Chapter 11.2 Notes

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ARITHMETIC SEQUENCE

Determine Whether a Sequence is Arithmetic

$$a_1 = a, \qquad a_n = a_{n-1} + d$$

Note:-

In the equation above, the variable d, is refered to as the *common difference*, the difference between consecutive terms in the sequence.

$$d = a_n - a_{n-1}$$

Example 1 Show that the sequence is arithmetic. List the first term and the common difference.

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a) $4, 2, 0, -2, \ldots$

$$d = (2-4) \to (0-2) \to (-2-0)$$
$$d = -2$$

The common difference shows that this sequence is arithmetic, not geometric.

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b) $\{s_n\} = \{4n-1\}$

$$s_1 = 4(1) - 1 = 3$$
$$s_2 = 4(2) - 1 = 7$$
$$s_3 = 4(3) - 1 = 11$$
$$s_4 = 4(4) - 1 = 15$$

Here we can see that the common difference between the terms in the sequence is 4.

Alternatively,

$$\begin{aligned} S_n - S_{n-1} &= 4n - 1 - (4(n-1) - 1) \\ &= 4n - 1 - [4n - 4 - 1] \\ &= 4n - 1 - [4n - 5] \\ &= 4n - 1 - 4n + 5 \\ &= 4 \quad \text{common differnece} \end{aligned}$$

Find a Formula for an Arithmetic Sequence

nth Term of an Arithmetic Sequence

For an arithmetic sequence, $\{a_n\}$ whose first term is a and whose common difference is d, the nth term is determined by the formula

$$a_n = a + (n-1)d$$

Note:-

$$\begin{aligned} a_1 &= a \\ a_2 &= a_1 + d = a + d \\ a_3 &= a_2 + d = a + d + d = a + 2d \\ a_4 &= a_3 + d = a + 2d + d = a + 3d \\ &\vdots \\ a_n &= a_{n-1} + d = a + (n-1)d \end{aligned}$$

Example 2 Find the twenty fourth term of the arithmetic sequence:

$$-3, 0, 3, 6, \ldots$$

$$a = -3 \qquad d = 3$$

$$a_n = a + (n-1)d$$

$$-3 + (n-1) \cdot 3$$

$$-3 + 3n - 3 \to 3n - 6$$

$$a_{24} = 3(24) - 6$$

$$= \boxed{66}$$

Example 3: The sixth term of an arithmetic sequence is 26, and the nineteenth term is 78. Find the first term and the common difference. Give a recursive formula for the sequence. What is the nth term of the sequence?

$$a_6 = 26 \qquad a_{19} = 78$$

$$a_n = a + (n-1)d$$

$$\begin{aligned} 26 &= a + (6-1)d \\ 78 &= a + (19-1)d \end{aligned} \Rightarrow \begin{cases} 26 = a + 5d \\ 78 = a + 18d. \end{cases}$$

$$52 = 13d$$

$$4 = d$$

So,

$$26 = a + 5(4) = 6$$
$$a_1 = 6$$

Sum of n Terms of an Arithmetic Sequence

Let $\{a_n\}$ be an arithmetic sequence with first term a and common difference d. The sum s_n of the first n terms of $\{a_n\}$ is

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + a_n)$$

Find the sum of the first n terms of the sequence

$$\{4n + 2\}$$

$a_1 = 4(1) + 2 = 6$
$a_2 = 10$
$a_3 = 14$
$a_4 = 18$
$a_5 = 22$
$a_6 = 26$
$a_7 = 30$

$s_n = 6 + 10 + 14 + 18 + \ldots + 4n + 2$
$= \frac{n}{2}(a_1 + a_n)$
$= \frac{n}{2}(8 + 4n)$
$= \frac{4n(2+n)}{2}$
$= \boxed{2n(2+n)}$

Example 5: The conrner section of a stadium has 20 seats in the first row and 40 rows in all. Each successive row contains two additional seats. How many seats are in this section?

Solution: ☺

Given that each row has 2 more seats then the previous rows.

$$s_{40} = 20 + 22 + 24 + \ldots$$

$$a_n = a + (n-1)d$$

$$= 20 + 39 \cdot 2$$

$$a_n = 98$$

$$s_{40} = \frac{n}{2}(a + a_n)$$

$$\frac{40}{2}(20 + 98)$$

$$= \boxed{2360}$$

More Sum Examples

Example 0.0.1 (Find the Sum)

$$5 + 7 + 9 + \ldots + 75$$

$$a_1 = 5 \qquad a_n = 75 \qquad d = 2$$

$$a_n = 5 + (n-1)2$$

$$75 = 3 + 2n$$

$$72 = 2n$$

$$n = 36$$

So, Using the formula $\longrightarrow \frac{n}{2}(a_1 + a_n)$

$$\frac{36}{2}(5 + 75)$$

$$= 1440$$

Example 0.0.2 (Find the Sum)

$$7 + 7.5 + 8 + 8.5 + \ldots + 103.5$$

$$a_1 = 7 \qquad a_n = 103.5 \qquad d = 0.5$$

$$a_n = 7 + (n-1)0.5$$

$$103.5 = 7 + \left(0.5n - \frac{1}{2}\right)$$

$$97 = 0.5n$$

$$194 = n$$

So,

$$\frac{194}{2}(7 + 103.5)$$

$$= 10718.5$$