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Übungen zur Vorlesung

Graph Theory

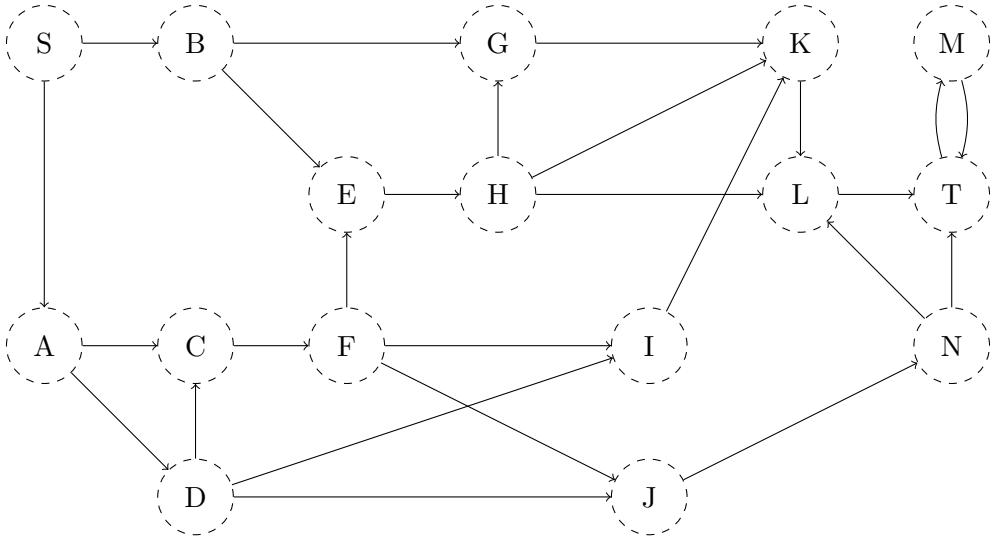
Summer 2023

Blatt 6

Please upload exactly one PDF file in Ilias. The submissions will **not** be graded. They serve only as feedback for the tutor.

Aufgabe 1: Graph Coloring, Cliques and Independant Sets (5 Punkte)

You are still stuck on the same train as in exercise 1 of the last exercise sheet and the train still has not moved from its spot in past few hours. You watch as the train driver, wearing a black coat and a hat pulled low over his face, walks out of the driver's cabin of the train and drives off in a car parked next to the rails. You decide to file a complaint about this train driver in a office of the railroad company. You also want to report the train driver in a police station because he got into his car with a half-drunk alcoholic cocktail in his hand. Furthermore you want to buy a ticket at a bus ticket office to take a bus to your destination.



To plan ahead, you look at the map of the railwork network from the last exercise sheet again. You want to visit all of the just mentioned places. You know that 2 neighboring cities never have 2 offices of the railroad company, 2 police stations or 2 bus ticket offices.

a) Run the greedy algorithm for node coloring from the lecture on the railwork network, coloring the cities in alphabetical order.

b) Is your found coloring the optimal solution to the problem? If yes, explain. If no, show a better coloring that requires fewer colors.

c) Add an edge to the graph of the rail network such that the algorithm generates a coloring that requires one additional color.

You observe that other passengers outside are carrying draisines from a nearby storehouse onto the rails in order to still somehow get to their destination. You decide to join these passengers and with

the help of a draisine and your knowledge from the previous suberxercies, you want to visit all of the just mentioned places.

d) Fortunetely you know that two neighboring cities always together only have at most one storehouse where you can return your draisine. Determine the maximum number of cities with a storehouses.

Because the city you want to visit last fortunetily has such a storehouse, you could start your journey back home immediately. With joy you look forward to the next approaching day.

Aufgabe 2: Planarity and Hyper-Cubes (5 Punkte)

A hypercube Q_i is an undirected graph with the vertex set V of bit strings $(x_1, \ldots, x_n) \in \{0, 1\}$ in which two nodes are connected by an edge if they exactly differ in one bit.

a) Draw Q_1, Q_2, Q_3 as planar graphs if possible.

b) Determine the chromatic number of Q_i for $i \geq 1$.

c) Investigate, whether for $i \leq 3$ the complete graph K_{i+1} is a subdivision of Q_i.

d) *Draw* Q_4.

e) Determine, whether Q_4 is planar and prove your statement.