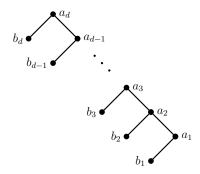MATH 420/720   One Last Poset Exercise

Consider the poset P_d on $2d$ elements $a_1, a_2, \ldots, a_d, b_1, b_2, \ldots, b_d$, defined by the relations

$$a_1 \prec a_2 \prec \cdots \prec a_d \qquad \text{and} \qquad a_j \succ b_j \text{ for } 1 \leq j \leq d.$$

Show that the order polynomial $\Omega_d(n)$ of P_d satisfies the relation

$$\Omega_{d+1}(n+1) = \Omega_d(n) + (n+1)\,\Omega_{d-1}(n).$$

Conclude that $\Omega_d(n) = S(n+d, n)$, a Stirling number of the second kind.