Section 3.1 Notes

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Definition:

A polynomial function is a function of form $f(x) = a_n x^n + a_{n-1} + a_1 x + a_0$

where $a_n, a_{n-1}, ..., a_1, a_0$ are real numbers and n is a nonnegative integer. The degree of a polynomial is the highest degree of its terms
Recall: The domain of a polynomial function is all real numbers.

Example 1 Determine whether the functions are polynomials or not. For those who are state the degree of the polynomia. For those who are not state why not.

a) $f(x) = x(x + 5)$

Answer: 😃

factored form $= f(x) = x^2 + 5x$

is a polynomial

Degree: 2

b) $f(x) = \sqrt{x}(\sqrt{x} - 2)$

Answer: 😃

Not a polynomial, not an integer exponent

c) $f(x) = \frac{x^2 - 5x}{x^3}$

Answer: 😃

Can be re-written as: $\frac{x^2}{x^3}$ - $\frac{5x}{x^3}$

$= x^{-1} - 5x^{-2}$

since leading coeficient has negative exponent, it is **not** a polynomial

d) $f(x) = -3x^2(x + 5)^3$

Answer: 😃

$= -3x^2(x^3 + ...)$

This is a polynomial

degree: 5

Power Functions

A **Power Function** of degree n is a **monomial** of the form $f(x) = ax^n$ where a is a real number, $a \neq 0$, and n is a **positive integer** if n is an **even integer**, then the following are true about the power functions:

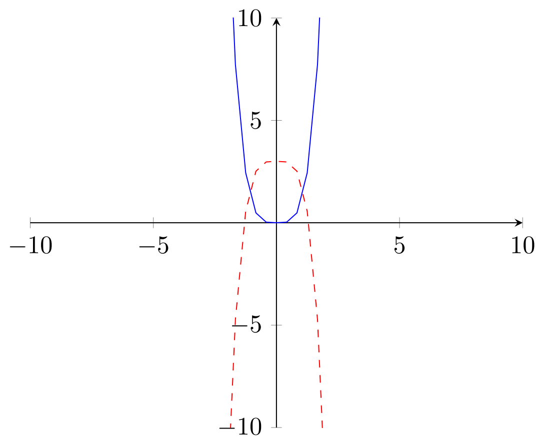
1. The graph of the function is symmetric over the **y-axis**
2. $D =$ all real numbers; $R = [0, \infty]$
3. The graph resembles the graph of $y = x^2$

if n is an **odd integer**, then the following are true about the power function:

1. The graph of the function is symmetric over the origin
2. $D =$ all real numbers; $R =$ all real numbers
3. The graph resembles the graph of $y = x^3$

Example 2 Use Transformations to graph each function

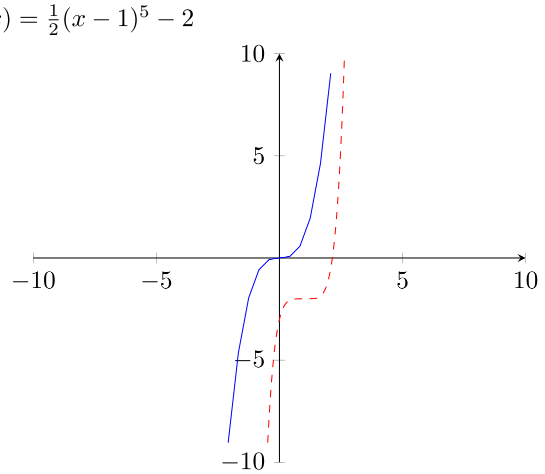
a) $f(x) = -x^4 + 3$



Steps: 😄

Parent Function: $y = x^4$ (Blue line)
negative sign infront of the leading coeficient represents: **Reflect over the x-axis**
constant is +3 which reprents: **shift up 3 units**

b) $f(x) = \frac{1}{2}(x-1)^5 - 2$



Steps: 😄

Parent Function: $y = x^5$ (Blue line)
1/2 represents: **compress by 1/2** (multiply y by 1/2)
-1 represents: **Shift to the right 1**
-2 represents: **shift down 2**

Definition

if f is a function and r is a real number for which $f(r) = 0$, then r is called a real zero of f.
if $(x - r)^m$ is a factor of a polynomial f and $(x - r)^{m+1}$ is **not** a factor, then r is called a zero of multiplicity m of f

That is:

1. r is a real zero of a polynomial function f.
2. r is an x-intercept of the graph of f.
3. $x - r$ is a factor of f.

Example 3 Form a polynomial whose real zeros and degree are given.

a) zeros: -2,2,3; degree: 3

$p(x) = (x + 2)(x - 2)(x - 3)$

$= p(x) = x^3 - 3x^2 - 4x + 12$

b) zeros: -3,-1,2,5 degree: 4

$p(x) = (x + 3)(x + 1)(x - 2)(x + 5)$
$= p(x) = x^4 - 3x^3 - 15x^2 + 19x + 30$

Multiplicity and Turning Points

If the zero is of **even multiplicity**; the graph of f **touches** the x-axis at that zero
If the zero is of **odd multiplicity**; the graph of f **crosses** the x-axis at that zero

Definition

The points at which a graph changes direction are called **turning points** (each turning point yields a local maximum or local maximum)

if f is a polynomial of degree n, then f has at most $n - 1$ turning points.
i.e. **maximum number of turning points = degree -1**

For large values of x, either positive or negative, the graph of the polynomial
$f(x) = a_n x^n + a_{n-1} + \ldots\ldots + a_n x + a + 0$
$y = a_n x^n$. The behavior of the graph of a function for large values of x, either positive and negative, is referred to as its end behavior

Example 4 For each polynomial function

1. List each real zero and its multiplicity
2. Find the x-axis and y-intercepts
3. Determine whether the graph crosses or touches the x-axis at each x-intercept
4. Determine the maximum number of turning points on the graph
5. Determine the end behavior of the function
6. Sketch the graph of the polynomial

a) $f(x) = (x - \frac{1}{3})^2(x-1)^3$

Solution: 😄

Zeros: $\frac{1}{3}, 1$

Multiplicity = 2, 3

x-intercepts: $\frac{1}{3}$ (touches), 1 (crosses)

y-intercepts = $-\frac{1}{9}$

Max number of turning points: 4

end-behavior: as $x \to \infty$, $f(x) \to \infty$ as $x \to -\infty$, $f(x) \to -\infty$

b) $f(x) = -2(x^2+1)^3$

Solution: 😄

Zeros: None

Multiplicity: None

x-intercepts: None

y-intercepts: -2

Max number of turning points: 5

end-behavior: as $x \to \infty$, $f(x) \to -\infty$ as $x \to -\infty$, $f(x) \to -\infty$

c) $f(x) = 5x(x+3)^3$

Solution: 😄

Zeros: 0, -3

Multiplicity: 3, 1

x-intercepts: 0,-3

y-intercepts: (0,0)

Max number of turning points: 3

end-behavior:

$$\lim_{x\to\infty} f(x) = \infty, \lim_{x\to-\infty} f(x) = -\infty$$

d) $f(x) = -x^2(x^2-1)(x+1)^3$

Solution: 😄

Zeros: 0, -1,1

Multiplicity: 2, 1, 4

x-intercepts: 0,-1,1

y-intercepts: 0

Max number of turning points: 6

End behavior:

$$\lim_{x\to\infty} f(x) = -\infty, \lim_{x\to-\infty} f(x) = \infty$$