Project 3.23. *In this chapter we've encountered our first proofs by contradiction. The first step of such a proof is to assume the* negation *of a statement, and the remainder of the proof is devoted to leading this assumption to a contradiction. It is, therefore, important to practice negating statements. Try your luck by negating the following statements:*

(i) *G is normal and H is regular.*

(ii) *Any cubic polynomial has a real root.*

(iii) *The newspaper article was neither accurate nor entertaining.*

(iv) *A sequence of real numbers is convergent only if it is bounded.*

(v) *If x is a real number then x^2 is positive or zero.*

(vi) *H/N is a normal subgroup of G/N if and only if H is a normal subgroup of G.*

(vii) *For all $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that $n \geq N$ implies $|a_n - L| < \epsilon$.*

Here are the negations:

(i) G is not normal or H is not regular.

(ii) There exists a cubic polynomial that does not have a real root.

(iii) The newspaper article was accurate or entertaining.

(iv) A sequence of real numbers is convergent and not bounded.

(v) x is a real number and x^2 is not positive and not zero.

(vi) H/N is a normal subgroup of G/N and H is not a normal subgroup of G, or H is a normal subgroup of G and H/N is not a normal subgroup of G/N.

(vii) There exists $\epsilon > 0$ such that for all $N \in \mathbb{N}$, (there exists) $n \geq N$ and $|a_n - L| \geq \epsilon$.