MAE 200 Final project

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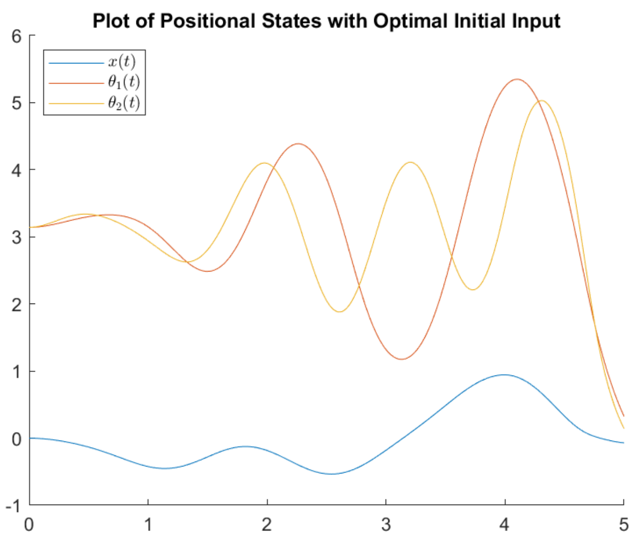
Based on code from github.com/matthew-d-stringer/Mae200-Final-Project

Step 1: Computing $u(t)$ on $t \in [0, T]$

For step 1, we must use adjoint-based optimization in order to compute the optimal input u_k. To do this, we use the provided code from Numerical Renaissance to calculate a more optimal input given a previous "guess" input. In order to get a satisfactory input, 2 things were done: First, an initial optimal input was generated given no input at all. Second, several more inputs were generated based on the previous computed optimal input. This results in a more optimal input. While generating inputs, I found that it was easier to generate satisfactory system responses when given a larger time horizon of 5 seconds rather than the default 3 second time horizon. This program in contained in `Dual_Pendulum_Input.m`



Step 2: State Estimation on $[0, T]$ based on noisy measurements

Depending on the covariance matrix of system noise and process noise, you can define Q and R weighting matrices. Then plugging these into equation 22.30 of Numerical Renaissance, you can march backwards with RK4 to find the steady state value of the P matrix that enables you answer to calculate an optimal L matrix. Using these values we can calculate the value of L with

$$L = -PC^H Q_2^{-1}$$

Step 3: Feedback Control

Using the Algebraic Ricatti Equation we can march backwards using RK4 to determine the X from equation 22.13a of Numerical Renaissance. By calculating this X matrix we can determine an optimal K matrix. This is done by solving the equation,

$$K = -R^{-1} B^H X.$$

Step 4: State Estimation based on ∞-horizon

I began with constructing my model of my system based on the linearized model around equation 22.34 of Numerical Renaissance.

Since E is invertible around $\vec{q} = \vec{0}$, we can solve for the A and B matrices from the standard form,

$$\dot{q} = Aq + Bu,$$

by inverting the E matrix.

After inputting these matrices into matlab, we are left with the following system:

```
sys =
    A =
             x1       x2       x3       x4       x5       x6
    x1        0        0        0        1        0        0
    x2        0        0        0        0        1        0
    x3        0        0        0        0        0        1
    x4        0    0.491    0.982        0        0        0
    x5        0    5.175   0.9428        0        0        0
    x6        0   0.9428     20.7        0        0        0

    B =
             u1
    x1        0
    x2        0
    x3        0
    x4   0.1044
    x5   0.1002
    x6   0.2004

    C =
         x1  x2  x3  x4  x5  x6
    y1    1   0   0   0   0   0
    y2    0   1   0   0   0   0
    y3    0   0   1   0   0   0

    D =
         u1
    y1    0
    y2    0
    y3    0

    Continuous-time state-space model.
```

Setting our Q matrix to 1, and our R matrix to 1, we create a Kalman filter using the `kalman` function. This results in an L matrix

```
L =
    0.4575    0.3394    0.3830
    0.3394    4.5172    0.2583
    0.3830    0.2583    9.0800
    0.2356    0.9622    1.9510
    0.8249   10.2934    1.8422
    1.7895    1.7995   41.3303
```

Step 5: Optimal Control

Using the linearized model described in Step 5, it is possible to utilize MATLAB's `lqr` function. Since we primarily care about the positional states of our system, we set their weights to be twice as large as their derivatives.

```
Q = diag([1 1 1 0.5 0.5 0.5]);
```

Since we only have one input, we can set its weight to 1.

```
R = 1;
```

This results in the following K matrix:

```
K =
    1.0000 -376.2629  680.7842    6.1759 -175.6396  154.7618
```