Physics Notes

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1 Kinematics

1.1 Describing Motion 1

1.1.1 Average Speed

- Average speed is the distance traveled over change in time
- It is a scaler
- Measured in meters/second.
- Magnitude of Velocity Vector

1.1.2 Average Velocity

- Average velocity is a vector.
- Measured in meters/second.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

1.1.3 Average Velocity

- Rate that velocity changes
- Is a vector
- Units are meters/second/second

$$a = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

1.1.4 Displacement

The displacement from t_0 to t_1 of a position function $x(t)$ with velocity function $v(t)$ is

$$\int_{t_0}^{t_1} v(t)dt$$

1.2 Describing Motion 2

1.2.1 Kinematic Equations

Variables

v_0	Initial velocity
v	Final velocity
Δx	Displacement
a	Acceleration
t	Time

- $v = v_0 + at$
- $x = x_0 + v_0 t + \frac{1}{2}at^2$
- $v^2 = v_0^2 + 2a\Delta x$

1.2.2 Acceleration Due to Gravity

- Near the surface of Earth, objects accelerate at a rate of $9.8\frac{m}{s^2}$
- This is acceleration due to gravity (g)
- This can be approximated to $10\frac{m}{s^2}$
- As you move from Earth, acceleration decreases.

1.2.3 Objects Falling From Rest

- Objects starting from rest have $v_0 = 0$
- Typically down is the positive direction
- Acceleration is $+g$.

1.2.4 Objects Launched Upward

- Must examine the motion of object going up and down.
- Since object is going up, that is the positive direction.
- Acceleration is $-g$.
- At the highest point, $v = 0$.

1.3 Projectile Motion

A **projectile** is an object that is acted upon only gravity.

1.3.1 Independence of Motion

- Projectiles launched at an angle have motion in 2 dimensions.
  - Vertical - acceleration is gravity
  - Horizontal - 0 acceleration
- Vertical and Horizontal motion are treated separately

> **Note:** An object will travel the maximum horizontal distance with a launch angle of 45°

1.3.2 Steps for any Projectile Motion Problem

1. First, know that
$$a = \begin{bmatrix} 0 \\ -g \end{bmatrix}$$
2. Then find your v_0 as a vector
3. Find your x_0 as a vector
4. Substitute your vectors into the following formula
$$x(t) = -\frac{1}{2}at^2 + v_0 t + x_0$$

1.3.3 Graphing Projectile Motion

In order to graph a path, solve for $y = f(x)$. Do this by solving for t in relation to x and then substitute into the y component.
For example:

$$\begin{aligned} x &= f(t) \\ y &= g(t) \\ \text{Find } y &= h(x) \\ t &= f^{-1}(x) \\ y = g(f^{-1}(x)) &= h(x) \\ \text{so } h(x) &= g(f^{-1}(x)) \end{aligned}$$

1.4 Circular And Relative Motion

1.4.1 Converting Linear to Angular Velocity

If we have an object moving counter clockwise around a point, let $\omega = \frac{d\theta}{dt}$. If we know that the object has velocity v and position s, we know that $s = r\theta$ where r is the radius of the circular path. By taking the derivative of both sides, $\dot{s} = r\dot{\theta}$. Now we can substitute to find the angular velocity.

$$\dot{s} = r\omega$$

2 Dynamics

2.1 Newton's First Law and Free Body diagrams

Newton's First Law

> An object at rest will remain at rest, and an object in motion will remain in motion, at constant velocity and in a straight line, unless acted upon by a net force.

2.1.1 Force

- A force is a push or pull on an object.
- Units of force are in Newtons (N).
- A newton is roughly the weight of an apple

$$1N = 1\frac{kg * m}{s^2}$$

Contract Force
A force that arises that from direct contact between objects.

- Tension
- Applied Force
- Friction

Field Force
Forces that act at a distance.

- Gravity
- Electrical

- Magnetic

2.1.2 Net Force

A net force is the vector sum of all the forces acting on an object.

$$F_{net} = \sum F$$

2.1.3 Equilibrium

- Static Equilibrium
  - Net force is 0
  - Net torque is 0
  - Object is at rest
- Mechanical Equilibrium
  - Net force is 0
  - Net torque is 0
- Translational Equilibrium
  - Net force is 0

2.1.4 Free Body Diagram

A Free Body Diagram (FBD) is a diagram that maps all of the forces that are applied to a single object.

2.2 Newton's 2nd and 3rd Laws of Motion

2.2.1 Newton's 2nd Law of Motion

- The acceleration of an object is in the direction of and directly proportional to the the net force applied, and inversely proportional to the object's mass.
- Valid only in *inertial reference frames*.

$$F_{net} = \sum F = ma$$

2.2.2 Mass vs. Weight

- Mass is the amount of stuff that something is made up of (independent of gravity)
- Weight is the force of gravity on an object. (dependent on gravity)

2.2.3 Newton's 3rd Law of Motion

- All forces come in pairs
- If Object 1 exerts a force on Object 2, then Object 2 must exert a force back on Object 2.
- This counter force is equal in magnitude and opposite in direction.

$$F_{1on2} = -F_{2on1}$$

2.3 Friction

2.3.1 Coefficient of Friction

- Ratio of the frictional force and the normal force
- 2 kinds:
  1. Kinetic (when 2 objects are rubbing)
  2. Static (when 2 objects are not sliding)

$$\mu = \frac{F_f}{F_N}$$

which results in

$$F_f = \mu F_N$$

where F_f is the force of friction, F_N is the normal force, and μ is the coefficient of friction.

2.4 Retarding or Drag Forces

2.4.1 Retarding Forces

- Frictional forces can be functions of the object's velocity
- These forces are called drag or retarding forces

2.4.2 The Skydiver

Typically drag forces on a free-falling object take the form of

$$F_{drag} = bv$$

or

$$F_{drag} = cv^2$$

By using Newton's 2nd Law, create a differential equation. Then use separation of variables to solve for velocity, then acceleration, then position.

2.5 Ramps and Inclines

2.5.1 Drawing FBD for Ramps

1. Choose the object and draw it as a dot or box
2. Draw and Label all the External Forces
3. Sketch a Coordinate System

2.6 Atwood Machine

2.6.1 What is an Atwood Machine

Two objects connected by a light string over a massless pulley

2.6.2 Properties of Atwood Machines

- Ideal pulleys are frictionless and massless
- Tension is constant in a light string passing over an ideal pulley

2.6.3 Setup for Atwood Machines

1. Adopt a sign convention for positive and negative motion
2. Analyze each mass separately using Newton's 2nd Law.

2.6.4 Solution

$$\begin{aligned} F_y = m_1 g - m_2 g &= (m_1 + m_2)a \\ (m_1 - m_2)g &= (m_1 + m_2)a \\ a &= g\frac{(m_1 - m_2)}{(m_1 + m_2)} \end{aligned}$$

3 Work, Energy, and Power

3.1 Work

3.1.1 What is Work

- Work is the process of moving an object by applying a force
- The object must move
- The force must cause the movement

- Work is measured in Joules

$$W = F \cdot \Delta x = F \Delta x \cos\theta$$

3.1.2 Non-Constant Forces

- Work done is the area under the force vs. displacement graph.

$$W = \int_{x_i}^{x_f} F(x)dx$$

3.1.3 Hook's Law

- The more you stretch or compress a spring, the greater the force of the spring.
- The spring's force is opposite the direction of its displacement from equilibrium.
- This is modeled as a linear relationship.

$$F_s = -kx$$

3.1.4 Determining the Spring Constant

- Graph Force vs Displacement for the Spring
- The slope of this graph is the Spring Constant

$$k = \frac{\Delta F}{\Delta x}$$

3.1.5 Work in Multiple Dimensions

$$W = \int dW$$
$$W = \int_{r_1}^{r_2} F \cdot dr$$

3.1.6 Work-Energy Theorem

$$W = \int_{x_i}^{x_f} F(x)dx$$
$$F = ma = m\frac{dv}{dt} \qquad v = \frac{dx}{dt} \quad dx = vdt$$
$$W = \int m\frac{dv}{dt}v\ dt$$
$$W = \int_{v_i}^{v_f} mv\ dv$$

$$\begin{aligned}\mathbf{W} &= \mathbf{m}\int_{\mathbf{v_i}}^{\mathbf{v_f}} \mathbf{v}\ \mathbf{dv} \\ &= m\left.\frac{v^2}{2}\right|_{v_i}^{v_f} \\ &= m\left(\frac{v_f^2 - v_i^2}{2}\right) \\ K = \frac{1}{2}mv^2 &\quad K \text{ is kinetic energy} \\ W &= K_f - K_i = \Delta K\end{aligned}$$

Energy Formula:

$$\mathbf{K} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{mv^2}$$

3.2 Energy and Conservative Forces

3.2.1 What is Energy?

- Energy is the ability to do work
- in other words, Energy is the ability to move an object

3.2.2 Kinetic Energy

- Kinetic Energy is energy of motion.
  - The ability or capacity of moving object move another object.

$$K = \frac{1}{2}mv^2$$

3.2.3 Potential Energy

- Potential Energy (U) is energy an object possesses due to its position or state of being.
- A single object can only have kinetic energy, because potential energy requires an interaction between objects.

3.2.4 Internal Energy

- The internal energy of a system includes the kinetic energy and potential energy.
- By changing a system's internal structure, you can change its internal energy.

3.2.5 Gravitational Potential Energy (U_g)

$$U_g = mg\Delta h$$

3.2.6 Conservative Forces

- A force in which the work done on an object is independent of the path taken
- A force in which the work done moving along a closed path is zero
- A force in which the work done is directly related to the change in potential energy

$$W = -\Delta U$$

Conservative Forces

- Gravity
- Elastic Forces
- Coulumbic

Non-Conservative Forces

- Friction
- Drag
- Air Resistance

3.2.7 Work Done by Conservative Forces

$$\begin{aligned} W = -\Delta U \implies \Delta U &= -W \\ \Delta U &= -\int_{r_i}^{r_f} F \cdot dr \end{aligned}$$

3.2.8 Newton's Law of Universal Gravitation

$$F_g = \frac{-Gm_1m_2}{r^2}\hat{r}$$

3.2.9 Gravitational Potential Energy

$$\begin{aligned} U_g &= -\int_{\infty}^{r} F_g \cdot dr \\ U_g &= Gm_1m_2 \int_{\infty}^{r} r^{-2} dr \\ U_g &= Gm_1m_2 \left[-r^{-1}\right]_{\infty}^{r} \\ U_g &= Gm_1m_2 \left(-r^{-1} - 0\right) \\ U_g &= -\frac{Gm_1m_2}{r} \end{aligned}$$

3.2.10 Elastic Potential Energy

$$U_s = -\int_0^x F_s \cdot dx$$
$$U_s = -\int_0^x -kx dx$$
$$U_s = k\left[\frac{x^2}{2}\right]_0^x$$
$$U_s = \frac{kx^2}{2}$$

3.2.11 Force from Potential Energy

$$\begin{aligned} dU = -dW_f &= -F \cdot dl \\ &= -F\cos\theta dl \\ &= -F_l dl \\ F_l = -\frac{dU}{dl} \end{aligned}$$

Where F_l is the force in the direction of potential energy

3.2.12 Gravitational Force from the Gravitational Potential Energy

$$F_r = -\frac{dU}{dr}$$
$$F_r = \frac{d}{dr}\frac{Gm_1m_2}{r}$$
$$F_r = Gm_1m_2\frac{d}{dr}r^{-1}$$
$$F_r = Gm_1m_2\frac{-1}{r^2}$$
$$F_r = -\frac{Gm_1m_2}{r^2}$$
$$F = -\frac{Gm_1m_2}{r^2}\hat{r}$$

3.3 Conservation of Energy

3.3.1 Conservation of Mechanical Energy

$$\begin{aligned} W_f = \Delta K \quad W_f &= -\Delta U \\ \Delta K &= -\Delta U \\ \Delta K + \Delta U &= 0 \end{aligned}$$

3.3.2 Non-Conservative Forces

- change total mechanical energy of a system
- Work done is typically converted to internal (thermal) energy.

$$E_{TOTAL} = K + U + W_{NC}$$
$$E_{MECHANICAL} = K + U$$

Where W_{NC} is work done by non-conservative forces

3.4 Power

3.4.1 Definition

- Power is the rate at which work is done.
- Units of power are joules/second, or watts
- Average power:

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

3.4.2 Instantaneous Power

$$\begin{aligned} P &= \frac{dW}{dt} \\ &= \frac{Fdr}{dt} \\ &= F \cdot v \end{aligned}$$

4 Momentum And Impulse

4.1 Definition of Momentum

- vector describing how difficult it is to stop a moving object
- Total Momentum is the sum of individual momenta

$$p = mv$$

4.2 Impulse

- Impulse is change in Momentum (J)

$$J = \Delta p$$

4.3 Relationship between Force and Δp

$$F = ma$$
$$F = m\frac{dv}{dt}$$
$$F = \frac{d}{dt}(mv)$$
$$F = \frac{dp}{dt}$$

4.4 Impulse-Momentum Theorem

$$F = \frac{dp}{dt}$$
$$\int_0^t F \cdot dt = \int_{p_i}^{p_f} dp$$
$$J = F\Delta t = \Delta p$$

5 Rotational Motion

5.1 Rotational Kinematics

5.1.1 Linear vs Angular Velocity

- Linear speed given by v
- Angular speed given by ω
- Direction is perpendicular to the path based on right hand rule

5.1.2 Converting Linear to angular velocity

$$v = r\frac{d\theta}{dt} = r\omega$$
$$\omega = \frac{v}{r}$$

5.1.3 Linear vs Angular acceleration

- Linear acceleration given by a
- Angular acceleration given by α

5.1.4 Centripetal Acceleration

$$a = -\frac{v^2}{r}$$

5.2 Moment of Inertia

5.2.1 Types of Inertia

- Inertial mass (linear inertia) is an object's ability to resist linear acceleration.
- Moment of Inertia (rotational inertia) is an object's ability to resist rotational acceleration.

5.2.2 Kinetic Energy of Rotating Disk

$$K_{TOT} = \frac{\omega^2}{2}\int_0^r r^2 dr = \frac{1}{2}J\omega^2$$

5.2.3 Common Moments of Inertia

- Disk: $J = \frac{1}{2}mr^2$
- Hoop: $J = ml^2$
- Sphere: $J = \frac{2}{5}mr^2$
- Hollow Sphere: $J = \frac{2}{3}mr^2$
- Rod(around center point): $J = \frac{1}{12}ml^2$
- Rod(around end point): $J = \frac{1}{3}ml^2$

5.3 Torque

$$\tau = r \times F$$
$$|\tau| = rF\sin\theta$$

5.3.1 Direction of Torque Vector

- Torque vector is perpendicular to both force and position vector
- Use the right hand rule
- Positive Torques cause counter-clockwise rotations

5.3.2 Translational vs Rotational

$$F = ma \quad \tau = J\alpha$$

5.4 Angular Momentum (L)

$$L_Q = r \times p = (r \times v)m$$

where r is a vector from point Q to the force