Chapter 4 - Probability

Matt Warner

4.1

Experiment

An experiment is any activity in which there are at least two possible outcomes, and the result of the activity cannot be predicted with absolute certainty.

Sample Space

The sample space, denoted as S, is the set of all possible outcomes from an experiment.

Example

Give the sample space for each of the following experiments:

(a) Roll a regular six-sided die once and record the number of spots on the top face.

Solution:
The sample space for rolling a regular six-sided die once is:

$$S_a = \{1, 2, 3, 4, 5, 6\}$$

(b) Flip a coin three times and record the sequence of tosses.

Solution:
The sample space for flipping a coin three times and recording the sequence of tosses is:

$$S_b = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

(c) Pick a student at random and record their gender and grade level.

Solution:
The sample space for picking a student at random and recording their gender and grade level can be represented as:

(d) Select parts from an assembly line until you find a bad part. Record the sequence of G's and B's (for good parts and bad parts, respectively).

Solution:
The sample space for selecting parts from an assembly line until a bad part is found and recording the sequence of G's (good parts) and B's (bad parts) can be represented as:

Events and Operations

Event

Any collection or subset of outcomes from a sample space

(a) For each of the following give the event described.

(i) Let **A** be the event of an even die roll.

(ii) Let **B** be the event of at least two heads.

(iii) Let **C** be the event that the first and last flips are the same.

(iv) Let **D** be the event that a sophomore or junior is selected.

(b) For Part (c) — Describe the following events in words.

(i) $\mathbf{A} = \{(\text{Male, Fresh.}), (\text{Male, Soph.}), (\text{Male, Junior}), (\text{Male, Senior})\}$

(ii) $\mathbf{B} = \{(\text{Male, Fresh.}), (\text{Female, Fresh.})\}$

(iii) $\mathbf{C} = \{(\text{Male, Junior}), (\text{Male, Senior}), (\text{Female, Junior}), (\text{Female, Senior})\}$

Operations for Creating New Events

- **Union $\mathbf{A} \cup \mathbf{B}$** — **A** or **B** (all outcomes from **A** or from **B** or from both)
- **Intersection $\mathbf{A} \cap \mathbf{B}$** — **A** and **B** (all outcomes shared by both **A** and **B**)
- **Complement $\mathbf{A}'$** — not **A** (all outcomes from **S** that are not in **A**)

Note:-

Two events whose intersection is empty (i.e., $\mathbf{A} \cap \mathbf{B} = \{\}$ or $\mathbf{A} \cap \mathbf{B} = \emptyset$) are said to be disjoint or mutually exclusive.

Example

Suppose that a sample space $\mathbf{S} = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}, \mathbf{g}, \mathbf{h}\}$. Use the events

$$A = \{a, b, c\} \quad B = \{b, c, e, g\}, \quad C = \{f, g, h\}, \quad D = \{c, f, h\}$$

To find each of the following.

1. $A \cup B$
2. $A \cap B$
3. $A \cap C$
4. A'

1. $(A \cap B)'$
2. $A \cup B \cup C$
3. $A \cap B \cap D$

Solutions

a $\{a, b, c, e, g\}$

b$\{b, c\}$

c$\emptyset$

d$\{d, e, g, f, h\}$

e$\{a, d, e, f, g, h\}$

f$\{a, b, c, e, g, f, h\}$

g$\{c\}$

4.2 - An Introduction to Probability

Question

We often say "The probability of flipping a coin and getting a head is $\frac{1}{2}$ or 50%"

What precisely is meant by this? Use the table below to help give an interpretation of the probability.

#Tosses	#Heads	%Heads
10	4	40%
100	44	44%
500	265	53%
1000	485	48.5%
5000	2533	50.66%
10000	5025	50.25%

Since tossing a coin is repeatable. In the long run, we flip the coin many times, independently and under similar conditions. approximately half the flips will be heads.

Example

An experiment consists of rolling a 6-sided die once. Suppose that the die has been rigged or tampered with so that the faces are not equally likely (i.e. it's unfair die). Suppose that the sample space and corresponding probabilities are given in the following table.

Roll	1	2	3	4	5	6
Probability	0.30	0.25	0.10	0.15	0.05	0.15

a) what two conditions must be true (or checked) *for* this to be a *legitimate distribution?*

- $\sum$ (probability) = 1.0
- $0 \leq$ each probability ≤ 1

b) Find the probabilites of the following events.

1. A = the event that the roll is an even number
   event $= \{2, 4, 6\}$ $P(A) = .25 + .15 + .15 = .55$
2. B = the event that the roll is at most 3
   event $= \{1, 2, 3\}$ $P(B) = .30 + .25 + .10 = .65$
3. C = the event that the roll is at least 5
   event $= \{5, 6\}$ $.05 + .15 = .20$

Important Rules

- **Complement Rule:** $P(A') = 1 - P(A)$
- **Addition Rule:** $P(A \cup B) = P(A) + P(B) - P(A \cup B)$

Example 0.1

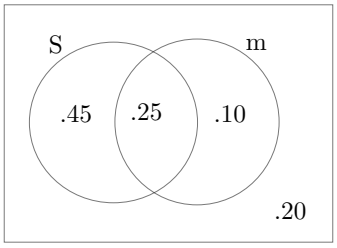
let a and b be events with $P(A) = .30, p(B) = .40$, $P(A \cap B) = .10$. Find the probability that

1. a) A or B occurs
   $.30 + .40 - .10 = .60$
2. A and B occurs
   $P(A \cap B) = .10$
3. c) neither A nor B occur
   1 - $P(A \cup B) = 1 - .60 = .40$
4. d) just A (and not B) occurs
   $P(A \cup B') = .20$

Example 0.2

At a particular coffee shop, suppose that 70% of customers put sugar in their coffee, 35% add milk, and 25% use both. Suppose that a customer of this ocffee shop is selcted at random.

1. Draw a venn diagram to illustrate the events in this problem
2. What is the probability that the customer uses at least one of these two items?
3. What is the probability that the customer uses just sugar?
4. What is the probability that the customer uses just one of these two items?



Solution:

b)A $P(S \cup m) = p(S) = P(m) - p(S \cap m) = .70 + .35 - .25 = .80$

c) .20

d) $(S \cap m') = .45$

e) $= P(s \cap m') \cup P(m \cap s') = .45 + 10 = .55$

---

1 4.4 - Conditional Probability

TThe probability of an event A may be affected by, or depend on the occurrence of another event B.

$$P(A) = \text{ original or unconditional probability of A happening}$$

$$P(A|B) = \text{ Conditional probability of A happening given that B has occurred}$$

Example 1.1
A class contains