Mrf461
Hw7
*** Given 24h extension ***

Question 4:
a) Exercise 5.1.1, sections b, c

b.) **2.69e14** = (40^7) + (40^8) + (40^9)
There are 10 digits, 26 letters, and 4 symbols. Total there are 40 options. Each position can be any of the 40. Since it's an or statement we add together the possible combinations for length 7 (40^7), length 8 (40^8), and length 9 (40^9)

c.) **9.41e13** = (14^1 * 40^6) + (14^1 * 40^7) + (14^1 * 40^8)
Since the first character cannot be a letter, there are 14 options for position one, 14 * 1. This is multiplied by the remaining positions left in the sequences.

b) Exercise 5.3.2, section a

a.) **1536**
There are 3 possibilities. Position one can be any of the 3, but since all other positions cannot be the same as the preceding position this means there only two options. Therefore, the total = 3*2*2*2*2*2*2*2*2*2 = 1536

c) Exercise 5.3.3, sections b, c

b.) ((10*9*8) * 26^4)
There are 26 ways to fill the 4 letters (26^4). If each digit has to be unique, we subtract one from the total, 10, for each digit. Therefore 10 * (10 -1) * (10 - 2). Multiply the possibilities for digits times the possibility for letters

c.) (10*9*8) * (26*25*24*23)
Similar to b., but now for each letter, we have to subtract a letter each time one is used, therefore if there are 4 letters, the possibilities for letters would be: 26 * (26-1) * (26-2) * (26-3). The total combinations would be this multiplied by having a unique digit which is calculated above (10 * (10 -1) * (10 - 2)).

Question 5:
a) Exercise 5.4.2, sections a, b

a.) 20,000
From the seven total positions, 3 positions are locked in as 2 different options. There are 10 options for the other four positions (10 * 10 * 10 * 10), which is true for both options (* 2).

b.) 10,080
If the last 4 digits are different, the calculation for the last 4 positions is instead (10*9*8*7) and then multiplied (*2) for each of the possible first 3-digit options

b) Exercise 5.5.3, sections a-g

a.) 1024 = 2^10
If there are no restrictions, the number of combinations is equal to the total number of combinations of binary strings. Given there are 10 positions to account for and its either a 0 or a 1 (2 choices), we have two choices, across 10 sets so, 2^10.

b.) 128 = 2^7
Different from a.), we have to account for 3 positions being fixed, therefore there are only 7 positions that each have 2 possibilities for outcomes, or 2^7.

c.) 284 = (2^7 + 2^8)
Since it's an or statement we find the number of combinations for having 3 positions fixed == 2^7 and the number of combinations that have 2 positions fixed == 2^8 and add them together.

d.) 256 = (2^6 * 2^2)
There's 2^2 possible outcomes for the first and last two bits. There are 2^6 possible outcomes for the middle 6 bits. Since they are in combination together, we multiply 2^2 * 2^6.

e.) 16 =2^4
6 of the 10 positions have to be 0 which leaves 4 positions which are variable, therefore 2^4.

f.) 8 = 2^3
6 of the 10 positions are fixed to be 0 and position 1 has to be 1, therefore 3 positions are still variable, or 2^3

g.) 64= 2^6

Four positions are locked in to be 1's, therefore 6 positions can still be variable, or 2^6

c) Exercise 5.5.5, section a

a.) (35 choose 10) * (30 choose 10)

10 selections must be made from 2 groups. The 'boy' group has 30 options and the 'girl' group has 35 options. Order doesn't matter since there isn't mention of them performing differentiating roles. Therefore, from the girl group, 35 choose 10 and from the boy group 30 choose 10.

d) Exercise 5.5.8, sections c-f

c.) ((13 choose 1) * (13 choose 1) * (24 choose 3))

13 ranks in a suit and selecting at least one from two separate suits. If 26 other cards are in both suits and we've selected 2 cards from that total (26 - 2), then there are 24 cards to choose 3 others from.

d.) ((13 choose 1)* (4 choose 4) * (48 choose 1))

If we're selecting 4 cards from the same suit, we are selecting 4 cards from 13 cards, but there are 4 possibilities for suits to multiply into that. After we've selected 4 cards, 48 cards remain, from which we are selecting any 1 card from the 48.

e.) ((13 choose 1) * (4 choose 3) * (12 choose 1) * (4 choose 2))

We have to select 3 cards from the same suit first. 13 cards per suit, 3 cards of same rank from 4 possible suits. After we've selected those, there are only 12 options for rank (3 + 2 > 4) from which we have to choose 2 cards of same rank from 4 possible suits.

e) Exercise 5.6.6, sections a, b

a.) (44 choose 10) * (56 choose 10)

Each party has to select a committee, in which there aren't specific roles listed for each of the 10 committee members, thus order doesn't matter. There are 44 members in one party to choose 10 from (44 choose 10) and 56 members in the other party to select 10 from (56 choose 10)

b.) P(44, 2) * P(56, 2)

Each party has to select two members to serve specific roles. The roles are different, therefore order matters. From one party there are 44 options for 2 roles (P(44,2)) and from the other party there are 56 options to select from for the 2 roles (P(56,2)).

Question 6

a) Exercise 5.7.2, sections a, b

a.) 13 choose 1 * 51 choose 4

There are 13 clubs, thus 13 choose 1. Since the question says ‘at least’, this means there can be additional clubs in the 5 card hand so we account for the remaining 12 clubs in the total and therefore only remove a single card, so 51 left from which we select 4 or 51 choose 4.

b.) 26 * 4 choose 2 * 50 choose 3

To select 2 cards of the same rank they have to be from different suits. From two suits there are 26 cards, 4 suits total, and we need to select 2. Then for the remaining three in the hand we select any 3, 50 choose 3.