Junior Sophomore Experimental Physics Laboratory Report

Trinity College Dublin, School of Physics

# Group A Laboratory 2: The Hall Effect

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October 25, 2023


## 1 Abstract

In this experiment, the characteristics of an electromagnet, a Halbach magic cylinder, and a lock-in amplifier (LIA) were investigated as useful apparatus when observing and measuring the classical Hall effect for different Germanium crystal semiconductors. The Hall effect is the build up of charge on one side of a semiconductor with current flowing through it due to an external magnetic field. The electromagnet displayed hysteresis behaviour and a narrow hysteresis loop was measured and plotted. It was then established that the Hall voltage was directly proportional to the strength of the magnetic field being applied to the semiconductor. The charge carriers were found to be electrons for the sample used in the first two experiments and holes for the other sample used in subsequent experiments. For n-type germanium, it was determined that the hall coefficient was $R_H = -14.89\ (\pm 0.08) \times 10^{-3} m^3 C^{-1}$ for circuit current $I = 20mA$, the electron concentration was $N_- = 4.12\ (\pm 0.03) \times 10^{20} m^{-3}$, and the electron mobility was $\mu_- = 1814.50\ (\pm 5.51) cm^2/Vs$. For p-type Germanium the same parameters were calculated as $R_H = 12.4\ (\pm 1.8) \times 10^{-3} m^3 C^{-1}$, $N_+ = 5.030\ (\pm 0.04) \times 10^{20} m^{-3}$, $\mu_+ = 2884\ (\pm 53)\ cm^2/Vs$. All of the calculated values were of the expected orders of magnitude when compared with accepted values. In the final experiment, it was found that for a rotating magnetic field, the LIA is able to make measurements which are even more accurate than for the static magnetic field. Measurements of time varying voltage using the LIA showed that the peak value would differ from the minimum value by a 90° phase shift as the signal would be destructed by the LIA without resonance with the chosen reference frequency.

# 2 Introduction and Theory

The Hall effect, discovered by Edwin Hall in 1879[1], is a phenomenon in which an electric field is generated by the separation of equal and opposite charges in a metal or semiconductor due to an external magnetic field. The Hall effect can be understood in general using the laws of electromagnetism. Consider a semiconductor of dimensions $(l, w, t)$ with a current flowing directly along the direction of the object's length, say $\hat{l}$. Then the object-specific current density $\vec{J}$ (of units Amps per square-metre) is given by

$$\vec{J} = \frac{\vec{I}}{wt}, \tag{2.1}$$

where the product $wt$ defines the cross-sectional area of the material. If a magnetic field $\vec{B} \perp \vec{J}$ is applied to the material, then the charge carriers (electrons or holes) will experience a Lorentz force given by

$$\vec{F} = q\vec{v} \times \vec{B}, \tag{2.2}$$

where $\vec{v}$ is the velocity of the material's charge carriers in the $\hat{l}$ direction. This force causes an equal and opposite build up of charge on both sides of the semiconductor leading to a potential difference known as the *Hall voltage* $(V_H)$. The electric field associated with this charge distribution is the *Hall field* $(\vec{E}_H)$ and we see it can be written as $|\vec{E}_H| = V_H/w$, or in vector form

$$\vec{E}_H = R_H \vec{B} \times \vec{J}, \tag{2.3}$$

where $R_H$ is the Hall coefficient of units $m^3/C$. Taking the magnitude of the vector equation (2.3) and relating it to the Hall voltage, we see that

$$V_H = R_H BJw \implies V_H = \frac{R_H BI}{t} \implies R_H = \frac{V_H t}{BI}, \tag{2.4}$$

since $\vec{B} \perp \vec{J}$. The current density can be expressed in a different form $\vec{J} = Nq\vec{v}$, where $q$ is the charge of the carriers in the material and $N$ is the carrier concentration per unit volume. Given that nature strives for equilibrium, the Hall field $\vec{E}_H$ will continue to increase until its force $\vec{F}_E = q\vec{E}_H$ on the charge carriers is equal to the Lorentz force so that the carriers are once again able to move through the material with no deflection. Once this balance is reached we may write

$$\vec{F}_E = -\vec{F}_B \implies q\vec{E}_H = -q\vec{v} \times \vec{B} \implies \vec{E}_H = \vec{B} \times \vec{v} = \frac{1}{Nq}\vec{B} \times \vec{J}, \tag{2.5}$$

which when compared with equation (2.3), reveals that the Hall coefficient can be written as

$$R_H = \frac{1}{Nq}. \tag{2.6}$$

Therefore the sign of the hall coefficient (and thus the direction of $\vec{E}_H$) depends on whether the charge carriers are holes (+) or electrons (−). In experiment, we can determine $R_H$ using equation (2.4) and use the result to determine whether electrons or holes are the primary charge carriers of the sample.

When the value of $R_H$ is known then the carrier concentration $N$ can also be calculated, one can obtain the conductivity of the sample $\sigma$ of units ($Sm^{-1}$) through measurement, then the carrier mobility $\mu$ of units ($m^2V^{-1}s^{-1}$) is given by

$$\mu = \frac{\sigma}{Nq} \implies \mu = \frac{1}{Nq\rho}, \tag{2.7}$$

where $\rho$ is the sample-specific resistivity of units ($\Omega m$) given by $\rho = Rwt/l$ where $R$ is the resistance of the sample.

# 3 Experimental Setup

## Electromagnet and Germanium Circuit

The Hall effect was investigated for a Germanium crystal circuit under the influence of two types of magnetic fields. In the first case, the magnetic field was generated by an electromagnet and the setup can be seen in Figure (3.1).

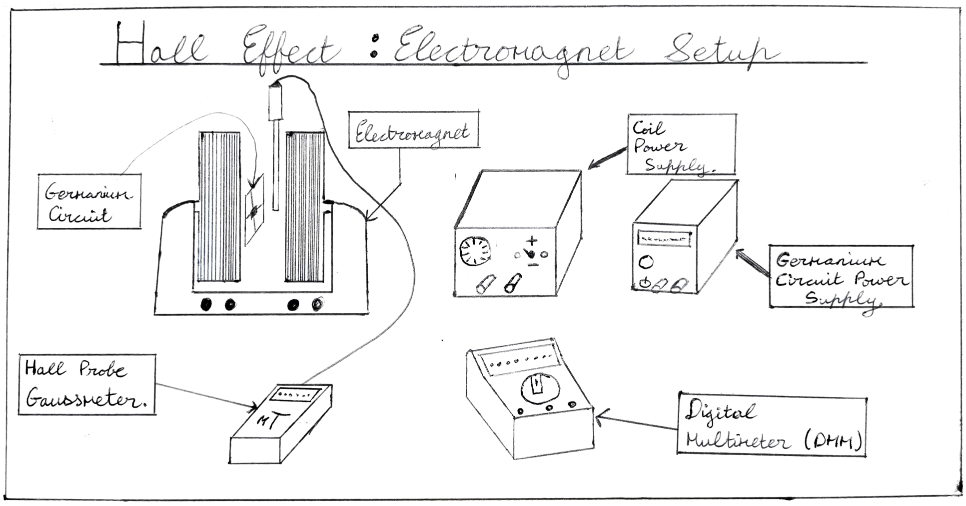
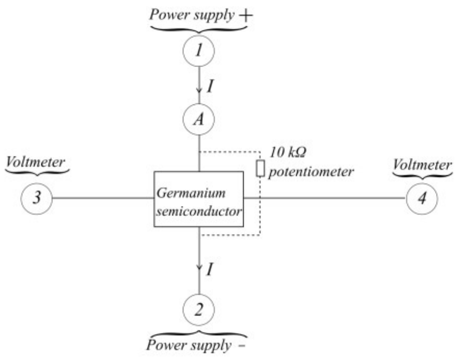



Figure 3.1: A sketch of the equipment used in the first two sections of the experiment where an electromagnet is used to produce a constant field $\vec{B}$.

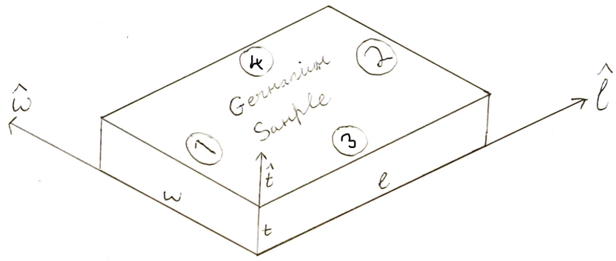
The electromagnet creates a magnetic field directed perpendicular to the Germanium circuit as desired and the value of $B$ is subsequently measured in $mT$ by the Hall probe Gauss-meter. Digital multi-meters are used to measure the current through the electromagnet $I_C$, the Hall voltage $V_H$ and the current being supplied to the Germanium crystal $I$.

The electromagnet used has an iron core which is a soft ferromagnetic material. When the current $I_C$ is increased through the coil an imposed field $H$ is applied to the iron core, and the magnetisation $M$ of the core increases irreversibly and non-linearly until it reaches a maximum value[3]. The contribution of the magnetised core to the external field $B$ is non-trivial and since $M$ responds to $H$ and not $B$, when $H$ is reduced to zero, a small remnant contribution from the magnetised core will remain due to the non-linear response. A plot of $M$ as a function of $H$ in this case is called a hysteresis loop. For a soft material such as iron, the loop is narrow which means that the response of $M$ is not so delayed and the magnetisation can be saturated easily. For a hard ferromagnetic such as the material used in the Halbach cylinder, the hysteresis loop is significantly wider (almost square shaped). The hard material can be used as a permanent magnet since once it is magnetised, $M$ remains at a maximum value when the imposed field is turned off.

The chosen Germanium crystal circuit has a structure common to the analogous circuit used in later sections of the experiment. A schematic diagram of the circuit can be seen below in Figure (3.2)[2].




(a) A diagram of the circuit as seen from above. Note that the current travels parallel to the direction $1 \to 2$.




(b) Orthonormal coordinate system defined by the geometry of the sample. We will use these basis vectors when dealing with vector quantities in the experiment.

Figure 3.2: Germanium Circuit and its natural coordinates.

The dimensions of the sample are $(l, w, t) = (10.00, 5.00, 1.00) \pm 0.02mm$. When measuring the voltage $V_{34}$ between terminals (3) and (4), one must account for the fact that they are slightly misaligned due to a vertical displacement in the circuit. To ensure that $V_{34} = V_H$, it must be considered that some of the potential difference between terminals (1) and (2) will leak and contribute to $V_{34}$ such that

$$V_{34} = V_H + V_0, \tag{3.1}$$

where $V_0$ denotes the misalignment voltage contribution. In the first set of experiments,

$V_0$ can be balanced out using the potentiometer in the circuit.

**Magic Cylinder and Lock in Amplifier**

In later parts of the experiment the same circuit is used but with the Halbach magic cylinder replacing the electromagnet as the source of magnetic field. The cylinder allows $V_0$ to be found relatively easily since it is free to rotate. To see this we consider that rotating the cylinder allows us to go from $\vec{B}$ to $-\vec{B}$ easily and when the polarity is flipped the sign of $V_H$ also changes but the sign of $V_0$ does not therefore

$$V_H = \frac{1}{2}[V_{34}(B, I) - V_{34}(-B, I)], \quad V_0 = \frac{1}{2}[V_{34}(B, I) + V_{34}(-B, I)]. \tag{3.2}$$

A magic cylinder is constructed from eight wedge shaped elements which are permanently magnetised in a certain direction such that the sum of their contributions creates a constant magnetic field within the cylinder. The magnetic field within the cylinder is transversely oriented relative to the axis of rotation.

The lock-in amplifier (LIA) operates by providing a D.C. output as the root-mean squared value of an incident A.C. signal. The LIA has the ability to *lock in* on a frequency of interest since it acts by reversing the polarity of the incoming signal at intervals specified by the chosen frequency. It is very sensitive in this respect because the readout will be skewed heavily (or zero) if the chosen reference frequency does not match the input signal. For this reason measurements made with the LIA are highly resolved by a large signal to noise ratio.

# 4 Methodology

**1: Characteristics of an Electromagnet**

A digital multi-meter was connected in series with the electromagnet and the coil power supply. The Hall probe Gauss-meter was turned on to measure the magnetic field between the poles, to calibrate the Gauss-meter, it was rotated until the maximum value of $B$ was obtained and then fixed in place. Measurements of $B$ were then recorded for $I_C$ ranging from $-2.5A$ up to $2.5A$ in the increasing and decreasing direction. These measurements were then tabulated and plotted to determine whether the increasing and decreasing directions of current are equivalent in the electromagnet.

**2: Measuring the Hall Voltage $V_H$**

To prepare to measure the Hall voltage, the circuit was altered to include a power supply which supplied the current $I$ to the sample. This was connected in series with a multi-meter to measure $I$ and a separate multi-meter was connected in parallel with terminals (3) and (4) to measure $V_{34}$. To compensate for this misalignment voltage ($V_0$),

$I$ was set to $10mA$ and $B$ was set to $0T$. The potentiometer connected across (1) and (2) was then adjusted until $V_{34} \approx 0$. Measurements of $B$ and $V_{34}$ were then made for $I = (10, 20, 28)mA$ where $B$ was only adjusted by increasing $I_C$ from $-2.5A$. A plot was made for each case and $R_H$ was determined using equation (2.4) which shows that the slope of the graph would be $IR_H/t$. A compass was used to determine the direction of $\vec{B}$. T he sign of $R_H$ was then applied to determine the charge carriers present in the Germanium crystal and equation (2.6) was used to find the carrier concentration ($N$). The voltage drop along the $\hat{l}$ direction ($V_{12}$) was then measured and the resistance of the sample ($R$) was computed by applying Ohm's law directly. Lastly, the conductivity and hence the carrier mobility were calculated using equation (2.7).

## 3: The Magic Cylinder

The Germanium power source was connected at terminals (1) and (2) and the current $I$ was set to an arbitrary value chosen to be $10mA$. To set $\vec{B}$ perpendicular to the Germanium circuit a multi-meter was connected to measure $V_{34}$, and the cylinder was rotated until the voltage reached a maximum value. (It is known from the cross product in equation (2.3) that the voltage reaches a maximum when $\vec{B} \perp \vec{J}$ but zero when parallel). $V_{34}$ and $V_{12}$ were then measured for $I = (5, 10, 15, 20, 25)mA$ and the measurements were repeated with the cylinder rotated through 180° so that the magnetic field was still perpendicular but transversely negative. This allowed the misalignment voltage $V_0$ and Hall voltage $V_H$ to be calculated using the formulae (3.2). The same quantities from section (2) were once again calculated and a table was made including the values $(V_H, V_0, V_0/V_{12}, R_H, R)$. The displacement responsible for $V_0$ was then determined based on the average of the values $V_0/V_{12}$. The cylinder was then set to rotate at a constant angular velocity and the resulting AC signal of $V_{34}$ was presented on an oscilloscope. The oscilloscope was used to measure $V_{34}$ for equal and opposite magnetic fields over the same range of $I$ again and an analogous table of results was constructed.

## 4: The Lock in Amplifier

The signal of $V_{34}$ was filtered through the LIA and again presented on the oscilloscope. The rotating cylinder was connected to the LIA through the *reference* cable which resulted in the reference signal appearing as a $1V$ square wave as measured from peak-to-peak. With $I = 10mA$, the time constant was set to $0.3s$ and sensitivity was set to $250mV$. The oscilloscope range was adjusted until a clear view of the output signal was obtained. The output voltage, now being the root mean square value for $V_H$ was recorded as the phase angle $\theta$ of the LIA was varied over 180°. Then for a set of values of current $I = (5, 10, 15, 20)mA$, the peak value of $V_H$ and its dependence on $\theta$ were taken note of. This was repeated for a set of three very small values for the current $I = (0.1, 0.5, 1)mA$ and graphs of $V_H$ against both current and phase angle were produced.

# 5 Results and Discussion

## 1: The Characteristics of an Electromagnet

After measuring the magnetic field strength $B$ for increasing and decreasing values of the current supplied to the coil $I_C$, the results were compiled in Tables (5.1)

Table 5.1: Measurements of magnetic field strength made using the Hall-probe Gaussmeter for increasing and decreasing current variations.

| $0A \rightarrow 2.5A$ | | $2.5A \rightarrow 0A$ | |
|---|---|---|---|
| $I_C \pm 0.1(A)$ | $B \pm 0.1(mT)$ | $I_C \pm 0.1(A)$ | $B \pm 0.1(mT)$ |
| 0.0 | 6.1 | 0.0 | 0.4 |
| 0.5 | 21.4 | 0.5 | 31.6 |
| 1.0 | 51.6 | 1.0 | 62.5 |
| 1.5 | 82.6 | 1.5 | 91.7 |
| 2.0 | 114.4 | 2.0 | 119.7 |
| 2.5 | 145.5 | 2.5 | 145.5 |
| $0A \rightarrow -2.5A$ | | $-2.5A \rightarrow 0A$ | |
| $I_C \pm 0.1(A)$ | $B \pm 0.1(mT)$ | $I_C \pm 0.1(A)$ | $B \pm 0.1(mT)$ |
| 0.0 | 11.6 | 0.0 | 15.9 |
| -0.5 | -42.9 | -0.5 | -48.9 |
| -1.0 | -71.8 | -1.0 | -80.6 |
| -1.5 | -102.4 | -1.5 | -110.5 |
| -2.0 | -133.0 | -2.0 | -139.5 |
| -2.5 | -162.8 | -2.5 | -166.0 |

The results of the table were then plotted and this resulted in Figure (5.1)

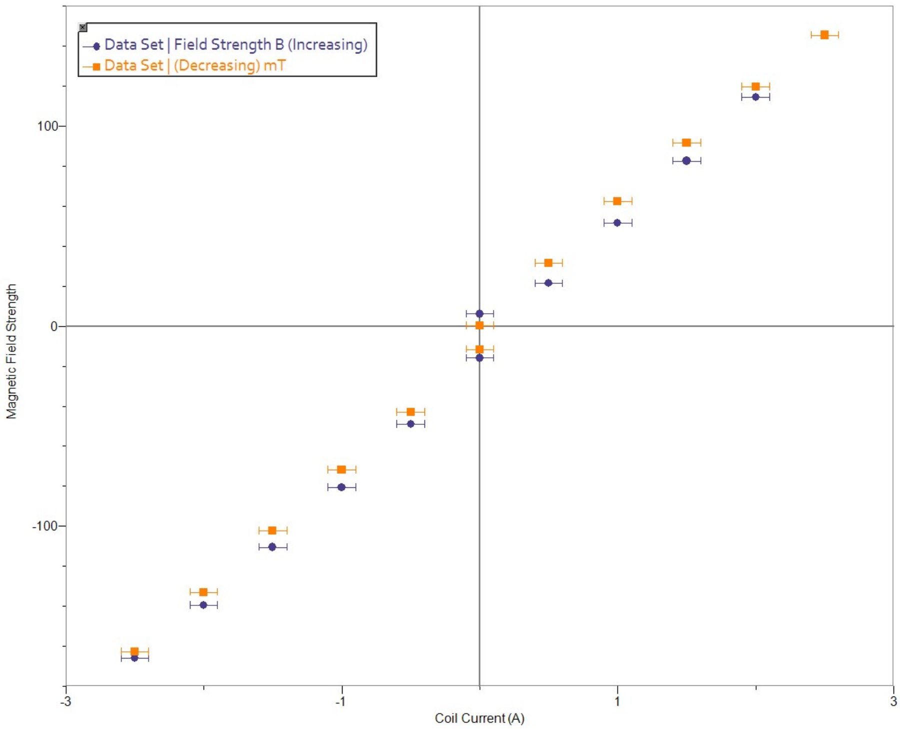



Figure 5.1: A graph showing $B$ versus $I_C$ for the electromagnet used in experiment 1. The orange points are measurements made as the current was decreased and the indigo points are for increasing current.

In this graph we see that values of the magnetic field $B$ produced by the electromagnet are not uniquely defined by the current $I_C$ through the coil. As mentioned above this is an example of a hysteresis loop and the narrow shape of this loop is evident in the overlapping data sets and is a signature of **soft** ferromagnetic materials. This result is verified by the fact that the core of the electromagnet is made of iron which acts as a temporary magnet.

## 2: Measurements of the Hall Voltage $V_H$ Using the Electromagnet

The voltage $V_{34}$ was reduced to $0.2 \pm 0.1mV$ in a zero magnetic field using the potentiometer. Then with $V_{34} = V_H$ the results of measuring $V_H, B$ and $I_C$ for circuit current $I = \{10, 20, 28(\pm 2)mA\}$ are shown in Table (5.2):

Table 5.2: Measurements of coil current, the Hall voltage and the magnetic field for the Hall effect using an electromagnet.

$I = 10 \pm 2mA$

| $I_C \pm 0.1(A)$ | $B \pm 0.1(mT)$ | $V_H \pm 0.1(mV)$ |
|---|---|---|
| -2.5 | -182.5 | -15.5 |
| -2.0 | -153.3 | -12.9 |
| -1.5 | -122.3 | -10.0 |
| -1.0 | -89.1 | -6.8 |
| -0.5 | -54.1 | -3.3 |
| 0.0 | -17.1 | 0.4 |
| 0.5 | 19.3 | 4.1 |
| 1.0 | 56.6 | 7.8 |
| 1.5 | 93.6 | 11.5 |
| 2.0 | 129.5 | 14.9 |
| 2.5 | 164.7 | 18.1 |

$I = 20 \pm 2mA$

| $I_C \pm 0.1(A)$ | $B \pm 0.1(mT)$ | $V_H \pm 0.1(mV)$ |
|---|---|---|
| -2.5 | -158.3 | -43.7 |
| -2.0 | -133.6 | -36.9 |
| -1.5 | -106.8 | -29.2 |
| -1.0 | -77.7 | -20.7 |
| -0.5 | -47.2 | -11.4 |
| 0.0 | -15.3 | -1.7 |
| 0.5 | 17.0 | 8.2 |
| 1.0 | 48.7 | 17.8 |
| 1.5 | 80.2 | 27.2 |
| 2.0 | 111.1 | 36.1 |
| 2.5 | 140.7 | 44.3 |

$I = 28 \pm 2mA$

| $I_C \pm 0.1(A)$ | $B \pm 0.1(mT)$ | $V_H \pm 0.1(mV)$ |
|---|---|---|
| -2.5 | -158.3 | -63.7 |
| -2.0 | -132.5 | -53.7 |
| -1.5 | -106.7 | -42.5 |
| -1.0 | -77.6 | -30.0 |
| -0.5 | -47.2 | -16.8 |
| 0.0 | -15.3 | -2.6 |
| 0.5 | 16.6 | 11.6 |
| 1.0 | 48.2 | 25.5 |
| 1.5 | 80.4 | 39.4 |
| 2.0 | 111.1 | 52.2 |
| 2.5 | 140.4 | 64.1 |

Each data set was used to plot a graph of the Hall voltage against magnetic field strength and these are shown below in Figure (5.2 - 5.4). **Note**, the polarity of the magnetic field was purposely flipped in the following three figures to present a positively sloped graph but as will be seen the slope is indeed negative as a consequence of the sign of the Hall coefficient.

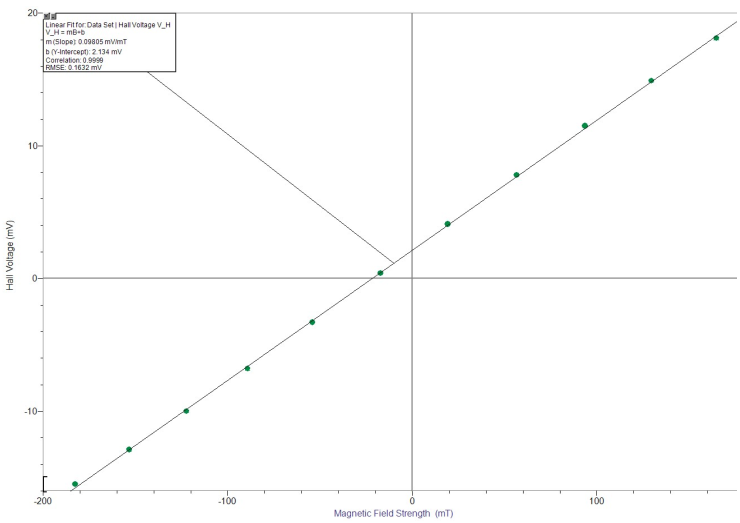



Figure 5.2: $V_H$ plotted against $B$ for $I = 10mA$. Note that these measurements were made in the second of two laboratory session where the exact setup could not be replicated from the first. Hence the values for $B$ have a wider range. The value of the slope was found to be $M_{10mA} = 0.09805 \pm 0.0004$ using built in regression tools.

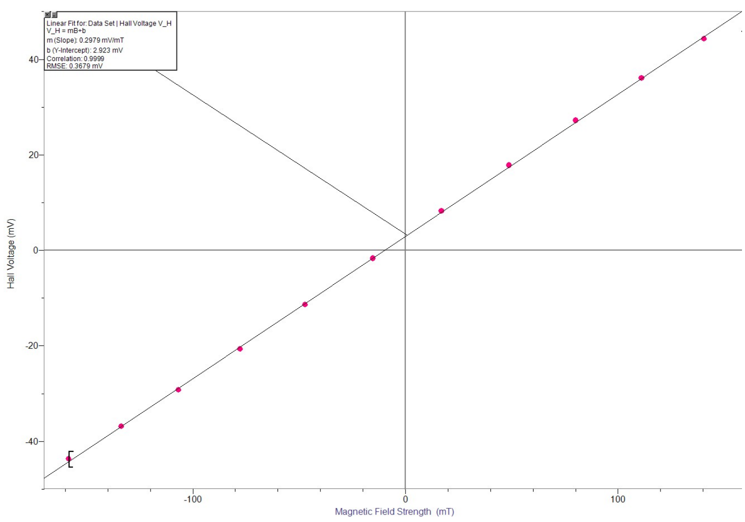



Figure 5.3: $V_H$ plotted against $B$ for $I = 20mA$. The value of the slope was found to be $M_{20mA} = 0.2978 \pm 0.0012$

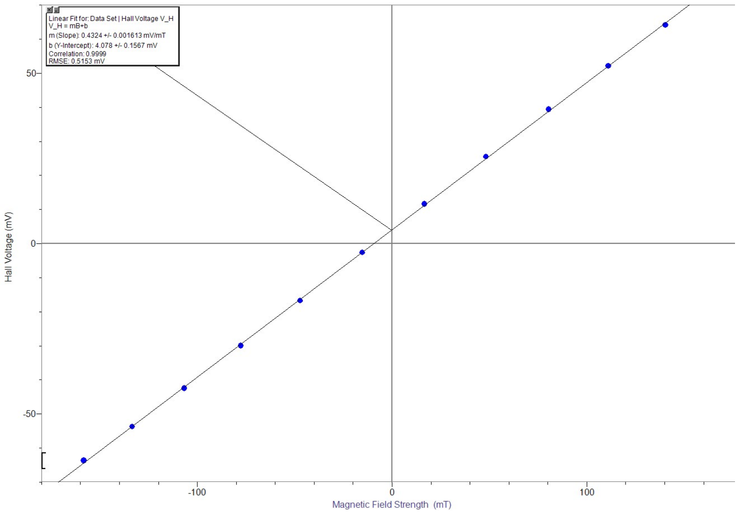



Figure 5.4: $V_H$ plotted against $B$ for $I = 28mA$. The value of the slope was found to be $M_{28mA} = 0.4324 \pm 0.0016$

In each case a straight line intersecting near the origin is observed and it can be concluded that $B \propto V_H$, $R_H$ was then calculated from the slopes of the graphs. After placing a compass between the poles of the electromagnet it was observed that $\vec{B}$ was proportional to $-\hat{t}$. Then since $\vec{J}$ is proportional to $\hat{l}$, the cross product of equation (2.3) reveals that

$$|\vec{E}_H| = -R_H| - J_l B_t \hat{w}| \implies \frac{|\vec{E}_H|}{R_H} = -|JB|. \tag{5.1}$$

The absolute value quantities are positive which shows that the sign of $R_H$ is negative and the charge carriers are **electrons**, the Germanium crystal used in this section is therefore an n-type semiconductor. The tabulated values obtained for $R_H$ are shown in Table (5.3).

Table 5.3: Calculated values with negative sign applied for the Hall coefficient $R_H$

| $I \pm 2(mA)$ | $R_H \quad m^3C^{-1}$ | $\Delta R_H \quad m^3C^{-1}$ |
|---|---|---|
| 10 | $-0.009805$ | $1.92 \times 10^{-5}$ |
| 20 | $-0.01489$ | $8.87 \times 10^{-5}$ |
| 28 | $-0.01544$ | $1.34 \times 10^{-4}$ |

The propagation of uncertainty was calculated using the Gaussian formula

$$R_H = \frac{Mt}{I} \implies \frac{\Delta R_H}{R_H} = \sqrt{\left(\frac{\partial R_H}{\partial M}\Delta M\right)^2 + \left(\frac{\partial R_H}{\partial I}\Delta I\right)^2 + \left(\frac{\partial R_H}{\partial I}\Delta I\right)^2} \tag{5.2}$$

The coefficients for $I = \{20, 28\}mA$ are within experimental uncertainty of each other but for $10mA$ this is not the case due to an inaccuracy explained beneath Figure (5.2) we shall allow this value to be an outlier which we do not include in the average. We expect that they would be the same from the proportionality of $V_H$ and $B$ since it is linear. The electron concentration was then computed:

$$N_{-} = \frac{1}{\langle R_H \rangle q_e} = 4.12 \times 10^{20} m^{-3} \pm 0.03 \times 10^{20} m^{-3}. \tag{5.3}$$

This was satisfactory as the concentration is expected to be in the order of $10^{20} m^{-3}$[4]. Measurements were made for how $V_{12}$ responds to a change in $I$ and of course by Ohm's law the slope of these measurements (shown below) is the resistance $R$.

Table 5.4: Measurements of the voltage drop through terminals 1 and 2 along the Germanium circuit.

| $I \pm 2(mA)$ | $V_{12} \pm 0.1(V)$ | $R(\Omega)$ |
|---|---|---|
| 10 | 1.7 | 170 |
| 15 | 2.5 | 167 |
| 20 | 3.3 | 165 |
| 25 | 4.2 | 168 |
| 30 | 5.0 | 167 |

Then calculating the slope for this basic data set we find $\langle R \rangle = 167 \pm 3(\Omega)$. Then the conductivity is the inverse of the resistivity

$$\sigma_{-} = \frac{l}{Rwt} = 11.98 \pm 2.40 Sm^{-1}. \tag{5.4}$$

Then finally the electron mobility of the Germanium sample is $\mu_{-} = \sigma / N_e q_e = 0.18145 \pm 5.51 \times 10^{-4} (m^2/Vs)$ or in more commonly used units $\mu_{-} = 1814.5 \pm 5.51 (cm^2/Vs)$. In comparison with literature values the conductivity and mobility in the Germanium sample were not within experimental error of accepted values however the values had the correct order of magnitude.

**The Magic Cylinder**

For the magic cylinder $B = 170 \pm 10 mT$, after finding the perpendicular orientation of $\vec{B}$, $V_{34}$ and $V_{12}$ were measured for various values of $I$ as shown in Table (5.5).

Table 5.5: Measurements of $V_{12}$ and $V_{34}$ for a germanium crystal circuit in a magnetic field ($B = 170mT$) produced by the magic cylinder apparatus.

| $I(mA)$ | $V_{12}(V)$ | $V_{34}(B, I)mV$ | $V_{34}(-B, I)mV$ |
|---|---|---|---|
| 5 | $0.37 \pm 0.01$ | $13.5 \pm 0.1$ | $-4.7 \pm 0.1$ |
| 10 | $0.74 \pm 0.01$ | $27.0 \pm 0.1$ | $-9.5 \pm 0.1$ |
| 15 | $1.32 \pm 0.01$ | $47.9 \pm 0.1$ | $-16.9 \pm 0.1$ |
| 20 | $1.90 \pm 0.01$ | $68.8 \pm 0.1$ | $-24.2 \pm 0.1$ |
| 25 | $2.49 \pm 0.01$ | $89.7 \pm 0.1$ | $-31.6 \pm 0.1$ |

Then applying the results to compute $R_H$,$R$,$V_0$ and $V_H$ we obtain Table 5.6

Table 5.6: Calculated values of the circuit resistance, Hall voltage, misalignment voltage and Hall coefficient for a range of values for current in the magic cylinder experiment.

| $I(mA)$ | $R$ $(\Omega)$ | $V_H$ $(V)$ | $V_0$ $(V)$ | $\lvert R_H\rvert$ $(m^3C^-1)$ |
|---|---|---|---|---|
| 5 | 74.0 | $0.0091 \pm 0.0001$ | $0.0044 \pm 0.0001$ | $0.0107 \pm 0.0005$ |
| 10 | 74.0 | $0.0183 \pm 0.0001$ | $0.0088 \pm 0.0001$ | $0.0107 \pm 0.0009$ |
| 15 | 88.0 | $0.0324 \pm 0.0001$ | $0.0155 \pm 0.0001$ | $0.0127 \pm 0.0022$ |
| 20 | 95.0 | $0.0465 \pm 0.0001$ | $0.0223 \pm 0.0001$ | $0.0137 \pm 0.0030$ |
| 25 | 99.6 | $0.0607 \pm 0.0001$ | $0.0290 \pm 0.0001$ | $0.0143 \pm 0.0041$ |

To determine the sign of $R_H$ we note that $V_H$ is positive, then by equation (2.4) we see that the sign of $R_H$ is positive when $\vec{B} \perp \vec{J}$ telling us that the charge carriers are now **holes**. The crystal used in experiments (3) and (4) is a p-type Germanium crystal and the electric field due to $V_H$ will be oriented from terminal 4 to terminal 3, holes will accumulate at terminal 3 leading to a build up of negative charges at terminal 4 to return the system to equilibrium.

We can calculate the resistivity $\rho$ as $\langle R\rangle wt/l$ where $\langle R\rangle$ is the average of $R$ from Table (5.6). We find that $\rho = (4306 \pm 8) \times 10^{-5}\Omega m$. Then computing the average of the ratio $V_0/V_{12}$ gave a value of 0.01178. An estimate for the displacement between contacts 3 and 4 can then be made from the resistivity.

$$\frac{\rho}{\langle V_0\rangle/I} = \frac{V_{12}wt}{V_0 l} \approx 4cm \tag{5.5}$$

Once again we calculated the conductivity and carrier mobility and concentration of the sample. $\sigma_+ = 23.22\ (\pm 1.00)\ Sm^{-1}$, $N_+ = 5.030\ (\pm 0.04) \times 10^{20} m^{-3}$ and $\mu_+ = 0.2884 (\pm 0.0053)\ m^2/Vs$, where (+) has been used to denote the carriers being holes. Once again the values found were of the correct order of magnitude but not within experimental error of the accepted values[4].

The measurements were repeated with the magic cylinder rotating at a constant angular velocity. The voltage $V_{34}$ was displayed on an oscilloscope and it appeared as an AC signal varying sinusoidally with peak values for both angles where $\vec{B} \perp \vec{J}$. Measuring these peak values $V_{34}(170mT, I)$ and $V_{34}(-170mT, I)$ provided the results of Table (5.7).

Table 5.7: Measurements of $V_{12}$ and $\max(V_{34})$ for a germanium crystal circuit in a rotating magnetic field ($B = 170mT$).

| $I(mA)$ | $V_{12}(V)$ | $V_{34}(B, I)mV$ | $V_{34}(-B, I)mV$ |
|---|---|---|---|
| 5 | $0.37 \pm 0.01$ | $14.8 \pm 1$ | $-4.4 \pm 1$ |
| 10 | $0.74 \pm 0.01$ | $28.8 \pm 1$ | $-9.2 \pm 1$ |
| 15 | $1.32 \pm 0.01$ | $48.8 \pm 1$ | $-17.6 \pm 1$ |
| 20 | $1.90 \pm 0.01$ | $72.8 \pm 1$ | $-25.0 \pm 1$ |
| 25 | $2.49 \pm 0.01$ | $91.2 \pm 1$ | $-31.2 \pm 1$ |

We can then see that the measurements made using the oscilloscope are within experimental error of the previous measurements in the non-rotating case therefore the sign of $R_H$ could be determined using the same method. As the cylinder rotated we were able to observe that the Hall voltage $V_H$ varied with time. This implies that the distribution of charges in the crystal due to the hall effect was constantly changing. The importance of not grounding either of the terminals 1 and 2 is apparent here since there is a large build up of charges in motion on the crystal which would experience a strong force to the grounded terminal potentially blowing the fuse.

## 4: The Lock-In Amplifier

With the lock-in amplifier and oscilloscope initialised as described in the methodology the AC signal of $V_{34}$ in a rotating magnetic field was filtered through the lock-in amplifier. The output voltage on the oscilloscope was then a DC value for the maximum of $V_{34}$. With $I$ set to $10mA$, the output voltage was then measured as the phase angle of the LIA was varied from $0° \rightarrow 180°$. The results are shown in Table (5.8).

Table 5.8: Measurements of the output voltage $V_{out}$ from the lock-in amplifier for different phase angles, using the oscillation of $V_{34}$ under a rotating magnetic field at $I = 10mA$ as the input signal.

| Phase: $\theta(\pm 2)°$ | Output $V_{out}(\pm 2)mV$ | $\theta(\pm 2)°$ | $V_{out}(\pm 2)mV$ |
|---|---|---|---|
| 0 | 26 | 100 | 4 |
| 10 | 28 | 110 | 0 |
| 20 | 24 | 120 | $-4$ |
| 30 | 22 | 130 | $-8$ |
| 40 | 20 | 140 | $-12$ |
| 50 | 18 | 150 | $-16$ |
| 60 | 16 | 160 | $-20$ |
| 70 | 14 | 170 | $-24$ |
| 80 | 10 | 180 | $-28$ |
| 90 | 6 | | |

To better visualise the phase dependence, the contents of the table were then plotted to obtain Figure (5.5).

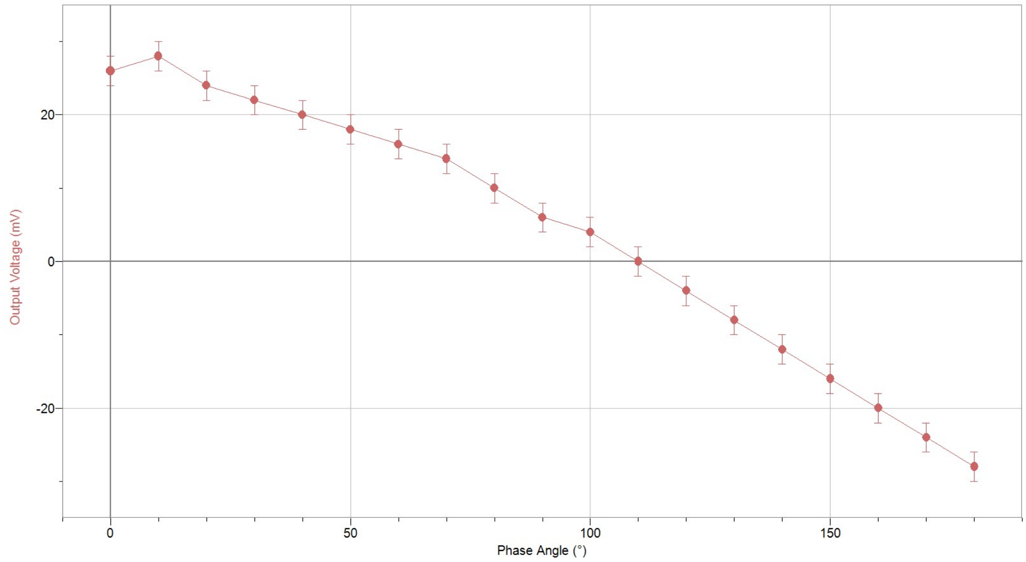



Figure 5.5: The output voltage $V_{out}$ from the lock-in amplifier plotted against the phase angle $\theta$

We see that the difference in phase between the peak values and the zero values is $\approx 90°$. This can be explained by the operation of the lock-in amplifier. For an AC signal such as the one produced by the rotating magnetic field, the LIA reverses the polarity at every half cycle of the reference frequency determined by the angular frequency of the rotating cylinder. The phase angle is used to synchronise the reference and input signal so that the output can be selected from a narrow band of frequencies and amplified as an RMS DC signal with a very high signal to noise ratio. In the case of Figure (5.5), we see that the signal is *locked in* when $\theta = 10°$ but when the phase angle has been shifted by $90°$ the switching of polarity completely destructs the wave giving a zero output.

To use the lock-in amplifier to measure the Hall effect we measured the output $V_H$ for different values of current. The measurements were only valid when the signal was locked in so for each value of $I$, the phase angle which gave the maximum output had to be found although this turned out to be zero for all values of current in this case. The results obtained are shown below in Table (5.9).

Table 5.9: Measurements of the Hall Voltage for a range of $I$ using the lock-in amplifier and an oscilloscope.

| $I \pm 2(mA)$ | $V_H \pm 0.1(mV)$ |
|---|---|
| 5 | 23.2 |
| 10 | 45.6 |
| 15 | 83.2 |
| 20 | 120.0 |

Further calculations result in $R_H = 0.03050\ (\pm 0.0060) m^3 C^{-1}$, again the sign of the Hall coefficient is positive. $N = 2.046(\pm 0.015) \times 10^{20} m^{-3}$.
To test the ability of the lock-in amplifier further the previous measurements were repeated for very small values of current and the results are shown in Table (5.10).

Table 5.10: Measurements of the Hall voltage for $I = (0.1, 0.5, 1) mA$ using the lock-in amplifier and an oscilloscope. $\theta_{sync}$ denotes the angle at which $V_H$ was maximally amplified by the LIA.

| $I \pm 0.1 (mA)$ | $V_H \pm 0.2 \times 10^{-4} (V)$ | $\theta_{sync} \pm 2(°)$ |
|---|---|---|
| 0.1 | $6.4 \times 10^{-4}$ | 0 |
| 0.5 | $6.4 \times 10^{-4}$ | 90 |
| 1.0 | $6.9 \times 10^{-3}$ | 0 |

Unfortunately it seemed to be the case that for the two smaller values of $I$ the values of $V_H$ were not able to be resolved as they were too small. The isolated point for $I = 1mA$ which was measured accurately is not enough to draw any conclusions overall but the ability of the lock-in amplifier to measure such a small signal through experimental noise is evident. In static measurements of a small quantity, there is no AC signal which can be used to selectively recover the measurement above noise which may be comparable in size to the quantity itself.

# 6 Conclusions

Throughout this series of experiments, the classical Hall effect was observed using a Germanium crystal sample. In the first experiment an electromagnet was used to generate the magnetic field. The magnetic field produced by the electromagnet was shown to display hysteresis behaviour based on whether the current through its coils was increasing or decreasing. The hysteresis loop was plotted and it was determined that the core of the electromagnet was a soft ferromagnetic material since the loop was narrow.

In the second experiment it was found that the Hall voltage ($V_H$) is linearly proportional to the magnetic field strength ($B$). The Hall coefficient was calculated from each of the three graphs showing this relationship and a value of $R_H = -14.89\ (\pm 0.08) \times 10^{-3} m^3 C^{-1}$ was obtained for $I = 20mA$. The sign of $R_H$ was deemed to be negative which means that in experiments (1) and (2), the charge carriers were electrons and the crystal used was n-type Germanium. The concentration of electrons in the sample was $N_- = 4.12\ (\pm 0.03) \times 10^{20} m^{-3}$. The conductivity and carrier mobility of the sample were found to be $\sigma_- = 11.98\ (\pm 2.40) Sm^{-1}$ and $\mu_- = 1814.50\ (\pm 5.51) cm^2/Vs$.

For the third experiment, a permanently magnetised cylinder $B = 170\ (\pm 10) mT$ known as the magic cylinder was used to produce the Hall effect on a different Germanium circuit. The misalignment voltage $V_0$ due to a vertical displacement of terminals (3) and (4) was determined based on the fact that it remains constant when the direction of the magnetic field is reversed for each different value of current the relation $V_0 \approx V_H/2$ held to a good approximation. The hall coefficient was once again calculated and in this case it remained roughly constant as the current was varied since the magnetic field was also a constant, the value obtained was $R_H = 12.4(\pm 1.8) \times 10^{-3} m^3 C^{-1}$. $R_H$ had a positive sign therefore the crystal used in experiments 3 and 4 was p-type Germanium. From the ratio $V_0/V_{12}$ it was estimated that the misalignment of terminals (3) and (4) was approximately $4cm$. The semiconductor parameters were once again calculated giving $\sigma_+ = 23.22\ (\pm 1.00)\ Sm^{-1}$, $N_+ = 5.030\ (\pm 0.04) \times 10^{20} m^{-3}$ and $\mu_+ = 0.2884(\pm 0.0053)\ m^2/Vs$. The cylinder was then set rotating and the measurements were repeated using an oscilloscope to measure the signal of $V_{34}$ as it varied in time. With the rotating cylinder the values obtained at the peaks of the AC signal were identical to the static measurements previously made showing that the oscilloscope can be used to measure the same quantities accurately in a rotating magnetic field.

In the fourth experiment the lock-in amplifier was used to make highly accurate measurements of the Hall voltage for different values of current $I$. It was found that the maximum output voltage and the minimum output voltage were separated by a 90° phase shift of the lock-in amplifier. This stems from the function of the amplifier to flip the polarity of the incoming signal at specified intervals. The Hall voltage was measured for the Germanium circuit in a rotating magnetic field. Subsequent calculations revealed that contrary to the calculations made in the static case, $R_H = 0.03050(\pm 0.0060) m^3 C^{-1}$ and $N_+ = 2.046(\pm 0.015) \times 10^{20} m^{-3}$. These values were substantially closer to those found in literature which can be accredited to the signal to noise ratio associated with the LIA.

# 7 Appendices

## List of Figures

## 7.1 List of References

[1]: Edwin Hall (1879). "On a New Action of the Magnet on Electric Currents". American Journal of Mathematics.

[2]: "The Hall Effect". Junior Sophomore Laboratory Handbook. Trinity College Dublin, School of Physics

[3] J. M. D. Coey, Magnetism and Magnetic Materials, Cambridge University Press, 1st, 2010

[4]: "Properties of Si, Ge and GaAs at 300K" - `https://eesemi.com/sigegaas.htm`, a comprehensive reference on semiconductor manufacturing.