Networked Embedded Systems Lab
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Introduction to Embedded Systems – WS 2022/23

Sample Solution to Exercise 6: Low Power II

Task 1: Energy Harvesting

Consider a processor with negligible leakage power dissipation and a dynamic power dissipation that depends as follows on the operating frequency f in Hz:

$$P_{\text{dynamic}}(f) = \left(\frac{f}{1\,\text{MHz}}\right)^3 \text{mW}$$

Whenever the processor is idle it enters the zero power state without additional overhead. The set of hard real-time tasks τ_i listed in Table 1 shall be executed on the processor.

Task	τ_1	τ_2	τ_3
Arrival Time [ms]	0	0	0
Period [ms]	6	4	12
Relative Deadline [ms]	6	4	12
Cycles [$\times 10^3$]	2	1	2

Table 1: Characteristics of the set of hard real-time tasks τ_i to be executed.

The system has a battery with an energy level $E_{\text{bat}}(t)$ and it is replenished by a constant power source with $P(t) = P_{\text{in}}$.

1. Assume an initial battery charge of $E_{\text{bat}}(t = 0) = 6\,\mu\text{J}$, a constant input power $P_{\text{in}} = 0.5\,\mu\text{J/ms}$, and a constant clock frequency of $f = 1\,\text{MHz}$ for task processing. Schedule the tasks $\tau_1, \ldots, \tau_3$ according to the Earliest-Deadline-First (EDF) algorithm and draw the evolution of the battery energy level $E_{\text{bat}}(t)$ in Figure 2 for the time interval $[0\,\text{ms}, 12\,\text{ms}]$.

   Solution:

   The execution times C_i of tasks τ_i are: $C_1 = 2\,\text{ms}$, $C_2 = 1\,\text{ms}$, and $C_3 = 2\,\text{ms}$.

   Applying EDF scheduling as shown in Figure 1, the processor is busy in $[0\,\text{ms}, 9\,\text{ms}]$. The resulting battery diagram is given in Figure 2.

   □

2. Assume the battery does not run out of charge. Prove or disprove the following statement:

   To maximize the energy stored in the battery at the end of each hyper-period (12 ms), all tasks τ_i have to be executed at the same frequency and this frequency leads to a utilization of 1.0.

   Hint: either provide the main arguments or a formal proof, both are accepted.

   Solution:

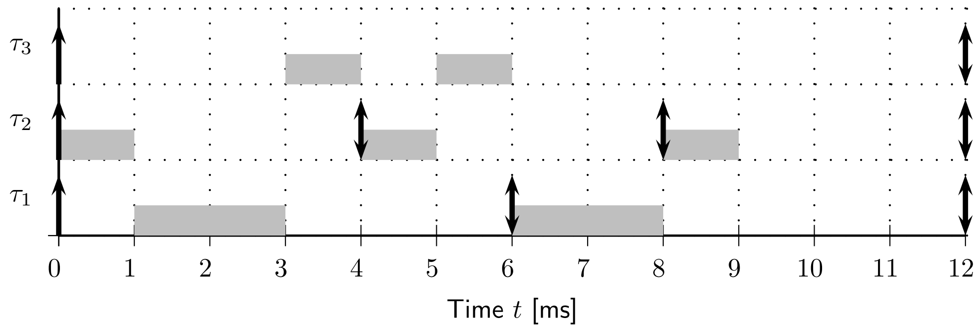

Figure 1: The resulting schedule for tasks τ_i when applying EDF algorithm.

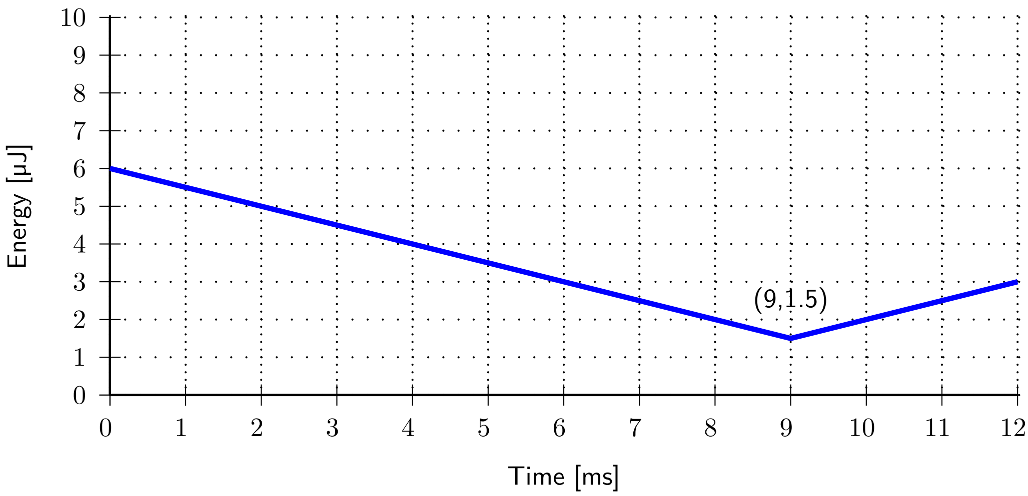

Figure 2: The resulting evolution of the battery charge state $E_{\text{bat}}(t)$.

- First, note that the given power consumption $P_{\text{dynamic}}(f)$ is a strictly convex function of the frequency f.
- The employed EDF scheduling algorithm where deadlines of tasks equal their periods guarantees a feasible schedule as long as the utilization of the processor is $U \leq 1.0$. The execution of all tasks with a constant frequency allows for a schedule where the utilization satisfies $U = 1.0$, i.e., the processor is constantly busy. For the given task set, this frequency is $f = 0.75$ MHz.
- The optimality of processing at a constant frequency during the whole time follows the same argumentation used to derive the optimal Dynamic Voltage and Frequency Scaling (DVFS) in the lecture [*9. Power and Energy*, slides 9-28f]. Due to the strict convexity of the power consumption $P_{\text{dynamic}}(f)$ the increase in the average power consumption of the higher frequency task always dominates the savings in the average power consumption of the lower frequency task execution.

  The above conditions (constant frequency during execution, processor executing all the time) lead to a schedule, where the maximum frequency of task execution can not be further reduced as the whole time interval where tasks can be executed is filled with execution and all tasks are executed with the same frequency.

□

Task 2: Solar Cell Characteristics and Maximum Power Point Tracking

In this task we consider a solar energy harvesting system that performs maximum power point tracking for optimal harvesting. Specifically, the system employs the power point tracking algorithm introduced in the lecture. The algorithm dynamically adapts the operating point in discrete voltage steps Δ to match the maximum power point P^* as close as possible. The flow chart describing the algorithm is shown in Figure 3. The solar cell connected to the systems has an I-V-characteristic that is described by the following model:

$$I(V) = G \cdot 1\,\mathrm{A} - \left(\exp\left(\frac{V}{0.1\,\mathrm{V}}\right) - 1\right) \cdot 0.01\,\mathrm{mA}, \quad (1)$$

where I is the solar cell's output current, V its output voltage and G is the relative solar irradiance (unit-less with $0 \leq G \leq 1$).

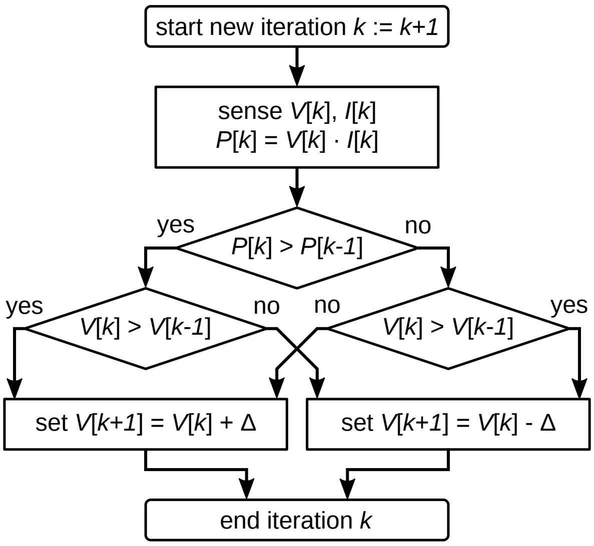

Figure 3: Flow chart describing the Maximum Power Point Tracking algorithm.

1. Compute the power extracted from the solar cell when operating at solar cell voltage $V = 0.7\,\mathrm{V}$ for relative irradiances of $G = \{0.1, 0.2, 0.5, 1.0\}$.

   Solution:

Relative Irradiance G	**0.1**	**0.2**	**0.5**	**1.0**
Voltage V [V]	0.7	0.7	0.7	0.7
Current I [mA]	89.0	189.0	489.0	989.0
Power P [mW]	62.3	132.3	342.3	692.3

$\square$

2. Execute the power point tracking algorithm by hand, once for $G = 0.1$ and once for $G = 1.0$. Use a step-size $\Delta = 0.05\,\mathrm{V}$ for the voltage adjustments and start with iteration $k = 1$, $V[0] = 0.7\,\mathrm{V}$ and $V[1] = 0.75\,\mathrm{V}$. Based on these result, determine a lower bound on the maximum power point P^*.

   Solution:

   Note that the power P extracted at a given operating point V, calculated as

$$P(V) = V \cdot I(V) = V \cdot G - V \cdot \left(\exp\left(\frac{V}{0.1\,\mathrm{V}}\right) - 1\right) \cdot 10^{-5}$$

is a concave function of V. As the algorithm adjusts the operating point in discrete voltage steps Δ, the maximum of the power $P[k]$ observed by the algorithm presents a lower bound of the actual maximum power point P^*. Only in the special case where the voltage of the maximum power point is a multiple of Δ, it is matched exactly by the algorithm.

- $G = 0.1$:

Iteration k	**0**	**1**	**2**	**3**	**4**	**5**	**...**
Voltage $V[k]$ [V]	0.7	0.75	0.7	0.65	0.7	0.75	...
Current $I[k]$ [mA]	89.0	81.9	89.0	93.4	89.0	81.9	...
Power $P[k]$ [mW]	62.3	61.4	62.3	60.7	62.3	61.4	...

  Lower bound: $P^* \geq 62.3$ mW

- $G = 1.0$:

Iteration k	**0**	**1**	**2**	**3**	**4**	**5**	**...**
Voltage $V[k]$ [V]	0.7	0.75	0.8	0.85	0.9	0.95	...
Current $I[k]$ [mA]	989.0	981.9	970.2	950.9	919.0	866.4	...
Power $P[k]$ [mW]	692.3	736.4	776.2	808.2	827.1	823.1	...

  Lower bound: $P^* \geq 827.1$ mW

□

3. Consider now a photovoltaic panel (PV) consisting of two identical cells arranged in series, as illustrated in Figure 4. Due to partial shading of the PV panel, cell 1 receives the full relative irradiance of $G_1 = 1.0$, while cell 2 receives only $G_2 = 0.1$. Find a reasonable upper bound on the power that can be generated using this PV panel. How does the extracted power compare to cell 1, if it is used standalone with the full relative irradiance of $G_1 = 1.0$?

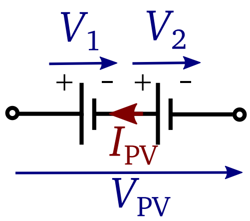

Figure 4: A photovoltaic panel consisting of two cells connected in series.

Solution: *(for ease of presentation units are omitted in intermediate steps)*

For reference and a better understanding, we start with a standalone cell 1 with relative irradiance $G_1 = 1.0$. Based on our first observation in the previous subtask (b), we find the voltage V_1^* of the maximum power point P_1^* by numerically solving

$$\frac{d}{dV_1} P_1(V_1) = \frac{d}{dV_1} V_1 \cdot I_1(V_1),$$

resulting in a voltage $V_1^* \approx 0.919$ V at the maximum power point. The (maximum) power generated at this operating point is $P_1^* = V_1^* \cdot I_1(V_1^*) \approx 829$ mW. If both cells of the PV panel would have a relative irradiance of $G_1 = G_2 = 1.0$, then the optimal generated power would be $P_{PV}^* \approx 1658$ mW.

Similarly, the voltage of the maximum power transfer for cell 2 with $G_2 = 0.1$ is calculated as $V_2^* \approx 0.712$ V with $P_2^* \approx 62.4$ mW. If both cells would have a relative irradiance $G_1 = G_2 = 0.1$, then the optimal generated power would be $P_{PV}^* \approx 125$ mW.

There are several options to determine suitable bounds on the power that can be generated in such a situation. In the following, we first determine an upper bound, i.e., a value which is larger than what can be generated by the partly shaded photovoltaic panel.

The series connection of cell 1 and cell 2 as shown in Figure 4 leads the following conditions:

$$V_{PV} = V_1 + V_2 \quad \text{and} \quad I_1 = I_2 = I_{PV}$$

The above current equality leads to the following condition when considering the I-V-characteristics from (1):

$$G_1 - \left(\exp\left(\frac{V_1}{0.1}\right) - 1\right) \cdot 10^{-5} = G_2 - \left(\exp\left(\frac{V_2}{0.1}\right) - 1\right) \cdot 10^{-5} \Leftrightarrow$$
$$\Leftrightarrow (G_1 - G_2) \cdot 10^5 = \exp\left(\frac{V_1}{0.1}\right) - \exp\left(\frac{V_2}{0.1}\right) \Leftrightarrow$$
$$\Leftrightarrow \exp\left(\frac{V_1}{0.1}\right) = (G_1 - G_2) \cdot 10^5 + \exp\left(\frac{V_2}{0.1}\right)$$

Note that an increase in V_2 leads to an increasing V_1. Now, let us start with $V_2 = 0$, which leads to:

$$V_1 = 0.1 \cdot \log\big((G_1 - G_2) \cdot 10^5 + \exp(0)\big) \approx 1.14\,\text{V}$$

The current at this operating point is $I_{PV} = 0.1\,\text{A}$ and the power $P_{PV} = I_{PV} \cdot (V_1 + V_2) \approx 114.1\,\text{mW}$. The voltage $V_1 \approx 1.14\,\text{V}$ at this operating point is already above $V_1^* \approx 0.919\,\text{V}$ the maximum power point of cell 1. Therefore, an increase in V_1 (and V_2) results in a decrease in the output power for cell 1.

When adding the maximum power that can be extracted from panel 2, i.e. $P_2^* \approx 62.4\,\text{mW}$, to the power extracted at this operating point, we get the following safe upper bound on the panel's output power:

$$P_{PV}^* \leq 176.5\,\text{mW} = 114.1\,\text{mW} + 62.4\,\text{mW} \tag{2}$$

We note that this is significantly lower than $P_1^* \approx 829.0\,\text{mW}$ that can be generated with a single cell 1 at the same operating point. In addition, it is only slightly better than a situation, where both cells receive the low irradiance, namely 125 mW.

An alternative to get an estimation of the generated power of the PV panel would be to generate a table starting from increasing currents I_{PV} and calculating the resulting individual voltages V_1 and V_2, the total voltage $V_1 + V_2$ as well as the resulting power $I_{PV} \cdot (V_1 + V_2)$. To this end, the I-V-characteristics needs to be solved for V. Such an approach leads to approximately optimal currents, voltages and generated power of $I_{PV}^* \approx 0.0946\,\text{A}$, $V_1 \approx 0.629\,\text{V}$, $V_2 \approx 1.141\,\text{V}$ and $P_{PV}^* \approx 167.5\,\text{mW}$, respectively.

□

Task 3: Application Control

We consider an application control scenario with the harvested energy in time interval $[t, t+1]$ of $p(t)$, used energy of $u(t)$, battery capacity B and battery charge level $b(t) \in [0, B]$. The utility function used for application control is defined as $\mu(u) = \sqrt{u}$. Where not specified otherwise, the energy values of $p(t)$, $u(t)$, $b(t)$, and B are given in watt-hours (Wh) and the time t is given in hours (h).

1. Consider the energy harvesting profile $p(t)$ given in Figure 5 that repeats daily. What is the maximum average power $\widehat{u_{\text{max}}}$ that can be used by the system?

   Solution:

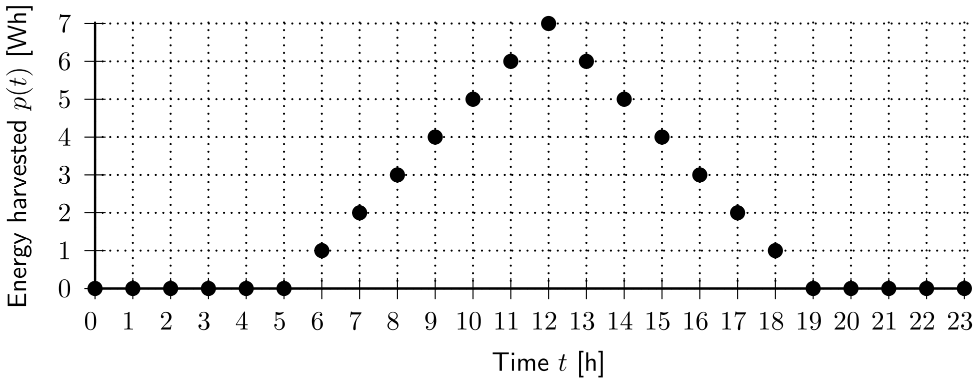

Figure 5: Energy harvesting profile $p(t)$ for a daily cycle. The values of $p(t)$ are all integers.

The daily harvested energy is calculated by (read the integer values of $p(t)$ from the diagram):

$$(1\,\text{Wh} + 2\,\text{Wh} + 3\,\text{Wh} + 4\,\text{Wh} + 5\,\text{Wh} + 6\,\text{Wh}) \cdot 2 + 7\,\text{Wh} = 49\,\text{Wh}$$

This results in the following maximum average harvesting power:

$$\widehat{u_{\text{max}}} = \frac{49\,\text{Wh}}{24\,\text{h}} = 2.04\,\text{W}$$

A possibility to reach this maximum average harvesting power would be to start with an arbitrary initial energy charge level and use $u(t) = p(t)$ during the whole day.

□

2. Given the knowledge of the daily energy input profile $p(t)$ in Figure 5, calculate the minimal battery size B_{min} such that the used energy satisfies $u(t) = 2$ for every time interval during a day. Complete the diagram in Figure 6 with the daily evolution of the used energy $u(t)$ and the battery charge state $b(t)$ at the beginning of the interval for the found battery size B_{min}.

   Solution:

   The energy used during the night, where the harvested energy is lower than the consumed energy is:

   $$1\,\text{Wh} + 11 \cdot 2\,\text{Wh} + 1\,\text{Wh} = 24\,\text{Wh}$$

   This corresponds to the amount of energy that the battery needs to store. Therefore, $B \geq 24\,\text{Wh} \Rightarrow B_{\text{min}} = 24\,\text{Wh}$.

   The resulting evolution of the energy usage $u(t)$ and battery charge state $b(t)$ for $B = 24\,\text{Wh}$ is given in Figure 6. We observe that in intervals $t \in [0, 6]$ and $t \in [18, 23]$ of a day $u(t) > p(t)$ and therefore energy is used from the battery. In intervals $t \in [7, 17]$ the battery is not used and the extra input power of intervals $t \in [8, 16]$ is stored to the battery, with the battery overflowing in interval $t = 16$.

□

In the following tasks we assume a system with a battery size that is limited to $B = 11\,\text{Wh}$. For application control we assume an observation interval of $T = 24\,\text{h}$ and require that the current battery level is reached at the end of this observation interval, *i.e.*, $b(t + T) = b(t) \quad \forall t \geq 0$.

3. Determine an optimal energy usage function $u^*(t)$ that maximizes the minimal used energy and maximizes the total utility.

   Solution:

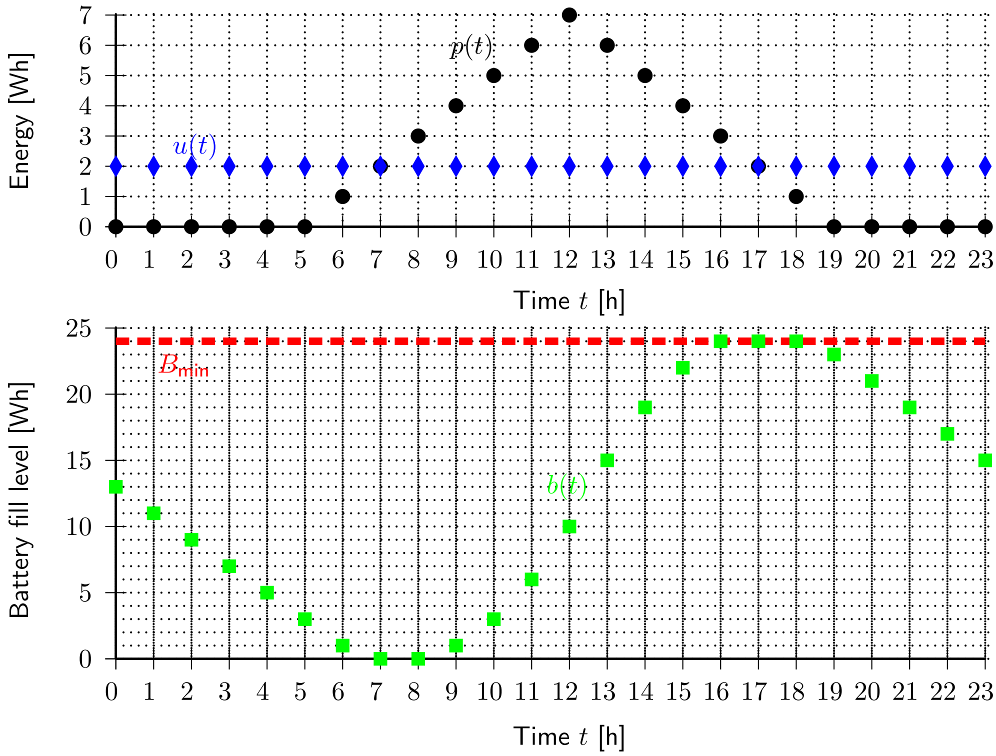

Figure 6: The energy harvested $p(t)$ and used $u(t)$ during the day on top, and the battery fill level $b(t)$ and battery capacity $B = 24$ Wh on the bottom.

We use the explanations and theorems explained in the lecture, see [*9. Power and Energy*, slide 9-58ff]. In particular, we increase the usage level step-by-step, fixing what we determined after each of these steps. Finally, it can be validated that the optimal energy use $u^*(t)$ satisfies the conditions in the main theorem.

- In the first step, we maximize the minimum used energy by increasing all used energy values until we can not satisfy the battery size constraints as well as the correctness constraints anymore. When choosing $u(t) = 1$ for all time intervals during a day, then we find $b(18) = 11 = B$ to be required to service the night. The battery is depleted completely in between, i.e., $b(6) = b(7) = 0$. Finally, we can observe that the battery already reaches its full capacity of 11 Wh in time interval starting at time 11 and therefore, energy will be wasted as some harvested energy arriving in time intervals starting at 11, 12, 13, 14, 15, 16 and 17 can not be stored anymore.
- In order to increase the overall utility, we therefore increase the used energy $u(t)$ for $t \in [7, 17]$ such that (a) there is sufficient energy available in each time interval and (b) there is no superfluous energy as long as the battery is full. In other words, we at first set the used energy for $t \in [7, 17]$ to $u(t) = 2$. As a result, we satisfy our conditions for $t \in [7, 17]$. Then we set the used energy for $t \in [8, 16]$ to $u(t) = 3$. As a result, we satisfy our conditions for $t \in [8, 16]$.
- If we set in the next step the used energy for $t \in [9, 15]$ to $u(t) = 4$, we see that we do not reach a full battery anymore at $t = 16$ and therefore, we can not ensure the used energy levels determined before without violating constraints. As a result, increasing the used energy for $t \in [9, 15]$ to $u(t) = 4$ is not feasible. We just need to increase it up to a value that guarantees $b(16) = B$. What is now the value of $u(t)$ to get the battery fully charged at this time? We just need to solve a simple equation that calculates the stored energy in the battery for time intervals starting at 9,

10, 11, 12, 13, 14 and 15, with the required battery level u as a variable.

$$(7-u)+2\cdot(6-u)+2\cdot(5-u)+2\cdot(4-u)=11\,\text{Wh} \Rightarrow u=\frac{26}{7}\,\text{Wh}$$

The resulting battery level $b(t)$ is summarized in Table 2.

t [h]	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
$b(t)$ [Wh]	0	$\frac{2}{7}$	$\frac{11}{7}$	$\frac{27}{7}$	$\frac{50}{7}$	$\frac{66}{7}$	$\frac{75}{7}$	11

Table 2: Battery level with optimal energy use $u^*(t)$.

The resulting diagram of the optimal energy use $u^*(t)$ and the battery level $b(t)$ is shown in Figure 7.

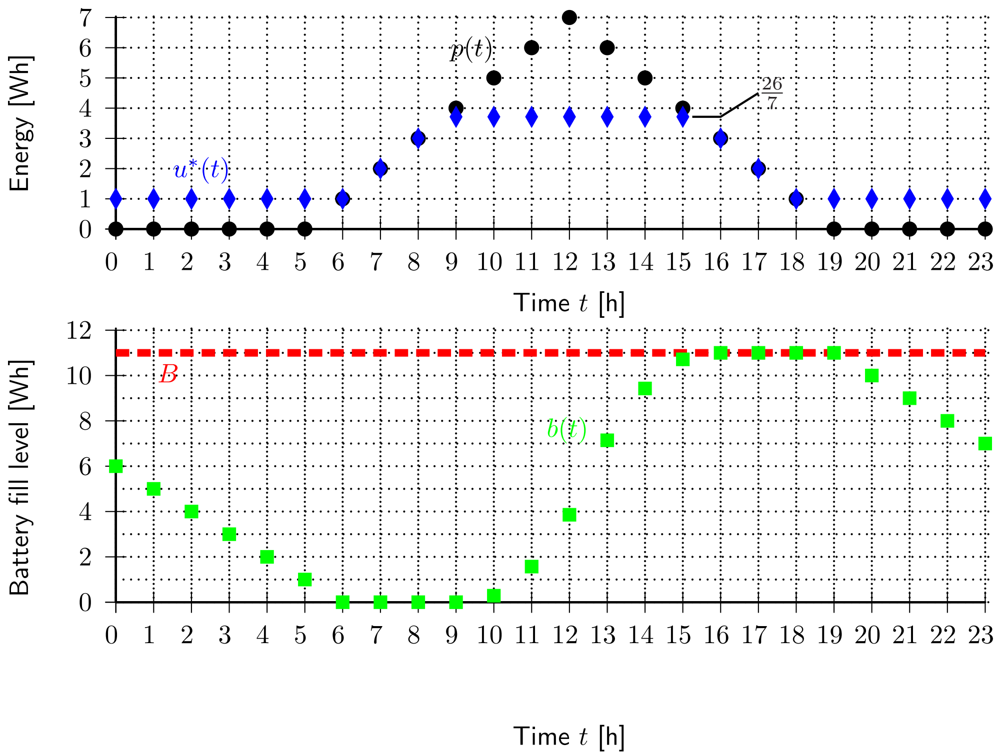

Figure 7: The energy harvested $p(t)$ and used $u(t)$ during the day on top, and the battery fill level $b(t)$ and battery capacity $B = 24\,\text{Wh}$ on the bottom.

□

4. Unexpectedly, we do not harvest any energy, *i.e.* $p(t) = 0$, during one of the intervals $t \in [6, 18]$. The exact interval during which no energy is harvested is unknown. The energy use is still $u^*(t)$ as computed above. Explain the consequences, considering all possible scenarios. Are there any system failures happening and at what time of the day?

Solution:

- If $p(t) = 0$ for any interval $t \in [6, 11]$, then there will be an immediate energy failure as the demand surpasses the available energy in the interval $[t, t+1]$.
- If $p(t) = 0$ for any interval $t \in [12, 18]$, the violation will happen some time during the night as the battery is not completely full in the afternoon.

□

5. In this task we use a finite horizon control scheme for application control. Suppose that we unexpectedly have zero energy in interval $t = 12$, i.e., $p(12) = 0$. What is the computed energy use in this case?

Solution:

- For all times $t \leq 12$ we have $\hat{u}^*(t) = u^*(t)$, where $u^*(t)$ is as before: The function $\hat{u}^*(t)$ is computed for every time step t, but as there is no deviation from assumptions ($t \leq 12$) and assumptions remain the same for the whole future, we have $\hat{u}^*(t) = u^*(t)$ for $t \leq 12$.
- At $t = 13$ we have now $\hat{b}(t) = \frac{1}{7}$ instead of $\frac{50}{7}$. Based on this, a new use function $u^*(t)$ is computed which will determine the function $\hat{u}^*(t)$ from now on (as long as no other deviations between estimated future harvested energy and actual harvested energy occurs). In order to achieve the minimal used energy of 1 Wh we need a full battery at $t = 18$. The new value of the used energy that guarantees a full battery level at $t = 18$ is computed while considering the energy balance equations for $t \in [13, 17]$:

$$\frac{1}{7} + (2 - u) + (3 - u) + (4 - u) + (5 - u) + (6 - u) = 11 \Rightarrow \hat{u} = \frac{64}{35} \approx 1.82\,\text{Wh} \tag{3}$$

  Therefore, we find $\hat{u}^*(t) = \frac{64}{35}$ Wh for $t \in [13, 17]$.
- Afterwards, $\hat{u}^*(t)$ follows $u^*(t)$ as the battery state is identical to the one for use function $u^*(t)$ and there are no further mispredictions of the harvested energy $p(t)$.

The resulting energy use function $\hat{u}^*(t)$ and battery state $\hat{b}(t)$ are shown in Figure 8.

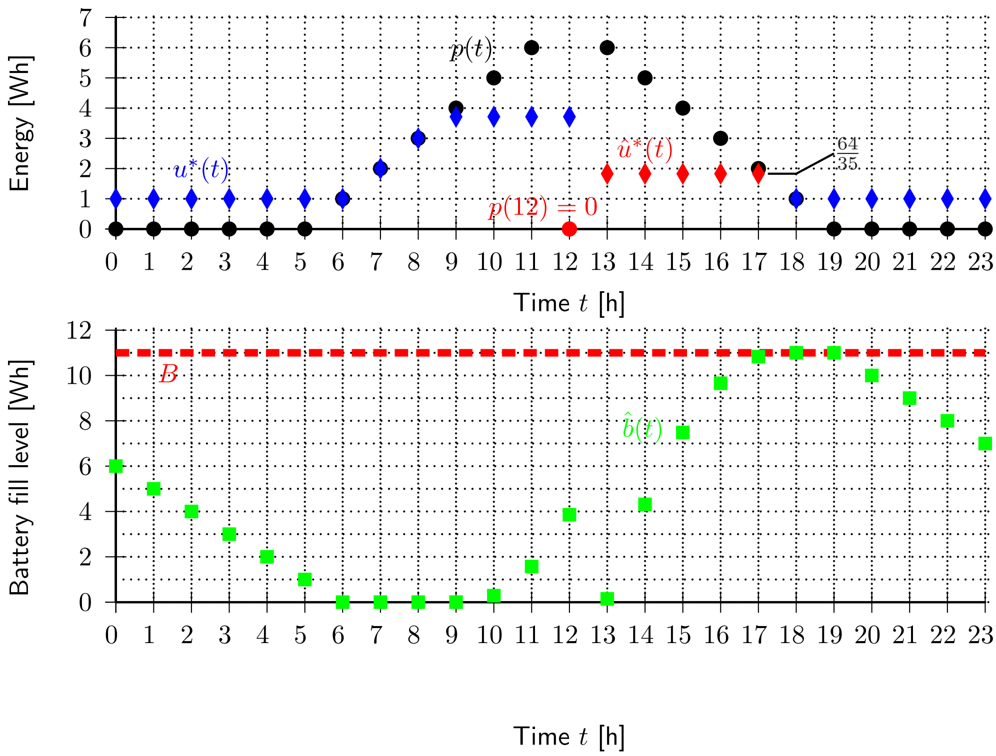

Figure 8: The zero energy input at $t = 12$ results in a depleted battery at time $t = 13$. Consequently, the control scheme calculates an $\hat{u}^*(t)$ that is different from the initial $u^*(t)$ between $t \in [13, 17]$.

$\square$