MATH 420/720 Homework Quiz 1 (5 February 2025)

(a) Define a permutation of $[n]$.

(b) The *Lucas numbers* are defined by $L_0 = 2$, $L_1 = 1$, and

$$L_n = L_{n-1} + L_{n-2} \qquad \text{for } n \geq 2\,.$$

Let C_n be the set of tilings of n boxes arranged in a circle with dominos and monominos. Show that $\#C_n = L_n$ for $n \geq 1$.

MATH 420/720 Homework Quiz 2 (12 February 2025)

(a) Define the Stirling numbers $S(n,k)$ of the second kind.

(b) Show that

(i) $S(n,n) = 1$

(ii) $S(n,n-1) = \binom{n}{2}$

(iii) $S(n,n-2) = \binom{n}{3} + 3\binom{n}{4}$.

MATH 420/720 Homework Quiz 3 (19 February 2025)

(a) Define a partition λ of n.

(b) Denote by $p_e(n,k)$ the number of partitions of n having exactly k parts. Prove that

$$p_e(n,k) = p(n-k,k)\,.$$