9.2 Notes

Matt Warner

THE PARABOLA

A **parabola** is the set of all points in a plane equidistant from a fixed point F (the **focus** and a fixed line l (th **directrix** that lie in the plane.

The **axis** of the parabola is the line through F that is perpendicular to the directrix.

The **vertex** of the parabola is the point V on the axis halfway from F to l.

Parabola with Vertex $V(h, k)$

Standard equation, focus, directrix	Graph for $p > 0$	Graph for $p < 0$
$(x-h)^2 = 4p(y-k)$ Focus: $F(h, k+p)$ Directrix: $y = k - p$ Length of latus rectum: $4p$		
$(y-k)^2 = 4p(x-h)$ Focus: $F(h+p, k)$ Directrix: $x = h - p$ Length of latus rectum: $4p$		

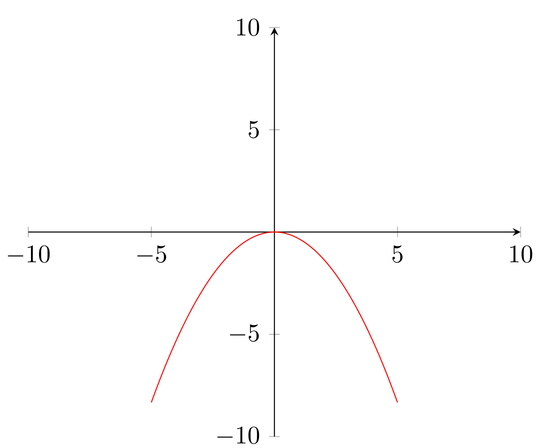
Ex 1 Sketch the graph of the following functions. Find the vertex, focus, and directrix

Vertex	Focus	Directrix	Equation
$(0,0)$	$(a,0)$	$x=-a$	$y^2 = 4ax$
$(0,0)$	$(-a,0)$	$x=a$	$y^2 = -4ax$
$(0,0)$	$(0,a)$	$y=-a$	$x^2 = 4ay$
$(0,0)$	$(0,-a)$	$y=a$	$x^2 = -4ay$

a) $x^2 = -3y$

$$4p = -3$$
$$p = \frac{-3}{4}$$

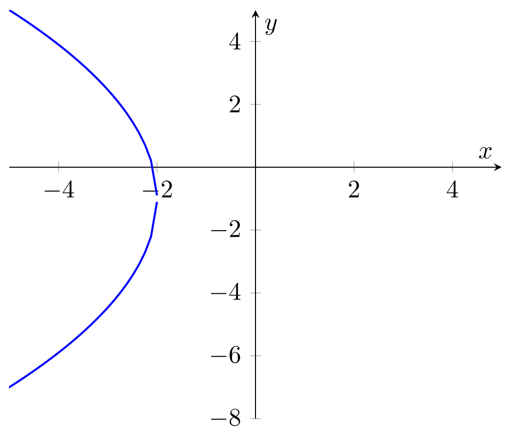
Vertex: $(0,0)$, Focus: $(0, \frac{-3}{4})$



b) $(y+1) = -12\,(x+2)$

$$\text{Vertex} = (-2,-1)$$
$$4p = -12 \rightarrow \text{focal width} = 12 \rightarrow \text{split in half: } 6$$
$$p = -3$$
$$\text{Focus: } (-5,-1) \rightarrow \text{(The focus is p, or -3 away from the vertex.) (x-axis)}$$
$$\text{Focus: } (-5,5) \rightarrow \text{(This focus is 6 units from our vertex) (y-axis)}$$

Thus,



Ex 2: Find the equation of the parabola that satisfies the given conditions.

a) Focus $F(-3,-2)$, directrix $y = 1$

$$\text{Vertex: } (-3, \tfrac{-1}{2})$$
$$(x+3)^2 = 4p(y+\tfrac{1}{2})$$
$$(x+3)^2 = 4(-1.5)\left(y+\tfrac{1}{2}\right)$$
$$(x+3)^2 = -6\left(y+\tfrac{1}{2}\right)$$

b) Vertex $V(3,-2)$, axis (axis of symmetry) parallel to the x-axis, and y-intercept 1.

$$h = 3$$
$$k = -2$$
$$(y-k)^2 = 4p(x-h)$$
$$(y+2)^2 = 4p(x-3)$$
$$(1+2)^2 = 4p(0-3)$$
$$-3 = 4p$$
$$\boxed{(\text{y+2})^2 = -3(x-3)}$$