**MATH 435/735** **Module Homework II**

(1) Consider $\mathbb{Z}[x]$ as a module over itself. Let $M = \langle 2, x \rangle$ considered as a submodule of $\mathbb{Z}[x]$. Show that $\{2, x\}$ is a not a basis of $M$ and conclude that $M$ is not free.

(2) An $R$-module $M$ is *irreducible* if $M \neq 0$ and 0 and $M$ are the only submodules of $M$. Prove that $M$ is irreducible if and only if $M \neq 0$ and $M$ is cyclic with any nonzero element as generator.

(3) (Grad students) Show that similar linear transformations of a finite-dimensional vector space (or, equivalently, similar $n \times n$ matrices) have the same characteristic and the same minimal polynomial.

(4) Prove Lemma 19 in Section 12.2.