

# Physics Notes

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# 1 Kinematics

## 1.1 Describing Motion 1

### 1.1.1 Average Speed

- Average speed is the distance traveled over change in time
- It is a scalar
- Measured in meters/second.
- Magnitude of Velocity Vector

### 1.1.2 Average Velocity

- Average velocity is a vector.
- Measured in meters/second.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

### 1.1.3 Average Velocity

- Rate that velocity changes
- Is a vector
- Units are meters/second/second

$$a = \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

### 1.1.4 Displacement

The displacement from  $t_0$  to  $t_1$  of a position function  $x(t)$  with velocity function  $v(t)$  is

$$\int_{t_0}^{t_1} v(t) dt$$

## 1.2 Describing Motion 2

### 1.2.1 Kinematic Equations

Variables	
$v_0$	Initial velocity
$v$	Final velocity
$\Delta x$	Displacement
$a$	Acceleration
$t$	Time

- $v = v_0 + at$
- $x = x_0 + v_0t + \frac{1}{2}at^2$
- $v^2 = v_0^2 + 2a\Delta x$

### 1.2.2 Acceleration Due to Gravity

- Near the surface of Earth, objects accelerate at a rate of  $9.8 \frac{m}{s^2}$
- This is acceleration due to gravity ( $g$ )
- This can be approximated to  $10 \frac{m}{s^2}$
- As you move from Earth, acceleration decreases.

### 1.2.3 Objects Falling From Rest

- Objects starting from rest have  $v_0 = 0$
- Typically down is the positive direction
- Acceleration is  $+g$ .

### 1.2.4 Objects Launched Upward

- Must examine the motion of object going up and down.
- Since object is going up, that is the positive direction.
- Acceleration is  $-g$ .
- At the highest point,  $v = 0$ .

## 1.3 Projectile Motion

A **projectile** is an object that is acted upon only gravity.

### 1.3.1 Independence of Motion

- Projectiles launched at an angle have motion in 2 dimensions.
  - Vertical - acceleration is gravity
  - Horizontal - 0 acceleration
- Vertical and Horizontal motion are treated separately

**Note:** An object will travel the maximum horizontal distance with a launch angle of  $45^\circ$

### 1.3.2 Steps for any Projectile Motion Problem

1. First, know that

$$a = \begin{bmatrix} 0 \\ -g \end{bmatrix}$$

2. Then find your  $v_0$  as a vector
3. Find your  $x_0$  as a vector
4. Substitute your vectors into the following formula

$$x(t) = -\frac{1}{2}at^2 + v_0t + x_0$$

### 1.3.3 Graphing Projectile Motion

In order to graph a path, solve for  $y = f(x)$ . Do this by solving for  $t$  in relation to  $x$  and then substitute into the  $y$  component.

For example:

$$x = f(t)$$

$$y = g(t)$$

$$\text{Find } y = h(x)$$

$$t = f^{-1}(x)$$

$$y = g(f^{-1}(x)) = h(x)$$

$$\text{so } h(x) = g(f^{-1}(x))$$

## 1.4 Circular And Relative Motion

### 1.4.1 Converting Linear to Angular Velocity

If we have an object moving counter clockwise around a point, let  $\omega = \frac{d\theta}{dt}$ . If we know that the object has velocity  $v$  and position  $s$ , we know that  $s = r\theta$  where  $r$  is the radius of the circular path. By taking the derivative of both sides,  $\dot{s} = r\dot{\theta}$ . Now we can substitute to find the angular velocity.

$$\dot{s} = r\omega$$

## 2 Dynamics

### 2.1 Newton's First Law and Free Body diagrams

#### Newton's First Law

An object at rest will remain at rest, and an object in motion will remain in motion, at constant velocity and in a straight line, unless acted upon by a net force.

#### 2.1.1 Force

- A force is a push or pull on an object.
- Units of force are in Newtons (N).
- A newton is roughly the weight of an apple

$$1N = 1 \frac{kg * m}{s^2}$$

#### Contract Force

A force that arises that from direct contact between objects.

- Tension
- Applied Force
- Friction

#### Field Force

Forces that act at a distance.

- Gravity
- Electrical

- Magnetic

### 2.1.2 Net Force

A net force is the vector sum of all the forces acting on an object.

$$F_{net} = \sum F$$

### 2.1.3 Equilibrium

- Static Equilibrium
  - Net force is 0
  - Net torque is 0
  - Object is at rest
- Mechanical Equilibrium
  - Net force is 0
  - Net torque is 0
- Translational Equilibrium
  - Net force is 0

### 2.1.4 Free Body Diagram

A Free Body Diagram (FBD) is a diagram that maps all of the forces that are applied to a single object.

## 2.2 Newton's 2nd and 3rd Laws of Motion

### 2.2.1 Newton's 2nd Law of Motion

- The acceleration of an object is in the direction of and directly proportional to the the net force applied, and inversely proportional to the object's mass.
- Valid only in *inertial reference frames*.

$$F_{net} = \sum F = ma$$

### 2.2.2 Mass vs. Weight

- Mass is the amount of stuff that something is made up of (independent of gravity)
- Weight is the force of gravity on an object. (dependent on gravity)



### 2.2.3 Newton's 3rd Law of Motion

- All forces come in pairs
- If Object 1 exerts a force on Object 2, then Object 2 must exert a force back on Object 1.
- This counter force is equal in magnitude and opposite in direction.

$$F_{1on2} = -F_{2on1}$$

## 2.3 Friction

### 2.3.1 Coefficient of Friction

- Ratio of the frictional force and the normal force
- 2 kinds:
  1. Kinetic (when 2 objects are rubbing)
  2. Static (when 2 objects are not sliding)

$$\mu = \frac{F_f}{F_N}$$

which results in

$$F_f = \mu F_N$$

where  $F_f$  is the force of friction,  $F_N$  is the normal force, and  $\mu$  is the coefficient of friction.

## 2.4 Retarding or Drag Forces

### 2.4.1 Retarding Forces

- Frictional forces can be functions of the object's velocity
- These forces are called drag or retarding forces

### 2.4.2 The Skydiver

Typically drag forces on a free-falling object take the form of

$$F_{drag} = bv$$

or

$$F_{drag} = cv^2$$

By using Newton's 2nd Law, create a differential equation. Then use separation of variables to solve for velocity, then acceleration, then position.

## 2.5 Ramps and Inclines

### 2.5.1 Drawing FBD for Ramps

1. Choose the object and draw it as a dot or box
2. Draw and Label all the External Forces
3. Sketch a Coordinate System

## 2.6 Atwood Machine

### 2.6.1 What is an Atwood Machine

Two objects connected by a light string over a massless pulley

### 2.6.2 Properties of Atwood Machines

- Ideal pulleys are frictionless and massless
- Tension is constant in a light string passing over an ideal pulley

### 2.6.3 Setup for Atwood Machines

1. Adopt a sign convention for positive and negative motion
2. Analyze each mass separately using Newton's 2nd Law.

### 2.6.4 Solution

$$\begin{aligned}F_y &= m_1g - m_2g = (m_1 + m_2)a \\(m_1 - m_2)g &= (m_1 + m_2)a \\a &= g \frac{(m_1 - m_2)}{(m_1 + m_2)}\end{aligned}$$

## 3 Work, Energy, and Power

### 3.1 Work

#### 3.1.1 What is Work

- Work is the process of moving an object by applying a force
- The object must move
- The force must cause the movement

- Work is measured in Joules

$$W = F \cdot \Delta x = F \Delta x \cos \theta$$

### 3.1.2 Non-Constant Forces

- Work done is the area under the force vs. displacement graph.

$$W = \int_{x_i}^{x_f} F(x) dx$$

### 3.1.3 Hook's Law

- The more you stretch or compress a spring, the greater the force of the spring.
- The spring's force is opposite the direction of its displacement from equilibrium.
- This is modeled as a linear relationship.

$$F_s = -kx$$

### 3.1.4 Determining the Spring Constant

- Graph Force vs Displacement for the Spring
- The slope of this graph is the Spring Constant

$$k = \frac{\Delta F}{\Delta x}$$

### 3.1.5 Work in Multiple Dimensions

$$W = \int dW$$

$$W = \int_{r_1}^{r_2} F \cdot dr$$

### 3.1.6 Work-Energy Theorem

$$W = \int_{x_i}^{x_f} F(x) dx$$

$$F = ma = m \frac{dv}{dt} \quad v = \frac{dx}{dt} \quad dx = v dt$$

$$W = \int m \frac{dv}{dt} v dt$$

$$W = \int_{v_i}^{v_f} mv dv$$

$$\begin{aligned}
 W &= m \int_{v_i}^{v_f} v \, dv \\
 &= m \left. \frac{v^2}{2} \right|_{v_i}^{v_f} \\
 &= m \left( \frac{v_f^2 - v_i^2}{2} \right)
 \end{aligned}$$

$$K = \frac{1}{2}mv^2 \quad K \text{ is kinetic energy}$$

$$W = K_f - K_i = \Delta K$$

Energy Formula:

$$K = \frac{1}{2}mv^2$$

## 3.2 Energy and Conservative Forces

### 3.2.1 What is Energy?

- Energy is the ability to do work
- in other words, Energy is the ability to move an object

### 3.2.2 Kinetic Energy

- Kinetic Energy is energy of motion.
  - The ability or capacity of moving object move another object.

$$K = \frac{1}{2}mv^2$$

### 3.2.3 Potential Energy

- Potential Energy (U) is energy an object possesses due to its position or state of being.
- A single object can only have kinetic energy, because potential energy requires an interaction between objects.

### 3.2.4 Internal Energy

- The internal energy of a system includes the kinetic energy and potential energy.
- By changing a system's internal structure, you can change its internal energy.

### 3.2.5 Gravitational Potential Energy ( $U_g$ )

$$U_g = mg\Delta h$$

### 3.2.6 Conservative Forces

- A force in which the work done on an object is independent of the path taken
- A force in which the work done moving along a closed path is zero
- A force in which the work done is directly related to the change in potential energy

$$W = -\Delta U$$

Conservative Forces

- Gravity
- Elastic Forces
- Coulumbic

Non-Conservative Forces

- Friction
- Drag
- Air Resistance

### 3.2.7 Work Done by Conservative Forces

$$W = -\Delta U \implies \Delta U = -W$$

$$\Delta U = - \int_{r_i}^{r_f} F \cdot dr$$

### 3.2.8 Newton's Law of Universal Gravitation

$$F_g = \frac{-Gm_1m_2}{r^2} \hat{r}$$

### 3.2.9 Gravitational Potential Energy

$$U_g = - \int_{\infty}^r F_g \cdot dr$$

$$U_g = Gm_1m_2 \int_{\infty}^r r^{-2} dr$$

$$U_g = Gm_1m_2 [-r^{-1}]_{\infty}^r$$

$$U_g = Gm_1m_2 (-r^{-1} - 0)$$

$$U_g = -\frac{Gm_1m_2}{r}$$

### 3.2.10 Elastic Potential Energy

$$\begin{aligned}
 U_s &= - \int_0^x F_s \cdot dx \\
 U_s &= - \int_0^x -kx dx \\
 U_s &= k \left[ \frac{x^2}{2} \right]_0^x \\
 U_s &= \frac{kx^2}{2}
 \end{aligned}$$

### 3.2.11 Force from Potential Energy

$$\begin{aligned}
 dU &= -dW_f = -F \cdot dl \\
 &= -F \cos \theta dl \\
 &= -F_l dl \\
 F_l &= -\frac{dU}{dl}
 \end{aligned}$$

Where  $F_l$  is the force in the direction of potential energy

### 3.2.12 Gravitational Force from the Gravitational Potential Energy

$$\begin{aligned}
 F_r &= -\frac{dU}{dr} \\
 F_r &= \frac{d}{dr} \frac{Gm_1m_2}{r} \\
 F_r &= Gm_1m_2 \frac{d}{dr} r^{-1} \\
 F_r &= Gm_1m_2 \frac{-1}{r^2} \\
 F_r &= -\frac{Gm_1m_2}{r^2} \\
 F &= -\frac{Gm_1m_2}{r^2} \hat{r}
 \end{aligned}$$

## 3.3 Conservation of Energy

### 3.3.1 Conservation of Mechanical Energy

$$\begin{aligned}
 W_f &= \Delta K \quad W_f = -\Delta U \\
 \Delta K &= -\Delta U \\
 \Delta K + \Delta U &= 0
 \end{aligned}$$

### 3.3.2 Non-Conservative Forces

- change total mechanical energy of a system
- Work done is typically converted to internal (thermal) energy.

$$E_{TOTAL} = K + U + W_{NC}$$

$$E_{MECHANICAL} = K + U$$

Where  $W_{NC}$  is work done by non-conservative forces

## 3.4 Power