MATH 850 Combinatorial Commutative Algebra Homework

(1) Let Δ be a simplicial complex with corresponding Stanley–Reisner ideal I_Δ, and let

$$m^\tau \ := \ \langle x_j : j \in \tau \rangle \, ,$$

the monomial (prime) ideal corresponding to $\tau \subseteq [n]$. Show that

$$I_\Delta \ = \ \bigcap_{\sigma \in \Delta} m^{[n] \setminus \sigma} .$$

(2) Let $R := \mathbb{F}[x_1, x_2, x_3, x_4]$ and $I := \langle x_1, x_2, x_3, x_4 \rangle$. Compute a finite free resolution for the R-module R/I.

(3) Let Δ be the boundary of a pentagon. Compute I_Δ and one of its finite free resolutions.

(4) Let Δ be the boundary of an octahedron. Compute I_Δ and one of its finite free resolutions.

(5) Let $I := \langle x_1x_3, x_1x_4, x_2x_4 \rangle \subset \mathbb{F}[x_1, x_2, x_3, x_4]$, and let I_d denote the $\mathbb{F}$-vector space of homogeneous polynomials in I of degree d. Compute the *Hilbert function* $h_I(n) := \dim_{\mathbb{F}}(I_n)$ and the *Hilbert series*

$$H_I(x) \ := \ \sum_{n \geq 0} h_I(n) \, x^n .$$