

Energy and Power

Energy Harvesting, Solar Cell Characteristics and Maximum Power Point Tracking, Application Control

Exercise class 8

Presenter:
Jürgen Mattheis

In cooperation with:
Pascal Walter

Based on the lecture of:
Marco Zimmerling

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University of Freiburg, Chair for Embedded Systems



Gliederung

Organisation

Task 1

Task 2

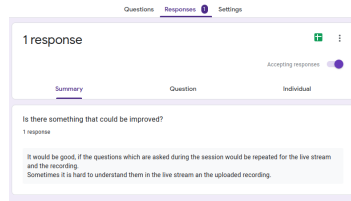
Task 3

Organisation



Organisation I

► feedback for me: <https://forms.gle/f3YN8EFrZ1vsfPoC6>



The screenshot shows the 'Responses' tab of a Google Form. At the top, there are tabs for 'Questions', 'Responses' (which is active and has a '1' next to it), and 'Settings'. Below the tabs, it says '1 response'. To the right of this text is a green icon of a document with a plus sign and a vertical ellipsis menu. Below this is a toggle switch labeled 'Accepting responses' which is currently turned on. Below the toggle are three tabs: 'Summary' (which is active), 'Question', and 'Individual'. Under the 'Summary' tab, the question 'Is there something that could be improved?' is displayed. Below the question, it says '1 response'. The response itself is shown in a light blue box: 'It would be good, if the questions which are asked during the session would be repeated for the live stream and the recording. Sometimes it is hard to understand them in the live stream an the uploaded recording.'

Task 1

Task 1 I

Task 1.1:



<i>Task</i>	τ_1	τ_2	τ_3
<i>Arrival Time</i> [ms]	0	0	0
<i>Period</i> [ms]	6	4	12
<i>Relative Deadline</i> [ms]	6	4	12
<i>Cycles</i> [$\times 10^3$]	2	1	2

$$P_{\text{dynamic}}(f) = \left(\frac{f}{1 \text{ MHz}} \right)^3 \text{ mW}$$

Task 1 II

Solution 1.1:



- ▶ The execution times C_i of tasks τ_i are: $C_1 = \frac{1 \cdot 10^3 \text{ Cycles}}{1 \cdot 10^6 \frac{\text{Cycles}}{\text{s}}} = 2\text{ms}$,
 $C_2 = \frac{1 \cdot 10^3 \text{ Cycles}}{1 \cdot 10^6 \frac{\text{Cycles}}{\text{s}}} = 1\text{ms}$, and $C_3 = \frac{1 \cdot 10^3 \text{ Cycles}}{1 \cdot 10^6 \frac{\text{Cycles}}{\text{s}}} = 2\text{ms}$ ($T = \frac{N}{f}$).
- ▶ Applying EDF schedule, the processor is busy in $[0\text{ms}, 9\text{ms}]$
- ▶ constant input power: $P_{in} = 0.5\mu\text{J}$
- ▶ We use 1MHz for processing tasks, leading to $P_{dynamic}(1\text{MHz}) = 1\text{mW}$
- ▶ This means we consume $1\text{mW} \cdot 1\text{ms} = 1\mu\text{J}$ per millisecond.
- ▶ Since all our tasks take a multiple of 1ms, we know that while processing tasks, our battery empties with $1\mu\text{J}/\text{ms} - 1\mu\text{J}/\text{ms} = -0.5\mu\text{J}/\text{ms}$

Task 1 III

Solution 1.1:

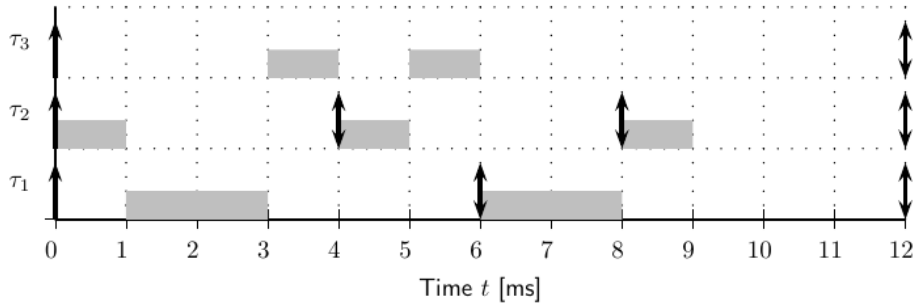


Figure 1: The resulting schedule for tasks τ_i when applying EDF algorithm.

Task 1 IV

Solution 1.1:

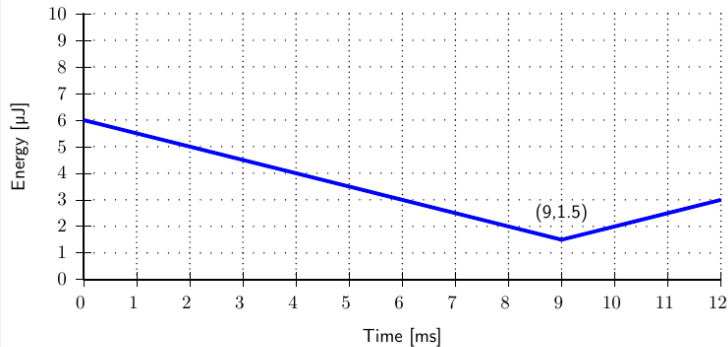


Figure 2: The resulting evolution of the battery charge state $E_{\text{bat}}(t)$.

Task 1 V

Task 1.2:



- ▶ To maximize the energy stored in the battery at the end of each hyper-period ($12ms$), all tasks τ_i have to be executed at the same frequency and this frequency leads to a utilization of 1.0

Task 1 VI

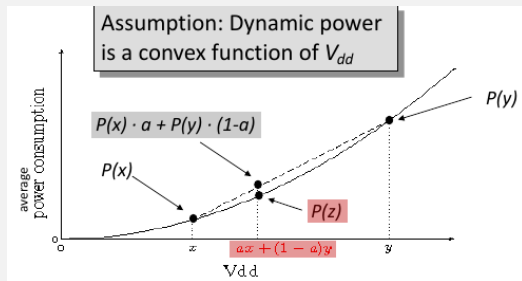
Requirements 1.2:



- ▶ *processor utilization* $U_p = \sum_{i=1}^n \frac{C_i}{T_i}$
- ▶ the given power consumption $P_{\text{dynamic}}(f)$ is a *strictly convex* function of the frequency f
- ▶ The EDF scheduling algorithm where deadlines of tasks equal their periods guarantees a feasible schedule as long as the utilization of the processor is $U \leq 1.0$
 - ▶ this schedulability test is *necessary* and *sufficient*, i.e. if one reaches a utilization of exactly $U = 1.0$, then it has to be schedulable

Task 1 VII

Requirements 1.2:



- The same argument applies if the power is a convex function of the **frequency**!

Task 1 VIII

Solution 1.2:



- ▶ *The execution of all tasks with a constant frequency allows for a schedule where the utilization satisfies $U = 1.0$, i.e., the processor is constantly busy*
 - ▶ *determine right frequency by looking at schedule: $\frac{9}{12} = 0.75$*
 - ▶ *determine right frequency by calculation:*
 - ▶
$$1 = \frac{\frac{2kCycles}{f}}{6ms} + \frac{\frac{1kCycles}{f}}{4ms} + \frac{\frac{2kCycles}{f}}{12ms} = \frac{4kCycles+3kCycles+2kCycles}{12ms} \Leftrightarrow f = \frac{9 \cdot 10^3 Cycles}{12 \cdot 10^{-3}s} = 0.75 \cdot 10^{3+3} \cdot \frac{1}{s} = 0.75MHz$$
- ▶ *the optimality of processing at a constant frequency during the whole time follows the same argumentation used to derive the optimal Dynamic Voltage and Frequency Scaling (DVFS).*

Task 1 IX

Solution 1.2:



- ▶ *Due to the strict convexity of the power consumption $P_{dynamic}(f)$ the increase in the average power consumption of the higher frequency task always dominates the savings in the average power consumption of the lower frequency task execution.*
- ▶ *the above conditions (constant frequency during execution, processor executing all the time) lead to a schedule, where the maximum frequency of task execution can not be further reduced as the whole time interval where tasks can be executed is filled with execution and all tasks are executed with the same frequency.*

Task 2



Task 2 I

Power Point Tracking Algorithm

Requirements 2.1:

- ▶ $I(U) = G \cdot 1 \text{ A} - \left(\exp\left(\frac{U}{0.1 \text{ V}}\right) - 1 \right) \cdot 0.01 \text{ mA}$
- ▶ $P = I \cdot U$

Task 2 II

Power Point Tracking Algorithm

Solution 2.1:



Relative Irradiance G	0.1	0.2	0.5	1.0
Voltage V [V]	0.7	0.7	0.7	0.7
Current I [mA]	89.0	189.0	489.0	989.0
Power P [mW]	62.3	132.3	342.3	692.3

Task 2 I

Task 2.2:

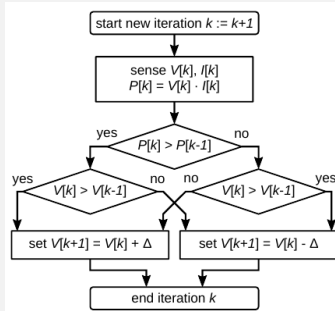


- ▶ for $G = 0.1$ and $G = 1.0$
- ▶ $\Delta = 0.05V$
- ▶ iteration $k = 1$, $V[0] = 0.7V$ and $V[1] = 0.75V$

Task 2 II

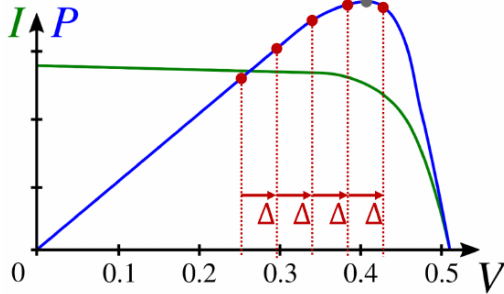
Requirements 2.2:

- ▶ $I(U) = G \cdot 1 \text{ A} - \left(\exp\left(\frac{U}{0.1 \text{ V}}\right) - 1 \right) \cdot 0.01 \text{ mA}$
- ▶ $P = I \cdot U$



Task 2 III

Requirements 2.2:



Task 2 IV

Requirements 2.2:



- ▶ $P(V) = V \cdot I(V) = V \cdot G - V \cdot (\exp(\frac{V}{0.1V}) - 1) \cdot 10^{-5}$
- ▶ *the power P extracted at a given operating point V , calculated as is a concave function of that V*
- ▶ *As the algorithm adjusts the operating point in discrete voltage steps Δ , the maximum of the power $P[k]$ observed by the algorithm presents a lower bound of the actual maximum power point P^* .*
 - ▶ *Only in the special case where the voltage of the maximum power point is a multiple of Δ , it is matched exactly by the algorithm.*

Task 2 V

Solution 2.2:



$G = 0.1$:

Iteration k	0	1	2	3	4	5	...
Voltage $V[k]$ [V]	0.7	0.75	0.7	0.65	0.7	0.75	...
Current $I[k]$ [mA]	89.0	81.9	89.0	93.4	89.0	81.9	...
Power $P[k]$ [mW]	62.3	61.4	62.3	60.7	62.3	61.4	...

Task 2 VI

Solution 2.2:

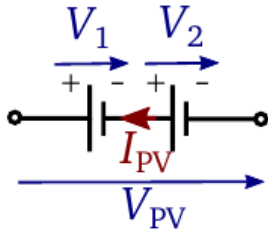


$G = 1.0$:

Iteration k	0	1	2	3	4	5	...
Voltage $V[k]$ [V]	0.7	0.75	0.8	0.85	0.9	0.95	...
Current $I[k]$ [mA]	989.0	981.9	970.2	950.9	919.0	866.4	...
Power $P[k]$ [mW]	692.3	736.4	776.2	808.2	827.1	823.1	...

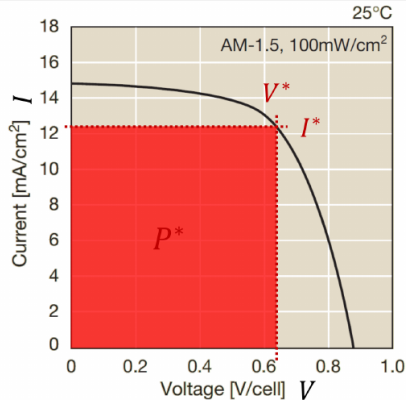
Task 2 VII

Task 2.3:



Task 2 VIII

Requirements 2.3:



Task 2 IX

Solution 2.3:



- ▶ find the **voltage** V_1^* of the **maximum power point** P_1^* by numerically solving:

$$\frac{d}{dU_1} P_1(U_1) = \frac{d}{dU_1} U_1 \cdot I_1(U_1) = 0$$
- ▶ cell 1 with $G = 1.0$: voltage at the maximum power point: $U_1^* \approx 0.919 \text{ V}$ and the maximum power generated: $P_1^* = U_1^* \cdot I_1(U_1^*) \approx 829 \text{ mW}$
- ▶ cell 2 with $G = 0.1$: $U_2^* = 0.712 \text{ V}$ and $P_2^* = 62.4 \text{ mW}$
- ▶ we determine an upper bound, i.e., a value which is larger than what can be generated by the partly shaded photovoltaic panel
- ▶ conditions: $U_{PV} = U_1 + U_2$ and $I_{PV} = I_1 = I_2$

Task 2 X

Solution 2.3:



$$\blacktriangleright G_1 - \left(\exp\left(\frac{U_1}{0.1}\right) - 1 \right) \cdot 10^{-5} = G_2 - \left(\exp\left(\frac{U_2}{0.1}\right) - 1 \right) \cdot 10^{-5}$$

$$\Leftrightarrow (G_1 - G_2) \cdot 10^5 = \exp\left(\frac{U_1}{0.1}\right) - \exp\left(\frac{U_2}{0.1}\right)$$

$$\Leftrightarrow \exp\left(\frac{U_1}{0.1}\right) = (G_1 - G_2) \cdot 10^5 + \exp\left(\frac{U_2}{0.1}\right)$$

\blacktriangleright Note that an increase in U_2 leads to an increasing U_1 ! First we assume $U_2 = 0$:

$$\blacktriangleright U_1 = 0.1 \cdot \log\left((G_1 - G_2) \cdot 10^5 + \exp(0)\right) \approx 1.14 \text{ V}$$

$$\blacktriangleright P_{PV} = I_{PV} \cdot (U_1 + U_2) \approx 114.1 \text{ mW} = 0.1 \text{ A} \cdot (1.14 \text{ V} + 0 \text{ V}) \approx 0.1141 \text{ W} = 114.1 \text{ mW}$$

Task 2 XI

Solution 2.3:



- ▶ The voltage $U_1 \approx 1.14V$ at this operating point is already above $U_1^* \approx 0.919V$ the maximum power point of cell 1. Therefore, an increase in U_1 (and U_2) results in a decrease in the output power for cell 1.
- ▶ safe upper bound: $P_{PV}^* \leq 176.5 \text{ mW} = 114.1 \text{ mW} + 62.4 \text{ mW}$
- ▶ We note that this is significantly lower than $P_1^* \approx 829.0 \text{ mW}$ that can be generated with a single cell 1 at the same operating point. In addition, it is only slightly better than a situation, where both cells receive the low irradiance, namely 125mW

Task 3

Task 3

Application control

Task 3.1:

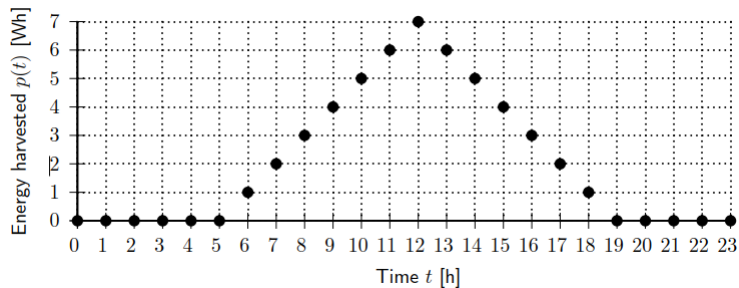


Consider the energy harvesting profile $p(t)$ given in Figure 4 that repeats daily. What is the maximum average power u_{max} that can be used by the system?

Task 3

Application control

Solution 3.1:



Task 3

Application control

Solution 3.1:



- ▶ *The daily harvested energy is calculated by:*
$$(1Wh + 2Wh + 3Wh + 4Wh + 5H + 6Wh) * 2 + 7Wh = 49Wh$$
- ▶ *This results in the following maximum average harvesting power:*
$$u_{max} = \frac{49Wh}{24h} = 2.04W$$

Task 3

Application control

Task 3.2:



Given the knowledge of the daily energy input profile $p(t)$ in Figure 5, calculate the minimal battery size B_{min} such that the used energy satisfies $u(t) = 2$ for every time interval during a day. Complete the diagram in Figure 6 with the daily evolution of the used energy $u(t)$ and the battery charge state $b(t)$ at the beginning of the interval for the found battery size B_{min} .

Task 3

Application control

Solution 3.2:



- ▶ The energy used during night, where the harvested energy is *lower* than the consumed energy, is: $1\text{Wh} + 11 \cdot 2\text{Wh} + 1\text{Wh} = 24\text{Wh}$
- ▶ We need at least this much energy in our battery to compensate this deficit, meaning $B \geq 24\text{Wh} \Rightarrow B_{\min} = 24\text{Wh}$.

Task 3

Application control

