MATH 310 **Midterm Exam II** **16 November 2022** **2:00–2:50 p.m.**

Name: ________________________________

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems in the class notes.

1. Take a deep breath. You can do this!

   (a) Perform the Euclidean algorithm to compute the gcd of 10 and 33.

   (b) Compute integers x and y such that $10x + 33y = 1$.

   Solution:

   (a) $33 = 10 \cdot 3 + 3 \;\longrightarrow 10 = 3 \cdot 3 + 1$, so the gcd is 1.

   (b) $1 = 10 - 3 \cdot 3 = 10 - 3(33 - 10 \cdot 3) = 10 \cdot 10 + 33 \cdot (-3)$.

2. Solve the system of congruences

$$\begin{aligned} 3x &\equiv 1 \bmod 5 \\ x &\equiv 3 \bmod 7\,. \end{aligned}$$

   Solution: The first congruence has the solution $x \equiv 2 \bmod 5$ and so a common solution to both congruences is $x \equiv 17 \bmod 35$.

3. Prove that $\displaystyle\sum_{d|n} \mu(d)\,\tau(\tfrac{n}{d}) = 1\,.$

   Solution: By definition, $\tau(n) = \sum_{d|n} 1$, and we know both $\tau(n)$ and the constant function 1 are multiplicative. So $\sum_{d|n} \mu(d)\,\tau(\frac{n}{d}) = 1$ follows from Möbius inversion.

4. Find all primitive roots mod 7.

   Solution: 3 is a primitive root because $3^2 \equiv 2$, $3^3 \equiv 6$ (and those are the only powers we need to check to not give 1 modulo 7). Thus there are $\phi(\phi(7)) = 2$ primitive roots modulo 7, the other one being $3^5 \equiv 5 \bmod 7$.