All problem sets are optional. If you turn in solutions to problems, I will read and return them to you with feedback.

**Problem D:** Prove that the 1-dimensional Hausdorff content $\mathcal{H}^1_\infty$ is not a Borel measure on $\mathbb{R}^n$ when $n \geq 2$. (One approach is to show $0 < \mathcal{H}^1_\infty(B(0,1)) = \mathcal{H}^1_\infty(U(0,1)) = \mathcal{H}^1_\infty(\partial B(0,1)) < \infty$ in $\mathbb{R}^2$.)

**Problem E:** Let $n \geq 1$ and $0 \leq s < \infty$. The *$s$-dimensional spherical measure* $\mathcal{S}^s$ on $\mathbb{R}^n$ is defined by the rule

$$\mathcal{S}^s(E) = \lim_{\delta \downarrow 0} \mathcal{S}^s_\delta(E); \quad \mathcal{S}^s_\delta(E) := \inf \left\{ \sum_{i=1}^\infty (2r_i)^s : E \text{ is covered by balls } B(x_i, r_i) \text{ with } 2r_i \leq \delta \right\}.$$

(a) Describe the difference in the definition of $\mathcal{S}^s$ and the $s$-dimensional Hausdorff measure $\mathcal{H}^s$. (b) Prove that $\mathcal{S}^s$ is a Borel regular measure. (c) Find constants $c_1$ and $c_2$ depending on at most $n$ and $s$ such that $c_1\mathcal{H}^s(E) \leq \mathcal{S}^s(E) \leq c_2\mathcal{H}^s(E)$ for all $E \subset \mathbb{R}^n$.

**Problem F:** Let $0 < \lambda < 1/2$ be given. Let $C_\lambda \subset [0,1]$ be the generalized Cantor set in $\mathbb{R}$ that is obtained by replacing $[0,1]$ by intervals $[0,\lambda] \cup [1-\lambda, 1]$ and iterating. Find the Hausdorff dimension $s$ of $C_\lambda$ and compute $\mathcal{H}^s(C_\lambda)$.