Chapter 4.5 Notes

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Properties of Logarithms

Laws of Logarithms

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a a^x = x$
4. $a^{\log_a x} = x$
5. $\log_a(u \cdot w) = \log_a u + \log_a w$
6. $\log_a\left(\frac{u}{w}\right) = \log_a u - \log_a w$
7. $\log_a\left(u^c\right) = c \log_a u$

Change of Base Formula

If $u > 0$ and if a and b are positive real numbers different from 1, then $\log_b u = \frac{\log_a u}{\log_a b}$

Note:-

- $\log_a(u + w) \neq \log_a u + \log_a w$
- $\log_a(u - w) \neq \log_a u - \log_a w$

Example 1:
Use the properties of logarithms to find the exact value of each expression

1. $2^{\log_2 x}$
2. $\log_3 8 \cdot \log_8 9$
3. $e^{\log_{e^2} 9}$

Soultion to Question 1: ☺

Using the Laws of logarithms,

$2^{\log_2 x} = x$

Solution to Question 2: ☺

The change of base formula states that $\log_3 8 \cdot \log_8 9$

Can be rewritten as: $\frac{\log 8}{\log 3} \cdot \frac{\log 9}{\log 8}$

Now,

$\left(\frac{\log 8}{\log 3}\right)\left(\frac{\log 9}{\log 8}\right)$

$= \frac{\log 9}{\log 3}$

Soultion to Question 3: 😊

Using the Change of Base Formula,

$e^{\log_{e^2} 9} = e^{\frac{\log_e 9}{\log_e e^2}}$

$= e^{\frac{\log_e 9}{2}}$

$= e^{\frac{1}{2}\log_e 9}$

$= e^{\log_e 9^{\frac{1}{2}}}$

Using the Law of exponents, this becomes,

$\sqrt{9}$ which $= 3$

Example 2:
Express in terms of logarithms of x,y or z.

1. $\log \frac{\sqrt{x}}{y^4 \sqrt[3]{z}}$
2. $\ln x\sqrt{\frac{y^4}{z^5}}$
3. $\log\left[\frac{x^2\sqrt{x+1}}{(x-2)^2}\right]$

Solution to Question 1: 😊

$$\log\sqrt{x} - \log y^4 - \log\sqrt[3]{z}$$
$$\log x^{\frac{1}{2}} - \log y^4 - \log z^{\frac{1}{3}}$$
$$\boxed{\frac{1}{2}\log x - 4\log y - \frac{1}{3}\log z}$$

Soultion to Question 2: 😊

$$\ln\left(x \cdot \frac{y^2}{z^{5/2}}\right)$$
$$\ln x + \ln y^2 - \ln z^{\frac{5}{2}}$$
$$\boxed{\ln x + 2\ln y - \frac{5}{2}\ln z}$$

Solution to Question 3: ☺

$$\log x^2 + \log\sqrt{x+1} - \log(x-2)^2$$

$$\boxed{2\log x + \frac{1}{2}\log(x+1) - 2\log(x-2)}$$

Example 3:
Write the expressions as one logarithm

1. $5\log_a x - \frac{1}{2}\log_a(3x-4) - 3\log_a(5x+1)$
2. $2\ln x - 4\ln(1/y) - 3\ln(xy)$
3. $\log\left(\frac{x^2+2x-3}{x^2-4}\right) - \log\left(\frac{x^2+7x+6}{x+2}\right)$

Solution to Question 1: ☺

$$\log_a x^5 - \log_a\sqrt{3x-4} - \log_a(5x+1)^3$$

$$\boxed{\log_a\left(\frac{x^5}{\sqrt{3x-4}(5x+1)^3}\right)}$$

Solution to Question 2: ☺

$$2\ln x + 4\ln y - 3\ln(xy)$$
$$= \ln x^2 + \ln y^4 - \ln(xy)^3$$
$$\ln\left(\frac{x^2y^4}{x^3y^3}\right)$$
$$\boxed{\ln\left(\frac{y}{x}\right)}$$

Solution to Question 3: ☺

$$\log\left(\frac{\frac{x^2+2x-3}{x^2-4}}{\frac{x^2+7x+6}{x+2}}\right)$$
$$= \log\left(\frac{(x+3)(x-1)}{(x+2)(x-2)}\cdot\frac{x+2}{(x+6)(x+1)}\right)$$
$$\log\left(\frac{(x+3)(x-1)}{(x-2)(x+6)(x+1)}\right)$$