Networked Embedded Systems Lab
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Introduction to Embedded Systems – WS 2022/23

Exercise 6: Low Power II

Task 1: Energy Harvesting

Consider a processor with negligible leakage power dissipation and a dynamic power dissipation that depends as follows on the operating frequency f in Hz:

$$P_{\text{dynamic}}(f) = \left(\frac{f}{1\,\text{MHz}}\right)^3 \text{mW}$$

Whenever the processor is idle it enters the zero power state without additional overhead. The set of hard real-time tasks τ_i listed in Table 1 shall be executed on the processor.

Task	τ_1	τ_2	τ_3
Arrival Time [ms]	0	0	0
Period [ms]	6	4	12
Relative Deadline [ms]	6	4	12
Cycles [$\times 10^3$]	2	1	2

Table 1: Characteristics of the set of hard real-time tasks τ_i to be executed.

The system has a battery with an energy level $E_{\text{bat}}(t)$ and it is replenished by a constant power source with $P(t) = P_{\text{in}}$.

1. Assume an initial battery charge of $E_{\text{bat}}(t = 0) = 6\,\mu\text{J}$, a constant input power $P_{\text{in}} = 0.5\,\mu\text{J/ms}$, and a constant clock frequency of $f = 1\,\text{MHz}$ for task processing. Schedule the tasks $\tau_1, \ldots, \tau_3$ according to the Earliest-Deadline-First (EDF) algorithm and draw the evolution of the battery energy level $E_{\text{bat}}(t)$ in Figure 1 for the time interval $[0\,\text{ms}, 12\,\text{ms}]$.

2. Assume the battery does not run out of charge. Prove or disprove the following statement:

   To maximize the energy stored in the battery at the end of each hyper-period (12 ms), all tasks τ_i have to be executed at the same frequency and this frequency leads to a utilization of 1.0.

   Hint: either provide the main arguments or a formal proof, both are accepted.

Task 2: Solar Cell Characteristics and Maximum Power Point Tracking

In this task we consider a solar energy harvesting system that performs maximum power point tracking for optimal harvesting. Specifically, the system employs the power point tracking algorithm introduced in the lecture. The algorithm dynamically adapts the operating point in discrete voltage steps Δ to match the

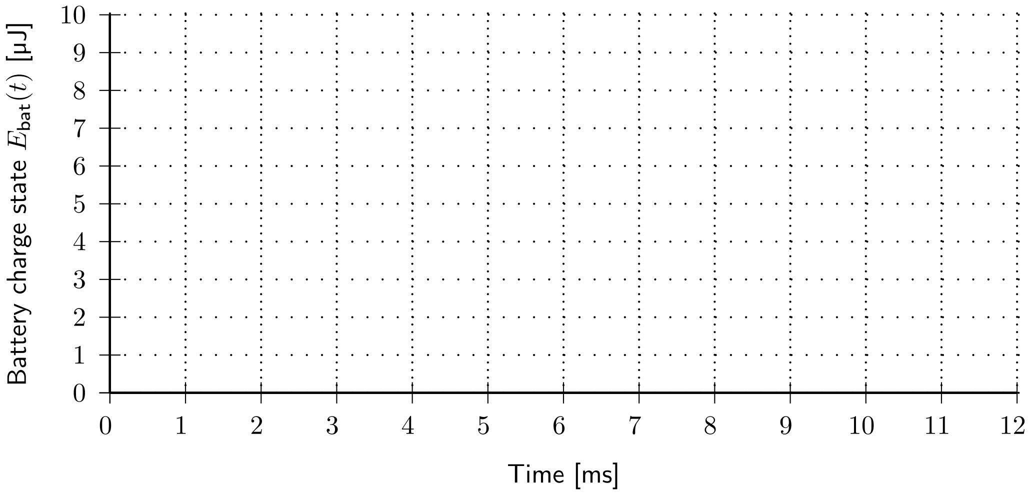

Figure 1: Evolution of the battery charge state $E_{\text{bat}}(t)$.

maximum power point P^* as close as possible. The flow chart describing the algorithm is shown in Figure 2. The solar cell connected to the systems has an I-V-characteristic that is described by the following model:

$$I(V) = G \cdot 1\,\mathrm{A} - \left(\exp\left(\frac{V}{0.1\,\mathrm{V}}\right) - 1\right) \cdot 0.01\,\mathrm{mA}, \tag{1}$$

where I is the solar cell's output current, V its output voltage and G is the relative solar irradiance (unit-less with $0 \leq G \leq 1$).

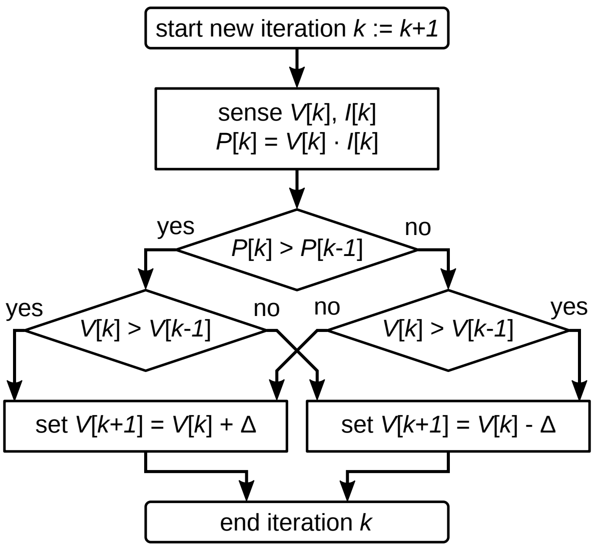

Figure 2: Flow chart describing the Maximum Power Point Tracking algorithm.

1. Compute the power extracted from the solar cell when operating at solar cell voltage $V = 0.7\,\mathrm{V}$ for relative irradiances of $G = \{0.1, 0.2, 0.5, 1.0\}$.

2. Execute the power point tracking algorithm by hand, once for $G = 0.1$ and once for $G = 1.0$. Use a step-size $\Delta = 0.05\,\mathrm{V}$ for the voltage adjustments and start with iteration $k = 1$, $V[0] = 0.7\,\mathrm{V}$ and $V[1] = 0.75\,\mathrm{V}$. Based on these result, determine a lower bound on the maximum power point P^*.

3. Consider now a photovoltaic panel (PV) consisting of two identical cells arranged in series, as illustrated in Figure 3. Due to partial shading of the PV panel, cell 1 receives the full relative irradiance of $G_1 = 1.0$, while cell 2 receives only $G_2 = 0.1$. Find a reasonable upper bound on the power that can be generated

using this PV panel. How does the extracted power compare to cell 1, if it is used standalone with the full relative irradiance of $G_1 = 1.0$?

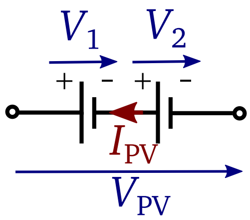

Figure 3: A photovoltaic panel consisting of two cells connected in series.

Task 3: Application Control

We consider an application control scenario with the harvested energy in time interval $[t, t+1]$ of $p(t)$, used energy of $u(t)$, battery capacity B and battery charge level $b(t) \in [0, B]$. The utility function used for application control is defined as $\mu(u) = \sqrt{u}$. Where not specified otherwise, the energy values of $p(t)$, $u(t)$, $b(t)$, and B are given in watt-hours (Wh) and the time t is given in hours (h).

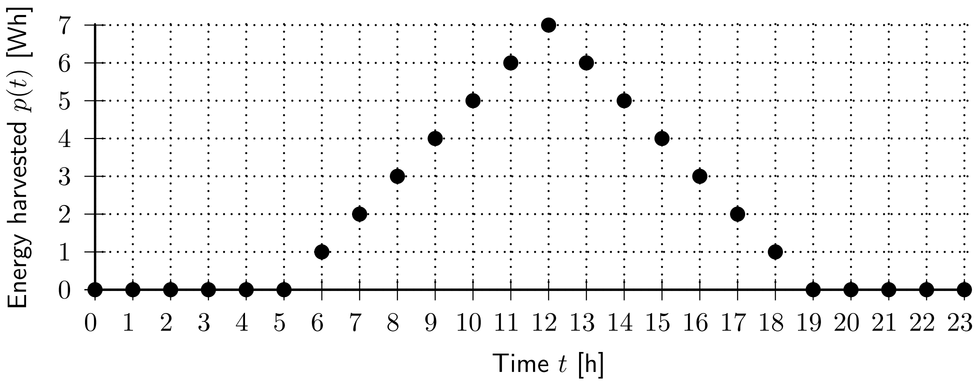

Figure 4: Energy harvesting profile $p(t)$ for a daily cycle. The values of $p(t)$ are all integers.

1. Consider the energy harvesting profile $p(t)$ given in Figure 4 that repeats daily. What is the maximum average power $\widehat{u_{\mathsf{max}}}$ that can be used by the system?
2. Given the knowledge of the daily energy input profile $p(t)$ in Figure 4, calculate the minimal battery size B_{min} such that the used energy satisfies $u(t) = 2$ for every time interval during a day. Complete the diagram in Figure 5 with the daily evolution of the used energy $u(t)$ and the battery charge state $b(t)$ at the beginning of the interval for the found battery size B_{min}.

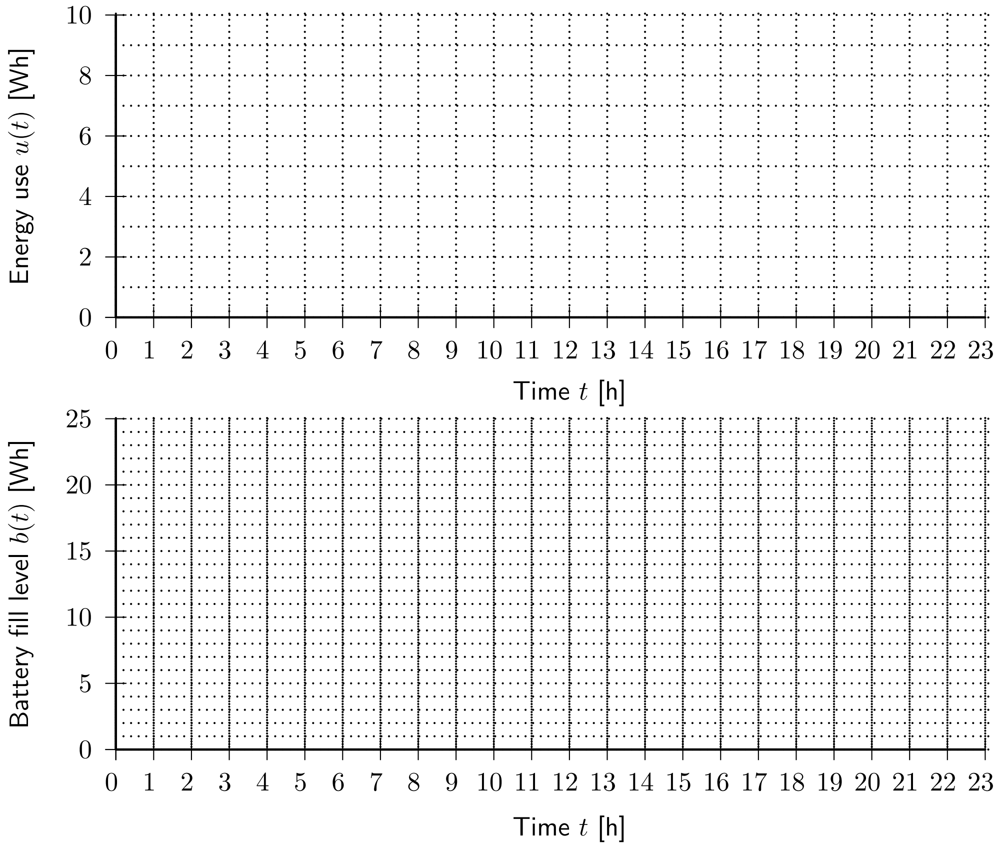

Figure 5: Energy harvesting profile $p(t)$ for a daily cycle. The values of $p(t)$ are all integers.

In the following tasks we assume a system with a battery size that is limited to $B = 11\,\text{Wh}$. For application control we assume an observation interval of $T = 24\,\text{h}$ and require that the current battery level is reached at the end of this observation interval, *i.e.*, $b(t+T) = b(t) \quad \forall t \geq 0$.

3. Determine an optimal energy usage function $u^*(t)$ that maximizes the minimal used energy and maximizes the total utility.
4. Unexpectedly, we do not harvest any energy, *i.e.* $p(t) = 0$, during one of the intervals $t \in [6, 18]$. The exact interval during which no energy is harvested is unknown. The energy use is still $u^*(t)$ as computed above. Explain the consequences, considering all possible scenarios. Are there any system failures happening and at what time of the day?
5. In this task we use a finite horizon control scheme for application control. Suppose that we unexpectedly have zero energy in interval $t = 12$, i.e., $p(12) = 0$. What is the computed energy use in this case?