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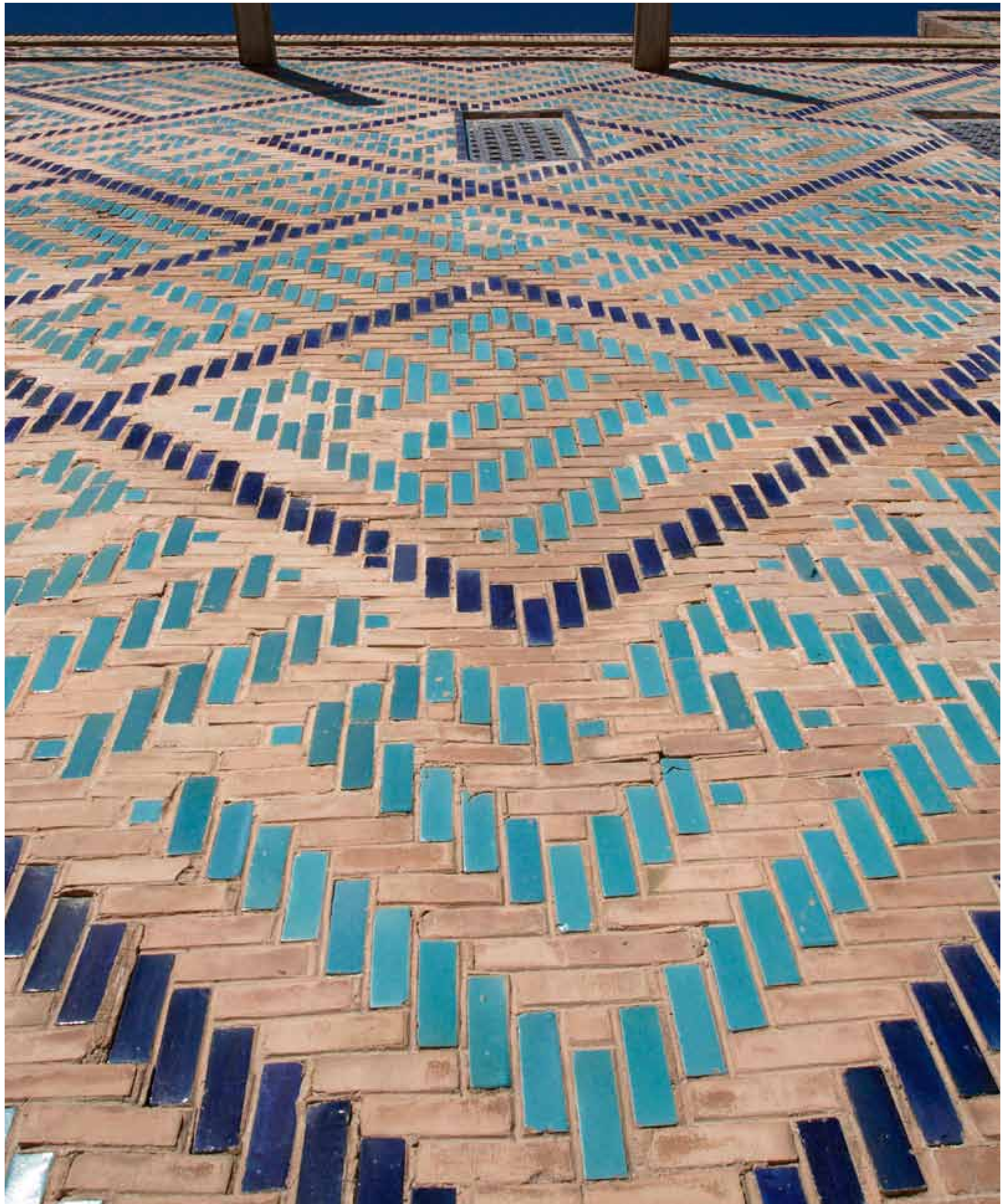


MEDIEVAL MOSAICS
Reveal Advanced Mathematics

By Samantha Holmes and Peter J. Lu

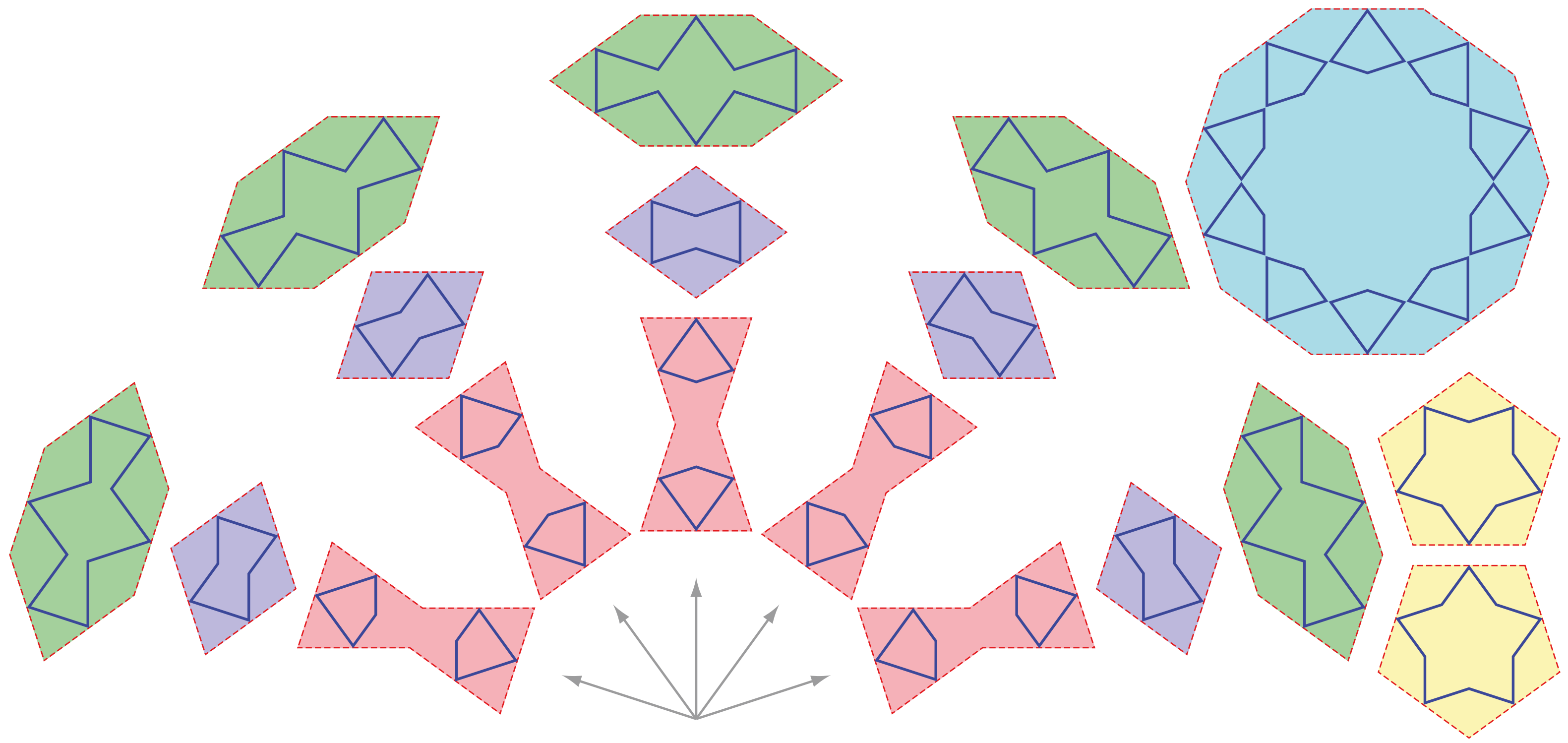
The 2011 Nobel Prize in Chemistry was awarded to Israeli researcher Dan Shechtman for his discovery of quasicrystals, materials whose atoms occur in complex patterns that can be expanded infinitely yet never repeat. Such a structure was considered impossible until Schechtman synthesized it for the first time in the early 1980s, changing our understanding of what kinds of formal symmetries can exist in nature. Immediately after, Tthe atoms in quasicrystals were immediately shownrecognized by Paul J. Steinhardt and Dov Levine to occur at the corners of tiles in Penrose patternstilings, non-repeating patterns with five-fold symmetry first theorized by British mathematician Roger Penrose in the 1970s, that have five fold-symmetry (that cannot occur in ordinary crystals) and never repeat.. Yet recent studies have found this same geometry and underlying symmetry in medieval Islamic mosaic tilings dating back as far as the 15th century.

It has traditionally been assumed in the West that the intricate geometrical patterns found in Islamic architecture and mosaic were created using elaborate manipulations of two simple tools, the straightedge and the compass, following constructions established by the ancient Greeks. Complex patterns would be traced across the entirety of large architectural surfaces. Yet such a mode of execution, carried out at the scale and complexity of grand mosques, shrines and palaces, would have been extremely difficult to draft accurately, resulting in geometric distortions – distortions that aren't found in the finished work.

In 2006, with the support of Harvard and Princeton universities, Steinhardt and Peter J. Lu set out to analyzed mosaics from the medieval Islamic period that haverepresent five- and ten-fold motifssymmetries, those whichat are highlighted also in quasicrystalline Penrose patterns. After examining hundreds of the geometric star-and-polygon patterns adorning boththe surfaces of architectural monuments across the medieval Islamic world, andas well as contemporaneous scrolls that documented architectural practice from that era, they found that, rather than requiring the recreation of each linear element *in situ*, decagonal Islamic mosaic patterns were likely created by the repeated juxtaposition of five larger units, or "girih tiles" (figure 1). Each of these tiles represents a single portion of the overall design, ornamented with specific linear patterns and drawn using the established methods; when assembled together like a jigsaw puzzle, the linear markings of these larger tiles connect to form a single unified pattern across the entire architectural surface. This method enabled a new level of complexity in Islamic geometric mosaics, generating the kind of five- and tenfold symmetries discovered in the non-repeating patterns of Shechtman's quasicrystals half a millennium later.

Entrance of the Abdullah Khan Madrasa, in Bukhara, Uzbekistan (1588 AD). The band of decoration at the left side of the portal, as well as the wooden door, show tessellations of five- and ten-fold motifs that can be reconstructed as tessellations of the girih tiles.

Entrance portal of the Guri Amir, mausoleum of Timurid ruler Amir Timur (Tamerlane) in Samarkand, Uzbekistan (1403-4 AD), with bands of decagonal decoration on the left and right that can be reconstructed as tessellations of the girih tiles.



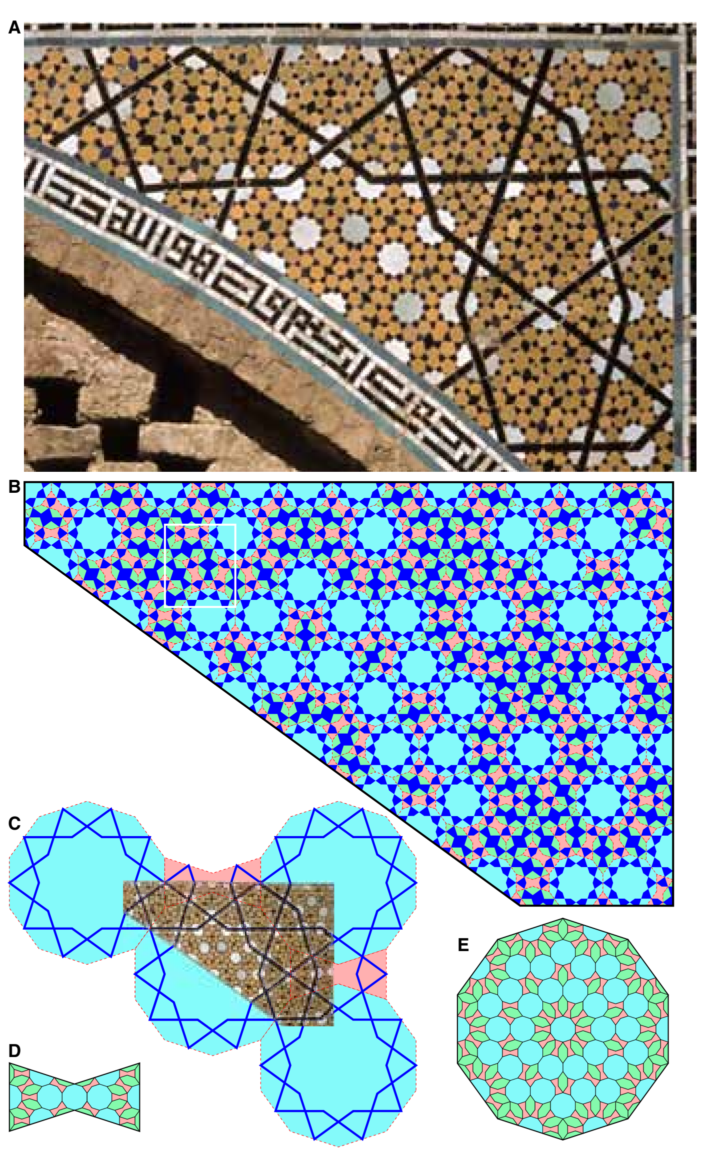
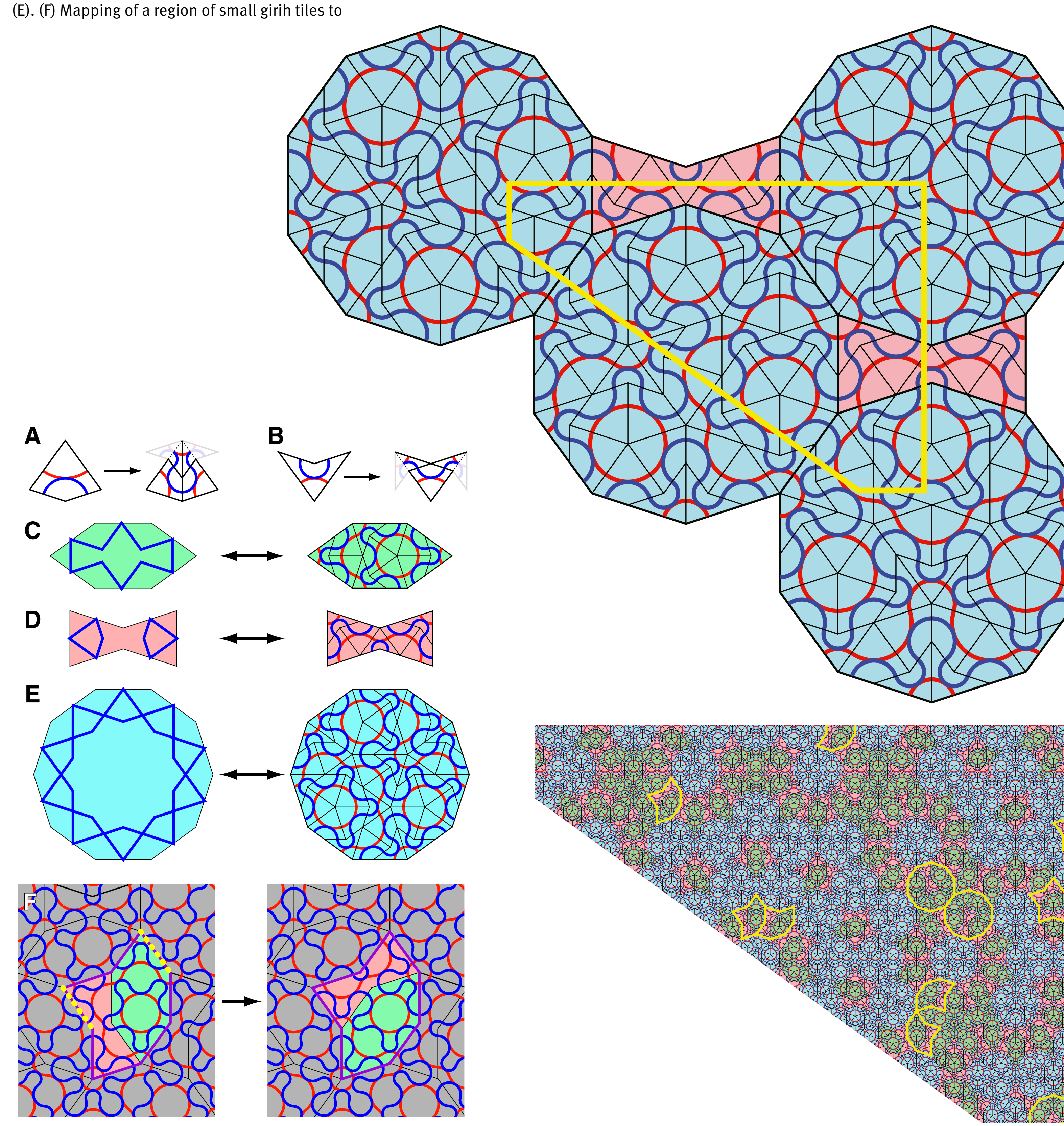


Fig. 1: The complete set of girih tiles: decagon, pentagon, hexagon, bowtie, and rhombus. The tiles would be assembled as a jigsaw puzzle, with the blue-line pattern transferred to the building, forming the physical decorative pattern with decagonal motifs. The edges of the tiles themselves were not preserved in the vast majority of patterns. Figure reproduced from Lu and Steinhardt 2007.

Fig. 2: Girih-tile subdivision found in the decagonal girih pattern on a spandrel from the Darb-I Imam shrine, Isfahan, Iran (1453 C.E.). (A) Photograph of the right half of the spandrel. (B) Reconstruction of the smaller-scale pattern using girih tiles where the blue-line decoration in Fig. 1 has been filled in with solid color. (C) Reconstruction of the larger-scale thick line pattern with larger girih tiles, overlaid on the building photograph. (D and E) Graphical depiction of the subdivision rules transforming the large bowtie (D) and decagon (E) girih-tile pattern into the small girih-tile pattern on tilings from the Darb-i Imam shrine and Friday Mosque of Isfahan. Figure reproduced from Lu and Steinhardt 2007.

Fig. 3: (A and B) The kite (A) and dart (B) Penrose tile shapes are shown at the left of the arrows with red and blue ribbons that match continuously across the edges in a perfect Penrose tiling. Given a finite tiling fragment, each tile can be subdivided according to the “inflation rules” into smaller kites and darts (at the right of the arrows) that join together to form a perfect fragment with more tiles. (C to E) Mappings between girih tiles and Penrose tiles for elongated hexagon (C), bowtie (D), and decagon (E). (F) Mapping of a region of small girih tiles to Penrose tiles, corresponding to the area marked by the white rectangle in Fig. 3B, from the Darb-i Imam shrine. At the left is a region mapped to Penrose tiles following the rules in (C) to (E). The pair of colored tiles outlined in purple have a point defect (the Penrose edge mismatches are indicated with yellow dotted lines) that can be removed by flipping positions of the bowtie and hexagon, as shown on the right, yielding a perfect, defect-free Penrose tiling. Figure reproduced from Lu and Steinhardt 2007.



The five girih tiles used to generate these patterns — decagon, pentagon, hexagon, bowtie, and rhombus — all bear edges of equal lengths, allowing them to be juxtaposed at will. Each individual unit contributes to the complexity of the overall pattern with its own internal symmetry: the decagon, 10-fold symmetry; the pentagon, five-fold; and the hexagon, bowtie, and rhombus, two-fold.

The girih tiles were used to create a wide range of architectural tilings throughout the medieval Islamic world, and reached an artistic, technical and geometric zenith during the Timurid dynasty. Notable examples from major Timurid sites in Uzbekistan include the Guri Amir (1403-4 AD) and Ulugh Beg Madrasa (1417-21) in Samarkand, and the Abdullah Khan madrasa (1588 AD) in Bukhara. The tiles were used in a particularly sophisticated way when their geometrical structure was applied at different scales within the same mosaic — each larger girih tile comprised of a collection of smaller ones — to create the overlapping patterns typical of Islamic mosaic. This layering can be seen in the Darb-i Imam Shrine and the Isfahan Friday Mosque in Iran.

At the Darb-i Imam shrine, a small-scale girih-tile tesselation is formed of filled-in black motifs (figs. 2A-B), while a second, larger pattern of thick black lines is created from the same girih tiles, but at a much greater length scale (fig 2C).

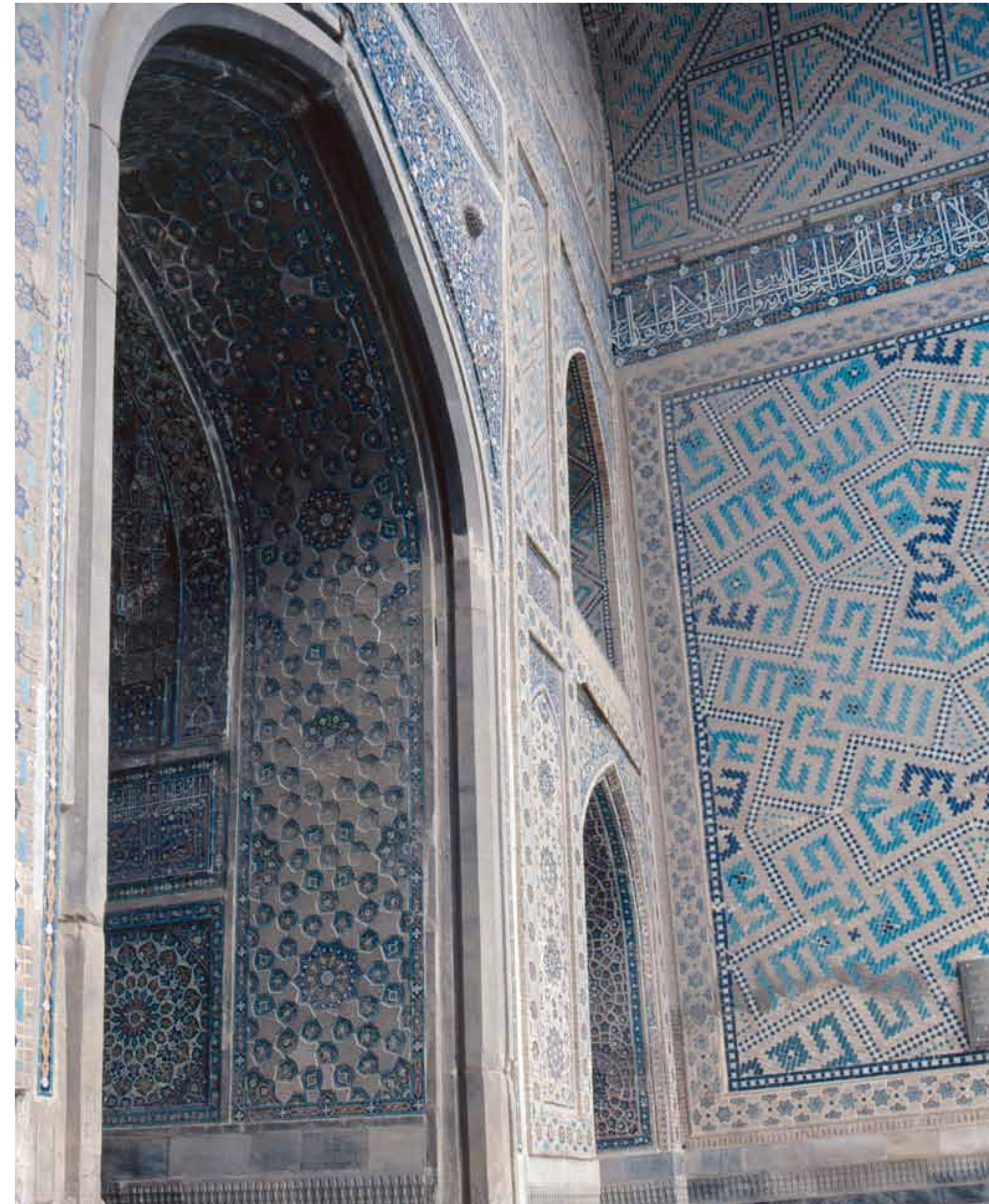
At the Darb-i Imam shrine, a small-scale girih-tile tesselation has filled in black motifs (figs. 2A-B). But there is a thick black line pattern which forms the decoration of a tessellation created from the same girih tiles, but at a much larger length scale (fig 2C). Interestingly, the combination of little girih tiles that fits into the outlines of each biglarge tile is the same in all cases, and reflects a deliberate subdivision of the large pattern into the small: that is, the designers started with a simple combination of large tiles, and subdivided them in a specific way to generate the pattern of small tiles (fig. 2D-E). Moreover, each girih tile can be replaced by a combination of Penrose tiles---strikingly, at both length scales---demonstrating that the tiling dating from 1450 AD is in fact a fragment of a large Penrose pattern.

Of the 3700 Penrose tiles in the Darb-i Imam shrine, only 11 are out of place, a figure that seems accounted for by human error in production or subsequent restoration (figure 3). The overwhelming consistency of the pattern suggests that advanced mathematical knowledge was required to lay these tiles in this particular combination, a hypothesis confirmed by documented discussions between practicing architects and mathematicians. And the outlines of the girih tiles found within the Topkaki scroll, which documents the architectural patterns used on buildings at that time, indicate that the tiles are not simply a pattern *recognizable* in Islamic decoration, but a tool utilized in their creation — a conceptual breakthrough applied in mosaic many centuries before theirits widespread usewidespread acceptance in the West.

Additional information can be found in the paper «Decagonal and Quasi-crystalline Tilings in Medieval Islamic Architecture» Science (2007), as well as on the author's website at http://www.peterlu.org.

Photograph by Peter J. Lu.

Peter J. Lu received his AB summa cum laude in physics (2000) from Princeton University, and AM (2002) and PhD (2008) in physics from Harvard University. He is presently a post-doctoral research fellow in the department of Physics and SEAS at Harvard University, where his main focus is on the physics of attractive colloids and the integration of high-performance imaging and analysis techniques. He conducts experiments aboard the International Space Station, examining phase separation of colloid mixtures in the absence of gravity. He has published his discoveries of modern quasicrystal geometry in medieval Islamic architectural tilings; the first precision compound machines, from ancient China; the first use of diamond, in prehistoric China; and the first quasicrystalline mineral found in nature.



Entrance portal of the madrasa of Ulugh Beg, grandson of Amir Timur, one of three large madrasas that forms the central Registan square in Samarkand, Uzbekistan (1417-21 AD). The main archway has a large band of decoration with decagonal motifs in deep relief, which can be reconstructed as tessellations of the girih tiles.