Probability — WS 6 — Due: September 18^{th} start of class

Name:______________________________

Example: Consider the experiment of choosing 2 marbles from a bag with 2 red marbles, 2 blue marble, and 1 green marble. We will describe the probability of all events from the sample space $S = \{RR, RB, RG, BB, BG\}$

Definition: A table of all probabilities for a sample space is called a

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Properties:

- Each outcome must be disjoint
- The probabilities must be between 0 and 1
- The probabilities add to 1.

Definition: When all of the outcomes are equally likely we say that the probability distribution is

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Recall that when all outcomes are equally likely the probability of an event occurring is

$$P(E) = \frac{\#\text{ favorable outcomes}}{\#\text{ possible outcomes}}$$

Definition: The **complement** of a subset are all elements in the set that are not in the subset.

Example: Let $S = \{a, b, c, d, e, f, g\}$ and $A = \{a, b\}$ then what is A^c (the complement of A)?

The complement of a set has the property that

$$P(A \text{ or } A^c) = \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_.$$

That is, a subset is always disjoint from its complement and the probability that one or the other happens is 1.

Sometimes it is easier to compute the probability of the complement of an event and then use the above property to compute the probability of the event.

Example: What is the probability of two people in our class having the same birthday? (there are 20 people in our class)

Example: There is a 50% chance of rain on Saturday and a 50% chance of rain on Sunday. Someone (not in our class) concludes that it will definitely rain this weekend since

$$P(\text{rain on Saturday or rain on Sunday}) = P(\text{rain on Saturday}) + P(\text{rain on Sunday}) = 100\%.$$

Why is their calculation wrong? How can we correctly compute the probability?

Definition: Two events are said to be **independent** if the outcome of one does not affect the probability of the other. The probability of independent events A and B occurring is

$$P(A \text{ and } B) = \underline{\hspace{4cm}}.$$

Independent events can sometimes be difficult to recognize, consider the following:

Example: What is the probability of drawing a card that it is both $A = \{\text{ a heart }\}$ and $B = \{\text{ an ace }\}$? Are the events independent? What if we remove the 2 of spades from the deck?

Class Activity:

1. Describe the probability distributions for the following random experiments:
   - Flipping a fair coin twice.
   - Counting the number of heads from flipping a coin three times.
   - The sum of rolling two six sided dice.
2. What is the probability of rolling doubles from three dice?
3. Assume that there is a one in a million chance that someone is struck by lightning if they are outside during a storm. If there are $50,000$ people outside in a city during a storm, what is the chance that someone is struck by lightning?

4. Let A and B be events with $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Find $P(A \text{ or } B)$ and $P(A \text{ and } B)$ if:
   - A and B are disjoint.

   - A and B are independent.

   - A^c and B are independent

5. Roll a die twice and consider the events $A = \{\text{first roll gives at least } 4\}$, $B = \{\text{second roll gives at most } 4\}$, and $C = \{\text{the sum of the rolls is } 10\}$. Find $P(A)$, $P(B)$, $P(C)$, and $P(A \cap B \cap C)$. Are A and B independent? What about B and C? Finally, are A and C independent? (we call this **pairwise independence**)