Calculus 3 Notes

Matthew Stringer

Course Page

Contents

1 Chapter 15 Double Integral

1.1 Definition of the Double Integral

Double Integral = volume below the graph $z = f(x, y)$ over a region R in xy-plane.

$$\int\int_R f(x,y)dA$$

Definition: Cut R into small pieces of area ΔA Lets divide the region R into i x and y components The height of each rectangular prism is $f(x_i, y_i)$ and the area of the region underneath is ΔA_i

$$\sum_i f(x_i, y_i)\Delta A_i$$

Finally, you get take the limit as $\Delta A_i \to 0$ to get the double integral.

1.2 Calculating the Double Integral

To compute the double integral, take **slices**. Let $S(x)$ = area of slice by plane parallel to the yz-plane. Then,

$$\text{volume} = \int_{x_{min}}^{x_{max}} S(x)dx.$$

$$S(x) = \int_{y_{min}(x)}^{y_{max}(x)} f(x,y)dy$$

$$\text{Finally, } \int\int_{\mathbf{R}} \mathbf{f(x,y)dA} = \int_{\mathbf{x_{min}}}^{\mathbf{x_{max}}} \left[\int_{\mathbf{y_{min}(x)}}^{\mathbf{y_{max}(x)}} \mathbf{f(x,y)dy}\right] \mathbf{dx}$$

This is called an **Iterated Integral** because one integral is integrated over the next. BTW: dA becomes $dy \cdot dx$ because dA is the area of each infinitesimal rectangle which equals $dy \cdot dx$

1.3 Double Integral in Polar Coordinate

Slices of circles on the polar coordinate plane are approximately rectangles when they are very tiny. Therefore,

$$\Delta A \approx \Delta r \Delta\theta$$

and therefore,

$$dA = dr d\theta$$

If you have the function $f(\theta, r)$, then in order to find the volume under the curve, use the formula

$$V = \int\int_R r f(\theta, r)dA$$

1.4 Applications of Double Integrals

1. Find area of a region R

$$A = \int\int_R 1 dA$$

2. Finding mass of a flat object with density δ =mass per unit area

$$\Delta m = \delta \Delta A$$
$$m = \int\int_R \delta dA$$

3. Finding average value of f in region R

$$\bar{f} = \frac{1}{\text{Area}} \int\int_R f dA$$

Weighted average of f with density δ:

$$\frac{1}{\text{Mass}(R)} \int\int_R f \delta dA$$

4. Center of Mass of a (planar) object (with density δ)?
Center of mass is at $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{1}{\text{Mass}} \int\int_R x \delta dA$$
$$\bar{y} = \frac{1}{\text{Mass}} \int\int_R y \delta dA$$

5. Moment of inertia of an object about the origin

$$\int\int_R r^2 \delta \, dA = \int\int_R (x^2 + y^2) \delta \, dA$$

Left off here