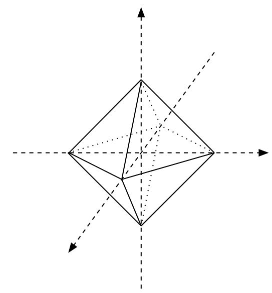
MATH 890.03
Combinatorics:
Seminar in Polytopes
Fall 2006



Meeting times: MWF 1:10–2:00

Prerequisites: Advanced Linear Algebra (it's ok to take MATH 725 concurrently) or consent of the instructor

Instructor: Matthias Beck (TH 933, 415.405.3473, `beck@math.sfsu.edu`)

Course Objectives: Polytopes are the natural generalizations of line segments and polygons to higher dimensions. Examples of polytopes in three dimensions include crystals, boxes, tetrahedra, and any convex object whose faces are all flat. One way to define a polytope is to consider the *convex hull* of a finite collection of points in Euclidean space $\mathbb{R}^d$. That is, suppose someone gives us a set of points $v_1, \dots, v_n$ in $\mathbb{R}^d$. The polytope determined by the given points v_j is defined by all linear combinations $c_1v_1 + c_2v_2 + \cdots + c_nv_n$, where the coefficients c_j are nonnegative real numbers that satisfy the relation $c_1 + c_2 + \cdots + c_n = 1$. This construction is called the *vertex description* of the polytope.

There is another equivalent definition, called the *hyperplane description* of the polytope. Namely, if someone hands us the linear inequalities that define a finite collection of half-spaces in $\mathbb{R}^d$, we can define the associated polytope as the simultaneous intersection of the half-spaces defined by the given inequalities.

It is amusing to see how many problems in combinatorics, number theory, and many other mathematical areas can be recast in the language of polytopes that exist in some Euclidean space. Conversely, the versatile structure of polytopes gives us number-theoretic and combinatorial information that flows naturally from their geometry.

Our goal in this seminar is to understand the combinatorial structure of polytopes. We will start by proving the equivalence of the two descriptions of polytopes given above and then proceed to think about questions such as 'which polytopes have the maximal number of faces given a fixed dimension and number of vertices (extreme points)?'

Textbook: G. Ziegler, *Lectures on Polytopes*, Springer.

Evaluation of Students: This is a seminar course, that is, the students will present the material, in approximately two lectures at a time. I will also assign some (very light) homework each week. Students will be graded on their lectures, homework assignments, and class participation.