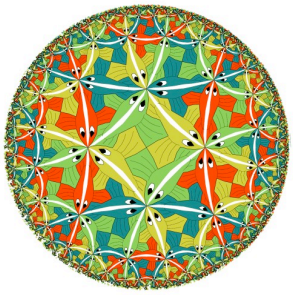
MATH 890.01
Advanced Number Theory:
Seminar in Modular Forms
Spring 2006



Meeting times: MWF 12:10–1:00

Prerequisites: Undergraduate Number Theory & Complex Analysis (i.e., the equivalent of MATH 310 & 380) or consent of the instructor

Instructor: Matthias Beck (TH 933, (415) 405-3473, `beck@math.sfsu.edu`)

Course Objectives: Modular forms are meromorphic functions on the upper half plane that satisfy fundamental symmetry and growth conditions. Roughly speaking, you can think about them as functions that are invariant under Möbius transformations. (The Escher picture above essentially illustrates this invariance, after the upper half plane gets conformally mapped to the unit disk.) Modular forms are essential tools for the study of many elementary number-theoretic functions, such as the partition function (how many ways are there to write n as a sum of integers?), the sum-of-divisors function ($\sum_{d|n} d^a$ for some fixed $a \in \mathbb{R}$), and the sum-of-squares function (how many ways are there to write n as a sum of k squares?), they form a powerful connection between algebraic and analytic number theory (as exemplified by Wiles' proof of Fermat's Last Theorem), but they also have a beautiful life of their own. In this seminar, we will study classical aspects of modular forms and some of their applications to elementary (but hard) problems in Number Theory.

Textbook: T. Apostol, *Modular Functions and Dirichlet Series in Number Theory*, 2nd edition, Springer.

Syllabus: I plan to cover Chapters 1, 2, 3, and 6 in the text book, that is:

- Elliptic functions, Weierstrass $\wp$-function, Eisenstein series, discriminant function Δ, Klein's modular function J, Fourier expansions
- Möbius transformations, modular group, fundamental regions, modular functions, Picard's Theorem
- Dedekind η-function and functional equation, Dedekind sum and its reciprocity law
- Modular forms of weight k, entire forms, Hecke operators, eigenforms, Dirichlet series

Evaluation of Students: This is a seminar course, that is, the students will present the material, in approximately two lectures at a time. I will also assign some (very light) homework each week. Students will be graded on their lectures, homework assignments, and class participation.