chapter 8

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Chapter 8 - Confidence Intervals Based on a Single Sample

Satistical inference - using a sample statistic (e.g. $\overline{x}$ = sample mean) to make some statement or some conclusion about a population parameter (e.g. μ = population mean) There are two types.

- **Confidence interval**
  - Uses a sample statistic to *estimate* the unknown value of a parameter.
  - We write the estimate in the form of an interval that we belive captures the actual or true parameter value.
  - The confidence interval has a specified level of confidence.
- **Hypothesis Test** (or significance test)

Section 8.2 - A Confidence Interval for a Population Mean when σ is Known

- Provided
  - Underlying population has a normal distribution or n is large ($n \geq 30$)
  - σ (population standard deviation) has a known value
- A $100(1-a)\%$ confidence interval to estimate μ is:

$$\overline{x} \pm \left(Z_{\frac{a}{z}} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

- The **critical value** ($Z_a z$) is chosen based on the **level of confidence**

  $\pm Z_{\frac{a}{z}}$ = values from a standard normal distribution that capture the middle $100(1-a)\%$
- Simplified form
  - estimate $\pm$ margin of error
  - **margin of error** = (critical value x **standard error of $\overline{\mathbf{x}}$**)

Question 1

An administrator at a large university wants to estimate μ, the mean GPA of all students on campus. A random sample of $n = 50$ students is selected and the GPA of each student is recorded. The resulting mean is $\overline{x} = 2.60$. Assume that the GPAs in the population are normally distributed with $\sigma = 0.75$

Problem 1. Calculate a 95% confidence interval

its asking for 95%, so our tails are

$$\alpha = .05 \rightarrow \frac{\alpha}{2} = 0.25$$

our z scores are then

$$\pm 1.96$$

So, using the formula

$$\overline{x} \pm \left(Z_{\frac{a}{z}} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

We have

$$2.60 \pm (1.96 \cdot \frac{.75}{\sqrt{50}})$$

our confidence intervals are then

$$2.60 \pm .21$$

$$= 2.39, 2.81$$

Problem 2. Calculate a 90% confidence interval to estimate μ

If its asking for 90% are tails are

$$0.05$$

Our z scores are then

$$\pm 1.6499$$

So, using the formula

$$\overline{x} \pm \left(Z_{\frac{a}{z}} \cdot \frac{\sigma}{\sqrt{n}} \right)$$

We have

$$2.60 \pm (1.6449 \cdot \frac{.75}{\sqrt{50}})$$

So, our confidence intervals are

$$2.60 \pm .17$$

$$= 2.43, 2.77$$

Problem 3. Calculate a 99% confidence interval to estimate μ

99% means are tails are

$$.005$$

our z scores would then be

$$\pm 2.5758$$

So

$$2.60 \pm (2.5758 \cdot \frac{.75}{\sqrt{50}})$$

So, our confidence intervals are

$$2.60 \pm .27$$

$$2.33, 2.87$$

The effect of choosing the confidence level and sample size

The confidence level (90%, 95%, 99% etc.) and the sample size (n) are chosen by the statistician or researcher. It is important to understand how these choices affect the overall length of the confidence interval.

- If the confidence level **increases** then the interval gets **wider** and **less** precise
- If the confidence level **decreases**, then the interval gets **narrower** and **more** precise
- If the sample size n **increases** then the interval gets **narrower** and **more** precise
- If the sample size n **decreases**, then the interval gets **wider** and **less** precise

Question 2

Which confidence interval would be longer and which would be shorter?

(a) 90% confidence and $n = 50$
 95% confidence and $n = 50$

Solution:

95% confidence interval would be longer than 90%

(b) 95% confidence and $n = 50$
 95% confidence and $n = 100$

Solution:

95% confidence and $n = 50$ would be longer

Meaning of confidence

In the previous example

- μ (= mean GPA of the entire campus) is **unknown** but it has a **fixed value**
- After getting the sample and making our calculation, our 95% confidence interval is also **fixed** with endpoints 2.39 to 2.81
- But, if we took another sample we would likely get a different $\bar{x}$ and **different** endpoints than before, and we would still state that we are confident the new interval captures μ.

Sample size calculation

Before the sample is picked

- we specify the desired
  - confidence level
  - bound for the margin of error (B)
- we ask: What size sample is needed?

$$n = \left(\frac{\sigma \cdot Z_{\frac{a}{z}}}{B} \right)^Z$$

Note:-

ALways round n up to a whole number

8-3 A Confidence Interval for a Population Mean when σ is Unknown

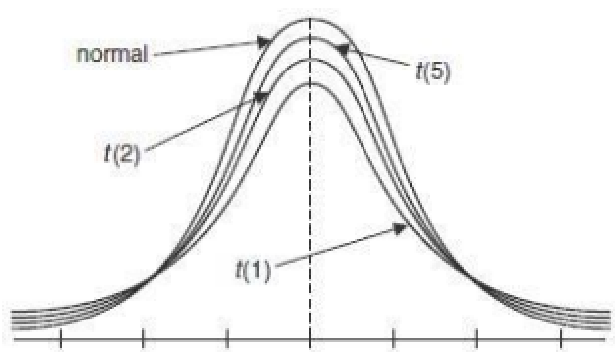
- Provided
  - Underlying population has a normal distribution
  - σ (population standard deviation) has an **unknown value**
  - s (sample standard deviation) has a **known value**
- A 100(1 - a)% confidence interval to estimate μ is

$$\bar{x} \pm \left(t_{\frac{a}{z}} \cdot \frac{s}{\sqrt{n}} \right)$$

- $\pm t_{\frac{a}{z}}$ = critical values from a **t distribution** (with **degree of freedom** $df = n - 1$) that capture the middle 100(1 - a)%

Facts about the t distribution

- It is described by a symmetric, bell-shaped curve that is centered at 0.
- There are many t distributions. Each is identified by giving its degree of freedom (df).
- It is wider and has more spread than the standard normal distribution
- As the df increases the t distribution looks more like the standard normal distribution.



Question 3

In each of the following, find the appropriate t critical value for use in constructing a condidence interval

Problem 1. n = 10, 90% confident

$$df = n - 1$$

$$df = 9$$

$$t_{.05,9} = 1.8331$$

Problem 2. n = 15, 95% confidence

$$df = 14$$

95% confidence means our tails are both .025

$$t_{.025,14} = 2.1448$$

n = 21, 99% confidence

$$df = 20$$

our tails our .005, so looking up 20df and .005, we get

$$t_{.005,20} = 2.8453$$

Question 4

Oil obtained from orange blossoms through distillation is used in perfume. Suppose the oil yield is normally distributed. In a random sample of 11 distillations, the sample mean oil yield was $\bar{x} = 980.2$ g with standard deviation $s = 27.6$ g.

Find a 95% confidence interval for the true mean oil yield per batch.

$$\overline{x} \pm \left(T\frac{s}{\sqrt{n}}\right)$$

$$980.2 \pm \left(T\frac{27.6}{\sqrt{11}}\right)$$

$$df = 10$$

$$\text{tails} = .025$$

$$t_{.025,10} = 2.2281$$

So,

$$980.2 \pm \left(2.2281\frac{27.6}{\sqrt{11}}\right)$$

$$980.2 \pm 18.5$$

We are 95% confident that μ is between 961.7, and 998.7

Question 5

The earth is structured...

using,

$$\overline{x} \pm \left(T\frac{s}{\sqrt{n}}\right)$$

$$127.5 \pm \left(T\frac{21.3}{\sqrt{26}}\right)$$

$$df = 25 \qquad \text{our tails are .05}$$

finding the value in the t-table

$$t_{.05,25} = 1.7081$$

So,

$$127.5 \pm \left(1.7081\frac{21.3}{\sqrt{26}}\right)$$

$$127.5 \pm 1.7081$$

We are 90% confident that μ is between 120.4, and 134.6