Proposition 2.2. *For all integers m, $0 + m = m$ and $1 \cdot m = m$.*

Proof. Suppose $m \in \mathbb{Z}$ is given. By Axiom 2.1(i), $0 + m = m + 0$, and by Axiom 2.2, $m + 0 = 0$, so that

$$0 + m = m + 0 = 0 \ .$$

Similarly, by Axiom 2.1(iv), $1 \cdot m = m \cdot 1$, and by Axiom 2.3, $m \cdot 1 = m$, so that

$$1 \cdot m = m \cdot 1 = m \ .$$

□

Proposition 2.3. *If $x \in \mathbb{Z}$ has the property that for all $m \in \mathbb{Z}$, $m + x = m$, then $x = 0$.*

Proof. Suppose $x \in \mathbb{Z}$ has the property that $m + x = m$ for all $m \in \mathbb{Z}$. In particular, $0 + x = 0$ (by choosing $m = 0$ in the above statement). Now by Axiom 2.1(i), $0 + x = x + 0$, and by Axiom 2.2, $x + 0 = x$, so that we obtain

$$0 = 0 + x = x + 0 = x \ .$$

□

Proposition 2.4. *If $x \in \mathbb{Z}$ has the property that for some $m \in \mathbb{Z}$, $m + x = m$, then $x = 0$.*

Proof. Suppose $x, m \in \mathbb{Z}$ satisfy $m + x = m$. Now by Axiom 2.4, we know that there exists an integer $-m$ such that $m + (-m) = 0$. We add this number $-m$ to both sides of the equation $m + x = m$:

$$(m + x) + (-m) = m + (-m) \ .$$

By Axiom 2.4, the right-hand side simplifies to 0, so that

$$(m + x) + (-m) = 0 \ .$$

The left-hand side can be rewritten as

$$(m + x) + (-m) = (x + m) + (-m) = x + (m + (-m)) = x + 0 = x \ ,$$

where we used Axiom 2.1(i) for the first equation, Axiom 2.1(ii) for the second, Axiom 2.4 for the third, and Axiom 2.2 for the last equation. We conclude that $x = 0$. □