Energy and Power

Energy Harvesting, Solar Cell Characteristics and Maximum Power Point Tracking, Application Control

Exercise class 8

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Based on the lecture of:
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Gliederung

Organisation

Overview

Task 1

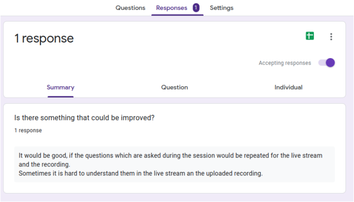
Task 2

Task 3

Organisation

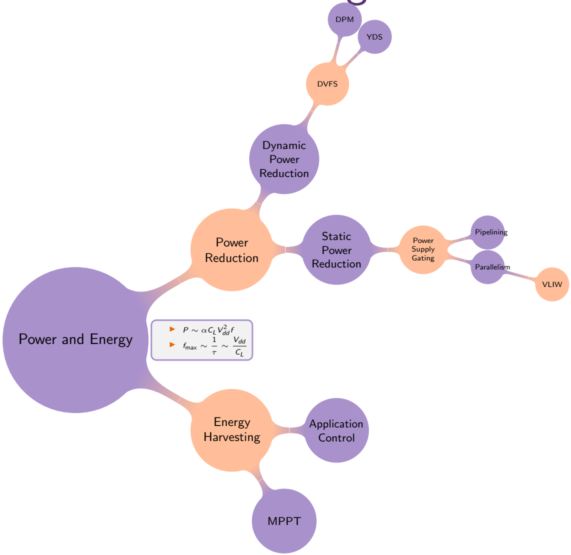
Organisation I

- feedback for me: `https://forms.gle/f3YN8EFrZ1vsfPoC6`



Overview

Overview Scheduling



- $P \sim \alpha C_L V_{dd}^2 f$
- $f_{\max} \sim \frac{1}{\tau} \sim \frac{V_{dd}}{C_L}$

Task 1

Task 1 I

Task 1.1:

Task	τ_1	τ_2	τ_3
Arrival Time [ms]	0	0	0
Period [ms]	6	4	12
Relative Deadline [ms]	6	4	12
Cycles [$\times 10^3$]	2	1	2

$$P_{\text{dynamic}}(f) = \left(\frac{f}{1\,\text{MHz}}\right)^3 \text{mW}$$

Task 1 II

Solution 1.1:

- *The execution times C_i of tasks τ_i are:* $C_1 = \frac{1 \cdot 10^3 \, Cyles}{1 \cdot 10^6 \, \frac{Cycles}{s}} = 2ms$, $C_2 = \frac{1 \cdot 10^3 \, Cyles}{1 \cdot 10^6 \, \frac{Cycles}{s}} = 1ms$, *and* $C_3 = \frac{1 \cdot 10^3 \, Cyles}{1 \cdot 10^6 \, \frac{Cycles}{s}} = 2ms$ $(T = \frac{N}{f})$.
- *Applying EDF schedule, the processor is busy in* $[0ms, 9ms]$
- *constant input power:* $P_{in} = 0.5\mu J$
- *We use* $1MHz$ *for processing tasks, leading to* $P_{dynamic}(1MHz) = 1mW$
- *This means we consume* $1mW \cdot 1ms = 1\mu J$ *per milisecond.*
- *Since all our tasks take a multiple of 1ms, we know that while processing tasks, our battery empties with* $0.5\mu J/ms - 1\mu J/ms = -0.5\mu J/ms$

Task 1 III

Solution 1.1:

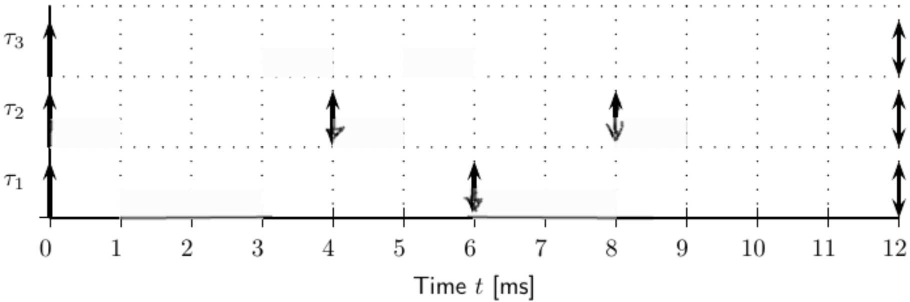


Figure 1: The resulting schedule for tasks τ_i when applying EDF algorithm.

Task 1 IV

Solution 1.1:

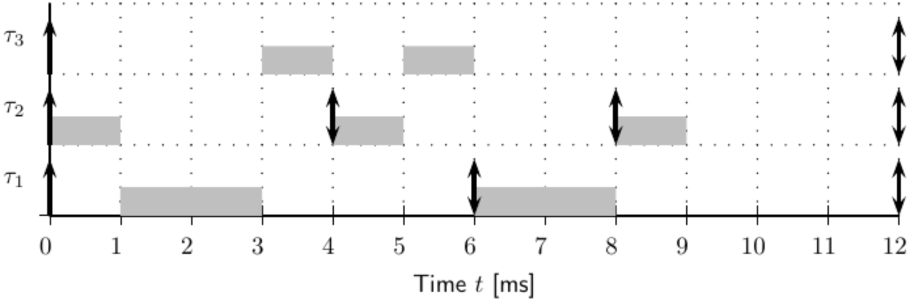


Figure 1: The resulting schedule for tasks τ_i when applying EDF algorithm.

Task 1 V

Solution 1.1:

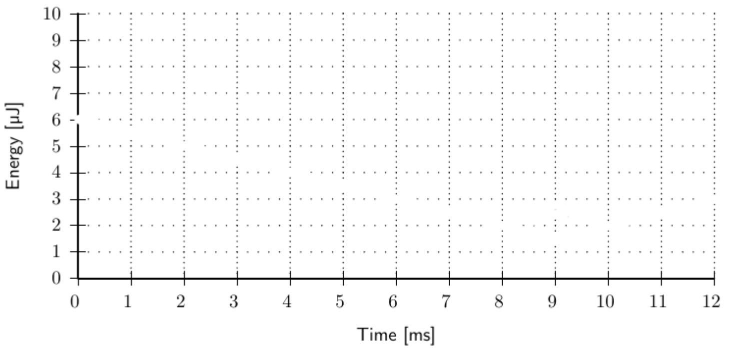


Figure 2: The resulting evolution of the battery charge state $E_{\text{bat}}(t)$.

Task 1 VI

Solution 1.1:

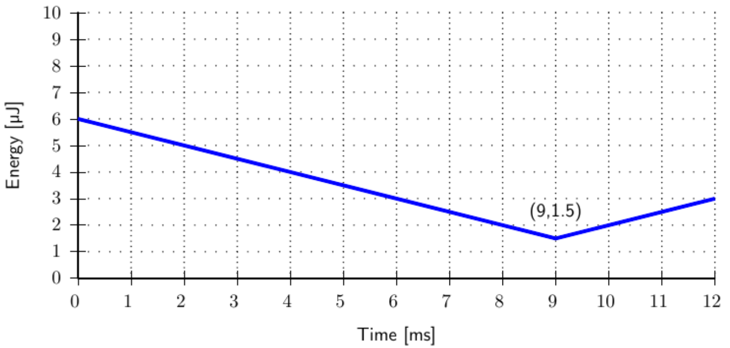


Figure 2: The resulting evolution of the battery charge state $E_{\mathrm{bat}}(t)$.

Task 1 VII

Task 1.2:

- To maximize the energy stored in the battery at the end of each hyper-period ($12ms$), all tasks τ_i have to be executed at the same frequency and this frequency leads to a utilization of 1.0

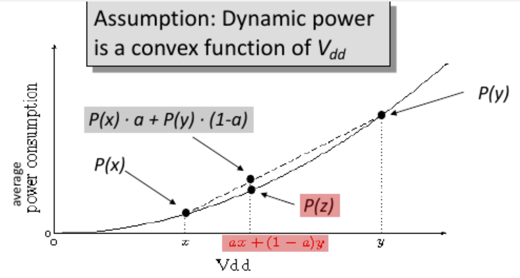
Task 1 VIII

Requirements 1.2:

- *processor utilization* $U_p = \sum_{i=1}^{n} \frac{C_i}{T_i}$
- *the given power consumption* $P_{dynamic}(f)$ *is a strictly convex function* $(f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y))$ *of the frequency* f
- *The EDF scheduling algorithm where deadlines of tasks equal their periods guarantees a feasible schedule as long as the utilization of the processor is* $U \leq 1.0$
  - *this schedulability test is necessary and sufficient, i.e. if one reaches a utilization of exactly* $U = 1.0$, *then it has to be schedulable*

Task 1 IX

Requirements 1.2:



- *The same argument applies if the power is a convex function of the frequency!*

Task 1 X

Solution 1.2:

- *The execution of all tasks with a constant frequency allows for a schedule where the utilization satisfies $U = 1.0$, i.e., the processor is constantly busy*
  - *determine right frequency by looking at schedule:* $\frac{9}{12} = 0.75$
  - *determine right frequency by calculation:*
    - $1 = \frac{\frac{2kCycles}{f}}{6ms} + \frac{\frac{1kCycles}{f}}{4ms} + \frac{\frac{2kCycles}{f}}{12ms} = \frac{\frac{4kCycles+3kCycles+2kCycles}{f}}{12ms} \Leftrightarrow f = \frac{9 \cdot 10^3\, Cycles}{12 \cdot 10^{-3} s} = 0.75 \cdot 10^{3+3} \cdot \frac{1}{s} = 0.75MHz$
- *the optimality of processing at a constant frequency during the whole time follows the same argumentation used to derive the optimal Dynamic Voltage and Frequency Scaling (DVFS).*
  - *Due to the strict convexity of the power consumption $P_{dynamic}(f)$ the increase in the average power consumption of the higher frequency task always dominates the savings in the average power consumption of the lower frequency task execution.*

Task 1 XI

Solution 1.2:

- *the above conditions (constant frequency during execution, processor executing all the time) lead to a schedule, where the maximum frequency of task execution can not be further reduced as the whole time interval where tasks can be executed is filled with execution and all tasks are executed with the same frequency.*

Task 1 XII

Sidenote

- Power is polynomial if you plot over voltage or frequency. So tasks with high frequency will consume polynomially more power than those with low frequency.
- The example from the lecture assumes only two different frequencies, but with more different frequencies the principle is similar.
- The line we see is caused by the fact that there is a certain time share of tasks with the low frequency, namely α, and a certain time share of tasks with the high frequency, namely $(1 - \alpha)$. Depending on how the time share α is composed, more or less time units of the tasks with the high or low frequency flow into the averaged total power.

Task 1 XIII

Sidenote

- If you leave the frequency constant, you don't have this situation with a certain time share of tasks with the high or low frequency, but you lie somewhere on this polynomial power curve, depending on how high or low the selected frequency is.
- If, on the other hand, you have a mixture with a certain time share of the task with the high frequency and the task with the low frequency, then the time share of tasks with the high frequency dominates, which then ensures that you end up somewhere on this line between the task with the high frequency y and the task with the low frequency x, i.e.:
  $\alpha \cdot P(x) + (1 - \alpha) \cdot P(y)$

Task 2

Task 2 I

Power Point Tracking Algorithm

Requirements 2.1:

- $I(U) = G \cdot 1\ \mathrm{A} - \left(\exp\left(\frac{U}{0.1\ \mathrm{V}}\right) - 1\right) \cdot 0.01\ \mathrm{mA}$
- $P = I \cdot U$

Task 2 II

Power Point Tracking Algorithm

Solution 2.1:

Relative Irradiance G	**0.1**	**0.2**	**0.5**	**1.0**
Voltage V [V]	0.7	0.7	0.7	0.7
Current I [mA]				
Power P [mW]				

Task 2 III

Power Point Tracking Algorithm

Solution 2.1:

Relative Irradiance G	**0.1**	**0.2**	**0.5**	**1.0**
Voltage V [V]	0.7	0.7	0.7	0.7
Current I [mA]	89.0	189.0	489.0	989.0
Power P [mW]	62.3	132.3	342.3	692.3

Task 2 I

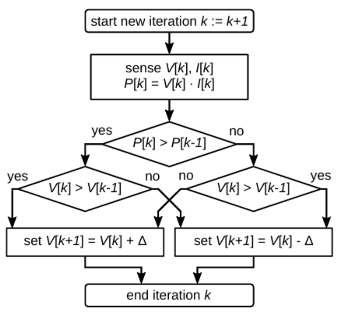
Task 2.2:

- for $G = 0.1$ and $G = 1.0$
- $\triangle = 0.05\,V$
- iteration $k = 1$, $V[0] = 0.7\,V$ and $V[1] = 0.75\,V$

Task 2 II

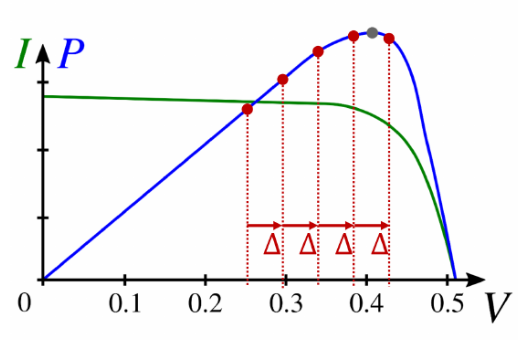
Requirements 2.2:

- $I(U) = G \cdot 1\ \mathrm{A} - \left(\exp\left(\frac{U}{0.1\ \mathrm{V}}\right) - 1\right) \cdot 0.01\ \mathrm{mA}$
- $P = I \cdot U$



Task 2 III

Requirements 2.2:



Task 2 IV

Requirements 2.2:

- $P(V) = V \cdot I(V) = V \cdot G - V \cdot \left(\exp\left(\frac{V}{0.1\ \mathrm{V}}\right) - 1\right) \cdot 10^{-5}$
- *the power P extracted at a given operating point V, calculated as is a concave function of that V*
- *As the algorithm adjusts the operating point in discrete voltage steps $\triangle$, the maximum of the power $P[k]$ observed by the algorithm presents a lower bound of the actual maximum power point P^*.*
  - *Only in the special case where the voltage of the maximum power point is a multiple of $\triangle$, it is matched exactly by the algorithm.*

Task 2 V

Solution 2.2:

$G = 0.1$:

Iteration k	0	1	2	3	4	5	...
Voltage $V[k]$ [V]	0.7	0.75					...
Current $I[k]$ [mA]	89.0						...
Power $P[k]$ [mW]	62.3						...

Task 2 VI

Solution 2.2:

$G = 0.1$:

Iteration k	**0**	**1**	**2**	**3**	**4**	**5**	**...**
Voltage $V[k]$ [V]	0.7	0.75	0.7	0.65	0.7	0.75	...
Current $I[k]$ [mA]	89.0	81.9	89.0	93.4	89.0	81.9	...
Power $P[k]$ [mW]	62.3	61.4	62.3	60.7	62.3	61.4	...

Task 2 VII

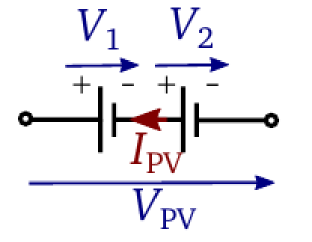
Solution 2.2:

$G = 1.0$:

Iteration k	**0**	**1**	**2**	**3**	**4**	**5**	**...**
Voltage $V[k]$ [V]	0.7	0.75	0.8	0.85	0.9	0.95	...
Current $I[k]$ [mA]	989.0	981.9	970.2	950.9	919.0	866.4	...
Power $P[k]$ [mW]	692.3	736.4	776.2	808.2	827.1	823.1	...

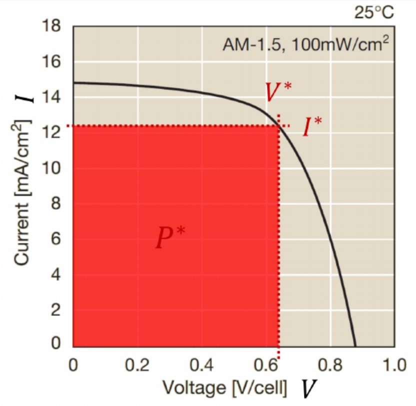
Task 2 VIII

Task 2.3:



Task 2 IX

Requirements 2.3:



Task 2 X

Solution 2.3:

- *find the voltage* V_1^* *of the maximum power point* P_1^* *by numerically solving:*
$$\frac{d}{dU_1} P_1\left(U_1\right) = \frac{d}{dU_1} U_1 \cdot I_1\left(U_1\right) = 0$$
- *cell 1 with* $G = 1.0$: *voltage at the maximum power point:* $U_1^* \approx 0.919\ \mathrm{V}$ *and the maximum power generated:* $P_1^* = U_1^* \cdot I_1\left(U_1^*\right) \approx 829\ \mathrm{mW}$
- *cell 2 with* $G = 0.1$: $U_2^* = 0.712V$ *and* $P_2^* = 62.4mW$
- *we determine an upper bound, i.e., a value which is larger than what can be generated by the partly shaded photovoltaic panel*
- *conditions:* $U_{PV} = U_1 + U_2 \quad$ *and* $\quad I_{PV} = I_1 = I_2$

Task 2 XI

Solution 2.3:

- $$G_1 - \left(\exp\left(\frac{U_1}{0.1}\right) - 1\right) \cdot 10^{-5} = G_2 - \left(\exp\left(\frac{U_2}{0.1}\right) - 1\right) \cdot 10^{-5}$$
$$\Leftrightarrow (G_1 - G_2) \cdot 10^5 = \exp\left(\frac{U_1}{0.1}\right) - \exp\left(\frac{U_2}{0.1}\right)$$
$$\Leftrightarrow \exp\left(\frac{U_1}{0.1}\right) = (G_1 - G_2) \cdot 10^5 + \exp\left(\frac{U_2}{0.1}\right)$$
- *Note that an increase in U_2 leads to an increasing U_1! First we assume $U_2 = 0$:*
- $U_1 = 0.1 \cdot \log\left((G_1 - G_2) \cdot 10^5 + \exp(0)\right) \approx 1.14 \text{ V}$
- $P_{PV} = I_{PV} \cdot (U_1 + U_2) \approx 114.1 \text{ mW} = 0.1A \cdot (1.14V + 0V) \approx 0.1141 \text{ W} = 114.1 \text{ mW}$

Task 2 XII

Solution 2.3:

- *The voltage* $U_1 \approx 1.14V$ *at this operating point is already above* $U_1^* \approx 0.919V$ *the maximum power point of cell 1. Therefore, an increase in* U_1 *(and* U_2*) results in a decrease in the output power for cell 1.*
- *safe upper bound:* $P_{PV}^* \leq 176.5 \text{ mW} = 114.1 \text{ mW} + 62.4 \text{ mW}$
- *We note that this is significantly lower than* $P_1^* \approx 829.0$ *mW that can be generated with a single cell 1 at the same operating point. In addition, it is only slightly better than a situation, where both cells receive the low irradiance, namely* $125mW$

Task 3

Task 3

Application control

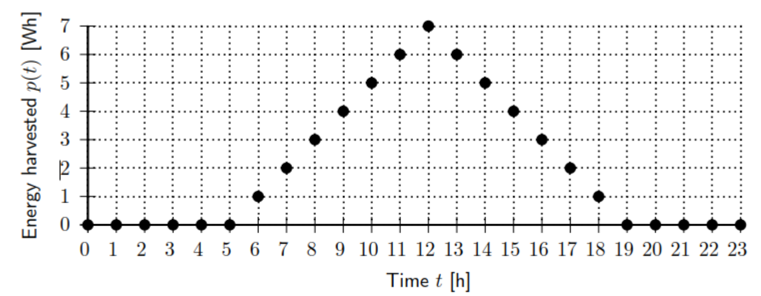
Task 3.1:

Consider the energy harvesting profile p(t) given in Figure 4 that repeats daily. What is the maximum average power u_{max} that can be used by the system?

Task 3

Application control

Solution 3.1:



Task 3

Application control

Solution 3.1:

- *The daily harvested energy is calculated by:* $(1Wh + 2Wh + 3Wh + 4Wh + 5H + 6Wh) * 2 + 7Wh = 49Wh$
- *This results in the following maximum average harvesting power:* $u_{max} = \frac{49Wh}{24h} = 2.04W$

Task 3

Application control

Task 3.2:

Given the knowledge of the daily energy input profile $p(t)$ in Figure 5, calculate the minimal battery size B_{min} such that the used energy satisfies $u(t) = 2$ for every time interval during a day. Complete the diagram in Figure 6 with the daily evolution of the used energy $u(t)$ and the battery charge state $b(t)$ at the beginning of the interval for the found battery size B_min.

Task 3

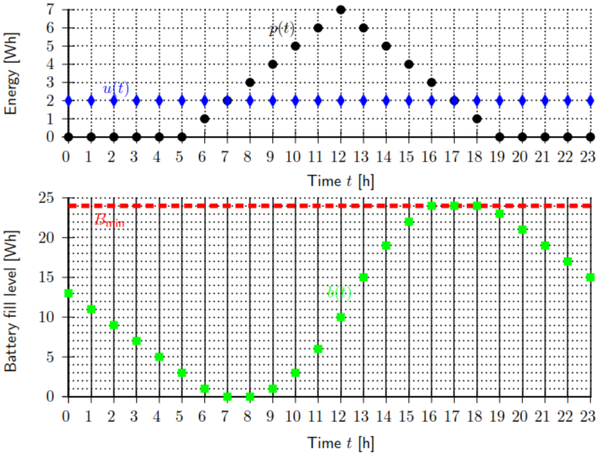
Application control

Solution 3.2:

- *The energy used during night, where the harvested energy is lower than the consumed energy, is:* $1Wh + 11 \cdot 2Wh + 1Wh = 24Wh$
- *We need at least this much energy in our battery to compensate this deficit, meaning* $B \geq 24Wh \Rightarrow B_{min} = 24Wh$.

Task 3 I

Application control



Task 3 II

Application control

Sidenote

- We observe that in intervals $t \in [0, 6]$ and $t \in [18, 23]$ of a day $u(t) > p(t)$ and therefore energy is used from the battery. In intervals $t \in [7, 17]$ the battery is not used and the extra input power of intervals $t \in [8, 16]$ is stored to the battery, with the battery overflowing in interval $t = 16$.