Chapter 9 Notes

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Chapter 9 – Hypothesis Tests Based on a Single Sample

Section 9.1 – The Parts of a Hypothesis Test and Choosing the Alternative Hypothesis

Statistical inference – using a sample statistic (e.g. $\bar{x}$ = sample mean or $\hat{p}$ = sample proportion) to make some statement or some conclusion about a population parameter (e.g. μ = population mean or p = population proportion). There are 2 types.

- **Confidence interval**
  - Uses a sample statistic to estimate the unknown value of a population parameter.
  - We write the estimate in the form of an interval that we believe captures the actual or true parameter value.
- **Hypothesis test (or significance test)**
  - There are 2 hypotheses.
    - Null hypothesis – labeled H_0
    - Alternative hypothesis (or research hypothesis) – labeled H_A or H_1
  - Each hypothesis states one or more possible values for a parameter.
  - The purpose of the hypothesis test is to decide whether or not the sample data gives evidence against H_0 and in favor of H_A.

Example (Examples 9.1 – 9.3, pp. 392 – 393)

For each of the following, state the null and alternative hypotheses.

(a) Trust, in general, has been declining for many years; most concerning, trust has been decreasing in public health and safety institutions and workers. According to a recent survey, only 58% of Americans trust doctors. Suppose a national advertising campaign is conducted to address confidence in doctors and medical leaders, and an experiment is conducted to determine whether it has been effective.

$$H_o : p = .58$$

$$H_A : p > .58$$

(b) Even though the roads are crowded, it was reported that the mean number of kilometers driven per year by each Australian driver is 13,716. Over the past year, more public transportation has been made available in most large cities to encourage people to drive less and to use buses and trains instead. An observational study is conducted to determine whether the mean number of kilometers driven each year has decreased

$$H_o : \mu = 13716$$

$$H_A : \mu < 13716$$

(c) The variance in thickness for 20-lb printer paper at a manufacturing plant is known to be 0.0007. A new process is developed that uses more recycled fiber, and an experiment is conducted to detect any difference in the variance in paper thickness.

$$H_o : \sigma^2 = .0007$$
$$H_A : \sigma^2 \neq .0007$$

Note:-

We always believe the null hypothosies untill we can show the alternative is true

1 General Approach or Overview

When conducting hypothesis testing, it is essential to follow a structured approach to ensure clarity and rigor in the analysis. The general approach consists of the following steps:

1. **State the Null and Alternative Hypotheses:** Begin by clearly stating the null hypothesis (H_0) and the alternative hypothesis (H_A). Additionally, decide on the level of significance, denoted as α.
2. **Calculate the Test Statistic:** Calculate the test statistic. This statistic provides a measure of how close or consistent the sample data is to the hypothesized value of the parameter.
3. **Give the Rejection Region:** Define the rejection region, which is the range of values of the test statistic that would lead to the rejection of H_0.
4. **Give a Decision and Write a Conclusion:**
   - If the test statistic falls within the rejection region:
     - Reject H_0 and conclude in favor of H_A.
     - The data or results are considered statistically significant.
     - For the conclusion, write: "The sample does have enough evidence, at level α, to support H_A."
   - If the test statistic does not fall within the rejection region:
     - Do not reject H_0.
     - The data or results are not considered statistically significant.
     - For the conclusion, write: "The sample does not have enough evidence, at level α, to support H_A."

   IMPORTANT NOTES:

- The primary goal is to provide support for H_A.
- Initially, we assume that H_0 is true until the sample data provides sufficient evidence to support H_A. This concept is analogous to the legal system's presumption of innocence until guilt is proven.
- If the sample data fails to support H_A, it does not imply that H_0 is proven true. This is akin to the legal system, where failure to prove guilt does not equate to proving innocence.
- Avoid using phrases like "Accept H_0" or "There is evidence to support H_0."

Section 9.2 - Hypothesis Test Errors

Our hypothesis testing procedure could lead to an incorrect decision

	Reject H_0	**Do not reject H_0**
H_0 is true	Type I Error - Rejecting H_0 even though H_0 is true.	Correct
H_A is true	Correct	Type II Error - Not Rejecting the null when H_A is true.

2 Type I and Type II Errors in Hypothesis Testing

In the context of statistical hypothesis testing, it's essential to understand the concepts of Type I and Type II errors, which play a crucial role in decision-making and drawing conclusions. These errors are often associated with the acceptance or rejection of null hypotheses.

2.1 Type I Error (False Positive)

Definition: Type I error, also known as a false positive, occurs when a statistical test incorrectly rejects a null hypothesis that is actually true. In other words, it represents a false alarm or an erroneous positive result.

Symbol: Type I error is typically denoted as α (alpha).

Example: An example of Type I error is when a study concludes that a new drug is effective in treating a medical condition when, in reality, the drug has no therapeutic effect.

2.2 Type II Error (False Negative)

Definition: Type II error, also known as a false negative, occurs when a statistical test fails to reject a null hypothesis that is actually false. In this case, the test overlooks a genuine effect or difference.

Symbol: Type II error is often denoted as β (beta).

Example: An example of Type II error is when a security system fails to detect unauthorized access to a computer network, missing a legitimate security breach.

Example (Exercise 9.47, p.401) – Hypothesis Testing for Highway 405

Highway 405, running from northern to southern California, is famously known as the busiest interstate road in the United States, with approximately 374,000 vehicles using it daily, often referred to as "Carmageddon." Transportation officials are considering raising tolls in California to fund planned repairs. To make this decision, they will conduct a hypothesis test to determine whether there is evidence suggesting that the mean number of vehicles per day on Highway 405 has increased.

State the Null and Alternative hypothesis

$$H_o : \mu = 374,000$$
$$H_A : \mu > 374,000$$

Describe the type I and type II errors.

Type I - deciding traffic increased when it really didnt

Type II - Deciding traffic didnt increase when it in fact has

If a type I error is committed, who is more angry?

Deciding higher traffic brings higher tolls even though traffic really didn't increase

Therefore, The drivers are more angry

If a type II error is committed, who is more angry?

Deciding traffic didn't increase means state misses out on toll money.

So, the transportation officials are more angry

The Logic of Hypothesis Testing

Suppose that we want to test the following hypotheses.

$$H_0: \text{ coin is fair}$$

$$H_A: \text{ coin is unfair}$$

Sample Data - Suppose that 20 tosses of the coin result in 11 heads and 9 tails

What would be the decision?

Reject H_0
Do not reject $H_0 \rightarrow$ This one

Why?

- When H_0 is true we would expect to get around 10 heads and 10 tails
- The observed 11 heads and 9 tails is
  - Close to what we expect
  - likely to happen

Sample Data - Suppose the 20 tosses of the coin results in 19 heads and 1 tail

What would be the decision?

Reject $H_0 \rightarrow$ This one
Dont reject H_0

Why?

- When H_0 is true we would expect to get around 10 heads and 10 tails
- The observed 19 heads and 1 tail is
  - Not close to what we expect
  - Unlikely to happen

Note:-

The observed data being close to/far from what we expect will be determined by the rejection region of the test

The observed data being likely/unlikely to happen will be determined by the p value of the test.

Section 9.3 - Hypothesis Tests Concerning a Population Mean When σ is Known

Provided

- Underlying population has a normal distribution or n is large ($n \geqslant 30$)
- σ (population standard deviation) has a known value

To test the null hypothesis

$$H_0 : \mu = \mu_0$$

versus one of these alternative hypotheses

$$H_A : \mu < \mu_0$$
$$H_A : \mu > \mu_0$$
$$H_A : \mu \neq \mu_0$$

Use the test statistic

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Note:-

The standard deviation

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Is often called the standard error of $\bar{x}$

Rejection Region

If (i): Reject H_0 if $Z \leqslant -Z_a$

If (ii): Reject H_0 if $Z \geqslant Z_a$

If (iii): Reject H_0 if $Z \leqslant -Z_{\frac{a}{2}}$ or if $Z \geqslant Z_{\frac{a}{2}}$

Example

The principal of a high school claims that μ, the mean SAT score for all graduates of his school, is higher than the national average of 500. To prove his claim, he selects a sample of 75 recent graduates and calculates a sample mean of $\bar{x} = 520$. Does the sample provide enough evidence to support his claim? We may assume that the SAT scores for all graduates of his high school follow a normal distribution with a population standard deviation of $\sigma = 100$. Test using a level of significance $\alpha = 0.05$.

Problem 1. Give the population and the parameter

Our population:

all grads of this highschool who took the sat

our parameter:

$$\mu = \text{ Mean SAT score for this population}$$

Problem 2. State the null and alternative hypotheses

$$H_0 : \mu = 500$$

$$H_A : \mu > 500$$

Problem 3. Calculate the test statistic

Using the formula

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

We get

$$\frac{520 - 500}{\frac{100}{\sqrt{75}}} = 1.73$$

Note:-

When $H_0 : \mu = 500$ is true, $\bar{x}$ should be around 500

Is $\bar{x} = 520$ close to or far from 500?

Problem 4. Find the rejection region

Problem 5. Give the decision and write a conclusion to the problem

Question 1

Buiness and management (in the book)

$$H_0 : \mu = 51500$$
$$H_A : \mu < 51500$$
$$\alpha = .01$$

Big sample and population standard deviation tells us this is a Z test

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

So,

$$\frac{49762 - 51500}{\frac{3750}{\sqrt{38}}}$$
$$= -2.86$$

Looking up the z score we get in the table we get

$$Z = -2.3263$$

Reject H_0 ***if Z*** $\leq$ ***-2.3263*** So, we reject H_0, there is enough evidence at the 1% level, to conclude mean salaray has decreased.

Note:-

if $<$, we use the left end of the tail

if $>$, we use the right end of the tail

9.4 - p Values

The **p value** of a hypotheses test

- measures the strength of the evidence against H_0
- indicates how likely the sample data was to have occured when H_0 is true
- p value = probability of obtaining another test statistic value at least as extreme as the actual (or observed) value of the test statistic, when H_0 is true
- the **smaller** the p value the **more evidence** there is **against** H_0 and **in favor** of H_A
    - If p value $\leqslant \alpha$ then **reject** H_0
    - If p value $> \alpha$ then **do not reject** H_0

Example (SAT problem)

Hypothesis

$$H_0 : \mu = 500$$
$$H_A : \mu > 500$$

Find the area beyond 1.73

$$1 - .9582$$
$$= .0418$$

Since the p value is less than alpha, we reject the null

9.5 - Hypotheses Tests Concerning a Population Mean when σ is unknown

Provided

- Underlying population has a normal distribtution
- σ has an unknown value
- s has a known value

To test the null hypothesis

$$H_0 : \mu = \mu_0$$

Versus one of these alternative hypothesis

(i) $H_A : \mu < \mu_0$

(ii) $H_A : \mu > \mu_0$

(iii) $H_A : \mu \neq \mu_0$

Use the test statistic

$$T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Note:-

the standard deviation $\hat{\sigma}_{\bar{X}} = \frac{s}{\sqrt{n}}$ is often called the (estimated) standard error of $\bar{X}$.)

Rejection Region:

- Use the t distribution with degrees of freedom $df = n - 1$.
- If (i): Reject H_0 if $T \leqslant -t_\alpha$.
- If (ii): Reject H_0 if $T \geqslant t_\alpha$.
- If (iii): Reject H_0 if $T \leqslant -t_{\alpha/2}$ or if $T \geqslant t_{\alpha/2}$.

Question 2

A major car manufacturer wants to test a new engine to determine whether it meets air pollution standards. The mean emission, μ, of all engines of this type must be less than 20 parts per million of carbon. Ten engines are manufactured for testing purposes and their emission levels yield a sample mean of $\bar{x} = 17.57$ and a sample standard deviation of $s = 2.95$. Does the sample give enough evidence to conclude that this type of engine meets the pollution standard? Assume emission levels for all engines of this type are normally distributed. Test using the level of significance $\alpha = 0.05$.

Problem 1. Give the population and the parameter

Population: All engines of this type produced by the manufacturer

Parameter: The mean emission level

Problem 2. State the null and alternative hypotheses

$$H_0 : \mu = 20$$

$$H_A : \mu < 20$$

Problem 3. Calculate the test statistic

Using the formula

$$T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

We have

$$T = \frac{17.57 - 20}{\frac{2.95}{\sqrt{10}}}$$

$$T = -2.60$$

Problem 4. Find the rejection region

Finding our rejection region, we have

$$\alpha = .05$$

$$t_{.05,9} = -1.8331$$

So,

Since, $T \leqslant t_a$, we reject the null

Question 3

Most water treatment facilities monitor the pH of their drinking water on an hourly basis. One such water treatment facility has a target pH of 8.5. During one particular hour, 17 water specimens were selected and yielded a sample mean of $\bar{x} = 8.42$ and a sample standard deviation of $s = 0.16$. Does the sample give enough evidence to conclude that the mean pH level differs from 8.5? Assume the pH levels vary according to a normal distribution. Test using the level of significance $\alpha = 0.05$.

Problem 1. Give the population and parameter

Population: All water at this facility

Parameter: Mean pH levels

$$H_0 : \mu = 8.5$$

$$H_A : \mu \neq 8.5$$

This will be a two tailed test

$$T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

So,

$$\frac{8.42 - 8.5}{\frac{.16}{\sqrt{17}}} = -2.06$$

$$df = n - 1$$

$$17 - 1$$

$$= 16$$

$$\alpha = .025$$

critical test statistic:

$$-2.1199, 2.1199$$

Reject H_0 if

$$T \geqslant 2.1199$$

$$T \leqslant -2.1199$$

So, we reject, the sample does not have enough evidence, at $\alpha = .05$ to show μ differes

The p value is between .025&.05, and on the right .025&.05

$$.05 < \text{ p-value } < .10$$

Since p is bigger than alpha, we reject H_0

Question 4

Biology and Enviromental Science

We have

$$n = 26 \quad \mu = 31.9 \quad \bar{x} = 30.088 \quad s = 4.433 \quad \alpha = .01$$

$$H_0 : \mu = 3.19$$

$$H_A : \mu < 31.9$$

No σ, so its a t test

$$\frac{30.088 - 31.9}{\frac{4.433}{\sqrt{26}}} = -2.08$$

$$t_{25,.01} = -2.4851$$

Reject H_0 if

$$T \leqslant -2.4851$$

Do not reject H_0, The sample does not have evidence at the $\alpha = 1\%$ to show the mean yield decreased For p value, we have

$$T_{25,?} = -2.08$$

looking in the table, we see that the p-value is between .01, and .025

We can conclude that the p value is bigger than alpha, so we reject H_0

Question 5

overweight study....

problem 1.

Population: All 8th-graders from this district

Parameter: percent of population that is overweight

9.6 - Large-Sample Hypothesis Tests Concerning a Population Proportion

Previously:

- Measurements or data values were quantitative.
- Population parameter was $\mu =$ population mean.
- Sample statistic was $\bar{x} =$ sample mean.

Now:

- Measurements or data values will be qualitative.
- Population parameter: $p =$ proportion of individuals in the population with a specified characteristic.
- Sample statistic: $\hat{p} =$ proportion of individuals in the sample with the specified characteristic.

Provided:

- n is large enough that both $np_0 \geqslant 5$* and $n(1-p_0) \geqslant 5$* are true.
  *Comparing each against the values 10 or 15 are common alternative conditions.

To test the null hypothesis

$$H_0 : p = p_0$$

Versus one of these alternative hypotheses:

(i) $H_A : p < p_0$

(ii) $H_A : p > p_0$

(iii) $H_A : p \neq p_0$

Use the test statistic

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

(Note – the standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}}$ is often called the standard error of $\hat{p}$.)

Rejection Region:

- If (i): Reject H_0 if $Z \leqslant -z_\alpha$.
- If (ii): Reject H_0 if $Z \geqslant z_\alpha$.
- If (iii): Reject H_0 if $Z \leqslant -z_{\alpha/2}$ or if $Z \geqslant z_{\alpha/2}$.

Question 6

A recent study claimed that 15% of all 8th grade students are overweight. Health administrators in a particular school district selected a random sample of 120 8th grade students and found 10 to be overweight. Does the sample give enough evidence to conclude that this district has a lower percent of overweight 8th grade students? Test using the level of significance $\alpha = 0.05$.