Chapter 1 - Differentiation

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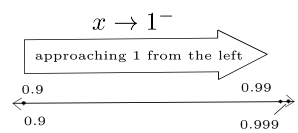
Limits: A numerical and Graphical Approach

Consider the pattern formed by the following sequence of numbers

$$0.9, 0.99, 0.999, 0.9999, \text{ and so on}$$

The numbers appear to be getting closer to the number 1, yet never equal 1 exactly. Assuming that the sequence continues in the same manner, we could say that the *limit* of this sequence of numbers is 1. Note that this sequence of numbers approaches 1 "from the left," meaning that all numbers in the sequence are less than the limit, 1. We write $x \to a$, read "x approaches a from the left", to represent a sequence of numbers that approaches a from the left.

Thus, the sequence 0.9, 0.99,0.999, and so on, is written $x \to 1$



The following sequence approaches 1 "from the right":

$$1.1, 1.01, 1.001, 1.0001, 1.00001, \text{ and so on}$$

We write $x \to a^+$, read "x approaches a from the right" to represent a sequence of numbers that approaches a from the right.

Thus, the sequence 1.1, 1.01,1.001,1.0001,1.00001, and so on, is written $x \to 1^+$

Example 0.1

For each sequence, determine its limit, and rewrite the sequence in the form $x \to a^-$ or $x \to a+$

1. $2.24, 2.249, 2.2499, 2.24999, \ldots$
2. $5.51, 5.501, 5.5001, 5.50001, \ldots$
3. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{31}{32}, \frac{63}{64}$

Solution:

a) $x \to 2.25^-$

b) $x \to 5.5^+$

$c)x \to 1^-$