Section 4.4 Notes

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Logarithmic Functions

Recall:

1. Exponential functions are one-to-one with H.A. $y = 0$
2. One-to-one functions have inverse functions

Note:-

The inverse of exponential functions is the logarithmic function.

Definition 0.0.1

Leta be a positive real number different from 1. The logarithm of **x** with base a is defined by $\log_a x = y$ if and only if $x = a^y$ for every $x > 0$ and every real number y.

Example 1: Change to logarithmic form

a) $3^{-4} = \frac{1}{81}$

Soultion: ☺

$$\log_3 \frac{1}{81} = -4$$

b) $(0.9)^t = \frac{1}{2}$

Solution: ☺

$$\log_{0.9} \frac{1}{2} = t$$

Example 2: Change to exponential form

a) $\log_a \frac{1}{256} = -4$

Solution: ☺

$$a^{-4} = \frac{1}{256}$$

b) $\log_a 343 = \frac{3}{4}$

Solution: ☺

$$a^{\frac{3}{4}} = 343$$

Example 3: Find the exact value of each logarithm

a) $\log_5 \sqrt[3]{25}$

Solution:

$$5^x = 25^{1/3}$$
$$5^x = \left(5^2\right)^{1/3}$$
$$5^x = 5^{2/3}$$
$$\frac{2}{3} \rightarrow solution$$

b) $\log_{\sqrt{3}} 9$

Solution:

$$(\sqrt{3}^3) = 9$$
$$(3^{\frac{1}{2}})^x = 3^2$$
$$\frac{1}{2}x = 2$$
$$x = 4$$

Example 4: Find the domain of each function

a) $f(x) = 6 + 4\log_3(2x - 1)$

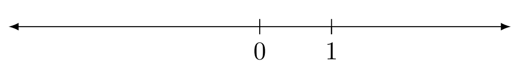
Solution:

$$2x - 1 > 0$$
$$2x > 1$$
$$x > \frac{1}{2}$$

b) $f(x)\log_2\left(\frac{x}{x-1}\right)$

Solution:

$\frac{x}{x-1} > 0$



$$(-\infty, 0) \quad (0, 1) \quad (1, \infty)$$
$$\frac{(-)}{(-)} \quad \frac{(+)}{(-)} \quad \frac{(+)}{(+)}$$

Only the Positive Outputs will be included in our Solution set.

So,

$D : (-\infty, 0) \cup (1, \infty)$

Definition 0.0.2

Common Logarithm: (Logarithm with base 10) $\log x = \log_{10} x$ for every $x > 0$ Natural Logarithm: (Logarithm base e) $\ln x = \log_e x$ for every $x > 0$

Note:-

Properties of logarithms
1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a a^x = x$
4. $a^{\log_a x} = x$

Example 5: Find the number if possible

a) $10^{\log 3}$

Solution: 😄

$$10^{\log_{10} 3}$$

$$= 3$$

b) $\log 0.00001$

Solution: 😄

$$\log_{10} 10^{-5}$$
$$= -5$$

c) $\log_5 0$

Solution: 😄

Undefined

d) $5^{\log_5 4}$

Solution: 😄

$$5^{\log_5 4} = 4$$

e) $e^{1 \ln 5}$

Solution: 😄

$$e^1 \cdot e^{\ln 5}$$
$$e \cdot e^{\log_e 5}$$
$$e \cdot 5$$
$$= 5e$$

Example 6: Find the domain. Sketch the graph. From the graph determine the range and any asymptotes of f. Find f^{-1}. Graph f^{-1}.

a) $f(x) = \log_5(3 - x)$

Solution: 😃