Chapter 3.3 Lecture Notes

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Complex Zeros

Fundamental Theorem of Algebra

If a polynomial $f(x)$ has positive degrees and complex coefficients, then $f(x)$ has a least one complex zero.

Complete Factorization Theorem for polynomials

if $f(x)$ is a polynomial of degree $n > 0$, then there exists n complex numbers $c_1, c_2, ..., c_n$ such that $f(x) = a(x - c_1)(x - c_2)...(x - c_n)$ where a is the leading coefficient of $f(x)$. Each number c_k is a zero of $f(x)$.

Conjugate Pairs theorem

Let $f(x)$ be a polynomial whose coefficients are real numbers. if $r = a + bi$ is a zero of f, then the Conjugate $\bar{r} = a - bi$ is also a zero of f

Example 1 find $f(x)$ given the zeros.

a) -3, 1, -7i: degree 3

Solution: 😃

1) $f(x) = (x + 3)(x - 1 + 7i)(x - 1 - 7i)$

2) $f(x) = (x + 3)((x - 1)^2(-49i^2))$

3) $f(x) = (x + 3)(x - 2x + 1 + 49)$

4)$f(x) = x^3 + x^2 + 44x + 150$

b) 0, 3i, 4+i

Solution: 😃

1) $f(x) = x(x - 3i)(x + 3i)(x - 4 + i)(x - 4 - i)$

2) $f(x) = x(x^2 - 9i^2)((x - 4)^2 - i^2)$

3) $f(x) = (x^3 + 9x)(x^2 - 8x + 17)$

4) $f(x) = x^5 - 8x^4 + 26x^3 - 72x^2 + 153x$

Example 2 Use the given zero to find the remaining zeros of the function

a) $f(x) = x^3 + 3x^2 + 25x + 65$

Zero: -5i

Solution: 😃

1) Zeros:(x+5i)(x-5i)(x-c)

2) $\frac{f(x)}{(x+5i)(x-5i)}$

3) $\frac{f(x)}{x^2-25i^2}$

$=\frac{f(x)}{x^2+25}$

4) Using polynomial long division we get: $(x+3)$ as our 3^{rd} zero

b) $f(x) = 3x^4 + 5x^3 + 25x^2 + 45x - 18$; Zero: 3i

Solution: 😄

1) Known zeros are: $(3i, -3i)$

2) $\frac{f(x)}{(x+3i)(x-3i)}$

3) $\frac{f(x)}{(x^2+9)}$

4) Using polynomial long division we get: $\frac{1}{3}$, -2 as the remaining zeros

<u>Example 3</u> Find all complex zeros of $f(x)$

Solution: 😄

$$f(x) = x^3 + 13x^2 + 57x + 85$$

$$\frac{1, 85, 17, 5}{1}$$

$$\pm 1, 85, 17, 5$$

Using long division we get -5 as our first zero

Using the quadratic equation with our remainder we get $-4 \pm i$ as our remaining zeros