Proposition 8.6. *Suppose $x, y \in \mathbb{R}$ with $x < y$. Then there exists $z \in \mathbb{R}$ such that $x < z < y$.*

Proof. We claim that $z = \frac{x+y}{2}$ will satisfy $x < z < y$. First,

$$z - x = \frac{x+y}{2} - \frac{2x}{2} = \frac{1}{2}(y-x) \in \mathbb{R}_{>0}$$

because $1/2 \in \mathbb{R}_{>0}$ (by Proposition 8.4) and $y - x \in \mathbb{R}_{>0}$ (by assumption). Hence $x < z$. Second,

$$y - z = \frac{2y}{2} - \frac{x+y}{2} = \frac{1}{2}(y-x) \in \mathbb{R}_{>0} \ ,$$

whence $z < y$. $\square$