MATH 335 Midterm Exam 3/18/16 9:10–10:00 a.m.

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to theorems that we proved in class.

1. (5 points) Name your favorite prime number.

2. Let $m, n \in \mathbb{Z}$.

   a. (10 points) Carefully define $\gcd(m, n)$.

   b. (15 points) Find $x, y \in \mathbb{Z}$ such that $29\,x + 34\,y = 1$.

   Solution:

   (b) The Euclidean algorithm gives

   $$\begin{aligned} 34 &= 29 + 5 \\ 29 &= 5 \cdot 5 + 4 \\ 5 &= 4 \cdot 1 + 1 \end{aligned}$$

   and so

   $$1 = 5 - 4 = 6 \cdot 5 - 29 = 6(34 - 29) - 29 = (-7) \cdot 29 + 6 \cdot 34$$

   that is, $x = -7$ and $y = 6$ will do.

3. Let $m \in \mathbb{Z}_{>0}$.

   a. (10 points) Carefully define an mth root of unity and a primitive mth root of unity.

   b. (15 points) Find $z \in \mathbb{C}$ such that $z^{34} = i$.

   Solution:

   (b) $z = e^{\pi i/68}$ will do, since $\left(e^{\pi i/68}\right)^{34} = e^{\pi i/2} = i$.

4. Let $a, b, m \in \mathbb{Z}$.with $m > 0$.

   a. (10 points) Carefully define $a \equiv b \pmod{m}$.

   b. (15 points) Find $x \in \mathbb{Z}$ such that $29\,x \equiv 1 \pmod{34}$.

   Solution:

   (b) In 1(b) we saw that $(-7) \cdot 29 + 6 \cdot 34 = 1$, which implies $(-7) \cdot 29 \equiv 1 \pmod{34}$, and so $x \equiv -7 \equiv 27 \pmod{34}$ will do.

5. Let R be a ring.

   a. (10 points) Carefully define what it means for $a \in R$ to be a unit.

   b. (10 points) Show that $[29]$ is a unit in $\mathbb{Z}_{34}$.

   Solution:

   (b) In 3(b) we saw that $27 \cdot 29 \equiv 1 \pmod{34}$, which means $[27]$ and $[29]$ are inverses in $\mathbb{Z}_{34}$; in particular, $[29]$ is a unit in $\mathbb{Z}_{34}$.