Chapter 9.4 Notes

Matt Warner

THE HYPERBOLA

Definition

A **Hyperbola** is the set of all points in a plane, the difference of whose distance from two fixed points (the **foci**) in the plane is a positive constant. The foci are a distance c from the center, where $c^2 = a^2 + b^2$.

The **vertices** of the hyperbola are the points obtained at the intersection of the graph and the x-axis or y-axis. Vertices are a distance a from the center

The **transverse axis** of the hyperbola is the line segment passing through the center and the vertices. The length of the transverse axis is $2a$

The **conjugate axis** of the hyperbola is the line segment passing through the cetner and the points that are not on the hyperbola intersecting the other axis. The length of the conjugate axis is $2b$.

Standard Equations of hyperbola with center (h, k) and $c^2 = a^2 + b^2$

Standard equation, foci, vertices	Standard equation, foci, vertices
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Transversal axis: parallel to the x-axis Foci: $F(h \pm c, k)$ Vertices: $(h \pm a, k)$	Transversal axis: parallel to the y-axis Foci: $F(h, k \pm c)$ Vertices: $(h, k \pm a)$
Asymptotes: $y - k = \pm\frac{b}{a}(x - h)$	Asymptotes: $y - k = \pm\frac{a}{b}(x - h)$

Note:-

Ellipses have the equation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, where there is a addition sign in between.
hyperbolas have the equation $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, where there is a subtraction sign in between

Ex 1: Sketch the graphs. Find the vertices and foci.

$$\text{a) } y^2 - \frac{x^2}{15} = 1$$

Note:-

Locatating $a^2 \to a^2$ is under the (first) postive variable.

Finding the value for $c \to c^2 = a^2 + b^2$

Center: $(0,0)$

$a = 1$

$b = \sqrt{15} \approx 3.9$

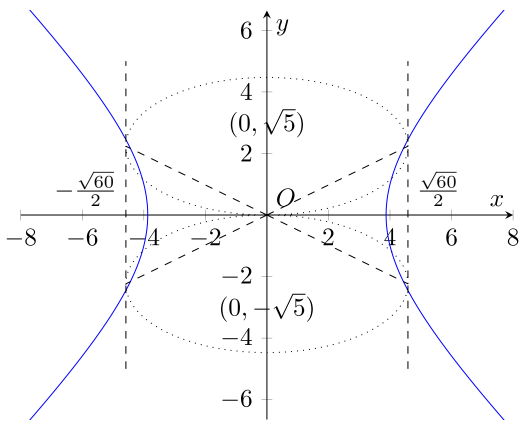
$c^2 = 1^2 + \left(\sqrt{15}\right)^2 \to c = 4$

Vertices: $(0, \pm 1)$

Foci: $(0, \pm 4)$

Asymp: $y = \pm \frac{1}{\sqrt{15}} x$

Graph:



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b) $\frac{(x-3)^2}{25} - \frac{(y-1)^2}{4} = 1$

Center: $(3,1)$

$a = 5$

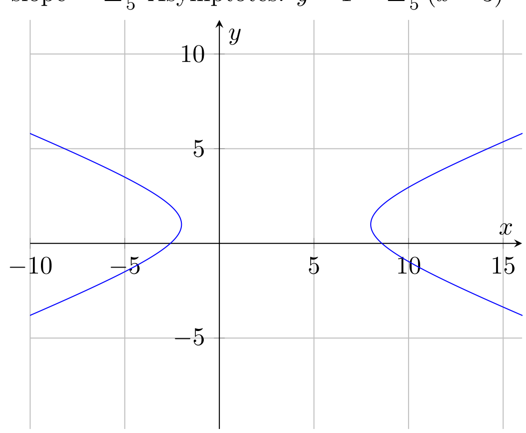
$b = 2$

$c = \sqrt{29}$

Vertices: (8,1),(-2,1)

Foci: $(3 \pm \sqrt{29}, 1)$

slope $= \pm\frac{2}{5}$ Asymptotes: $y - 1 = \pm\frac{2}{5}(x-3)$



c) $2y^2 - x^2 + 2x + 8y + 3 = 0$