MATH 301 Exploration and Proof Fall 2006

Meeting times: MWF 9:10–10:00

Prerequisites: Grade of C or better in MATH 226 (Calculus I) or equivalent

Instructor: Matthias Beck (TH 933, 415.405.3473, `beck@math.sfsu.edu`)

Course Objectives: Mathematics is a powerful and flexible language that can be adapted to solve a wide array of problems from many areas. Mathematics connects seeming disparate subjects and provides a unifying philosophy behind logical thinking. Calculus students are acquainted with mathematics as a computational tool but are usually not familiar with mathematics as a language that helps us to explore possibilities, discover conjectures, and prove or refute them. MATH 301 is designed to help students make the transition to more advanced mathematics, where discourse and proof are emphasized. The goals of the course are to improve students' abilities to

- understand the difference between a definition, an axiom, and a theorem;
- read and understand proofs;
- generate and test conjectures when confronted with a new mathematical problem;
- use concepts from logic and set theory in mathematical discussions and proofs;
- present clearly written solutions and proofs.

We will discuss basic mathematical objects (such as the integers, which will form the starting point for the course) that seem familiar to us; however, we will define them axiomatically, pretending that "we don't know anything" aside from the definitions. This will lead us, for example, to *proving* that $n \cdot 0 = 0$ for any integer n. The proof of such a seemingly obvious statement requires some care in handling and manipulating the axioms that define the integers. Good proofs require good writing, and writing is difficult. In this course, you will get a lot of practice writing.

Evaluation of Students: Students will be graded on their ability to write clear proofs, to work with new definitions, and to apply known theorems. Grades will be based on weekly homework assignments (50%), quizzes (20%), a term paper (10%), and a final exam (20%).

Textbook: M. Beck, *Axioms, theorems, proofs, and all that: Variations on a theme of Ross Geoghegan.* These lecture notes will be available on the `Blackboard` course webpage.