Scheduling Periodic and Mixed Task Sets

Total Bandwidth Server (TBS), Rate Monotonic (RM), Polling Server

Exercise class 5

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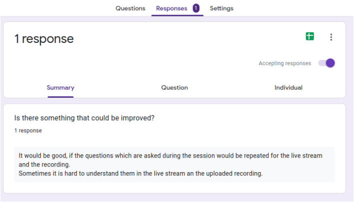
Gliederung

Organisation

Organisation I

- feedback for me: `https://forms.gle/f3YN8EFrZ1vsfPoC6`



- get the slides before the exercise class: `https://github.com/matthejue/Einfuehrung_in_ESE_Tutoratsfolien_out`
  - warning: the slides often get changed just shortly before or after the exercise class. Both the lecture and the exercise classes are pretty running edge

Overview

Overview Scheduling I

Aperiodic Task Scheduling

- **event-driven**
- **can arrive at any point in time**
- **Optimality:** Minimize the maximum lateness of the task set
- **precedence relations** between tasks can be described through an acyclic directed graph G where tasks are represented by nodes and precedence relations by arrows. G induces a partial order on the task set
- **2 types of aperiodic tasks:**
  - **sporadic task:** has a minimum inter-arrival time between consecutive instances of the task
  - **firm task:** minimum inter-arrival time between consecutive instances of the task cannot be bounded

	equal arrival times, non-preempive	**arbitrary arrival times, preemptive**
independant tasks	**Earliest Deadline Due (EDD)** • priority determined by $min(D_i)$ for all remaining J_i • Schedulability Test: $\sum_{k=1}^{i} C_k \le d_i$ for each task J_i	**Earliest Deadline First (EDF)** • priority determined by $min(d_i)$ for all remaining J_i that have already arrived (are ready) and not finished every time the arrival time of a task is reached • Schedulability Test: $t + \sum_{k=1}^{i} c_k(t) \le d_i$ for all active tasks J_i ∘ $c_k(t)$ is the remaining worst-case execution time of task J_k
dependant tasks	**Latest Deadline First (LDF)** • priority determined by $max(D_i)$ for all J_i without successors or whose successors have been all selected in the precedence graph inserted into the queue to be executed last • at runtime, tasks are extracted from the head of the queue: the first task inserted in the queue will be executed last	**Earliest Deadline First - Star (EDF*)** • release time and deadline of individual tasks are modified such that all the precedence constraints are satisfied ∘ $r_j^* = max(r_j, max(r_i^* + C_i : J_i \rightarrow J_j))$ ∘ $d_i^* = min(d_i, min(d_j^* - C_j : J_i \rightarrow J_j))$ • scheduling problem is transformed into a problem without precedence constraints, which can then be handled by a "normal" EDF scheduler

Overview Scheduling II

Periodic Task Scheduling + Mixed Tasks Sets (Periodic + Aperiodic)

- **time-driven**
- **regularly activated with a constant period**
- **Optimality of fixed-priority assignments:** every task set schedulable by another fixed-priority algorithm can also be scheduled by RM and DM

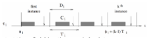
	$D_i = T_i$	$D_i \leq T_i$
static priority	**Rate-Monotonic (RM)** • priority determined by $min(T_i)$ for all remaining J_i • Schedulability Test 1: $U = \sum_{i=1}^{n} \frac{C_i}{T_i} \leq n\left(2^{1/n} - 1\right)$ (sufficient but not necessary) • Schedulability Test 2: same as Schedulability Test 2 for DM **Background Scheduling (BS):** • aperiodic tasks scheduled after FCFS when no periodic task ready to execute **Polling Server (PS):** • PS scheduled as periodic task, it serves any pending aperiodic requests until its capacity (execution time) C_S is exhausted • If no aperiodic requests are pending, PS suspends itself until the beginning of its next period, and the budget originally allocated for aperiodic service is freed up and assigned to periodic tasks • Schedulability Test 1: $\sum_{i=1}^{n} \frac{C_i}{T_i} + \frac{C_s}{T_s} \leq (n+1)\left[2^{1/(n+1)} - 1\right]$ (sufficient but not necessary) • Schedulability Test 2: If $C_a \leq C_s$ and $2T_s \leq D_a$ (necessary and sufficient, fitting computation time) • Schedulability Test 3: $T_S + \left\lceil \frac{C_a}{C_s} \right\rceil T_s \leq D_a$ (necessary and sufficient, not fitting computation time)	**Deadline-Monotonic (DM)** • priority determined by $min(D_i)$ for all remaining J_i • Schedulability Test 1: $\sum_{i=1}^{n} \frac{C_i}{D_i} \leq n\left(2^{1/n} - 1\right)$ (sufficient but not necessary) • Schedulability Test 2: for all tasks τ_i smallest R_i that satisfies $R_i = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$ and $R_i \leq D_i$ (necessary and sufficient)
dynamic priority	**Earliest Deadline First (EDF)** • Schedulability Test: $\sum_{i=1}^{n} \frac{C_i}{T_i} = U \leq 1$ **Total Bandwidth Server (TBS):** • $d_k = \max(r_k, d_{k-1}) + \frac{C_k}{U_s}$ • Schedulability Test: $U_p + U_s \leq 1$ (necessary and sufficient)	**Earliest Deadline First (EDF)** • Schedulability Test: Buttazzo, Hard real-time computing systems: predictable scheduling

Overview Scheduling III

Important Parameters

Periodic Tasks

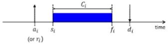
- Γ : set of periodic tasks
- τ_i : periodic task
- $\tau_{i,j}$: j-th instance of task τ_i
- $r_{i,j}, s_{i,j}, f_{i,j}, d_{i,j}$: release time, start time, finishing time, and absolute deadline of the j-th instance of task τ_i



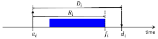
- Φ_i : phase of task τ_i (release time of first instance)
- C_i : worst-case execution time of task τ_i
- T_i : period of task τ_i
- D_i : relative deadline of task τ_i

Real-Time Tasks

- *Arrival time* a_i or *release time* r_i is the time at which a task becomes ready for execution.
- *Start time* s_i is the time at which a task starts its execution.
- *Execution time* C_i is the time needed by the processor to execute a task without interruption.
- *Finishing time* f_i is the time at which a task finishes its execution.
- *Absolute deadline* d_i is the time by which a task should be completed.

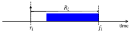


- *Relative deadline* D_i is the difference between the absolute deadline and the arrival time of a task, that is, $D_i = d_i - a_i$.
- *Response time* R_i is the difference between the finishing time and the arrival time of a task, that is, $R_i = f_i - a_i$.
- *Lateness* $L_i = f_i - d_i$ represents the delay of a task completion with respect to its deadline, that is, $L_i \leq 0$ if a task completes within its deadline.
- *Tardiness* $E_i = \max(0, L_i)$ is the time a task stays active after its deadline.
- *Laxity* $X_i = d_i - a_i - C_i$ is the maximum time a task can be delayed on its execution in order to complete within its deadline.



Critical Instant

- *Definition*: A critical instant of a task is the release time r_i that produces the largest response time R_i, that is, the largest difference between release time r_i and finishing time f_i.
- *Lemma*: For any task, a critical instant occurs whenever the task is released simultaneously with all higher-priority tasks.



Overview Scheduling IV

Important Formulas

- **release time:**

$$r_{i,j} = \Phi_i + (j-1)T_i$$

- **absolute deadline:**

$$d_{i,j} = r_{i,j} + D_i = \Phi_i + (j-1)T_i + D_i$$

and if

$$D_i = T_i$$

then

$$d_{i,j} = \Phi_i + jT_i$$

- **rate:**

$$\frac{1}{T_i}$$

- **processor utilization:**

$$U_p = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

Metrics to compare schedules

- Average response time: $\bar{t}_r = \frac{1}{n}\sum_{i=1}^{n}(f_i - a_i)$
- Total completion time: $t_c = \max_i(f_i) - \min_i(a_i)$
- Weighted sum of response times: $t_w = \frac{\sum_{i=1}^{n} w_i(f_i - a_i)}{\sum_{i=1}^{n} w_i}$
- Maximum lateness: $L_{max} = \max_i(f_i - d_i)$
- Number of late tasks: $N_{late} = \sum_{i=1}^{n} miss(f_i)$

  where $miss(f_i) = \begin{cases} 0 \text{ if } f_i \leq d_i \\ 1 \text{ otherwise} \end{cases}$

Only these metrics are useful to evaluate real-time schedules, as they involve task deadlines.

Task 1

Task 1 I

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Task 1.1:

	τ_1	τ_2	τ_3
C_i	1	1	2
T_i	3	5	13

- what can be the maximum value of U_s such that the whole set (i.e. periodic tasks and the TBS) is schedulable with EDF?

Task 1 II

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Requirements 1.1:

Schedulability test:

Given a set of *n* periodic tasks with processor utilization U_p and a total bandwidth server with utilization U_s, the whole set is schedulable by EDF if and only if

$$U_p + U_s \leq 1$$

- *processor utilization factor:* $U = \sum_{i=1}^{n} \frac{C_i}{T_i}$

Task 1 III

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Solution 1.1:

- *Maximum utilization of the Total Bandwidth Server:*
  $U_{s,\text{max}} = 1 - U_p = 1 - (\frac{1}{3} + \frac{1}{5} + \frac{2}{13}) = \frac{61}{195} \approx 0.3128$

Task 1 I

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Task 1.2:

- construct EDF-Schedule
- assume $U_s = 0.25$
- three aperiodic requests served by TBS:

	J_4	J_5	J_6
r_i	0	15	10
C_i	2	1	1

- arrival time of first instance is 0:

	τ_1	τ_2	τ_3
a_i	0	0	0
C_i	1	1	2
T_i	3	5	13

Task 1 II

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Requirements 1.2:

- $d_i = \max(r_i, d_{k-1}) + \frac{C_k}{U_s}$
  - d_{k-1} *denotes the previously calculated deadline* ($k-1$ *means the predecessor in the ordering according to the release time*)

Solution 1.2:

- *order the aperiodic tasks by increasing release time* $r_i : J_4, J_6, J_5$
- *calculate the deadlines:*
  - $d_4 = \max(r_4, d_0) + \frac{2}{0.25} = \max(0, 0) + 8 = 8$
  - $d_6 = \max(r_6, d_4) + \frac{1}{0.25} = \max(10, 8) + 4 = 14$
  - $d_5 = \max(r_5, d_6) + \frac{1}{0.25} = \max(15, 14) + 4 = 19$
- *periodic tasks already ordererd by increasing period:* $t_i : \tau_1, \tau_2, \tau_3$

Task 1 III

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

Solution 1.2:

	τ_1	τ_2	τ_3
a_i	0	0	0
C_i	1	1	2
T_i	3	5	13

	J_4	J_5	J_6
r_i	0	15	10
d_i	8	19	14
C_i	2	1	1

Task 1 IV

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

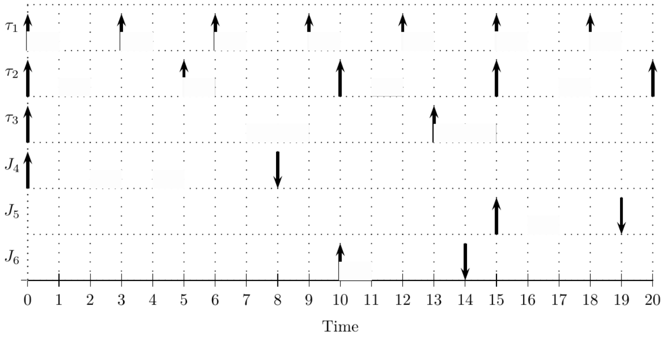


Figure 1: EDF schedule solution for Task 1

Task 1 V

Earliest Deadline First (EDF) and Total Bandwidth Server (TBS)

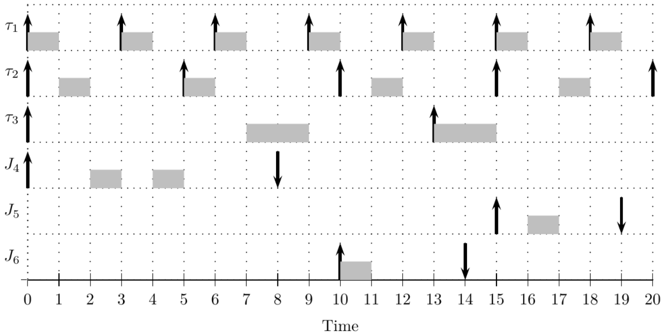


Figure 1: EDF schedule solution for Task 1

Task 2

Task 2 I

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Task 2.1:

- task-set schedulable with RM
- using sufficient test

	τ_1	τ_2	τ_3
C_i	1	3	2
T_i	3	8	9

Requirements 2.1:

- $U = \sum_{i=1}^{n} \frac{C_i}{T_i} \leq n\left(2^{1/n} - 1\right)$, *U is the fraction of the processor time spent on executing task set*

Task 2 II

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Solution 2.1:

- $\frac{1}{3} + \frac{3}{8} + \frac{2}{9} = 0.93 \leq 3(2^{\frac{1}{3}} - 1) = 0.78 \quad \times$
- *condition is not necessary, hence we don't know whether the task set is schedulable with RM or not*

Task 2 III

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Task 2.2:

- task-set schedulable with RM
- using necessary test

	τ_1	τ_2	τ_3
C_i	1	3	2
T_i	3	8	9

Task 2 IV

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Requirements 2.2:

- *guarantee that all the tasks can be scheduled in any possible instance*
- *in particular, if a task can be scheduled in its critical instances, then the schedulability guarantee condition holds*
  - *a critical instance of a task occurs whenever the task is released simultaneously with all higher priority tasks*
- *Schedulability Test: For all tasks τ_i smallest R_i that satisfies*

$$R_i = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i}{T_j} \right\rceil C_j \text{ and } R_i \leq D_i \text{ (necessary and sufficient)}$$

Task 2 V

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Requirements 2.2:

DM_guarantee (Γ) {
    for (each $\tau_i \in \Gamma$) {
        $I_i = \sum_{k=1}^{i-1} C_k$;
        do {
            $R_i = I_i + C_i$;
            if ($R_i > D_i$) return(UNSCHEDULABLE);
            $I_i = \sum_{k=1}^{i-1} \left\lceil \frac{R_i}{T_k} \right\rceil C_k$;
        } **while** ($I_i + C_i > R_i$);
    }
    return(SCHEDULABLE);
}

Task 2 VI

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Solution 2.2:

- *The tasks are first ordered by their priorities:* τ_1, τ_2 *and* τ_3
  - *In this case the tasks are already ordered*
- τ_3 :
  - $R_3^0 = C_3 = 2 \quad I_3^0 = \left\lceil \frac{2}{3} \right\rceil 1 + \left\lceil \frac{2}{8} \right\rceil 3 = 1 + 3 = 4 \quad 4 + 2 \neq 2$
  - $R_3^1 = 4 + 2 = 6 \quad I_3^1 = \left\lceil \frac{6}{3} \right\rceil 1 + \left\lceil \frac{6}{8} \right\rceil 3 = 2 + 3 = 5 \quad 5 + 2 \neq 6$
  - $R_3^2 = 5 + 2 = 7 \quad I_3^2 = \left\lceil \frac{7}{3} \right\rceil 1 + \left\lceil \frac{7}{8} \right\rceil 3 = 3 + 3 = 6 \quad 6 + 2 \neq 7$
  - $R_3^3 = 6 + 2 = 8 \quad I_3^3 = \left\lceil \frac{8}{3} \right\rceil 1 + \left\lceil \frac{8}{8} \right\rceil 3 = 3 + 3 = 6 \quad 6 + 2 = 8 \ldots \checkmark$
    (since $R_3 = 8 \leq T_3 = 9$ *)*

Task 2 VII

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Solution 2.2:

- τ_2:
  - $R_2^0 = C_2 = 3 \quad I_2^0 = \lceil \frac{3}{3} \rceil 1 = 1 \quad 1 + 3 \neq 3$
  - $R_2^1 = 1 + 3 = 4 \quad I_2^1 = \lceil \frac{4}{3} \rceil 1 = 2 \quad 2 + 3 \neq 4$
  - $R_2^2 = 2 + 3 = 5 \quad I_2^2 = \lceil \frac{5}{3} \rceil 1 = 2 \quad 2 + 3 = 5 \ldots \checkmark$

  (since $R_2 = 5 \leq T_2 = 8$)
- τ_1:
  - $R_1^0 = C_1 = 1 \quad I_1^0 = 0 \quad 0 + 1 = 1 \ldots \checkmark$

  (since $R_1 = 1 \leq T_1 = 3$)
- *The necessary and sufficient test succeeds. This means that the task set is schedulable with RM.*

Task 2 VIII

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Task 2.3:

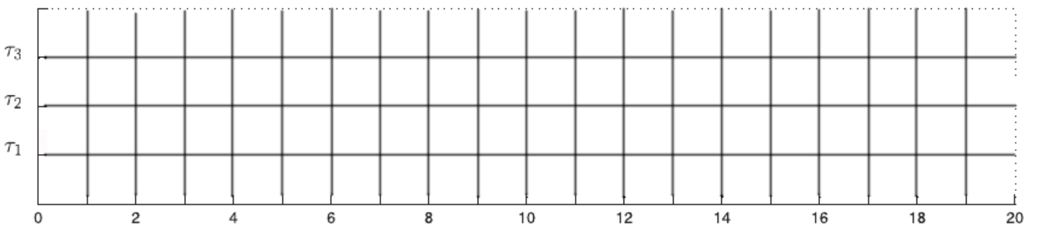
- first job of each task arrives at time 0
- construct schedule for interval $[0, 20]$

	τ_1	τ_2	τ_3
r_i	0	0	0
C_i	1	3	2
T_i	3	8	9

Task 2 IX

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

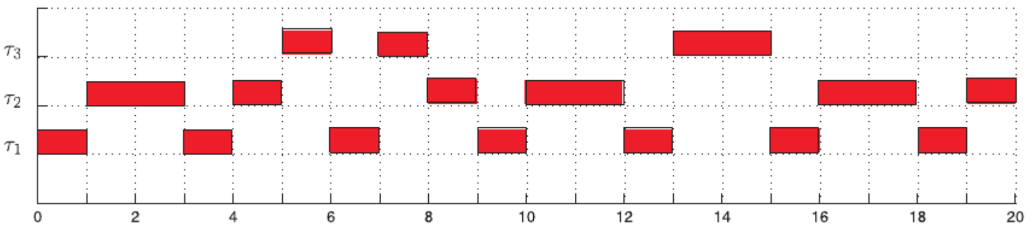
Solution 2.3:



Task 2 X

Schedulability Test for Fixed Priorities – Rate Monotonic (RM)

Solution 2.3:



Task 3

Task 3

Scheduling with Polling Server

Task 3.1:

	τ_1	τ_2	τ_3
C_i	2	2	2
D_i	6	8	16
T_i	6	8	16

- In addition to the above periodic tasks, we have an aperiodic job J_a with $C_a = 1$ and relative deadline D_a. Let $T_s = 25$ and $C_s = 1$ respectively, where T_s denotes the period and C_s the computing time (or capacity) of the polling server (PS)
- Compute the minimum relative deadline of J_a which is guaranteed not to be missed, that is, its aperiodic guarantee.

Task 3 I

Scheduling with Polling Server

Solution 3.1:

- *As reminder: The polling server itself acts like a periodic task, that uses its capacity to serve aperiodic tasks. If the polling server has the current highest priority it begins to serve any pending aperiodic requests within the limits of its capacity.*
- *If there are no pending aperioidic tasks at that time it suspends its entire capacity! (until the beginning of the next period)*
- *Therefore, the worst possible case occurs when J_a appears slightly later than the polling server checks for pending aperiodic tasks. Meaning, the polling server suspends its capacity and J_a has to wait* $T_s + \lceil \frac{C_a}{C_s} \rceil T_s = (1 + \lceil \frac{C_a}{C_s} \rceil) T_s$

Task 3 II

Scheduling with Polling Server

Solution 3.1:

- *To guarantee to not miss the deadline D_a the condition $(1 + \lceil \frac{C_a}{C_s} \rceil) T_s \leq D_a$ needs to hold.*
- *Entering the values for the exercise gives us $D_a = (1 + \frac{1}{1}) \cdot 25 = 50$*

Sidenote

Note that the above computation of course only holds if the RM schedule meets all the deadlines.

Task 3

Scheduling with Polling Server

Task 3.2:

Using the sufficient test of RM, test if the polling server of task 3.1 is schedulable along with the periodic task-set.

Requirements 3.2:

- *Schedulability Test:* $\sum_{i=1}^{n} \frac{C_i}{T_i} + \frac{C_s}{T_s} \leq (n+1)\left[2^{1/(n+1)} - 1\right]$

Task 3

Scheduling with Polling Server

Solution 3.2:

- *We have already seen the sufficient but not necessary condition of* $\sum_{i=1}^{n} \frac{C_i}{T_i} \leq n(2^{1/n} - 1)$ *for rate monotonic scheduling.*
- *The convenient part about our polling server: we can just treat it as an additional periodic task.*
- *Therefore, the same test offers us a sufficient condition for rate monotonic scheduling with a polling server! Simply increase from n tasks to (n+1)*
- *Check:* $\sum_{i=1}^{n} \frac{C_i}{T_i} + \underbrace{\frac{C_s}{T_s}}_{\text{server task}} \leq (n+1)(2^{1/(n+1)} - 1)$

Task 3

Scheduling with Polling Server

Solution 3.2:

- *Putting in values we obtain:*

$\frac{2}{6} + \frac{2}{8} + \frac{2}{16} + \frac{1}{25} \leq (3+1)(2^{\frac{1}{3+1}} - 1) \Leftrightarrow \frac{449}{600} \leq \sqrt[4]{256} \cdot \sqrt[4]{2} - 4 \Leftrightarrow 0.75 \leq 0.76.$
Since this is true, we know (as this is a sufficient condition) that the RM schedule meets all deadlines!

Appendix

Overview periodic Task Scheduling

	Deadline equals period	Deadline smaller than period
static priority	RM (rate-monotonic)	DM (deadline-monotonic)
dynamic priority	EDF	EDF

Overview Aperiodic Task Scheduling

Schedulability test

	Deadline equals period ($D_i = T_i$)	Deadline smaller than period ($D_i \leq T_i$)
static priority	(1) $\sum_{i=1}^{n} \frac{C_i}{T_i} \leq n\left(2^{1/n} - 1\right)$ (sufficient but not necessary) (2) same as Schedulability Test 2 for DM	(1) $\sum_{i=1}^{n} \frac{C_i}{D_i} \leq n\left(2^{1/n} - 1\right)$ (sufficient but not necessary) (2) smallest R_i that satisfies $R_i = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$ and $R_i \leq D_i$ for all tasks τ_i (necessary and sufficient)
dynamic priority	$\sum_{i=1}^{n} \frac{C_i}{T_i} = U \leq 1$ (necessary and sufficient)	$\rightarrow$ Buttazzo, *Hard real-time computing systems: predictable scheduling algorithms and applications*

Mixed Task Sets

- So far: we differentiated between periodic and aperiodic tasks.
- Now: Consider a mixed task set!
- We want to be able to find a schedule when there's both periodic and aperiodic tasks.

Schedulability tests

Sufficient? Necesarry?

- We're interested in whether a given problem can be scheduled by algorithms.
- Depending on the algorithm we can derive sufficient and necesarry conditions.

  Sufficient: If $A \implies B$ then A is a sufficient condition for B.

  Necesarry: If $B \implies A$ then A is a necesarry condition for B.
- A necesarry and sufficient condition means, both statements are logically equivalent.

Schedulability tests

Utilization

Different kind of utilizations also play a big role in our analysis. We introduced the processor utilization factor $U = \sum_{i=1}^{n} \frac{C_i}{T_i}$ and later on U_s as the server utilization.

(More about servers later)

RM - Rate Monotonic Scheduling

Schedulability

- RM is optimal among all fixed-priority assignments in the sense that no other fixed-priority algorithm can schedule a task set that cannot be scheduled by RM.
- As in the lecture, we have $\sum_{i=1}^{n} \frac{C_i}{T_i} \leq n(2^{1/n} - 1)$ as a sufficient but not necesarry condition.

RM(PS) - Rate Monotonic Polling Server

- One way to handle both periodic and aperiodic tasks is to use a so called server.
- This PS (Polling Server) acts as a periodic task (meaning it is instantiated at regular intervals T_s) whose job it is to, once it has the highest priority, serve any pending aperiodic requests within the limits of a server capacity C_s.
- Since we introduce yet another periodic task, the schedulability analysis simply is the same as normal *RM* with one additional task. Again, we have the sufficient but not necesarry condition: $\frac{C_s}{T_s} + \sum_{i=1}^{n} \frac{C_i}{T_i} \leq (n+1)(2^{1/(n+1)} - 1)$

Literature

Bücher

Buttazzo, Giorgio C. *Hard real-time computing systems: predictable scheduling algorithms and applications*. Vol. 24. Springer Science & Business Media, 2011.