Worksheet 4: Chinese Remainder Theorem

1. Let a by the day (of the month) you were born and b the month.[1] Find $x \in \mathbb{Z}$ such that

$$x \equiv a \bmod 31 \qquad \text{and} \qquad x \equiv b \bmod 12\,.$$

2. Suppose $m, n \in \mathbb{Z}_{>0}$ are relatively prime, and $a, b \in \mathbb{Z}$. Prove that

$$x \equiv a \bmod m \qquad \text{and} \qquad x \equiv b \bmod n$$

has a solution $x \in \mathbb{Z}$ and that x is unique modulo mn.

3. Generalize the statement (and your proof) of 2. to a system of k congruences.

4. Andrews 5.3.1. (Feel free to use `sage`.)

5. Write down a precise statement for each definition we have given this week. For each definition, give an example and a non-example.

[1] If $a = 26$ and $b = 9$... happy birthday!!