Calculus 3 Notes

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1 Section 14.6

1.1 Standard Linear Approximation

For $f(x, y)$ at (x_0, y_0), the Standard Linear Approximation of $f(x, y)$ is:

$$L(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \tag{1.1.1}$$

1.2 The Error in the Standard Linear Approximation

If f has continuous first and second partial derivative throughout an open set containing a rectangle R centered at (x_0, y_0) and if M is any upper bound for the values of $|f_{xx}|, |f_{yy}|$, and $|f_{xy}|$ on R, then the error $E(x, y)$ incurred in replacing $f(x, y)$ on R by its linearizion satisfies the inequality

$$|E(x, y)| \leq \frac{1}{2} M(|x - x_0| + |y - y_0|)^2. \tag{1.2.1}$$

1.3 Tangent Plane

For $f(x, y)$ at (x_0, y_0), the tangent plane of $f(x, y)$ is:

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \tag{1.3.1}$$

If you have a surface $z = f(x, y)$ at $P(x_0, y_0, z_0)$ use

$$f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) - (z - z_0) = 0 \tag{1.3.2}$$

1.4 Normal Line

The normal line to $f(x, y, z)$ at $P_0(x_0, y_0, z_0)$ has the following equations:

$$\begin{aligned} x &= x_0 + f_x(P_0)t \\ y &= y_0 + f_y(P_0)t \\ z &= z_0 + f_z(P_0)t \end{aligned}$$

2 Section 14.7

2.1 Definitions of local maximums and minimums

If $f(x, y)$ is defined on a region R containing the point (a, b), then:

1. $f(a, b)$ is a **local maximum** value of f if $f(a, b) \geq f(x, y)$ for all domain points (x, y) in an open disk centered at (a, b).
2. $f(a, b)$ is a **local minimum** value of f if $f(a, b) \leq f(x, y)$ for all domain points (x, y) in an open disk centered at (a, b).

2.2 First Derivative Test for Local Extreme Values

If all partial derivatives are equal to zero or undefined, then they are critical points. In order to find local extrema, you must set all partial derivatives to zero and solve the system of equations.

2.3 Second Derivative Test for Local Extreme Values

Let D be the **discriminant** or **Hessian** of f so that

$$D = f_{xx}f_{yy} - f_{xy}^2$$

then

1. f has a **local maximum** at (a, b) if $f_{xx} < 0$ and $D > 0$ at (a, b).
2. f has a **local minimum** at (a, b) if $f_{xx} > 0$ and $D > 0$ at (a, b).
3. f has a **saddle point** at (a, b) if $D < 0$ at (a, b).
4. **the test is inconclusive** at (a, b) if $D = 0$ at (a, b) and another method must be in order to determine the behavior at (a, b).

2.4 Finding Absolute Maxima and Minima on Closed Bounded Regions

In order to find absolute extrema for $f(x, y)$ on a closed and bounded region R,

1. *List the interior points of* R where f may have local maxima or minima and evaluate f at these points. These are critical points of f.
2. *List the boundary points of* R where f has local maxima and minima and evaluate f at these points. For every boundary, fix one or more of the variables in order to create a function of a single variable and find its local maxima and minima.
3. *Look through the lists* for the maximum and minimum values of f. These will be the absolute maximum and minimum values of f on R.