Proposition 2.9. *If $m, x_1, x_2 \in \mathbb{Z}$ satisfy the equations $m + x_1 = 0$ and $m + x_2 = 0$, then $x_1 = x_2$.*

Proof. Suppose m, x_1 and x_2 are elements of $\mathbb{Z}$ which satisfy the equation $m + x = 0$, that is,

$$m + x_1 = 0 \qquad \text{and} \qquad m + x_2 = 0 \ .$$

Equating both zeros yields $m + x_1 = m + x_2$, and we can add $-m$ on both sides:

$$-m + (m + x_1) = -m + (m + x_2)\,.$$

It remains to use Axioms 2.1(ii), 2.4, 2.1(i), and 2.2 on both sides of this equation:

$$\begin{aligned} (-m + m) + x_1 &= (-m + m) + x_2 \\ 0 + x_1 &= 0 + x_2 \\ x_1 + 0 &= x_2 + 0 \\ x_1 &= x_2 \ . \end{aligned}$$

□