

## Worksheet 7: The Möbius Function

1. Show that  $\mu(n)$  is multiplicative.

2. Prove that

$$\sum_{d|n} \mu(d) = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{if } n > 1. \end{cases}$$

*Hint:* for  $n > 1$ , try induction on the number of prime factors of  $n$ .

3. Prove the *Möbius Inversion Formula*:

$$f(n) = \sum_{d|n} g(d) \quad \text{if and only if} \quad g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right).$$

*Hint:* write sums like the one on the right-hand side as

$$\sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) = \sum_{de=n} \mu(d) f(e).$$

4. Andrews 6.4.1, 6.4.3, 6.4.7, 6.4.8, 6.4.11.

5. Write down a precise statement for each definition we have given this week. For each definition, give an example and a non-example.