

Ehrhart Polynomials

VIII Encuentro Colombiano De Combinatoria

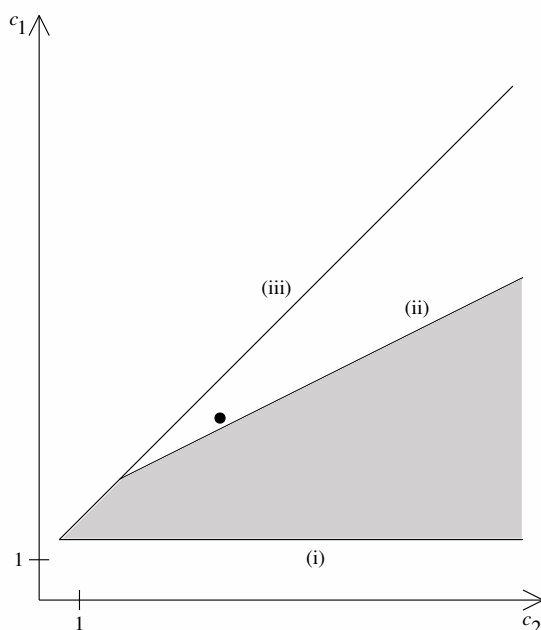
Day I: Appetizers

- (1) Pick five points in \mathbb{Z}^3 and let \mathcal{P} be their convex hull (in \mathbb{R}^3). Compute the Ehrhart polynomial of \mathcal{P} .
- (2) [sage] Plot the roots of the Ehrhart polynomials of cross polytopes in different dimensions. What's going on here?
- (3) Show that a sequence $f(n)$ is given by a polynomial of degree $\leq d$ if and only if

$$\sum_{n \geq 0} f(n) z^n = \frac{h(z)}{(1-z)^{d+1}}$$

for some polynomial $h(z)$ of degree $\leq d$. Furthermore, $f(n)$ has degree d if and only if $h(1) \neq 0$.

- (4) Verify (parts of) the classification picture of degree-2 Ehrhart polynomials $c_2 t^2 + c_1 t + 1$: every half-integral point in the figure below corresponds to an Ehrhart polynomial.



- (5) [research problem] Give the corresponding classification picture of degree-3 Ehrhart polynomials.