Energy and Power

Dynamic Voltage Scaling and Dynamic Power Management, Dynamic Voltage Scaling for Real-Time Tasks, Dynamic Power Management

Exercise class 7

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Based on the lecture of:
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Gliederung

Organisation

Overview

Task 1

Task 2

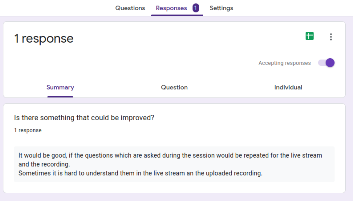
Task 3

Literature

Organisation

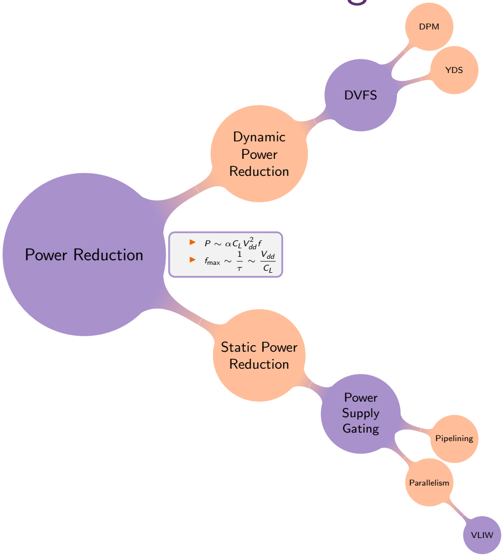
Organisation I

- feedback for me: `https://forms.gle/f3YN8EFrZ1vsfPoC6`



Overview

Overview Scheduling



Task 1

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Task 1.1:

What does the constant term in the power consumption $P(f)$ represent? Where does this term come from?

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.1:

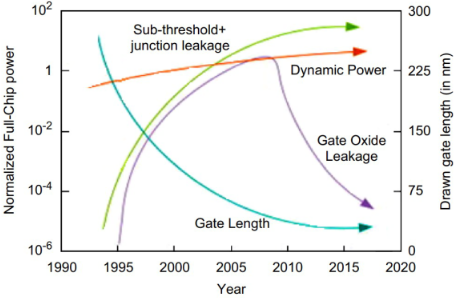
$$P(f) = \left(10\left(\frac{f}{100\,\mathrm{MHz}}\right)^3 + 20\right)\ \mathrm{mW}$$

- *The constant term in power consumption represents the minimal power the processor consumes while on.*
- *That is, power that is drawn even though no gates are being switched (the processor is idling).*
- *As seen in the lecture, a non-negligible threshold voltage V_t and junction leakage are significant contributors to the minimal power. Gate-oxide leakage also contributes to the static power but it is a lot smaller than the other two components.*

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.1:



[J. Xue, T. Li, Y. Deng, Z. Yu, Full-chip leakage analysis for 65 nm CMOS technology and beyond, Integration VLSI J. 43 (4) (2010) 353–364]

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Task 1.2:

The energy consumption to execute C cycles is $\frac{C}{f} \cdot P(f)$. There is a critical frequency f_{crit} between 50 MHz and 1000 MHz at which the energy consumption per cycle $\frac{P(f)}{f}$ is minimized. What is the critical frequency f_{crit} of this processor?

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.2:

- *We want to minimize the energy consumption per cycle* ($\frac{P(f)}{f}$) *with respect to the frequency* f. *By doing this, we will find* f_{crit}.
- *First, we define* $f_n := \frac{f}{100MHz}$ *to be the normalized frequency. We are doing this for easier calculation.*
- *We then define the energy consumption per cycle as* $Q(f_n) := \frac{P(f_n)}{f_n} = \frac{10f_n^3+20}{f_n} = 10f_n^2 + \frac{20}{f_n}$
- *The first term decreases, the second term increases when reducing* f_n*!*
- *The latter is a result of the increasing execution time of a cycle when decreasing the frequency, the processor has to be on for a longer time.*

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.2:

- *Calculating the derivative of* $Q(f_n)$ *we obtain* $20f_n - \frac{20}{f_n^2}$
- *This equals 0 for* $f_n = 1$, *meaning* $\frac{f_{crit}}{100MHz} = 1$. *Therefore we know, that the critical frequency is* $f_{crit} = 100MHz$.

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Task 1.3:

When the processor is idle at frequency f_{min} for t seconds, the consumed energy is $P(f_{min}) \cdot t$. The break-even time is defined as the minimum idle interval, for which it is worthwhile for the processor to go into sleep mode. What is the break-even time of the processor?

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.3:

- *To break even we need to consider both the costs for staying in a certain mode and the costs of switching between modes.*
- *The point we search means, that the consumed energy for switchting to sleep, staying in sleep and switching back to run needs to be at most* $P(f_{min}) \cdot t$.
- *Since we do not consider switching times, and we assume that modes can be switched without any time passing, we can neglect that time factor.*
- *In total, this means we get the following inequality:*
  $P(f_{min}) \cdot t \geq E_{RunToSleep} + E_{sleep} + E_{SleepToRun}$

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.3:

- $P(f_{min}) \cdot t \geq E_{RunToSleep} + E_{sleep} + E_{SleepToRun}$
- *We know that the energy consumption from switching to run to sleep is 0, same for the energy consumption when idling in sleep. The energy consumption for switching from sleep to run is* $30\mu J$.
- *Thus, the above inequality is*

  $P(f_{min}) \cdot t \geq 30\mu J$

  $t \geq \frac{30\mu J}{(10*0.5^3+20)mW}$

  $t \geq 1.412ms$

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.3:

- *The break-even time is the minimal time for which the inequality holds and therefore* $t_{break\text{-}even} = 1.412ms$.

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Task 1.4:

A workload-conserving schedule is defined as a schedule that is always executing a job when the ready queue is not empty. For the three jobs above, provide the workload-conserving schedule that minimizes the energy consumption without violating the timing constraints. For this subquestion, all tasks are executed at critical frequency $f_{crit} = 100MHz$. What is the energy consumption of this schedule?

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.4:

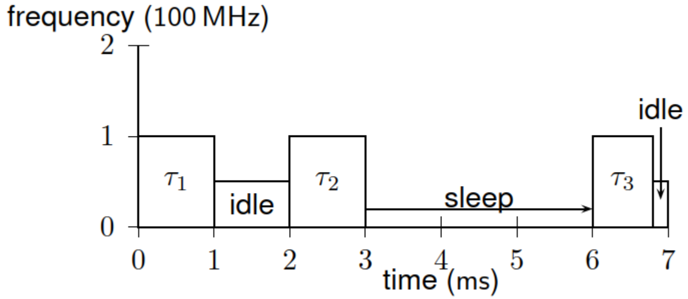
	arrival time	deadline	execution cycles
τ_1	0 ms	2 ms	100000
τ_2	2 ms	6 ms	100000
τ_3	6 ms	7 ms	80000

- *For this question it is important to check if the processor should switch to sleep mode between tasks or not. Frequency switching has negligible overhead, meaning when not executing a task we can let the processor idle with frequency f_{min}.*
- *This means we can use the break-even point as calculated in task (c).*
- *The execution times for τ_1, τ_2 and τ_3 are $1ms, 1ms$ and $0.8ms$ respectively.*

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.4:



- *The energy consumption of the schedule is*

$$E = \frac{C_{\tau_1}}{f_{crit}} \cdot P(f_{crit}) + t_{idle,1} \cdot P(f_{min}) + \frac{C_{\tau_2}}{f_{crit}} \cdot P(f_{crit}) + E_{sleep} + E_{modechange} + \frac{C_{\tau_3}}{f_{crit}} \cdot P(f_{crit})$$

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.4:

- *Plugging in the values yields:* $E = 0.001s \cdot 30mW + 0.001s \cdot 21.25mW + 0.001s \cdot 30mW + 0J + 30J + 0.0008s \cdot 30mW + 0.0002s \cdot 21.25mW = 0.1395mJ$

Sidenote

Note that in the time interval [1, 2] we do not go into sleep mode as the interval is smaller than the break-even time. During the idle time intervals [1, 2] and [6.8, 7], we select the minimum frequency f_{min}. Since no task is executed then, this choice minimizes power consumption.

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Task 1.5:

Is there another workload-conserving schedule without timing constraints violations for the three jobs that has a lower energy consumption than the schedule in (d)? There are no restrictions at what frequency tasks have to be executed. If so, provide the schedule, otherwise prove the optimality of the schedule in (d).

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

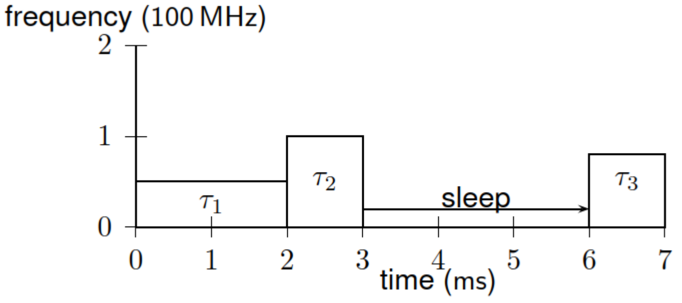
Solution 1.5:

- *Yes! The idea is to use the convex nature of the power consumption to slow down execution of tasks* τ_1 *and* τ_3 *to avoid idle times after their executions.*
- *Even though we execute tasks below the critical frequency, the overall consumption is lower.*
- *We calculate the frequencies for* τ_1 *and* τ_3 *to close the idle gaps. Thus, the new frequencies are* $f_{\tau_1} = \frac{C_{\tau 1}}{t_{\tau 1}}$ *and* $f_{\tau_3} = \frac{C_{\tau 3}}{t_{\tau 3}}$ *where* $t_{\tau_1} = 2ms$ *and* $t_{\tau_3} = 1ms$*.*
- $f_{\tau_1} = 50MHz$ *and* $f_{\tau_2} = 80MHz$

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.5:



- *The energy consumption of the schedule is*
  $E = t_{\tau_1} \cdot P(f_{\tau_1}) + \frac{C_{\tau_2}}{f_{crit}} \cdot P(f_{crit}) + E_{sleep} + E_{modechange} + t_{\tau_3} \cdot P(f_{\tau_3})$

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.5:

- *Plugging in the values yields:*
  $E = 0.002s \cdot 21.25mW + 0.001s \cdot 30mW + 0J + 30\mu J + 0.001s \cdot 25.12mW = 0.12762mJ$

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Task 1.6:

Does a schedule for the three jobs without timing constraints violations exist that is not workload-conserving but consumes less energy than the optimal workload-conserving schedule? If so, provide the schedule, otherwise prove the optimality of the workload-conserving schedules.

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

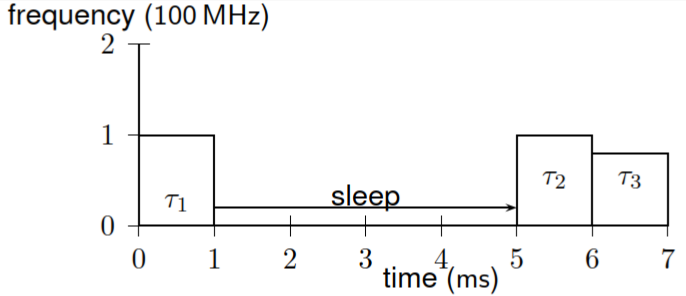
Solution 1.6:

- *Yes! The idea is to batch the inactive time into one block that the processor will spend in sleep mode and execute τ_1 with critical frequency.*
- *This shows: Workload-conserving strategies are not necessarily the best!*
- *By using this strategie, the processor sleeps a very long time, as we are allowed to delay τ_2, saving energy.*

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.6:



- *The energy consumption of the schedule is*
  $E = \frac{C_{\tau 1}}{f_{crit}} \cdot P(f_{crit}) + E_{sleep} + E_{mode\ change} + \frac{C_{\tau 2}}{f_{crit}} \cdot P(f_{crit}) + t_{\tau_3} \cdot P(f_{\tau_3})$

Task 1

Dynamic Voltage Scaling and Dynamic Power Management

Solution 1.6:

- *Plugging in the values yields:*
  $E = 0.001s \cdot 30mW + 0J + 30\mu J + 0.001s \cdot 30mW + 0.001s \cdot 25.12mW = 0.115120mJ$

Task 2

Task 2 I

Task 2.1:

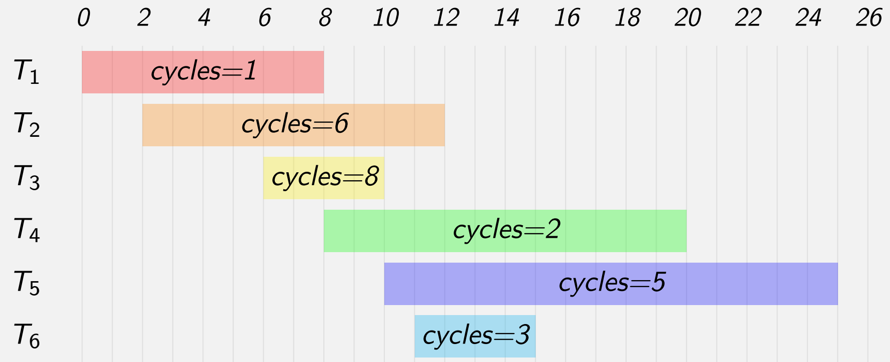
Job ID	1	2	3	4	5	6
arrival time (ms)	0	2	6	8	10	11
absolute deadline (ms)	8	12	10	20	25	15
cycles ($\times 10^3$)	1	6	8	2	5	3

Requirements 2.1:

- $V'([z, z']) = \{v_i \in V : z \leq a_i < d_i \leq z'\}$
- $G([z, z']) = \sum_{v_i \in V'([z,z'])} cycles_i / (z' - z)$

Task 2 I

Solution 2.1:



Task 2 II

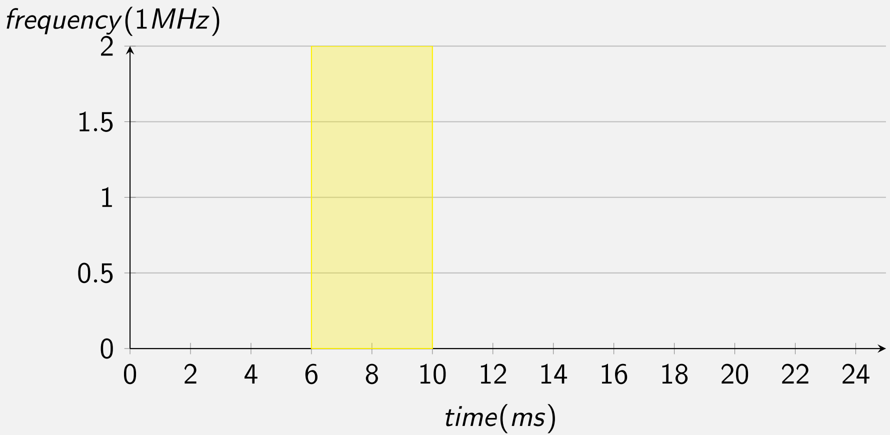
Solution 2.1:

- $G([0,8]) = \frac{1}{8} = 0.125$
- $G([0,12]) = \frac{1+6+8}{12} = 1.25$
- $G([0,10]) = \frac{1+8}{10} = 0.9$
- $G([0,20]) = \frac{1+6+8+2+3}{20} = 1$
- $G([0,25]) = \frac{1+6+8+2+3+5}{25} = 1$
- $G([0,15]) = \frac{1+6+8+3}{15} = 1.2$
- $G([2,12]) = \frac{6+8}{10} = 1.4$
- $G([2,10]) = \frac{8}{8} = 1$
- $G([2,20]) = \frac{6+8+2+3}{18} = 1.06$
- $G([2,25]) = \frac{6+8+2+3+5}{23} = 1.04$
- $G([2,15]) = \frac{6+8+3}{13} = 1.31$
- $\boxed{G([6,10]) = \frac{8 \cdot 10^3 cycles}{4 \cdot 10^{-3} s} = 2MHz}$,

  $C_3 = \frac{8 \cdot 10^3 cycles}{2 \cdot 10^6 \frac{cycles}{s}} = 4ms$
- $G([6,20]) = \frac{8+2+3}{14} = 0.93$
- $G([6,25]) = \frac{8+2+3+5}{19} = 0.95$
- $G([6,15]) = \frac{8+3}{9} = 1.22$
- $G([8,20]) = \frac{2+3}{12} = 0.42$
- $G([8,25]) = \frac{2+3+5}{17} = 0.59$
- $G([8,15]) = \frac{3}{7} = 0.43$
- $G([10,25]) = \frac{5+3}{15} = 5.3$
- $G([10,15]) = \frac{3}{5} = 0.6$
- $G([11,15]) = \frac{3}{4} = 0.75$

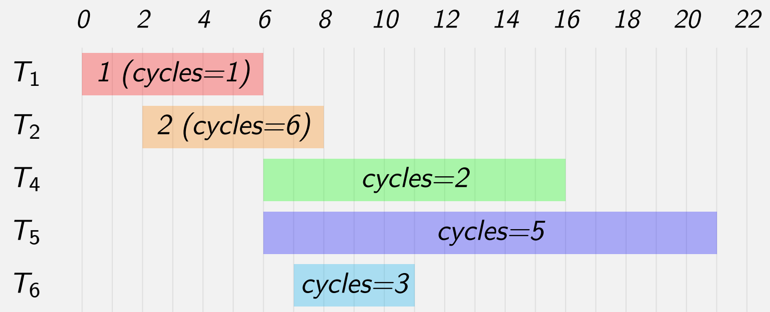
Task 2 III

Solution 2.1:



Task 2 IV

Solution 2.1:



Task 2 V

Solution 2.1:

- $G([0,6]) = \frac{1}{6} = 0.17$
- $G([0,8]) = \frac{1+6}{8} = 0.875$
- $G([0,16]) = \frac{1+6+2+3}{16} = 0.75$
- $G([0,21]) = \frac{1+6+2+5+3}{21} = 0.81$
- $G([0,11]) = \frac{1+6+3}{11} = 0.91$
- $G([2,8]) = \frac{6}{6} = 1$
- $G([2,16]) = \frac{6+2+3}{14} = 0.79$
- $G([2,21]) = \frac{6+2+5+3}{19} = 0.84$

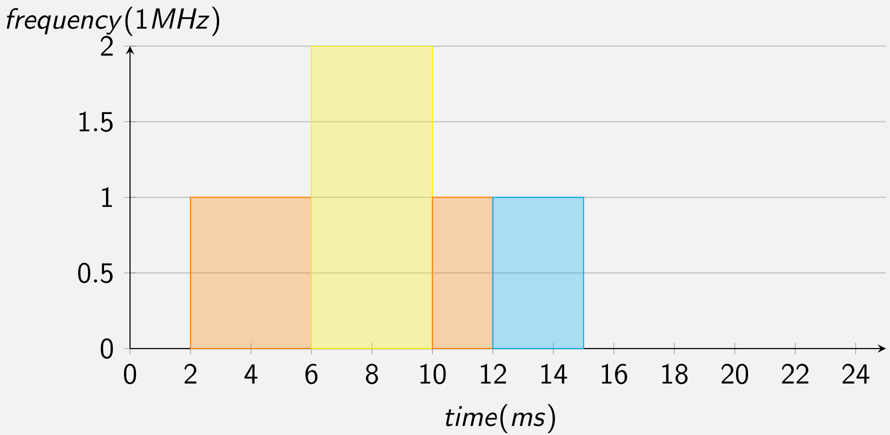
- $\boxed{G([2,11]) = \frac{6+3 \cdot 10^3 cycles}{9 \cdot 10^{-3} s} = 1MHz}$,

  $C_{2,6} = \frac{9 \cdot 10^3 cycles}{1 \cdot 10^6 \frac{cycles}{s}} = 9ms,\ d_2 = 12 < 15 = d_6 \Rightarrow$

  Task 2 first
- $G([6,16]) = \frac{2+3}{10} = 0.5$
- $G([6,21]) = \frac{2+5+3}{15} = 0.67$
- $G([6,11]) = \frac{3}{5} = 0.6$
- $G([7,11]) = \frac{3}{4} = 0.75$

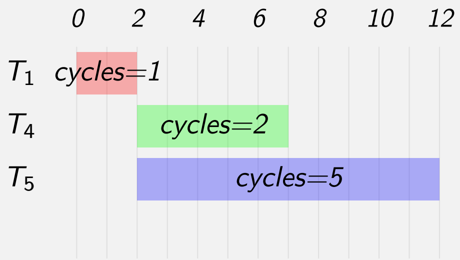
Task 2 VI

Solution 2.1:



Task 2 VII

Solution 2.1:



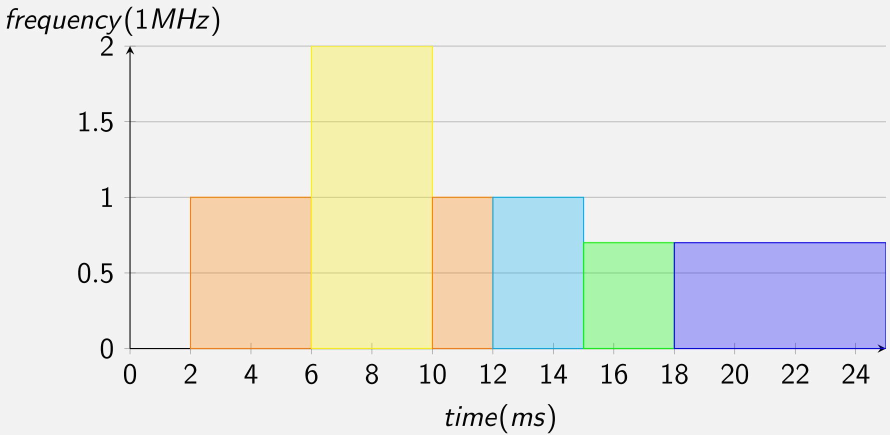
Task 2 VIII

Solution 2.1:

- $G([0,2]) = \frac{1}{2} = 0.5$
- $G([0,7]) = \frac{1+2}{7} = 0.43$
- $G([0,12]) = \frac{1+2+5}{12} = 0.67$
- $G([2,7]) = \frac{2}{5} = 0.4$
- $\boxed{G([2,12]) = \frac{2 + 5 \cdot 10^3 cycles}{10 \cdot 10^{-3} s} = 0.7MHz}$, $C_{4,5} = \frac{7 \cdot 10^3 cycles}{0.7 \cdot 10^6 \frac{cycles}{s}} = 10ms$, $d_4 = 20 < 25 = d_5 \Rightarrow \textit{Task 4 first}$

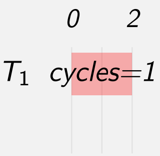
Task 2 IX

Solution 2.1:



Task 2 X

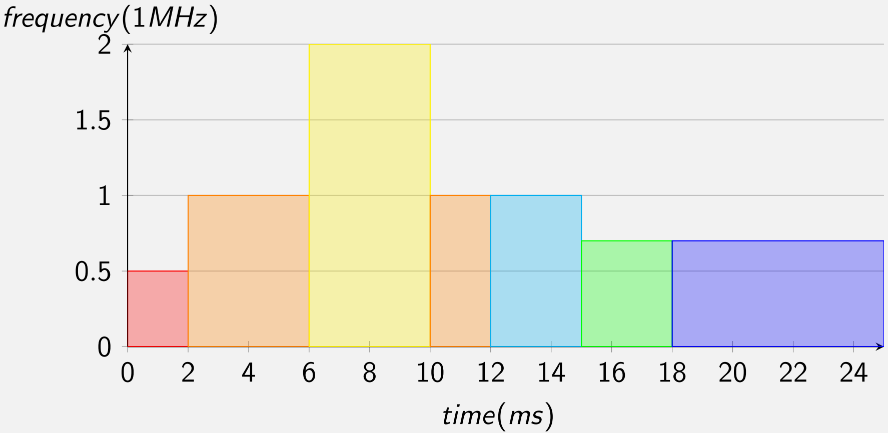
Solution 2.1:



- $\boxed{G([0,2]) = \frac{1 \cdot 10^3 cycles}{2 \cdot 10^{-3} s} = 0.5 MHz}$, $C_1 = \frac{1 \cdot 10^3 cycles}{0.5 \cdot 10^6 \frac{cycles}{s}} = 2ms$

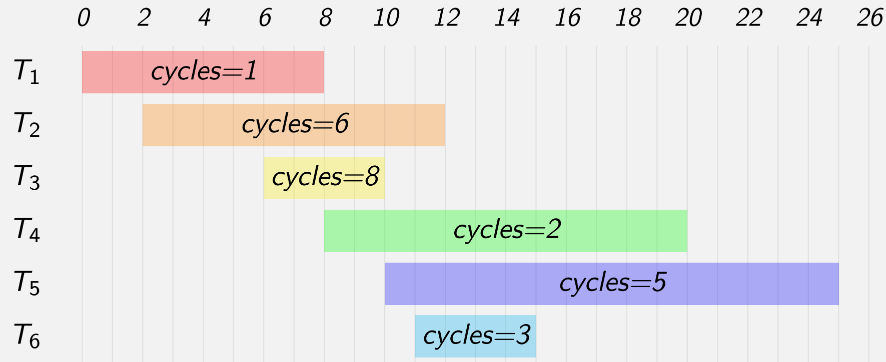
Task 2 I

Solution 2.1:



Task 2 I

Solution 2.2:



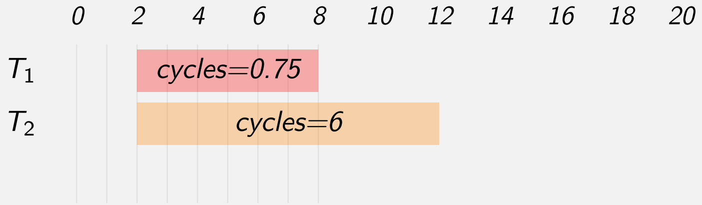
Task 2 II

Solution 2.2:

- $$\boxed{G([0,8]) = \frac{1}{8} = 0.125}$$

Task 2 III

Solution 2.2:



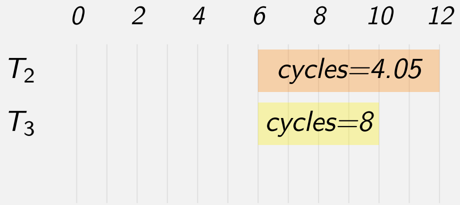
Task 2 IV

Solution 2.2:

- $G([2,8]) = \frac{1-0.125\cdot 2}{6} = 0.125$
- $\boxed{G([2,12]) = \frac{(1-0.125\cdot 2)+6}{10} = 0.675}$
- $d_1 = 8 > 12 = d_2$, *Task 1 has ealier dealine (EDF)*
- $T_1 = \frac{0.75}{0.675} \approx 1.11, d_1^* = 2 + 1.11 = 3.11$

Task 2 V

Solution 2.2:



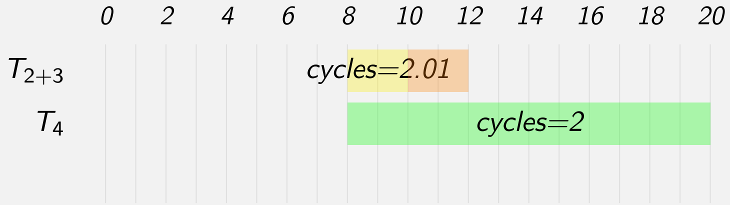
Task 2 VI

Solution 2.2:

- $G([6,10]) = \frac{6-(0.675\cdot(4-1.111))}{4} = 1.01$
- $\boxed{G([6,12]) = \frac{6-(0.675\cdot(4-1.111))+8}{6} = 2.01}$
- $d_2 = 12 > 10 = d_3$, *Task 3 has ealier dealine (EDF)*
- $T_3 = \frac{8}{2.01} \approx 3.98, d_3^* = 6 + 3.98 \approx 10$

Task 2 VII

Solution 2.2:



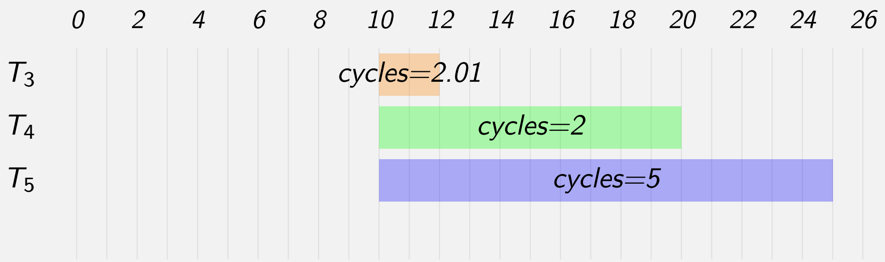
Task 2 VIII

Solution 2.2:

- $G([8,10]) = \frac{8-2.01\cdot 2}{2} \approx 1.99$
- $\boxed{G([8,12]) = \dfrac{6-0.675\cdot(4-1.111)+8-2.01\cdot 2}{4} \approx 2.01}$
- $G([8,20]) = \frac{6-0.675\cdot(4-1.111)+8-2.01\cdot 2+2}{12} \approx 0.84$

Task 2 IX

Solution 2.2:



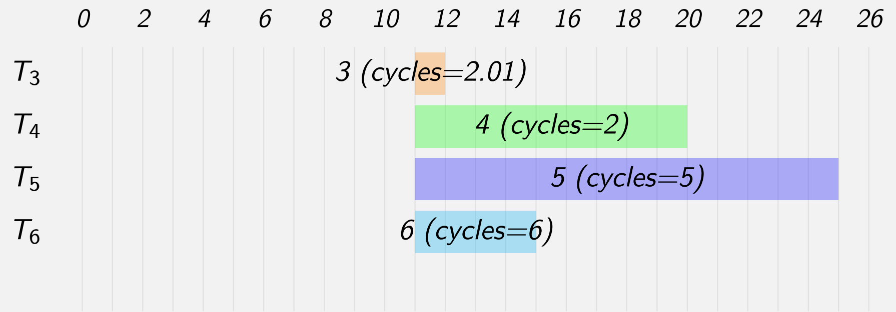
Task 2 X

Solution 2.2:

- $\boxed{G([10,12]) = \frac{6-0.675\cdot(4-1.111)}{2} \approx 2.02}$ *(rounding error)*
- $G([10,20]) = \frac{6-0.675\cdot(4-1.111)+2}{10} \approx 0.6$
- $G([10,25]) = \frac{6-0.675\cdot(4-1.111)+2+5}{15} \approx 0.74$

Task 2 XI

Solution 2.2:



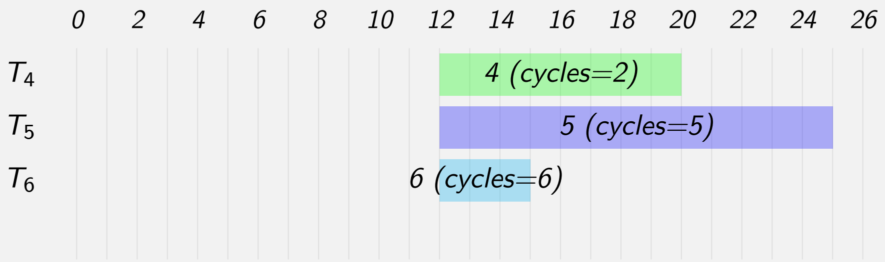
Task 2 XII

Solution 2.2:

- $\boxed{G([11,12]) = \frac{6 - 0.675 \cdot (4 - 1.111) - 1 \cdot 2.01}{1} \approx 2.04}$ *(rounding error)*
- $G([11,15]) = \frac{6-0.675\cdot(4-1.111)-1\cdot2.01+3}{4} \approx 1.26$
- $G([11,20]) = \frac{6-0.675\cdot(4-1.111)-1\cdot2.01+3+2}{9} \approx 0.78$
- $G([11,25]) = \frac{6-0.675\cdot(4-1.111)-1\cdot2.01+3+5+2}{14} \approx 0.86$

Task 2 XIII

Solution 2.2:



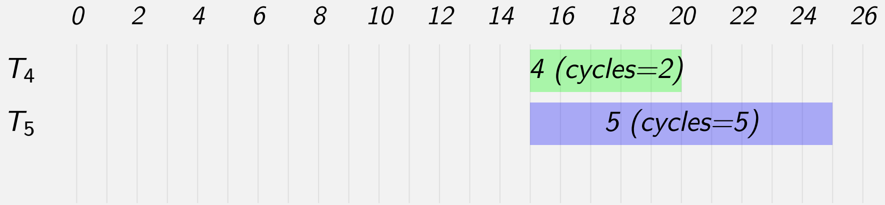
Task 2 XIV

Solution 2.2:

- $\boxed{G([12,15]) = \frac{3}{3} \approx 1}$
- $G([12,20]) = \frac{3+2}{8} \approx 0.63$
- $G([12,25]) = \frac{3+2+5}{13} \approx 0.62$

Task 2 XV

Solution 2.2:



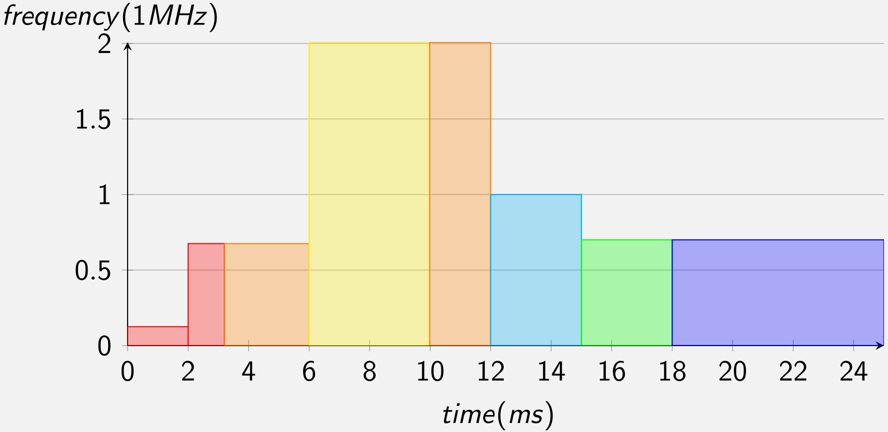
Task 2 XVI

Solution 2.2:

- $G([15,20]) = \frac{2}{5} \approx 0.4$
- $\boxed{G([15,25]) = \frac{2+5}{10} \approx 0.7}$
- $d_4 = 20 < 25 = d_5$, *Task 4 has ealier dealine (EDF)*

Task 2 XVII

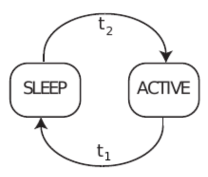
Solution 2.2:



Task 3

Task 3 I

Task 3.1:



Task 3 II

Solution 3.1:

First we express the life time L in terms of the maximal number of task executions N_{max} and period T :

$$L = N_{\text{max}} \cdot T$$

Then we set up an equation for the total energy that is consumed during the lifetime L :

$$\begin{aligned}
E &= N_{max} \cdot (t_{task} \cdot P_{active} + (T - t_{task}) \cdot P_{sleep}) \\
&= N_{max} \cdot \left(t_{task} \cdot P_{active} + \left(\frac{L}{N_{max}} - t_{task}\right) \cdot P_{sleep}\right) \\
&= N_{max} \cdot (t_{task} \cdot P_{active} - t_{task} \cdot P_{sleep}) + L \cdot P_{sleep}
\end{aligned}$$

Task 3 III

Solution 3.1:

Solving for N_{max} results in:

$$N_{\text{max}} = \frac{E - L * P_{sleep}}{t_{task} * (P_{active} - P_{sleep})} = 4.87 * 10^6$$

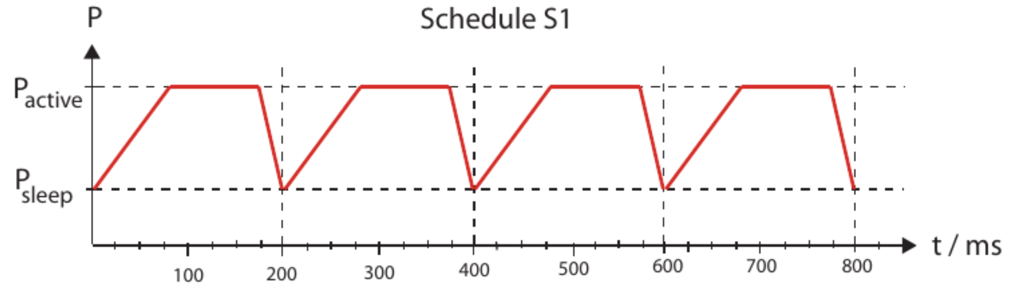
To support up to N_{max} executions, we can deduce that for T the following must hold:

$$T \geq \frac{L}{N_{\text{max}}} = 32.39 \text{ s}$$

Task 3 I

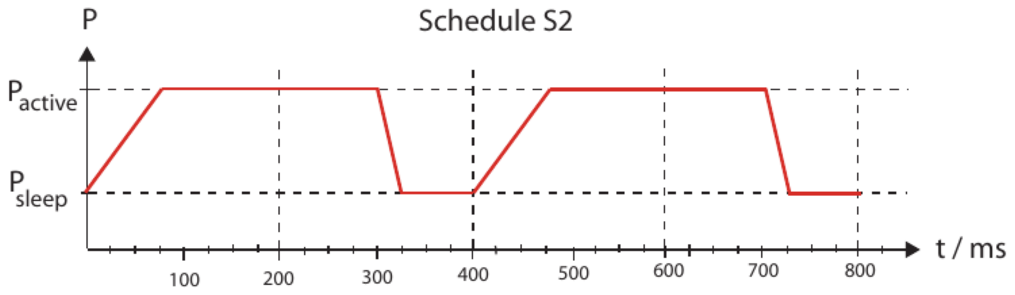
Solution 3.2:

Function $P(t)$ for schedules $\mathrm{S1}$ *and* $\mathrm{S2}$*, for* $t \in [0, 800]$ *is shown in the following diagram:*



Task 3 II

Solution 3.2:



Task 3 I

Solution 3.3:

The energy consumption of the Schedule $\mathrm{S}2$*, has a periodicity of* $2 \cdot T$*. Therefore we compute the energy difference for the first two periods and then average those values to get the average energy difference per period* T*. In the 1st period:*

$$\Delta E_1 = E_{S1} - E_{S2} = t_1 \cdot \frac{P_{sleep} + P_{active}}{2} - t_1 \cdot P_{active}$$

In the 2nd period:

$$\Delta E_2 = E_{S1} - E_{S2} = t_2 \cdot \frac{P_{active} + P_{sleep}}{2} - t_2 \cdot P_{sleep}$$

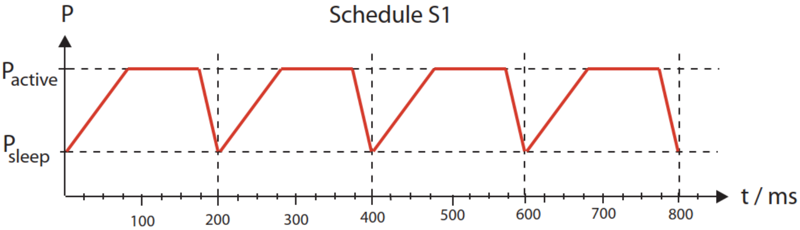
Task 3 II

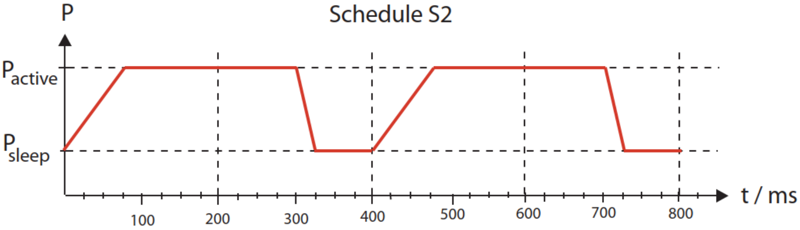
Solution 3.3:

On average, the energy difference per period between $\mathrm{S1}$ *and S2 is:*

$$\Delta E = \frac{\Delta E_1 + \Delta E_2}{2} = \frac{(t_2 - t_1)\,(P_{active} - P_{sleep})}{4} \approx 13.875\mu\mathrm{J}$$

Task 3 III

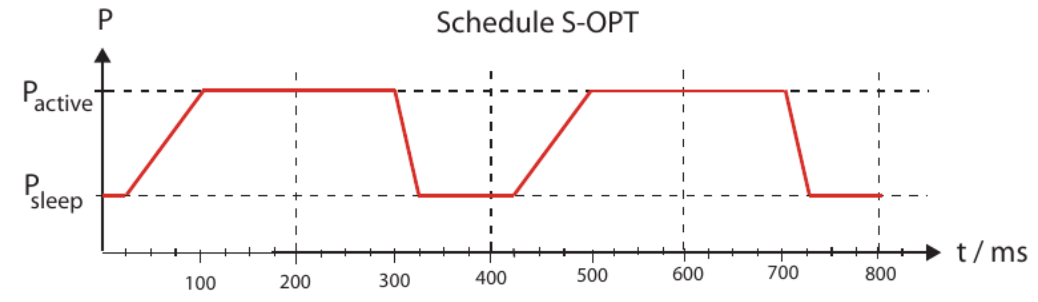

P
Schedule S1
P_{active}
P_{sleep}
t / ms
100
200
300
400
500
600
700
800


P
Schedule S2
P_{active}
P_{sleep}
t / ms
100
200
300
400
500
600
700
800

Task 3 I

Solution 3.4:

Function $P(t)$ for schedule S-OPT, for $t \in [0, 800]$ is shown in the following diagram:



Task 3 II

Solution 3.4:

The energy consumption of the Schedule S-OPT, has a periodicity of $2 \cdot T$. Therefore we compute the energy difference for the first two periods and then average those values to get the average energy difference per period T. In the 1st period:

$$\Delta E_1' = t_1 \cdot \frac{P_{active} + P_{sleep}}{2} - t_1 \cdot P_{sleep}$$

In the 2nd period:

$$\Delta E_2' = t_2 \cdot \frac{P_{active} + P_{sleep}}{2} - t_2 \cdot P_{sleep}$$

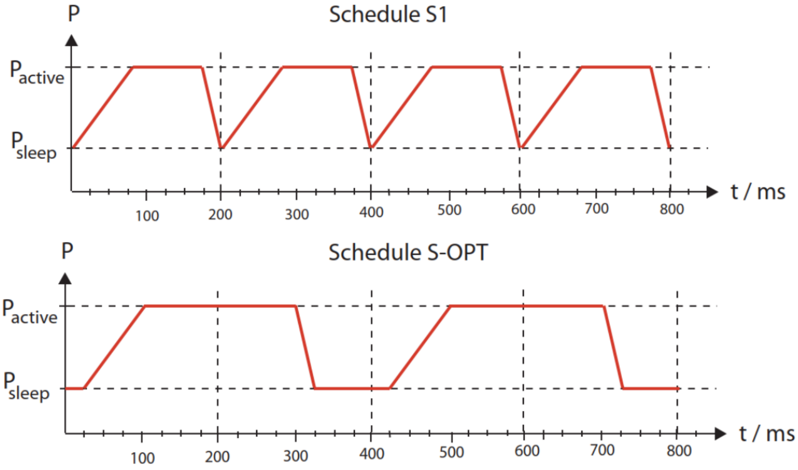
Task 3 III

Solution 3.4:

On average, the energy difference per period between $\mathrm{S1}$ *and S-OPT is:*

$$\Delta E' = \frac{\Delta E_1 + \Delta E_2}{2} = \frac{(t_2 + t_1)\left(P_{active} - P_{sleep}\right)}{4} \approx 27.75\mu\mathrm{J}$$

Task 3 IV


P
Schedule S1
P
active
P
sleep
t / ms
100
200
300
400
500
600
700
800
P
Schedule S-OPT
P
active
P
sleep
t / ms
100
200
300
400
500
600
700
800

Literature

Bücher