Chapter 4.1 Notes

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COMPOSITE FUNCTIONS:

Definition 0.0.1

Given two functions f and g the composite function, denoted by $f \circ g$ (read as "f composed with g"), is defined by $f \circ g = f(g(x))$

Note:-

The Domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

Example 1 - Find: a. $f \circ g(4)$ **b.** $g \circ f(2)$ **c.** $f \circ f(1)$ **d.** $g \circ g(0)$

1) $f(x) = \sqrt{x+1}$ and $g(x) = 3x$

solution: 😊

a) $f(g(4)) = \sqrt{13}$

b) $g(f(2)) = 3\sqrt{3}$

c) $f(f(1)) = \sqrt{\sqrt{2}+1}$

d) $g(g(0)) = 0$

2) $f(x) = |x-2|$ and $g(x) = \frac{3}{x^2+4}$

Solution: 😊

a) $f(g(4)) = \frac{37}{20}$

b) $g(f(2)) = \frac{3}{4}$

c) $f(f(1)) = 1$

d) $g(g(0)) = \frac{48}{73}$

Example 2: Find the domain of the composite function $f \circ g$

Note:-

when finding the domain of f(g(x)), you include the restrictions for g(x) and any new restrictions in $f \circ g$

Question 1

$$f(x) = \frac{x}{x+3}$$
$$g(x) = \frac{2}{x}$$

Solution *$(f \circ g)$:* ☺

domain of $g(x)$*:* $x \neq 0$

$f \circ g : \frac{\frac{2}{x}}{\frac{2}{x}+3} \rightarrow \frac{2}{2+3x}$

domain of $f(g(x))$*:* $x \neq -\frac{2}{3}$

so, domain: $\{x \neq 0, x \neq -\frac{2}{3}\}$

Question 2

$$f(x) = x^2 + 4$$
$$g(x) = \sqrt{x-2}$$

solution *$(f \circ g)$:* ☺

Domain of g(x): $x|x \geq 2$

$f(\sqrt{x-2}) = (\sqrt{x-2})^2 + 4$

$x - 2 + 4 \rightarrow x + 2$ (we can ignore this)

so, Domain: $\{x|x \geq 2\}$

Example 3. Find: i. $f \circ g$ **ii.** $g \circ f$ **iii.** $f \circ f$ **iv.** $g \circ g$

For the given functions f and g. State the domain of each composite function.

Question 3

$$f(x) = x^2 + 1$$
$$g(x) = 2x^2 + 3$$

Solution i. *$(f \circ g)$:* ☺

Domain of $g(x) \rightarrow D : \mathbb{R}$

$f(2x^2 + 3) = (2x^3 + 3)^2 + 1$

$\rightarrow 4x^4 + 12x^2 + 10$ (No restrictions in domain)

So, $D : \mathbb{R}$

Solution ii. *$(g \circ f)$:* 😊

Domain of $f(x) \rightarrow D : \mathbb{R}$

$g(x^2 + 1) = 2(x^2 + 1)^2 + 3$

$= 2(x^4 + 2x^2 + 1) + 3$

$= 2x^4 + 4x^2 + 5$ (No restrictions in domain)

So, $D : \mathbb{R}$

Solution iii. *$(f \circ f)$:* 😊

Domain of $f(x) \rightarrow D : \mathbb{R}$

$f(x^2 + 1) = (x^2 + 1)^2 + 1$

$= x^4 + 2x^2 + 1$ (No restrictions in Domain)

So, $D : \mathbb{R}$

Solution iv. *$(g \circ g)$:* 😊

Domain of $g(x) \rightarrow D : \mathbb{R}$

$g(2x^2 + 3) = 2(2x^2 + 3)^2 + 3$

$= 8x^4 + 24x^2 + 21$ (no domain restrictions)

So, $D : \mathbb{R}$

Question 4

$$f(x) = \sqrt{x - 2}$$
$$g(x) = 1 - 2x$$

Solution i. *$(f \circ g)$:* 😊

Domain of $g(x) \rightarrow D : \mathbb{R}$

$f(1 - 2x) = \sqrt{1 - 2x - 2} \rightarrow \sqrt{-2x - 1}$

$\sqrt{-2x - 1} \geq 0$

$-2x - 1 \geq 0 \rightarrow -2x \geq 1$

$x \leq -\frac{1}{2}$

So, $D : \{x | x \leq -\frac{1}{2}\}$

Solution ii. *($g \circ f$):* ☺

Domain of $f(x) \rightarrow D : \{x|x \geq 2\}$

$g(\sqrt{x-2}) = 1 - 2\sqrt{x-2}$

So, $D : \{x|x \geq 2\}$