Section 9.3 Notes

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9.3 THE ELLIPSE

Definition

An **ellipse** is the set of all points in a plane, the sum of whose distances from two fixed points (the **foci**) in the plane is a positive constant. The foci are a distance c from the center, where $c^2 = a^2 - b^2$.

The **major axis** of the ellipse is the longest line segment passing through the center and foci. The end points of the major axis are called the vertices of the ellipse. Vertices are a distance of a from the center.

The **minor axis** of the ellipse is the shortest line segment passing through the center.
The length of the major axis is $2a$, and the length of the minor axis is $2b$.

Standard Equation of an ellipse with center (h, k) and $c^2 = a^2 - b^2$

Standard equation, foci, vertices	Standard equation, foci, vertices
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where $a > b > 0$ Major axis: parallel to the x-axis Foci: $F(h \pm c, k)$ Vertices: $(h \pm a, k)$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ where $a > b > 0$ Major axis: parallel to the y-axis Foci: $F(h, k \pm c)$ Vertices: $(h, k \pm a)$

Example 1: Sketch the graph of the functions. Find the vertices and foci.

a) $y^2 + 9x^2 = 9$

$$\frac{y^2 + 9x^2}{9} = \frac{9}{9}$$

$$x^2 + \left(\frac{y}{3}\right)^2 = 1$$

Center: $(0, 0)$
$a = 3$
$b = 1$

Note:-

a is always larger than b, so $a = 3$

Now, we use the formula, $c^2 = a^2 - b^2$ to find c. so, $c = \sqrt{8} \to 2\sqrt{2}$

vertices $= (0, \pm 3)$

Foci $= (0, \pm 2\sqrt{2})$

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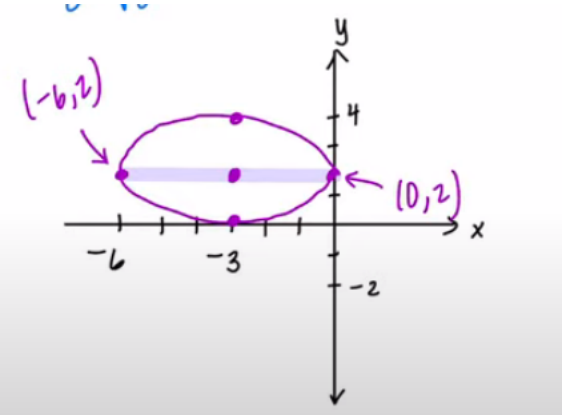
b) $\frac{(x+3)^2}{9} + \frac{(y-2)^2}{4} = 1$

Center = (-3,2)

$a = 3$

$b = 2$

$$c^2 = 3^2 - 2^2 \rightarrow c = \sqrt{5}$$



As you can see in the figure, the center is located at $(-3, 2)$, and we can create the ellipse by addding ± 2 to the y-axis at the center, and adding ± -3 to the x-axis at our center point. (These are our a and b values.)

Thus, The vertices are listed as $\rightarrow (-6, 2), (0, 2)$

Foci = $(-3 \pm \sqrt{5}, 2)$, (x-value in the center point $\pm c$, y-value of the center point.)

c) $9x^2 + 4y^2 - 18x + 16y - 11 = 0$

Note:-

Compleating the Sqaure:
$\left(\frac{b}{2}\right)^2$ = new value for c in the equation

$$9\left(x^2 - 2x + 1\right) + 4\left(y^2 + 4y + 4\right) = 11 + 9 + 16$$

$$\frac{(x-1)^2}{36} + \frac{4(y+2)^2}{36} = \frac{36}{36}$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$$

Center = $(1, -2)$
$a = 3$
$b = 2$
$c = \sqrt{5}$