Proposition 2.15.

(i) $-(m+n) = (-m) + (-n)$.

(ii) $-m = (-1)m$.

(iii) $(-m)n = m(-n) = -(mn)$.

Proof. We prove (ii) first and use this to prove (i) and (iii).

(ii) By Axioms 2.3, 2.1(i), 2.1(iii), 2.4, and Proposition 2.5,

$$m + (-1)m = m \cdot 1 + (-1)m = 1 \cdot m + (-1)m = (1 + (-1))\, m = 0 \cdot m = 0 \ .$$

But $m + (-1)m = 0$ means by definition (Axiom 2.4) that $(-1)m$ is an additive inverse of m, that is $-m = (-1)m$. (One might also recall Proposition 2.9, which says that this additive inverse is unique.)

(i) By part (ii), Axiom 2.1(iii), and part (ii) again,

$$-(m+n) = (-1)(m+n) = (-1)m + (-1)n = (-m) + (-n) \ .$$

(iii) By part (ii), Axiom 2.1(iv), Axiom 2.1(v), and part (ii) again,

$$(-m)n = ((-1)m)\, n = (m(-1))\, n = m\, ((-1)n) = m(-n) \ .$$

Finally, by part (ii), Axiom 2.1(v), Axiom 2.1(iv), Axiom 2.1(v), and part (ii),

$$m(-n) = m\, ((-1)n) = (m(-1))\, n = ((-1)m)\, n = (-1)(mn) = -(mn) \ .$$ □