Math 4110/5110, Introduction to Modern Analysis, Fall 2014 **Homework #1**

Due In Class: Thursday, September 10

Reading: Read *Some Remarks on Writing Mathematical Proofs* by John M. Lee, available at

`http://www.math.washington.edu/~lee/Writing/writing-proofs.pdf`

Read Chapter 1 in Rudin's *Principles of Mathematical Analysis, 3rd Edition* ("*the textbook*").

A nonempty set $A \subseteq \mathbb{R}$ is *bounded below* if there exists $x \in \mathbb{R}$ such that $x \leq a$ for all $a \in A$. In this case, the number x is called a *lower bound* for A. If x is a lower bound for A with the property that $x' \leq x$ for every lower bound x' for A, then we say x is the *infimum* of A and write $x = \inf A$.

One proof of the following statement is given in the textbook (see the proof of Theorem 1.11). Give a different proof.

Theorem 1. *If $A \subseteq \mathbb{R}$ is bounded below, then $\inf A$ exists. (Give a different proof than the proof of Theorem 1.11 in the textbook.)*

Prove the following statements.

Theorem 2 (Approximation Property)**.** *Let $A \subseteq \mathbb{R}$ be a nonempty set bounded below. For all $\varepsilon > 0$, there exists $a \in A$ such that $a < \inf A + \varepsilon$.*

Theorem 3. *If $A, B \subseteq \mathbb{R}$ are nonempty sets bounded below, then $A + B$ is bounded below and $\inf(A + B) = \inf A + \inf B$.*

Theorem 4 (Exercise 1.8)**.** *There is no order relation on $\mathbb{C}$ (the complex numbers) that makes $\mathbb{C}$ an ordered field.*

Theorem 5 (Exercise 1.9)**.** *For any $z, w \in \mathbb{C}$, say $z = a + bi$ and $w = c + di$, define the relation $z < w$ if either $a < c$ or $a = c$ and $b < d$. Then $(\mathbb{C}, <)$ is an ordered set.*

You may cite the following fact, which we proved in class, without further justification: $|x + y| \leq |x| + |y|$ for all $x, y \in \mathbb{R}$. Prove the next statement rigorously using induction.

Theorem 6. *For all $n \geq 2$, for all $x_1, \ldots, x_n \in \mathbb{R}$,*

$$|x_1 + \cdots + x_n| \leq |x_1| + \cdots + |x_n|.$$