Chapter 11.3 Notes

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GEOMETRIC SEQUENCE

Determine Whether a Sequence is Geometric

Defintion

The formula for a Geometric Sequence is as follows:

$$a_1 = a \qquad a_n = r a_{n-1}$$

where r = the common ratio

Formula for r

$$r = \frac{a_n}{a_{n-1}}$$

Formula for sum of a geometric sequence:

$$\text{sum: } s_n = \frac{a_1 \left(1 - r^n\right)}{(1 - r)}$$

Formula for nth term

$$\text{nth term: } a_n = a_1 r^{(n-1)}$$

Examples

Example 0.1 (Show that the sequence is geometric. List the first term and the common ratio.)

a) $2, 8, 32, 128$

Using the formula

$$r = \frac{a_n}{a_{n-1}}$$

We find that

$$r = \frac{8}{2} = 4$$

So,

$$a_1 = 2 \quad r = 4$$

Example 0.2 (Show that the sequence is geometric. List the first term and the common ratio.)

b) $\{s_n\} = \{3^{n+1}\}$

Given that $s_n = 3^{n+1}$ we can use the formula

$$r = \frac{a_n}{a_{n-1}}$$

So,

$$\frac{3^{n+1}}{3^{n-1+1}} = \frac{3^{n+1}}{3^n}$$

Note:-

Recall rules of exponents:

When you have the same base and you are dividing, you subtract the exponents

So,

$$\frac{3^{n+1}}{3^n} = 3^{n+1-n}$$

After simplifing we get

$$r = 3$$

Therefore,

$$s_1 = 9 \qquad r = 3$$

Example 0.3 (Show that the sequence is geometric. List the first term and common ratio.)

(c) $\{t_n\} = \{3(2)^n\}$

Given that $s_n = 3(2)^2$, we can use the formula

$$r = \frac{s_n}{s_{n-1}}$$

So,

$$r = \frac{3\left(2^n\right)}{3\left(2^{n-1}\right)} = 2^{n-(n-1)}$$

Therefore,

$$s_1 = 6 \qquad r = 2$$

Example 0.4 (Find the ninth term of the geometric sequence and find a recursive formula for the sequence)

$3, 2, \frac{4}{3}, \frac{8}{9}$

Given that $s_1 = 3$ we need to find r,

$$r = \frac{s_n}{s_{n-1}} = \frac{2}{3} = r$$

Now that we have s_1 and r, we can use the formula

$$s_n = s_1(r)^{n-1}$$

So,

$$a_9 = 3\left(\frac{2}{3}\right)^8$$

Recursive formula

$$a_n = r \cdot a_{n-1}$$

So,

$$a_n = \frac{2}{3} \cdot a_{n-1}$$

Sum of n Terms of a Geometric Sequence

Let $\{a_n\}$ be a geometric sequence with first term a and common ratio r, where $r \neq 0, r \neq 1$. The sum s_n of the first terms of $\{a_n\}$ is

$$S_n = a \cdot \frac{1 - r^n}{1 - r}, r \neq 0, 1$$

Example 0.5 (Find the sum of the first n terms of the sequence)

$\{3^n\}$

$$r = \frac{3^n}{3^{n-1}}$$

$$r = 3^{n-(n-1)} = 3$$

$$r = 3 \qquad s_1 = 3$$

So by using the formula

$$s_n = \frac{a_1\left(1 - r^n\right)}{(1 - r)}$$

$$\frac{3(1 - 3^n)}{-2} = \boxed{-\frac{3}{2}\left(1 - 3^n\right)}$$

Determine Whether a Geometric Series Converges or Diverges

Defintion

An infinite sum of the form

$$a + ar + ar^2 + \cdots + ar^{n-1} +$$

with the first term a and a common ratio r, is called an infinite geometric series and is denoted by

$$\sum_{k=1}^{\infty} ar^{k-1}$$

Sum of an Infinite Geometric Series

If $|r| < 1$, the sum of the infinite geometric series $\sum_{k=1}^{\infty} ar^{k-1}$ is

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$$

Example 0.6 (Find the sum of the geometric series)

$1 + \frac{1}{3} + \frac{1}{9} + \ldots$

$$a = 1 \qquad r = \frac{\frac{1}{3}}{1} = \frac{1}{3}$$

Using the formula

$$\frac{a}{1-r}$$

$$\frac{1}{1-\frac{1}{3}}$$

So,

$$\text{sum} = \frac{3}{2}$$

Example 0.7 (Find the fraction representation of the repeating decimal)

$0.8888\ldots$

$$0.8 + 0.08 + 0.008 + \cdots$$

So,

$$r = \frac{0.08}{0.8} = 0.1$$

$$s_1 = 0.8 \qquad r = 0.1$$

$$= \frac{8}{9}$$