Proposition 8.4. *If $a \in \mathbb{R}_{>0}$ then $1/a \in \mathbb{R}_{>0}$.*

Proof. Suppose $a \in \mathbb{R}_{>0}$. Axiom 8.6(iii) says that exactly one of the possibilities $\frac{1}{a} \in \mathbb{R}_{>0}$, $\frac{1}{a} = 0$, and $-\frac{1}{a} \in \mathbb{R}_{>0}$ holds. We will show that the latter two lead to contradictions, and hence we must have $\frac{1}{a} \in \mathbb{R}_{>0}$.

If $\frac{1}{a} = 0$ then $1 = a \cdot \frac{1}{a} = a \cdot 0 = 0$, contradicting Axiom 8.3.

The other possibility $-\frac{1}{a} = 0 - \frac{1}{a} \in \mathbb{R}_{>0}$ means that $\frac{1}{a} < 0$. On the other hand, we know that $a \in \mathbb{R}_{>0}$, i.e., $0 < a$. Hence we can apply Proposition 8.2(iv) to obtain $0 = \frac{1}{a} \cdot 0 > \frac{1}{a} \cdot a = 1$. This means that $0 - 1 = -1 \in \mathbb{R}_{>0}$, which contradicts Proposition 8.3(i). $\square$