Calculus 3 Notes

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1 Chapter 15 Double Integral

1.1 Definition of the Double Integral

Double Integral = volume below the graph $z = f(x, y)$ over a region R in xy-plane.

$$\iint_R f(x,y)dA$$

Definition: Cut R into small pieces of area ΔA Lets divide the region R into i x and y components The height of each rectangular prism is $f(x_i, y_i)$ and the area of the region underneath is ΔA_i

$$\sum_i f(x_i, y_i)\Delta A_i$$

Finally, you get take the limit as $\Delta A_i \to 0$ to get the double integral.

1.2 Calculating the Double Integral

To compute the double integral, take **slices**. Let $S(x)$ = area of slice by plane parallel to the yz-plane. Then,

$$\text{volume} = \int_{x_{min}}^{x_{max}} S(x)dx.$$

$$S(x) = \int_{y_{min}(x)}^{y_{max}(x)} f(x,y)dy$$

$$\text{Finally, } \iint_{\mathbf{R}} \mathbf{f(x,y)dA} = \int_{\mathbf{x_{min}}}^{\mathbf{x_{max}}} \left[\int_{\mathbf{y_{min}(x)}}^{\mathbf{y_{max}(x)}} \mathbf{f(x,y)dy} \right] \mathbf{dx}$$

This is called an **Iterated Integral** because one integral is integrated over the next. BTW: dA becomes $dy \cdot dx$ because dA is the area of each infinitesimal rectangle which equals $dy \cdot dx$

1.3 Double Integral in Polar Coordinate

Slices of circles on the polar coordinate plane are approximately rectangles when they are very tiny. Therefore,

$$\Delta A \approx \Delta r \Delta \theta$$

and therefore,

$$dA = dr d\theta$$

If you have the function $f(\theta, r)$, then in order to find the volume under the curve, use the formula

$$V = \iint_R r f(\theta, r) dA$$

1.4 Applications of Double Integrals

1. Find area of a region R
$$A = \iint_R 1dA$$
2. Finding mass of a flat object with density δ =mass per unit area
$$\Delta m = \delta \Delta A$$
$$m = \iint_R \delta dA$$
3. Finding average value of f in region R
$$\bar{f} = \frac{1}{\text{Area}} \iint_R f dA$$
Weighted average of f with density δ:
$$\frac{1}{\text{Mass}(R)} \iint_R f \delta dA$$
4. Center of Mass of a (planar) object (with density δ)?
Center of mass is at $(\bar{x}, \bar{y})$ where
$$\bar{x} = \frac{1}{\text{Mass}} \iint_R x \delta dA$$
$$\bar{y} = \frac{1}{\text{Mass}} \iint_R y \delta dA$$
5. Moment of inertia of an object about the origin
$$\iint_R r^2 \delta \ dA = \iint_R (x^2 + y^2) \delta \ dA$$

Left off here

1.5 Triple Integrals

- Really similar to double integrals
- Finds the summation of volumes of a 3 dimensional region
$$\iiint_R f \ dV$$

1.6 Applications of Triple Integrals

- Mass
$$\text{Mass} = \iint_R \delta \ dV$$

- Average Value
$$\bar{f} = \frac{1}{\text{Vol}(R)} \iiint_R f \, dV$$
or with density:
$$\bar{f} = \frac{1}{\text{Mass}(R)} \iiint_R f\delta \, dV$$
- Center of Mass: $(\bar{x}, \bar{y}, \bar{z})$
$$\bar{x} = \frac{1}{\text{Mass}(R)} \iiint_R x\delta \, dV$$
$$\bar{y} = \frac{1}{\text{Mass}(R)} \iiint_R y\delta \, dV$$
$$\bar{z} = \frac{1}{\text{Mass}(R)} \iiint_R z\delta \, dV$$
- Moment of Inertia:
$$\iiint_R (\text{distance to axis})^2 \delta \, dV$$
$$I_z = \iiint_R r^2\delta \, dV = \iiint_R \delta(x^2 + y^2) \, dV$$
$$I_y = \iiint_R \delta(x^2 + z^2) \, dV$$
$$I_x = \iiint_R \delta(y^2 + z^2) \, dV$$