Northeastern University

Report for Experiment #14

Standing Waves

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Abstract

This report investigates principles of standing waves, specifically resonance, through two investigations. The first investigation focuses on standing wave formation in a vibrating string apparatus, analyzing the relationship between wave velocity, tension, and resonance frequency across multiple nodes. The second investigation examines resonance in a closed air column, using sound waves at various frequencies to determine the speed of sound through wavelength measurements. Both investigations confirm theoretical predictions with minor deviations attributed to measurement uncertainties, emphasizing the importance of resonance in understanding wave behavior.

Introduction

The study of wave mechanics provides fundamental insights into physical phenomena, such as the behavior of standing waves in strings and sound waves in air columns. These systems offer practical demonstrations of concepts like resonance, wave velocity, and the relationships between frequency, wavelength, and tension.

In the first investigation, a vibrating string apparatus is used to explore standing wave formation across multiple resonance modes. By adjusting the tension in the string and measuring the resulting wave velocity, the experiment aims to verify the theoretical relationship between these variables and determine the string's mass per unit length.

The second investigation focuses on sound wave resonance in a closed air column, where distances between resonances are measured for various frequencies to calculate the speed of sound. Together, these experiments aim to validate theoretical wave equations and highlight the role of resonance in practical systems.

Investigation 1

In the first investigation, the equipment included a 120 hz vibrator, 2 rods, 3 clamps, weights, a string, and a bucket. In addition, a digital scale and ruler were used for weight and distance measurements respectively.

First, the two rods were placed into the stands. From there, the clamps were used to connect the electric vibrator and the bucket. Next, the string was attached to both rods, with the end with the bucket strung over the pulley. The vibrator was placed adjacent to the string without it fully touching the string.

After this, the bucket was filled with weights and the vibrator was turned on. Initially, 900 g was placed into the bucket. The weights in the bucket were then adjusted in order for the three-node resonance to occur. Once the nodes were made apparent, the distance between the nodes were measured. The wavelength was then taken by calculating the average of the distances. The tension was then calculated by multiplication the mass inside of the bucket with the gravity. The error from the tension is then found by seeing how much weight it takes to make the standing waves disappear. Finally, the wave velocity was calculated using:

$$v = f\lambda \tag{1}$$

This velocity was then squared, with the error in the velocity squared found using the equation:

$$\delta v^2_{string} = 2v_{string}\delta v_{string} \tag{2}$$

These results were placed into Table 1 as shown below:

Table 1 - Three Node Resonance Data

Node	m (kg)	δm (kg)	d (m)	d_avg (m)	λ (m)	δd (m)	δd_avg (m)	δλ (m)	T (N)	δT (N)	v_string (m/s)	δv_string (m/s)	v^2 (m^2/s^2)	δv^2 (m^2/s^2)
3	1.083	0.071	0.765	0.7675	1.535	0.018	0.018		10.62423	0.69651	184.2		33929.64	
			0.77											

From there, the same process was applied to the four, five, six, and seven node resonance structures, with their respective tables shown below:

Table 2 - Four Node Resonance Data

Node	m (kg)	δm (kg)	d (m)	d_avg (m)	λ (m)	δd (m)	δd_avg (m)	δλ (m)	T (N)	δT (N)	v_string (m/s)	δv_string (m/s)	v^2 (m^2/s^2)	δv^2 (m^2/s^2)
4	0.494	0.023	0.505	0.521666667	1.043333333	0.022	0.022		4.84614	0.22563	125.2		15675.04	
			0.54			0.022								
			0.52											

Table 3 - Five Node Resonance Data

Node	m (kg)	δm (kg)	d (m)	d_avg (m)	λ (m)	δd (m)	δd_avg (m)	δλ (m)	T (N)	δT (N)	v_string (m/s)	δv_string (m/s)	v^2 (m^2/s^2)	δv^2 (m^2/s^2)
5	0.281	0.018	0.378	0.404	0.808	0.017	0.015333333		2.75661	0.17658	96.96		9401.2416	
			0.417			0.014								
			0.421			0.015								
			0.4											

Table 4 - Six Node Resonance Data

Node	m (kg)	δm (kg)	d (m)	d_avg (m)	λ (m)	δd (m)	δd_avg (m)	δλ (m)	T (N)	δT (N)	v_string (m/s)	δv_string (m/s)	v^2 (m^2/s^2)	δv^2 (m^2/s^2)

6	0.176	0.013	0.301	0.3206	0.6412	0.014	0.01625		1.72656	0.12753	76.944		5920.379136	
			0.332			0.018								
			0.327			0.017								
			0.328			0.016								
			0.315											

Table 5 - Seven Node Resonance Data

Node	m (kg)	δm (kg)	d (m)	d_avg (m)	λ (m)	δd (m)	δd_avg (m)	δλ (m)	T (N)	δT (N)	v_string (m/s)	δv_string (m/s)	v^2 (m^2/s^2)	δv^2 (m^2/s^2)
7	0.121	0.011	0.247	0.2625	0.525	0.012	0.0138		1.18701	0.10791	63		3969	
			0.268			0.015								
			0.264			0.013								
			0.268			0.014								
			0.27			0.015								
			0.258											

From there, the tension in the string and the velocity squared were plotted as shown in Figure 1:

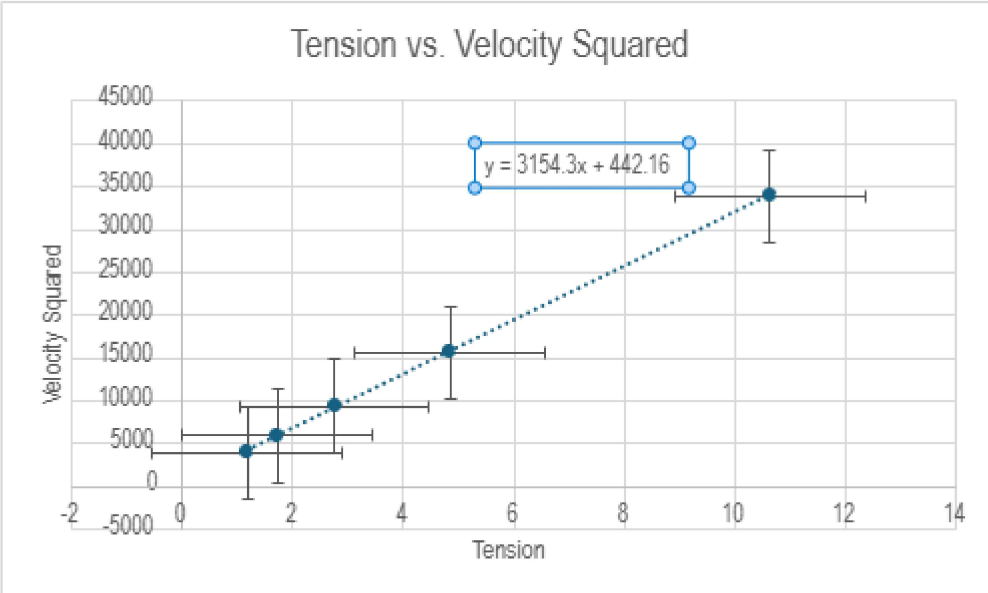

Figure 1 - Velocity Squared as a Function of Tension

Based on the slope, the mass per unit length of the string was found to be .000317 (T/v^2).

Investigation 2

In the second investigation, the equipment included a sound wave apparatus and speaker. In addition a ruler was used for measuring the distance.

First, the signal generator was used to pick a value between the range of 400 and 500 hz. Next, the plug in the tube was moved away from the speaker in order to change the length of the air column inside a tube. Once an increase in volume was heard, the distance was recorded as the resonance was found. This plug was then adjusted twice more for that resonance, with the distance recorded once again. This procedure was then repeated for the third and fifth resonance, with the wavelength found by subtracting the distance between the resonances and multiplying by 2. The results of this are shown in Table 6:

Table 6 - 453 Hz Resonance Data

f (Hz)	δf (Hz)	n	L (m)	L_avg (m)	δL_avg (m)
453		1	0.15	0.165	0.006236096
			0.175		
			0.17		

n	L (m)	L_avg (m)	δL_avg (m)

3	0.538	0.540666667	
	0.54		
	0.544		

n	L (m)	L_avg (m)	δL_avg (m)
5	0.914	0.915666667	
	0.919		
	0.914		

λ (m)	δλ (m)
0.751333333	

Next, the same process was applied with a frequency between 600 - 900 hz and 1000 - 1200 hz. The data for this was recorded in Tables 7 and 8 as shown below:

Table 7 - 736 Hz Resonance Data

f (Hz)	δf (Hz)	n	L (m)	L_avg (m)	δL_avg (m)
736		1	0.108	0.110666667	
			0.112		
			0.112		

n	L (m)	L_avg (m)	δL_avg (m)
3	0.331	0.335666667	
	0.339		
	0.337		

n	L (m)	L_avg (m)	δL_avg (m)
5	0.561	0.559666667	

				0.546	
				0.572	

λ (m)	δλ (m)
0.45	

Table 8 - 1112 Hz Resonance Data

f (Hz)	δf (Hz)	n	L (m)	L_avg (m)	δL_avg (m)
1112		1	0.059	0.059666667	
			0.059		
			0.061		

n	L (m)	L_avg (m)	δL_avg (m)
3	0.218	0.22	
	0.225		
	0.217		

n	L (m)	L_avg (m)	δL_avg (m)
5	0.368	0.372	
	0.38		
	0.368		

λ (m)	δλ (m)
0.320666667	

From there, the wavelength and 1/f were plotted:

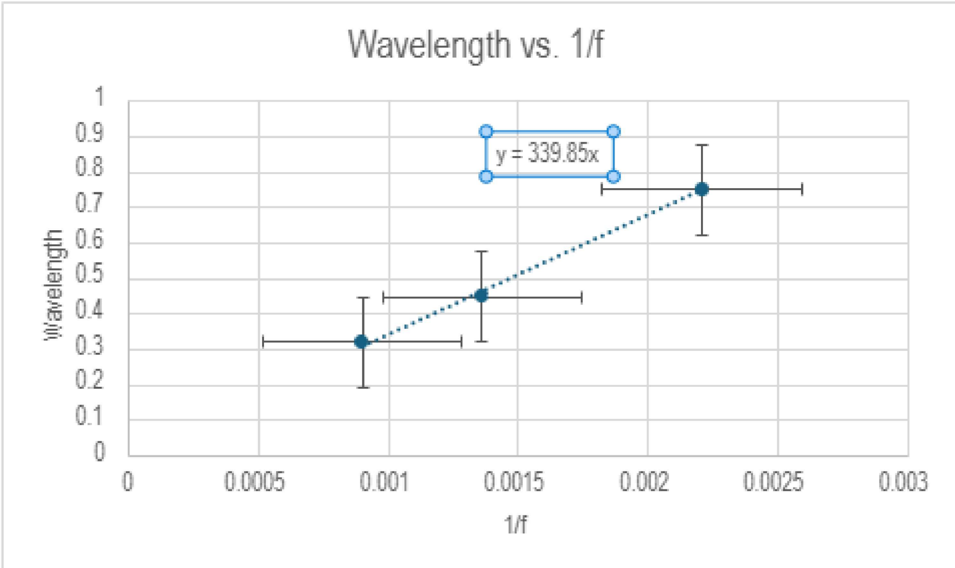

Figure 2 - Wavelength As A Function of 1/f

Based on the slope, the measured speed of sound came out to 339.85 m/s. This did not deviate too much from the actual speed of sound, which is 343 m/s.

Conclusion

The experiments successfully demonstrated the principles of wave mechanics through two distinct investigations. In the first experiment, standing wave patterns on a vibrating string were analyzed, confirming a linear relationship between wave velocity squared and string tension. The calculated mass per unit length of the string aligned closely with theoretical expectations, showcasing the reliability of the experimental setup.

In the second experiment, resonance in a closed air column was observed across various frequencies. The measured wavelengths and frequencies provided accurate calculations for the speed of sound, aligning well with standard values. Minor discrepancies in both experiments were attributed to experimental uncertainties, such as measurement errors in distance and tension.

Overall, these investigations validate theoretical models of wave behavior and demonstrate the utility of experimental methods in understanding resonance and wave mechanics. This work underscores the importance of precise measurements and controlled conditions in accurately studying physical phenomena.

Questions

1. The speed of the wave can be found using the equation:

$$f = \frac{v}{2L}$$

$$248 = \frac{v}{2(.65)}$$

$$248 = \frac{v}{1.3}$$

$$322.4\,\frac{m}{s} = v$$

2. The tension in the string can be found using the equation:
$v = \sqrt{\frac{T}{\mu}}$
$322.4 = \sqrt{\frac{T}{.0005}}$
$103941.76 = \frac{T}{.0005}$
$51.97\, N = T$
3. First, the wavelength can be found using the equation:
$\Delta x = \frac{Wavelength}{2}$
$.48 = \frac{Wavelength}{2}$
$.96\, m = Wavelength$
From there, the frequency can be found using the equation:
$f = \frac{v}{Wavelength}$
$f = \frac{343}{.96}$
$f = 357.29\, Hz$
4. First, the wavelength can be found using the equation:
$Wavelength = \frac{v}{f}$
$Wavelength = \frac{1000}{512}$
$Wavelength = 1.95\, m$
From there, the separation can be found using the equation:
$L_{3/4} - L_{1/4} = \frac{Wavelength}{2}$
$L_{3/4} - L_{1/4} = \frac{1.95}{2}$
$L_{3/4} - L_{1/4} =.98\, m$
5. First, the frequency can be found using the equation:
$f = \frac{v}{Wavelength}$
From there, velocity can be substituted as follows:
$f = \sqrt{\frac{\gamma p}{\rho}} \cdot \frac{1}{Wavelength}$
In addition, pressure can be substituted for temperature using:
$f = \sqrt{\frac{\gamma \rho RT}{\rho}} \cdot \frac{1}{Wavelength}$
$f = \sqrt{\gamma RT} \cdot \frac{1}{Wavelength}$
Therefore, frequency is dependent on the square root of temperature.

Honors Questions

3. The frets in a guitar allow for the precise pitches to play, while having no frets in a violin lets it play microtones between the standard pitches.