Section 4.3 Notes

Matt Warner

EXPONENTIAL FUNCTIONS

Terminology

Exponential functions f with base a
$\rightarrow$ Domain = $(-\infty, \infty)$
$\rightarrow$ Range = $(0, \infty)$

Definition 0.0.1

$f(x) = a^x$ for every x in R where $a > 0$ and $a \neq 1$

Theorem

The exponential function f given by $f(x) = a^x$ for $0 < a < 1$ or $a > 1$ is one-to-one. Thus, the following conditions are satisfied for real numbers x_1 and x_2.

1. if $x_1 \neq x_2$, then $a^{x}1 \neq a^{x}2$
2. if $a^{x}1 = a^{x}2$, then $x_1 = x^2$

Recall: Laws of Exponents

1. $a^m \cdot a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $a^{-n} = \frac{1}{a^n}$
4. $(a \cdot b)^n = a^n \cdot b^n$
5. $\left(\frac{a}{b}\right)^{-n} = \frac{b^n}{a^n}$
6. $\left(a^m\right)^n = a^{m \cdot n}$
7. $a^0 = 1$

Example 1 Solve the equations.

a) $9^{\left(x^2\right)} = 3^{3x+2}$

b) $\left(\frac{1}{2}\right)^{6-2x} = 2$

c) $9^{2x} \cdot 27^{x^2} = 3^{-1}$

d) $\left(e^4\right)^x \cdot e^{x^2} = e^{12}$

Solution to Question 1: 😃

$$\begin{aligned} (3x^2)^{x^2} &= 3^{3x+2} \\ 3^{2x^2} &= 3^{3x+2} \\ 2x^2 &= 3x+2 \\ 2x^2 - 3x - 2 &= 0 \\ x &= -\frac{1}{2}, x = 2. \end{aligned}$$

Soultion to Question 2: 😃

$$\begin{aligned} \left(2^{-1}\right)^{6-2x} &= 2 \\ 2^{-6+2x} &= 2^1 \\ -6+2x &= 1 \\ 2x &= 7 \\ x &= \frac{7}{2} \end{aligned}$$

Solution to Question 3: 😃

$$\begin{gathered} \left(3^2\right)^{2x} \cdot \left(3^3\right)^{x^2} = 3^{-1} \\ 3^{4x} \cdot 3^{3x^2} = 3^{-1} \\ 3^{4x+3x^2} = 3^{-1} \\ 4x + 3x^2 = -1 \\ 3x^2 + 4x + 1 = 0 \\ (3x+1)(x+1) = 0 \\ x = -\tfrac{1}{3}, -1 \end{gathered}$$

Soultion to Question 4: 😃

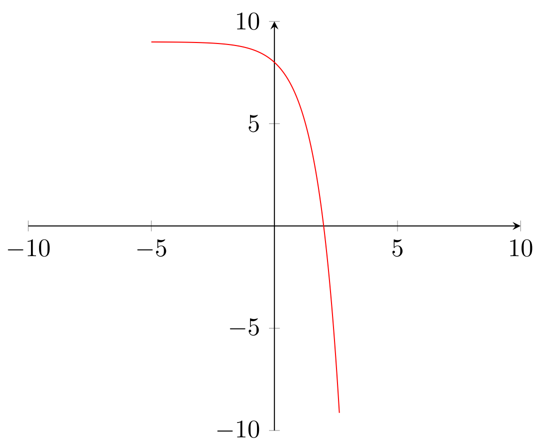
$$e^{4x} \cdot e^{x^2} = e^{12}$$
$$e^{4x+x^2} = e^{12}$$
$$4x + x^2 = 12$$
$$x^2 + 4x - 12 = 0$$
$$(x+6)(x-2) = 0$$
$$x = -6, 2$$

Example 2 Sketch the graph of f if

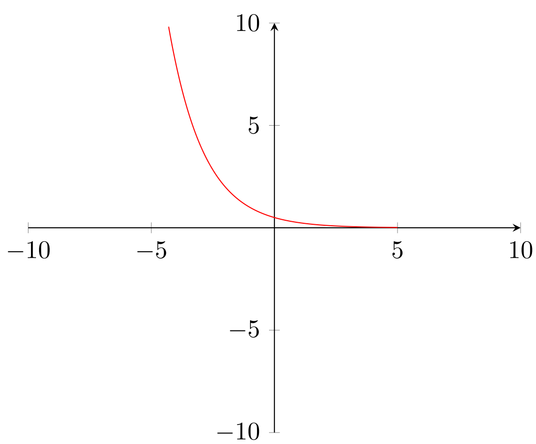
a) $f(x) = -3$^$x + 9$

b) $f(x) = 2$^$\{-(x+1)\}$

Soultion to Question 1:



Soultion to Question 2:



Definition 0.0.2: number e

If n is a positive integer, then $\left(1+\frac{1}{n}\right)^n \to e \approx 2.71828$ as $n \to \infty$.

Note:-

The natural exponential function f is defined by $f(x) = e^x$ for every real number x.

Example 3 Graph $f(x) = e^x + 4$

Soultion: 😄