Calculus 3 Notes

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1   Section 13.2

1.1   Ideal Projectile Motion

If v_0 makes an angle α with the horizon, then

$$v_0 = (|v_0| \cos \alpha)\hat{\imath} + (|v_0| \sin \alpha)\hat{\jmath}$$

Also, assume $r_0 = 0\hat{\imath} + 0\hat{\jmath}$. With these formulas it can be derived that

$$r(t) = -\frac{1}{2}gt^2\hat{\jmath} + v_0 t$$

1.1.1   Range of a Projectile

$$R = \frac{v_0^2}{g} \sin(2\alpha)$$

Where R is the range, v_0 is the initial velocity, g is acceleration due to gravity, and α is the launch angle

1.1.2   Maximum Height of a Projectile

Lets say that y_{max} is the maximum height, then:

$$y_{max} = \frac{v_0 \sin \alpha}{2g}$$

2   Section 13.3

2.1   Finding Tangent Vector to a Path

If $r(t)$ is the function for the path, then $r'(t)$ represents the tangent vector for the path.

2.2   Length of Path

If $r(t)$ is the function for the path and D represents the length of the path from $t \in (t_0, t_1)$, then

$$D = \int_{t_0}^{t_1} |r'(t)|dt$$

3   Section 13.4

3.1   Curvature

If T is the tangent unit vector of a smooth curve, then the curvature of the curve is

$$\kappa = \left| \frac{dT}{ds} \right|$$

where s is arc length.

3.2   Formula for Calculating Curvature

If $s(t)$ is a smooth curve, then curvature is

$$\kappa = \left| \frac{\frac{dT}{dt}}{\frac{ds}{dt}} \right|$$

where

$$T = \frac{s'(t)}{|s'(t)|}$$

and $v = r'(t)$

3.3   Principal Unit Normal

At any point $\kappa \neq 0$, the principal unit normal vector for a curve is

$$N = \frac{1}{\kappa} \frac{dT}{ds}$$

3.4   Formula for Calculating N

If $r(t)$ is a smooth curve, then the principal unit normal is

$$N = \frac{\frac{dT}{dt}}{\left| \frac{dT}{dt} \right|}$$

4   Section 13.5

4.1   Binormal Vector

The binormal vector (B) is a vector that is orthogonal to both T and N.

$$B = T \times N$$

4.2   Acceleration Vector

If the acceleration vector is
$$a = a_T T + a_N N$$
then
$$a_T = s''(t) = \frac{d}{dt}|s'(t)| \text{ and } a_N = \kappa \left(\frac{ds}{dt}\right)^2 = \kappa |s'(t)|^2$$
also
$$a_N = \sqrt{|s''(t)|^2 - a_T^2}$$

5   Section 14.6

5.1   Standard Linear Approximation

For $f(x, y)$ at (x_0, y_0), the Standard Linear Approximation of $f(x, y)$ is
$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \tag{5.1.1}$$

5.2   The Error in the Standard Linear Approximation

If f has continuous first and second partial derivative throughout an open set containing a rectangle R centered at (x_0, y_0) and if M is any upper bound for the values of $|f_{xx}|, |f_{yy}|$, and $|f_{xy}|$ on R, then the error $E(x, y)$ incurred in replacing $f(x, y)$ on R by its linearizion satisfies the inequality
$$|E(x, y)| \le \frac{1}{2}M(|x - x_0| + |y - y_0|)^2. \tag{5.2.1}$$

5.3   Tangent Plane

For $f(x, y)$ at (x_0, y_0), the tangent plane of $f(x, y)$ is:
$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \tag{5.3.1}$$
If you have a surface $z = f(x, y)$ at $P(x_0, y_0, z_0)$ use
$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0 \tag{5.3.2}$$

5.4   Normal Line

The normal line to $f(x, y, z)$ at $P_0(x_0, y_0, z_0)$ has the following equations:
$$x = x_0 + f_x(P_0)t$$
$$y = y_0 + f_y(P_0)t$$
$$z = z_0 + f_z(P_0)t$$

6 Section 14.7

6.1 Definitions of local maximums and minimums

If $f(x, y)$ is defined on a region R containing the point (a, b), then:

1. $f(a, b)$ is a **local maximum** value of f if $f(a, b) \geq f(x, y)$ for all domain points (x, y) in an open disk centered at (a, b).

2. $f(a, b)$ is a **local minimum** value of f if $f(a, b) \leq f(x, y)$ for all domain points (x, y) in an open disk centered at (a, b).

6.2 First Derivative Test for Local Extreme Values

If all partial derivatives are equal to zero or undefined, then they are critical points. In order to find local extrema, you must set all partial derivatives to zero and solve the system of equations.

6.3 Second Derivative Test for Local Extreme Values

Let D be the **discriminant** or **Hessian** of f so that

$$D = f_{xx}f_{yy} - f_{xy}^2$$

then

1. f has a **local maximum** at (a, b) if $f_{xx} < 0$ and $D > 0$ at (a, b).

2. f has a **local minimum** at (a, b) if $f_{xx} > 0$ and $D > 0$ at (a, b).

3. f has a **saddle point** at (a, b) if $D < 0$ at (a, b).

4. **the test is inconclusive** at (a, b) if $D = 0$ at (a, b) and another method must be in order to determine the behavior at (a, b).

6.4 Finding Absolute Maxima and Minima on Closed Bounded Regions

In order to find absolute extrema for $f(x, y)$ on a closed and bounded region R,

1. *List the interior points of R* where f may have local maxima or minima and evaluate f at these points. These are critical points of f.

2. *List the boundary points of R* where f has local maxima and minima and evaluate f at these points. For every boundary, fix one or more of the variables in order to create a function of a single variable and find its local maxima and minima.

3. *Look through the lists* for the maximum and minimum values of f. These will be the absolute maximum and minimum values of f on R.