**Proposition 11.7.** *For any $x, y \in \mathbb{R}$:*

(i) $|x-y| = 0$ *if and only if* $x = y$

(ii) $|x-y| = |y-x|$

(iii) $|x-z| \leq |x-y| + |y-z|$.

(iv) $|x-y| \geq ||x| - |y||$

*Proof.* Suppose $x, y \in \mathbb{R}$.

(i) follows directly from Proposition 11.6(i) and the observation that $x - y = 0$ is equivalent to $x = y$.

(ii) Since $|-1| = -(-1) = 1$, we get with Proposition 11.6(ii)

$$|x-y| = |(-1)(y-x)| = |-1|\,|y-x| = |y-x| \ .$$

(iii) follows with Proposition 11.6(iv):

$$|x-z| = |(x-y)+(y-z)| \leq |x-y| + |y-z| \ .$$

(iv) We may assume that $|x| \geq |y|$, since we can always switch the role of $x$ and $y$. Then $||x| - |y|| = |x| - |y|$, and we need to prove that $|x-y| \geq |x| - |y|$. But this follows from choosing $z = 0$ in (iii):

$$|x| = |x-0| \leq |x-y| + |y-0| = |x-y| + |y| \ ,$$

whence

$$|x-y| \geq |x| - |y| \ .$$

□