REAL ANALYSIS

MATH 131, HARVEY MUDD COLLEGE

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TODAY: LIMIT POINTS, OPEN & CLOSED SETS

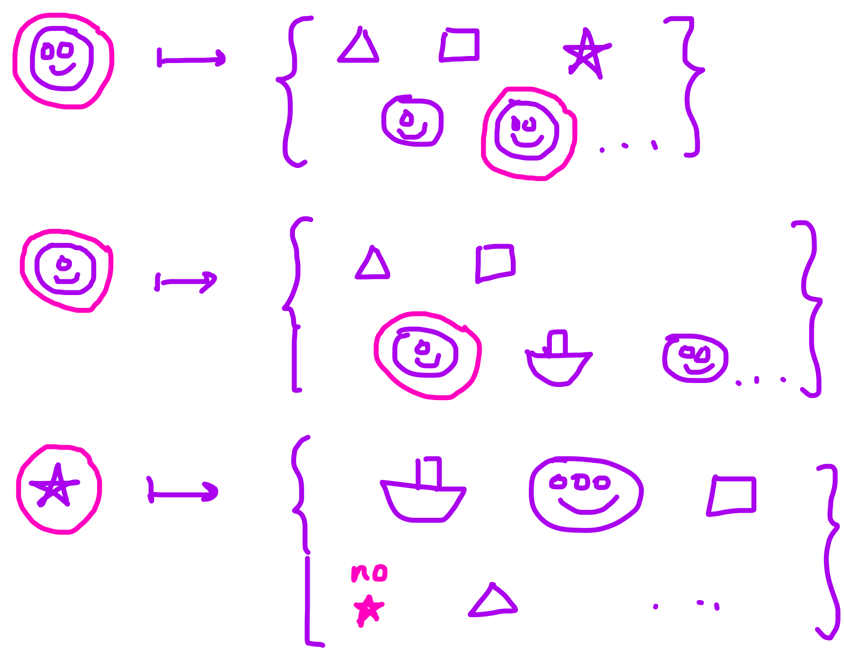
Return: to Cantor's Thm: $A \not\sim 2^A$.

proof. Suppose $\exists$ bijection $f: A \leftrightarrow 2^A$.

Then $\underset{\text{elt}}{a} \mapsto \underbrace{f(a)}_{\text{subset of } A}$

Goal: show $\exists$ subset B that's not $f(a)$, for all a.

Idea: $a \in A$ $\quad$ $f(a)$



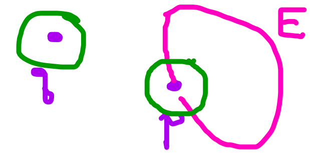
construct

$B =$

Let $B = \{a : a \notin f(a)\}$. Suppose $B = f(x)$ for some x.

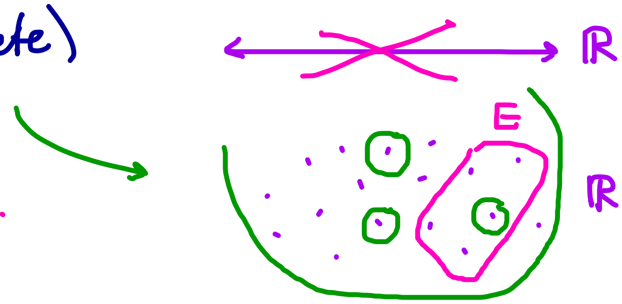
Q. Is $x \in B$? ...

Recall: Point $p \in X$ is a lim pt. of E if every nbhd of p contains a point of E different from p.



Notice: in discrete metric, balls are one pt or all X.

Ex. $(\mathbb{R}, \text{discrete})$



E has no lim. pts.

Notice: p is not a lim pt of E means $\exists$ some nbhd $\mathcal{U}$ of p s.t. $\mathcal{U}$ contains no pt of E other than p.

LOGIC PRACTICE:

- A horse is superior if $\underbrace{\text{every leg is strong}}$. ($\forall x$, $A(x)$ is true)

So: a " " not superior if $\underbrace{\exists \text{ leg that's not strong}}_{\exists x \text{ s.t. } A(x) \text{ not true.}}$

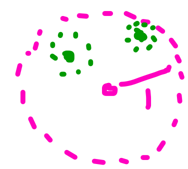
- A horse is lame if $\underbrace{\text{it has broken leg}}$. ($\exists x$ s.t. $A(x)$ true)

So: " " " not lame if $\underbrace{\text{every leg not broken}}_{\forall x,\ A(x) \text{ is not true.}}$

Def'n. Call p an <u>isolated pt</u> of E if
$p \in E$ <u>and</u> p is <u>not</u> a <u>l.p.</u> of E.

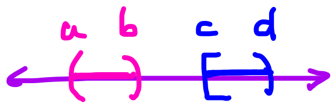
Def'n. Call p an <u>interior pt.</u> of E if
$\exists$ nbhd N of p s.t. $N \subset E$.

So: p is not an <u>int. pt</u> of E if
$\forall$ nbhd N of p, $N \not\subset E$.



Def'n. A set E is <u>open</u> if
<u>every</u> pt of E is an <u>interior pt</u> of E.

Ex. $\mathbb{R}$

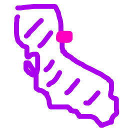


(a,b) open $\quad$ $[c,d)$ not open b/c c is not <u>interior</u>.

$\emptyset$ empty set is <u>open</u>.
$\mathbb{R}$ is <u>open</u>.

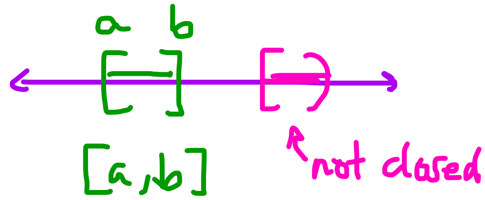
Ex. $(\mathbb{R}, \text{discrete})$

<u>all</u> subsets are open.



• <u>Def'n</u> A set E is <u>closed</u> if
E contains <u>all</u> its <u>lim. pts</u>.

<u>EX</u>. $\mathbb{R}$



$\emptyset$ <u>closed</u>.

$\mathbb{R}$ <u>closed</u>.