CENG 5333

Dr. Pete Rogers

Final Project Report

Matt Alexander

3 May 2010

Measuring Reservoir Reliability Using Extended Drought Records

Introduction

This paper is an investigation of a method to analyze historical drought reconstructions for the goal of giving water supply managers a better idea of how to anticipate future drought events. Drought is a difficult thing to define and describe quantitatively. Flooding, because of the generally short durations of flood events, are easier to measure. Droughts are a longer “creeping disaster” that people often don’t realize the magnitude of until the lack of water is evident in the death of crops, trees, and dramatic water level declines in their recreational and water supply reservoirs. In a time when people of the world are increasingly aware about climate shifts and future drought cause by “climate change”, an important first practical step in understanding the problem could be to analyze current water supplies with respect to what we know about historic drought conditions and variability. This investigation is in two parts: first, historical drought will be described mathematically and historic drought events will be assigned a recurrence interval; second, the drought index reconstruction will be used to simulate reservoir inflows, evaporation and precipitation for years before those data were recorded. Reservoir models will be run to determine the firm yield of a particular reservoir during the period of record and then with the extended data, to compare between the data, and to see if extending the period of simulation could improve current models and estimates of firm yield.

Background

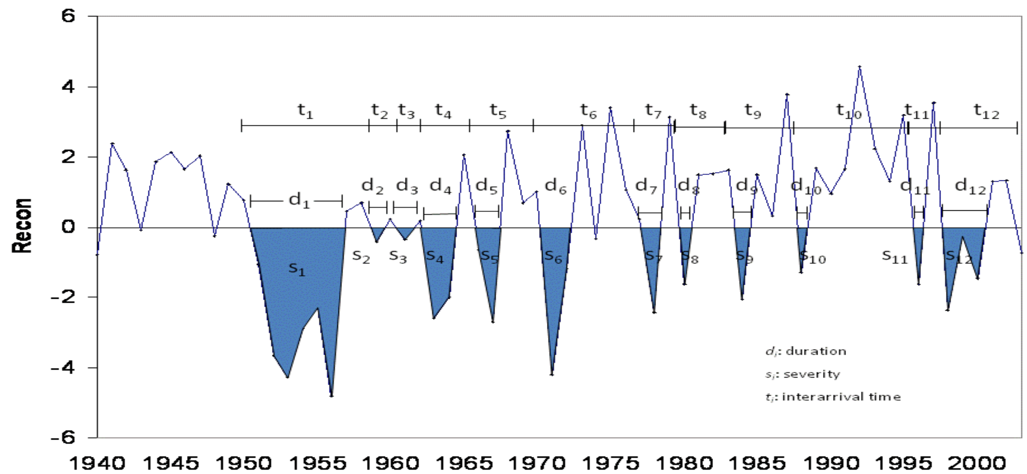
The design of reservoirs in Texas is done by using the longest precipitation, evaporation and inflow data available. Reservoir models are then run using these data to determine what the firm yield of the reservoir is so that water rights can be allocated. Tree rings have been studied for nearly 100 years as indicators of drought. In the past 20 years, researchers have been using the tree ring data to correlate with other climate indices. One of these approaches has used the Palmer Drought Index (PDI). The PDI is essentially “reconstructed” for the years where tree ring information exists, but measured climate data do not. In the Dallas region, there are several reconstructions that may prove useful to water supply managers, since the available tree ring data and thus the drought reconstructions go back all the way to the year 998. This paper’s intent is to use the correlated tree ring/PDI data made available by Cook (Cook, et al., 1999) with available precipitation and evaporation data (called net evaporation) and historical stream flows, to run standard reservoir models and determine the reservoir’s firm yield.

Model Formulation

Drought is a function of two variables: duration and severity. Figure 1 is a schematic that shows this relationship. Droughts are extreme events, and both variables can be described by a probability distribution. These distributions are combined in a formulation that describes the frequency of drought events. Modeling this data involves 2 steps:

1) Describing the Severity, and the Duration components of drought individually, with probability distributions (henceforth, *marginal* distributions)
2) Combining the two marginal distributions into a single bivariate distribution that accounts for the dependence of the two variables on each other.

Figure 1. Schematic for Duration (d), Severity (s), and Interarrival Time (t).



The data for the drought reconstruction comes from Cook et al., 1999, and is shown in Figure 2. The reconstruction is combined with actual measured data for the period of record, which is shown in Figure 3, and clearly shows what water suppliers have called the “drought of record” from 1951 – 1957.

Figure 2. Parmer Drought Index Reconstruction

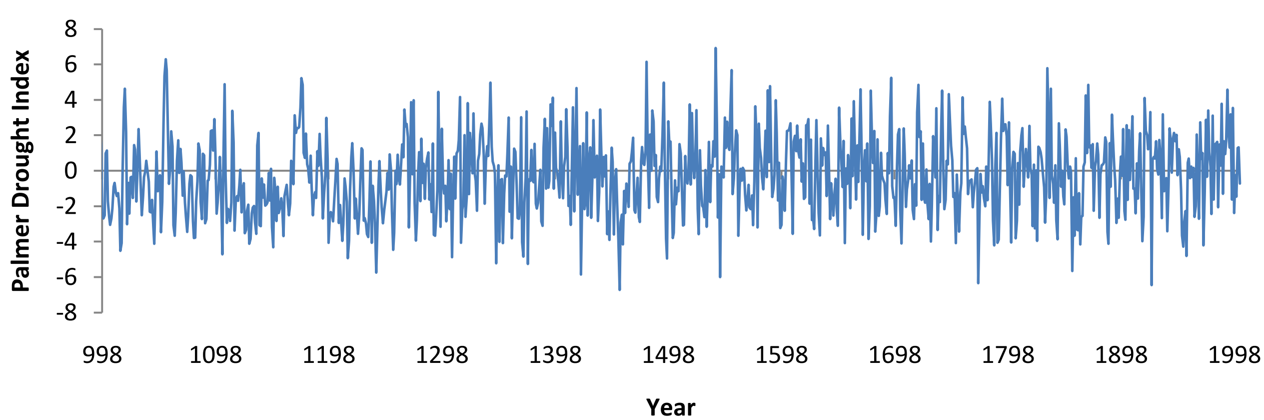
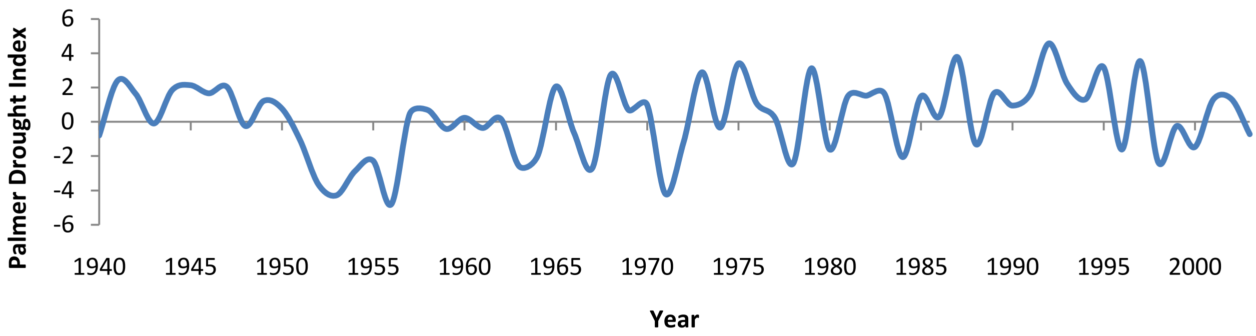


Figure 3. Palmer Drought Index (Period of Record)



Solution Approach

In order to actually pull a list of droughts with their severity and duration information from the 1000 year record, a small piece of code was needed. This code is described in Appendix A. The code parses the reconstruction for years where the PDI was less than 0, and if a year or more were less than 0, it consolidated the years into a single "drought event." The code returned 195 drought events with durations 1 year or greater, with their corresponding durations, severities, and interarrival times (t_n in Figure 1).

Univariate Descriptions (Marginal Distributions for Severity and Duration)

Once the drought record was established the droughts were analyzed using gamma distributions and fitted gamma distributions were developed using the method of least squares. The cumulative distribution functions for the drought's duration and severity are shown below in Figures 4 and 5.

The gamma distribution (cdf):

$$p = F(x \mid a, b) = \frac{1}{b^a \Gamma(a)} \int_0^x t^{a-1} e^{\frac{-t}{b}} \, dt$$

P is the probability that a single from the gamma distribution (with parameters a and b) fall in the interval [0,x] (Mathworks, 2010).

Since both duration and severity use the same distribution, only the parameters change for each:

Table 1. Parameters Used for Respective Marginal Distributions

	a	b
Duration	0.82	2.82
Severity	0.86	6.3

Figure 4. Observed Drought Severity with Fitted Gamma Distribution

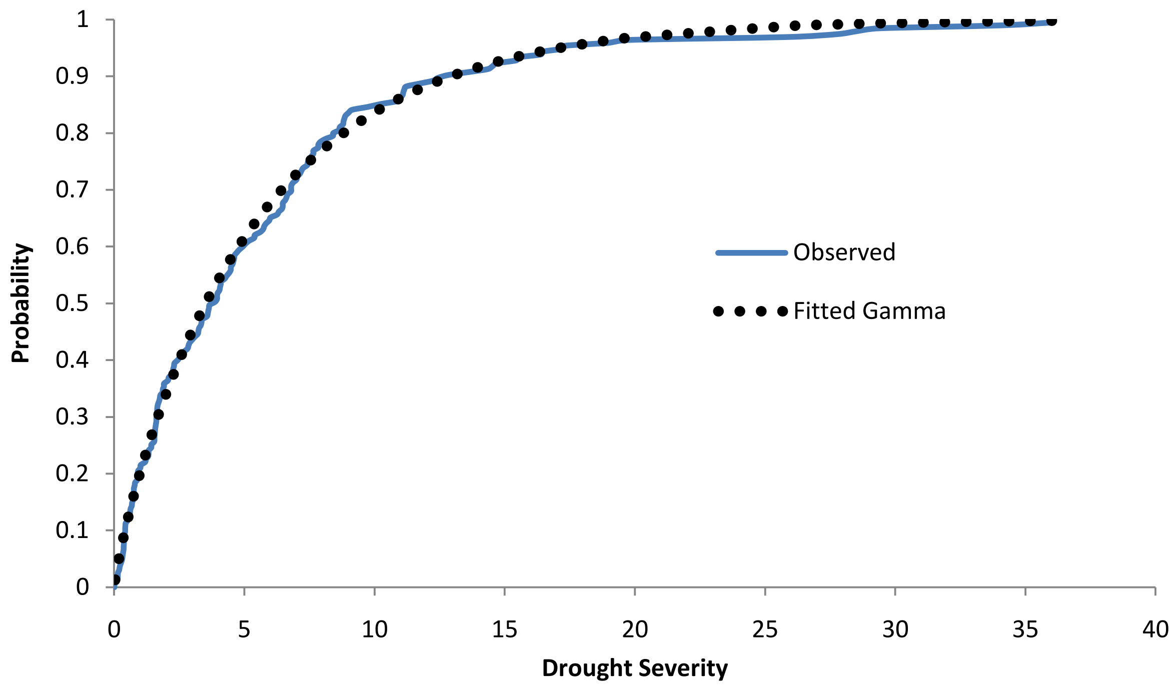
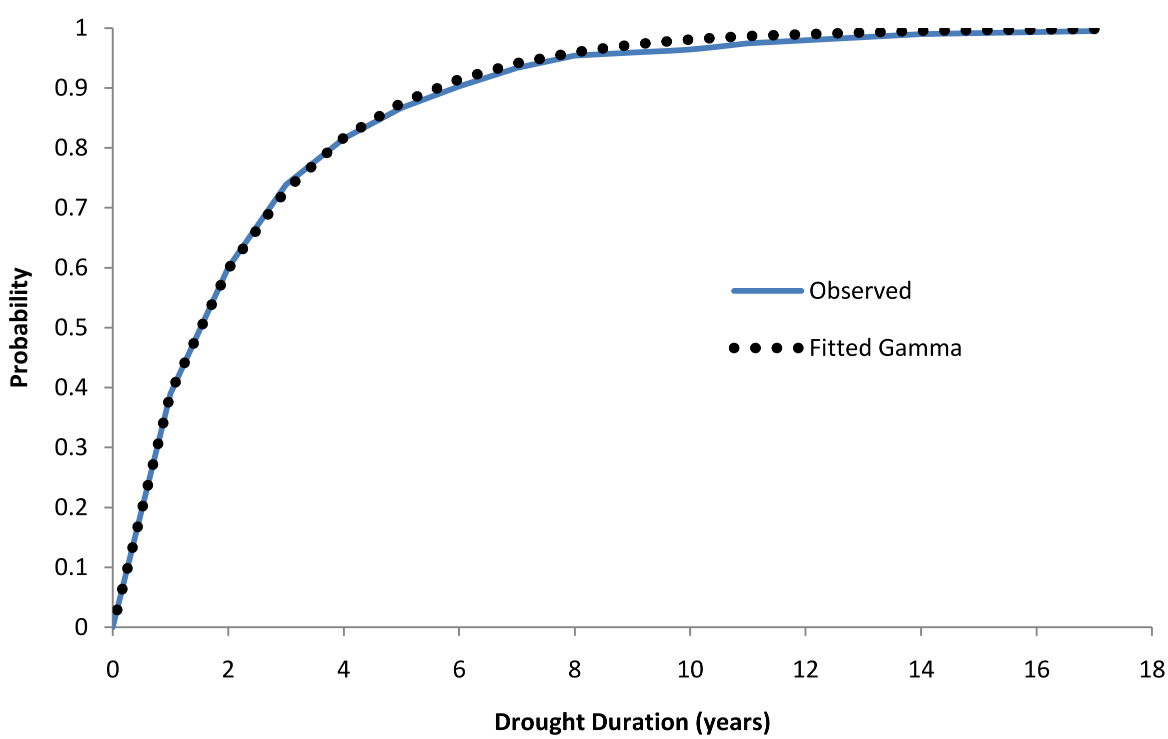


Figure 5. Observed Drought Duration with Fitted Gamma Distribution



Multivariate Description of Drought

The choice for the bivariate description of drought was done by evaluating various different copulas in the statistical software R. There are various methods to determine fit, and the routines within the program determine the "log-likelihood" that a particular distribution fits the data. The copula that fit the best was described by Shiau (2003) and used for the modeling of drought in this paper. The joint cumulative distribution function used is:

$$F(x,y) = e^{-[(-lnF(x)^{m})+(-lnF(y))^{m}]^{\frac{1}{m}}}$$

where $F(x)$ and $F(y)$ are the marginal distributions for random variables *X* (duration) and *Y* (severity);

$$m \geq 1$$

$$\text{and } m = \frac{1}{\sqrt{1-\rho}}$$

where ρ is the correlation coefficient for *X* between *Y*.

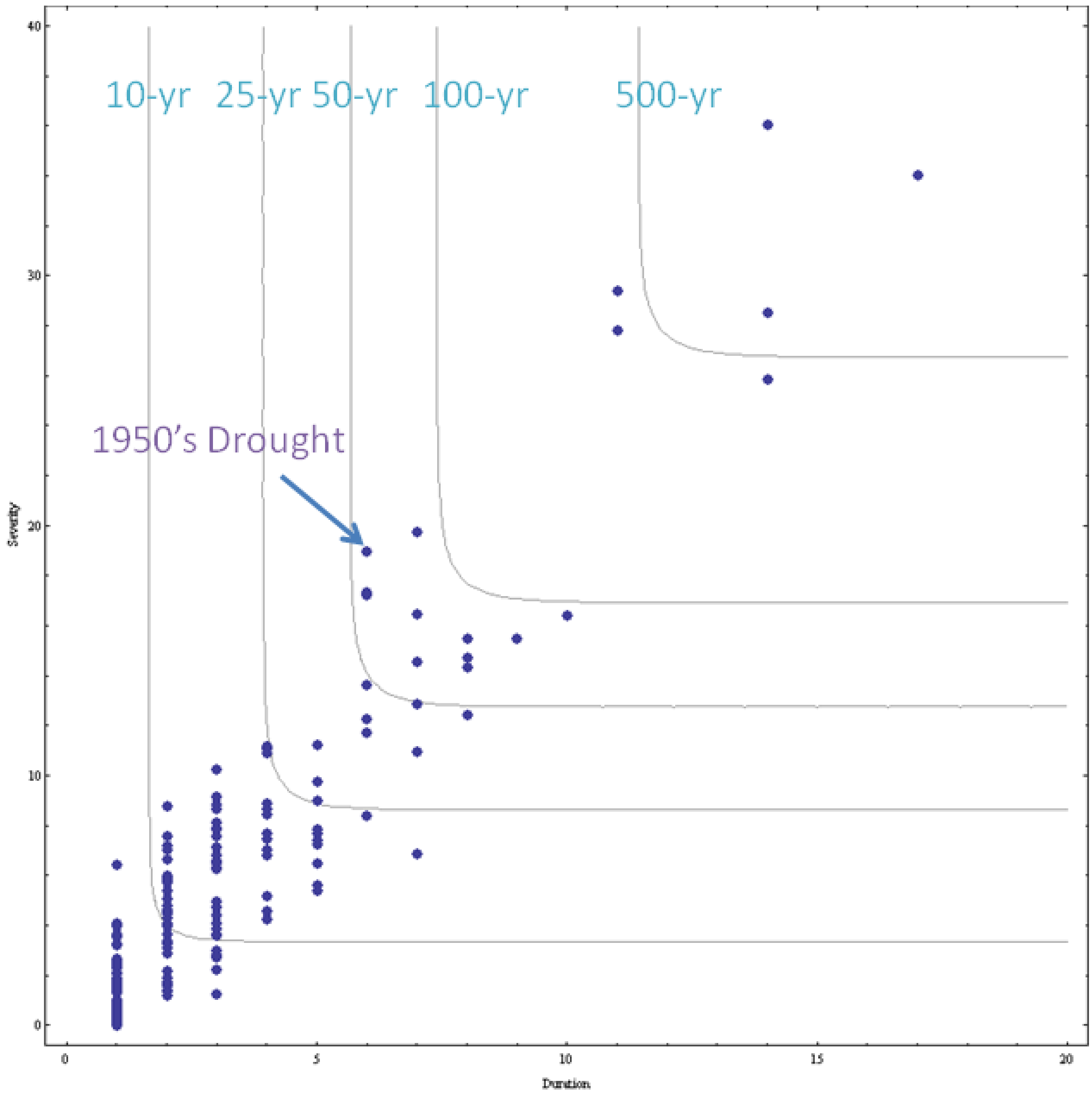
The recurrence interval of the drought can be calculated from the above bivariate distribution and:

$$E(T) = \frac{E(L)}{1 - F(x,y)}$$

$E(T)$ is the recurrence interval and $E(L)$ is the expected (average) interarrival time (t_n from Figure 1).

Figure 6 below shows the contours created by the above bivariate distribution for duration (x) and severity (y).

Figure 6. Drought Recurrence Intervals with Historical Drought Events

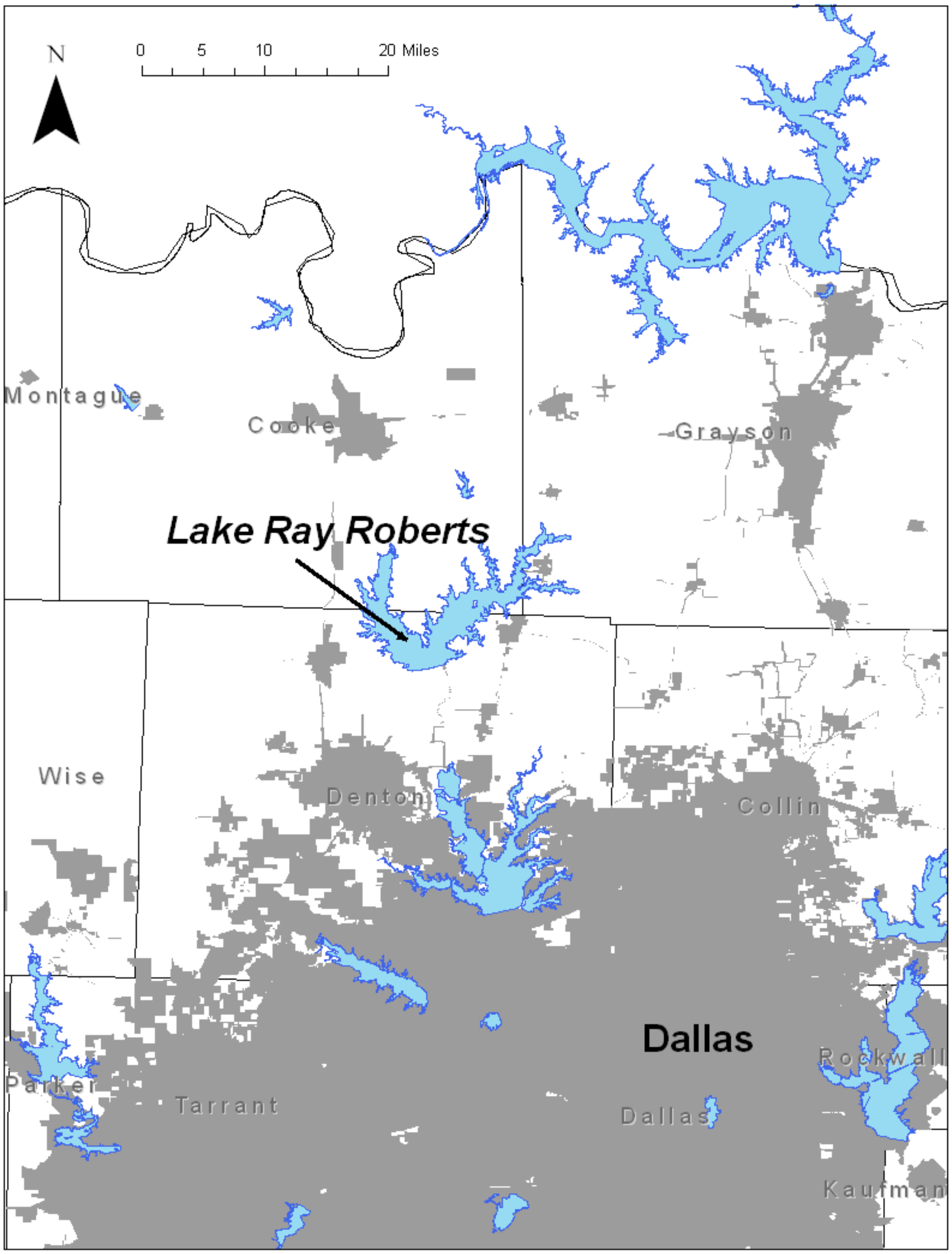


The significance of knowing the recurrence interval is that with multiple drought events to study, a water supplier can use the proper magnitude event on which to model his system, to ensure that the quantity of supplies are sufficient. The drought of record (1951 – 1957) falls near the 50 year drought recurrence interval. This may not be a good choice for a river master, or municipality. Instead a 100 year even might be preferred, or larger. The remaining of this paper considers the use of the entire 1005-year PDI as a basis for testing a known reservoir's reliability.

Using the Drought Reconstruction to Measure Reservoir Reliability

To get an idea of the drought reconstruction's usefulness in determining reservoir reliability, a specific reservoir was chosen to model. Lake Ray Roberts, a reservoir that supplies most of the water for the City of Denton, is located north of Dallas.

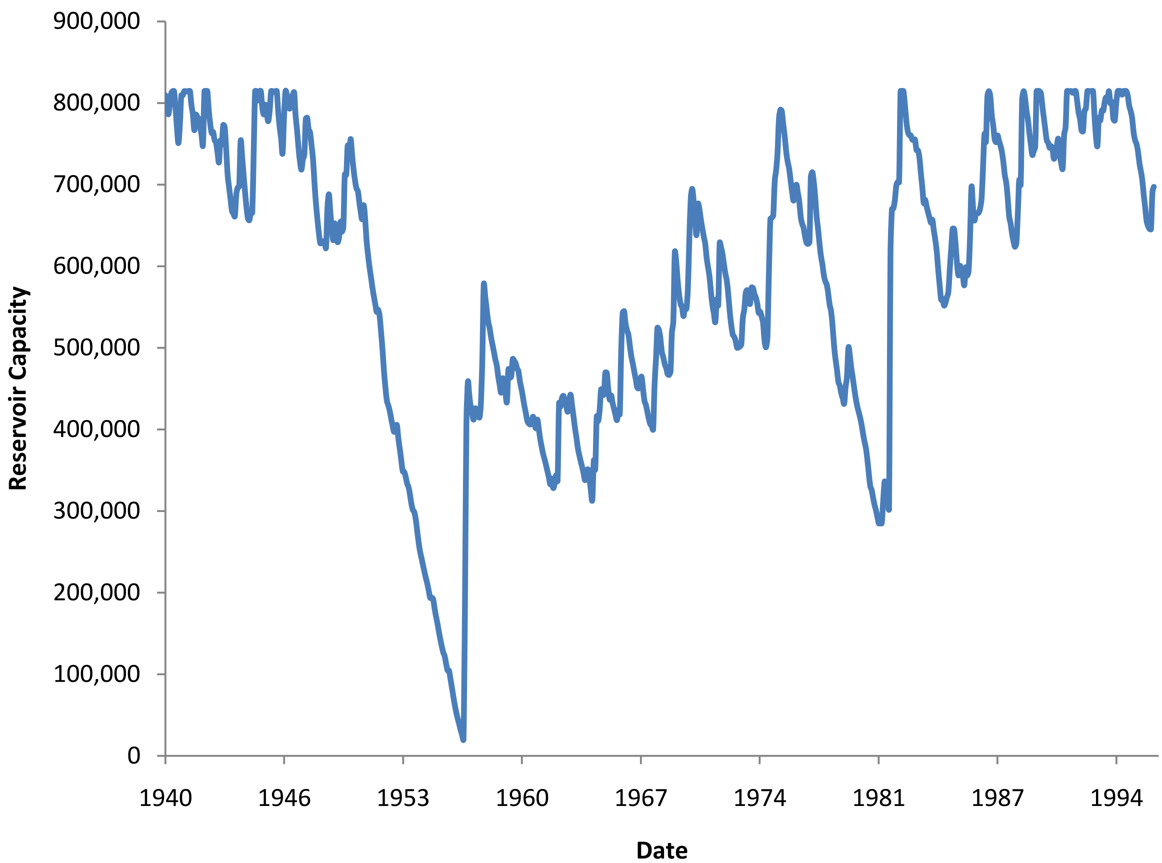
Figure 7. Lake Ray Roberts



The method for determining reliability in a reservoir in Texas involved simply developing a model of reservoir inflows and outflows, then maximizing the reservoir supply until the capacity almost goes to zero during the drought of record, which is seen in Figures 3 and 8. The Texas Commission on Environmental Quality, TCEQ, maintains surface water accounting models, called Water Availability Models or WAMs, which run this simulation to develop a reservoir's firm yield, a number that is then used to allocate water rights to various entities.

Using the WAM for the Trinity River Basin, a spreadsheet model of Lake Ray Roberts was developed, using the WAM information for reservoir inflows and net evaporation (TCEQ, 2010). The Ray Roberts storage-elevation-capacity information used came from the Texas Water Development Board database (TWDB, 2010). Maximizing the available supply from the reservoir using this model indicated that the firm yield was just over 90,000 acre-ft/ yr. Figure 9 shows the reservoir storage capacity for the period of record during the simulation.

Figure 8. Lake Ray Roberts Performance During the Period of Record
90,000 acre-ft/yr Firm Yield



Regression

In order to extend the reservoir simulation to capture the information provided by the drought reconstruction a linear regression was performed using the Cook PDI dataset with both the inflows and net evaporation datasets (from the period of record) used in the WAM. Figure 9 and 10 show the regressions of inflow and net evaporation.

The two regressions' R-squared values were low, suggesting that the correlation between the two variables during the period of record could be improved. Something to consider is that the tree ring width data that Cook et al. used to construct the PDI could be used directly to perform the linear regressions with the reservoir inflow and net evaporation.

Figure 9. Linear Regression of Inflow

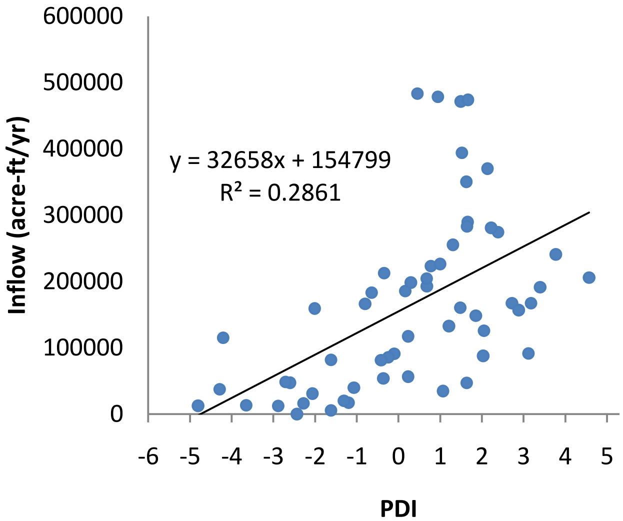
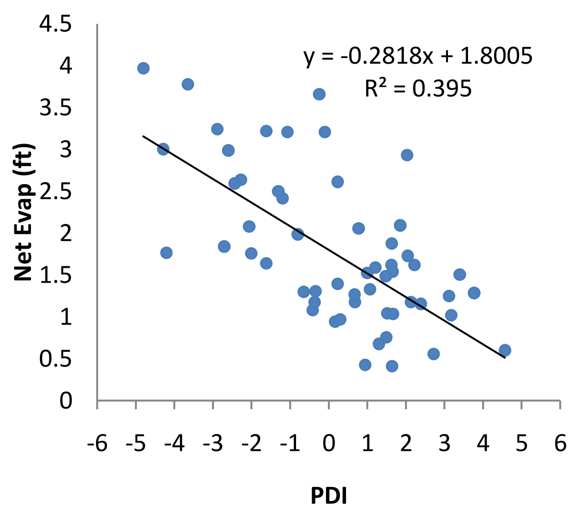


Figure 10. Linear Regression of Net Evaporation



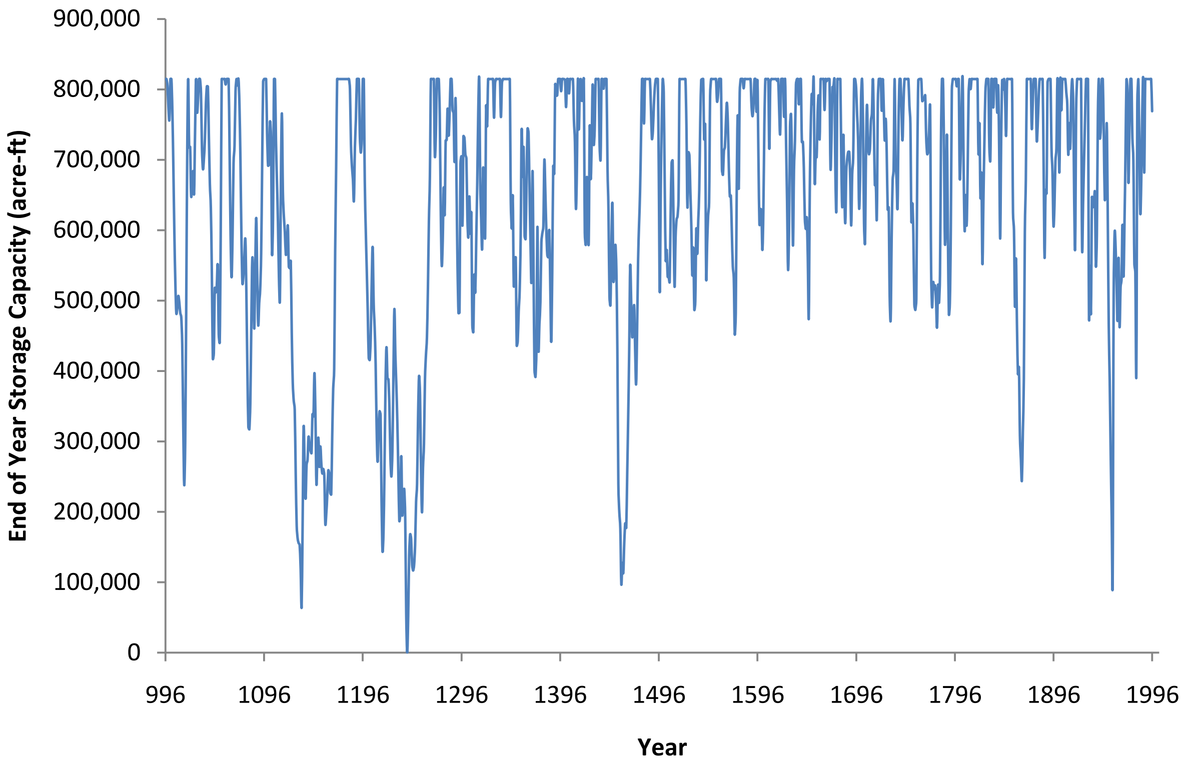
Results

Once the two input variables for the reservoir model were extended, the model used to determine the firm yield during the period of record was run again, this time with the 1005 years of data. The available water supply was maximized for the 1005 year period until the storage capacity just about dropped to zero. The firm yield using the 1005 year record came to be 78,500 acre-ft/yr. Table 2 compares the two model runs and Figure 11 shows the performance of Lake Ray Roberts during the simulation:

Table 2. Model Runs and Firm Yields

Model Run	Firm Yield
Period of Record	90,000
Extended Drought Record (1005 years)	78,500

Figure 11. Lake Ray Roberts Performance, 1005 Year Reconstructed Period



Conclusions

By looking at the droughts that are apparent in the 1005 year PDI data set (see Appendix B), it was expected that the huge droughts of the 1050 – 1300 year period would diminish the firm yields of a lake. How much was uncertain, however, and the modeling showing that the lake yield would be reduced by around 13% was interesting in that the reservoir seems to be fairly resilient to enormous potential droughts. A reduction of reservoir allocation of 13% is a significant reduction to municipal water suppliers, who would have to source this water elsewhere, whether it be future reservoirs, or tapping over-extended groundwater sources.

Further research that could be useful would be to use the mathematical description developed above and formulate a stochastic model, generating random future drought events and getting probabilities of failure for reservoirs. Further analysis could include the comparison of reservoir performance with the most recent drought in memory (2007-2008) and why that drought caused such a scare for water suppliers in Texas.

In future work a rigorous look at the errors this data generate could be done, and could be used to perform more sophisticated risk analysis for water suppliers, so that better decisions about investments and pricing structures could be developed.

References

Cook, E. R., D. M. Meko, D. W. Stahle, and M. K. Cleaveland (1999), Drought reconstructions for the continental United States, J. Clim., 12, 1145–1162. Data developed by Cook, et al was downloaded from http://www.ncdc.noaa.gov/

Mathworks, (2010), MATLAB Documentation for Statistics Toolbox: Gamma Distribution. Accessed at http://www.mathworks.com/access/helpdesk/help/toolbox/stats/brn2ivz-39.html

Shiau, J.T. (2003), Return period of bivariate distributed extreme hydrological events. Stochastic Environmental Research and Risk Assessment, 17, 42-57.

Texas Commission on Environmental Quality, TCEQ (2010), http://www.tceq.state.tx.us/permitting/water_supply/water_rights/wam.html

Texas Water Development Board, TWDB (2010), http://www.twdb.state.tx.us/data/surfacewater/surfacewater_toc.asp (see link for "Comprehensive Surface Water Information.")

Appendix A. Code for Extraction of Drought Events

```
Sub Find_droughts()
is_drought = False
numrecon = 1008
j = 0
ReconTally = 0
PrintMe = False
yearTally = 0
interarrival = 0
Dim recon As Range
  Set recon = Worksheets("recon").Range("B3:B1010")
   For i = 1 To numrecon
       If recon(i) < 0 And Not is_drought Then
           is_drought = True
           j = j + 1
           ReconTally = recon(i)
           yearTally = 1
           interarrivalTime = i - interarrival
           interarrival = i
       ElseIf recon(i) < 0 And is_drought Then
           ReconTally = ReconTally + recon(i)
           yearTally = yearTally + 1
       ElseIf recon(i) >= 0 And is_drought Then
           is_drought = False
           DroughtArray(j) = ReconTally
           yearArray(j) = yearTally
           ReconTally = 0
           PrintMe = True

       End If

           If PrintMe Then
               Cells(j + 1, 4) = DroughtArray(j)
               Cells(j + 1, 5) = yearArray(j)
               Cells(j + 1, 6) = interarrivalTime
               PrintMe = False
           End If
   Next i
End Sub
```

Appendix B. List of Drought Events

Start Year	End Year	Severity	Duration	Interarrival Time
998	1000	-7.545	3	2
1003	1016	-28.51	14	5
1020	1025	-8.395	6	17
1028	1029	-1.914	2	8
1032	1036	-5.376	5	4
1038	1045	-15.449	8	6
1047	1051	-7.414	5	9
1057	1057	-0.74	1	10
1061	1063	-9.135	3	4
1066	1066	-0.132	1	5
1069	1082	-25.859	14	3
1085	1086	-4.481	2	16
1089	1092	-6.792	4	4
1099	1101	-4.94	3	10
1103	1104	-6.003	2	4
1107	1112	-12.244	6	4
1115	1119	-9.001	5	8
1121	1134	-36.018	14	6
1137	1146	-16.383	10	16
1148	1164	-34.043	17	11
1167	1167	-0.755	1	19
1181	1181	-0.676	1	14
1183	1187	-7.652	5	2
1192	1195	-4.546	4	9
1198	1204	-16.47	7	6
1207	1217	-27.78	11	9
1221	1226	-13.637	6	14
1229	1234	-17.312	6	8
1236	1242	-19.757	7	7
1244	1251	-12.442	8	8
1254	1256	-10.241	3	10
1258	1258	-0.753	1	4
1260	1261	-1.322	2	2
1269	1270	-3.234	2	9
1272	1272	-0.192	1	3
1274	1277	-8.634	4	2
1279	1279	-1.725	1	5
1281	1281	-0.686	1	2
1283	1283	-1.583	1	2
1287	1289	-4.709	3	4
1291	1294	-11.071	4	4
1296	1297	-3.64	2	5
1299	1303	-7.254	5	3
1305	1308	-8.884	4	6
1310	1310	-1.564	1	5
1315	1317	-8.111	3	5
1319	1320	-4.061	2	4

1322	1322	-1.32	1	3
1325	1325	-0.138	1	3
1327	1329	-2.992	3	2
1336	1336	-1.866	1	9
1345	1347	-7.128	3	9
1349	1355	-10.96	7	4
1358	1358	-2.315	1	9
1360	1361	-4.808	2	2
1364	1367	-8.413	4	4
1369	1372	-11.102	4	5
1374	1376	-6.322	3	5
1378	1379	-1.187	2	4
1381	1384	-6.801	4	3
1386	1387	-5.956	2	5
1390	1390	-1.003	1	4
1392	1393	-1.549	2	2
1397	1397	-1.018	1	5
1400	1402	-2.246	3	3
1404	1405	-1.6	2	4
1410	1412	-6.477	3	6
1415	1415	-2.296	1	5
1418	1419	-1.544	2	3
1421	1422	-7.187	2	3
1424	1425	-3.948	2	3
1427	1427	-2.522	1	3
1430	1431	-2.897	2	3
1433	1433	-0.371	1	3
1436	1437	-4.04	2	3
1440	1440	-0.886	1	4
1442	1442	-0.324	1	2
1444	1447	-11.141	4	2
1450	1451	-4.321	2	6
1453	1463	-29.377	11	3
1468	1473	-11.721	6	15
1481	1481	-2.089	1	13
1483	1483	-0.007	1	2
1488	1490	-3.663	3	5
1492	1492	-0.009	1	4
1496	1497	-8.786	2	4
1501	1507	-12.855	7	5
1510	1512	-6.793	3	9
1514	1516	-1.221	3	4
1518	1518	-0.785	1	4
1521	1521	-0.42	1	3
1524	1525	-5.721	2	3
1527	1530	-7.438	4	3
1532	1533	-3.969	2	5
1535	1535	-1.773	1	3
1542	1544	-8.839	3	7
1547	1547	-0.375	1	5

1555	1556	-1.752	2	8
1560	1561	-4.619	2	5
1563	1563	-0.561	1	3
1566	1574	-15.485	9	3
1577	1577	-3.207	1	11
1581	1582	-1.424	2	4
1585	1585	-0.36	1	4
1590	1592	-2.697	3	5
1595	1595	-1.684	1	5
1597	1601	-9.741	5	2
1608	1608	-3.556	1	11
1616	1616	-0.612	1	8
1618	1619	-3.09	2	2
1623	1623	-1.859	1	5
1625	1627	-8.805	3	2
1630	1632	-6.251	3	5
1638	1638	-1.426	1	8
1641	1648	-14.694	8	3
1652	1652	-0.919	1	11
1654	1654	-4.089	1	2
1656	1656	-0.743	1	2
1658	1659	-2.186	2	2
1661	1661	-0.312	1	3
1664	1664	-1.623	1	3
1666	1666	-0.629	1	2
1670	1670	-3.613	1	4
1673	1673	-0.598	1	3
1675	1676	-5.836	2	2
1681	1682	-5.76	2	6
1684	1685	-4.578	2	3
1687	1691	-5.608	5	3
1693	1693	-0.146	1	6
1696	1699	-7.019	4	3
1703	1705	-7.836	3	7
1708	1710	-3.594	3	5
1712	1712	-0.427	1	4
1714	1717	-7.654	4	2
1722	1722	-1.649	1	8
1724	1725	-3.326	2	2
1727	1731	-11.206	5	3
1734	1734	-0.376	1	7
1736	1738	-6.569	3	2
1741	1743	-3.869	3	5
1750	1757	-14.353	8	9
1763	1769	-6.87	7	13
1772	1778	-14.531	7	9
1780	1780	-1.657	1	8
1785	1786	-7	2	5
1789	1791	-8.659	3	4
1798	1798	-0.804	1	9

1800	1801	-4.476	2	2
1805	1806	-6.619	2	5
1808	1808	-1.911	1	3
1812	1812	-0.9	1	4
1816	1816	-0.373	1	4
1819	1820	-3.367	2	3
1822	1822	-3.258	1	3
1824	1824	-3.95	1	2
1829	1832	-5.176	4	5
1835	1835	-1.535	1	6
1837	1839	-2.818	3	2
1841	1842	-6.662	2	4
1845	1848	-4.245	4	4
1852	1852	-0.441	1	7
1854	1857	-10.877	4	2
1859	1864	-17.239	6	5
1874	1874	-2.265	1	15
1879	1880	-2.853	2	5
1886	1887	-7.578	2	7
1889	1889	-0.9	1	3
1893	1897	-7.837	5	4
1899	1899	-0.285	1	6
1901	1902	-5.042	2	2
1904	1904	-1.646	1	3
1906	1906	-0.518	1	2
1909	1911	-4.416	3	3
1913	1913	-1.569	1	4
1916	1918	-7.919	3	3
1925	1925	-6.444	1	9
1931	1931	-0.209	1	6
1934	1934	-2.673	1	3
1936	1940	-6.475	5	2
1943	1943	-0.102	1	7
1948	1948	-0.246	1	5
1951	1956	-18.978	6	3
1959	1959	-0.417	1	8
1961	1961	-0.363	1	2
1963	1964	-4.609	2	2
1966	1967	-3.354	2	3
1971	1972	-5.399	2	5
1974	1974	-0.342	1	3
1978	1978	-2.435	1	4
1980	1980	-1.617	1	2
1984	1984	-2.062	1	4
1988	1988	-1.306	1	4
1996	1996	-1.619	1	8
1998	2000	-4.101	3	2
2003	2003	-0.726	1	5