Architecture Synthesis I

Scheduling with Pipeline Resources, Integer Linear Programming, Iterative Algorithms

Exercise class 10

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Based on the lecture of:
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University of Freiburg, Chair for Embedded Systems

Gliederung

Organisation

Task 1

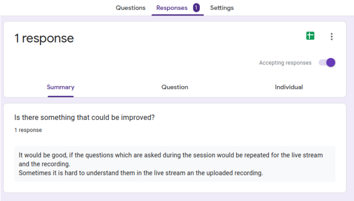
Task 2

Task 3

Organisation

Organisation I

- feedback for lecture until February 29
- feedback for me: `https://forms.gle/f3YN8EFrZ1vsfPoC6`

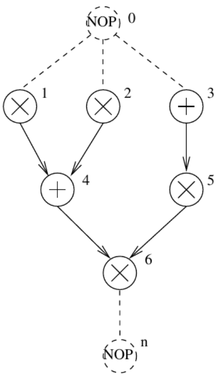


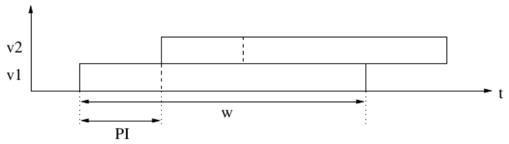
Task 1

Task 1 I

Scheduling with Pipeline Resources

Task 1.1:





Task 1

Scheduling with Pipeline Resources

Solution 1.1:

[...];
Determine candidates $U_{t,k}$ to be scheduled;
Determine set of occupied resources $O_{t,k}$;
Choose subset $S_t \subseteq U_{t,k}$ with maximal priority and $|S_{t,k}| + |O_{t,k}| \leq \alpha(v_k)$;
[...];

- *$O_{t,k}$ is the set of resources of type k that are occupied in the time slot t and are not yet available for the following operation. On each of these resources exactly one operation is executed in a pipeline-interval.*

Task 1

Scheduling with Pipeline Resources

Solution 1.1:

t	k	$U_{t,k}$	$O_{t,k}$	$S_{t,k}$
0	r_1			
	r_2			
1	r_1			
	r_2			
2	r_1			
	r_2			
3	r_1			
	r_2			
4	r_1			
	r_2			
5	r_1			
	r_2			
6	r_1			
	r_2			
7	r_1			
	r_2			
8	r_1			
	r_2			
9	r_1			
	r_2			
10	r_1			
	r_2			
11	r_1			
	r_2			
12	r_1			
	r_2			

Task 1

Scheduling with Pipeline Resources

Solution 1.1:

t	k	$U_{t,k}$	$O_{t,k}$	$S_{t,k}$
0	r_1	v_3	—	v_3
	r_2	v_1, v_2	—	v_1
1	r_1	—	v_3	—
	r_2	v_2	v_1	—
2	r_1	—	—	—
	r_2	v_2, v_5	—	v_2
3	r_1	—	—	—
	r_2	v_5	v_2	—
4	r_1	—	—	—
	r_2	v_5	—	v_5
5	r_1	—	—	—
	r_2	—	—	—
6	r_1	v_4	—	—
	r_2	—	—	—
7	r_1	—	—	—
	r_2	—	—	—
8	r_1	—	—	—
	r_2	v_6	—	—
9	r_1	—	—	—
	r_2	—	—	—
10	r_1	—	—	—
	r_2	—	—	—
11	r_1	—	—	—
	r_2	—	—	—
12	r_1	—	—	—
	r_2	—	—	—

Solution 1.1:

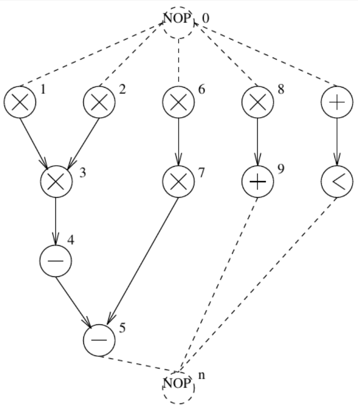
- *the resulting latency is* 12

Task 2

Task 2 I

Integer Linear Programming

Task 2.1:



- resource type r_1: multiplication operation takes 2 time units and 2 units of this resource type are allocated
- resource type r_2: all other ALU Operations take 1 time unit and 2 units of this resource type are allocated

Task 2 II

Integer Linear Programming

Solution 2.1:

	l_i (ASAP)	h_i(ALAP)
v_1		
v_2		
v_3		
v_4		
v_5		
v_6		
v_7		
v_8		
v_9		
v_{10}		
v_{11}		

Task 2 III

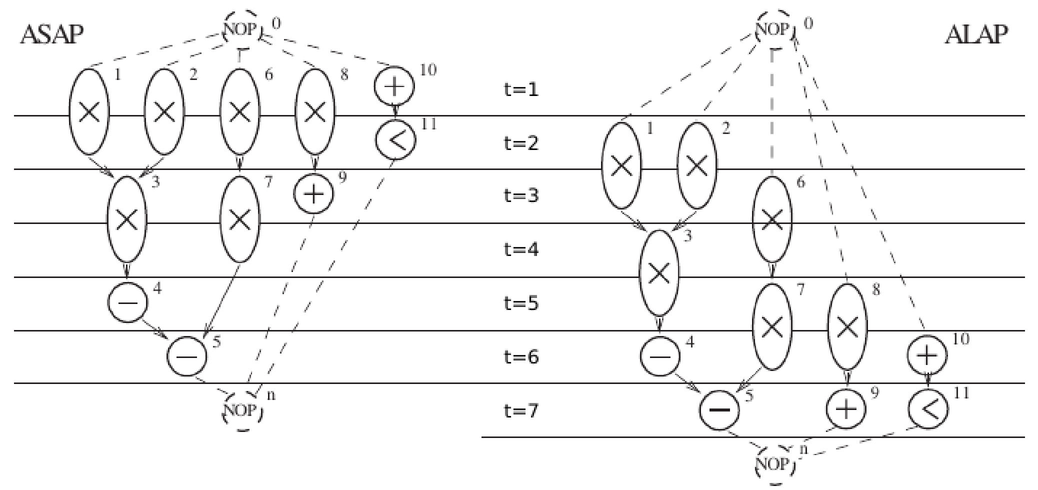
Integer Linear Programming

Solution 2.1:

	l_i (ASAP)	h_i(ALAP)
v_1	1	2
v_2	1	2
v_3	3	4
v_4	5	6
v_5	6	7
v_6	1	3
v_7	3	5
v_8	1	5
v_9	3	7
v_{10}	1	6
v_{11}	2	7

Task 2 IV

Solution 2.2:



Task 2 I

Integer Linear Programming

Solution 2.2:

- *(i) Objective function:*

$$\min. \quad L = \tau(v_n) - \tau(v_0)$$

- *(ii) Introduction of binary variables:*

$$\begin{aligned}
x_{1,1} + x_{1,2} &= 1 & 1 \cdot x_{1,1} + 2 \cdot x_{1,2} &= \tau(v_1) \\
x_{2,1} + x_{2,2} &= 1 & 1 \cdot x_{2,1} + 2 \cdot x_{2,2} &= \tau(v_2) \\
x_{3,3} + x_{3,4} &= 1 & 3 \cdot x_{3,3} + 4 \cdot x_{3,4} &= \tau(v_3) \\
x_{4,5} + x_{4,6} &= 1 & 5 \cdot x_{4,5} + 6 \cdot x_{4,6} &= \tau(v_4) \\
x_{5,6} + x_{5,7} &= 1 & 6 \cdot x_{5,6} + 7 \cdot x_{5,7} &= \tau(v_5)
\end{aligned}$$

Task 2 II

Integer Linear Programming

Solution 2.2:

$$x_{6,1} + x_{6,2} + x_{6,3} = 1 \quad 1 \cdot x_{6,1} + 2 \cdot x_{6,2} + 3 \cdot x_{6,3} = \tau(v_6)$$
$$x_{7,3} + x_{7,4} + x_{7,5} = 1 \quad 3 \cdot x_{7,3} + 4 \cdot x_{7,4} + 5 \cdot x_{7,5} = \tau(v_7)$$
$$x_{8,1} + \ldots + x_{8,5} = 1 \quad 1 \cdot x_{8,1} + \ldots + 5 \cdot x_{8,5} = \tau(v_8)$$
$$x_{9,3} + \ldots + x_{9,7} = 1 \quad 3 \cdot x_{9,3} + \ldots + 7 \cdot x_{9,7} = \tau(v_9)$$
$$x_{10,1} + \ldots + x_{10,6} = 1 \quad 1 \cdot x_{10,1} + \ldots + 6 \cdot x_{10,6} = \tau(v_{10})$$
$$x_{11,2} + \ldots + x_{11,7} = 1 \quad 2 \cdot x_{11,2} + \ldots + 7 \cdot x_{11,7} = \tau(v_{11})$$

Task 2 III

Integer Linear Programming

Solution 2.2:

- *(iii) Data dependencies:*

$$
\begin{aligned}
&\tau(v_3) - \tau(v_1) \geq 2 \quad && \tau(v_3) - \tau(v_2) \geq 2 \\
&\tau(v_4) - \tau(v_3) \geq 2 \quad && \tau(v_5) - \tau(v_4) \geq 1 \\
&\tau(v_7) - \tau(v_6) \geq 2 \quad && \tau(v_5) - \tau(v_7) \geq 2 \\
&\tau(v_9) - \tau(v_8) \geq 2 \quad && \tau(v_{11}) - \tau(v_{10}) \geq 1 \\
&\tau(v_n) - \tau(v_5) \geq 1 \quad && \tau(v_n) - \tau(v_9) \geq 1 \\
&\tau(v_n) - \tau(v_{11}) \geq 1 && \\
&\tau(v_1), \tau(v_2), \tau(v_6), \tau(v_8), \tau(v_{10}) \geq \tau(v_0) \geq 1 &&
\end{aligned}
$$

Task 2 IV

Integer Linear Programming

Solution 2.2:

- *(iv) Resource limitations:*
  - $t = 1$:

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} \leq 2$$
$$x_{10,1} \leq 2$$

  - $t = 2$:

$$x_{1,1} + x_{1,2} + x_{2,1} + x_{2,2} + x_{6,1} + x_{6,2} + x_{8,1} + x_{8,2} \leq 2$$
$$x_{10,2} + x_{11,2} \leq 2$$

  - $t = 3$:

$$x_{1,2} + x_{2,2} + x_{6,2} + x_{6,3} + x_{8,2} + x_{8,3} + x_{3,3} + x_{7,3} \leq 2$$
$$x_{10,3} + x_{11,3} + x_{9,3} \leq 2$$

Task 2 V

Solution 2.2:

- $t = 4$:

$$x_{6,3} + x_{8,3} + x_{8,4} + x_{3,3} + x_{3,4} + x_{7,3} + x_{7,4} \leq 2$$
$$x_{10,4} + x_{11,4} + x_{9,4} \leq 2$$

- $t = 5$

$$x_{8,4} + x_{8,5} + x_{3,4} + x_{7,4} + x_{7,5} \leq 2$$
$$x_{10,5} + x_{11,5} + x_{9,5} + x_{4,5} \leq 2$$

- $t = 6$:

$$x_{8,5} + x_{7,5} \leq 2$$
$$x_{10,6} + x_{11,6} + x_{9,6} + x_{4,6} + x_{5,6} \leq 2$$

- $t = 7$

$$(0 \leq 2)$$
$$x_{11,7} + x_{9,7} + x_{5,7} \leq 2$$

Integer Linear Programming I

Solution 2.3:

- *Restating the resource limitations, and introducing additional variables:*
  - $t = 1$:

$$x_{1,1} + x_{2,1} + x_{6,1} + x_{8,1} - \alpha(r_1) \leq 0$$
$$x_{10,1} - \alpha(r_2) \leq 0$$

  - *Latency limitations:*

$$L = \tau(v_n) - \tau(v_0) \leq \bar{L} = 6$$

  - *New objective function:*

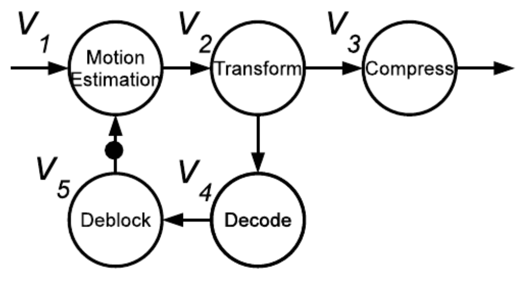
$$min.\ \ C = \alpha(r_1) \cdot c(r_1) + \alpha(r_2) \cdot c(r_2) = 2 \cdot \alpha(r_1) + \alpha(r_2)$$

Task 3

Tasks 3 I

Iterative Algorithms

Task 3.1:



	ν_1	ν_2	ν_3	ν_4	ν_5
$w(\nu_i)$	10	10	10	5	5

Tasks 3 II

Iterative Algorithms

Solution 3.1:

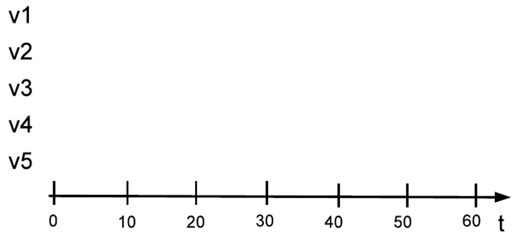
- $\tau(\nu_2) - \tau(\nu_1) \geq 10$
- $\tau(\nu_3) - \tau(\nu_2) \geq 10$
- $\tau(\nu_4) - \tau(\nu_3) \geq 10$
- $\tau(\nu_5) - \tau(\nu_4) \geq 5$
- $\tau(\nu_1) - \tau(\nu_5) \geq 5 - 1 \cdot P$

Tasks 3 III

Iterative Algorithms

Solution 3.2:

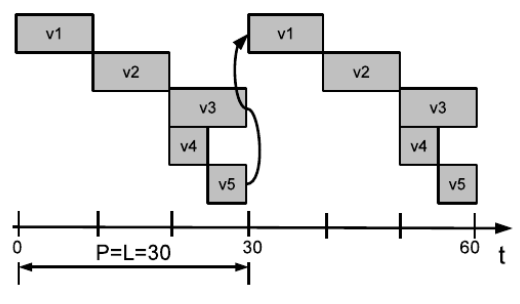
- *solve system of inequalities for P:*
  - $0 - 25 \geq 5 - P \Leftrightarrow P_{min} = 30$
  - $L = 30$



Tasks 3 IV

Iterative Algorithms

Solution 3.2:

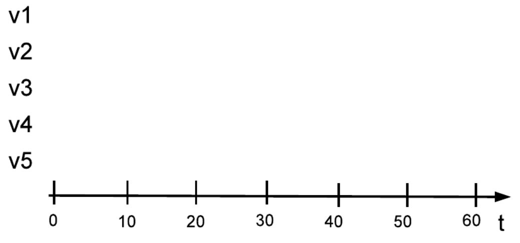


Tasks 3 V

Iterative Algorithms

Solution 3.3:

- $\tau(\nu_1) - \tau(\nu_5) \geq 5 - n \cdot 10 \Leftrightarrow n_{min} = 3$
- *we have to add 2 more tokens on the edge between* v_5 *and* v_1
- $L = 30$



Tasks 3 VI

Iterative Algorithms

Solution 3.3: