MAE 200 Final project

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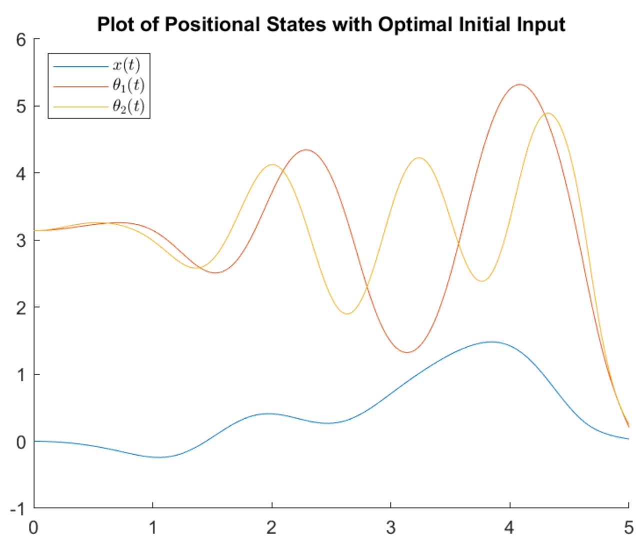
Step 1: Computing $u(t)$ on $t \in [0, T]$

For step 1, we must use adjoint-based optimization in order to compute the optimal input u_k. To do this, we use the provided code from Numerical Renaissance to calculate a more optimal input given a previous "guess" input. In order to get a satisfactory input, 2 things were done: First, an initial optimal input was generated given no input at all. Second, several more inputs were generated based on the previous computed optimal input. This results in a more optimal input. While generating inputs, I found that it was easier to generate satisfactory system responses when given a larger time horizon of 5 seconds rather than the default 3 second time horizon. This resulted in the following program from `Dual_Pendulum_Input.m`

```
T = 5;
u_k = zeros(T/0.01+1,1);
[u_k x_k] = NR_Dual_Pendulum(T, u_k);

t = 0:0.01:T;

for i = 1:3
    [u_k x_k] = NR_Dual_Pendulum(T, u_k);
end
close all
hold on
plot(t, x_k(1,:))
plot(t, x_k(2,:))
plot(t, x_k(3,:))
hold off
```



Step 2

Step 3

Step 4: State Estimation based on α-horizon

I began with constructing my model of my system based on the linearized model around equation 22.34 of Numerical Renaissance. This resulted in the following code:

```
E = [
    mc+m1+m2  -m1*l1       -m2*l1;
    -m1*l1   I1+m1*l1^2      0  ;
    -m2*l2    0          (I2 + m2*l2^2);
];

E = [
    eye(3) zeros(3)
    zeros(3) E
];

A_bar = [
    0   0       0
    0 m1*g*l2   0
    0   0      m2*g*l2
];

A_bar = [
    zeros(3)  eye(3)
    A_bar zeros(3)
];

B_bar = [
    0
    0
    0
    1
    0
    0
];
```

Since E is invertible around $\vec{q} = \vec{0}$, we can solve for the A and B matrices from the standard form,

$$\dot{q} = Aq + Bu,$$

by inverting the E matrix. Thus, we create the following code

```
A = inv(E)*A_bar;
B = inv(E)*B_bar;

C = eye(3, 6);
D = 0;

sys = ss(A,B,C,D);
```

After running this code, we are left with the following system

```
sys =
    A =
```

```
         x1      x2      x3      x4      x5      x6
x1        0       0       0       1       0       0
x2        0       0       0       0       1       0
x3        0       0       0       0       0       1
x4        0   0.491   0.982       0       0       0
x5        0   5.175  0.9428       0       0       0
x6        0  0.9428    20.7       0       0       0

B =
         u1
x1        0
x2        0
x3        0
x4   0.1044
x5   0.1002
x6   0.2004

C =
     x1  x2  x3  x4  x5  x6
y1    1   0   0   0   0   0
y2    0   1   0   0   0   0
y3    0   0   1   0   0   0

D =
     u1
y1    0
y2    0
y3    0

Continuous-time state-space model.
```

Setting our Q matrix to 1, and our R matrix to 1, we create a Kalman filter using the following code,

```
Q = 1;
R = 1;
[kalmf,L,~,Mx,Z] = kalman(sys,Q,R);
```

This results in an L matrix

```
L =
    0.4575    0.3394    0.3830
    0.3394    4.5172    0.2583
    0.3830    0.2583    9.0800
    0.2356    0.9622    1.9510
    0.8249   10.2934    1.8422
    1.7895    1.7995   41.3303
```

Step 5: Optimal Control

Using the linearized model described in Step 5, it is possible to utilize MATLAB's `lqr` function. Since we primarily care about the positional states of our system, we set their weights to be twice as large as their derivatives.

```
Q = diag([1 1 1 0.5 0.5 0.5]);
```

Since we only have one input, we can set its weight to 1.

```
R = 1;
```