

Architecture Synthesis I

Scheduling with Pipeline Resources, Integer Linear Programming, Iterative Algorithms

Exercise class 10

Presenter:
Jürgen Mattheis

In cooperation with:
Pascal Walter

Based on the lecture of:
Marco Zimmerling

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University of Freiburg, Chair for Embedded Systems



Gliederung

Organisation

Overview

Task 1

Task 2

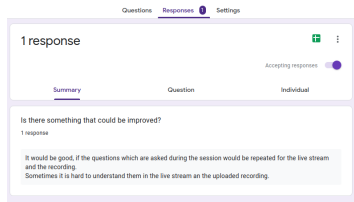
Task 3

Organisation



Organisation I

- ▶ feedback for me: <https://forms.gle/f3YN8EFrZ1vsfPoC6>

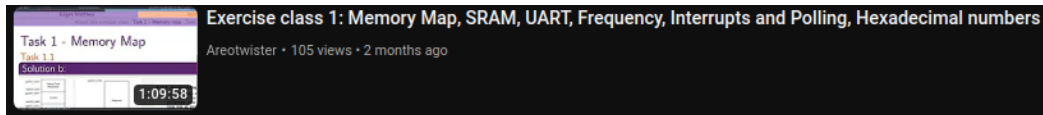


The screenshot shows the 'Responses' tab of a Google Form. At the top, there are tabs for 'Questions', 'Responses' (which is active and has a '1' next to it), and 'Settings'. Below the tabs, it says '1 response'. There are three sub-tabs: 'Summary', 'Question', and 'Individual'. The 'Summary' tab is selected. It shows the question 'Is there something that could be improved?' with '1 response'. The response text is: 'It would be good, if the questions which are asked during the session would be repeated for the live stream and the recording. Sometimes it is hard to understand them in the live stream an the uploaded recording.'

- ▶ feedback from students in the last exercise class: don't overexplain tasks at the beginning so there's enough time for the tasks at the end

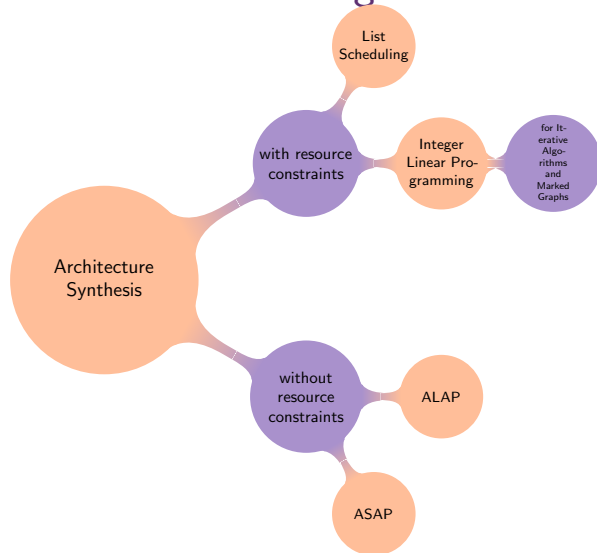
Organisation I

- ▶ the exercise class videos on youtube do now have a title containing the topics covered in the exercises of the corresponding exercise class



Overview

Overview Scheduling

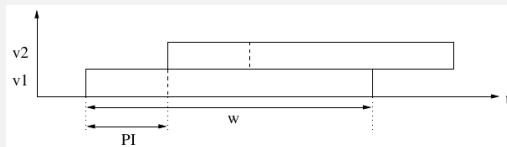
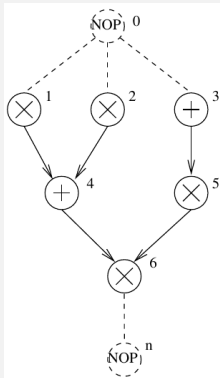


Task 1

Task 1 I

Scheduling with Pipeline Resources

Task 1.1:



Task 1

Scheduling with Pipeline Resources

Solution 1.1:



```

LIST( $G_S(V_S, E_S), G_R(V_R, E_R), \alpha, \beta, priorities$ ) {
   $t = 1$ ;
  REPEAT {
    FORALL  $v_k \in V_T$  {
      Determine candidates  $U_{t,k}$  to be scheduled;
      Determine set of occupied resources  $O_{t,k}$ ;
      Choose subset  $S_t \subseteq U_{t,k}$  with maximal priority
      and  $|S_{t,k}| + |O_{t,k}| \leq \alpha(v_k)$ ;
       $\tau(v_i) = t \ \forall v_i \in S_k; \ }$ 
       $t = t + 1$ ;
    } UNTIL ( $v_n$  planned)
  } RETURN ( $\tau$ ); }

```

Task 1

Scheduling with Pipeline Resources

Solution 1.1:



- ▶ $O_{t,k}$ is the set of resources of type k that are occupied in the time slot t and are not yet available for the following operation. On each of these resources exactly one operation is executed in a pipeline-interval.
- ▶ $O_{t,k} = \{v_s : \beta(v_s) = v_t \wedge \tau(v_s) < t < \tau(v_s) + PI\}$ instead of
 $T_{t,k} = \{v_s : \beta(v_s) = v_t \wedge \tau(v_s) < t < \tau(v_s) + w(v_s, v_t)\}$

Task 1

Scheduling with Pipeline Resources

Solution 1.2:



► *without pipelining:*

t	k	$U_{t,k}$	$T_{t,k}$	$S_{t,k}$
0	r_1	v3	-	v3
	r_2	v1 v2	-	v1
1	r_1	-	v3	-
	r_2	v2	v1	-
2	r_1	-	-	-
	r_2	v2 v5	v1	-
3	r_1	-	-	-
	r_2	v2 v5	v1	-
4	r_1	-	-	-
	r_2	v5	-	v2
5	r_1	-	-	-
	r_2	v5	v2	-
6	r_1	-	-	-
	r_2	v5	v2	-
7	r_1	-	-	-
	r_2	v5	v2	-
8	r_1	v4	-	v4
	r_2	v5	-	v5
9	r_1	-	v4	-
	r_2	-	v5	-
10	r_1	-	-	-
	r_2	-	v5	-
11	r_1	-	-	-
	r_2	-	v5	-
12	r_1	-	-	-
	r_2	v6	-	v6
13	r_1	-	-	-
	r_2	-	v6	-
14	r_1	-	-	-
	r_2	-	v6	-
15	r_1	-	-	-
	r_2	-	v6	-
15	r_1	-	-	-
	r_2	-	-	-

Task 1

Scheduling with Pipeline Resources

Solution 1.2:

t	k	$U_{t,k}$	$O_{t,k}$	$S_{t,k}$
0	r_1			
	r_2			
1	r_1			
	r_2			
2	r_1			
	r_2			
3	r_1			
	r_2			
4	r_1			
	r_2			
5	r_1			
	r_2			
6	r_1			
	r_2			
7	r_1			
	r_2			
8	r_1			
	r_2			
9	r_1			
	r_2			
10	r_1			
	r_2			
11	r_1			
	r_2			
12	r_1			
	r_2			

Task 1

Scheduling with Pipeline Resources

Solution 1.2:



t	k	$U_{t,k}$	$O_{t,k}$	$S_{t,k}$
0	r_1	v_3	—	v_3
	r_2	v_1, v_2	—	v_1
1	r_1	—	v_3	—
	r_2	v_2	v_1	—
2	r_1	—	—	—
	r_2	v_2, v_5	—	v_2
3	r_1	—	—	—
	r_2	v_5	v_2	—
4	r_1	—	—	—
	r_2	v_5	—	v_5
5	r_1	—	—	—
	r_2	—	v_5	—
6	r_1	v_4	—	v_4
	r_2	—	—	—
7	r_1	—	v_4	—
	r_2	—	—	—
8	r_1	—	—	—
	r_2	v_6	—	v_6
9	r_1	—	—	—
	r_2	—	v_6	—
10	r_1	—	—	—
	r_2	—	—	—
11	r_1	—	—	—
	r_2	—	—	—
12	r_1	—	—	—
	r_2	—	—	—

Solution 1.2:



- ▶ *the resulting latency is 12*

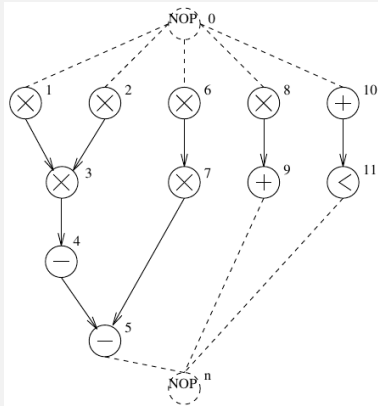
Task 2



Task 2 I

Integer Linear Programming

Task 2.1:



- ▶ **resource type r_1 :** multiplication operation takes 2 time units and 2 units of this resource type are allocated
- ▶ **resource type r_2 :** all other ALU Operations take 1 time unit and 2 units of this resource type are allocated

Task 2 II

Integer Linear Programming

Requirements 2.1:



```
ASAP( $G_S(V_S, E_S), w$ ) {  
   $\tau(v_0) = 1$ ;  
  REPEAT {  
    Determine  $v_i$  whose predec. are planed;  
     $\tau(v_i) = \max\{\tau(v_j) + w(v_j) \mid \forall (v_j, v_i) \in E_S\}$   
  } UNTIL ( $v_n$  is planned);  
  RETURN ( $\tau$ );  
}
```

Task 2 III

Integer Linear Programming

Requirements 2.1:

```
ALAP( $G_S(V_S, E_S), w, L_{max}$ ) {  
   $\tau(v_n) = L_{max} + 1$ ;  
  REPEAT {  
    Determine  $v_i$  whose succ. are planed;  
     $\tau(v_i) = \min\{\tau(v_j) \mid (v_i, v_j) \in E_S\} - w(v_i)$   
  } UNTIL ( $v_0$  is planned);  
  RETURN ( $\tau$ );  
}
```

Task 2 IV

Integer Linear Programming

Solution 2.1:



	l_i (ASAP)	h_i (ALAP)
v_1		
v_2		
v_3		
v_4		
v_5		
v_6		
v_7		
v_8		
v_9		
v_{10}		
v_{11}		

Task 2 V

Integer Linear Programming

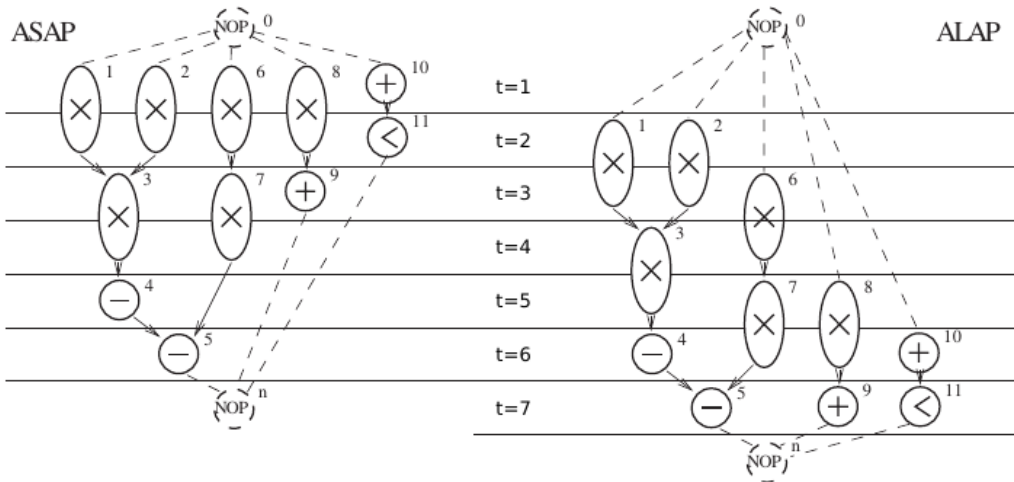
Solution 2.1:



	l_i (ASAP)	h_i (ALAP)
v_1	1	2
v_2	1	2
v_3	3	4
v_4	5	6
v_5	6	7
v_6	1	3
v_7	3	5
v_8	1	5
v_9	3	7
v_{10}	1	6
v_{11}	2	7

Task 2 VI

Solution 2.2:



Task 2 I

Integer Linear Programming

Solution 2.2:



► (i) *Objective function:*

$$\min \quad L = \tau(v_n) - \tau(v_0)$$

► (ii) *Introduction of binary variables:*

$$x_{1,1} + x_{1,2} = 1 \quad 1 \cdot x_{1,1} + 2 \cdot x_{1,2} = \tau(v_1)$$

$$x_{2,1} + x_{2,2} = 1 \quad 1 \cdot x_{2,1} + 2 \cdot x_{2,2} = \tau(v_2)$$

$$x_{3,3} + x_{3,4} = 1 \quad 3 \cdot x_{3,3} + 4 \cdot x_{3,4} = \tau(v_3)$$

$$x_{4,5} + x_{4,6} = 1 \quad 5 \cdot x_{4,5} + 6 \cdot x_{4,6} = \tau(v_4)$$

$$x_{5,6} + x_{5,7} = 1 \quad 6 \cdot x_{5,6} + 7 \cdot x_{5,7} = \tau(v_5)$$

Task 2 II

Integer Linear Programming

Solution 2.2:



$$x_{6,1} + x_{6,2} + x_{6,3} = 1 \quad 1 \cdot x_{6,1} + 2 \cdot x_{6,2} + 3 \cdot x_{6,3} = \tau(v_6)$$

$$x_{7,3} + x_{7,4} + x_{7,5} = 1 \quad 3 \cdot x_{7,3} + 4 \cdot x_{7,4} + 5 \cdot x_{7,5} = \tau(v_7)$$

$$x_{8,1} + \dots + x_{8,5} = 1 \quad 1 \cdot x_{8,1} + \dots + 5 \cdot x_{8,5} = \tau(v_8)$$

$$x_{9,3} + \dots + x_{9,7} = 1 \quad 3 \cdot x_{9,3} + \dots + 7 \cdot x_{9,7} = \tau(v_9)$$

$$x_{10,1} + \dots + x_{10,6} = 1 \quad 1 \cdot x_{10,1} + \dots + 6 \cdot x_{10,6} = \tau(v_{10})$$

$$x_{11,2} + \dots + x_{11,7} = 1 \quad 2 \cdot x_{11,2} + \dots + 7 \cdot x_{11,7} = \tau(v_{11})$$

Task 2 III

Integer Linear Programming

Solution 2.2:



► (iii) *Data dependencies:*

$$\tau(v_3) - \tau(v_1) \geq 2 \quad \tau(v_3) - \tau(v_2) \geq 2$$

$$\tau(v_4) - \tau(v_3) \geq 2 \quad \tau(v_5) - \tau(v_4) \geq 1$$

$$\tau(v_7) - \tau(v_6) \geq 2 \quad \tau(v_5) - \tau(v_7) \geq 2$$

$$\tau(v_9) - \tau(v_8) \geq 2 \quad \tau(v_{11}) - \tau(v_{10}) \geq 1$$

$$\tau(v_n) - \tau(v_5) \geq 1 \quad \tau(v_n) - \tau(v_9) \geq 1$$

$$\tau(v_n) - \tau(v_{11}) \geq 1$$

$$\tau(v_1), \tau(v_2), \tau(v_6), \tau(v_8), \tau(v_{10}) \geq \tau(v_0) \geq 1$$

Task 2 IV

Integer Linear Programming

Solution 2.2:

▶ (iv) *Resource limitations:*

▶ $t = 1$:

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} \leq 2$$

$$x_{10,1} \leq 2$$

▶ $t = 2$:

$$x_{1,1} + x_{1,2} + x_{2,1} + x_{2,2} + x_{6,1} + x_{6,2} + x_{8,1} + x_{8,2} \leq 2$$

$$x_{10,2} + x_{11,2} \leq 2$$

▶ $t = 3$:

$$x_{1,2} + x_{2,2} + x_{6,2} + x_{6,3} + x_{8,2} + x_{8,3} + x_{3,3} + x_{7,3} \leq 2$$

$$x_{10,3} + x_{11,3} + x_{9,3} \leq 2$$

Task 2 V

Solution 2.2:



▶ $t = 4$:

$$x_{6,3} + x_{8,3} + x_{8,4} + x_{3,3} + x_{3,4} + x_{7,3} + x_{7,4} \leq 2$$

$$x_{10,4} + x_{11,4} + x_{9,4} \leq 2$$

▶ $t = 5$

$$x_{8,4} + x_{8,5} + x_{3,4} + x_{7,4} + x_{7,5} \leq 2$$

$$x_{10,5} + x_{11,5} + x_{9,5} + x_{4,5} \leq 2$$

▶ $t = 6$:

$$x_{8,5} + x_{7,5} \leq 2$$

$$x_{10,6} + x_{11,6} + x_{9,6} + x_{4,6} + x_{5,6} \leq 2$$

▶ $t = 7$

$$(0 \leq 2)$$

$$x_{11,7} + x_{9,7} + x_{5,7} \leq 2$$

Integer Linear Programming I

Solution 2.3:



- ▶ *Restating the resource limitations, and introducing additional variables:*

- ▶ $t = 1$:

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} \leq \alpha(r_1)$$

$$x_{10,1} \leq \alpha(r_2)$$

[...]

- ▶ *Latency limitations:*

$$L = \tau(v_n) - \tau(v_0) \leq \bar{L} = 6$$

- ▶ *New objective function:*

$$\min C = \alpha(r_1) \cdot c(r_1) + \alpha(r_2) \cdot c(r_2) = 2 \cdot \alpha(r_1) + \alpha(r_2)$$

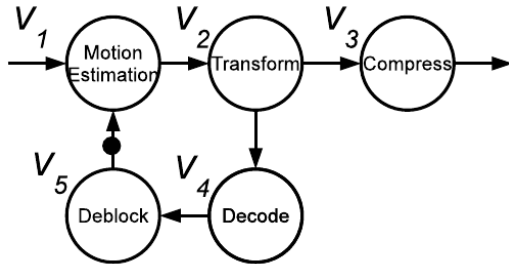
Task 3



Tasks 3 I

Iterative Algorithms

Task 3.1:



	ν_1	ν_2	ν_3	ν_4	ν_5
$w(\nu_i)$	10	10	10	5	5

Tasks 3 II

Iterative Algorithms

Solution 3.1:



- ▶ $\tau(\nu_2) - \tau(\nu_1) \geq 10$
- ▶ $\tau(\nu_3) - \tau(\nu_2) \geq 10$
- ▶ $\tau(\nu_4) - \tau(\nu_3) \geq 10$
- ▶ $\tau(\nu_5) - \tau(\nu_4) \geq 5$
- ▶ $\tau(\nu_1) - \tau(\nu_5) \geq 5 - 1 \cdot P$

Tasks 3 III

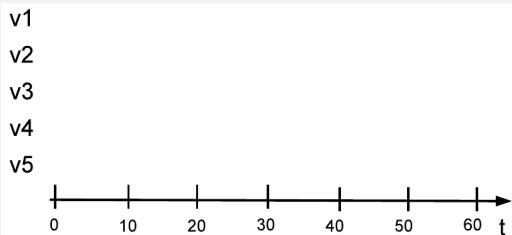
Iterative Algorithms

Solution 3.2:

▶ *solve system of inequalities for P :*

▶ $0 - 25 \geq 5 - P \Leftrightarrow P_{\min} = 30$

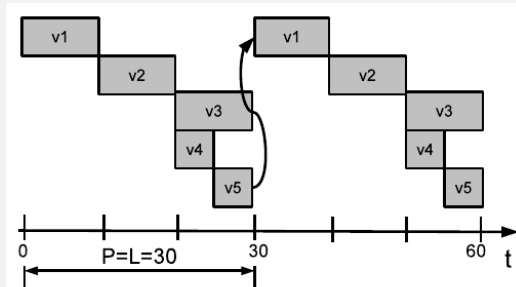
▶ $L = 30$



Tasks 3 IV

Iterative Algorithms

Solution 3.2:



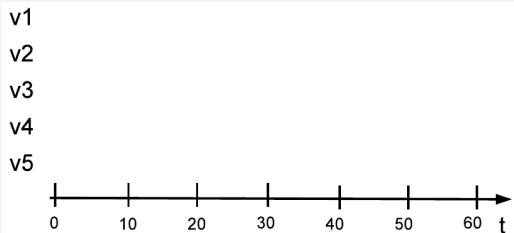
Tasks 3 V

Iterative Algorithms

Solution 3.3:



- ▶ $\tau(v_1) - \tau(v_5) \geq 5 - n \cdot 10 \Leftrightarrow n_{min} = 3$
- ▶ *we have to add 2 more tokens on the edge between v_5 and v_1*
- ▶ $L = 30$



Tasks 3 VI

Iterative Algorithms

Solution 3.3:

