wspec Documentation

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This is the Wolbachia speciation (wspec) module for numerical simulations of speciation events driven by Wolbachia-induced cytoplasmic incompatibility (CI) and reinforcement processes.

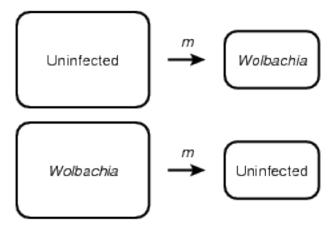
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ANALYTICAL

Analytical solutions for stability of cytoplasmic incompatibility (CI) patterns, gene flow, and reproductive values of migrants between populations with different infection states.



wspec.analytical.dynamics_IM(f, ci, m, t, x)

Infection dynamics of Wolbachia for the scenario with an infected mainland (IM), iterative map.

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- m: float in interval [0, 1] migration rate
- t: float in interval [0, 1] transmission rate of Wolbachia
- x: float in interval [0, 1] infection frequency of Wolbachia

Returns:

out: float infection frequency in the next generation

wspec.analytical.dynamics_SP (f, ci, t, x)

Infection dynamics of Wolbachia in a single host population (SP), iterative map returns the infection frequency in the next generation.

Args:

- **f:** float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

```
x: float in interval [0, 1] infection frequency of Wolbachia
      Returns:
           out: float infection frequency in the next generation
wspec.analytical.dynamics_UM (f, ci, m, t, x)
      Infection dynamics of Wolbachia for the scenario with an uninfected mainland (UM), iterative map returns the
      infection frequency in the next generation.
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           m: float in interval [0, 1] migration rate
           t: float in interval [0, 1] transmission rate of Wolbachia
           x: float in interval [0, 1] infection frequency of Wolbachia
      Returns:
           out: float infection frequency in the next generation
wspec.analytical.fix1_IM (f, ci, m, t)
      Fixpoint x_1^* for the scenario with an infected mainland (IM).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           m: float in interval [0, 1] migration rate
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: float critical migration rate
wspec.analytical.fix1_SP (f, ci, t)
      Infection frequency fixpoint x_1^* for a single host population (SP).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: float equilibrium frequency of Wolbachia
wspec.analytical.fix1_UM (f, ci, m, t)
      Fixpoint x_1^* for the scenario with an uninfected mainland (UM).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           m: float in interval [0, 1] migration rate
           t: float in interval [0, 1] transmission rate of Wolbachia
```

Returns: out: float equilibrium frequency of Wolbachia wspec.analytical.fix2_IM (f, ci, m, t)Fixpoint x_2^* for the scenario with an infected mainland (IM). Args: **f: float in interval [0, 1]** fecundity reduction in infected females ci: float in interval [0, 1] level of CI m: float in interval [0, 1] migration rate t: float in interval [0, 1] transmission rate of Wolbachia **Returns:** out: float critical migration rate wspec.analytical.fix2_SP (f, ci, t)Infection frequency fixpoint x_2^* for a single host population (SP). Args: **f: float in interval [0, 1**] fecundity reduction in infected females ci: float in interval [0, 1] level of CI t: float in interval [0, 1] transmission rate of Wolbachia **Returns:** out: float equilibrium frequency of Wolbachia wspec.analytical.fix2_UM (f, ci, m, t)Fixpoint x_2^* for the scenario with an uninfected mainland (UM). Args: f: float in interval [0, 1] fecundity reduction in infected females ci: float in interval [0, 1] level of CI m: float in interval [0, 1] migration rate t: float in interval [0, 1] transmission rate of Wolbachia **Returns:** out: float equilibrium frequency of Wolbachia wspec.analytical.fix3_IM (f, ci, m, t)Fixpoint x_3^* for the scenario with an infected mainland (IM). **Args:** f: float in interval [0, 1] fecundity reduction in infected females ci: float in interval [0, 1] level of CI m: float in interval [0, 1] migration rate t: float in interval [0, 1] transmission rate of Wolbachia **Returns:**

out: float critical migration rate

```
wspec.analytical.fix3_SP (f, ci, t)
      Infection frequency fixpoint x_3^* for a single host population (SP).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: float equilibrium frequency of Wolbachia
wspec.analytical.fix3_UM (f, ci, m, t)
      Fixpoint x_3^* for the scenario with an uninfected mainland (UM).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           m: float in interval [0, 1] migration rate
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: float equilibrium frequency of Wolbachia
wspec.analytical.fix SP(f, ci, t)
      Infection frequency fixpoints for a single host population (SP).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: tuple of three floats equilibrium frequencies x_1^*, x_2^*, and x_3^* of Wolbachia
wspec.analytical.fix UM(f, ci, m, t)
      Infection frequency fixpoints for the scenario with an infected mainland (IM).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           m: float in interval [0, 1] migration rate
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: tuple of three floats equilibrium frequencies x_1^*, x_2^*, and x_3^* of Wolbachia
wspec.analytical.gff_DS(m, s)
      Gene flow factor (gff) for the case of divergent selection (DS). Residents have a viability advantage of s over
      migrants, equivalent to migrants having a viability cost of \frac{s}{1+s}. Derived from fitness graph for the reproductive
      value of a migrant.
```

```
Args:
           m: float in interval [0, 1] migration rate
           s: positive float selection coefficient
      Returns:
           out: float gene flow factor
wspec.analytical.lcrit SP(f, t)
      Critical CI level (lcrit) for a single host population (SP).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: float critical CI level
wspec.analytical.lcrit UM(f, m, t)
      Critical CI level (lcrit) for the scenario with an uninfected mainland (UM).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           m: float in interval [0, 1] migration rate
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: float critical CI level
wspec.analytical.mcrit_IM(f, ci, t)
      Critical migration rate (mcrit) for the scenario with an infected mainland (IM).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: float critical migration rate
wspec.analytical.mcrit_IMA (f, ci, s, t)
      Critical migration rate (mcrit) for the scenario with an infected mainland (IM) with local host adaptation (A).
      Args:
           f: float in interval [0, 1] fecundity reduction in infected females
           ci: float in interval [0, 1] level of CI
           s: positive float selection coefficient
           t: float in interval [0, 1] transmission rate of Wolbachia
      Returns:
           out: float critical migration rate
```

```
wspec.analytical.mcrit_UM (f, ci, t)
Critical migration rate (mcrit) for the sc
```

Critical migration rate (mcrit) for the scenario with an uninfected mainland (UM).

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float critical migration rate

```
wspec.analytical.mcrit_UMA (f, ci, s, t)
```

Critical migration rate (mcrit) for the scenario with an uninfected mainland (UM) with local host adaptation (A).

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- s: positive float selection coefficient
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float critical migration rate

```
wspec.analytical.meff DS(m, s)
```

Effective migration rate (meff) for the case of divergent selection (DS). Residents have a viability advantage of s over migrants, equivalent to migrants having a viability cost of $\frac{s}{1+s}$. Derived from fitness graph for the reproductive value of a migrant.

Args:

- m: float in interval [0, 1] migration rate
- s: positive float selection coefficient

Returns:

out: float effective migration rate

```
wspec.analytical.prcrit (ci, q, s)
```

Minimal rejection probability (pr) of a mutant allele at the locus for female mating preference to spread in an uninfected island receiving migration from an infected mainland.

Args:

- ci: float in interval [0, 1] level of CI
- q: float in interval [0, 1] transition probability to a new mating round
- s: positive float selection coefficient

Returns:

out: float minimal rejection probability

```
wspec.analytical.reproval_DS(s)
```

Reproductive value of a migrant for the case of divergent selection (DS). Residents have a viability advantage of s over migrants, equivalent to migrants having a viability cost of $\frac{s}{1+s}$. Derived from fitness graph.

Args:

```
s: positive float selection coefficient
```

Returns:

```
out: float reproductive value
```

```
wspec.analytical.reproval_FUHETCL(f, ci, t)
```

Reproductive value of an uninfected female (FU) in a heterogenous host population (HET), *per class* (CL) reproductive value (as opposed to *per capita*). Derived from fitness graph.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

```
out: float reproductive value
```

```
wspec.analytical.reproval_FUHETCP (f, ci, t)
```

Reproductive value of an uninfected female (FU) in a heterogenous host population (HET), *per capita* (CP) reproductive value (as opposed to *per class*). Derived from fitness graph.

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

```
out: float reproductive value
```

```
wspec.analytical.reproval_FUHOMW (f, ci)
```

Reproductive value of an uninfected female migrant (FU) in a homogenous Wolbachia infected population (HOMW). This applies to the case of perfect Wolbachia transmission, t = 1. Derived from fitness graph:

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI

Returns:

out: float reproductive value

```
wspec.analytical.reproval_FWHETCL(f, ci, t)
```

Reproductive value of an infected female (FW) in a heterogenous host population (HET), *per class* (CL) reproductive value (as opposed to *per capita*). Derived from fitness graph.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.reproval FWHETCP (f, ci, t)
```

Reproductive value of a Wolbachia infected female (FW) in a heterogenous host population (HET), *per capita* (CP) reproductive value (as opposed to *per class*). Derived from fitness graph.

Args:

- **f: float in interval [0, 1**] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.reproval_FWHOMU (f, ci, t)
```

Reproductive value of a Wolbachia infected female migrant (FW) in a homogenous uninfected population (HOMU). Derived from fitness graph.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.reproval IPHOMU (f, ci, t)
```

Average reproductive value of a migrant from an infected population (IP), i.e. where the Wolbachia infection is at a stable frequency below 1 due to imperfect transmission, in a homogenous uninfected population (HOU). Averaged over sexes (F/M) and cytotypes (U/W). Derived from fitness graph.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
\verb|wspec.analytical.reproval_MUHETCL| (f, ci, t)
```

Reproductive value of an uninfected male (MU) in a heterogenous host population (HET), *per class* (CL) reproductive value (as opposed to *per capita*). Derived from fitness graph.

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.reproval_MUHETCP (f, ci, t)
```

Reproductive value of an uninfected male (MU) in a heterogenous host population (HET), *per capita* (CP) reproductive value (as opposed to *per class*). Derived from fitness graph.

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.reproval_MUHOMW (f, ci)
```

Reproductive value of an uninfected male migrant (MU) in a homogenous Wolbachia infected population (HOMW). This applies to the case of perfect Wolbachia transmission, t=1. Derived from fitness graph.

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI

Returns:

out: float reproductive value

```
wspec.analytical.reproval_MWHETCL(f, ci, t)
```

Reproductive value of an infected male (MW) in a heterogenous host population (HET), *per class* (CL) reproductive value (as opposed to *per capita*). Derived from fitness graph.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.reproval_MWHETCP (f, ci, t)
```

Reproductive value of a Wolbachia infected male (MW) in a heterogenous host population (HET), *per capita* (CP) reproductive value (as opposed to *per class*). Derived from fitness graph.

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
\verb|wspec.analytical.reproval_MWHOMU| (f, ci, t)
```

Reproductive value of a Wolbachia infected male migrant (MW) in a homogenous uninfected population (HOMU). Derived from fitness graph.

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

```
out: float reproductive value
```

```
wspec.analytical.reproval_UHETCL(f, ci, t)
```

Average reproductive value of an uninfected host (U) in a heterogenous population (HET), *per class* (CL) reproductive value (as opposed to *per capita*). Derived from fitness graph. Averaged over the two sexes.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

```
out: float reproductive value
```

```
wspec.analytical.reproval_UHETCP (f, ci, t)
```

Average reproductive value of an uninfected host (U) in a heterogenous population (HET), *per capita* (CP) reproductive value (as opposed to *per class*). Derived from fitness graph. Averaged over the two sexes.

Args:

- **f: float in interval [0, 1**] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.reproval_UHOMW (f, ci)
```

Reproductive value of an uninfected migrant (U) in a a homogenous Wolbachia infected population (HOMW). This applies to the case of perfect Wolbachia transmission, t=1. Derived from fitness graph. Averaged over the two sexes.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI

Returns:

out: float reproductive value

```
wspec.analytical.reproval WHETCL (f, ci, t)
```

Reproductive value of an infected host (W) in a heterogenous host population (HET), *per class* (CL) reproductive value (as opposed to *per capita*). Derived from fitness graph.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

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out: float reproductive value

```
wspec.analytical.reproval WHETCP (f, ci, t)
```

Average reproductive value of a Wolbachia infected host (W) in a heterogenous population (HET), *per capita* (CP) reproductive value (as opposed to *per class*). Derived from fitness graph. Averaged over the two sexes.

Args:

- f: float in interval [0, 1] fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.reproval_WHOMU (f, ci, t)
```

Reproductive value of a Wolbachia infected migrant (W) in a homogenous uninfected population (HOU). Derived from fitness graph. Averaged over the two sexes.

Args:

- **f: float in interval [0, 1]** fecundity reduction in infected females
- ci: float in interval [0, 1] level of CI
- t: float in interval [0, 1] transmission rate of Wolbachia

Returns:

out: float reproductive value

```
wspec.analytical.xPref (ci, pr, q, s)
```

Approximated equilibrium frequency of a mutant allele at the locus for female mating preference in an uninfected island receiving migration from an infected mainland.

Args:

- ci: float in interval [0, 1] level of CI
- pr: float in interval [0, 1] rejection probability of mating preference mutant allele
- q: float in interval [0, 1] transition probability to a new mating round
- s: positive float selection coefficient

Returns:

out: float equilibrium frequency of mutant preference allele

```
wspec.analytical.xTprefEQ(pr, s, xPref)
```

Modified from Kirkpatrick (1982), equation (2).

Args:

- pr: float in interval [0, 1] rejection probability of mating preference mutant allele
- s: positive float selection coefficient
- **xPref:** float in interval [0, 1] frequency of preference allele in Kirkpatrick's model

Returns:

out: float equilibrium frequency of preferred trait

CHAPTER

TWO

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