Chapter 4.2 Notes

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Definition 0.0.1

A function f with domain D and range R is a one-to-one function if either of the following equivalent conditions is satisfied.

1. Whenever $a \neq b$ in D, then $f(a) \neq f(b) in R$.

2. Whenever $f(a) = f(b) in R$, then $a = b$ in D.

Example 1 Determine whether the function $f(x) = 2x^3 - 4$ is one-to-one

Solution For Example 1:

$$f(a) = f(b)$$
$$2a^3 - 4 = 2b^3 - 4$$
$$2a^3 = 2b^3$$
$$a^3 = b^3$$
$$a = b.$$

Since $a = b$, the function is one-to-one. ☺

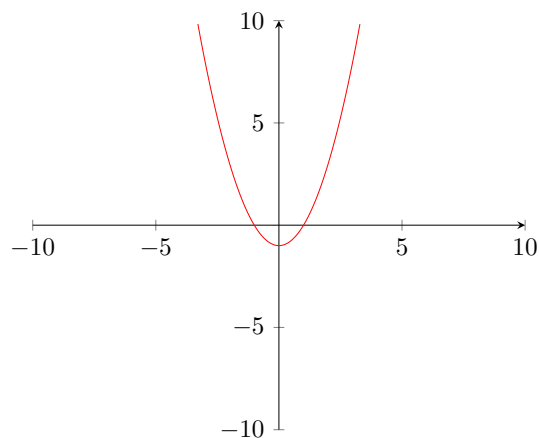**Note:-**

Horizontal Line Test
a function f is one-to-one is and only if every horizontal line intersects the graph of f in at most one point.

Example 2 Use the Horizontal line test to determine if the following are one-to-one functions.
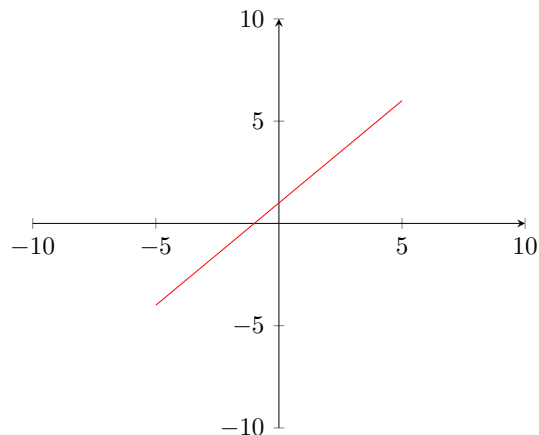
Question 1

$$f(x) = x^2 - 1$$

Solution to Question 1: ☺

Not one-to-one

Question 2

$$f(x) = \frac{2}{3}x + 1$$

Yes, the graphs shows the function to be one-to-one

Note:-

1. A function that is increasing throughout its domain is one-to-one.

2. A function that is decreasing throughout its domain is one-to-one.

Definition 0.0.2

Let f be a one-to-one function with domain D and range R. A function g with domain R and range D is the inverse function of f, provided the following conditions is true for every x in D and every y in R:.