Biomedical Imaging Fall 2020

Signals and Systems

Use Matlab for programming and display.

Please prepare solutions in pdf format and upload them to the Moodle platform (https://moodle-app2.let.ethz.ch/).

Exercises

1. Plane waves

A plane wave in two dimensions is described by $f(x,y) = e^{i(k_x x + k_y y)}$ with $k_x, k_y \in \mathbb{R}$

- Write a program that calculates 2D plane waves for any given k_x, k_y.
- Display and examine Re(f), Im(f), Abs(f), Phase(f) for a selection of k_x, k_y.
- What determines the direction of each wave?
- What is the wavelength as a function of k_x, k_y?
- Think of some linear, shift-invariant (LSI) operation and apply it.
- What happens to the waves?

2. Fast Fourier Transform (FFT)

This exercise studies the behavior of FFT, using a rectangle input as an example.

The prepared code works with a 1D input vector of length 256, with a rectangle of length 16 at the center. Run the code and answer the following questions. Along with the questions, review the related parts of the code.

a) The code first calculates and displays the FFT of the input vector straightforwardly (Figure 1). Why is the sinc-shaped Fourier transform split in two halves?

b) The code then performs 'fftshift' on the transform, which swaps its first and second halves (Figure 2). Why does the phase of the transform oscillate rapidly? Isn't the rectangle symmetric so that the transform should be purely real?

c) The code now includes phase correction, shifting the origin in the original domain to the center of the rectangle (Figure 3). Review the related lecture notes.

d) Next the code attempts to shift the origin simply by fftshift instead, similar to what it does in the Fourier domain (Figure 4). As you see, this works only almost. Why doesn't it work fully?

3. Building a Comb

e) Build a 1D comb function by starting with a single point impulse in the center and successively adding impulses on the left and right
    - At each stage, calculate and examine the Fourier transform. What happens?
    - What happens when you vary Δx of the original comb?