Chapter 3.4 Notes

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Definition

A rational function is a function of the form $f(x) = \frac{g(x)}{h(x)}$ where $g(x)$ and $h(x)$ are polynomials and $h(x) \neq 0$.

Recall Domain of rational functions is all real numbers except those that make denominator 0.

Example 1 find the domain of each rational function

a) $f(x) = \frac{x}{(x-6)(2-x)}$

Solution:

$$D : \{x | x \neq 6, 2\}$$

b) $k(x) = \frac{x-1}{x^2+3}$

Solution:

$$D : \mathbb{R}$$

c) $f(x) = \frac{-2(x^2-1)}{(x^2+2x+1)}$

Solution:

$$D : \{x | x \neq -2\}$$

Vertical Asymptotes

The Vertical Asymptotes of a rational function can be found by setting the denominator to 0 and solve for x. That is, let $h(x) = 0$.

Horizontal Asymptotes

$$Let f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots\dots + a_1 + a_0}{b_k x^k + b_{k-1} x^{k-1} + \dots\dots + b_1 + b_0}$$

1. if $n < k$, then the x-axis (the line $y = 0$) is the Horizontal Asymptote for the graph of f.
   i.e. degree of numerator = degree of denominator implies H.A. is $y = 0$
2. if $n = k$, then the line $y = a_n/b_k$ (the ratio of leading coefficients) is the Horizontal Asymptote for the graph of f.
3. if $n > k$, the graph has no Horizontal Asymptote.
   Instead, either $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to \infty$ or as $x \to -\infty$

   i.e. degree of numerator ¿ degree of denominator implies no H.A.

There is an O.A. (oblique asymptote) is degree numerator = degree denominatorThere is an O.A. (oblique asymptote) is degree numerator = degree denominator +1

Oblique Asymptote

An oblique asymptote for a graph in a line $y = ax + b$, with $a \neq 0$, such that the graph approaches this line a $x \rightarrow \infty$ or as $x \rightarrow -\infty$

Note: The degree of the numerator must be one greater than the degree of the denominator. To find the oblique asymptote, do long division (or synthetic division)

Example 2 Find the vertical, horizontal, and oblique asymptotes, if any, of each rational function

Question 1

$$f(x) = \frac{2x+7}{x^2-3x+2}$$

Solution for question 1:

$$f(x) = \frac{2x+7}{(x-1)(x-2)}$$

Vertical Asymptote: x = 1, x = 2

Horizontal Asymptotes: y = 0

Oblique Asymptote: None

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Question 2

$$f(x) = \frac{5x^2+3x-1}{4x+1}$$

Solution for question 2: ☺

Vertical Asymptote: x = $-\frac{1}{4}$

Horizontal Asymptotes: None

Oblique Asymptote: y = $\frac{5}{4}x + \frac{7}{16}$

Question 3

$$f(x) = \frac{6x^2-4x+3}{2x^2+x-1}$$

Solution to question 3:

$$f(x) = \frac{6x^2-4x+3}{(x+1)(2x-1)}$$

Vertical Asymptote: x = -1, x = $\frac{1}{2}$

Horizontal Asymptotes: y = 3

Oblique Asymptote: None

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Question 4

$$f(x) = \frac{x^2 + x - 12}{x^2 - 9}$$

Solution to question 4:

$$f(x) = \frac{(x-3)(x+4)}{(x+3)(x-3)}$$
$$f(x) = \frac{x+4}{x+3}$$

Vertical Asymptote: x = - 3

Horizontal Asymptotes: y = 1

Oblique Asymptote: None 😄

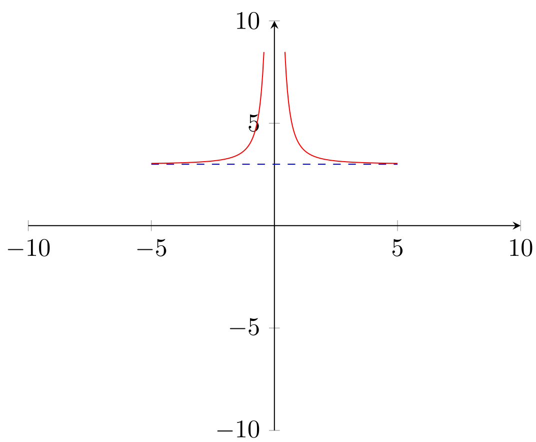
Example 3 Graph each rational function using transformations

Question 5

$$f(x) = 3 + \frac{1}{x^2}$$

Solution to question 5: 😄

Shift vertically by a factor of 3



Question 6

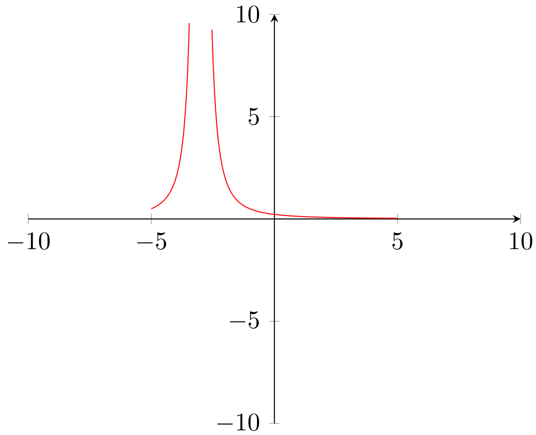
$$f(x) = \frac{2}{(x+3)^2}$$

Solution to question 6:

$$2 * \frac{1}{(x+3)^2}$$

1. Shift up 2

2. Shift left 3



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