Math Cram Packet

Unit 6: Rational Functions

- Factoring
  - $x^7 - 128$
  - $6x^4 + 11x^2 - 10$
  - $4x^2 - 12xy + 9y^2$
  - $x^2 + 10x + 25 - 4y^2$
  - $x^2 + 5$
  - $(x + 3)^2 - 81$
  - $3x^2 + 10x - 5$
  - $x^4y^2 - x^2y^4$
  - $x^3 + 27y^3$
  - $4x^3 + x^2y - 4xy^2 - y^3$

- Mult/Div/Complex
  - https://mcckc.edu/tutoring/docs/bt/algebra/Multiplying_Rational_Expressions.pdf

25. $\frac{a^3-b^3}{3a^2+9ab+6b^2} \cdot \frac{a^2+2ab+b^2}{a^2-b^2}$

26. $\frac{x^3+y^3}{x^2+2xy-3y^2} \cdot \frac{x^2-y^2}{3x^2+6xy+3y^2}$

27. $\frac{4x^2-9y^2}{8x^3-27y^3} \cdot \frac{4x^2+6xy+9y^2}{4x^2+12xy+9y^2}$

- 55. $\frac{8a^3+b^3}{2a^2+3ab+b^2} \div \frac{8a^2-4ab+2b^2}{4a^2+4ab+b^2}$

56. $\frac{x^3+8y^3}{2x^2+5xy+2y^2} \div \frac{x^3-2x^2y+4xy^2}{8x^2-2y^2}$

- Add/Subtract
  - http://www.kutasoftware.com/FreeWorksheets/Alg2Worksheets/Adding+Subtracting%20Rational%20Expressions.pdf
  - 6) $\frac{x+2}{2x^2+13x+20} - \frac{x+3}{2x^2+13x+20}$
  - 21) $\frac{5n+5}{5n^2+35n-40} + \frac{7n}{3n}$
  - 26) $\frac{\frac{5}{4}}{\frac{5}{m} - \frac{4}{m}}$
- Graph Rational Functions
  - How to find HA:
    - Degree numerator greater than degree denominator - no HA
    - Degree numerator less than degree denominator - HA @ y=0
    - Same degree - HA is y=ratio of leading coefficients
  - How to find VA: set the denominator equal to 0 and solve for x
  - How to find Hole: Holes in the graph of a rational function are generally produced by factors that are common to both the numerator and the denominator!
    - http://sites.csn.edu/istewart/Math126/graph_rational_func/graph_rational_func.htm
    - x-coordinate of the hole
      - set equal to 0 the factor that is common to both the numerator and the denominator of the function then solve

        - y-coordinate of the hole
            - First, we MUST create a new function by reducing the original function. Remember that we have to call it a "new" function since reducing changes the domain. Please note that functions are no longer equal if they have a different domain.
            - Finally, to find the y-coordinate of the hole, we simply replace x in the new function with the x-coordinate of the hole.

1. Given $f(x)=\frac{x+2}{x^2-4}=\frac{x+2}{(x+2)(x-2)}$, the function $g(x)=\frac{1}{x-2}$ is the "new" function and the y-coordinate of the hole is $g(-2)=\frac{1}{-2-2}=-\frac{1}{4}$.

   The coordinates of the hole are at $(-2,-\frac{1}{4})$.

- Solve Rational Equations
    - http://www.montereyinstitute.org/courses/Algebra1/COURSE_TEXT_RESOURCE/U11_L2_T1_text_final.html
    - Find common denominator!
    - $\frac{7}{x+2}+\frac{5}{x-2}=\frac{10x-2}{x^2-4}$

- Inverse/Direct

Unit 7: Exponential and Logarithmic Functions
http://www.wou.edu/mathcenter/files/2015/09/Exponents-and-Logarithms.pdf

- Simplify Exponential Expressions
    - $X^a+X^a=$
    - $2x^a-x^a=$
    - $x^a*x^b=$
    - $x^a/x^b=$
    - $(x^a)^b=$
    - $x^{-a}=$
    - $1/x^{-a}=$
    - $(xy)^n=$
    - $(x/y)^n=$
    - $x^{1/2}=$

- $x^1=$
- $1^a=$
- $x^0=$
- $x^a \neq$ negative number
- If $b^m=b^n$, $m=n$
- $81^{3/4}=$
- $64^{2/3}=$
- $9^{-½}=$
- $32^{-7/5}=$

- Simplify Log Expressions
  - $\text{Log}_b x+\text{Log}_b y=$
  - $\log_b x-\log_b y=$
  - $\log_b x^n=$
  - $\log_b b=$
  - $\log_b 1=$
  - $\log_b x=\log x / \log b$
  - $\text{Log } x = \log_{10} x$
  - $\text{Ln } x = \log_e x$
  - $b^x=y$ iff $\log_b y=x$ means $b^{\log b(y)}=y$ and $\log_b b^x=x$
  - $(\log_a b)(\log_b c)=$
  - $(\log_a b)(\log_b c)(\log_c d)=$
  - $\log_a b=1/\log_b a$
  - $\log_a b=\log b / \log a$
  - if $\log_b m=\log_b n$, $m=n$
  - *remember to check order of operations
  - Don't forget to check your work!
- Solve Exponential Equations
  - Use exponential properties!
  - Simplify as much as possible!
  - $X^{4/3}=81$
  - $4^{2X+3}=1$
  - $5^{3-2X}=5^{-X}$
  - $3^{1-2X}=243$

- $6^{3m} * 6^{-m} = 6^{-2m}$

- $2^x/2^x=2^{-2x}$

- $(\frac{1}{6})^{3x+2} * 216^{3x} = 1/216$

- Solve Log Equations
  - Logs on both sides
    - Combine both sides into single logs
    - Apply the theorem + burn the logs
    - Solve for x
    - Check solutions!

1. $2\log_4 x+\log_4 2x=\log_4 54$

2. $\log_8 x+\log_8(x+6)=\log_8(5x+12)$

  - Logs on one side only
    - Combine one side to single log
    - Convert to exponential form
    - Solve for x
    - Check solutions!

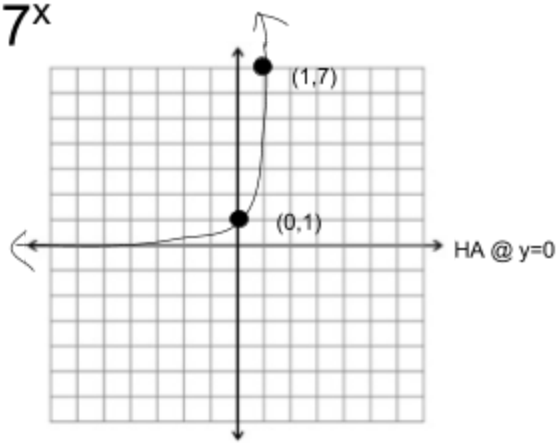
1. $\log_4 x+\log_4(x+3)=1$

2. $\log_2(x+5)-\log_2(2x-1)=5$

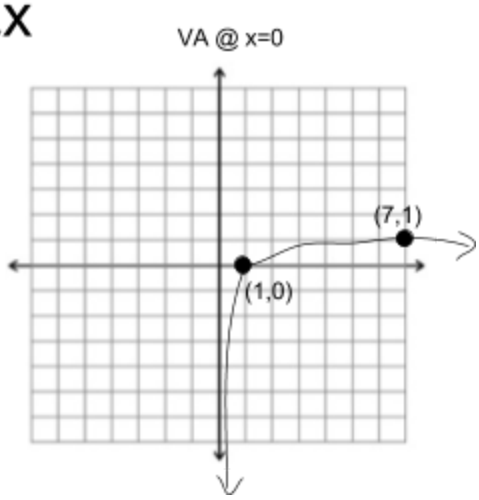
3. ln(6x-5)=3

- Exponential & Log Graphs
  - They are <u>inverses</u>
  - Characteristics (14)
    - Domain
    - Range
    - Fxn, 1-1, cont
    - X-int
    - Y-int
    - Max (abs+local)
    - Min (abs+local)
    - Incr interval
    - Decr interval
    - CCU
    - CCD
    - Asymptotes (HA, VA)
  - example:

$y=7^x$

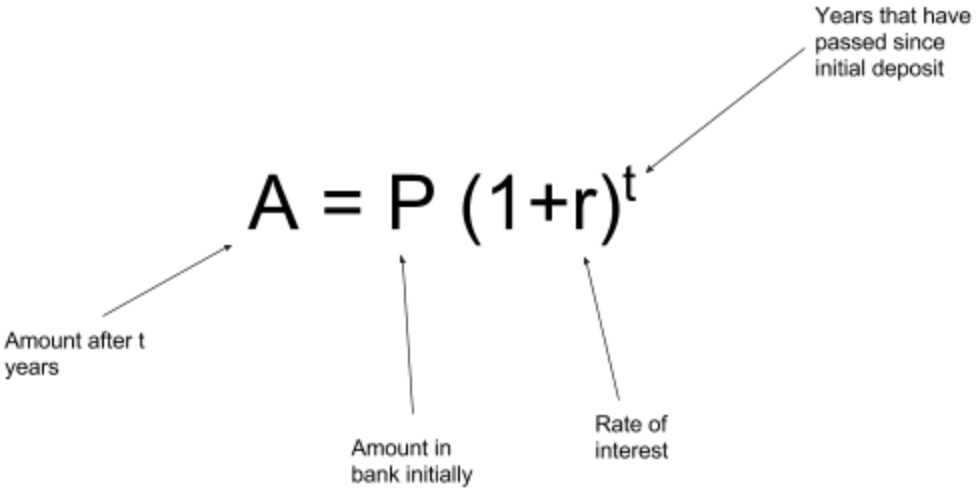


$y=\log_7 x$



- Exponential Modeling
  - $y=a * b^x$
  - Find points (0,c) (1,d)
  - Plug (0,c) into general equation to find particular equation
  - To find b, plug in (1,d) to the result of step #2
  - Use the particular equation to solve the other problems
  - *you <u>should not</u> round numbers while finding the particular equation, but when answering the questions, you <u>should</u> round
  - Ex. # of bacteria triples every minute - find particular equation

- Finance Applications
  - Compounded annually
    - $A = P(1+r/n)^{nt}$
      - n= # of times interest is compounded per year
      - Aka $A = P(1+r)^{t}$ in this case
  - Compounded Quarterly
    - $A=P(1+r/4)^{4t}$
  - Compounded daily
    - $A=(1+r/365)^{365t}$
  - Compounded continuously
    - $A=Pe^{rt}$



Unit 8: Radical Functions

- Simplify: Add/Subtract
  - *you can only combine like radicals of the same root
  - Simplifying - find the highest square that is a factor! + no radicals in denominator
  - *tip: factor trees are helpful!
  - $-3\sqrt{(12)} + 3\sqrt{(3)} + 3\sqrt{(20)}=$
  - $-3\sqrt[6]{(3)} - 2\sqrt[6]{(192)} + \sqrt[6]{(320)}=$

- $4\sqrt[6]{(3)} + 2\sqrt[4]{(32)} - 3\sqrt[6]{(192)} - 2\sqrt[6]{(192)} =$

- Multiply
  - *when multiplying, only the root has to match
  - $2\sqrt{(7)} * -3\sqrt{(3)} =$
  - $(4 + \sqrt{(6)})^2 =$
  - $(\sqrt{(7)} - 2)(\sqrt{(7)} + 2) =$
  - $(7 + \sqrt{(6)})(1 + \sqrt{(6)})$
  - $(\sqrt{(3)} + \sqrt{(5x)})(\sqrt{(3)} - 5\sqrt{(5x)})$

- Rationalize Denominator
  - Multiply the fraction by 1 to eliminate the radical in the denominator
  - $1/\sqrt{(2)}$
  - $2/5\sqrt{(3)}$
  - $2 / (5+\sqrt{(3)})$
  - $4/\sqrt[3]{(25)}$
  - $7/\sqrt[5]{(128)}$
  - $7\sqrt{(3)} + 3 / 7\sqrt{(3)} - 3$

- Solve Radical Equations
    - Type 1 - one radical
        - Isolate radical
        - Undo the radical
        - Solve for x
        - Check answer(s)

1. $5 + \sqrt[3]{(x-2)} = 7$

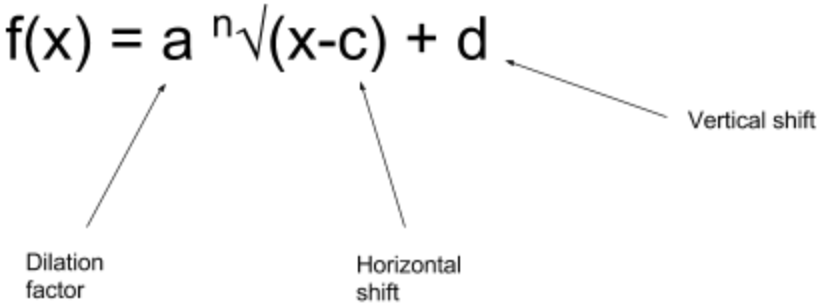
2. $\sqrt[3]{((x + 2)^2)} = 9$

    - Type 2 - more than one radical
        - Get 1 side with only 1 radical
        - Undo radical (square/cube/etc BOTH sides)
        - Repeat until all radicals are gone
        - Solve for x
        - Check answer(s)

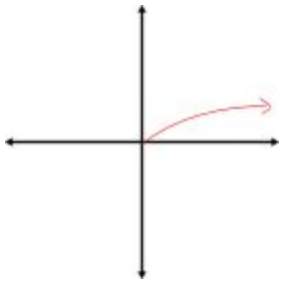
1. $\sqrt{(x + 4)} + \sqrt{(x - 4)} = 4$

2. $\sqrt{(x + 2)} + \sqrt{(x - 3)} = 5$

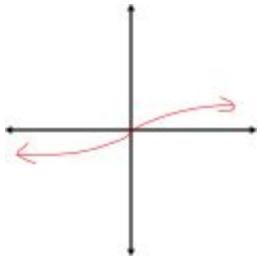
- Radical Graphs: Even/Odd



    - Even - the index of the radical is an even number (ex. 2)



- Odd - the index of the radical is an odd number (ex. 3)



1) Plug in numbers to graph points
2) Create graph
3) Use graph to fill in characteristics list

- Characteristics List:
  - Even (14)
    - Domain
    - Range
    - Fxn, cont, 1-1
    - Max
    - Min
    - Incr
    - Decr
    - CCU
    - CCD
    - Elbow
    - X-int
    - Y-int
  - Odd (14)
    - Domain
    - Range
    - Fxn, cont, 1-1
    - Max
    - Min
    - Incr
    - Decr
    - CCU
    - CD
    - POI (Point Of Inflection)

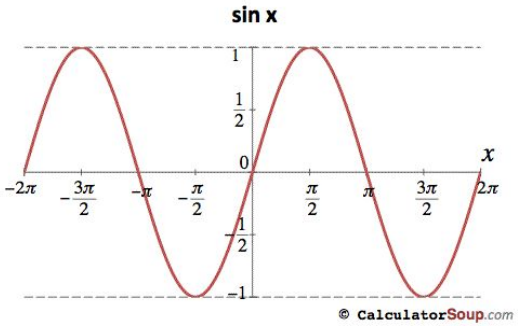
        - X-int
        - Y-int
- Modeling Problems
    - Define variables
    - General equation: $y = k * x^n$
    - Known data (data points)
    - Solve for k - store exact value in calculator!
    - Find particular equation
        - Plug in data points to find k first, then write particular equation
    - *answers can be rounded in the word problems, but not when finding k or the particular equation!
    - If the power (n) is unknown, you must solve for k as well as n!
        - Use either elimination or substitution to simplify the two log equations you made

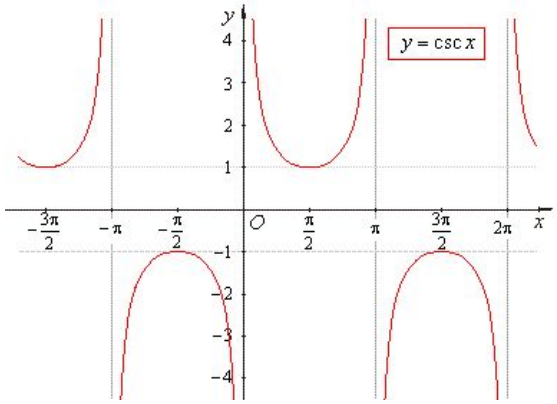
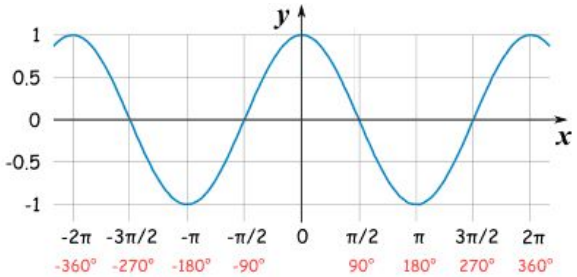
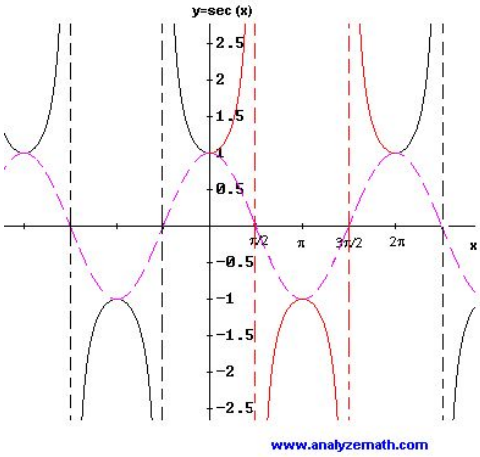
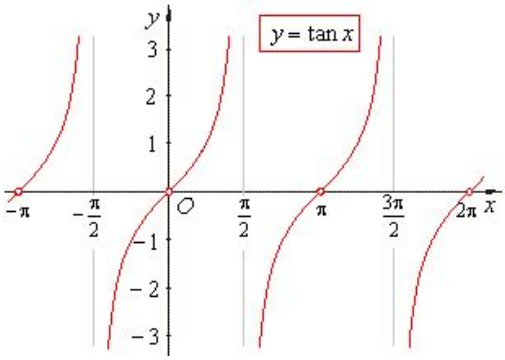
1. [insert practice problem here]

Unit 9: Trigonometry

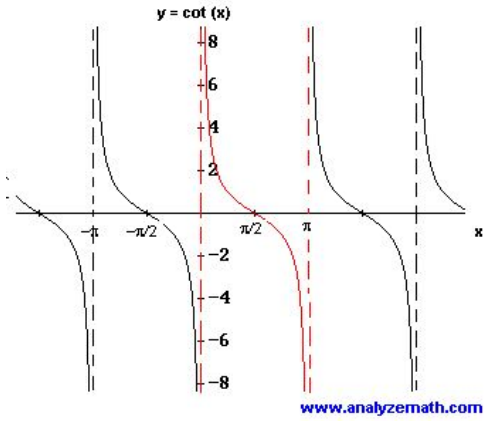
- Trig Graphs

Y = sin θ



Y = csc θ	
Y = cos θ	
Y = sec θ	
Y = tan θ	

Y = cot θ



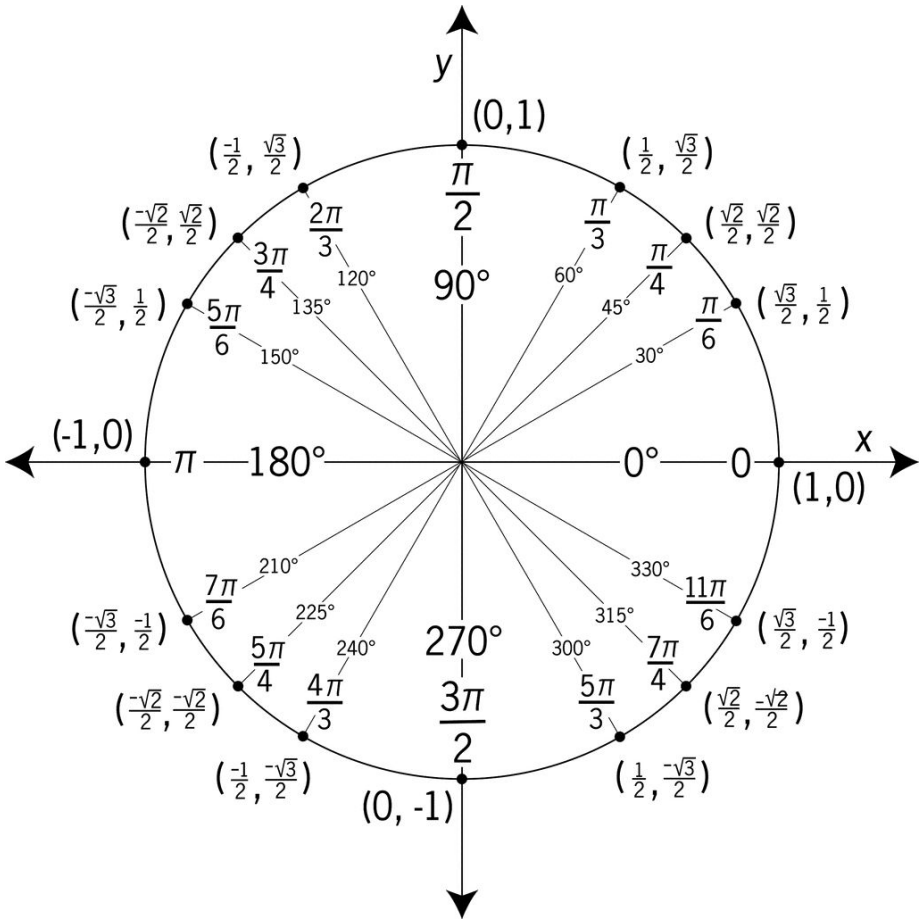
- Characteristics when domain is (-2π, 2π)
  - Sin (y = sin x)
    - Domain: all real numbers
    - Range: [-1, 1]
    - Fxn? Y
    - Cont? Y
    - 1-1? N
    - X-ints: -2π, -π, 0, π, 2π
    - Y-ints: 0
    - Max: 1
    - Min: -1
    - Incr: (-2π, -3π/2) (-π/2, π/2) (3π/2, 2π)
    - Decr: (-3π/2, -π/2) (π/2, 3π/2)
    - CCU: (-π, 0) (π, 2π)
    - CCD: (-2π, -π) (0, π)
    - POI: (-2π,0) (-π,0) (0,0) (π,0) (2π,0)
  - Cos (y = cos x)
    - Domain: all real numbers
    - Range: [-1, 1]
    - Fxn? Y
    - Cont? Y
    - 1-1? N
    - X-ints: -3π/2, -π/2, π/2, 3π/2
    - Y-ints: 1
    - Max: 1
    - Min: -1
    - Incr: (-π, 0) (π, 2π)
    - Decr: (-2π, -π) (0, π)
    - CCU: (-3π/2, -π/2) (π/2, 3π/2)
    - CCD: (-2π, -3π/2) (0, π/2) (3π/2, 2π)
    - POI: (-3π/2, 0) (-π/2, 0) (π/2, 0) 3π/2, 0)
- Coterminal and Reference Angles/Arcs
  - Coterminal angles: angles that share a terminal ray
    - To find positive coterminal angle: add 360°

    - To find negative coterminal angle: find angle that combined with original angle = 360°, then put negative sign in front
    - Find a positive and a negative coterminal arc: -2π/3

- Degrees/Radians
    - Degrees to Radians (multiply by π/180)
        - 36°

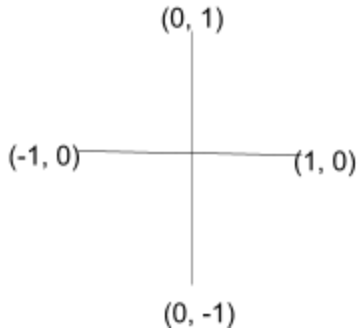
    - Radians to Degrees (multiply by 180/π)
        - 3π/2

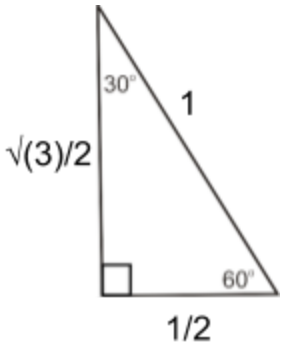
- Unit Circle/Special Triangles

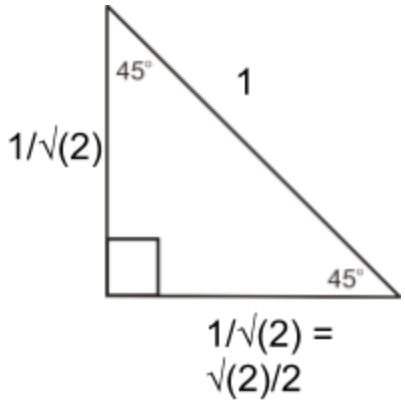


Other Helpful Things	(x, y) (cos, sin)

S	A
T	C



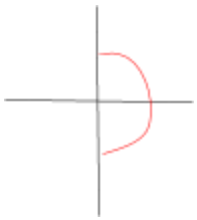
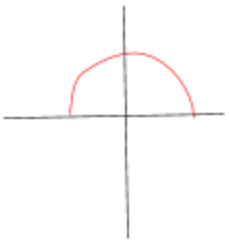
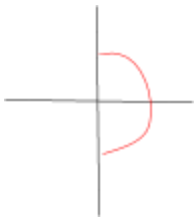




- SOHCAHTOA/CHOSHACAO: Evaluate Trig Expressions
  - *if it is not a special angle, use a calculator
  - tan 60°
  - sin 90°
  - cos 90°
  - sin 225°
  - cos 150°
  - tan 240°

- Inverse Trig Functions
- Ranges:

sin/csc	cos/sec	tan/cot

-90° ≤ θ ≤ 90°	0 ≤ θ ≤ 180°	-90° ≤ θ ≤ 90°
		

- arcsin (-1)
- $\cos^{-1}$
- arcsec (-2)
- $\tan^{-1}(\sqrt{3}/3)$
- $\tan(\tan^{-1}\sqrt{3})$
- $\csc(\tan^{-1}1)$
- $\cos(\sec^{-1}\sqrt{2})$

- Transformations: Sine
  - Y = a sin b (x - c) + d
  - A = amplitude (cannot be negative when listing, -1 is saying whether it is reflected over x-axis or not)
    - Impacts max/min
  - B = # of cycles in 2π
    - Period = 2π/b (length of 1 cycle)
  - C = phase shift (horizontal)
    - left/right shift
  - D = vertical shift
    - up/down shift that impacts the placement of the *sinusoidal axis*
- Transformations: Cosine
  - Y = a cos b (x - c) + d
- Modeling Problems