Guidelines for authors

Alan N. Jones[1] †, H.-C. Smith[1] and J.Q. Long[2]

[1]STM Journals, Cambridge University Press, The Printing House, Shaftesbury Road, Cambridge CB2 8BS, UK

[2]DAMTP, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WA, UK

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1. Introduction

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† Email address for correspondence: JFMEditorial@cambridge.org

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2. Methods

2.1. *Particle Transport*

2.1.1. *Translation and Rotation*

Three coordinate frames are generally used to describe the position and orientation of ellipsoidal particles,which can be referred to as the inertial frame $\mathbf{x}^{(in)} = [x^{(in)}, y^{(in)}, z^{(in)}]^T$, the co-moving frame $\mathbf{x}^{(cm)} = [x^{(cm)}, y^{(cm)}, z^{(cm)}]^T$ and the particle frame $\mathbf{x}^{(p)} = [x^{(p)}, y^{(p)}, z^{(p)}]^T$.The co-moving frame translates with the particle with its origin fixed at the particle centroid. The axises of the particle frame always coincide with the semi-axises of the ellipsoid. Thus, the particle frame also record particle rotation.

A point $\mathbf{x}^{(in)}$ in the inertial frame can be transformed to the co-moving frame by

$$\mathbf{x}^{(cm)} = \mathcal{T}\mathbf{x}^{(in)} \tag{2.1}$$

Here, the translation matrix $\mathcal{T}$ is defined as

$$\mathcal{T} = \begin{bmatrix} 1 & 0 & 0 & -x_p \\ 0 & 1 & 0 & -y_p \\ 0 & 0 & 1 & -z_p \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{2.2}$$

with (x_p, y_p, z_p) being the coordinates of the particle centroid in the inertial frame. Transformation between the co-moving frame and the particle can be given as

$$\mathbf{x}^{(cm)} = \mathcal{R}\mathbf{x}^{(in)}. \tag{2.3}$$

The rotation matrix $\mathcal{R}$ can be expressed by Euler angles (ϕ, θ, ψ) or quaternions $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \eta)$. In this study, we follow Chesnutt & Marshall (2009) and write $\mathcal{R}$ in the form of quaternions

$$\mathcal{R} = \begin{bmatrix} 1 - 2(\varepsilon_2^2 + \varepsilon_3^2) & 2(\varepsilon_1\varepsilon_2 + \varepsilon_3\eta) & 2(\varepsilon_1\varepsilon_3 - \varepsilon_2\eta) \\ 2(\varepsilon_2\varepsilon_1 - \varepsilon_3\eta)) & 1 - 2(\varepsilon_3^2 + \varepsilon_1^2) & 2(\varepsilon_2\varepsilon_3 + \varepsilon_1\eta) \\ 2(\varepsilon_3\varepsilon_1 + \varepsilon_2\eta)) & 2(\varepsilon_3\varepsilon_2 - \varepsilon_1\eta)) & 1 - 2(\varepsilon_1^2 + \varepsilon_1^2) \end{bmatrix}. \tag{2.4}$$

The initial values of quaternions are determined by

$$\varepsilon_1 = \cos\frac{\phi - \psi}{2}\sin\frac{\theta}{2},\ \varepsilon_2 = \sin\frac{\phi - \psi}{2}\sin\frac{\theta}{2},\ \varepsilon_3 = \sin\frac{\phi + \psi}{2}\cos\frac{\theta}{2},\ \eta = \cos\frac{\phi + \psi}{2}\cos\frac{\theta}{2}. \tag{2.5}$$

Then quaternions are evolved by the following equation

$$\begin{bmatrix} \mathrm{d}\varepsilon_1/\mathrm{d}t \\ \mathrm{d}\varepsilon_2/\mathrm{d}t \\ \mathrm{d}\varepsilon_3/\mathrm{d}t \\ \mathrm{d}\eta/\mathrm{d}t \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \eta\Omega_x^{(p)} - \varepsilon_3\Omega_y^{(p)} + \varepsilon_2\Omega_z^{(p)} \\ \varepsilon_3\Omega_x^{(p)} + \eta\Omega_y^{(p)} - \varepsilon_1\Omega_z^{(p)} \\ -\varepsilon_2\Omega_x^{(p)} + \varepsilon_1\Omega_y^{(p)} + \eta\Omega_z^{(p)} \\ -\varepsilon_1\Omega_x^{(p)} - \varepsilon_2\Omega_y^{(p)} - \varepsilon_3\Omega_z^{(p)} \end{bmatrix}, \tag{2.6}$$

where $\Omega_x^{(p)}$, $\Omega_y^{(p)}$ and $\Omega_z^{(p)}$ are the components of rotation rate in the particle frame.

The discrete element method (DEM) is employed to evolve particle movements. The governing equations of the linear and angular momentum are given as

$$m\frac{\mathrm{d}\mathbf{v}_i^{(in)}}{\mathrm{d}t} = \mathbf{F}_{E,i}^{(in)} + \sum_{j\neq i}\mathbf{F}_{C,j\to i}^{(in)}, \tag{2.7}$$

$$I_x^{(p)}\frac{\mathrm{d}\Omega_{x,i}^{(p)}}{\mathrm{d}t} - \Omega_{y,i}^{(p)}\Omega_{z,i}^{(p)}(I_y^{(p)} - I_z^{(p)}) = M_{E,i,x}^{(p)} + \sum_{j\neq i} M_{C,j\to i,x}^{(p)}, \tag{2.8}$$

$$I_y^{(p)}\frac{\mathrm{d}\Omega_{y,i}^{(p)}}{\mathrm{d}t} - \Omega_{z,i}^{(p)}\Omega_{x,i}^{(p)}(I_z^{(p)} - I_x^{(p)}) = M_{E,i,y}^{(p)} + \sum_{j\neq i} M_{C,j\to i,y}^{(p)}, \tag{2.9}$$

$$I_z^{(p)}\frac{\mathrm{d}\Omega_{z,i}^{(p)}}{\mathrm{d}t} - \Omega_{x,i}^{(p)}\Omega_{y,i}^{(p)}(I_x^{(p)} - I_y^{(p)}) = M_{E,i,z}^{(p)} + \sum_{j\neq i} M_{C,j\to i,z}^{(p)}. \tag{2.10}$$

Here, $\mathbf{v}_i^{(in)}$ and $\mathbf{\Omega}_i^{(p)} = [\Omega_{x,i}^{(p)}, \Omega_{y,i}^{(p)}, \Omega_{z,i}^{(p)}]^T$ are the velocity and rotation rate of particle i. m is the particle mass, $\mathbf{I}^p = [I_x^{(p)}, I_y^{(p)}, I_z^{(p)}]^T$ is the moment of inertia with $I_x^{(p)} = m(b^2+c^2)/5$, $I_y^{(p)} = m(c^2+a^2)/5$ and $I_z^{(p)} = m(a^2+b^2)/5$. $\mathbf{F}_{E,i}^{(in)}$ and $\mathbf{M}_{E,i}^{(in)}$ are the electrostic force and torque exerted on particle i. $\mathbf{F}_{C,j\to i}^{(in)}$ and $\mathbf{M}_{C,j\to i}^{(p)} = [M_{C,j\to i,x}^{(p)}, M_{C,j\to i,y}^{(p)}, M_{C,j\to i,z}^{(p)}]^T$ are the contact force and torque acting on particle i by particle j.

2.2. *Collision between Ellipsoidal Particles*

2.2.1. *Collision Detection*

In the particle frame of the ith particle, the ellipsoid can be written as

$$\mathbf{X}^{(p)T} Q_i^{(p)} \mathbf{X}^{(p)} = 0. \tag{2.11}$$

Here, $\mathbf{X}^{(p)} = [x^{(p)}, y^{(p)}, z^{(p)}, 1]^T$ is the generalized position vector in the particle frame, and the characteristic matrix of ellipsoid i is

$$Q_i^{(p)} = \begin{bmatrix} 1/a^2 & 0 & 0 & 0 \\ 0 & 1/b^2 & 0 & 0 \\ 0 & 0 & 1/c^2 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{2.12}$$

For points in the inertial frame, the coordinates can be transformed to the particle frame through $\mathbf{x}^{(p)} = \mathcal{R}\mathcal{T}\mathbf{x}^{(in)}$. Thus, the ellipsoid can be given in the inertial frame as

$$(\mathcal{RT}\mathbf{X}^{(in)})^T Q_i^{(p)} (\mathcal{RT}\mathbf{X}^{(in)}) = \mathbf{X}^{(in)^T} \mathcal{T}^T \mathcal{R}^T Q_i^{(p)} \mathcal{RT}\mathbf{X}^{(in)} = \mathbf{X}^{(in)^T} Q_i^{(in)} \mathbf{X}^{(in)} = 0, \tag{2.13}$$

where $Q_i^{(in)} = \mathcal{T}^T \mathcal{R}^T Q_i^{(p)} \mathcal{RT}$ is the 4×4 characteristic matrix of ellipsoid i in the inertial frame.

If a point $\mathbf{X}^{(in)}$ satisfies the equation of two different ellipsoids $Q_1^{(in)}$ and $Q_2^{(in)}$, then two ellipsoids intersect at $\mathbf{X}^{(in)}$. Multiplying 2.13 of ellipsoid 1 by λ and subtracting 2.13 of ellipsoid 2 thus yields

$$\mathbf{X}^{(in)^T} (\lambda Q_1^{(in)} - Q_2^{(in)}) \mathbf{X}^{(in)}. \tag{2.14}$$

When two ellipsoids overlap, a family of non-trival solutions $\mathbf{X}^{(in)}$ exist to describe the intersection. Since $Q_1^{(in)}$ is invertible, $Q_1^{(in)^{-1}} Q_2^{(in)}$ should be singular. Thus, if two eigen values of $Q_1^{(in)^{-1}} Q_2^{(in)}$ are complex conjugates, two ellipsoid intersect (Alfano & Greer 2003).

2.2.2. *Contact Point*

When two ellipsoid intersect, the contact point is identified to calculate the contact interactions. In this study, the method of level surfaces are applied for contact point identification (Schneider & Eberly 2002; Ting 1992). (Note: cite paper on geometric potential algorithms by Ning (1992)) 2.13 can be expressed in the quadratic form as

$$P_i(\mathbf{x}^{(in)}) = \mathbf{x}^{(in)^T} \mathcal{S}_i^{(in)} \mathbf{x}^{(in)} + \mathbf{b}_i^{(in)^T} \mathbf{x}^{(in)} + c_i^{(in)} = 0. \tag{2.15}$$

Here, $\mathcal{S}_i^{(in)}$, $\mathbf{b}_i^{(in)}$ and $c_i^{(in)}$ are defined by the components of $Q_i^{(in)}$

$$\mathcal{S}_i^{(in)} = \begin{bmatrix} q_{11}^{(in)} & q_{12}^{(in)} & q_{13}^{(in)} \\ q_{12}^{(in)} & q_{22}^{(in)} & q_{23}^{(in)} \\ q_{13}^{(in)} & q_{23}^{(in)} & q_{33}^{(in)} \end{bmatrix} \tag{2.16}$$

$$\mathbf{b}_i^{(in)} = 2[q_{14}^{(in)}, q_{24}^{(in)}, q_{34}^{(in)}]^T, \tag{2.17}$$

$$c_i^{(in)} = q_{44}^{(in)}. \tag{2.18}$$

The contact point on ellipsoid 1 is defined as the tangent point of ellipsoid 1 on the innermost level surface of ellipsoid 2. The level surfaces of ellipsoid 2 is given by

$$P_2(\mathbf{x}^{(in)}) = \alpha, \tag{2.19}$$

where $\alpha < 0$ and $\alpha > 0$ corresponds to the interior and exterior of ellipsoid 2. Then finding the contact point is equivalent to finding the minimum value of α in 2.19 under the constraint of $P_1(\mathbf{x}^{(in)}) = 0$. By defining the Lagrangian function

$$\mathcal{L}(\mathbf{x}^{(in)}) = P_2(\mathbf{x}^{(in)}) + \tau P_1(\mathbf{x}^{(in)}) \tag{2.20}$$

for optimization, the contact point $\mathbf{x}_{C,1}^{(in)}$ is given by

$$\mathbf{x}_{C,1}^{(in)} = -\frac{1}{2}(\mathcal{S}_2^{(in)} + \tau \mathcal{S}_1^{(in)})^{-1}(\mathbf{b}_2^{(in)} + \tau \mathbf{b}_1^{(in)}) = \frac{1}{\Phi(\tau)} \mathbf{y}(\tau), \tag{2.21}$$

where $\Phi(\tau)$ is the determinant of $(\mathcal{S}_2^{(in)} + \tau\mathcal{S}_1^{(in)})$. τ is the Larangian multiplier that can be obtained from the following six-order polynomial (see Chesnutt & Marshall 2009)

$$\mathbf{y}(\tau)^T \mathcal{S}_1^{(in)} \mathbf{y}(\tau) + \Phi(\tau) \mathbf{b}_1^{(in)^T} \mathbf{y}(\tau)^T + \Phi^2(\tau) c_1 = 0. \tag{2.22}$$

The above process can be repeated to identify the contact point of ellipsoid 2 on the level surfaces of ellipsoid 1.

2.2.3. *Contact Forces and Torques*

When two particles collide, the velocity at the contact point is

$$\mathbf{v}_{C,i}^{(in)} = \mathbf{v}_i^{(in)} + \mathbf{\Omega}_i^{(in)} \times \mathbf{r}_{C,i}^{(in)}, \tag{2.23}$$

where $\mathbf{r}_{C,i}^{(in)} = \mathbf{x}_{C,i}^{(in)} - \mathbf{x}_{p,i}^{(in)}$ points from the ellipsoid centroid to the contact point. The normal velocity $\mathbf{v}_{rel,n}^{(in)}$ ang tangential velocity $\mathbf{v}_{rel,t}^{(in)}$ are defined by

$$\mathbf{v}_{rel,n}^{(in)} = (\mathbf{v}_{C,i}^{(in)} - \mathbf{v}_{C,j}^{(in)}) \cdot \mathbf{n}, \tag{2.24}$$

and

$$\mathbf{v}_{rel,t}^{(in)} = (\mathbf{v}_{C,i}^{(in)} - \mathbf{v}_{C,j}^{(in)}) - (\mathbf{v}_{C,i}^{(in)} - \mathbf{v}_{C,j}^{(in)}) \cdot \mathbf{n}. \tag{2.25}$$

Here, the unit vector along the outward normal direction at the contact point are given by

$$\mathbf{n}(\mathbf{x}_{C,i}^{(in)}) = \nabla P_i(\mathbf{x}_{C,i}^{(in)}) / \left|\nabla P_i(\mathbf{x}_{C,i}^{(in)})\right|, \tag{2.26}$$

while the tangent unit vector equals

$$\mathbf{t} = \mathbf{v}_{rel,t}^{(in)} / \left|\mathbf{v}_{rel,t}^{(in)}\right|. \tag{2.27}$$

In each collision, particles are treated as soft spheres. The contact forces and torques are calculated according to the Hertz contact model (Marshall 2009).

$$\mathbf{F}_{C,j\to i}^{(in)} = (F_{ne} + F_{nd})\mathbf{n} + F_t \mathbf{t} \tag{2.28}$$

Here, the normal force consists of two terms, i.e., the normal elastic force F_{ne} and the normal damping force F_{nd}. The normal elastic force can be expressed as

$$F_{ne} = -k_N \delta_N. \tag{2.29}$$

$\delta_N = \left|\mathbf{x}_{C,i}^{(in)} - \mathbf{x}_{C,j}^{(in)}\right|$ is the normal overlap, and the stiffness k_N is written as

$$k_N = \frac{4}{3} E \sqrt{R\delta_N}, \tag{2.30}$$

The effective radius R is defined by the mean curvature of two ellipsoids at their contact points as

$$R = (K_{C,i} + K_{C,j})^{-1}, \tag{2.31}$$

with the local mean curvature K_i given by

$$K_i = \frac{h^3}{2}\left[\frac{1}{a^2b^2}\left(\frac{x_i^{(p)2}}{a^2} + \frac{y_i^{(p)2}}{b^2}\right) + \frac{1}{b^2c^2}\left(\frac{y_i^{(p)2}}{b^2} + \frac{z_i^{(p)2}}{c^2}\right) + \frac{1}{c^2a^2}\left(\frac{z_i^{(p)2}}{c^2} + \frac{x_i^{(p)2}}{a^2}\right)\right], \tag{2.32a}$$

$$h = [(x_i^{(p)})^2/a^4 + (y_i^{(p)})^2/b^4 + (z_i^{(p)})^2/c^4]^{-1/2}. \tag{2.32b}$$

The effective elastic modulus E is defined as

$$\frac{1}{E} = \frac{1-\nu_i^2}{E_i} + \frac{1-\nu_j^2}{E_j}, \tag{2.33}$$

where E_i and ν_i are the elastic modulus and Poisson ratio of particle i. The normal damping force is proportional to the normal relative velocity

$$F_{nd} = -\eta_N \mathbf{v}_{rel} \cdot \mathbf{n}, \tag{2.34}$$

where the normal damping coefficient is defined as

$$\eta_N = \alpha_N (m k_N)^{1/2}. \tag{2.35}$$

Here, m is the particle mass, and α_N is related to the coefficient of restitution e (Marshall 2009). The tangential force is calculated based on the static friction model and written as

$$F_t = -\mu_F |F_n| \tag{2.36}$$

where $\mu_F = 0.3$ is the friction coefficient.Once the full contact force $\mathbf{F}_{C,j\to i}^{(in)}$ is obtained, the corresponding rotation torque is computed by

$$\mathbf{M}_{C,j\to i}^{(in)} = \mathbf{r}_{C,ij}^{(in)} \times \mathbf{F}_{C,j\to i}^{(in)}. \tag{2.37}$$

2.3. *Induced Charge*

2.3.1. *Governing equation of surface charge*

The electrostatic interactions between dielectrical ellipsoidal particles are considered using the techniques introduced by Barros *et al.* (2014); Barros & Luijten (2014). The governing equation of the induced charge (or the bound charge) on a dielectric surface is given by

$$\mathcal{A}\sigma_b = b \tag{2.38}$$

The left term of 2.38 is

$$\mathcal{A}\sigma_b = \bar{\kappa}\sigma_b + \varepsilon_0 \Delta\kappa (\mathbf{E}_b + \mathbf{E}_{ext}) \cdot \mathbf{n}, \tag{2.39}$$

while the term on the right hand side writes

$$b = (1-\bar{\kappa})\sigma_f - \varepsilon_0 \Delta\kappa \mathbf{E}_f \cdot \mathbf{n}. \tag{2.40}$$

Here, σ_b and σ_f are the bound charge density and the free charge density, respectively. $\bar{\kappa} = \kappa_p + \kappa_m$ is the mean dielectric constant of the particle (κ_p) and the medium (κ_m). $\Delta\kappa = \kappa_p - \kappa_m$ is the difference of the dielectric constants. $\varepsilon_0 = 8.854 \times 10^{-12} F/m$ is the vacuum permittivity. $\mathbf{E}_{ext}$ is the external field. $\mathbf{E}_b$ is the electrical field induced by the bound charge at the particle surface, which reads

$$\mathbf{E}_b(\mathbf{r}_i) = \int_S \sigma_b(\mathbf{r}_i) \frac{\mathbf{r}_i - \mathbf{r}_j}{4\pi\varepsilon_0 |\mathbf{r}_i - \mathbf{r}_j|^3} \mathrm{d}S(\mathbf{r}_j). \tag{2.41}$$

$\mathbf{E}_f$ is the electric field induced by the free charge that can be obtained similar to 2.41.

2.3.2. *Surface discretization*

To implement 2.38, the ellipsoidal surface is discretized into uniform surface patches using the open-source code *DistMesh* developed by Persson & Strang (2004). Then the matrix in 2.38 can be written as

$$\mathcal{A}_{ij} = \overline{\kappa}_i \delta_{ij} + \Delta\kappa_i \mathbf{n}_i \cdot \mathcal{I}_{ij} a_j \tag{2.42}$$

where the components of $\mathcal{I}_{ij}$ is the Green function from the jth patch to the ith patch.

$$\mathcal{I}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/4\pi\left|\mathbf{r}_i - \mathbf{r}_j\right|^3 \tag{2.43}$$

The Green function becomes singular when considering the contribution of each patch to itself ($\mathcal{I}_{ii}$). In the present study, this contribution is omitted with

$$\mathcal{I}_{ii} = \mathbf{0}. \tag{2.44}$$

For approximations with higher-order accuracy, see Barros *et al.* (2014). Then the free charge σ_f is assumed to be uniformly distributed on the surface, and 2.38 is solved to get the bound charge density σ_b. The full charge density is then given as $\sigma(\mathbf{r}_i) = \sigma_f(\mathbf{r}_i) + \sigma_b(\mathbf{r}_i)$.

2.3.3. *Electrostatic Force and Torque*

The electrostatic force and toque can be directly computed by integrating the force and torque on each surface patch, which are shown as follows

$$\mathbf{F}_E = \int_S (\sigma_f + \sigma_b)\mathbf{E}\mathrm{d}S, \;\; \mathbf{M}_E = \int_S (\sigma_f + \sigma_b)\mathbf{r} \times \mathbf{E}\mathrm{d}S. \tag{2.45}$$

3. Results and Discusions

For results and discussions.

4. Conclusion

For conclusion.

Appendix A. Validation of the electrostatic interaction calculation

A.1. *Potential energy between a point charge and a dielectric sphere*

The electrical potential energy U between a point charge q and a dielectric sphere with the radius of R and the dielectric constant κ_p is given by (J.D.Jackson 1999; Barros & Luijten 2014)

$$U = \frac{q^2}{8\pi\varepsilon_0\kappa_m R}\sum_{n=0}^{\infty}\frac{(1-\tilde{\kappa})n}{(1+\tilde{\kappa})n+1}\frac{1}{1+(1+d/R)^{2(n+1)}}, \tag{A 1}$$

where κ_m is the dielectric constant of the medium, $\tilde{\kappa} = \kappa_p/\kappa_m$ is the dielectric constant ratio, d is the distance from the point charge to the sphere surface. The electrical energy normalized by $U_0 = q^2/\varepsilon_0\kappa_m R$ shows good agreement with the theoratical solution in 1. Minor deviation occurs when d/R becomes small, which is due to the limited number of surface patches used here ($N_{patch} = 956$). Increasing the total patch number or only refining the local surface patches could further increase the accuracy.

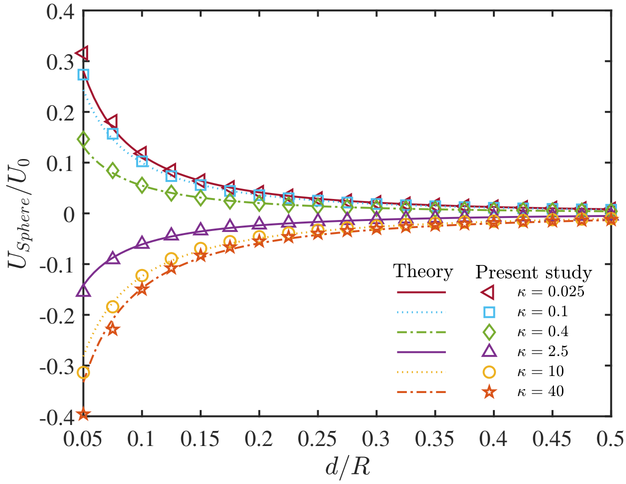

Figure 1: Normalized potential energy between a point charge and a dielectric sphere

A.2. *Electrostatic torque on ellipsoidal particle*

For a neutral dielectric ellipsoidal particle defined by $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$, the electrostatic torque $\mathbf{M}_E$ exerted by the external field $\mathbf{E} = [E_x, E_y, E_z]$ is given by (T.B.Jones 1995)

$$M_{E,x} = \frac{4\pi\varepsilon_m abc}{3} \cdot \frac{(\tilde{\kappa}-1)^2(L_z - L_y)E_z E_y}{[1+(\tilde{\kappa}-1)L_z][(\tilde{\kappa}-1)L_y]}, \tag{A 2a}$$

$$M_{E,y} = \frac{4\pi\varepsilon_m abc}{3} \cdot \frac{(\tilde{\kappa}-1)^2(L_x - L_z)E_x E_z}{[1+(\tilde{\kappa}-1)L_x][(\tilde{\kappa}-1)L_z]}, \tag{A 2b}$$

$$M_{E,z} = \frac{4\pi\varepsilon_m abc}{3} \cdot \frac{(\tilde{\kappa}-1)^2(L_y - L_x)E_y E_x}{[1+(\tilde{\kappa}-1)L_y][(\tilde{\kappa}-1)L_x]}. \tag{A 2c}$$

Here, the elliptical integrals are defined as

$$L_x = \frac{abc}{2}\int_0^\infty \frac{ds}{(s+a^2)R_s}, \tag{A 3a}$$

$$L_y = \frac{abc}{2}\int_0^\infty \frac{ds}{(s+b^2)R_s}, \tag{A 3b}$$

$$L_z = \frac{abc}{2}\int_0^\infty \frac{ds}{(s+c^2)R_s}, \tag{A 3c}$$

with $R_s = [(s+a^2)(s+b^2)(s+c^2)]^{1/2}$. We consider a spheroid with $b = c = 1$ and a ranging from 1 to 8 under an external field $\mathbf{E} = E_0[1, 1, 1]$ (Add schematic). The dielectric constant of the particle and the medium is $\kappa_p = 2.5$ and $\kappa_m = 1$. As shown in 2, the our calculation results coincide well with the theoratical solution in A 2.

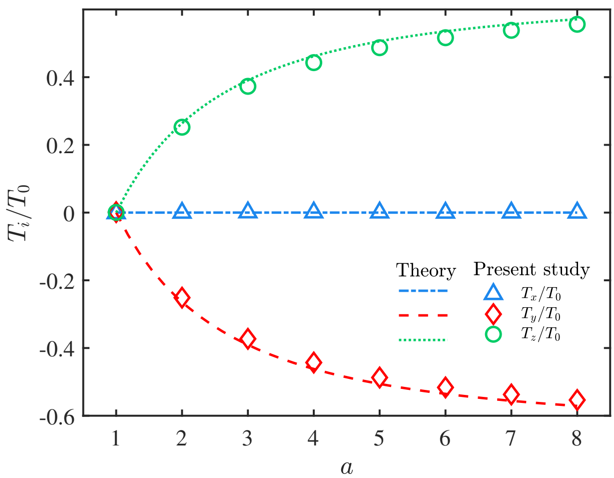

Figure 2: Normalized torque acting on the dielectric ellipsoid by the external field

a	1	2	3	4	5	6	7	8
N_{patch}	956	1564	2204	2812	3516	4092	4732	5340

Table 1: Number of patches N_{patch} of ellipoids with a different semi-axis length a.

Appendix B. Simulation acceleration using a reduced particle stiffness

In this appendix, the dimensionless linear momentum equation is derived in presence of the electrostic interaction. And guidelines of adjusting the electrostatic force are given when a reduced elastic modulus is used for acceleration. When a charged particle encounters a head-on collision, the linear momentum equation (2.7) is

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -(k_N\delta_N + \eta_N v_{rel,n})\mathbf{n} + \mathbf{F}_E. \tag{B 1}$$

We introduce the dimensionless velocity $\hat{\mathbf{v}}$ and the dimensionless overlap $\hat{\delta}_N$ as

$$\hat{\mathbf{v}} = \mathbf{v}/v_0, \tag{B 2a}$$

$$\hat{\delta}_N = \delta_N/\delta_0. \tag{B 2b}$$

Here, the approaching velocity between two colliding particles is chosen as the charateristic velocity v_0, while the characteristic overlap is given by

$$\delta_0 = \left(\frac{mv_0^2}{ER^{1/2}}\right)^{2/5}. \tag{B 3}$$

Then B 1 becomes

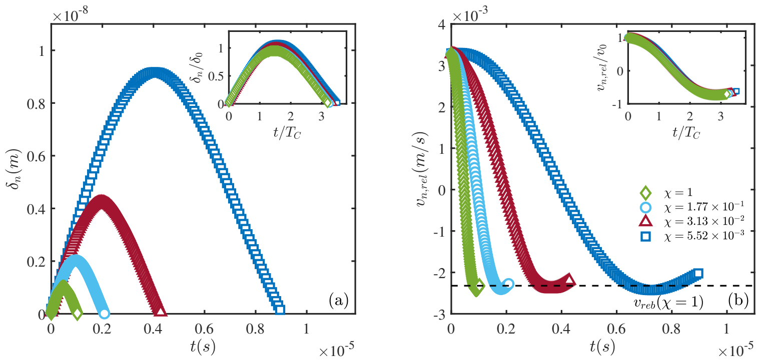

Figure 3: (a) Normal overlap and (b) normal relative velocity between two oppositely charged particles in a head-on collision with unmodified electrostatic force

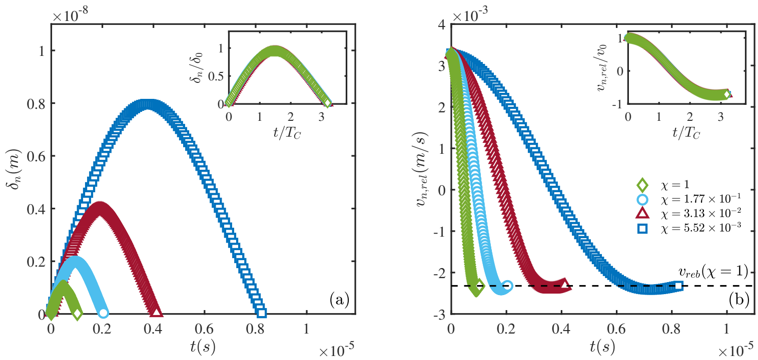

Figure 4: (a) Normal overlap and (b) normal relative velocity between two oppositely charged particles in a head-on collision with modified electrostatic force

$$\frac{\mathrm{d}\hat{\mathbf{v}}}{\mathrm{d}\hat{t}} + \frac{4}{3}\hat{\delta}_N^{3/2}\mathbf{n} + \frac{2}{\sqrt{3}}\alpha_N\hat{\delta}_N^{1/4}\hat{v}_{rel,n}\mathbf{n} = \left(\frac{1}{m^3 v_0^6 E^2 R}\right)^{1/5}\mathbf{F}_E. \tag{B 4}$$

After choosing a reduced elastic modulus E_R, the dimensionless electrostatic force

$$\hat{\mathbf{F}}_E = \left(\frac{1}{m^3 v_0^6 E^2 R}\right)^{1/5}\mathbf{F}_E \tag{B 5}$$

should remain unchanged, so that B 4 gives the correct results. Thus, the dimensional electrostatic force should be modified as

$$\mathbf{F}_{E,R} = \left(\frac{E_R}{E_O}\right)^{2/5}\mathbf{F}_{E,O} = \chi^{2/5}\mathbf{F}_{E,O}. \tag{B 6}$$

Here, the subscripts O and R denote variables in a case with the original stiffness and that in a reduced case, respectively. $\chi = E_R/E_O$ is the reduced ratio. (Add case discription here)

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