Exercise 1

Given is the following schedule. Note, the deadlines D_i are given relative to the arrival time a_i of a process.

process i	arrival time a_i	computation time C_i	deadline D_i
J_1	0	6	18
J_2	4	2	4
J_3	2	4	7
J_4	6	2	4

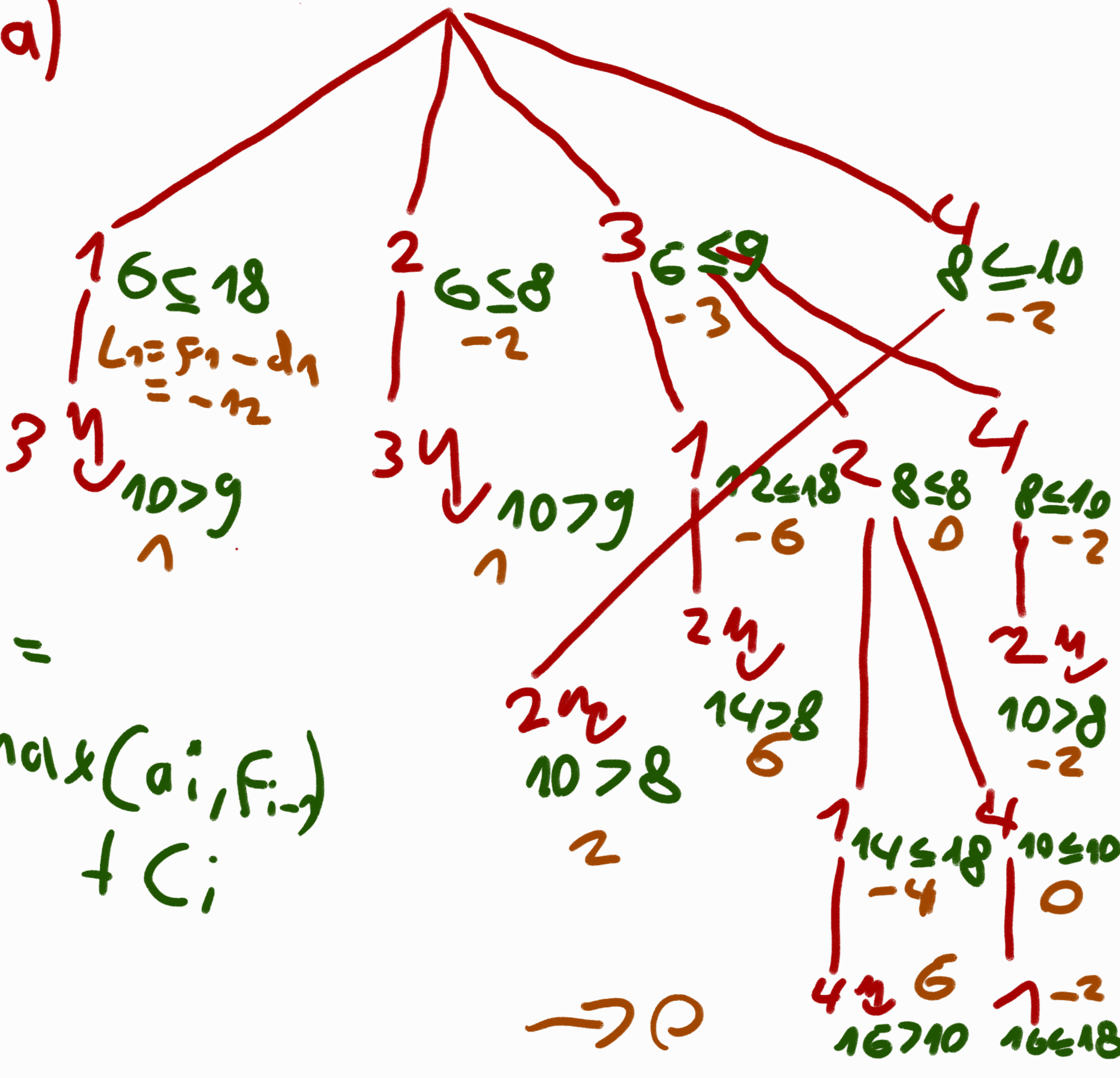
$18+0=18$
$4+4=8$
$7+2=9$
$4+6=10$

a) Draw the scheduling tree (with all feasible paths) for the given set of non-preemptive processes and mark the branches that are pruned by Bratley's algorithm. What is the resulting schedule with Bratley's algorithm?

b) Is there a feasible schedule if the algorithm must meet the precedence constraint $J_4 \prec J_2$?

c) Is the schedule feasible with the precedence constraint $J_4 \prec J_2$ if preemption is allowed?

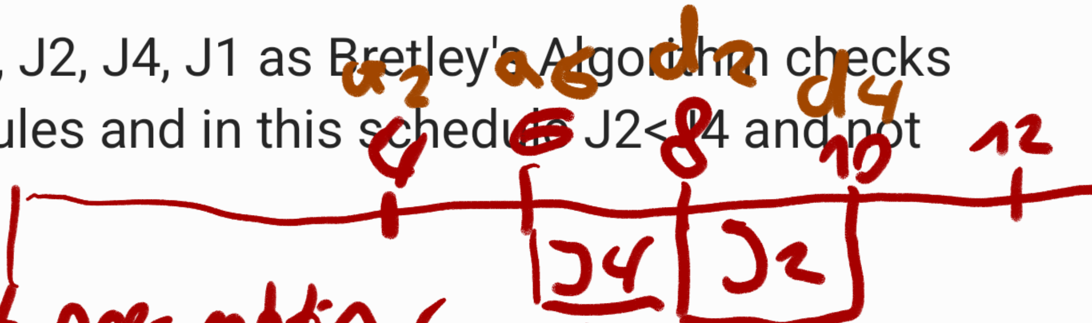
a)



$F_i = \max(a_i, F_{i-1}) + C_i$

b)

No, there's only J3, J2, J4, J1 as Bretley's Algorithm checks all possible schedules and in this schedule J2<J4 and not J4<J2 is the case.



c)

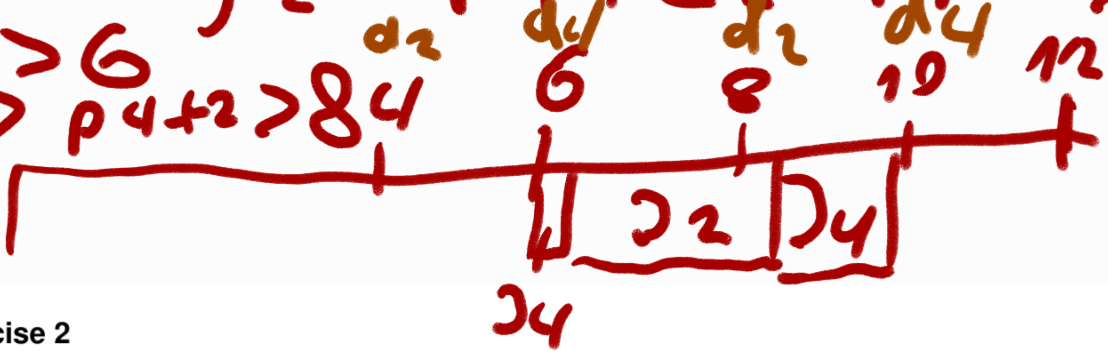
without preemption (worst case)

No, because $F_2 \geq F_4 + C_2 = 8 + 2 = 10 > 8 = d_2$

with best case preemption

$$F_2 \geq p_4 + C_2 = p_4 + 2 > 8 = d_2$$

$p_4 > 6 \Rightarrow p_4 + 2 > 8$



Exercise 2

Give a modified version of Bratley's algorithm which finds the solution with the minimum lateness. Which additional information has to be stored in the search tree? How can the bound condition be changed?

Initialize minimum lateness variable Lmin with infinity and minimum lateness schedule Smin with empty list.

Write lateness to every node and as soon as leave node reached update Lmin by maximum lateness over all nodes in this tree branch if this maximum lateness is smaller than current Lmin.

Improvement: Skip branch as soon as leave has lateness that is bigger than current Lmin.

Exercise 4

Given is the following periodic task set. Verify its schedulability by the Rate-Monotonic algorithm with $T_2 = D_2$ equals either a) 8 or b) 6. Use the criterion based on the processor utilization factor, if this is sufficient, or response time analysis otherwise. Moreover, draw the schedules until time a) 24 and b) 12.

process i	phase Φ_i	computation time C_i	period T_i	deadline D_i
τ_1	0	1	4	4
τ_2	0	2	a) 8 b) 6	a) 8 b) 6
τ_3	0	3	12	12

a) $U = \sum_{i=1}^{n} \frac{C_i}{T_i} = \frac{1}{4} + \frac{2}{8} + \frac{3}{12} = \frac{6+6+6}{24}$

$= \frac{18}{24} = \frac{3}{4} < 3 \cdot (\sqrt[3]{2} - 1) = 0{,}78$

$= n(2^{\frac{1}{n}} - 1) = U_{lub}$

$\rightarrow$ RM schedulable

$\left(\frac{1}{4} + \frac{2}{6} + \frac{3}{12} = \frac{3+4+3}{12} = \frac{10}{12} = \frac{5}{6} = 0{,}83 \right)$

$\frac{1}{4}$ $\quad$ $\frac{1}{8}$ $\quad$ $\frac{1}{12}$

$prio(\tau_1) > prio(\tau_2) > prio(\tau_3)$

(smallest period first)

b)

$R_i^{(0)} = C_i$

$R_i^{(j+1)} = C_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i^{(j)}}{T_k} \right\rceil \cdot C_k$

$R_1^{(0)} = C_1 = \underline{1} \leq 4 = D_1$

$R_2^{(0)} = C_2 = \underline{2} \leq 6 = D_2$

$R_2^{(1)} = 2 + \lceil \frac{2}{4} \rceil \cdot 1 = \underline{3} \leq 6$

$R_2^{(2)} = 2 + \lceil \frac{3}{4} \rceil \cdot 1 = \underline{3} \leq 6$

$\frac{1}{4} \qquad \frac{1}{6} \qquad \frac{1}{12}$

$prio(\tau_1) > prio(\tau_2) > prio(\tau_3)$

(smallest period first)

→ schedulable

$R_3^{(0)} = C_3 = \underline{3} \leq 12 = D_3$

$R_3^{(1)} = 3 + \lceil \frac{3}{4} \rceil \cdot 1 + \lceil \frac{3}{6} \rceil \cdot 2 = 6 \leq 12$

$R_3^{(2)} = 3 + \lceil \frac{6}{4} \rceil \cdot 1 + \lceil \frac{6}{6} \rceil \cdot 2 = 7 \leq 12$

$R_3^{(3)} = 3 + \lceil \frac{7}{4} \rceil \cdot 1 + \lceil \frac{7}{6} \rceil \cdot 2 = 9 \leq 12$

$R_3^{(4)} = 3 + \lceil \frac{9}{4} \rceil \cdot 1 + \lceil \frac{9}{6} \rceil \cdot 2 = 10 \leq 12$

$R_3^{(5)} = 3 + \lceil \frac{10}{4} \rceil \cdot 1 + \lceil \frac{10}{6} \rceil \cdot 2 = \underline{10} \leq 12$


2
4
6
8
10
12