

Worksheet 4: “Chinese Remainder Theorem”

1. Let a be the day (of the month) you were born and b the month.¹ Find $x \in \mathbb{Z}$ such that

$$x \equiv a \pmod{31} \quad \text{and} \quad x \equiv b \pmod{12}.$$

2. Suppose $m, n \in \mathbb{Z}_{>0}$ are relatively prime, and $a, b \in \mathbb{Z}$. Prove that

$$x \equiv a \pmod{m} \quad \text{and} \quad x \equiv b \pmod{n}$$

has a solution $x \in \mathbb{Z}$ and that x is unique modulo mn .

3. Generalize the statement (and your proof) of 2. to a system of k congruences.
4. Andrews 5.3.1. (Feel free to use `sage`.)
5. Write down a precise statement for each definition we have given this week. For each definition, give an example and a non-example.

¹If $a = 21$ and $b = 9$... happy birthday!!