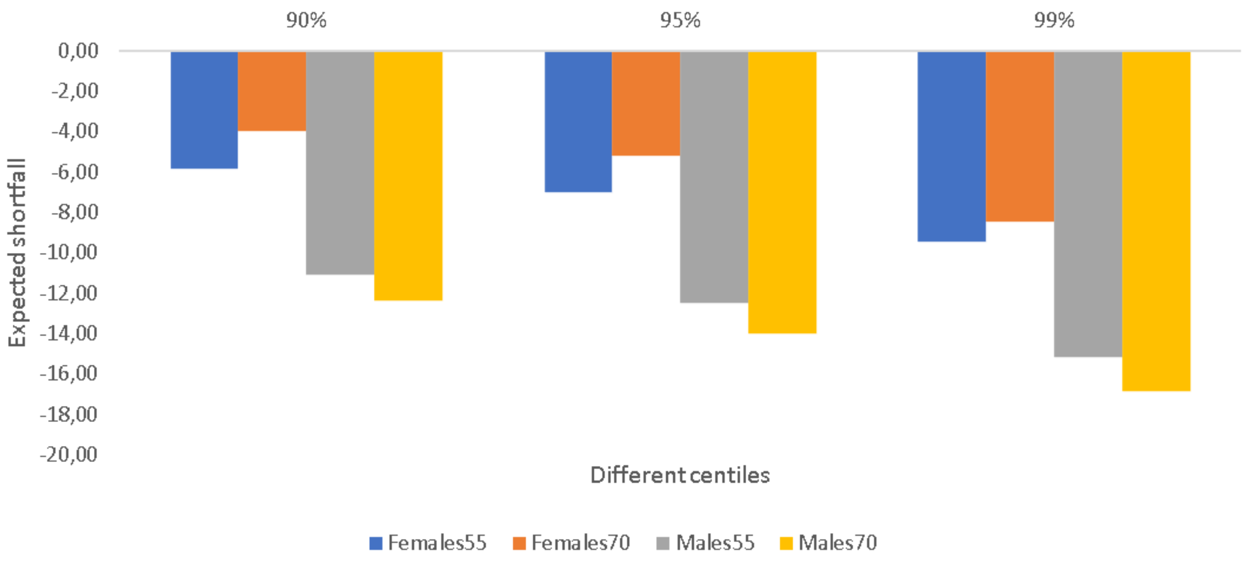

Males average sejour, depending on the age of beginning are generally shorter: however, their present value might be higher due to the lower delay
Males
Females
Years of capital conversion: from 55 to 70


The P&L surface for male insured of 60 years. From years x number of repetitions to the level of wealth
Level of wealth
number of repetitions
years from contract beginning

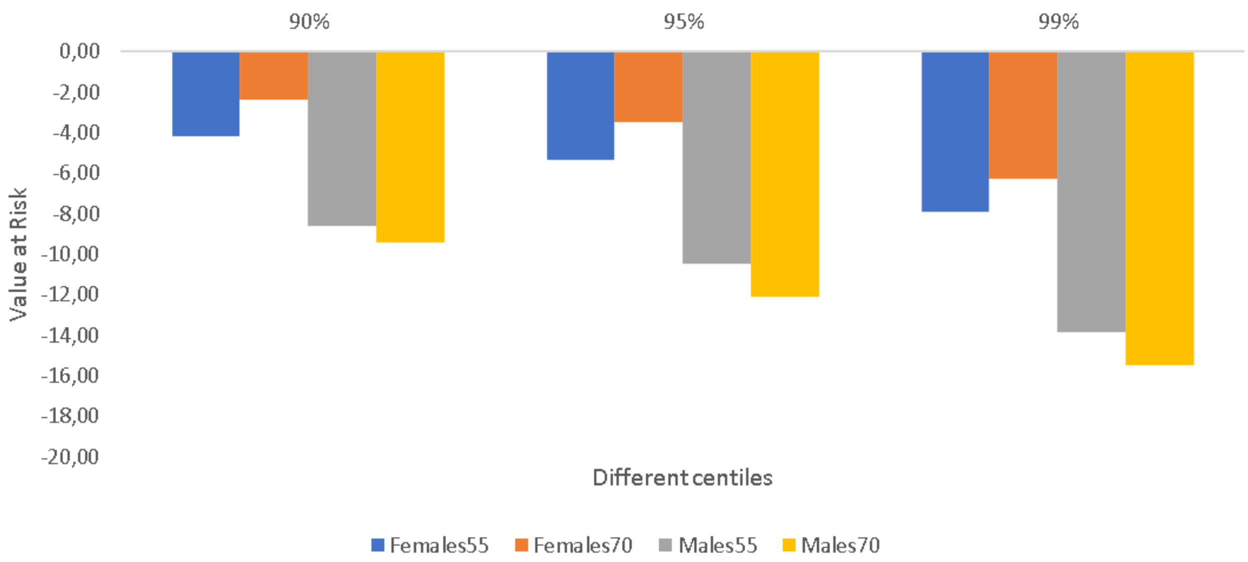

The P&L surface for female insured of 65 years. From years x number of repetitions to the level of wealth
Level of wealth
number of repetitions
years from contract beginning

	Expected shortfalls				Values at Risk		
Females55	-5,86	-7,00	-9,48	Females55	-4,16	-5,38	-7,92
Females70	-3,98	-5,24	-8,47	Females70	-2,38	-3,47	-6,27
Males55	-11,08	-12,49	-15,14	Males55	-8,62	-10,45	-13,85
Males70	-12,36	-13,99	-16,85	Males70	-9,42	-12,13	-15,47

The Expected shortfall of fair premium distribution.



The VaR of fair premium distribution.



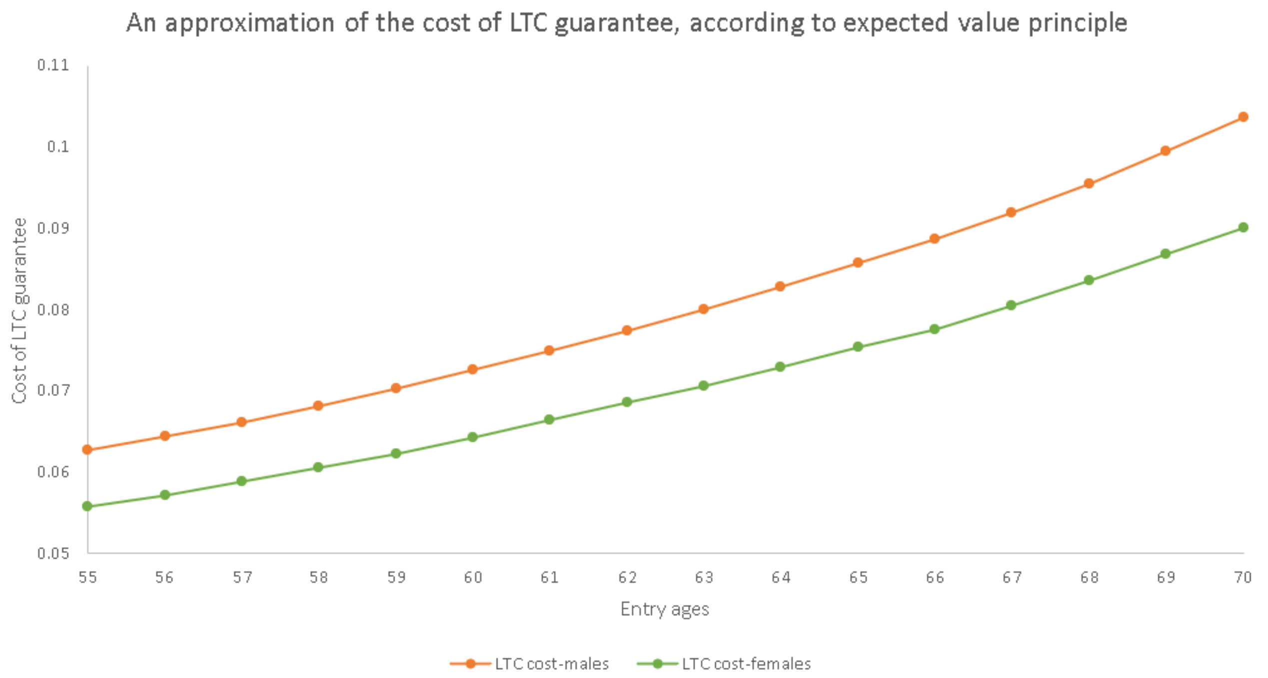


Figure 9- The estimation of Lee-Carter parameter α for the different transition probabilities and sexes. The function, as in the standard Lee-Carter plots the α as a function of the anagraphic age. Source: Levantesi and Menzietti, 2016. First row males, last row females.

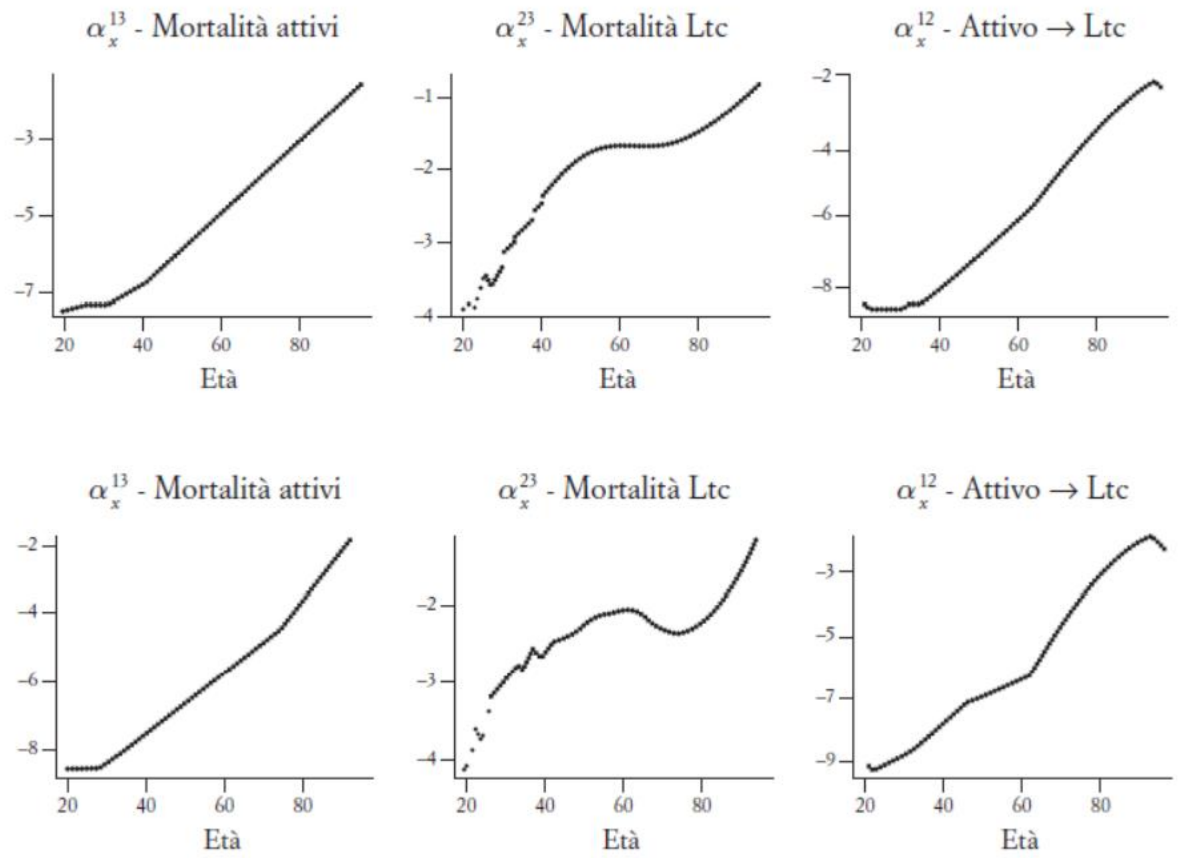


Figure 10-The estimation of Lee-Carter parameter β for the different transition probabilities and sexes. The function, as in the standard Lee-Carter plots the β as a function of the anagraphic age. Source: Levantesi and Menzietti, 2016. First row males, last row females.

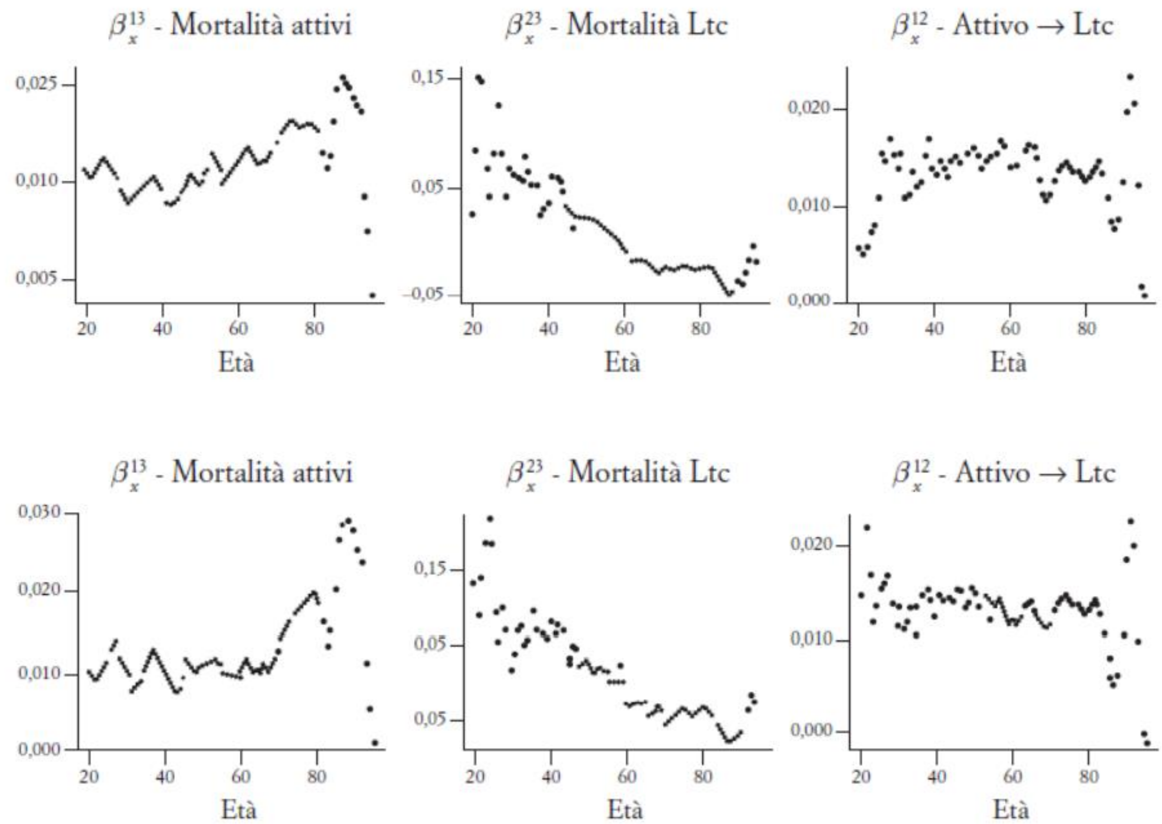
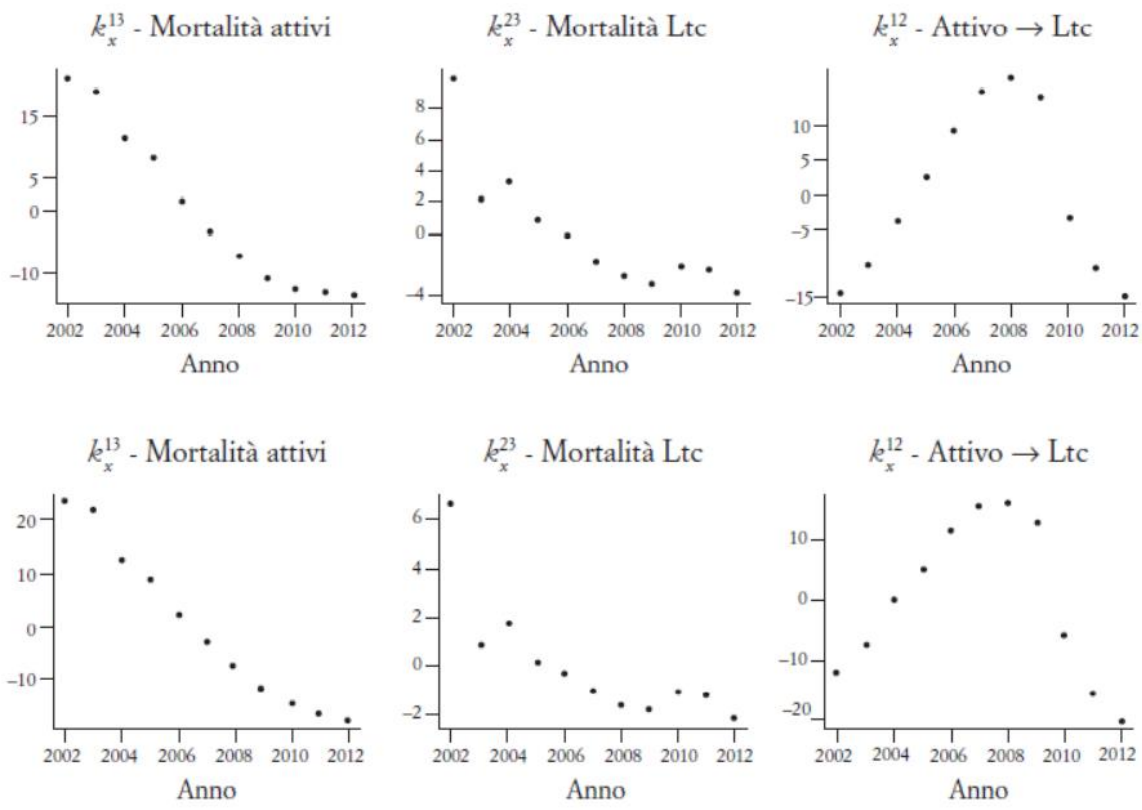


Figure 6-The estimation of Lee-Carter parameter K for the different transition probabilities and sexes. The function, as in the standard Lee-Carter plots the alpha as a function of the calendar year. Source: Levantesi and Menzietti, 2016. First row males, last row females.



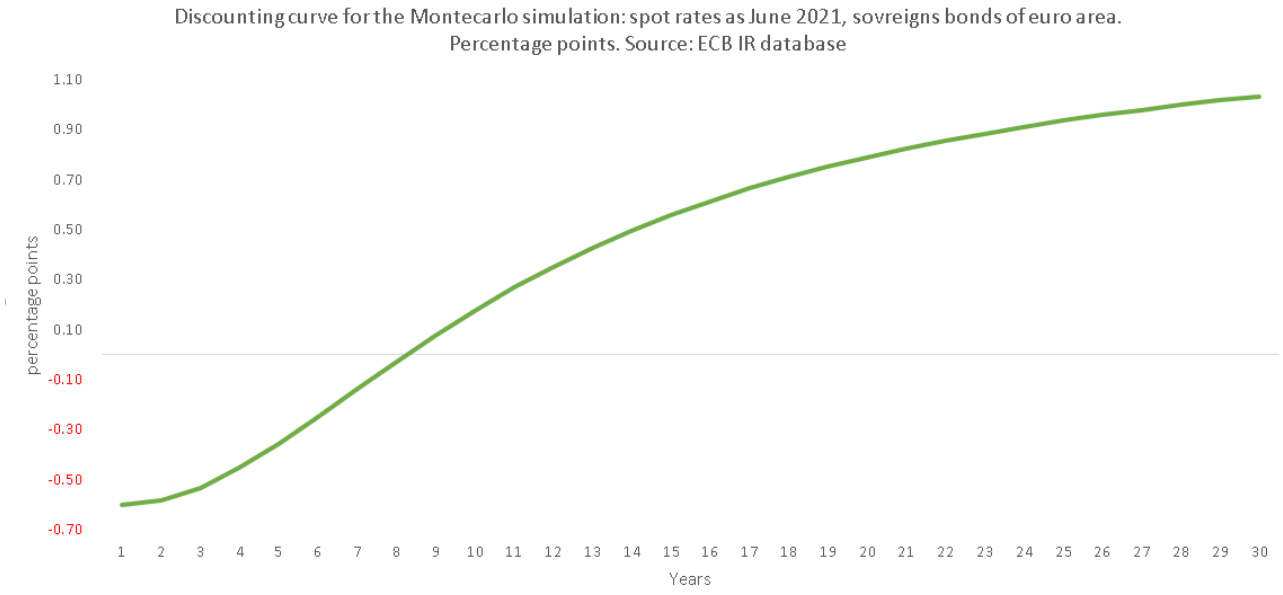


Figure 11-The interest rate curve used for the discount factor of the Montecarlo simulation. Spot rates as of June 2021. Source: https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_area_yield_curves/html/index.en.html. Timeslot: beginning June, Feature: All Bonds, spot rate

```
% This function provides the fair premium level for the enhanced annuity
% contract of our interest. Results are obtained through a montecarlo
% simulation. The inputs are the following: t is the age of contract
% beginning, i.e. the age of premium payment, between 55 and 70. N is the
% number of simulation, that possibly must be greater than 100.000. The
% payment of the annuity happens with annual frequency
% Consider that you must repeat the experiment for all the ages and both
% sexes, so the "overall" number of repetitions will be 30*N. The Boolean
% lumpsum, asks if you are interested in an additional guarantee that pays
% a lump sum of 5/12 of the yearly payment once the insured enters the LTC
% status. k is the number of years for additional death guarantee, if
% applicable, else put=0. By this way, it is possible to add both the
% additional guarantees, one for the death case and one for the LTC
% transition. However, it is not at market a contract selling both the
% guarantees: a possible way is to remove the boolean of imput and putting
% k=-1 in order to call the additional LTC lumpsum guarantee, in order to
% same memory.
function [fairpremiumlevel,yearlyaverage,scenario,se,sevector,sejours] =italianLTCannuity(t,n,lumpsum,k)
% the outputs are the followings:
%fairpremium= the fair premium level resulting from the simulation
%yealyaverage=the average outflow (for the insurer)/inflow(for the insured) year or year of such a contract
%scenario=the vector of all the fair premium level of the simulation, to
%build up a distribution. se=the standard error of the fair premium level
%sevector=the standard error of the outflow for each year of the contract
%duration. sejours=the distribution of sejour in LTC status, including the number of
%individuals hitting the LTC barrier.
load("Females.mat")
% instead ask load("Males.mat") to load the dataset of transition
% probabilities for males.
years=120-t;
% this is the number of years of life of the contract, as we assume the life
% horizon of our insured ends up at 120 years.
% then we preallocate the memory for the variables, defining when available
% their length
out=zeros(n,years);
yearlyaverage=zeros(years,1);
sejours=[];
% here starts the for loop in the number of repetitions
for j=1:n
    % the two flags died and ltc, representing if these conditions are
    % applicable to the individual. Of course, at the beginning of the
    % simulation they are false, as the contract is only sold to healthy
    % individuals
    died=false;
    ltc=false;
    sejour=0;
    % note that LTC flag is redudant, we can use sejour==0 as a flag
```

```
    % we refresh the sejour values up to 0 initially. This is redundant
    % additional control
for i=1:years
    %the loop for the years of length of the contract
    %now we control if the Boolean of the death guarantee is activated
    deathguarantee=(i<=k);
    % consider how the elseif structure speeds up the code
    if died
        break
    % the code stops if the insured is died, as the eventual death
    % guarantee has already been considered. This is the absorbing barrier
    % of the Markov chain. Let's call the LTC mortality as x
    elseif ltc
        x=morteltc(i+t,i);
        %now we correct the pure Markov mortality with the mortality spread
        %to correct the ltc mortality for the sejour duration. In this case
        %the elseif is not compulsory here, but just more efficient than
        %if..if...
        if sejour==1
            x=x+mortalityspread(i+t,1);
        % this is the semimarkov sejour correction for shorter sejour,
        % generally neutral to mortality
        elseif sejour==2
            x=x+mortalityspread(i+t,2);
        % this is the semi-Markov sejour correction for average sejour,
        % generally decreasing mortality
        else
            x=x+mortalityspread(i+t,3);
        % this is the semi-Markov sejour correction for longer sejour,
        % generally decreasing mortality
        end
        PID=random("binomial",1,x);
        % every transition probability is represented by a Bernoulli
        % distribution
        if PID
        % if mortality happens, and so we have transition from LTC to death
            died=true;
        % update the flag and store the sejour duration to the sejour
        % scenario memory, to consider a probability distribution of sejour
        % duration up to the death.
            sejours=[sejours sejour];
        % now we check if the additional guarantee was still effective once
        % the insured passed away.
            while deathguarantee
             out(j,i)=1;
             % the payment is the "healthy  payment" to the survivors even if
             % the insured passed from the LTC status. so we can observe an
             % outflow pattern decreasing, while in the ordinary contract it
             % was increasing (up to the death).
             i=i+1;
```

```
             % we increasing by one the year of the contract and the we
             % re-check if the death guarantee is still applicable
             deathguarantee=(i<=k);
            end
         % this is the end of the section where we assumed the LTC insured
         % passed away, now we consider what happens when the LTC insured
         % survives: we increase the sejour duration of one and we pay the
         % corresponding year benefit
        else
            sejour=sejour+1;
            out(j,i)=2;
         % end of the part where we consider a LTC insured that survives
         % this year. This is the end of LTC part in general, as we
         % considered PII and PID
        end
    else
        % finally, we consider an healthy insured. Since in one year one
        % transition could be made, the insured will end up being death or
        % LTC, but it is not possible in a single interval dt to shift of
        % two states. We first, obviously, consider PAD, so death
        % probability for healthy insured.
        PAD=random("binomial",1,morteattivi(i+t,i));
        % No semi-Markov correction needed, still a Bernoulli R.V.
        % modelling.
        if PAD
            died=true;
            % we update the flag and, as already seen, check for the
            % additional death guarantee, exactly as for PID.
            while deathguarantee
            out(j,i)=1;
            i=i+1;
            deathguarantee=(i<=k);
            end
        else
            % now, if the insured is not passed away this year, we
            % consider the probability that he becomes LTC. This is the
            % only logical and acceptable order in which computations must
            % be performed. Provided that only one transition is possible
            % in a time interval, we check first of all the insured is
            % passed away, and then if she is still autonomous or needs
            % help in performing ADLs. Since we know that our insured has
            % not passed away, we have to compute the transition
            % probability conditioning to the information of w belonging to
            % D-complement. We are considering the probability that w
            % belong to the instersection of D-complement with
            % A-complement, but since we already have informations about w
            % not belonging to D, we have rescale the probability, and so
            % we divide by 1-pAD, basically our conditioned version of
            % pAI=pAI/(pAI+pAA)<=1
            PAI=random("binomial",1,transizioneltc(i+t,i)/(1-morteattivi(i+t,↙
```

```
i)));
            % again, a Bernoulli R.V. with no Semimarkov correction.
            if PAI
                ltc=true;
                %we need to refresh the flag and to put the sejour equal to
                %one, as in the first time step the sejour status had been
                %spent.
                sejour=1;
                % as this is the first payment in LTC status, we check if
                % the additional LTC guarantee was applicable, in this case
                % we pay the first benefit of two (doubled than the base
                % amount) plus the additional guarantee of 5/12 of the
                % yearly LTC payment (5 monthly payments). Note that this
                % additional guarantee is not an anticipation, but an
                % higher insured amount in case LTC claims happens.
                if lumpsum
                  out(j,i)=2.833333;
                else
                  out(j,i)=2;
                end
                % the situation where transition to LTC happened terminates
                % here, now consider the scenario of the absence of an any
                % reformatio in peius, so when an healthy insured hopefully
                % remains healthy even the next year
            else
                out(j,i)=1;
                % we simply pay/receive the "ordinary annuity".
            end
            % we concluded the study of the transition PAI
        end
        % we concluded the study of the transition PAD, even in the else
        end
        % we finally managed to study the biometric history of our
        % individuals for a single year. Here we complete the study of the
        % semi-Markov chain for one year

end
end
% we then concluded the simulation for the different years and different
% individuals. We store now, apart from the whole history of the
% trajectories (out), something more immediately appreciable, as the
% time-series pattern of the average outflow. Remember than MATLAB by
% default performs the column operation on a matrix.
yearlyaverage=mean(out);
% now we consider the volatility of the yearly payment for the insurer
sevector=std(out)./sqrt(n-1);
% now we compute the fair premium level, as the discounted sum of the mean
% of each yearly payment. The dataset already stored discount factor to
% speed up the code. Interest rates are AAA bond interest rate in euro
% area, to use a more conservative forecast on the fair premium level.
```

```
fairpremiumlevel=yearlyaverage*discountfactor(:,years)';
% we then want to store the scenario distribution of the fair premium
% level, i.e. the vector of length n containing all the premium levels
% emerged by the simulation, their average is the fair premium one.
scenario=zeros(n,1);
% to obtain such a scenario distribution, we repeat the present value of
% each yearly payment but for the whole column of the outcome, so to
% obtaining a vector of length n (number of simulations). By linearity of
% the mean, we have mean(scenario)=fairpremiumlevel.
for i=1:years
    scenario=scenario+out(:,i).*discountfactor(i);
end
% this is the standard error of the scenario distribution
se=std(scenario)/sqrt(n-1);
% the code terminates here, in order to proper call the function and obtain
% a whole "enhanced pension" table for all the possible ages of conversion:
table_females=zeros(16,1);
for m=55:70
table_females(m-54)=italianLTCannuity(m,100000,false,0);
end
```

```
% We now want to assess the net gain in wealth by the insurance
% contract. This could be either positive (if the insured lives long enough,
% or in case he is LTC)
% Every individuals start with a negative value equal to the premium, since
% it is already paid and lost- financially speaking. Every year (or fraction),
% we add the amount paid by the insurance, adequately compounded.
% Here, we work only with
% 1000 simulations. We have chosen as a representative individual a male
% aged 60 and a woman aged 65, at the insurance beginning.
netwealth_submartingale=zeros(100000,60);
netwealth_submartingale(:,1)=-coefficiente60*ones(100000,1);
for i=2:54
    % Wealth is recursively defined as the wealth of the previous year and
    % the payment of the current (that could be 0,1 or 2).
    netwealth_submartingale(:,i)=+out(:,i-1).*compoundingfactor(i-1);
    % we adequately revalue the payments made by the insurer to preserve
    % the time value of money. We expressed the wealth in the value of
    % money of the contract beginning. It is possible to use other options,
    % provided consistency is preserved.
end
%this is the end of the code.
%
```