MATH 301 Fake Midterm Exam 10/22/06 9:10–10:00 a.m.

Show complete work—that is, all the steps needed to completely justify your answer. Simplify your answers as much as possible. You may refer to propositions in the class notes (without stating the exact number of the proposition).

This exam is modeled for final-exam time frame ($2\frac{1}{2}$ hours). I would consider any subset of three of these problems to be worthy of a 50-minute exam.

1. Suppose m is an integer.
   (a) Carefully define $-m$.
   (b) Prove that $-m = (-1)m$.
2. Suppose $m, n \in \mathbb{Z}$.
   (a) Carefully define the statement m divides n.
   (b) Show that for all $n \in \mathbb{N}$, 24 divides $5^{2n} - 1$.
3. Suppose $A, B \subseteq \mathbb{R}$ are sets.
   (a) Carefully define the statement $A = B$.
   (b) Recall that the *complement* of A (in $\mathbb{R}$) is defined as $A^c = \mathbb{R} - A$. Prove that $(A \cup B)^c = A^c \cap B^c$.
4. Suppose k and m are nonnegative integers such that $m \leq k$.
   (a) Carefully define the binomial coefficient $\binom{k}{m}$.
   (b) Show that $\displaystyle\sum_{m=0}^{k} \binom{k}{m} = 2^k$.
5. Suppose $m, n \in \mathbb{Z}$.
   (a) Carefully define the statement $m \equiv n \pmod{34}$.
   (b) Prove that the relation $\equiv \pmod{34}$ is an equivalence relation.