Scheduling Periodic and Mixed Task Sets

Total Bandwidth Server (TBS), Rate Monotonic (RM), Polling Server

Exercise class 6

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Gliederung

Organisation

Overview

Task 1

Task 2

Task 3

Appendix

Literature

Organisation



Organisation I

► feedback for me: https://forms.gle/f3YN8EFrZ1vsfPoC6



- ▶ get the slides before the exercise class: https: //github.com/matthejue/Einfuehrung_in_ESE_Tutoratsfolien_out
 - warning: the slides often get changed just shortly before or after the exercise class. Both the lecture and the exercise classes are pretty running edge

Organisation II

- A (offline) 1:47 PM

 Can one task start in one core be interrupted and resume later in anothr core?
- A (offline) 1:55 PM Thanks
- Ne Negin 1:57 PM so can one specific task be executed in two cores?

Organisation III

Error in sample solution of task 4.2 of sheet 3

Hello Students.

In the enligsh exercise a error in the sample solution was found by coincidence in exercise 4.2 of sheet 3. Some of the summands in the numerator of the sample solution were correct, but some were wrong. We just wanted to inform you that we updated the exercise class slides accordingly, but in the recordings the wrong solution can still be seen

Best regards and have a nice day,

Pascal and lürgen

Overview



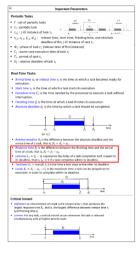
Overview Scheduling I

H Aperiodic Tank Scheduling			
* consideration * consideration provided in terms * consi			
	equal arrival times, non-preempive	arbitrary arrival times, preemptive	
	Earliest Deadline First (EDF)		
		 priority determined by 	
		$min(d_i)$	
	Earliest Deadline Due (EDD)	for all remaining	
	 priority determined by 	J,	
	$min(D_i)$ for all remaining	that have already arrived (are ready) and not finished every time the arrival time of a task is reached • <u>ischedulability.Test</u> :	
independant tasks	J _i <u>Schedulability Yest</u>	$t + \sum_{k=1}^{i} c_k(t) \le d_i$	
	$\sum_{i}^{i} C_{h} \leq d_{i}$	for all active tasks	
	for each task	J.	
	J.		
	J.	• c _h (t)	
		is the remaining worst-case execution time of task	
		J _k	
	Latest Deadline First (LDF)	Earliest Deadline First - Star (EDF*)	
	priority determined by	release time and deadline of individual tasks are modified such that all the precedence constraints are	
	$max(D_i)$	satisfied	
	tor all	• $r_j^* = max(r_j, max(r_i^* + C_i : J_i \rightarrow J_j))$	
dependant tasks	without successors or whose successors have been all selected in the procedence graph invented into the gaves to be excessed list. at natifire, tasks are extracted from the head of the gaves: the first task invented in the queue will be executed list.	$\label{eq:definition} \bullet d_i^* = min(d_i, min(d_j^* - C_j: J_i \to J_j))$ $\bullet scheduling problem is transformed into a problem without procedence constraints, which can then be handled by a 'normal' EDF scheduler$	

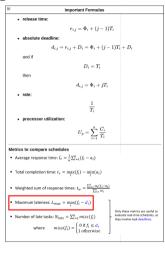
Overview Scheduling II



Overview Scheduling III



Overview Scheduling IV







Task 1 I

Task 1.1:			 ■
	$ au_1$	$ au_2$	$ au_3$
C_i	1	1	2
T_i	3	5	13

- ► TBS executes along with the periodic tasks above
- what can be the maximum value of U_s such that the whole set (i.e. periodic tasks and the TBS) is schedulable with EDF?

Task 1 II

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Requirements 1.1:

Z

Schedulability test:

Given a set of n periodic tasks with processor utilization U_p and a total bandwidth server with utilization U_s , the whole set is schedulable by EDF if and only if

$$U_p + U_s \le 1$$

▶ processor utilization factor:
$$U = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

Task 1 III

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Solution 1.1:

/

► Maximum utilization of the Total Bandwidth Server:

$$U_{s,\text{max}} = 1 - U_p = 1 - (\frac{1}{3} + \frac{1}{5} + \frac{2}{13}) = \frac{61}{195} \approx 0.3128$$

Task 1 I

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Task 1.2:

7

- construct EDF-Schedule
- ightharpoonup assume $U_s = 0.25$
- ► three aperiodic requests served by TBS:

	J_4	J_{5}	J_6
r_i	0	15	10
C_i	2	1	1

arrival time of first instance of each periodic task is 0:

	$ au_1$	$ au_2$	$ au_{3}$
a _i	0	0	0
C_i	1	1	2
T_i	3	5	13

Task 1 II

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Requirements 1.2:



- $ightharpoonup d_i = \max(r_i, d_{k-1}) + \frac{C_k}{U}$
 - \triangleright d_{k-1} denotes the previously calculated deadline (k-1) means the predecessor in the ordering according to the release time)

Solution 1.2:



- \triangleright order the aperiodic tasks by increasing release time $r_i: J_4, J_6, J_5$
- calculate the deadlines:

 - ▶ $d_4 = \max(r_4, d_0) + \frac{2}{0.25} = \max(0, 0) + 8 = 8$ ▶ $d_6 = \max(r_6, d_4) + \frac{1}{0.25} = \max(10, 8) + 4 = 14$ ▶ $d_5 = \max(r_5, d_6) + \frac{1}{0.25} = \max(15, 14) + 4 = 19$

Task 1 III

Solution 1.2:				
	$ au_1$	$ au_2$	$ au_3$	
a _i	0	0	0	
C_i	1	1	2	
T_i	3	5	13	
	J_4	J_5	J_6	
r_i	0	15	10	
d_i	8	19	14	
C_i	2	1	1	

Task 1 IV

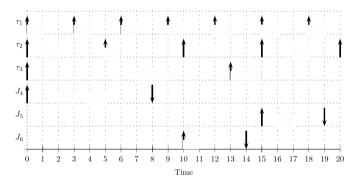


Figure 1: EDF schedule solution for Task 1

Task 1 V

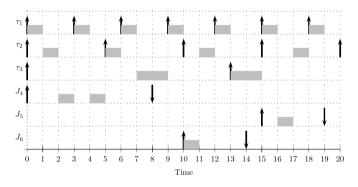


Figure 1: EDF schedule solution for Task 1

Task 2



Task 2 I

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Task 2.1:

1

- task-set schedulable with RM
- using sufficient test

	$ au_1$	$ au_2$	$ au_3$
C_i	1	3	2
T_i	3	8	9

Requirements 2.1:



▶ $U = \sum_{i=1}^{n} \frac{C_i}{T_i} \le n \left(2^{1/n} - 1\right)$, U is the fraction of the processor time spent on executing task set

Task 2 II

Schedulability Test for Fixed Priorities - Rate Monotonic (RM)

Solution 2.1:

<u>/</u>

- $ightharpoonup rac{1}{3} + rac{3}{8} + rac{2}{9} = 0.93 \le 3(2^{\frac{1}{3}} 1) = 0.78 imes 1$
- ► condition is not necessary, hence we don't know whether the task set is schedulable with RM or not

Task 2 III



Task 2 IV

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Requirements 2.2:



- guarantee that all the tasks can be scheduled in any possible instance
- ▶ in particular, if a task can be scheduled in its critical instances, then the schedulability guarantee condition holds
 - ► a critical instance of a task occurs whenever the task is released simultaneously with all higher priority tasks
- \triangleright Schedulability Test: For all tasks τ_i smallest R_i that satisfies

$$R_i = C_i + \sum_{i=1}^{i-1} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$
 and $R_i \leq D_i$ (necessary and sufficient)

Task 2 V

Schedulability Test for Fixed Priorities - Rate Monotonic (RM)

Requirements 2.2:

```
\begin{split} \mathbf{DM\_guarantee} & (\Gamma) \ \{ \\ & \mathbf{for} \ (\mathbf{each} \ \tau_i \in \Gamma) \ \{ \\ & I_i = \sum_{k=1}^{i-1} C_k; \\ & \mathbf{do} \ \{ \\ & R_i = I_i + C_i; \\ & \mathbf{if} \ (R_i > D_i) \ \mathbf{return} (\mathbf{UNSCHEDULABLE}); \\ & I_i = \sum_{k=1}^{i-1} \left\lceil \frac{R_k}{T_k} \right\rceil C_k; \\ & \} \ \mathbf{while} \ (I_i + C_i > R_i); \\ & \} \ \mathbf{return} (\mathbf{SCHEDULABLE}); \\ \} \end{split}
```

Task 2 VI

Schedulability Test for Fixed Priorities - Rate Monotonic (RM)

Solution 2.2:



- ▶ The tasks are first ordered by their priorities: τ_1 , τ_2 and τ_3
 - ► In this case the tasks are already ordered
- $ightharpoonup au_3$
 - $R_3^0 = 1 + 3 + 2 = 6$ $I_3^0 = \begin{bmatrix} \frac{6}{3} \end{bmatrix} 1 + \begin{bmatrix} \frac{6}{8} \end{bmatrix} 3 = 2 + 3 = 5$ $5 + 2 \le 6 \times 6$
 - $R_3^1 = 5 + 2 = 7$ $I_3^1 = \begin{bmatrix} \frac{7}{3} \end{bmatrix} 1 + \begin{bmatrix} \frac{7}{8} \end{bmatrix} 3 = 3 + 3 = 6$ $6 + 2 \le 7 \times 1$
 - $R_3^2 = 6 + 2 = 8 \quad I_3^2 = \left\lceil \frac{8}{3} \right\rceil 1 + \left\lceil \frac{8}{8} \right\rceil 3 = 3 + 3 = 6 \quad 6 + 2 \le 8 \checkmark$ $\Rightarrow R_2 = 8 < T_2 = 9 \checkmark$

Task 2 VII

Schedulability Test for Fixed Priorities - Rate Monotonic (RM)

Solution 2.2:



 \vdash τ_2 :

$$R_2^0 = 1 + 3 = 4$$
 $I_2^0 = \left[\frac{4}{3}\right] 1 = 2$ $2 + 3 \le 4 \times 4$

$$R_2^{\bar{1}} = 2 + 3 = 5$$
 $I_2^{\bar{1}} = \begin{bmatrix} \frac{5}{3} \\ \frac{1}{3} \end{bmatrix} 1 = 2$ $2 + 3 \le 5$ \checkmark

$$\Rightarrow R_2 = 5 \le T_2 = 8 \checkmark$$

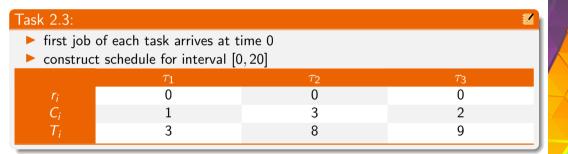
ightharpoonup

$$R_1^0 = C_1 = 1$$
 $I_1^0 = 0$ $0 + 1 \le 1$ \checkmark

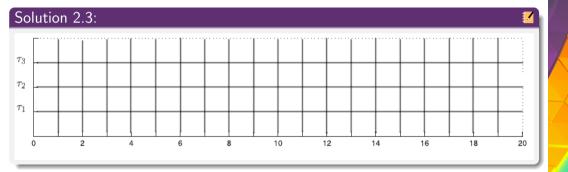
$$\Rightarrow R_1 = 1 \leq T_1 = 3 \checkmark$$

► The necessary and sufficient test succeeds. This means that the task set is schedulable with RM.

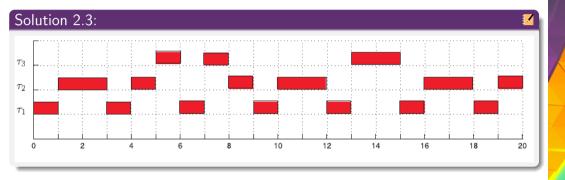
Task 2 VIII



Task 2 IX



Task 2 X



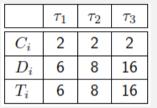
Task 3



Task 3

Scheduling with Polling Server

Task 3.1:



- In addition to the above periodic tasks, we have an aperiodic job J_a with $C_a = 1$ and relative deadline D_a . Let $T_s = 25$ and $C_s = 1$ respectively, where T_s denotes the period and C_s the computing time (or capacity) of the polling server (PS)
- ightharpoonup Compute the minimum relative deadline of J_a which is guaranteed not to be missed, that is, its aperiodic guarantee.

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Task 3 I

Scheduling with Polling Server

Solution 3.1:

- Z
- As reminder: The polling server itself acts like a periodic task, that uses its capacity to serve aperiodic tasks. If the polling server has the current highest priority it begins to serve any pending aperiodic requests within the limits of its capacity.
- ► If there are no pending aperioidic tasks at that time it suspends its entire capacity! (until the beginning of the next period)
- ► Therefore, the worst possible case occurs when J_a appears slightly later than the polling server checks for pending aperiodic tasks. Meaning, the polling server suspends its capacity and J_a has to wait $T_s + \lceil \frac{C_a}{C_s} \rceil T_s = (1 + \lceil \frac{C_a}{C_s} \rceil) T_s$

Task 3 II

Scheduling with Polling Server

Solution 3.1:



- ▶ To guarantee to not miss the deadline D_a the condition $(1 + \lceil \frac{C_a}{C_s} \rceil) T_s \leq D_a$ needs to hold.
- Entering the values for the exercise gives us $D_a = (1 + \frac{1}{1}) \cdot 25 = 50$

Sidenote 9

Note that the above computation of course only holds if the RM schedule meets all the deadlines.

Task 3

Scheduling with Polling Server

Task 3.2:

7

Using the sufficient test of RM, test if the polling server of task 3.1 is schedulable along with the periodic task-set.

Requirements 3.2:



Schedulability Test:
$$\sum_{i=1}^{n} \frac{C_i}{T_i} + \frac{C_s}{T_s} \leq (n+1) \left[2^{1/(n+1)} - 1 \right]$$

Task 3

Scheduling with Polling Server

Solution 3.2:



- We have already seen the sufficient but not necessary condition of $\sum_{i=1}^{n} \frac{C_i}{T_i} \le n(2^{1/n} 1) \text{ for rate monotonic scheduling.}$
- The convenient part about our polling server: we can just treat it as an additional periodic task.
- ▶ Therefore, the same test offers us a sufficient condition for rate monotonic scheduling with a polling server! Simply increase from n tasks to (n+1)

Task 3

Scheduling with Polling Server

Solution 3.2:



► Putting in values we obtain:

$$\frac{2}{6} + \frac{2}{8} + \frac{2}{16} + \frac{1}{25} \le (3+1)(2^{\frac{1}{3+1}} - 1) \Leftrightarrow \frac{449}{600} \le \sqrt[4]{256} \cdot \sqrt[4]{2} - 4 \Leftrightarrow 0.75 \le 0.76$$
. Since this is true, we know (as this is a sufficient condition) that the RM schedule meets all deadlines!

Appendix



Overview periodic Task Scheduling

	Deadline equals period	Deadline smaller than period
static priority	RM (rate-monotonic)	DM (deadline-monotonic)
dynamic priority	EDF	EDF

Overview Aperiodic Task Scheduling

Schedulability test

	Deadline equals period $(D_i = T_i)$	Deadline smaller than period $(D_i \leq T_i)$
static priority	$(1) \sum_{i=1}^n \frac{C_i}{T_i} \leq n \left(2^{1/n} - 1\right)$	$(1) \sum_{i=1}^n \frac{C_i}{D_i} \leq n \left(2^{1/n} - 1\right)$
	(sufficient but not necessary)	(sufficient but not necessary)
	(2) same as Schedulability Test 2 for	(2) smallest R_i that satisfies
	DM	$R_i = C_i + \sum_{j=1}^{i-1} \left[\frac{R_i}{T_j} \right] C_j$ and $R_i \le D_i$ for all tasks τ_i
	, ,	(necessary and sufficient)
dynamic priority	$\sum_{i=1}^{n} rac{C_i}{T_i} = U \leq 1$ (necessary and sufficient)	→ Buttazzo, Hard real-time computing systems: predictable scheduling algorithms and applications

Mixed Task Sets

- ► So far: we differentiated between periodic and aperiodic tasks.
- ► Now: Consider a mixed task set!
- ► We want to be able to find a schedule when there's both periodic and aperiodic tasks.

Schedulability tests

Sufficient? Necesarry?

- ▶ We're interested in whether a given problem can be scheduled by algorithms.
- Depending on the algorithm we can derive sufficient and necesarry conditions.

Sufficient: If $A \implies B$ then A is a sufficient condition for B.

Necesarry: If $B \implies A$ then A is a necesarry condition for B.

► A necesarry and sufficient condition means, both statements are logically equivalent.

Schedulability tests Utilization

Different kind of utilizations also play a big role in our analysis. We introduced the processor utilization factor $U = \sum_{i=1}^{n} \frac{C_i}{T_i}$ and later on U_s as the server utilization.

(More about servers later)

RM - Rate Monotonic Scheduling

Schedulability

- ▶ RM is optimal among all fixed-priority assignments in the sense that no other fixed-priority algorithm can schedule a task set that cannot be scheduled by RM.
- As in the lecture, we have $\sum_{i=1}^{n} \frac{C_i}{T_i} \le n(2^{1/n} 1)$ as a sufficient but not necessarry condition.

RM(PS) - Rate Monotonic Polling Server

- One way to handle both periodic and aperiodic tasks is to use a so called server.
- ▶ This PS (Polling Server) acts as a periodic task (meaning it is instantiated at regular intervals T_s) whose job it is to, once it has the highest priority, serve any pending aperiodic requests within the limits of a server capacity C_s .
- Since we introduce yet another periodic task, the schedulability analysis simply is the same as normal RM with one additional task. Again, we have the sufficient but not necessary condition: $\frac{C_s}{T_s} + \sum_{i=1}^{n} \frac{C_i}{T_i} \le (n+1)(2^{1/(n+1)}-1)$

Literature



Bücher



Buttazzo, Giorgio C. Hard real-time computing systems: predictable scheduling algorithms and applications. Vol. 24. Springer Science & Business Media, 2011.